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Investments in Energy Technological Change Under Uncertainty

Ekundayo Shittu
University of Massachusetts Amherst, dayiski@gmail.com

INVESTMENTS IN ENERGY TECHNOLOGICAL CHANGE UNDER UNCERTAINTY

A Dissertation Presented

by

EKUNDAYO SHITTU

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Approved as to style and content by:

Erin Baker, Chair

Ana Muriel, Member

John Stranlund, Member

Mario Rotea, Department Chair
Department of Mechanical and Industrial Engineering

To the Author and Finisher of my faith

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ABSTRACT

INVESTMENTS IN ENERGY TECHNOLOGICAL CHANGE UNDER UNCERTAINTY

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EKUNDAYO SHITTU

B.Eng., UNIVERSITY OF ILORIN, NIGERIA

M.S., AMERICAN UNIVERSITY IN CAIRO, EGYPT

Ph.D., UNIVERSITY OF MASSACHUSETTS AMHERST

Directed by: Professor Erin Baker

This dissertation addresses the crucial problem of how environmental policy uncertainty influences investments in energy technological change. The rising level of carbon emissions due to increasing global energy consumption calls for policy shift. In order to stem the negative consequences on the climate, policymakers are concerned with carving an optimal regulation that will encourage technology investments. However, decision makers are facing uncertainties surrounding future environmental policy.

The first part considers the treatment of technological change in theoretical models. This part has two purposes: (1) to show—through illustrative examples—that technological change can lead to quite different, and surprising, impacts on the marginal costs of pollution abatement. We demonstrate an intriguing and uncommon result that technological change can increase the marginal costs of pollution

abatement over some range of abatement; (2) to show the impact, on policy, of this uncommon observation. We find that under the assumption of technical change that can increase the marginal cost of pollution abatement over some range, the ranking of policy instruments is affected.

The second part builds on the first by considering the impact of uncertainty in the carbon tax on investments in a portfolio of technologies. We determine the response of energy R&D investments as the carbon tax increases both in terms of overall and technology-specific investments. We determine the impact of risk in the carbon tax on the portfolio. We find that the response of the optimal investment in a portfolio of technologies to an increasing carbon tax depends on the relative costs of the programs and the elasticity of substitution between fossil and non-fossil energy inputs.

In the third part, we zoom-in on the portfolio model above to consider how uncertainty in the magnitude and timing of a carbon tax influences investments. Under a two-stage continuous-time optimal control model, we consider the impact of these uncertainties on R&D spending that aims to lower the cost of non-fossil energy technology. We find that our results tally with the classical results because it discourages near-term investment. However, timing uncertainty increases near-term investment.

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CHAPTER 1
INTRODUCTION

It has become imperative and generally accepted that the approach to mitigating the negative trend of climate change lies in the development of a new cluster of alternative technologies while improving on the current economic options available. In fact, it can be credibly argued that drastic emissions reduction must be made to forestall a global climate change catastrophe. This calls for a consideration of investments in technologies that aim at near-zero emissions or high level emissions reduction. However, there are technology options that have the potential of either not adequately addressing these levels of abatement or do so at very high incremental abatement costs.

A key driver for these investments is the policy type adopted by the regulatory agency. Unfortunately, at this time, there is no well-defined operating global policy to motivate the level of investments to bring about an appreciable reduction or outright halt in carbon emissions. Thus, there are inherent uncertainties associated with the anticipation of a future policy to influence R&D spending in these technologies.

In view of the foregoing reasons, this research aims to address the question of how a firm should optimally allocate its technology R&D investments: first, given a portfolio of technologies with different impacts on emissions reduction and different incremental costs of abatements; and second, in the face of uncertainties about a future environmental policy.

In order to address this question, first, we acknowledge and establish that not all technical changes are the same in their effects on the cost of reducing emissions incrementally. Second, we show that this disparity affects the potency of regulatory policies to induce spending in these technologies. Third, we address the issue of how overall and individual R&D spending in a suite of technologies is influenced by an increasing carbon tax. Finally, we examine, individually, the impact of uncertainty in the magnitude and uncertainty in the timing of the carbon tax on R&D investment into non-fossil technologies under a continuous-time, *act-learn-act,* optimal control framework.

Due to its central relevance, the next part of this introductory chapter discusses the meaning and technological implication of the marginal abatement cost in Section 1.1. Sections 1.2 and 1.3 describe the motivation and objectives of this dissertation. This chapter ends with a preface to the rest of this dissertation in Section 1.4.

1.1 The marginal abatement cost

In this section, we present a simple model for a conceptual definition of the marginal abatement cost (MAC) curve. Abatement cost is the cost of reducing emissions below the business-as-usual level — we assume that in the absence of technical change and in the absence of carbon policy there exists a profit-maximizing level of emissions, $\bar{\varepsilon}$. Abatement is defined as the reduction in emissions below this level. For example, if actual emissions are ε, then abatement is $\mu = \bar{\varepsilon} - \varepsilon$. Some kinds of technical change may lead to a new profit maximizing level of emissions, say $\varepsilon^* < \bar{\varepsilon}$. In this case there will be abatement defined by $\mu = \bar{\varepsilon} - \varepsilon^*$ even in the absence of a carbon policy. We define MAC to be zero (rather than negative) for abatement levels less than this. The MAC is the change in abatement cost per unit change in abatement. The MAC is central to addressing some of the questions in this research because the optimal behavior of a firm is to choose its level of abatement so that the marginal abatement cost is just equal to the carbon tax. To illustrate this further, consider a firm with an abatement-dependent profit function, $\pi(\mu)$, such that $\pi'(\mu) < 0$ and

$\pi''(\mu) < 0$.[1] Without regulation, the unrestricted emissions level is $\bar{\varepsilon}$ such that $\mu = 0$, and $\pi(\mu) \leq \pi(0)$ for all μ, since $\bar{\varepsilon}$ or $\mu = 0$ maximizes the firm's profit. Thus, the firm's abatement cost consists of any costs incurred at reducing emissions and this is captured by the change in the profit,

$$c(\mu) = \pi(0) - \pi(\mu) \tag{1.1}$$

Taking first and second order derivatives of (1.1), we have

$$c'(\mu) = -\pi'(\mu) > 0 \tag{1.2}$$

$$c''(\mu) = -\pi''(\mu) > 0 \tag{1.3}$$

where $c'(\mu)$ is the marginal abatement cost, and according to (1.3), the abatement cost is convex and increasing in abatement. Now, suppose an emissions tax, t is imposed on emissions, the firm will choose an abatement level that minimizes the total cost,

$$\min_{\mu} c(\mu) + t(\bar{\varepsilon} - \mu) \tag{1.4}$$

The first order condition for (1.4), $c'(\mu) = t$, implies that the firm chooses an optimal abatement level, μ^*, that equates the tax with the marginal abatement cost as illustrated in Figure 1.1.

Now we illustrate that *how* technical change is modeled matters by considering an example of technical change that pivots down the cost curve. Consistent with any reasonable theory of technical change, the thick line in the left panel of Figure 1.2 shows that the cost of abatement, $c(\mu)$ is everywhere lower after technical change. This is because a firm, or society, could always choose to discontinue use of a new

[1] Here, we suppress output and input choices on the assumption that the level of outputs and inputs are optimally chosen.

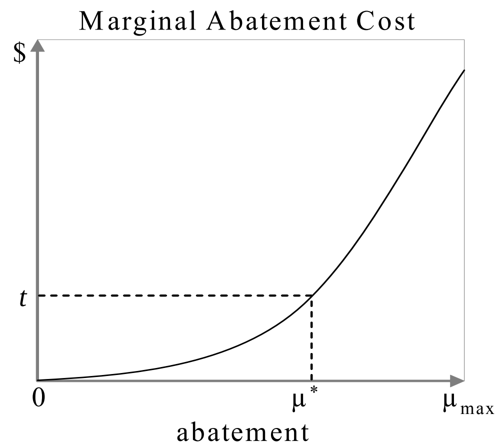

Figure 1.1. Marginal abatement cost curve.

technology if it increased costs. Thus with the same carbon tax, the firm will abate more after technical change than before technical change. Alternatively, a given level of abatement will cost less after technical change than before. The right panel of Figure 1.2 illustrates the associated MACs[2] where this property of abating more is carried over—since the MAC is simply the slope of the abatement cost curve, the MAC is also everywhere lower for this type of technical change representation. If the firm chooses its abatement level optimally by equating its marginal abatement cost with the tax, then for a given tax, t, optimal abatement level is higher after technical change, $\mu_2 > \mu_1$.

This illustration illuminates the importance of technical change representation in modeling because it has far reaching consequences on outcomes. Thus, how technical change influences this curve is crucial to answering some of the key questions we are addressing in this research.

[2]We have linear MACs here given assumption of a quadratic cost function.

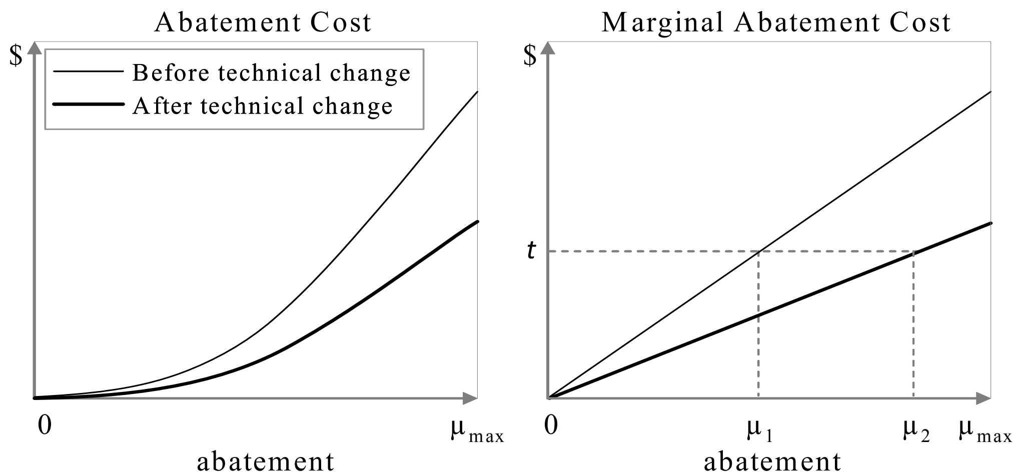

Figure 1.2. Illustrating the significance of technical change representation. The left hand panel shows the cost of abatement before and after technical change that pivots the cost curve to the right. The right hand panel shows the associated MACs.

1.2 Motivation

Some of the key points of interest in the arena of environmental economics include defining how an environmental policy induces technical change, what criteria defines an appropriate environmental policy, and how should investments in innovative alternatives be allocated given uncertainties in policy, marginal damages and technological success. However, researchers in this discipline have placed less emphasis on—or have outrightly ignored—a number of significant factors which are crucial to the determination of the optimal responses to these questions and others alike. The identified deficiencies in this literature include: (1) The representation of technical change as it affects the marginal cost of abatement is incomplete. This shortcoming, when considered, has the potential to influence the outcome of the relative ranking of regulatory tools or policies with regards to their incentives to firms to invest in innovation; (2) The role of uncertainty in a future carbon tax policy in shaping the investment decision of a firm has not been investigated under an *act-learn-act* continuous-time model with specific focus on non-fossil technologies. The way uncertainties in future carbon tax affect investment decisions has been modeled to target emissions reducing technologies, but we are not aware of any targeting non-fossil cost-reducing R&D programs; (3) The study of the effect of a random carbon tax on R&D spending has only considered stand-alone technologies, but not a portfolio of likely responsive technologies. The effect of the interaction—if it exists—between the programs in a portfolio, can only be identified in a portfolio setting. This dissertation focuses on these missing links and aims to cover these loopholes.

Therefore the motivation for this dissertation includes: (1) to influence the way that induced technical change is modeled by including multiple representations of technical change; (2) to provide an investment guiding template to firms on the best response to uncertainty about a future carbon tax policy; and (3) to inform policymakers of a robust climate technology policy.

1.3 Dissertation objectives

It is in view of the global need for the development of new technologies that understanding the array of effects technical change has on the marginal abatement cost is pertinent to understanding the implications of regulation. Thus, the objectives of this research are as follows:

1. The first general objective is to show that *how* technical change is modeled matters. This general objective can be broken down into the following specific objectives;

   (a) To show that technical change has the potential to increase the marginal abatement cost. It is widely implied in literature that technical change always reduces the marginal abatement cost and this has been the foundation for several key results including the outcomes of policy evaluations. The justification for this behavior is that some improvements in certain technologies are economically competitive at low level of emissions reduction, but become economically redundant when significant levels of emissions reductions are desired.

   (b) To identify how the ranking of environmental policies is impacted given this previously unidentified characteristic.

2. Given that *how* technical change is modeled matters, the second general objective is to investigate a firm's optimal investment in a portfolio of technologies, each with different impacts on the MAC. The specifics of this objective include;

   (a) To provide insights into what happens to the overall investment in R&D as the carbon tax increases.

   (b) To know what happens to the individual proportions of investment levels in the different technologies in the portfolio as the carbon tax increases.

   (c) To know the impacts of increasing risk in the carbon tax on portfolio investments.

3. Policy uncertainty in climate change modeling will influence the determination of near term investments in innovations and even the choice of an optimal technology policy. However, the outcomes of the effects of uncertainty are embedded in indistinct interpretations because in some instances, uncertainty increases near term R&D spending and in some others, it discourages it. Thus under a continuous time act-learn-act model of a firm's discounted cash flow, the specific objectives here include;

   (a) To determine the optimal investment in a continuous time model where there is an option to invest after learning.

(b) To determine the paths to the optimal capital stock in cost reducing alternative non-fossil technology given information about the firm's initial capital.

We examine these two objectives under the scenarios of tax magnitude and timing uncertainties existing individually, and we leave the insight into the *simultaneous* presence of these two uncertainties for future work.

1.4 Dissertation outline

The rest of this dissertation is outlined as follows. In Chapter 2, we present a comprehensive, but focused review of related work exploring the background and setting up the overall scope for this research. In Chapter 3, we show that it can *happen* for innovation to increase the marginal abatement cost. We illustrate that this *matters* through an example of innovation that increases marginal abatement costs over high abatement levels. Chapter 3 also shows the policy implications of such technologies by revisiting the seminal work of Milliman and Prince (1989) in a re-examination of the incentives to promote innovation, diffusion, and optimal agency response under five regulatory policies. Chapter 4 provides the theoretical framework for the representation of a set of four broadly defined R&D programs, and performs portfolio analysis. This chapter describes the levels of investments in each of the technologies in this portfolio and the overall investment under an uncertain carbon tax. Chapter 5 presents a continuous-time optimal control model to investigate the effect on a firm's investment of, first, uncertainty in tax magnitude, and second, uncertainty in tax timing. This chapter includes a numerical framework that is employed with the aid of computational tools for solving optimal control problems to illustrate the impacts of uncertainty in a carbon tax policy on a firm's level of capital stock and R&D spending. Chapter 6 concludes by providing a summary of the implications of the results in this dissertation on environmental policy. It also describes the overall contributions to the literature, and enunciates the avenues for future studies.

CHAPTER 2

RELATED WORK

There are three focal research interests in this dissertation. First, establishing a seminal and widely unnoticed significance of the effect of technical change representation on the marginal abatement cost; second, showing the impact of different representations of technical change on investment decisions under an increasing carbon tax; and third, with a target on alternative technology, show how investment responds to uncertainties in a carbon tax. In this chapter, we shape the scope of this dissertation by reviewing a number of the relevant, background studies in these areas, and then focus more on the specifics.

2.1 Endogenous technological change literature

In the context of climate change, there are several studies on endogenous technological advance, and this literature encompasses policy-induced technical change. This literature shows that technology development and deployment should be a critical part of climate change policy evaluation. In general, two approaches can be identified: the modeling and empirical approaches. In the modeling literature, Goulder and Schneider (1999) investigate the impact of the levying of carbon taxes on the level of R&D efforts in a setting of endogenous technological change. They find that induced R&D lowers the cost of achieving a given abatement target, but also increases the gross costs of the carbon tax. They conclude that the cost of achieving any given level of abatement is lower when induced innovation is included in the model. In a macro-economic model, Goulder and Mathai (2000) assess the implications of implementing endogenous technological change regarding the timing of greenhouse gas (GHG) abatement. They make a comparison of endogenous technological change through the simulation of explicit R&D activities with learning by doing where the stock of knowledge is a function of the cumulative level of abatement. Nordhaus (2002), in an updated version of the globally aggregated DICE model, analyses the timing and costs of climate change mitigation, and he concludes that induced innovation is important for reducing GHG emissions. Buonanno et al. (2003) use the RICE model to show that technological change is able to significantly lower abatement costs.

The empirical literature includes Newell (1997) and Newell et al. (1999). Newell (1997) shows, in an empirical framework, that an effect of changing energy prices on the direction of technological change can be derived from a model of the firm's optimal investment in research. Newell et al. (1999) further formalize the Hicksian notion of induced innovation to investigate whether government regulations have affected energy-efficiency innovation. They find evidence that both energy prices and government regulations have an effect on the energy efficiency of the models of general household items.

These and other related studies[1] find that the inclusion of endogenous technological change is important, and they also indicate that how technological change is endogenized—either through R&D channels or learning by doing—may have different implications for optimal environmental policies. This is because these two effects of achieving technical change lead to different interpretations on the optimal level of emissions reduction. For example, Goulder and Mathai (2000) show that when technical change comes by learning by doing, then near-term optimal abatement might be lower or higher depending on the specifications. However, when technical change results from R&D, then the presence of the option to induce more technical change than

[1] Aoki (1991), Jaffe and Palmer (1997), van der Zwaan et al.(2002) , Sue Wing (2003), and Popp (2004, 2006).

would otherwise be the case unambiguously calls for lower near-term abatement along with R&D expenditures. In general, the presence of endogenous technical change and assumptions about it can have implications for the optimal policy instrument (see Milliman and Prince (1989) and Montero (2002)).

2.2 Innovation and marginal abatement cost

The relationship between environmental policy and technical change has been the focus of the literature on endogenous technological change for quite some time. The strength of a given policy at inducing technical change is dependent on the response of the technology options available, and different technologies respond differently to a given policy. Thus researchers have developed different representations of technical change. For example, Baker, Clarke and Shittu (2008)[2] review a variety of approaches from the literature, and show that these representations have quite different, and sometimes surprising, effects on the marginal costs of pollution reductions. They demonstrate that theoretical and aggregate-level applied models have, indeed, used a number of different formulations for technical change.

For specific representations, Baker, Clarke, and Weyant (2006) consider three formulations of technical change that vary by how they impact the abatement cost curve: technical change that shifts the curve down, that pivots it down, and that pivots it to the right. They show that different representations of technical change have very different effects on the optimal societal investment in climate change technology R&D in the face of uncertainty. However, the empirical basis for this aspect of technical change—how it effects marginal abatement costs—has been largely overlooked in the development of these models.

[2] This paper is partly based on the work in this dissertation.

An underlying assumption throughout the theoretical economic literature on environmental innovation is that innovation reduces the marginal costs of emissions abatement (see, for example, Downing and White (1986), Fischer (2003), Jung (1996), Milliman and Prince (1989), Montero (2002), Parry (1998)). It is straightforward, however, to demonstrate examples where this is not true—where technical change aimed at reducing pollution and absolute abatement costs, may increase the marginal abatement cost (MAC). The purpose of Chapter 3 is to highlight this possibility and to demonstrate the potential for changed policy implications.

2.3 The basis for technology portfolio

An important policy implication based on the emerging dynamics and different directions of the development of energy systems, as observed by Gritsevskyi and Nakicenovic (2000), is that future research, development, and demonstration efforts and investments in new technologies should be distributed across "related" technologies rather than directed at only one technology from the cluster, even if that technology appears to be a "winner". In their approach, they represent the development of energy systems through a dynamic network. In this network, energy transformations correspond to energy technologies. They compare 520 alternative technological dynamics, which comprise 250 realizations or scenarios each. With an assumption of specific distribution functions for uncertainties, they find that approximately 10% of the alternative emergent dynamics are *optimal* in the sense that they meet demands at the lowest expected costs. In general, the results suggest that future energy research should aim at diversification.

Evidently, having a portfolio of R&D projects can minimize the adverse effects of GHG emissions because of the several potential ways that these projects aim at ameliorating the negative impacts on climate change. The portfolio decision considers both success in research and success in emissions mitigation irrespective of the regulatory policy in place, thus, the degree of technical success of a given R&D project can be taken as a measure of either the level of decline in emissions or as a measure of the decline in the demand for carbon, which could have resulted from the substitution of demand to non-carbon or non-fossil[3] inputs or an improvement in energy efficiency. In addition to this front end objective are the cost-related perspectives. For example, in the examination of several technologies in the context of a global integrated assessment model (IAM) of energy, agriculture, land-use, economics, and carbon cycle processes, Edmonds et al. (2004) discuss the significance of the development of an expanded suite of technologies including carbon capture and disposal, hydrogen systems and biotechnology because they hold the potential to dramatically reduce the cost of stabilizing GHG concentrations. Similarly, Baker, Chon and Keisler (2007) derive marginal abatement cost curves under different solar technologies using the IAM. Using an array of elicited expert definitions of technical success, they show that different technologies, if they achieve success as defined, have different impacts on the marginal cost of abatement. In addition, Fishelson and Kroetch (1989) show the possibility that the marginal and total costs are changing at different rates for different innovations justifies the use of more than one type of the technologies available. Although their work focuses on R&D into energy storage devices, but this result still holds in the general climate change arena.

Furthermore, there are ambiguities in the potency of environmental policies to induce technological innovation. For example, several earlier studies in this literature (Magat (1978), Downing and White (1986), Milliman and Prince (1989)) argue that emissions taxes and emissions permits generally provide more incentives for technological innovation than policies based on standards. On the other hand, and more recently, some other studies (Montero (2002), Sue Wing (2003)) show that this does

[3]Non-carbon and non-fossil are synonyms in this dissertation.

not always hold. We show that this ambiguity in the R&D planning problem arises because of the different R&D programs under consideration in these studies. For example, Baker, Clarke and Weyant (2006) point out that a research program to improve the efficiency of coal-fired electricity generation will create a different abatement cost profile from an R&D program into photovoltaic cells. While the first lowers the cost for moderate reductions in GHG emissions; the second will lower the costs of severely reducing emissions. As a consequence, the R&D problem becomes that of an efficient allocation of investment between a portfolio of available technologies.

2.4 Uncertainty and investment

The literature on firm's investment response to changes in environmental policy shows considerable efforts have been geared at this problem (e.g., Xepapadeas (1992), Kort (1995), Xepapadeas (1997, 1999)). Goulder and Schneider (1999), amongst others, show that the impact of carbon taxes in inducing technological change in the channel of R&D may lead to increased R&D expenditures, which in turn, could proportionally lead to technological progress. Despite this finding, there are no carbon-related policies at this time to induce technological change, and the anticipation of a policy in the near future is surrounded by uncertainties—these are uncertainties in climate change damages and technological success. For example, Baker and Adu-Bonnah (2008) combine uncertain technical change with uncertain damages to analyze the socially optimal portfolio of technology projects. In their model, they make R&D investment to impact the probability distribution over the outcome of technical change. First, they find that the socially optimal investment in alternative technologies is higher for riskier projects than less-risky projects, where the opposite is true for conventional technologies. Second, they find that less-risky alternative technologies and more-risky conventional technologies become more attractive when climate damages become riskier, in terms of a higher probability of a catastrophe.

Baker, Clarke, and Weyant (2006) show that the socially optimal investment in alternative technologies increases with some increases in risk in climate damages, while the socially optimal investment in conventional technologies decreases. Baker (2008) builds on the analytical results in the previous paper to show that in many cases abatement and alternative R&D act as "risk-substitutes": changes in risk that induce an increase in one, induce a decrease in the other. Specifically, alternative R&D tends to decrease in a Mean-Preserving Spread (MPS) that stretches the tail of the distribution; and increase in an MPS near the mean.

The different dimensions of uncertainty have quantitatively large impacts on optimal R&D investment, and qualitatively important impacts, such as reinforcing the benefits of diversification. In addition, the qualitative impact of uncertainty — whether optimal investment increases or decreases in uncertainty, for example — is ambiguous because it may depend on the specification of technical change.

A number of studies[4] have considered the general impacts of uncertainty on investment decisions. However, a few (e.g., Hassett and Metcalf (1999), Farzin and Kort (2000), Baker and Shittu (2006)) target the question of investment under a tax policy uncertainty as it relates investment in a given energy R&D. The uncertainty inherent in the expectation of a future carbon tax is twofold: (1) uncertainty about the magnitude of the tax; and (2) uncertainty about the timing of the tax. These uncertainties have been shown to independently influence the level of R&D spending. For example, Farzin and Kort (2000) model technology as reducing the carbon intensity of production; otherwise abatement is achieved through output reduction. This formulation leads to technical change that pivots the abatement cost curve to the right. They show that an increase in uncertainty about a future tax increase (at a known time) leads to optimally lower investment in abatement technology. They

[4]See Abel (1983), Hartman (1972), Pindyck (1988, 1991, 1993), and Dixit and Pindyck (1994).

also consider the impact of uncertainty about the timing of a (known) tax increase. Uncertainty about the timing leads firms to increase their investment in abatement capital. Their work suggests that uncertainty in the magnitude of a carbon tax is more important than uncertainty about the timing.

Baker and Shittu (2006) have a related model, but allow for substitution among carbon and non-carbon inputs; and model technical change in two ways—as a reduction in the cost of non-carbon inputs or, similar to Farzin and Kort (2000), as a reduction of the emission intensity of the carbon inputs. They also consider different types of mean-preserving-spreads in the carbon tax. In contrast to the results above, they show that firms that can flexibly substitute from carbon to non-carbon energy may increase R&D into non-carbon technologies when the uncertainty surrounding a carbon tax is increased. In other cases—if the firm is not flexible, or for technology that reduces the carbon intensity of output—firms will tend to decrease investment into R&D in an increase in uncertainty.

These studies indicate that if firms are acting optimally and considering uncertainty in a future carbon tax explicitly, then this will have an impact on their near term R&D investments. They show that uncertainty can lead to optimally higher investments in R&D in some cases. Whether this phenomenon is seen in actuality is an open empirical question that may have important implications for how endogenous technical change is modeled when environmental damages are uncertain.

CHAPTER 3

INSTRUMENT CHOICE AND TECHNICAL CHANGE

3.1 Introduction

In this chapter, we present examples of innovations that reduce the total cost of abatement yet increase marginal costs over some abatement levels. The examples are drawn from the climate change context, but the observation is general. In general, technological change can increase the MAC whenever an advance is made in a technology which will be substituted away from for high levels of abatement. In this case, the MAC may decrease for lower levels of abatement, but increase for higher levels of abatement. Investments in such innovations are not irrational, as long as they decrease the overall cost of abatement. To be clear, we do not argue that innovation *in general* will increase the MAC; only that it can happen and it could matter.

To illustrate this, consider the left hand side of Figure 3.1 for technical representation that *pivots right* the cost of abatement[1]. It will cost the firm less, $t_2 < t_1$, to achieve the same abatement level, μ', after technical change. Notice, however, that the abatement cost curve is steeper at high levels of abatement. The right panel of Figure 3.1 shows the MACs, and the MAC after technical change is higher where the slope is steeper. Thus, the cost of reducing emissions to the next unit, μ' is higher after technical change, $p_2 > p_1$.

[1]For this representation, the costs of abatement are everywhere lower after technical change except at zero and full abatements — this is still consistent with the theory of technical change. An example of this is improvements in the efficiency of coal-burning electricity production through increased CO_2 emissions capturing capability. This would provide a more efficient performance at moderate and intermediate levels of abatement, but with no impact at targeting zero emissions

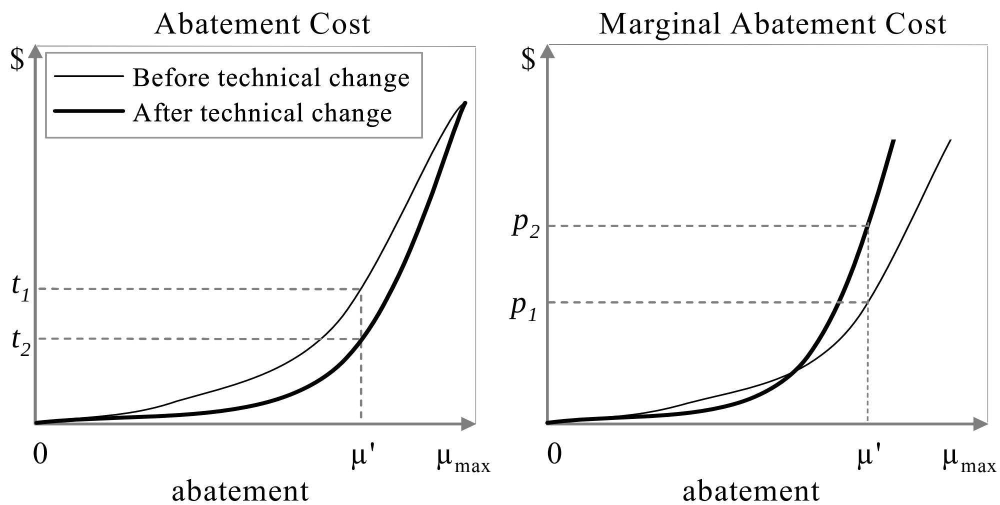

Figure 3.1. Technical change representation that pivots right the abatement cost curve. The left hand panel shows the cost of abatement before and after technical change that pivots the cost curve to the right. The right hand panel shows the associated MACs.

We also address the policy implications by revisiting the seminal work by Milliman and Prince (1989). They examine the incentives to promote innovation, diffusion, and optimal agency response under five regulatory policies—direct controls, emission subsidies, emission taxes, free marketable permits and auctioned marketable permits—under the assumption that innovation would *decrease* the MAC. We reconsider this analysis, assuming innovation increases the MAC. We find that many of the relative ranking of the policy instruments are changed under the assumption of increased MAC.

3.2 Can technical change increase the MAC?

In this section we argue that in fact technical change can increase the MAC, that this is not an anomalous case, but rather is a reasonable representation of many

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since this technology would have been switched away from to achieve 100% abatement. This is on assumption that 100% capturing is impossible.

improvements to intermediate technologies. We define intermediate technologies as technologies that have lower emissions than "Business as Usual" technologies, but will be substituted away from in the case of very low abatement. Examples of such improvements are increases in efficiency of coal-fired and gas-fired electricity generators, carbon capture and sequestration (of less than 100% of emissions), and cost reduction of efficient gas-fired generators. In the transportation sector, examples would be better and less expensive hybrid vehicles and bio-diesel. The salient features of these innovations are that (1) they will be beneficial for small and medium reductions in emissions, but (2) they will be substituted away from in the case of very high abatement.

The idea is this: if a firm improves an intermediate technology, say gas-fired electricity generation, but then wants to achieve an even higher level of abatement, then the firm will substitute away from the new and improved technology. Thus, the jump from the gas-fired technology to the very low-carbon technology will now be higher than it was before. One question that has been asked of Figure 1.2 is: can the firm (or the economy) simply choose the lower MAC if they end up in a high level of abatement? The answer is no – this logic can be applied to a cost curve, but not to a marginal cost curve. The only way to move back to the original MAC would be to pretend that the intermediate technology had not been improved; and thus ignore the extra pain of substituting away from it. But this, of course, is not rational.

3.2.1 A simple example

We illustrate this through a very simple electricity-sector, climate change example. Assume three electricity technologies are available: a high-emissions technology (pulverized coal), a moderate-emissions technology (a natural gas combined cycle plant), and a no-carbon power plant (nuclear). Table 3.1 shows the levelized cost of electricity (COE) and CO_2 emission rates for these plants.[2]

Table 3.1. Parameters for illustrative example

Technology	Plant CO_2 (kg/MWh)	Levelized COE (\$/MWh)	Total cost per MWh given tax of 7 ¢/kg
High-Emissions Plant (coal)	850	24	83.50
Moderate-Emissions Plant (natural gas)	370	57	82.90
Zero-Emissions Technology (nuclear)	0	74	74.00
Innovation 1 (lower cost natural gas)	370	30	55.90
Innovation 2 (carbon capture and storage)	37	66	68.59

We consider two forms of technological advance: Innovation 1, a reduction in the cost of the moderate-emissions technology that makes it a competitive option for intermediate levels of abatement, and Innovation 2, the development of technology that will allow for capture of 90 percent of the carbon emissions from the moderate-emissions, natural gas technology. The cost implications of the two advances are shown in the table.

For simplicity, we consider only abatement through substitution: we do not consider abatement through demand reduction.[3] We model and solve a linear program using the data from Table 3.1. For the base case, we assume that the first three plants

[2]The base case and the Innovation 2 data have been extracted from Narula et al. (2002) while the total cost of producing 1MWh given a tax of .07 \$/kg have been calculated using the plant CO_2 emissions and the COE.

[3]Note that this example is meant to be illustrative of a general principle and abstracts away from a range of issues associated with carbon emissions abatement in the electricity sector, including: issues associated with the relative cost basis of existing versus new power plants; indivisibility (all plants are assumed to be available at any size); the reality of a large and heterogenous set of electricity-generation options including a range of fossil technologies along with renewable technologies such as wind power, solar power, and biomass electricity; and regional heterogeneity in fuel costs. In addition, additional, non-climate environmental costs, such as those associated with the nuclear fuel cycle, are not considered in this illustrative example.

are available; for the two advanced-technology cases we replace the parameters for the moderate-emissions plant with the parameters for the respective innovation. We minimize the cost of electricity in \$/kWh subject to a specific limit on output and on CO_2 emissions. In order to derive the cost of abatement curve, we set the combined power output, P, of these technologies to 1000kWh, and vary the emissions limit, E, from zero to 8500kg, which represents the maximum level of emissions using the base case technology. Abatement is measured as the percentage reduction in emissions below the base case technology. When $P = 1000$, abatement $= (8500 - E)/8500$. The cost of abatement is measured as the cost differential from the baseline cost of 24 \$/MWh.

Figure 3.2 shows the absolute (left panel) and marginal (right panel) abatement costs for the base case and the two advanced-technology cases. In all cases, zero abatement corresponds to the use of the high-emissions coal technology, and full abatement corresponds to the use of the zero-emissions nuclear technology.

Prior to innovation, the abatement cost function traces out a changing mix of the high-emissions and zero-emissions technologies—the intermediate-emissions technology is not on the efficient frontier. After innovation, the first part, or leftward part of the abatement cost curve represents the cost of substituting from coal to the new, improved gas technology; while the second, steeper part of the curve represents the cost of substituting from gas to nuclear. In both cases of technological advance, the now-cost-effective intermediate-emissions technology lowers the *absolute* costs of abatement, except at full abatement. Hence, innovation can be considered environmentally-beneficial. In both cases the MAC is initially lower when the new improved technology is being substituted *toward*. However, the MAC is higher at high levels of abatement, when the new improved technology is being substituted *away from*. This property will generally hold for any improvement in technology that will be substituted away from at high levels of abatement. In other words, there is

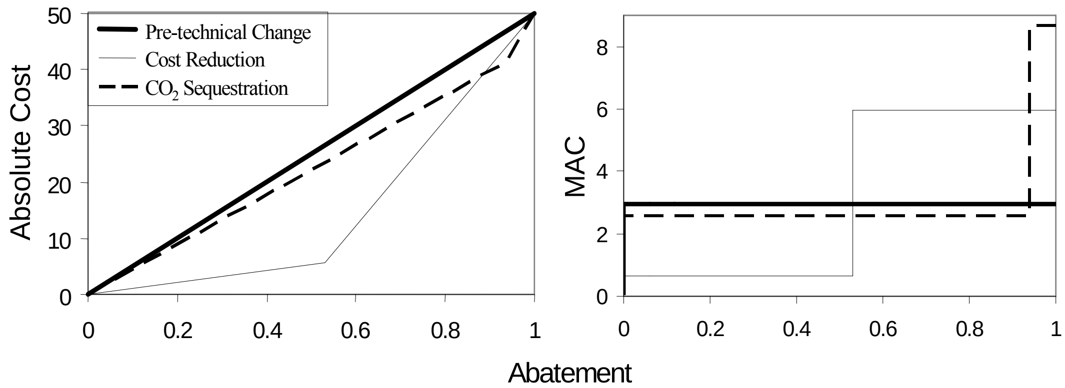

Figure 3.2. Impact of technical change on abatement cost

a functional difference between technologies that make partial abatement less costly and those that make full abatement less costly.

This implies that, for a given carbon tax, emissions may be higher after technical change than before. Again, we stress that the firm is strictly better off after technical change, but they may choose to emit more. Consider, for example, a tax of 7¢ per kg of emissions. The last column of Table 3.1 shows the total cost of producing 1MWh assuming a tax of 7¢ per kg. The table shows that before innovation the firm would choose to use the nuclear plant for a total cost of \$74 and emissions of 0; after innovation 1 the firm would use the gas plant for a total cost of \$55.90 $(= 30 + .07 * 370)$ and emissions of 370; or after innovation 2, a total cost of \$68.59 and emissions equal to 37. Thus, the total benefit to the firm is positive after innovation; but the firm will emit more after technical change for a given cost of carbon.

This result holds in more general cases where abatement is achieved through output reduction as well as substitution; it holds when the technologies are not perfect substitutes. This will happen any time an innovation is applied to a technology that will be substituted away from at high levels of abatement.

This result reiterates the phenomenon of a kinked MAC curve arising in models where numerous abatement activities can be combined. For example, Fullerton et al. (1997) discuss the case of electric utilities which use scrubbers, fuel switching, reallocation of production among plants, coal washing, and demand side management to reduce sulfur emissions. If the marginal cost of one technology depends on another and the intensity of use, the long run marginal abatement cost curves may exhibit kinks as above or even jumps as McKitrick (1999) discusses. These kinks hold significant implications for policy or instrument choice. Under uncertainty, Weitzman (1974) shows that the preference for a price-based or quantity-based instrument depends on the relative slopes of the marginal damages and the marginal abatement cost functions. Thus, if the slope of the MAC curve changes across a range of emissions, the choice of any instrument would be sensitive to the amount of emissions control required. For example, for a steeper MAC curve at low abatement levels, Baumol and Oates (1988) show that the regulator's choice of instrument will be price-based (tax or subsidy) control.

3.2.2 Review of past analyses

Very little work has been done to date comparing different representations of technical change within top-down models. Here we review three papers that indicate that the representations matter.

Baker and Adu-Bonnah (2008) consider uncertainty in the results of the R&D programs. They consider three possible outcomes of an R&D program: a breakthrough, a failure, or an incremental advancement. They investigate, both theoretically and computationally using a modified version of DICE, how the riskiness of the R&D program impacts the optimal level of investment in the program. They find that when technical change is represented as pivoting the cost curve down, then investment in a riskier program is considerably higher than in a certain program. When, however, technical change is represented as pivoting the cost curve to the right, the optimal investment is not significantly impacted by the riskiness of the program.

Baker and Shittu (2006) consider firms' incentives to adopt technologies as a function of a carbon tax. They compare technical change that reduces the carbon intensity of the carbon input with technical change that reduces the price of the non-carbon input. They show that the marginal benefits to adopting the first technology are proportional to the total carbon tax paid by the firm; the marginal benefits to adopting the second technology are proportional to the unconditional demand for non-carbon inputs. These two quantities – total carbon taxes and unconditional demand for non-carbon inputs – react differently to increases in a carbon tax. The total carbon tax paid by the firm follows a Laffer curve as the carbon tax increases – total carbon taxes first increase in an increase in the tax, but as the tax gets very high the firm substitutes away from carbon energy, and eventually the total tax paid gets very small. The unconditional demand for non-carbon inputs will monotonically increase in a carbon tax, as long as carbon and non-carbon are elastic substitutes. Thus, the incentive to adopt differs by technology.

In the only other work that we are familiar with that compares two representations of environmental technical change[4], Gerlagh and van der Zwaan (2006) compare decreases in the cost of non-fossil energy sources with improvements in CCS, represented as reducing the carbon intensity of fossil fuel. They use a learning curve approach, so the cost of non-fossil energy and the cost of CCS decrease as more of the technology gets put into play. They show that the time paths for the two technologies are qualitatively different, with the share of fossil technology that applies CCS first increasing and then decreasing to a stable level through time; whereas the share of

[4]Popp (2004) includes energy efficiency and reduction in the price of a non-carbon technology; however, the paper does not compare the investment in the two R&D programs. Also, as shown above, both of these representations lead to a lower MAC.

non-fossil technology monotonically increases through time. They also point out that in the absence of a representation for CCS, carbon taxes and fossil fuel taxes have an identical impact. They do not compare the share of CCS and of non-fossil fuel across more and more stringent targets.

3.3 Revisiting Milliman and Prince

In this section we present an illustrative example of how policy analysis is crucially impacted by assumptions about the impact of technical change on the MAC. We recreate the analysis from Milliman and Prince (MP from here on) under the assumption of increasing MAC, and show that incentives to innovate differ for different technologies. We compare a firm's incentive to innovate and promote diffusion; non-innovating firms' incentives to adopt the innovation, and all firms' incentives to promote optimal agency response, across five different policy instruments: direct emissions caps; emissions subsidies; free permits; auctioned permits; and emissions taxes. We focus on non-patented discoveries[5].

Figure 3.3 shows a single firm with an innovation which shifts its marginal cost curve from MAC to MAC'. The new marginal cost curve is lower over some range of abatement, but is higher at higher levels of abatement. We assume that the overall cost of abatement is always lower after technical change, thus the area bounded by $\varepsilon^m x$ is larger than the triangle xMI (note that ε^m is the business-as-usual emissions level). If we assume that the initial policy induces an emissions level that is to the left of the point x then the analysis from MP remains unchanged. Thus we assume that the initial emissions cap, ε^* is to the right of x, where the marginal cost of abatement has been increased by technical change.

[5]See Baker and Shittu (2004) for details. Also, see the Appendix.

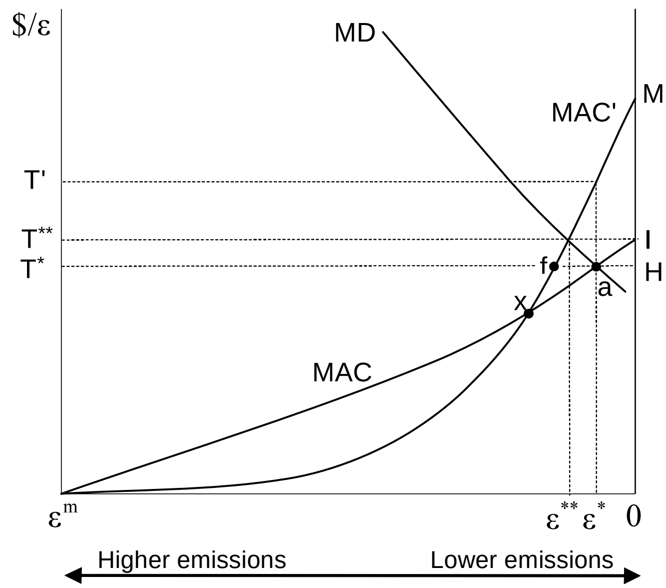

Figure 3.3. A model of technological change in pollution control

Figure 3.3 can also be interpreted as the marginal cost curves of a large number of identical firms, before and after the innovation has diffused. MD is the industry marginal damage cost associated with changes in the levels of emissions. Before technical change the emissions cap is set at ε^* and the equivalent tax or permit price is T^*. After technical change, but before diffusion, the innovating firm will produce emissions given either a direct cap of ε^* or a carbon tax, subsidy, or permit price of T^*. After diffusion and optimal policy response the new emissions cap is ε^{**} and the equivalent tax or permit price is T^{**}.

See Table 3.2 for the relative ranking of each instrument from the innovator's point of view for each step of the process: the top half of the table reviews the results from MP; the bottom half shows the results under our assumptions. The ranking of the instruments with respect to the firm's incentive to innovate remains unchanged from MP – direct controls under-perform the other instruments. Under all instruments except direct controls the firm gains the area within $\varepsilon^m x$ and loses the area within xfa.

Table 3.2. Instrument ranking comparison between increasing and decreasing MAC

MAC Decreasing	Direct Controls	Emissions Subsidy	Free Permits	Auctioned Permits	Emissions Taxes
Innovation Prom.	5th	1st	1st	**1st**	1st
Diffusion Prom.	2th	2nd	5th	**1st**	2nd
Optimal Ag. Res.	Oppose	Oppose	Oppose	**Oppose**	Favor
Overall Inno. Gain	Uncertain	Uncertain	Uncertain	**Gain**	Gain
MAC Increasing					
Innovation Prom.	5th	**1st**	1st	1st	1st
Diffusion Prom.	1st	**1st**	4th	5th	1st
Optimal Ag. Res.	Favor	**Favor**	Favor	Favor	Oppose
Overall Inno. Gain	Gain	**Gain**	Gain	uncertain	uncertain

Next we consider the incentives to promote diffusion for both the innovator and the non-innovator. MP found that auctioned permits provided the innovator with a positive diffusion incentive: the auctioned price of permits was lowered through diffusion. In our case, auctioned permits provide the innovator with the most negative incentive: when diffusion shifts MAC to MAC' for all firms, the auctioned permits increase in price from T^* to T'. This is because, here, technical change increases the marginal cost of abatement. The other instruments remain in the same order as MP: taxes, subsidies and direct control have no impact on the incentive; firms are always worse off after diffusion under free permits. Non-innovators profit unambiguously from diffusion under all instruments except auctioned permits. It is possible that the increase in the price of auctioned permits may outweigh the benefit of lowered abatement costs. This would not necessarily prevent diffusion, however – any individual firm, taking the auctioned price as given, would benefit from adopting the new technology (ignoring the cost of adoption).

Next, we consider the firm's incentive to promote optimal agency response. We find that, as noted in MP, the results for optimal agency response are exactly the opposite here as in MP. Given higher marginal costs, the optimal agency response is to increase the emissions limit (i.e. make it less stringent, from ε^* to ε^{**}) or increase the tax/subsidy (from T^* to T^{**}). Thus, unsurprisingly, the industry has an incentive to support optimal agency response in every case except emissions taxes. In MP, when technical change decreased marginal costs, the optimal agency response is to decrease the limit or decrease the tax/subsidy, thus the opposite results.

Finally, the 4th and 8th rows of Table 3.2 compare the overall innovator gains from the entire process of technical change. We find that direct controls, emission subsidies, and free permits guarantee positive gains to innovations that increase marginal abatement cost; for auctioned permits and emission taxes the result is ambiguous. This result is in contrast to the result in MP, where auctioned permits and taxes resulted in gains, and direct controls, subsidies, and free permits were ambiguous. The reason for the difference is that under an increasing MAC technical change reduces the stringency of the policy for direct controls and free permits, and increases subsidies. Taxes and auctioned permits, on the other hand, could lead to a loss if the transfer loss due to higher tax/price outweighs the savings in abatement cost. Note that if the marginal damages are constant then there is a clear gain for taxes and auctioned permits as well – it requires steeply sloped marginal damages to get a loss. However, all these calculations are net of the cost of technical change. More generally, this result, like the result in MP is heavily influenced by optimal agency response. If we only look at the combined incentives to innovate and promote diffusion, it can be shown that taxes and subsidies provide the greatest incentive, followed by free permits and direct controls, with auctioned permits last. In fact, it cannot be guaranteed that auctioned permits will lead to a gain after diffusion, because the loss from diffusion is potentially large. Taken altogether the dominant choice is emission subsidies: they tie for first in all ranking. Emission taxes, however, are not far behind, especially if the marginal damages are almost flat. In MP, auctioned permits are the dominant choice, but again, emissions taxes are not far behind. Thus, as long as marginal damages are not too steep, emission taxes may be the most robust instrument for promoting a variety of technologies. An interesting implication of this exercise is that different policy instruments may provide incentives for firms to move down different paths of innovation. If a firm faces a choice between two technologies that will lower overall costs, but one decreases the MAC while the other increases the MAC, then the presence of emission subsidies may cause firms to choose the technology that increases the MAC. This illustrates the importance of accurately representing technical change when evaluating policy instruments.

3.4 Discussion

This chapter has explored the possibility and implications of environmental technological advance that increases marginal abatement costs for higher levels of abatement. We first illustrated the possibility of such innovation using a simplified electricity-sector example[6]. We then considered the implications of such advance using the framework from MP. Not surprisingly, the analysis indicated that the policy implications can be substantively different when innovation increases marginal abatement costs. This indicates that the best framework for analyzing how policy instruments impact abatement technology and in particular to analyze which instrument is best, is a framework including a portfolio of technologies. Since different instruments have different incentives for different technologies, using the “wrong” instrument may promote the “wrong” technology. For example, if the costs of achieving technical change are similar for an efficiency improvement and cost reduction of non-carbon alternatives, direct controls may promote efficiency R&D while taxes may promote non-carbon alternatives. In fact, one interpretation of these results compared to MP is that taxes and auctioned permits provide more incentives for firms to choose technologies that

[6] An applied example based on a technologically-detailed integrated assessment model (IAM) can be found in Baker, Clarke and Shittu (2008).

lower marginal cost while direct controls, subsidies, and free permits provide more incentives for firms to choose technologies with higher marginal cost.

We close here by discussing the requirements for such a situation to occur. First, innovation must impact technologies associated with less than full abatement. By improving these technologies, the marginal cost of abatement must ultimately increase at some point as we move toward full abatement. In the climate context, examples would generally surround the development of new, lower-emissions fossil-based technologies, or the improvement of existing fossil technologies. In contrast, innovations that reduce the costs of full abatement—for example, lowering the costs of photovoltaic cells—will decrease the marginal costs of abatement.

Second, the optimal level of abatement must be in the range where marginal costs have increased. In the case of the carbon-capture example in Section 3.2.1, this would mean abatement of greater than 90 percent. On the other hand, cost reductions in intermediate-emissions technologies such as natural gas combined cycles or the development of integrated gasification combined cycles for coal would need substantively lower levels of abatement to obtain the increasing marginal cost range.

CHAPTER 4

ENERGY R&D PORTFOLIO UNDER UNCERTAIN CARBON TAX

4.1 Introduction

The negative impact on the climate of Greenhouse Gas (GHG) emissions confronts decision makers at the firm level, as well as policymakers, with the question of what steps should be taken to ameliorate this growing concern. On one hand, policymakers and regulators are wrestling with determining the optimal policy to spur technological change to improve carbon intensive technologies or develop non-carbon technologies. On the other hand, decision makers at the firm level are grappling with how to allocate their research and development (R&D) efforts in the face of several alternatives and under a future policy that is uncertain, but expected to increase in stringency.

The central theme of this chapter[1] is to address the optimal R&D investment response of a decision maker—at the firm level with a portfolio of alternative technologies—to a rising carbon tax. Understanding the optimal allocation of investment in these technologies is crucial for four reasons; (1) There are many new alternative technologies and potential improvements to currently economic technologies with different potentials to reduce GHG emissions. While some of these technologies have zero emissions, others improve on the current methods by reducing their emissions level. Thus, knowing which technology is optimally worthy of investment is important. (2) Like most economic resources, there is a limitation on the investment capabilities of

[1]This chapter is a version of the paper, under revise and resubmit, in IEEE Transactions on Engineering Managment.

a firm to undertake the research efforts on these improvements and innovative efforts. In addition to this, environmental R&D spending is irreversible. (3) Investment decisions made today have multi-period consequences on the future shape of energy technologies. (4) policymakers need a yardstick for evaluating the incentive effects, on firms, of a carbon tax regulation and the overall portfolio of technological change.

Regardless of the domain—product development, environment, or climate change—portfolio investment decisions do not come easy. For example, Loch and Kavadias (2002), in their analysis of the dynamic selection of new product development, underscore the combinatorial complexity of allocating a scarce budget over multiple periods. This is not just because decisions have multi-period consequences, but it is also due to the different return functions on the new product lines which are competing for a common pool of resources and are often interdependent (Dickinson, Thornton and Graves (2001)). This phenomenon of interdependence and having different return functions in new product development has similarities with the different ways the energy technologies influence the level of emissions, their effects on the demand for alternatives, and ultimately their interactions through complementary or substitution effects. On the climate change front, having a portfolio of R&D projects is important because of the several positive ways that each project impacts climate change. The climate change literature shows that a project portfolio has two advantages: (1) it diversifies uncertainty about the outcome of the technologies, and (2) it hedges against the uncertainty about how high the future carbon tax will be.

Understanding the interaction and interdependence properties between the technologies is one part of the discussion. The other part is the cost perspective of reducing GHG emissions by these technologies. For example, in the examination of several technologies in the context of a global integrated assessment model of energy, agriculture, land-use, economics, and carbon cycle processes, Edmonds et al. (2004) discuss the significance of the development of an expanded suite of technologies including carbon capture and disposal, hydrogen systems, and biotechnology, because they hold the potential to dramatically reduce the cost of stabilizing GHG concentrations. Similarly, Baker, Chon and Keisler (2007) derive marginal abatement cost curves under different solar technologies using the MiniCAM model[2]. Using an array of elicited expert definitions of technical success, they show that different technologies, if they achieve success as defined, have different impacts on the marginal cost of abatement. Fishelson and Kroetch (1989) show that the possibility that the marginal and total costs are changing at different rates for different innovations justifies the use of more than one type of the technologies available. Although their work focuses on R&D into energy storage devices, this result still holds in the general climate change arena.

This chapter has two objectives: to determine (1) how an increase in a carbon tax influences a firm's optimal energy R&D spending, in terms of overall investment level and in terms of the type of R&D in the portfolio; and (2) how parameters such as substitution elasticity and cost of technical change impact the optimal portfolio both in terms of overall investment size and technology specific investment. In addition, we explore the impact of riskiness in the carbon tax on the optimal portfolio in order to get insights into the effects of carbon tax uncertainty on investment. These objectives constitute a part of an important list of criteria guiding firms on decision making regarding their investments in response to an increasing carbon tax and uncertainty about climate policy in the presence of different available energy R&D technologies. The differences between these technologies have been widely ignored in the theoretical literature. Therefore, an important motivation is to find out whether the response of a given R&D program to an increasing carbon tax is independent of the consortium of options in the energy R&D portfolio. In other words, do the other programs in

[2] Brenkert et al. (2003) and Edmonds et al. (2004) give a complete description of the model.

the portfolio exert any influence in defining an optimal investment allocation to that technology? For example, how is the optimal investment in non-fossil fuel technologies impacted by the presence of carbon capture and sequestration technologies?

Closely related to this, Baker and Shittu (2006) examine a firm's profit-maximizing R&D response to an uncertain carbon tax for two R&D programs: cost reduction of non-fossil energy technologies and emissions reductions of currently economic technologies. They consider these different technologies independently, and conclude that the optimal investment in R&D does not always increase monotonically in a carbon tax. This chapter extends that analysis by considering four different energy R&D technologies in a portfolio setting. A two-stage theoretical model is developed to explore these issues on the four-project energy R&D portfolio: cost reduction of non-fossil energy technologies, emissions reduction of currently economic fossil-based technologies, fossil energy efficiency improvement, and total energy use efficiency program. Our model focuses on a firm that invests in technological improvements; for example, the American Electric Power (AEP), that both produces and uses R&D.

We proceed in Section 4.2 with a review of related literature. In Section 4.3, we provide the theoretical framework for the representation of a set of broadly defined R&D programs. Since the demand for energy inputs is central to the relevance of these technologies, Section 4.3.3 sets the framework for the overall optimal demand for fossil and non-fossil energy in this four-technology portfolio. Section 4.3.4 introduces the computational model. Section 4.4 delves further into the analysis of the developed framework with emphasis on the impact of increasing carbon tax on the levels of investments in each of the technologies in the portfolio and the overall investment. In this section, sensitivity analysis is carried out on the effect of R&D cost coefficient and substitution elasticity between fossil and non-fossil energy on investment. This section also discusses the impact of risk on investment. Section 4.5 concludes.

4.2 Background

The portfolio investment allocation problem has received significant research attention in the past because of its importance to managers and decision makers, and we observe that this problem exists in two relevant literatures—climate change and product development. We review both but with more emphasis on the climate change literature. In the climate change literature, the energy portfolio investment allocation problem is triggered by an exogenous factor—regulatory policy. In this analysis, we explore the influence of an increase in a carbon tax to spur investment in energy technologies. The role of policy uncertainty in inducing technological change in climate control has also attracted considerable research attention[3].

4.2.1 Overview of portfolio selection and the role of uncertainty

In the resource and portfolio allocation literature, Roussel et al. (1991) discuss the importance of portfolio selection for top management in organizations. They view general managers and R&D managers working as partners to pool their insights in deciding what to do and why and when to do it, by realistically assessing costs, benefits, and risk/reward, and they balance these variables within a portfolio of R&D activity that best fulfills the purposes of the corporation. This emphasizes several aspects in R&D portfolio management, but a number of papers focus on particular issues.[4] Two case studies, Loch et al. (2001) and Beaujon et al. (2001), describe the application of models to R&D project selection at BMW and GM, respectively.

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[3]The expectation of a tax policy shapes firms' investment decisions on energy R&D. Several earlier studies in this literature ranging from Magat (1978) through Milliman and Prince (1989) show that emissions taxes and emissions permits generally provide more incentives for technological innovation than policies based on standards.

[4]For example, appropriate project sequencing, Granot and Zuckaman (1991); simulating different portfolios to assess the value of information, Keisler (2004); optimal investment decisions when the return on investment is random, Berzinsh et. al. (2006).

Kavadias and Loch (2003) cover a wider range of issues concerning portfolio R&D, and more recent advances appear in Cooper et al. (1998).

In economics, the study of searching for the best alternative traces back to Weitzman (1979) in his focus on sequential investment. This research thrust extends into the climate change literature and energy technology R&D portfolios. For example, Gritsevskyi and Nakicenovic (2000) observe that an important policy implication is that future research, development, and demonstration efforts and investments in new technologies should be distributed across "related" technologies rather than directed at only one technology from the cluster. Closely related to this is the impact of R&D efforts on the cost of reducing GHG emissions, as this is important in determining the optimal portfolio. For example, Baker, Clarke and Weyant (2006) point out that a research program to improve the efficiency of coal-fired electricity generation will create a different abatement cost profile from an R&D program into photovoltaic cells. While the first lowers the cost for moderate reductions in GHG emissions, the second will lower the costs of severely reducing emissions. It is clear that having an energy R&D portfolio is the best strategy, but the question that arises is: in the face of these different technologies, what is the optimal level of spending on these technologies under an increasing emissions tax?

Pizer (1999) shows that uncertainty (without learning) is crucial to investment decisions because it raises the optimal level of emission reductions and leads to a preference for taxes over rate controls. This suggests that analysis that disregards the impact of uncertainty is likely to result in inefficient policy recommendations. For example, Grubler and Gritsevskyi (2002) consider the effects of uncertainties such as demand, technology costs, and the size of a carbon tax on technology choice. They find that the entry of an additional source of uncertainty makes the technology portfolio more diversified. While one group of previous efforts (Dixit and Pindyck (1994), Caballero (1991)) look at investment decisions by considering how optimal investment is impacted by uncertainty in prices or demand, others analyze the same question under technology uncertainty (Bosetti and Tavoni (2007)). Hasset and Metcalf (1999) argue that random changes in tax policy provide opportunities for firms to wait out high tax regimes and invest more heavily in low tax regimes. Other papers show how the optimal R&D investment changes with the risk-profile of the technologies and with uncertainty about climate damages[5].

4.3 Model

We model a firm's profit maximizing choice of energy R&D in the face of a tax on carbon emissions. We use a two-period theoretical model. Investments in R&D are made in the first period in anticipation of a future carbon tax. For simplicity, we ignore production in the first period. Optimal production is chosen in the second period after the firm learns about the carbon tax and technical change has been achieved, leading to a second period profit function. We take the market structure for output to be exogenous—the firm faces a known downward sloping demand curve. In the following subsections, we define the details of the model used to derive the optimal demand for the energy inputs, and then show how the marginal profit is influenced by investments in energy technology. We then present our computational model—with model parameters—using the defined representations of technical change.

4.3.1 Second period profits and optimal energy demand

The firm uses three inputs—non-energy inputs x, fossil energy inputs, ε_c and non-fossil energy inputs, ε_{nc}. Let ε_c be normalized so that, using the current technology, one unit of fossil energy produces one unit of emissions. Then the total firm-specific price paid for fossil energy is the cost of the fuel, P_c, plus the price of the carbon

[5] For a detailed review of uncertainty and investment in the context of climate change, see Baker and Shittu (2008).

emitted, equal to the carbon tax t. Assume that, under the current technology, non-fossil energy is more expensive than fossil energy: the firm-specific price of non-fossil energy equals $P_c + \eta$. The price of non-energy inputs is w. We consider a firm with a nested constant elasticity of substitution production function to produce output y. Thus, in the absence of technical change, the firm chooses the profit-maximizing inputs by solving

$$\begin{aligned} \pi &= \max_{\varepsilon_c, \varepsilon_{nc}, x} yp(y) - ((P_c + t)\varepsilon_c + (P_c + \eta)\varepsilon_{nc} + xw) \qquad (4.1) \\ \text{such that } y &= (x^\rho + (\varepsilon_c^\gamma + \varepsilon_{nc}^\gamma)^{\frac{\rho}{\gamma}})^{\frac{1}{\rho}} \end{aligned}$$

where $p(y)$ is the output price, $\zeta \equiv \frac{1}{1-\rho}$ is the elasticity of substitution between energy and non-energy inputs and $\sigma \equiv \frac{1}{1-\gamma}$ is the elasticity of substitution between fossil and non-fossil energy inputs. We assume the firm is facing a constant elasticity demand with inverse demand curve, $p(y) = Ay^{-\frac{1}{b}}$, where A is a constant and b is the price elasticity of demand. The solution to this problem (see the Appendix for details) gives the unconditional demand for fossil energy input and non-fossil energy input, ε_c^* and ε_{nc}^*, respectively, as

$$\varepsilon_c^* = P_c^{\frac{1}{\gamma-1}} P^{\frac{\gamma-\rho}{\gamma(\rho-1)}} \left[w^{\frac{\rho}{\rho-1}} + P^{\frac{\rho(\gamma-1)}{\gamma(\rho-1)}} \right]^{\frac{b(1-\rho)-1}{\rho}} \left(\frac{b}{b-1} \frac{1}{A} \right)^{-b} \qquad (4.2)$$

$$\varepsilon_{nc}^* = P_{nc}^{\frac{1}{\gamma-1}} P^{\frac{\gamma-\rho}{\gamma(\rho-1)}} \left[w^{\frac{\rho}{\rho-1}} + P^{\frac{\rho(\gamma-1)}{\gamma(\rho-1)}} \right]^{\frac{b(1-\rho)-1}{\rho}} \left(\frac{b}{b-1} \frac{1}{A} \right)^{-b} \qquad (4.3)$$

where $P = P_c^{\frac{\gamma}{\gamma-1}} + P_{nc}^{\frac{\gamma}{\gamma-1}}$.

4.3.2 Portfolio profit function

Now, we consider the firm's investment in technical change in the first period. We represent technical change as having an impact on the firm-specific cost of energy inputs such that the effective cost is reduced after technical change. Let technical change, $\overrightarrow{\boldsymbol{\alpha}}$, represent a vector of cost improvements in the technologies in the portfolio. For each technology, we have $0 \leq \alpha < 1$, with cost reduction in the inputs maximized as α tends to 1. The the firm's second period profit function, assuming that the carbon tax, t, is known, is $\pi\left(w, P_c + t, P_c + \eta; \overrightarrow{\boldsymbol{\alpha}}\right)$. Let $g\left(\overrightarrow{\boldsymbol{\alpha}}\right)$ represent the cost vector of the R&D programs aimed at achieving technical change of $\overrightarrow{\boldsymbol{\alpha}}$. We assume that the cost of technical change goes to infinity as $\overrightarrow{\boldsymbol{\alpha}}$ approaches 1: $\lim_{\overrightarrow{\alpha} \to 1} g(\overrightarrow{\boldsymbol{\alpha}}) = \infty$. We assume that $g\left(\overrightarrow{\boldsymbol{\alpha}}\right)$ is increasing and convex in each argument. Thus in the first period, the firm chooses $\overrightarrow{\boldsymbol{\alpha}}$, the level of technical change, when the carbon tax is still unknown by solving

$$\max_{\overrightarrow{\alpha}} -g\left(\overrightarrow{\boldsymbol{\alpha}}\right) + \mathrm{E}_t\, \pi\left(w, P_c + t, P_c + \eta; \overrightarrow{\boldsymbol{\alpha}}\right) \qquad \overrightarrow{\boldsymbol{\alpha}} = \alpha_A, \alpha_C, \alpha_E, \alpha_F \tag{4.4}$$

where the subscripts, A, C, E and F represent non-fossil, carbon capture and sequestration, energy efficiency, and fossil energy efficiency programs, respectively; and E_t refers to the expectation over the uncertain tax. The first order conditions for each $\boldsymbol{\alpha}_i$ are

$$g'\left(\alpha_i\right) = \mathrm{E}_t\left[\frac{\partial \pi}{\partial \alpha_i}\right] \qquad \forall i = A, C, E, F \tag{4.5}$$

It is clear that the optimal level of R&D spending increases if the right-hand side of (4.5) increases; this in turn depends on the probability distribution of t. Thus we will focus, computationally, on how a change in the probability distribution over the carbon tax, t, impacts the optimal investment in each technology in the portfolio. In particular, we will focus on how an increase in t impacts $\frac{\partial \pi}{\partial \alpha_i}$.

4.3.3 Representations of technical change

In this section, we show how we represent each type of technical change in the portfolio through their effects on the cost of inputs; and thus on the profit function of the firm in the second period. We use these definitions in a framework that captures the entire portfolio of technical change in the profit maximization problem. We define energy efficiency improvement as technical change that leads to a higher level of output for the same level of energy input. This in turn leads to a reduction in the effective price of both fossil and non-fossil energy inputs, per unit output. Examples of energy efficiency-improving R&D are general improvements in electricity generation, transmission and distribution efficiencies. We model this technical change such that both the cost of fossil and non-fossil energy inputs are effectively reduced by $(1 - \alpha_E)$. After technical change parameterized by α_E the second period profit function becomes $\pi\left[w, (1-\alpha_E)(P_c + t), (1-\alpha_E)(P_c + \eta)\right]$.

Fossil fuel R&D reduces the price of fossil energy by $(1 - \alpha_F)$ by increasing the per unit efficiency of fossil fuel use. For example, an increase in efficiency of a coal fired generator such that more output is produced per unit input. The reduction in the price of fossil energy as captured by this technology gives a second period profit given by $\pi\left[w, (1-\alpha_F)(P_c + t), P_c + \eta\right]$. The representations of R&D into non-fossil fuel technology and carbon capture and sequestration (CCS) technologies follow directly from the work by Baker and Shittu (2006). They model non-fossil fuel R&D as reducing the premium on non-fossil energy from η to $(1 - \alpha_A)\eta$. This program could represent, for example, a firm's research into minimizing the cost of their wind turbines or the development of less expensive solar power, which are non-fossil energy alternatives. They model CCS as reducing the carbon intensity of a unit of fossil energy from 1 to $(1 - \alpha_C)$. Thus the price of fossil energy is effectively reduced from $P_c + t$ to $P_c + (1 - \alpha_C)\, t$. This program represents an investment into technology that will capture a fraction α_C of the firm's fossil emissions. Under non-fossil fuel R&D and CCS, the second period profit functions are $\pi\left[w, (P_c + t), P_c + (1 - \alpha_A)\,\eta\right]$ and $\pi(w, P_c + (1 - \alpha_C)\, t, P_c + \eta)$, respectively.

Thus the firm's overall portfolio problem is

$$\max_{\vec{\alpha}} -g\left(\vec{\boldsymbol{\alpha}}\right) + \mathrm{E}_t\, \pi\left[w, (1-\alpha_E)(1-\alpha_F)(P_c + (1-\alpha_C)\, t), (1-\alpha_E)(P_c + (1-\alpha_A)\, \eta)\right] \tag{4.6}$$

4.3.4 Computational model

In this section, we describe the computational method applied. For a firm facing a constant demand elasticity with inverse demand curve defined in Section 4.3.1, the profit function follows

$$\pi = Ay^{*\left(\frac{b-1}{b}\right)} - (1-\alpha_E)(1-\alpha_F)(P_c + (1-\alpha_C)\, t)\varepsilon_c^* - (1-\alpha_E)(P_c + (1-\alpha_A)\, \eta)\varepsilon_{nc}^* - wx^* \tag{4.7}$$

where A is a constant and b is the price elasticity of demand; and asterisks denote the quantity is optimal. We maximize the firm's profits less overall cost of technical change, $g(\vec{\boldsymbol{\alpha}})$. In order to get computational results, we need to make an assumption about the cost of R&D. We represent the total cost of technical change as follows:

$$g(\vec{\boldsymbol{\alpha}}) = \sum_{i \in \{A,C,E,F\}} \frac{\kappa_i \alpha_i^2}{1-\alpha_i} \tag{4.8}$$

We have made the simplifying assumption that all the programs have the same functional form. This functional form has the advantage of simplicity, exhibits decreasing returns to scale in R&D, and ensures that R&D will not bring about zero-cost full abatement.[6] It also assumes that the technologies are not complements in terms of R&D programs. The R&D cost coefficients, κ_i, are subject to sensitivity analyses to determine their influence on the optimal levels of R&D. These programs are effectively different in the way they affect the abatement cost curve. Putting (4.7) and (4.8) into (4.6), we solve the deterministic maximization problem to determine the maximizing levels of α_i in each technology under different tax levels.

[6] In each case, if $\alpha_i = 1$ then emissions would be zero at the profit maximizing point.

Table 4.1. Model parameters

Parameter	Symbol	Value
Price of carbon inputs	P_c	1
Premium on non-carbon inputs	η	1
Price of non-carbon inputs	$P_{nc} = P_c + \eta$	2
Price of non-energy inputs	w	1
Output price coefficient	A	1
Demand price elasticity	b	1.1
Elasticity of substitution between energy and non-energy inputs	ζ	0.75
Elasticity of substitution between carbon and non-carbon energy inputs	σ	1.5; 6
Coefficient of investment cost	κ	1

4.3.5 Model parameters

In this section, we state our key assumptions and present the base-case values for model parameters. We start by assigning equal weights to the cost of technical change such that technology i has a cost coefficient $\kappa_i = 1$. This assumption presents an unbiased measure of the response to increasing tax of the demand for each technology in the portfolio. We also assume that the prices of the inputs are fixed.

Table 4.1 summarizes the baseline values assigned to the parameters. The assumptions about the prices of carbon and non-carbon inputs in this table make fossil and non-fossil technologies become economically equivalent when the tax is 1.

The elasticity values are converted to the parameters in the functions using $\zeta \equiv \frac{1}{1-\rho}$ and $\sigma \equiv \frac{1}{1-\gamma}$. Popp (2004), in his climate economy model including endogenous technological change, has calibrated the short-term elasticity of substitution between fossil and non-fossil energy as 1.6, implying that they are substitutes, but not very close substitutes. If we consider electricity generators, then in the long run, fossil and non-fossil energy are perfect substitutes. Goulder and Schneider (1999), in their analytical and numerical general equilibrium models in which technological change results from profit-maximizing investments in R&D, used 0.9. For non-electricity sector firms, we argue that substitution elasticities are even higher because an alternative to their energy supply is electricity which is readily obtainable. We present the following observations under two measures of elasticity of substitution between fossil and non-fossil energy inputs—a high value of 6 to represent the long term high substitutability between different sources of electricity generation, and a low value of 1.5, in the range of Popp's estimate.

4.4 Results of illustrative scenarios

In this section, we explore how a firm's optimal investments in these technologies respond to an increasing carbon tax under specific parameter values in order to illustrate general ideas. We discuss the experimental design steps required to understand the behavior of investments in these technologies, and we present results through the lens of how optimal investment in individual technologies respond to changes in parameters. We start by exploring the optimal portfolio reaction to an increasing tax under two specific estimates of the elasticity of substitution between fossil and non-fossil energy inputs—high and low. For each level of flexibility, we assume two types of distributions on the cost coefficients of the technologies. The first is uniform cost coefficients across the technologies, and the second is giving higher costs to the efficiency-improving technologies.

4.4.1 High elasticity of substitution

Using the parameters described above, the left hand side of Figure 4.1 shows the profit-maximizing response of total investment in the technologies to different levels of a carbon tax. The right hand panel shows the breakdown of individual investments across the four technologies in the portfolio. In these figures, the price elasticity of demand is 1.1, the elasticity of substitution between energy and non-energy input is 0.75, and between carbon and non-carbon energy is 6.

The total investment graph shows that the total optimal R&D investment first increases, then decreases, and finally flattens as the tax increases. The figure on the right shows how this breaks down technology by technology. We see that investment in all the programs increase in the tax when the tax is low. However, two of the programs—fossil fuel efficiency and CCS—decrease after the tax hits about 0.5 and 0.75, respectively. Energy efficiency and non-fossil fuel investments remain relatively high and unchanging at the high tax levels. To understand these patterns, consider how a carbon tax will effect the firm's demand for fossil and non-fossil inputs. As a carbon tax increases, the demand for fossil inputs will unambiguously decrease. The demand for non-fossil inputs, however, are more ambiguous. The substitution effect will lead to an increase in demand for non-fossil as the carbon tax increases. The output effect will lead to a decrease. It is clear that the demand for energy inputs changes with a carbon tax and drives the optimal level of R&D.

R&D spending in energy efficiency is higher than for other programs because investment in technology that improves on overall efficiency has a double-edged effect, as it affects the prices of both energy inputs simultaneously. Optimal investment in this technology is stable at high levels of the tax. This is because at high levels of the tax the firm will substitute away from fossil fuel energy inputs and rely mainly on non-fossil inputs. Once a firm has substituted completely away from fossil inputs, the carbon tax is no longer relevant. Thus, a firm that anticipates a high future tax must focus on technologies that use non-fossil inputs. Like energy efficiency, investments in non-fossil fuel technologies monotonically increase as the carbon tax increases. These results reflect the fact that firms substitute toward non-fossil technology as the tax increases; and therefore, invest more in improving non-fossil technologies. This will be true as long as the substitution effect is stronger than the output effect.

Investment in fossil fuel efficiency and CCS improvement programs first increase, and then decrease as the tax increases. The investment increases in the tax when the tax is small, because as long as the tax is small, fossil inputs are less expensive than non-fossil, and the firm will tend to use very large amounts of fossil fuel energy.

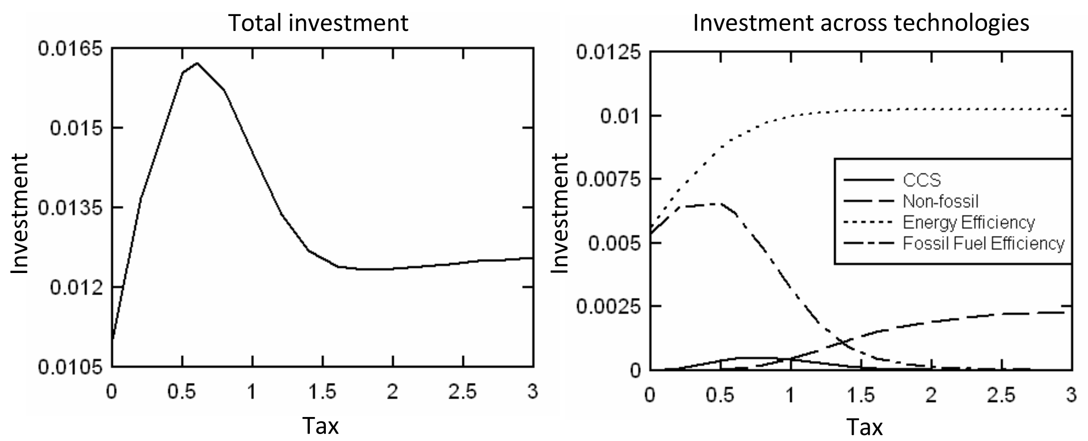

Figure 4.1. The left hand panel shows the response of total investment in the technologies as carbon tax increases. The right hand panel shows the breakdown of individual investments across the four technologies in the portfolio. In these figures, the price demand elasticity is 1.1, the elasticity of substitution between energy and non-energy input is 0.75, and between carbon and non-carbon energy is 6.

The R&D takes the edge off the carbon tax in reducing the carbon emissions per unit output. However, as the total price of fossil fuel rises further, and the firm substitutes away from fossil fuel, the benefits of fossil technology programs start to get small. It is interesting to note that the tax level, $tax = 0.5$, at which investments in fossil fuel efficiency program begin to decrease, is equivalent to the point when non-fossil fuel technology comes on stream.

A comparison of CCS and fossil fuel efficiency programs with non-fossil fuel R&D shows that initial investments are higher in fossil technologies—as the tax gets high, however, firms substitute more toward non-fossil energy, therefore investment in technologies that improve non-fossil energy become more attractive. Therefore, the economic interpretation of overall investment first increasing and then decreasing in the carbon tax is that investment appears to be highest when the carbon tax is high enough to provide incentives for using CCS, but not so high that firms start to substitute away from fossil energy significantly. The value of technologies which improve non-fossil increases as the carbon tax increases. For technologies that improve fossil

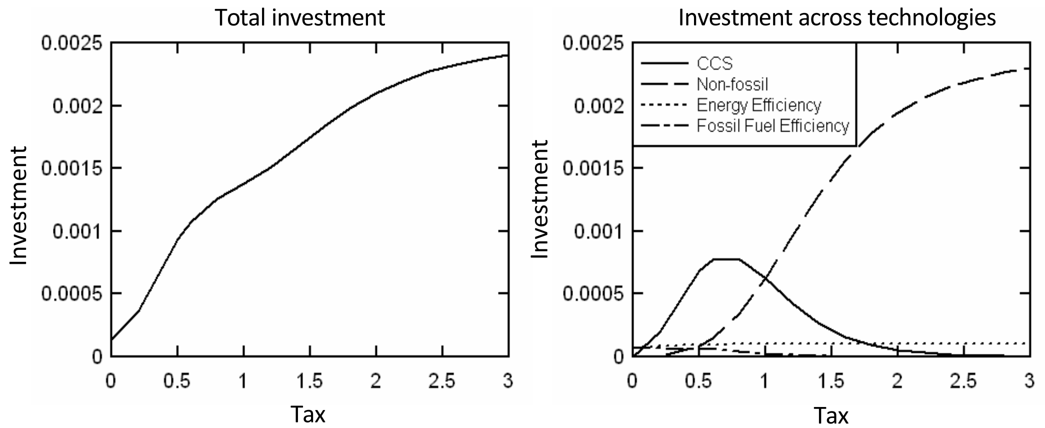

Figure 4.2. High cost efficiency programs / High elasticity of substitution: In these figures, cost coefficient, κ on efficiency programs is higher than for the non-efficiency improving programs. The elasticity of substitution is 6. The other parameters remain as presented in Table 4.1.

fuel technology (CCS and fossil fuel efficiency), the value of the technology follows a Laffer curve—first increases, and then decreases. For CCS in particular, the firm has no incentive to invest when the tax is zero, or when the tax is extremely high.

So far, we have assumed that the R&D cost coefficient, κ is equal across the technologies. In Figure 4.2, we explore the impact of having a high cost of investment in the efficiency programs. Here, we assume that the cost coefficients for both efficiency programs is a hundred times that of the other two programs. We see that in this case, the overall spending in the portfolio increases monotonically in a carbon tax. This is because the increase in the non-fossil program dominates the decrease in CCS and fossil fuel efficiency programs. It is evident here that as the carbon tax increases, the substitution effect driving the demand for fossil and non-fossil inputs mentioned earlier is transferred to CCS and non-fossil programs to act as substitutes. The interplay between CCS and non-fossil technologies acting as substitutes under increasing policy stringency increases the level of total investment and the optimal allocation of investments depends significantly on this interaction. Furthermore, the relatively high costs of R&D into the efficiency programs implies these programs have no observable influence on the substitution effect between CCS and non-fossil technologies because this interaction did not change.

4.4.2 Low elasticity of substitution

In this Section we present the results using a low value for the elasticity of substitution, $\sigma = 1.5$, with all other parameters as in Table 4.1 to gain further insights into the interactions between these technologies. The left hand panel of Figure 4.3 shows that in this case the overall investment increases in a carbon tax; this is in contrast to the results in Figure 4.1. Using the right hand panel of Figure 4.3, we focus on the drivers of this result. First, the optimal investment in non-fossil programs is relatively flat. This is because under the assumption of low substitutability between the energy inputs, the substitution effect is smaller and the output effect is larger than what we saw above. The firm doesn't increase the demand for non-fossil inputs to a great degree under an increasing carbon tax, therefore the optimal investment in non-fossil technology is also not very responsive. Second, investments in overall efficiency improvement program follow a pattern very similar to above, increasing steadily with a carbon tax. This is because it effects both technologies, and given low substitutability, this is of great benefit. Third, although the demand for CCS technology shows increasing investment, that of fossil fuel efficiency improvement is relatively stable and fairly higher. These three factors add up to cause the overall increase in total investment. Note that eventually output will decrease with the tax, leading to overall lower investment in technology. This effect will not be seen, however, until a very high carbon tax.

Now we use the same low elasticity, but different R&D cost coefficients to show the influence of the cost coefficients on investment decisions. Figure 4.4 illustrates an example of this with the total investment increasing in tax. In this figure, the cost

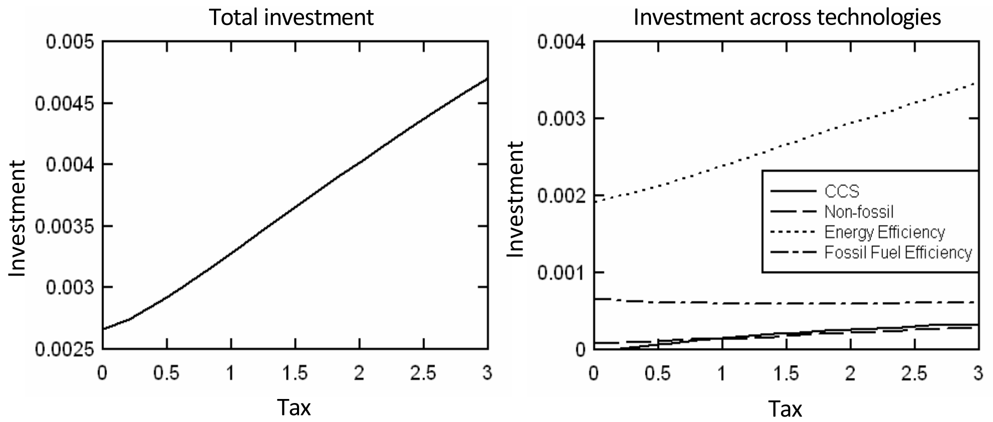

Figure 4.3. In these figures, the elasticity of substitution between fossil and non-fossil energy is 1.5 and the relative cost coefficients are equal.

coefficients for both efficiency programs ($\kappa = 100$) are higher than for the other two programs ($\kappa = 1$). We see that overall investment is increasing in the tax, similar to the results in Figures 4.2 and 4.3. A comparison of Figures 4.2 and 4.4 with Figures 4.1 and 4.3 shows that overall investment is lower when efficiency programs are costly. In a similar manner, a comparison between Figures 4.1 and 4.2 with Figures 4.3 and 4.4 show that the overall level of spending is significantly higher when elasticity is higher. Firms that are flexible (i.e. have a high elasticity) can substitute toward low carbon inputs as the carbon tax increases. But firms that are less flexible do not have this option, therefore an increasing carbon tax leads to reduced output. This, in turn, means that firms optimally invest less in any technology.

4.4.3 Observations

These specific examples imply two findings. First, the qualitative response of the optimal investment in a portfolio of technologies to an increasing carbon tax depends on the relative costs of the individual programs and the elasticity of substitution between fossil and non-fossil energy inputs. Second, non-fossil and CCS programs act as substitutes, with the investments in CCS first increasing in a carbon tax until

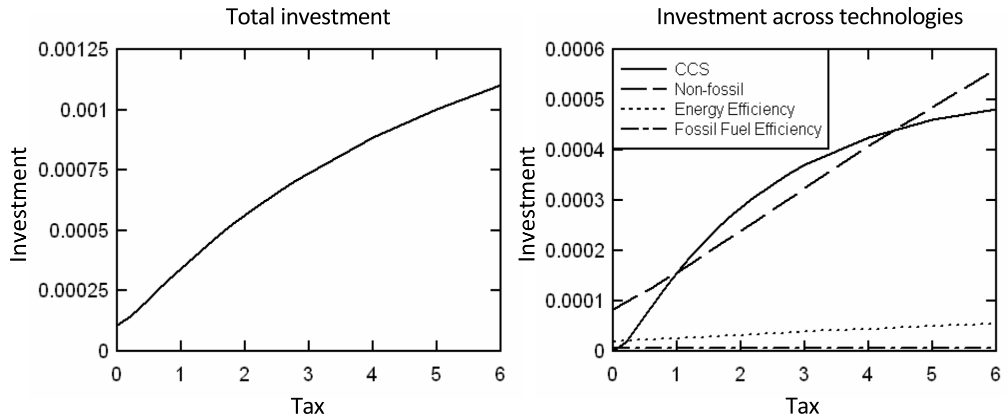

Figure 4.4. In these figures, the elasticity of substitution between fossil and non-fossil energy is 1.5 and the cost coefficients of the efficiency and fossil improvement technologies are 100 times the other programs.

the investment benefit is outweighed by the increasing tax. Beyond this threshold, it decreases in tax, and non-fossil increases at a more rapid rate, as the firm substitutes away from the use of fossil technology toward one that uses non-fossil input.

We capture investment behavior in these programs with the following propositions. In the first we show that, as long as the cost of R&D is equivalent for all programs, the investment in energy efficiency programs will always be higher than in fossil efficiency, which in turn will be higher than CCS.

Proposition 1 *Assume a portfolio of technologies with equal cost coefficients, κ, includes efficiency programs, fossil-fuel improvement, and CCS technologies (as defined in Section 4.3.3), and assume that the marginal cost of investment is finite at maximum abatement $g'(1) < \infty$; then the following relation holds between the optimal levels of R&D in energy efficiency, fossil fuel efficiency and CCS technologies; $\alpha_E^* \geq \alpha_F^* \geq \alpha_C^*$ for all t.*

Proof. Note that the following identities hold at the optimal investment levels assuming there is no corner point solution. From the first order conditions[7] in (4.5):

$$g'\left(\alpha_E^*\right) = \mathrm{E}_t\left[\left(P_c + t\right)\varepsilon_c^* + P_{nc}\varepsilon_{nc}^*\right] \quad (4.9)$$

$$g'(\alpha_F^*) = \mathrm{E}_t\left[\left(P_c + t\right)\varepsilon_c^*\right];\ g'(\alpha_C^*) = \mathrm{E}_t\left[t\varepsilon_c^*\right] \quad (4.10)$$

$$g'(\alpha_F^*) - g'(\alpha_C^*) = \mathrm{E}_t[P_c\varepsilon_c^*] \geq 0 \text{ and } g'(\alpha_E^*) - g'(\alpha_F^*) = \mathrm{E}_t\left[P_{nc}\varepsilon_{nc}^*\right] > 0 \quad (4.11)$$

which implies that $g'(\alpha_E^*) \geq g'(\alpha_F^*) \geq g'(\alpha_C^*)\ \forall t$. This implies $\alpha_E^* \geq \alpha_F^* \geq \alpha_C^*$.

If there is a corner point solution, i.e., $g'(\alpha) <$RHS when $\alpha = 1$, the same result still holds. ■

This result is consistent with our earlier observation that under equivalent cost structures for the technologies, it is optimal to target investments at efficiency programs. This is even more relevant if there is a limitation on investment budget. In the next proposition we show that optimal investments in non-fossil and CCS technologies are equal when the carbon tax is equal to the premium on non-fossil inputs.

Proposition 2 *Assume that the cost coefficient is the same across the technologies and the tax level, t is known. Then the optimal R&D investment in non-fossil and CCS technologies are equal when η, the premium on non-fossil energy, is equal to t.*

Proof. From the first order conditions in (4.5) and with known t:

$$g'\left(\alpha_C^*\right) = t\varepsilon_c^* \text{ and } g'(\alpha_A^*) = \eta\varepsilon_{nc}^* \quad (4.12)$$

If $t = \eta$, then $\varepsilon_c^* = \varepsilon_{nc}^*$; and the Right Hand Side of both equations in (4.12) are equal; implying, $\alpha_C^* = \alpha_A^*$. ■

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[7]See Baker and Shittu (Baker & Shittu 2006) for details of the first order conditions.

4.4.4 Design of experiments

The above analysis suggests that the qualitative impacts of an increase in the carbon tax on the overall portfolio depends precisely on the values of the parameters. Thus, in this section, we present an in-depth analysis of how the parameters influence the results. We vary five parameters, using a design of experiments approach: the R&D cost coefficients k_i of the four technologies, and the elasticity of substitution between fossil and non-fossil energy inputs. Each parameter has two levels, and thus, we have $2^5 = 32$ experiments. We create a design of experiments matrix for these factors. For the two levels of the cost coefficient factors, we use a high coefficient of 100 and a low coefficient of 1. For the two levels of elasticity, we use 1.5 for low and 6 for high.

Figure 4.5 show the results for the four programs in response to the experimental trials. For each experimental run, we find the percent change in optimal R&D investment given a change in the known tax from 0.5 to 1.5: $\frac{\alpha^*(1.5)-\alpha^*(0.5)}{\alpha^*(0.5)}$, where $\alpha^*(t)$ is the optimal level of R&D given tax, t. In the figure we show this quantity under a high elasticity on the vertical axis, and the quantity under a low elasticity on the horizontal axis. Each point represents a particular combination of cost coefficients. There are 16 possible combinations of high and low cost coefficients. Each of the panels shows these results for a different R&D program. These can be interpreted as follows. A point in the upper left hand corner of one of the graphs implies that the optimal investment increases significantly in a carbon tax when the elasticity is high, but has a small increase when the elasticity is low; and a point in the lower right hand corner implies that the optimal investment increases more in a carbon tax under low elasticity, but has a small increase when elasticity is high. Note that some of the scales are in the negative range, implying that in some cases the optimal investment decreases in the tax. Moreover, the range of the scales are quite different, providing information on which technologies are most strongly effected by changes

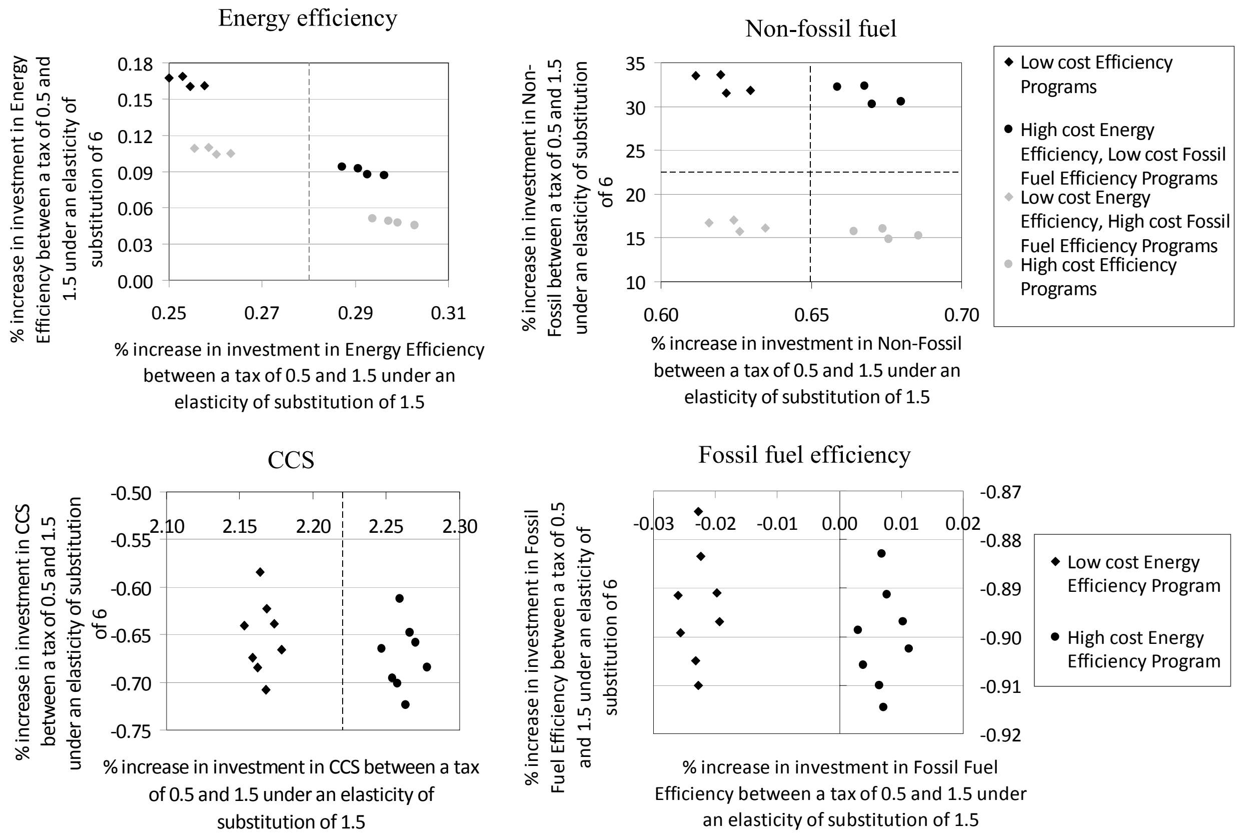

Figure 4.5. Percent increase in optimal investments. We use a high cost coefficient of 100 and a low of 1. For the two levels of elasticity, we use 1.5 for low and 6 for high.

in parameters. In particular, note that the highest percentage increases are seen for non-fossil R&D when the elasticity is high followed by CCS R&D when the elasticity is low. The biggest decreases are seen for fossil efficiency R&D when the elasticity is high followed by CCS R&D when the elasticity is low.

The axes in the energy efficiency graph in Figure 4.5 indicate that the percentage increase is always higher when the elasticity is low than when it is high. The intuition for this is that energy efficiency program is valuable when the tax is high and the flexibility is low since it reduces the cost of both inputs. This figure shows four distinct clusters. The two clusters on the left side represent cases where the increase in R&D investment is relatively higher under a high elasticity; the clusters on the right represent cases where the increase in R&D investment is relatively higher under a low elasticity. The points in the two clusters on the left all have a low R&D cost for energy efficiency technologies. An increase in the R&D cost of energy efficiency technologies leads energy efficiency R&D to be more responsive to an increase in the tax when elasticity is low; but less responsive when elasticity is high. Optimal investments in energy efficiency R&D increase most strongly with an increase in the tax when the cost of the R&D program is high, and firm flexibility is low.

As mentioned earlier, the non-fossil fuel graph on the top right of Figure 4.5 shows a significant increase in optimal investment in this technology when the firm is highly flexible; this is most pronounced, interestingly, when the R&D costs of fossil efficiency programs are low (the two clusters of black points). When fossil fuel efficiency is low cost, the investment in non-fossil is crowded out when the tax is low; when the tax increases fossil fuel efficiency investment drops off quickly, and is replaced by large investments in non-fossil. The R&D costs of energy efficiency influence the investment in non-fossil fuel program when substitution elasticity is low—leading to a larger increase when the cost of energy efficiency improvement is high—but not when elasticity is high.

The lower left panel shows that CCS has the greatest sensitivity to the elasticity, with significant *increases* in the tax when the elasticity is low, and large *decreases* in the tax when the elasticity is high. This can be explained as follows. When the firm is not flexible, it implies that they have a very hard time substituting away from fossil energy, even as the carbon tax gets very high. This makes CCS a very nice alternative as the tax gets high, allowing the firm to continue to use large amounts of fossil energy without paying the tax. On the other hand, when the firm is flexible, it will optimally substitute away from fossil toward non-fossil as the tax gets high. Thus, R&D into CCS gets less appealing with a very high tax. The two clusters that can be observed in the CCS graph of Figure 4.5 are differentiated by the R&D cost coefficient for energy efficiency technology. When energy efficiency R&D is more expensive (and flexibility is low), CCS is an even more attractive investment as the tax gets high. Comparing the CCS graph with the non-fossil graph, we see that an increasing tax is an incentive for investment in non-fossil fuel technology when input substitution is high, whereas it favors improvements in CCS technology when flexibility is low.

The fossil fuel efficiency graph in the bottom right of Figure 4.5 shows that a high elasticity generally reduces investment as the tax increases for this technology. On the other hand, optimal investment response is ambiguous when the elasticity is low: when the cost of the energy efficiency program is high, optimal investment in fossil fuel efficiency is increasing in tax; otherwise it decreases. Thus, fossil fuel R&D increases in a tax only when the firm is less flexible and the R&D cost of general energy efficiency is high.

The foregoing show that the key drivers of investment behavior are the elasticity of substitution and the R&D cost of the energy efficiency program, followed by the R&D cost of the fossil fuel efficiency program. The relative costs of non-fossil and CCS R&D programs have very little effect on the impact of an increasing carbon tax. When the cost of an energy efficiency program is low, then we are always on the left side of the graphs, meaning that when elasticity is low, all of the programs are more responsive to an increase in the carbon tax than if the cost of an energy efficiency program is high. In the case of the energy efficiency program, this can be explained by the relatively small investment into energy efficiency when the cost coefficient is high and the tax is low. Therefore, there is a large percentage increase when the tax goes up. For the other technologies, it is a substitution effect: when the R&D cost of energy efficiency is low, it makes up the bulk of R&D spending; when it is high, the other technologies can take up some of the slack.

All programs increase in a carbon tax when elasticity is low and the cost of energy efficiency is high, with CCS having by far the largest percentage increase. When elasticity is high, then CCS and fossil fuel efficiency decrease in a carbon tax, regardless of the relative cost coefficients. Energy efficiency improvement and non-fossil fuel technologies both increase more when fossil efficiency is expensive. However, when elasticity is low, investments in CCS and fossil fuel efficiency programs both increase more when energy efficiency program is costly.

4.4.5 R&D investments and increasing risk

In this section, we briefly discuss the effect of increasing risk on optimal investment. We define an increase in risk to be a mean-preserving spread (MPS). In Figure 4.6, we compare the optimal investment levels in each technology under a certain tax of 1 with the optimal investment levels given a probability of 0.25, 0.5, 0.25 over taxes of 0, 1, and 2, and given a $50-50$ chance of a carbon tax of 0 or 2. The first situation is least risky, the last, most risky. Rothschild and Stiglitz (1970) show that any increase in risk can be obtained by a sequence of such mean-preserving spreads.

In the figure, the slices show the percentage of the total investment that is in each technology; and the values in italics refer to the absolute amount invested in that technology. For these particular MPS, total optimal investment decreases with increasing risk under the set of parameters in Table 4.1 with elasticity equal to 6. The investment in the non-fossil fuel program, however, increases in risk, both in terms of the proportion of the total investment, and in terms of the absolute amount of the investment. All the other technologies decrease in absolute value in investment; with CCS decreasing the most. However, while the absolute value of the energy efficiency program decreases, its proportion of the portfolio becomes larger, mostly replacing CCS.

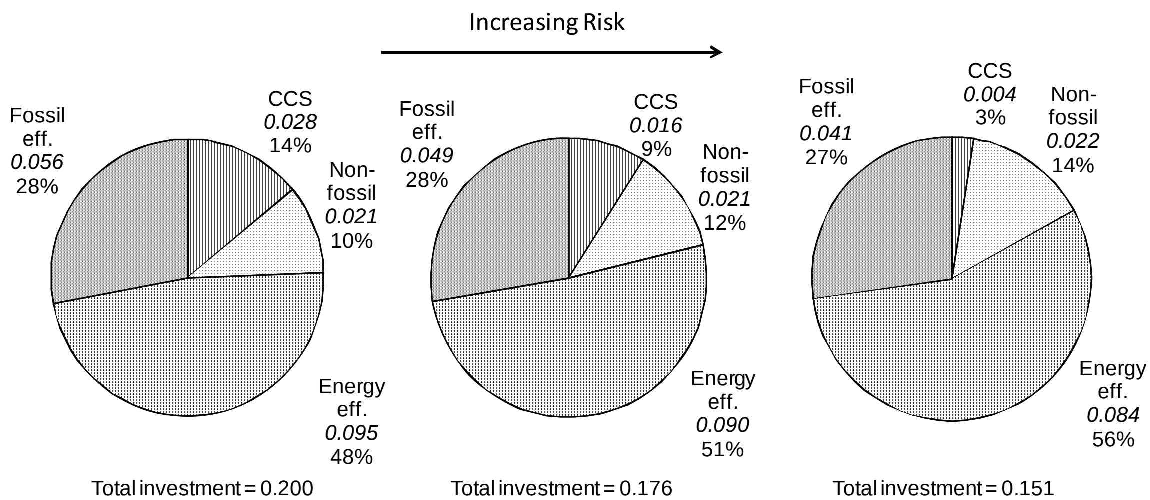

Figure 4.6. The left hand side shows the optimal investment distribution in the technologies at a tax of 1. The middle panel is an MPS of 0.25, 0.5, 0.25 over taxes of 0, 1, and 2 respectively. The right hand side is an MPS of 50-50 chance of tax=0 and tax=2. The elasticity of substitution between fossil and non-fossil energy is 6, while between energy and non-energy inputs is 0.75. The price demand elasticity is 1.1.

The effect of uncertainty depends on the elasticity of substitution. In Figure 4.7 we show the effect of risk on optimal portfolio investment for low elasticity, and observe that the optimal investment level in energy efficiency program is relatively flat riskiness, in contrast to higher proportional increases under a high elasticity in Figure 4.6. Similarly, in Figure 4.7, optimal investment in fossil fuel efficiency improvement increases slightly along with the non-fossil program under increasing risk. Only investment in CCS reduces in risk under low flexibility, but less than under high flexibility. It is also evident from these examples that risk has a much smaller overall effect on the optimal investment when the elasticity of substitution is low, than when it is high. This is because a firm that is less flexible is already confined to whatever technologies exist in its portfolio and thus there is a weak influence of tax uncertainty on the investment levels in the portfolio. However, this result assumes equal cost coefficients for the technologies.

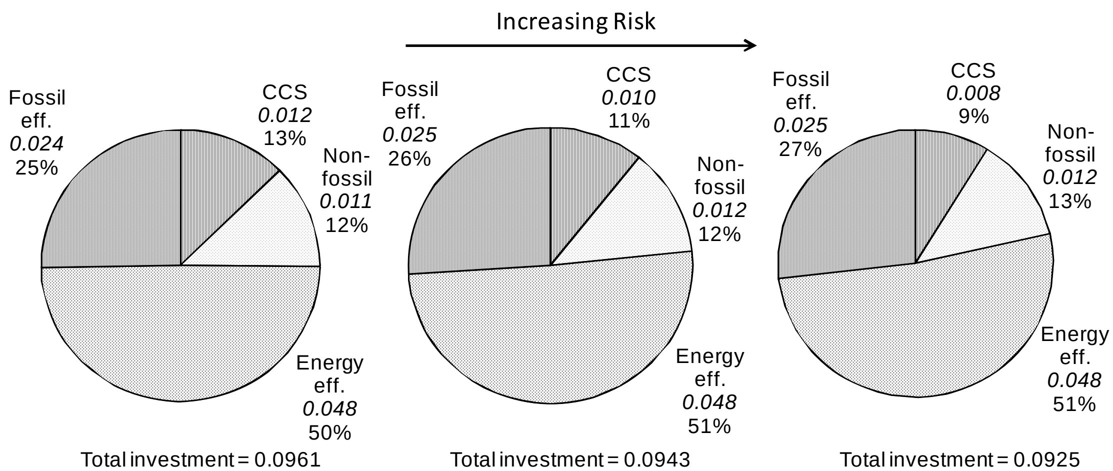

Figure 4.7. Effect of increasing risk on optimal portfolio investment. All parameters have same values as Figure 4.6 except substitution elasticity between fossil and non-fossil fuel is low here, 1.5.

In order to understand the impacts of risk, consider the shapes of the curves in Figures 4.1 - 4.4. Recall that the expected value of a concave function decreases in risk, while the expected value of a convex function increases; if a function is neither concave nor convex then the expected value will increase for some increases in risk and decrease for other decreases in risk (see Rothschild and Stiglitz (1971)). Note that in Figures 4.1, 4.2, and 4.4, overall investment is mostly concave in the tax, thus we would expect most MPS to lead to a decrease in investment. Similarly, all the programs with the exception of non-fossil fuel program are all mostly concave. Non-fossil fuel technology, however, is S-shaped, with a convex region between 0 and 1. This is what leads to the higher investment under uncertainty. Thus an MPS around a tax of 0.75 leads to an overall increase in investment, since the overall investment is convex in that region.

4.5 Conclusion

We consider a portfolio of R&D options in terms of reducing the effective cost of inputs. We distinguish between the R&D programs based on their influence on the demand for inputs, which in turn drives the effective price of inputs, and thus the optimal investment level in each of them. We find that R&D investment behavior is influenced by the relative cost of R&D programs into efficiency programs and the firm's flexibility in substituting between fossil and non-fossil energy inputs.

One of the key drivers of investments in these technologies are the costs of energy efficiency and fossil fuel efficiency programs. For example, when the cost of energy efficiency program is low and elasticity is low, the impact of an increasing tax is small. Moreover, we show that increasing tax is an incentive for investment in non-fossil fuel technology when firms are flexible, whereas it favors improvements in CCS technology when firms are less flexible. Overall investment appears to be highest when the carbon tax is high enough to provide incentives for using CCS, but not so high that firms start to substitute away from fossil fuel energy significantly.

The elasticity of substitution between energy inputs is crucial in determining the optimal investment profile. For a firm with a high substitution elasticity between fossil and non-fossil energy, the optimal investment in non-fossil and CCS programs exhibit ambiguous traits—initially, the investments in CCS exceeds that of non-fossil at low tax levels, but at higher levels, the converse is true. This ambiguous response in investment can be attributed to a number of factors. For example, investment in the CCS program increases in a carbon tax to offset the influence of the tax, but at high tax levels, these investments reduce since it is more economical to focus solely on non-fossil programs that are not influenced by the carbon tax.

On the other hand, when short term elasticity of substitution between fossil and non-fossil energy inputs is low, the carbon tax does not significantly influence the investment level in non-fossil energy programs. Overall portfolio investment reduces in the tax when the firm's ability to substitute away from fossil related inputs is limited. Uncertainty in the tax decreases the overall optimal investment in the portfolio for a firm with enough flexibility in its use of fossil and non-fossil energy—as observed with non-fossil technology substituting for CCS at considerably high tax levels. Surprisingly, an increase in risk has a much smaller effect on firms that are less flexible than highly flexible firms.

In summary, the contribution of this chapter is two-fold. First, it provides some insights to firms in terms of R&D investment in energy technologies. (1) Investments in efficiency that are independent of fuel type are the best. (2) In the case where efficiency is very expensive to attain, overall energy R&D investments should be fairly low when the expected tax is low, and only become very large as the expected tax gets large (in terms of making non-fossil fuel competitive with fossil energy. (3) Given the current uncertainty about a future tax, it looks like optimal R&D investments should be relatively small. However, it appears that non-fossil fuel program can be a hedge against uncertainty, and so more attention should be spent on this technology than would be under uncertainty. Second, this analysis provides insights to policymakers concerned about setting a carbon tax and crafting R&D policy. It appears that reducing uncertainty will increase investment. Also, it suggests that understanding the distribution of the elasticities of substitution within a firm that meets its energy demands from several sources may be very useful for predicting endogenous technical change in response to a carbon tax.

CHAPTER 5

OPTIMAL INVESTMENT DECISIONS UNDER RANDOM CARBON TAX AND TIMING

5.1 Introduction

In the previous chapter, we show that the key drivers of investments in energy technologies—at the firm level—are the relative costs of the technology options, the flexibility of the firm to use carbon and non-carbon energy inputs, and the size of the carbon tax. For example, when the cost of the energy efficiency program is low and elasticity is low, the impact of an increasing tax is small. However, we show that increasing tax is an incentive for investment in non-fossil fuel technology when firms are flexible, whereas it favors improvements in CCS technology when firms are less flexible. Overall investment appears to be highest when the carbon tax is high enough to provide incentives for using CCS, but not so high that firms start to substitute away from fossil fuel energy significantly. Thus, optimal investment in non-fossil can increase under some increases in uncertainty. In contrast, Farzin and Kort (2000)—in a model with end-of-pipe carbon capturing technology—show that an increase in uncertainty in the carbon tax at a known future time leads to optimally lower investment in abatement technology. In addition, they show that uncertainty about the timing of a known future tax increase increases a firm's investment in abatement capital. In concise form, uncertainty about the quantitative impact of a future tax influences a firm's level of R&D spending in a given technology.

Thus, there exists a significant difference in the prescription for optimal investment. This difference is underscored by the modeling approach. In the previous chapter, we assume that investment is made in the first stage preceding the advent of the tax change, while Farzin and Kort (2000) allow for investment before and after the tax change. This observation is crucial in setting up and understanding the results in this chapter.

This chapter explores this domain further and offers a different approach for three reasons. First, we consider the impact of carbon tax uncertainty under a two-stage continuous-time optimal control model on R&D spending that aims to lower the cost of non-carbon energy inputs. This differs from the previous chapter because we are only looking at one technology, and in an optimal control framework where we allow for R&D investment before and after the tax is realized. Second, we are adding to this literature the impact of technical change through input substitution, in addition to the existing output reduction. The incentive for this addition to the literature is from the results in Chapter 3 where we show that representing technical change in different ways can lead to different impacts and results. Third, we use a numerical approach to determine factors which influence investment behavior given uncertainty in the magnitude of a future tax. In particular, how is the result from the literature—that investment in an emissions abatement technology reduces with uncertainty in the carbon tax—different when consideration is given to R&D spending, strictly, to reduce the price of a non-carbon input under an uncertain carbon tax?

In this micro-economic framework, the analysis is based on two-stage optimal control techniques solved under cash flow discounting. In the first stage, the carbon tax is known and there is a probability distribution over the carbon tax for the second stage. We investigate what impact uncertainty about the second stage tax has on optimal abatement investment in the first stage. We find that, in this two stage model, R&D spending into non-fossil energy technology decreases as uncertainty in the future tax increases. This is in contrast to our findings in the previous chapter, in which R&D spending increased with some increases in uncertainty. Intuitively, this is due to the downward pressure on investment in this two-stage model allowing for near-term investment to be held back prior to the resolution of the future tax.

The rest of this chapter is organized as follows. In Sections 5.2 and 5.3, we present the basic model without and then with an adjustment cost on investment. In Section 5.4, we employ the numerical tools of solving optimal control problems to illustrate the impacts of uncertainty in tax magnitude on the firm's level of R&D spending. We conclude the analysis in Section 5.5 by exploring the effects on investment in non-fossil energy technology of uncertainty in the timing of a future carbon tax.

5.2 Basic model – no adjustment cost

In this model, we consider a monopolist that uses two energy inputs to produce its output. These energy inputs are carbon and non-carbon energy inputs. The firm utilizes these energy inputs in a production function with the form:

$$y = f(e_c, e_{nc}; \rho) \tag{5.1}$$

where:

e_c represents carbon energy inputs,

e_{nc} represents non-carbon energy inputs,

ρ is the substitution parameter between carbon and non-carbon energy inputs,

y denotes output.

The emissions level, E, of this production process is proportional to the carbon energy input,

$$E = \gamma e_c$$

where γ is emissions per unit input of carbon energy.

In this basic model, we assume the tax, $\tau \geq 0$, is known and is constant over time. Therefore, the firm's total tax payment on emissions at any given time is $\tau\gamma e_{c_t}$. We assume that the firm can reduce emissions at any given time, t, through input substitution and output reduction. The firm makes investments over time, $I_t \geq 0$, aimed at reducing the cost of non-carbon energy inputs by $(1-\alpha_t)$ where α_t is the level of improvement in the technology that uses non-carbon energy such that $0 < \alpha_t < 1$. The investment, I_t, impacts improvements in non-carbon technology, but we assume that the capital stock has a constant depreciation rate of δ over time. The differential equation for the capital stock of R&D, $\frac{dK}{dt}$ is

$$\frac{dK}{dt} = \dot{K} = I_t - \delta K_t; \qquad K(0) = K_0 \geq 0 \tag{5.2}$$

where:

δ is the nonnegative depreciation rate of capital stock,

K_t is the capital stock in time t

I_t represents the investment in time t

Integrating (5.2) shows that the stock of abatement capital follows:

$$K(t) = e^{-\delta t} K_0 + \int_0^t e^{-\delta(t-v)} I_v dv \tag{5.3}$$

The level of technical change in non-carbon technology is dependent on the capital stock such that

$$\lim_{K_t \to \infty} \alpha(K_t) = \theta \tag{5.4}$$

where θ is a nonnegative boundary less than 1 and $\alpha(0) = 0$ with $\alpha'(K_t) > 0$ and $\alpha''(K_t) < 0$.

The firm seeks to maximize the present value of its cash flow over an infinite planning horizon by determining its investments in the non-carbon technology according to:

$$\max_{I_t} \int_0^\infty \left[\pi_t(\tau, \alpha(K_t)) - I_t\right] e^{-rt} dt \tag{5.5}$$

$$s.t.\ \dot{K} = I_t - \delta K_t,\ K(0) = K_0 \tag{5.6}$$

where r is the firm's discount rate and assumed to be constant, and $\pi_t(\tau, \alpha_t)$ is the firm's profit function given as

$$\pi_t(\tau, \alpha_t)) = P(y^*)y^* - P_c e^*_{c_t} - (1 - \alpha_t)P_{nc}e^*_{nc_t} - \tau_t \gamma e^*_{c_t} \tag{5.7}$$

where P_c and P_{nc} are the prices of carbon and non-carbon energy inputs respectively. For a monopolist with a nested constant elasticity of production defined as

$$(e_c^\rho + e_{nc}^\rho)^{\frac{1}{\rho}} = y \tag{5.8}$$

with output price defined as $P(y) = Ay^{-\frac{1}{b}}$, the optimal demand for carbon and non-carbon energy inputs are as follows

$$e^*_{c_t} = (\tau\gamma + P_c)^{\frac{1}{\rho-1}} \left[(\tau\gamma + P_c)^{\frac{\rho}{\rho-1}} + ((1-\alpha_t)P_{nc})^{\frac{\rho}{\rho-1}}\right]^{\frac{-1-b(\rho-1)}{\rho}} \left[\frac{b}{A(b-1)}\right]^{-b} \tag{5.9}$$

$$e^*_{nc_t} = ((1-\alpha_t)P_{nc})^{\frac{1}{\rho-1}} \left[(\tau\gamma + P_c)^{\frac{\rho}{\rho-1}} + ((1-\alpha_t)P_{nc})^{\frac{\rho}{\rho-1}}\right]^{\frac{-1-b(\rho-1)}{\rho}} \times \ldots \left[\frac{b}{A(b-1)}\right]^{-b} \tag{5.10}$$

where A is a constant, b is the price elasticity of demand and ρ is the parameter of substitution between carbon and non-carbon energy inputs.

5.2.1 Model Analysis

We define the Hamiltonian as

$$H_t = (\pi_t(\tau, \alpha_t) - I_t)e^{-rt} + \lambda_t(I_t - \delta K_t)$$

where λ_t is the present value multiplier associated with the constraint relating the capital stocks between any two time intervals, for example, K_t and K_{t+1} in the discrete model. λ_t gives the marginal value of relaxing the constraint in (5.6), in other words, it is the marginal impact of an exogenous increase in capital at time t on the lifetime value of the firm's cash flows discounted to time zero. We define $\mu_t e^{-rt} = \lambda_t$ and we re-write the current value Hamiltonian, H_t^c. μ_t thus shows the value to the firm of an additional unit of capital at time t dollars

$$H_t^c = \pi_t(\tau, \alpha(K_t)) - I_t + \mu_t(I_t - \delta K_t) \tag{5.11}$$

The profit function is convex in the price of the outputs; but following the Mangasarian (1966) and Arrow and Kurtz (1970), the Hamiltonian must be concave in K for the necessary and sufficient conditions to hold. So, we assume that the Hamiltonian is concave in K, otherwise, the firm would choose an infinite investment, and this does not seem a likely outcome. Thus, H^c is concave in K and the state equation is linear in K; therefore, the following necessary conditions are also sufficient to solve (5.5); first the derivative of the current value Hamiltonian with respect to the control variable satisfies, $H_I^c = 0$, implying that

$$\mu^*(t) = 1 \tag{5.12}$$

μ_t is the shadow price of the investment in technical change in non-carbon technology. We interpret this as the value to the firm of acquiring additional unit of capital, which is fixed at 1 in this case. This shows that if there are no adjustment costs to capital stock acquisition, then the value of additional units of capital is fixed and does not depend on future marginal revenue or user costs. In other words, there are no opportunity costs to investing now. In the next section, we show the significance of investment adjustment costs which emphasize the real value of the capital.

The other necessary conditions that define the maximum principle are:

$$\frac{d(\mu_t e^{-rt})}{dt} = -H_K^c e^{-rt} \quad (5.13)$$

$$\dot{\mu}_t e^{-rt} - r\mu_t e^{-rt} = -\left(\frac{\partial \pi_t(\tau, \alpha(K_t))}{\partial K_t} - \mu_t \delta\right) e^{-rt} \quad (5.14)$$

$$\dot{\mu}_t = -\frac{\partial \pi_t(\tau, \alpha(K_t))}{\partial \alpha(K_t)} \cdot \frac{\partial \alpha(K_t)}{\partial K_t} + (r+\delta)\mu_t \quad (5.15)$$

$$H_\mu^c \Rightarrow \dot{K} = I_t - \delta K_t \quad (5.16)$$

and the transversality condition is

$$\lim_{t\to\infty} \mu_t e^{-rt} = 0 \quad (5.17)$$

where (5.13) and (5.17) govern the path of the costate variable which defines the rate of decrease of the shadow price over time toward the steady state. In (5.14), $\frac{\partial \pi(\tau,\alpha(K_t))}{\partial K_t}$ is the marginal contribution of the investment capital stock to the current profit, and $(r+\delta)\mu_t$ is the rate of depreciation of the cost of acquiring extra units of capital. The condition defined by equation (5.12) shows that at any time t the value of additional unit of capital investment in technical change in non-carbon technology is constant just as the change in the firm's marginal profit is constant to changes in investment capital which, as shown in (5.18), is equal to the sum of the firm's discount rate and capital stock depreciation; since $\mu_t = 1$, then $\dot{\mu}_t = 0$. Thus, from equation (5.16) we have

$$\frac{\partial \pi(\tau, \alpha(K_t))}{\partial \alpha(K_t)} \cdot \frac{\partial \alpha(K_t)}{\partial K_t} = (r+\delta) \quad (5.18)$$

If we let $I_0 = K^*$, i.e. the initial investment be the optimal capital outlay that satisfies (5.18), then the isocline at $\dot{K} = 0$, using (5.16) leads to,

$$I_t^* = \delta K_t^* \quad (5.19)$$

Thus, the optimal investment path to the steady-state consists of either an initial lump sum investment to get to the optimal capital K^* if $K_0 < K^*$ or not making any investments if $K_0 > K^*$; then after that simply investing at the depreciation rate to maintain K^*. The economic intuition of this solution is that in order for the firm to maintain the optimal profit level toward the steady state capital stock, instantaneous investments must be made just at the depreciation rate of the initial capital to maintain stability and this depends on the relationship between K_0 and K^*. This is essentially a *bang-bang* solution, and it is the result of not implementing adjustment costs or having a linear adjustment cost.

5.3 A model with adjustment costs

Some studies have highlighted the concept of adjustment costs in energy R&D investment theory (see Bohringer and Rutherford (2006)). The key assumption is that capital inputs are adjustable, but this adjustment comes at an expense, the adjustment cost. In the previous section, we assumed that a firm's level of investment does not influence the cost of investment. The problem with this assumption is that the model does not identify any mechanism through which expectations influence investment demand, whereas in practice, expectations about demand and costs are central to investment decisions. For example, the capital stocks of a producing firm are expected to expand under increasing sales with low costs of capital, and they are expected to contract with falling sales with high capital cost.

In the *adjustment cost theory*, developed by Eisner and Strotz (1963), Gould(1968), Lucas (1967), and Uzawa (1969), investment function is viewed as the demand function for capital accumulation of the users of capital. In this theory, increasing marginal costs of investment influences the rate at which the firm wishes to accumulate capital – investment costs rise with investment levels – and this justifies the need to modify the model in order to obtain a reasonable picture of actual investment decisions.

However, Mussa (1977) shows that these costs come in two forms, internal and external. Internal adjustment costs arise when firms face direct costs of changing their capital stocks as in Eisner and Strotz (1963), Gould(1968), and Lucas (1967). The emergence of these costs is due to installation of new machinery, training workers to operate new alternative production methods, and reorganization of production line which could lead to temporary decreases in productivity. External adjustment costs arise when a firm faces a perfectly elastic supply of capital, but where the price of capital goods relative to others adjusts such that investment or disinvestment are not plausible options the firm wants to consider.

Here, adjustment costs are assumed to be internal because these costs influence the demand for and use of alternative energy inputs. The internal cost of adjustment will be treated as a function of gross investment. Thus we set the adjustment cost as $C(I_t)$, where I_t represents the gross investment. In addition to introducing nonlinearities in the differential equations, it costs more for a small firm to invest I_t than for a large firm to act equally. Thus, the adjustment cost follows $C(I_t) > 0$, $C'(I_t) > 0$ and $C''(I_t) > 0$ for all $I_t > 0$ and $C(0) = 0$.

A critical issue is determining the functional form of the adjustment cost. Here we follow the Uzawa (1969) specification as applied in Bohringer and Rutherford (2006);

$$C(I_t) = I_t \left(1 + \frac{\phi I_t}{2K_t}\right) \tag{5.20}$$

where ϕ is the adjustment cost parameter to investment in the abatement program. In this representation, the cost has an edge over the usual generalized quadratic adjustment cost function since it depends on the relative rate of expansion rather than the absolute rate of expansion as defined by the quadratic function.

Given an adjustment cost, the basic model is reworked by solving (5.5) subject to (5.6), but replacing the investment I_t with $C(I_t)$ as follows;

$$\max_{I_t} \int_0^{\infty} \left[\pi_t(\tau, \alpha(K_t)) - C(I_t)\right] e^{-rt} dt \tag{5.21}$$

$$s.t.\ \dot{K} = I_t - \delta K_t,\ K(0) = K_0 \tag{5.22}$$

Here, the necessary and sufficient condition, $H_I^c = 0$, implies

$$\mu(t) = C'(I(t)) \tag{5.23}$$

At any time t, along the optimal investment path, (5.23) shows that the optimal shadow value of capital will be changing over time, implying that the optimal investment path is such that the marginal value of investment is just equal to the marginal cost of investment. The firm behavior is such that a higher marginal cost of investment leads to a lower optimal investment.[1] In other words, if the marginal cost is higher, then the firm will stop investing when the marginal value is higher; the marginal value is usually higher at a lower investment. Thus $\mu(t)$ is increasing in I_t[2]. The other conditions are

$$\dot{\mu} = -\frac{\partial\pi(\tau, \alpha(K_t))}{\partial\alpha(K_t)} \cdot \frac{\partial\alpha(K_t)}{\partial K_t} + (r+\delta)\mu_t \tag{5.24}$$

$$H_\mu^c \Rightarrow \dot{K} = I_t - \delta K_t \tag{5.25}$$

Therefore, the equation of motion for net investment can be derived by taking the time derivative of (5.23), substituting into (5.24), and reduce to

$$\dot{I}_t = \frac{1}{C''(I_t)}\left((r+\delta)C'(I_t) - \frac{\partial\pi(\tau, \alpha(K_t))}{\partial\alpha(K_t)} \cdot \frac{\partial\alpha(K_t)}{\partial K_t}\right) \tag{5.26}$$

---

[1] This is analogous to Romer (1996) that a direct economic interpretation of this is that a firm will increase its stock of capital if the market value of capital is at least what it costs to aquire it.

[2] Since $C(I_t)$ is increasing in I_t

Thus, at the steady state with $\dot{K} = 0$, the unique saddle point, K^* resulting from $I_t^* = \delta K^*$ implies that $\dot{I}_t = 0$. From (5.26), we have

$$\frac{\partial \pi(\tau, \alpha(K^*))}{\partial \alpha(K^*)} \cdot \frac{\partial \alpha(K^*)}{\partial K^*} = (r + \delta)C'(\delta K^*) \tag{5.27}$$

This illustrates that at the steady state, the marginal profit on abatement capital is constant and it strongly depends on the discount rates and the marginal cost of adjustment. It is useful to note that we can express the value of capital, μ_t, in terms of the future marginal revenue. Using (5.24), and an integrating factor $e^{-(r+\delta)t}$, we have,

$$\begin{aligned} e^{-(r+\delta)t} \left(\dot{\mu} - (r+\delta)\mu_t\right) &= \frac{d}{dt}\left(\mu_t e^{-(r+\delta)t}\right) \\ &= -\left(\frac{\partial \pi_t(\tau, \alpha(K_t))}{\partial \alpha(K_t)} \cdot \frac{\partial \alpha(K_t)}{\partial K}\right) e^{-(r+\delta)t} \end{aligned} \tag{5.28}$$

i.e.,

$$\frac{d}{dt}\left(\mu_t e^{-(r+\delta)t}\right) = -\left(\frac{\partial \pi_t(\tau, \alpha(K_t))}{\partial \alpha(K_t)} \cdot \frac{\partial \alpha(K_t)}{\partial K}\right) e^{-(r+\delta)t} \tag{5.29}$$

Taking the integral of both sides from $t = s$ to $t = \infty$, with the transversality condition $\lim_{t \to \infty} \mu_t e^{-rt} = 0$, we have

$$\mu(t) = \int_{s=t}^{\infty} \left(\frac{\partial \pi_s(\tau, \alpha(K_s))}{\partial \alpha(K_s)} \cdot \frac{\partial \alpha(K_s)}{\partial K_s}\right) e^{-(r+\delta)(s-t)} ds \tag{5.30}$$

This implies that, on the optimal path, the value of a unit of capital is equal to the discounted value of its future marginal revenue. This result stimulates the discussion surrounding the significance of the cost of adjustment. In essence, the firm will increase its capital stock if $\mu(t)$ is high, and reduce it if it is low. The value of capital, $\mu(t)$, is analogous to *Tobin's q* (Tobin 1969)–the ratio of the market value to the replenishment cost of capital.

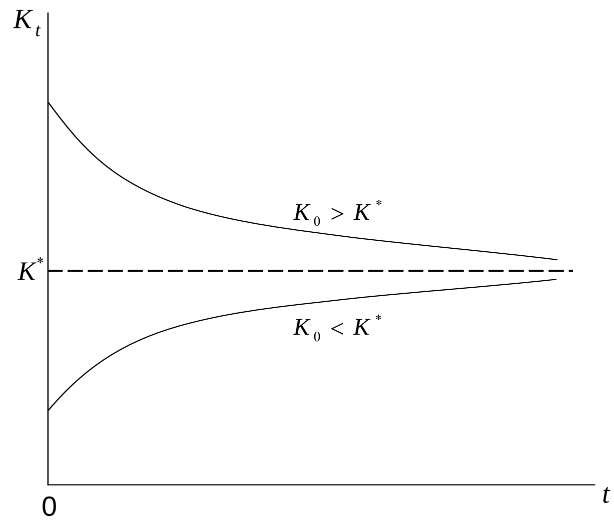

Figure 5.1. The time-path of capital stock

The steady state behavior of this model is similar to our discussion in Section 5.2, but here, the $\dot{K}$ isocline slopes upwards from left to right since $I_t = \delta K_t$ and there are no bounds on the investment level. Figure 5.1 illustrates the path of the capital stock through time depending on the relationship between the initial stock and the optimal stock at the steady state. If $K_0 < K^*$, I_t is positive and it approaches the steady state such that $I^* = \delta K^*$. This is attributed to the cost of adjustment—as the capital stock increases, the cost of investing gets smaller. However, if $K_0 > K^*$, $\mu(t)$ is zero, implying that no investment is made. However, at the steady state, investment becomes positive and equal to the value of capital depreciation. We sum up these results in the following;

Proposition 3 *For a given tax rate, τ, the optimal level of investment in alternative, non-carbon energy input technology only depends on the capital stock. If $K_0 > K^*$, then the firm will not make any investments to improve on the technology. If $K_0 < K^*$, the firm will make incremental investment in the technology.*

5.4 Uncertainty in the tax at known future time

We now extend this analysis by introducing uncertainty in the magnitude of the tax. While assuming that there is perfect information about the timing of the tax policy, i.e. the initial tax, τ is constant until a known time, T, but thereafter takes a jump, $d\tau$, following

$$d\tau = \begin{cases} 0 & \text{at } t \neq T \\ \tilde{\tau} & \text{at } t = T \end{cases} \tag{5.31}$$

where $\tilde{\tau}$ represents the random magnitude of the tax change and follows a probability distribution. Now, given the randomness in the tax, the firm's objective function originally defined in (5.5) becomes that of maximizing the expected discounted value of the stream of profits from investment net the investment cost defined as

$$\max_{I_t} \left(\int_0^T (\pi_t(\tau, \alpha(K_t)) - C(I_t))e^{-rt}dt + E_{\tilde{\tau}} \int_T^{\infty} (\pi_t(\tilde{\tau}, \alpha(K_t)) - C(I_t))e^{-rt}dt \right) \tag{5.32}$$

$$s.t.\ \dot{K} = I_t - \delta K_t,\ K(0) = K_0$$

Applying the recursion of dynamic programming to this problem makes the solution approach to be time-phased, i.e., the problem is two-stage since we consider how the post time T value of investment impacts on the investment decisions prior to time T. We start by solving the sub-problem defined over the interval $t \geq T$ where

$$V(K(T)) = E_{\tilde{\tau}} F(K_T, \tilde{\tau}) \tag{5.33}$$

is the firm's expected payoff after period T given the randomness in the tax, and assuming optimal investment thereafter. Now given that the realized tax is $\tilde{\tau} = \tau_T$, for this interval, we seek to solve for the optimal decision on abatement investment in alternative cost improvement through

$$F(K_T, \tau_T) = \max_{I_t} \int_T^{\infty} (\pi_t(\tau_T, \alpha(K_t)) - C(I_t))e^{-r(t-T)}dt \tag{5.34}$$

$$s.t.\dot{K} = I_t - \delta K_t, K(T) = K_T \geq 0 \tag{5.35}$$

The maximum conditions for (5.34) are the same as we derived earlier in (5.23)–(5.25), and the same dynamic defined by (5.26). Similarly, at the steady state, i.e. $\dot{I} = \dot{K} = 0$, there exists a unique path towards that optimal steady state (I^*, K^*). However, the path to this point depends on the relationship between the initial capital stock, K_T and the steady state capital stock, K^*. If $K_T < K^*$, the capital stock will increase towards the steady state through increasing investment. If $K_T > K^*$, investment will be zero, but depreciation in the capital stock over time will bring the capital stock down to the the steady state value. This is true for $T < t < \infty$. Intuitively, we expect that if the tax is much lower than expected then it may be true that $K_T > K^*$. Whereas if the tax is much higher than expected we may expect the opposite, $K_T < K^*$.

We now focus attention on first period behavior in which the firm's problem is to solve the following problem given this investment path knowledge about post time T,

$$\max_{I_t} \int_0^T (\pi_t(\tau, \alpha(K_t)) - C(I_t))e^{-rt}dt + e^{-rT}V(K(T)) \tag{5.36}$$

$$\begin{aligned} s.t.\ \dot{K} &= I_t - \delta K_t, \\ K(0) &= K_0 \end{aligned} \tag{5.37}$$

For the problem structured as above, we follow Arrow and Kurz (1970) in the following propositions[3].

Proposition 4 *(Pontryagin Maximum Principle). Let $I^*(t)$ be a choice of investment levels ($0 \leqq t \leqq T$) which maximizes $\int_0^T (\pi_t(\tau, \alpha(K_t)) - C(I_t))e^{-rt}dt + e^{-rT}V(K(T))$, subject to the conditions,*

1. $\dot{K} = I_t - \delta K_t$, *some constraints on the choices investments, and initial conditions on the state variables. Then there exist auxiliary variables, functions of time, $\mu(t)$, such that, for each t,*

2. $I^*(t)$ *maximizes* $H((\pi(t) - c(I(t)), I(t), \mu(t), t)$ *where*

$$H((\pi - c(I)), I, \mu, t) = \pi_t(\tau, \alpha(K_t)) - C(I_t)) + \mu_t(I_t - \delta K_t)$$

   and the functions μ_t satisfy the differential equation

3. $\mu_t = -\frac{\partial H}{\partial K}$, *evaluated at* $K = K(t)$, $I = I^*(t)$.

The optimal path is the solution of the differential equations stated in conditions 1-3 above.

Proposition 5 *(Transversality conditions). The solution to Proposition 4 also satisfies the condition,* $\mu(T) = \partial V(K(T))/\partial K(t)$

[3] This proposition only differs from the preceeding analysis because it resembles a problem with random scrap value.

Using these propositions, we have,

$$H_t^c(\tau, \alpha; I_t) = \pi_t(\tau, \alpha(K_t)) - C(I_t) + \mu_t(I_t - \delta K_t)$$

$$\mu(t) = C'(I(t)) \tag{5.38}$$

$$\dot{\mu} = -\frac{\partial \pi_t(\tau, \alpha(K_t))}{\partial \alpha(K_t)} \cdot \frac{\partial \alpha(K_t)}{\partial K_t} + (r + \delta)\mu_t \tag{5.39}$$

$$\mu(T) = e^{-rT} \frac{\partial V(K_T)}{\partial K_T} \tag{5.40}$$

(5.38) implies that the optimal investment is such that the shadow price $\mu(t)$ is equal to the current value of the marginal cost of investment at time t.

We can determine the effect of uncertainty by looking at equation (5.40). An increase in uncertainty will change the value of V; this in turn will impact the optimal value of the shadow price μ at time T; which will impact the path up to that point. If the marginal change in value increases in uncertainty, then the RHS of (5.40) increases in uncertainty, implying that the path up to this point must lead to a higher shadow value at time T. Since this is not an equilibrium condition, an increase in marginal value increases the shadow value which will lead to increased investment.

In the following subsections, we investigate the impact of increasing uncertainty. First, we determine the trajectory of the optimal investment path as the derivative of V with respect to K_T increases; that is, under increasing $\frac{\partial V(K_T)}{\partial K_T}$, by implementing a numerical example using well-defined functional forms in Section 5.4.2. Second, we determine whether $\frac{\partial V(K_T)}{\partial K_T}$ increases or decreases in increases in risk. We do this by investigating whether it is concave or convex in the random variable, $\tilde{\tau}$.

5.4.1 Effects of uncertainty

One of the central questions of this chapter is: how does uncertainty in the tax influence investment? In a three-step approach, we begin with the first step of determining the derivative of (5.33) with respect to K_T. Any change in the distribution of the tax, $\tilde{\tau}$ that causes the right-hand side of (5.41) to increase will increase the marginal firm value;

$$\frac{\partial V(K_T)}{\partial K_T} = E_{\tilde{\tau}}\left[\frac{\partial F(K_T, \tau_T)}{\partial K_T}\right] \tag{5.41}$$

Does this the right-hand side of (5.41) increase or decrease in risk? To know this we need to determine if the amount inside the expectation is convex or concave (or neither) in $\tilde{\tau}$. According to Rothschild and Stiglitz (1971)—if it is convex then it increases in risk; if it is concave, then it decreases in risk; and if it is neither then it increases with some increases in risk and decreases with other increases in risk or vice-versa. Thus, we would like to sign the second derivative of $\frac{\partial F(K_T,\tau_T)}{\partial K_T}$ with respect to τ. We start by looking at the first derivative using the expression in the brackets in equation (5.41) after substituting (5.34), we have,

$$\begin{aligned}
\frac{\partial F(K_T, \tau_T)}{\partial K_T} &= \int_T^\infty e^{-r(t-T)} \frac{\partial}{\partial K_T}\left[\pi_t(\tau_T, \alpha(K_t^*(K_T, \tau_T)))\right] dt \\
&= \int_T^\infty e^{-r(t-T)} \frac{\partial \pi_t(\tilde{\tau}, \alpha(K_t^*(K_T, \tau_T)))}{\partial K_t^*(K_T, \tau_T)} \frac{\partial K_t^*(K_T, \tau_T)}{\partial K_T} \\
&= \int_T^\infty e^{-r(t-T)} \left[\alpha'(K_t^*(K_T, \tau_T)) P_{nc} e^*_{nc_t}\right] \frac{\partial K_t^*(K_T, \tau_T)}{\partial K_T} dt
\end{aligned} \tag{5.42}$$

In the second step, we seek to know how the marginal firm value, or specifically, the derivative of the value function with capital at T changes with tax. We find the derivative of (5.42) with respect to τ_T,

$$\begin{aligned}
\frac{\partial^2 F(K_T)}{\partial K_T \partial \tau_T} &= \int_T^\infty e^{-r(t-T)} \frac{\partial}{\partial \tau_T}\left(\alpha'(K_t^*(K_T, \tau_T)) P_{nc} e^*_{nc_t} \frac{\partial K_t^*(K_T, \tau_T)}{\partial K_T}\right) dt \\
&= \int_T^\infty e^{-r(t-T)} P_{nc} e^*_{nc_t} \times \\
&\quad \left(\begin{array}{c} \frac{\partial \alpha'(K_t^*(K_T,\tau_T))}{\partial K_t^*(K_T,\tau_T)} \frac{\partial K_t^*(K_T,\tau_T)}{\partial \tau_T} \frac{\partial K_t^*(K_T,\tau_T)}{\partial K_T} + \\ \alpha'(K_t^*(K_T, \tau_T)) \frac{\partial K_t^*(K_T,\tau_T)}{\partial K_T \partial \tau_T} \end{array}\right) dt
\end{aligned} \tag{5.43}$$

By the definition from Section 5.2, on the right hand side of (5.43), the upper expression inside the brackets (before the +) is positive. This is because the first fraction of the upper expression is negative, the second fraction is also negative and the third one is positive. The lower expression (after the +) has the first partial positive, but there is insufficient knowledge about the sign of the fraction.

Therefore, the third step—finding the derivative of (5.43) with respect to the tax that explicitly describes the effect of uncertain tax change on the marginal value of the firm value through capital stock changes—is difficult to characterize even when completely derived. In order to get more insights, we apply numerical examples in Subsection 5.4.2.2. We use Rothschild and Stiglitz (1970) mean-preserving spread definition[4] to analyze how an increase in the riskiness of the carbon tax impacts near-term investment.

5.4.2 Numerical Approach

This section is motivated by the complexity of the preceding analytical approach. The traditional techniques of analyzing the problem with the aid of phase diagrams rely on differential equations, which in this model have forms with limited information about the paths that constitute a phase diagram. However, numerical methods are not restricted by this limitation provided functional forms that represent the basic characteristics of the model parameters are defined.

The cases presented in Section 5.4 show that the initial boundary condition lies on the capital stock, while the terminal boundary condition is for the flow variable—investment. We adopt the following functional forms with their corresponding derivatives to form the basis of the examples in this numerical approach.

The impact of capital stock on technical change is represented as

[4]The expected value of a convex function increases in risk; the expected value of a concave function decreases in risk.

$$\begin{aligned}
\alpha(K(t)) &= \theta(1-e^{-c_1K(t)}) \\
&\Rightarrow \alpha'(K(t)) = c_1\theta e^{-c_1K(t)} && (5.44) \\
\pi(.) &= ph(.) - p_c e_c - (1-\alpha(K(t))P_{nc}e^*_{nc} - \tau\gamma e^*_c \\
&\Rightarrow \frac{\partial \pi}{\partial \alpha} = p_{nc}e^*_{nc} && (5.45)
\end{aligned}$$

The cost of investment, following the definition in Section 5.3, is defined as

$$C(I(t)) = I(t)\left(c_2 + \frac{\phi I(t)}{c_3K(t)}\right)$$

which at optimality implies

$$\mu(t) = c_2 + \frac{2\phi I(t)}{c_3K(t)} \tag{5.46}$$

Thus, substituting (5.44) and (5.45) into (5.39), we have

$$\dot{\mu} = (r+\delta)\mu_t - p_{nc}e^*_{nc} \cdot c_1\theta e^{-c_1K(t)} \tag{5.47}$$

Likewise, solving for $I(t)$ in (5.46) and substituting into (5.37) yields;

$$\dot{K} = \frac{c_3K(t)}{2\phi}\left(\mu(t) - c_2\right) - \delta K(t) \tag{5.48}$$

In order to solve the problem defined by (5.47) and (5.48), we apply the fourth-order Runge-Kutta algorithm in a Visual Basic Application as illustrated in Naevdal (2003). The procedures follow the *shooting* algorithm[5]. In this method, with known boundary values, guesses are made for unknown initial starting values, and the differential equations are solved from a given initial time, t_0 to a given final time t_f using Runge-Kutta solver iteratively.

[5]Refer to Goffe(1993) for a guide to the numerical solution of two-point boundary value problems.

For this analysis, the iterative procedure starts at $t = 0$, $K(0) = K_0$ and is bounded by (5.40). We obtain estimates for $K(t + \Delta)$, and this estimate is used to obtain an estimate for $K(t + 2\Delta)$ and so on, where Δ is a small step increment in time. Similarly, we estimate $\mu(t + \Delta)$. The formulae for estimating $K(t + \Delta)$ and $\mu(t + \Delta)$ are given by

$$K(t + \Delta) = K_t + \frac{\Delta}{6}(x_1 + 2x_2 + 2x_3 + x_4) \quad (5.49)$$

$$\mu(t + \Delta) = \mu_t + \frac{\Delta}{6}(y_1 + 2y_2 + 2y_3 + y_4) \quad (5.50)$$

where x_i is given by;

$$\begin{aligned} x_1 &= f(t, K_t, \mu_t) \\ x_2 &= f(t + \Delta/2, K_t + x_1\Delta/2, \mu_t + y_1\Delta/2) \\ x_3 &= f(t + \Delta/2, K_t + x_2\Delta/2, \mu_t + y_1\Delta/2) \\ x_4 &= f(t + \Delta, K_t + x_2\Delta, \mu_t + y_1\Delta) \end{aligned}$$

and y_i is given by (5.47)

$$\begin{aligned} y_1 &= g(t, K_t, \mu_t) \\ y_2 &= g(t + \Delta/2, K_t + x_1\Delta/2, \mu_t + y_1\Delta/2) \\ y_3 &= g(t + \Delta/2, K_t + x_2\Delta/2, \mu_t + y_1\Delta/2) \\ y_4 &= g(t + \Delta, K_t + x_2\Delta, \mu_t + y_1\Delta) \end{aligned}$$

where the functions, f and g follow from (5.48) and (5.47) respectively, i.e.,

$$\begin{aligned} f(\cdot) &= \frac{c_3 K(t)}{2\phi}\left(\mu(t) - c_2\right) - \delta K(t) \\ g(\cdot) &= (r + \delta)\mu_t - p_{nc}e^*_{nc} \cdot c_1\theta e^{-c_1 K(t)} \end{aligned}$$

5.4.2.1 Example 1: Effect of second stage expected firm value on optimal capital stock

In this Subsection, we investigate the effect on investment before time T of increases in the right hand side of (5.40) at time T. To do this, we calibrate the endpoint boundary condition and give a means to observe it as an increasing function. We put a multiplier, β, on K_T through an assumed value function defined by

$$V(K_T) = (\beta K_T)^n$$

such that the firm marginal value function post time T increases in K_T as β is increased,

$$\frac{\partial V(K_T)}{\partial K_T} = n\beta^n K_T^{(n-1)}$$

The optimal paths generated by (5.49) and (5.50) in Figure 5.2 show that the trajectory of capital accumulation in the first period is everywhere increased with increasing marginal firm value function post time T.

Thus, tying this back with Section 5.4.1, if uncertainty increases the expected marginal value, it will increase first stage investment; if uncertainty decreases the expected marginal value, it will decrease first stage investment.

5.4.2.2 Example 2: Effect of tax uncertainty on firm value

In this example, we apply the original problem defined by (5.33)–(5.35) to the numerical framework. Using the functions defined in Section 5.4.2, we solve the control problem by assigning values to the tax—the tax ranges from 0 to 8 with increment of 2. For each of these tax values, the capital stock, K_T is varied incrementally from 1 to 5 to determine the optimal firm value (the optimal F given K_T and given a tax τ) for each capital starting at the initial time. The left panel of Figure 5.3 shows the effect of an increasing tax on the profile of the optimal marginal value of the firm given the starting capital, that is, how a given tax impacts $\frac{\partial F}{\partial K}$. This figure projects

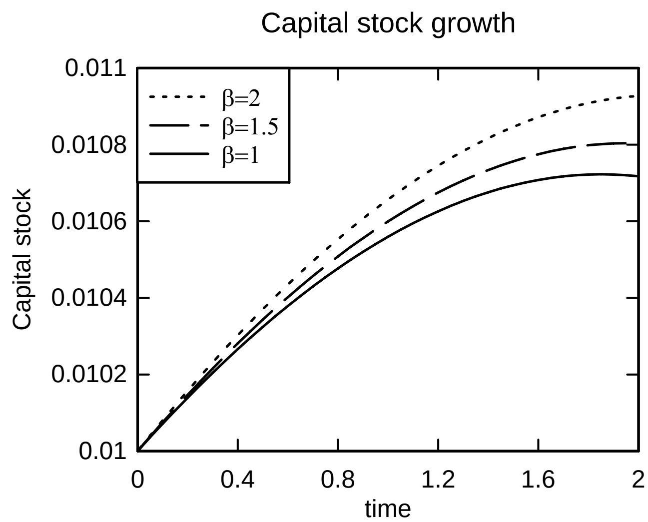

Figure 5.2. Here increasing firm marginal value is captured by the multiplier onK_T with increments of 0.5 from a low of 1. Thus, any change that increases the marginal firm value in the second period leads to an increase in the optimal capital stock everywhere in the first period.

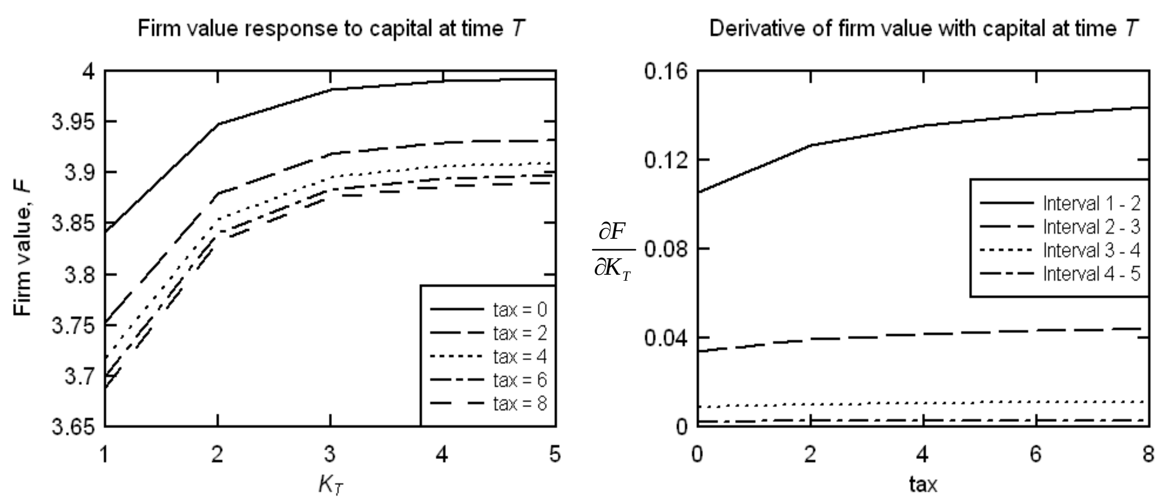

Figure 5.3. The left panel is the response of firm value to changes in initial capital under different tax expectations. The right panel is the derivative of firm value with respect to initial capital under increasing tax expectations.

two results: First, an increasing tax reduces firm value. The tax policy lowers the firm's profits, and thus, the value of the firm. The second result shows firm value increasing in the capital stock at time T, but at a decreasing rate. The interpretation here is that there is diminishing marginal firm value to capital stock accumulation by a firm. The right hand panel of Figure 5.3 illustrates how $\frac{\partial F}{\partial K}$ changes with tax—we have plotted the slope of the curves in the left panel against the tax. This shows that having a high capital stock at time T makes the firm robust to changes in the tax. Note that when the capital stock starts very high, like in *Interval 4-5*, the slope does not change in the tax, implying robust (in this sense). On the other hand, when the capital stock starts low, *Interval 1-2* curve, $\frac{\partial F}{\partial K}$ is concave in the tax. This implies that the expected value of $\frac{\partial F}{\partial K}$ decreases in risk.

5.4.3 Useful insight

The examples above expose an interesting insight. For example, if the capital stock is already high, then uncertainty will have very little effect on optimal investment in the second period. This is because optimal investment is already zero. On the other end of the spectrum, however, is a more interesting observation that if the initial capital stock is low, then the firm may or may not invest. Therefore, uncertainty plays a role—in this case, the optimal investment will decrease in risk. These show that there are marked differences in optimal investment in response to the capital stock level, and thus, raises the question: why does this happen? Revisiting our investment cost function—in Section 5.3—shows that these differences exist because investment cost is a decreasing function of the capital stock. In explicit terms, investment is very costly when the capital stock is low, and it costs less when the capital stock is high. In a future work, we will investigate the possibility of a turn around if the investment cost was very high.

In Chapter 4 and in Baker and Shittu (2006), we showed that optimal investment in non-carbon technology can increase in risk. Here, we have added another issue—second period investment. Clearly, these analyses have a push-pull interaction because of the possibility of investment in both stages versus the one-period investment in the previous works. Our numerical example show that the possibility of being able to invest in the future puts a downward pressure on near-term investment. Thus, the firm will benefit from the inherent advantages of the *option value* that comes with waiting before getting to invest after the tax level has been resolved.

In the next section, we focus on the effect of carbon tax timing uncertainty on investment decisions in this technology.

5.5 Random timing of a known tax change

We start this section by referring to our earlier two-stage problem defined in Section 5.4. However, we begin with the case with certainty in the timing of an upward and known tax increase. We analyze the impact of two scenarios; short and long term horizons under timing certainty before the tax change. We re-write the problem in (5.32) without uncertainty in the tax post time T as;

$$V(K_0) = \max_{I_t} \begin{pmatrix} \int_0^T (\pi_t(\tau_1, \alpha(K_t)) - C(I_t))e^{-rt}dt + ... \\ \int_T^\infty (\pi_t(\tau_2, \alpha(K_t)) - C(I_t))e^{-rt}dt \end{pmatrix} \tag{5.51}$$

$$s.t.\ \dot{K} = I_t - \delta K_t,\ K(0) = K_0 \tag{5.52}$$

where τ_1 and τ_2 are the tax values before and after time T. Thus, in this base case, in addition to a known time, T, we assume that $\tau_1 < \tau_2$ with a boundary and simple case that $\tau_1 = 0$. Transforming this problem for solution ease, we define the second stage cash flow stream as,

$$F(K_T) = \max_{I_t} \int_T^{\infty} (\pi_t(\tau_2, \alpha(K_t)) - C(I_t))e^{-r(t-T)}dt \quad (5.53)$$

$$s.t.\dot{K} = I_t - \delta K_t, \; K(T) = K_T \geq 0 \quad (5.54)$$

so that (5.51) becomes,

$$V(K_0) = \max_{I_t} \int_0^T (\pi_t(\tau_1, \alpha(K_t)) - C(I_t))e^{-rt}dt + e^{-rT}F(K_T) \quad (5.55)$$

subject to (5.52). It should be noted that this is different from (5.36) because here there is no expectation due to either a random tax or random time on the value function, $F(K_T)$. However, transferring the characteristics of the solution to (5.34) from Section 5.4 to describe (5.53), and using Proposition 4 to solve (5.55), the firm's first period investment path can be described. This solution, where it exists for $I^*(t)$ must satisfy the differential equation,

$$\dot{I}^*(t) = (r+\delta)\frac{C'(I_t^*)}{C''(I_t^*)} - \frac{\pi_K(\tau_1, \alpha(K_t))}{C''(I_t^*)} \quad (5.56)$$

$$\dot{\mu} = (r+\delta)C'(I_t^*) - \pi_K(\tau_1, \alpha(K_t)) \quad (5.57)$$

where $\pi_K(\tau_1, \alpha(K_t)) = \frac{\partial \pi_t(\tau_1, \alpha(K_t))}{\partial \alpha(K_t)} \cdot \frac{\partial \alpha(K_t)}{\partial K_t}$. (5.57) is bounded by $\mu(T) = e^{-rT}\frac{\partial F(K_T)}{\partial K_T}$. Now, we use this boundary to characterize the terminal condition at time T of the optimal first period investment plan of the firm as follows;

$$\lim_{T\to\infty} \mu(T) = 0 \quad (5.58)$$

$$\lim_{T\to 0} \mu(T) = \frac{\partial F(K_0)}{\partial K_0} \quad (5.59)$$

(5.58) says that if the expected time to a tax increase is infinitely distant, then the shadow value of optimal investment is zero. Therefore, there is no investment—implying that the tax change does not matter if it is in the future, and has no effect on the firm's level of investment. On the other hand, (5.59) shows that if the tax change is now, the marginal value of investment is positive and it depends on the current capital stock in the technology. Combining these boundary conditions with the dynamics of investment in (5.57), the firm's optimal investment behavior under timing certainty depends on the initial capital stock, K_0 and the timing of the tax change, T.

The firm's initial capital stock (if it exists) does not change for a tax change that is expected in the distant future because the marginal value of investment is zero, while the firm's capital stock follows Figure 5.1 towards the steady state capital stock for a near term tax change depending on the relationship between the starting capital, K_0 and the steady state capital, K^*.

5.5.1 Effects of uncertain timing

Now we explore when the time, T is a random variable with a known probability distribution. In this case, $V(T)$—the present value of the cash flow when the tax increase occurs—is also random. We cannot discuss the maximization of $V(T)$ because it will be different for every random variable T. This is peculiar to most problems under uncertainty and thus, we adopt the expected value function hypothesis. The firm will choose an investment profile which will make the expected value of $V(T)$ greatest. The generic present value of the cash flow for a given T is

$$V(T) = \int_0^T (\pi_t(\tau_1, \alpha(K_t)) - C(I_t))e^{-rt}dt + \int_T^\infty (\pi_t(\tau_2, \alpha(K_t)) - C(I_t))e^{-rt}dt \quad (5.60)$$

In order to address this timing uncertainty, we define $p(T)$ as the firm's subjective probability density function of the random time such that $T \in (0, \infty)$. We re-cast (5.60) as follows

$$E[V(T)] = \int_0^\infty p(T) \begin{pmatrix} \int_0^T (\pi_t(\tau_1, \alpha(K_t)) - C(I_t))e^{-rt}dt + ... \\ \int_T^\infty (\pi_t(\tau_2, \alpha(K_t)) - C(I_t))e^{-rt}dt \end{pmatrix} dT \tag{5.61}$$

Letting $W_i = (\pi_t(\tau_i, \alpha(K_t)) - C(I_t))$ for $i = 1, 2$, and using these definitions, (5.61) can be reduced to

$$\begin{aligned} E[V(T)] &= \int_0^\infty p(T) \left(\int_0^T W_1 e^{-rt}dt + \int_T^\infty W_2 e^{-rt}dt \right) dT \\ &= \int_0^\infty \int_0^T p(T)W_1 e^{-rt}dtdT + \int_0^\infty \int_T^\infty p(T)W_2 e^{-rt}dtdT \end{aligned} \tag{5.62}$$

We define

$$\Omega(t) = \int_t^\infty p(s)ds \tag{5.63}$$

as the probability that the tax increase only happens after time $t \in [0, \infty)$ such that $p(t) = -\dot{\Omega}(t)$. When T is a random variable which assumes values in the interval $[0, \infty]$ and it follows a probability density function $p(t)$, with properties $p(t) \geq 0$ for all t, and $\int_0^\infty p(t)dt = 1$. By reversing the order of integration with notable changes in the limits in (5.62), and using (5.63), we have

$$\begin{aligned} E[V(T)] &= \int_0^\infty W_1 e^{-rt} \left(\int_t^\infty p(T)dT \right) dt + \int_0^\infty \int_T^\infty p(T)W_2 e^{-rt}dtdT \\ &= \int_0^\infty \Omega(t)W_1 e^{-rt}dt + \int_0^\infty W_2 e^{-rt} \left(\int_0^t p(T)dT \right) dt \\ &= \int_0^\infty \Omega(t)W_1 e^{-rt}dt + \int_0^\infty W_2 e^{-rt} \left(\int_0^\infty p(T)dT - \int_t^\infty p(T)dT \right) dt \\ &= \int_0^\infty \Omega(t)W_1 e^{-rt}dt + \int_0^\infty (1 - \Omega(t))\, W_2 e^{-rt}dt \\ &= \int_0^\infty [\Omega(t)W_1 + (1 - \Omega(t))\, W_2]\, e^{-rt}dt \end{aligned} \tag{5.64}$$

As long as the timing of the increase in tax is uncertain, the problem becomes that of choosing the investment path that maximizes the expected present value of the cash flow. Thus, re-inserting the profit less cost of investment function into (5.64), we have

$$\max_{I_t} E\left[V(T)\right] = \int_0^\infty \left[\begin{array}{c} \Omega(t)(\pi_t(\tau_1, \alpha(K_t)) - C(I_t)) + ... \\ (1 - \Omega(t))(\pi_t(\tau_2, \alpha(K_t)) - C(I_t)) \end{array} \right] e^{-rt} dt \tag{5.65}$$

subject to (5.52). The admissible $I^*(t)$ for this problem where it exists must satisfy the differential equation[6],

$$\dot{I}^*(t) = (r+\delta)\frac{C'(I_t^*)}{C''(I_t^*)} - \left(\begin{array}{c} \Omega(t)\frac{\pi_K(\tau_1,\alpha(K_t))}{C''(I_t^*)} + ... \\ (1-\Omega(t))\frac{\pi_K(\tau_2,\alpha(K_t))}{C''(I_t^*)} \end{array} \right) \tag{5.66}$$

$$\dot{\mu} = (r+\delta)C'(I_t^*) - \left[\Omega(t)\pi_K(\tau_1, \alpha(K_t)) + (1-\Omega(t))\pi_K(\tau_2, \alpha(K_t))\right] \tag{5.67}$$

Comparing (5.67) with the case without uncertainty, (5.57), the squared brackets on the right-hand side is a *weighted average* over the marginal profit of the firm. (5.67) reduces to (5.57) if the probability that the tax change will not occur before any time t, $\Omega(t)$, is 1—in other words, the tax will not change, but remains at the current value, τ_1, for all $t \in [0, \infty)$. This implies that the firm's optimal capital stock, K_1^*, follows the solution to (5.56) when $\Omega(t) = 1$. On the other hand, the marginal profit due to the new tax replaces that due to the old tax if the probability that the tax change will occur corresponds to $\Omega(t) = 0$ implying that the firm's optimal capital stock, K_2^*, results from the solution to (5.66) when $\Omega(t) = 0$.

Comparing the investment paths under certainty in the timing of tax change with the uncertainty case, we explore the key optimality condition at the steady state. First, rewriting (5.66), and introducing the steady state property, $\dot{I} = \dot{K} = 0$, we have

$$C'(\delta K_t^*) = \frac{\Omega(t)\pi_K(\tau_1, \alpha(K_t^*)) + (1-\Omega(t))\pi_K(\tau_2, \alpha(K_t^*))}{r+\delta} \tag{5.68}$$

[6]Applying the faster Euler-Langrange differential equation which yields the same reesult as the Pontryagin principle.

The solution to this problem gives the optimal capital stock, K^* to be achieved under uncertainty before the new tax comes into effect. We follow this with a description of the optimal path and a comparison between the certain and uncertain timing optimal paths.

Under certainty in the timing of the tax increase, the firm will raise its stock of capital from the optimal low tax level, K_1^*, before the tax increase to the optimal high tax level, K_2^*, after the tax increase has been realized. However, the precise optimal path of investment between these levels of capital stock is not readily characterized. Thus, the left hand panel of Figure 5.4 illustrates three postulates of the paths between the steady state levels of the capital stock. Case (a) illustrates when the path is such that investment increases to make the steady state optimal capital stock be achieved at the time of the tax change. This case requires that the increase in investment is steady to increase the capital stock from K_1^* to K_2^*. In path (b), the marginal increase in the capital stock is lower. We infer that this case yields more to the influence of adjustment cost on the level of investment increase. The path described by (c) is a less likely scenario since a forward-looking firm will not wait to realize the tax before increasing the capital stock. This path may not be optimal because the value to early investments in the technology is lost and the influence of adjustment cost will make achieving the steep gradient[7] in investment difficult to realize. In the next section, we hypothesize on the path represented by (a).

Uncertainty in the timing leads to the optimal capital stock, K^*, from (5.68). This is higher everywhere before the tax increase is realized than the range in the certainty case—implying higher near-term investments. This is because the steady state capital stock, K^*, that solves (5.68) is a function of the *weighted average* over the marginal profit of the firm while the certainty case corresponds to the transition

[7] A bound on this path is for the capital stock to exhibit a *jump* at time T—the vertical long dash line—to the higher steady state value.

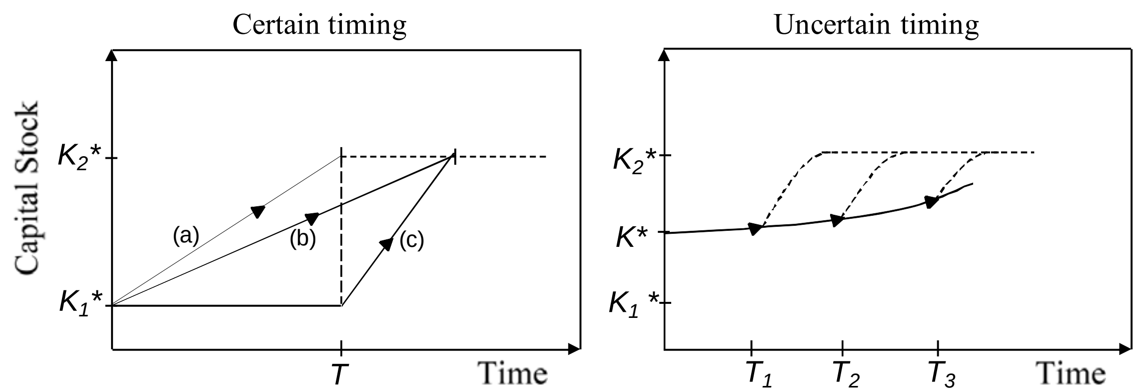

Figure 5.4. Investment response to timing of an increase in tax. On the left hand panel are three path postulates toward the steady state for known timing of the tax change. The right hand panel illustrates how timing uncertainty has the optimal capital stock path higher.

between K_1^* to K_2^*. Thus, the capital stock transition under uncertainty satisfies the relationship such that $K_1^* < K^* < K_2^*$. The right hand panel of Figure 5.4 illustrates a particular path to the saddlepoint associated with the high tax. In this figure, there are three potential possibilities of the time, T_1, T_2, and T_3 when the tax increase will be realized. In the next section, we hypothesize another possibility based on the conditional probability of the tax change, $(1 - \Omega(t))$, increasing with time. This raises the optimal path of the capital stock with the likelihood of higher *jumps* in the near term, T_1, than later term, T_3 of the tax increase. Stated differently, if—on average—the firm anticipates the tax increase to come sooner, then near-term investments will be higher.

5.5.1.1 Hypothesis

In this subsection, we hypothesize on the optimal transition between the capital stock steady states, first, under certainty; and second, under uncertainty. We use these hypotheses to underline the scope of future work that aims to adequately support the assumptions guiding these hypotheses.

We derive the optimal investment path to be followed under uncertainty by using (5.67) with the transversality condition. Introducing an integrating factor, $e^{-(r+\delta)t}$, (5.67) becomes

$$\begin{aligned} e^{-(r+\delta)t}\left(\dot{\mu} - (r+\delta)\mu_t\right) &= -[\Omega(t)\pi_K(\tau_1, \alpha(K_t)) + (1-\Omega(t))\pi_K(\tau_2, \alpha(K_t))]e^{-(r+\delta)t} \\ &= \frac{d}{dt}\left(\mu_t e^{-(r+\delta)t}\right) \end{aligned}$$

implying that

$$\frac{d}{dt}\left(\mu_t e^{-(r+\delta)t}\right) = -[\Omega(t)\pi_K(\tau_1, \alpha(K_t)) + (1-\Omega(t))\pi_K(\tau_2, \alpha(K_t))]e^{-(r+\delta)t}$$

Taking the integral of both sides from $t = s$ to $t = \infty$, with the transversality condition $\lim_{t\to\infty} \mu_t e^{-rt} = 0$, we have

$$\mu^*(t) = \int_{s=t}^{\infty} [\Omega(t)\pi_K(\tau_1, \alpha(K_t)) + (1-\Omega(t))\pi_K(\tau_2, \alpha(K_t))]e^{-(r+\delta)(s-t)} ds \qquad (5.69)$$

This gives the investment path to be followed under uncertainty. We have stated already that under certainty in the timing of the tax increase, the firm will raise its stock of capital from the optimal low tax level, K_1^*, before the tax increase to the optimal high tax level, K_2^*, after the tax increase has been realized. The left hand panel of Figure 5.5 illustrates scenario (a) from Figure 5.4. Our choice of this path—steady rise in investment—is based on the belief that the sooner the firm raises its optimal capital stock to the steady state through increased investments at time T, the higher is the investment payoff after the tax increase.

Under uncertainty in timing, the optimal capital stock, K^*, is higher everywhere before the tax increase is realized. However, we hypothesize that the investment path *jumps* to the saddlepath described by the certain high tax, τ_2. We argue that the cost of adjustment to making this jump is lower since the steady state capital stock, K^*,

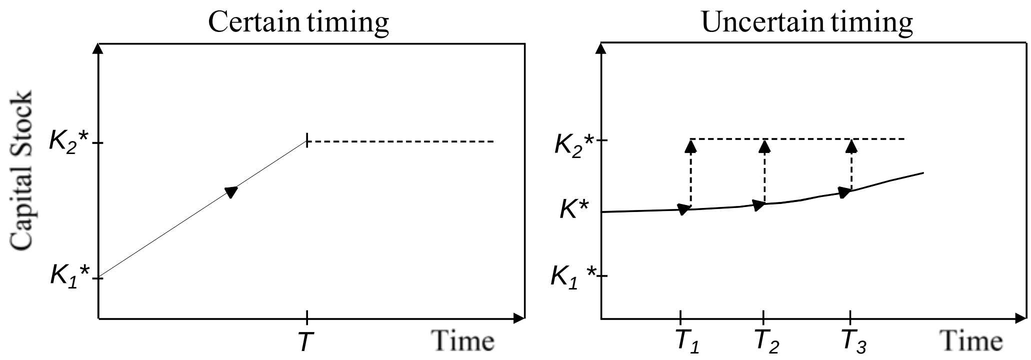

Figure 5.5. Hypothesis on optimal paths to the steady state.

prior to the tax change is already high, and the firm realizes higher benefits to early saddlepath entry. The right hand panel of Figure 5.5 illustrates three possibilities of this conjecture. Our belief about the conditional probability of the tax change, $(1 - \Omega(t))$, increasing with time thus raises the optimal path of the capital stock with the likelihood of higher *jumps* in the near term, T_1, than later term, T_3 of the tax increase—implying that investment will be higher the earlier the uncertainty is resolved than the later.

The saddlepath to the steady state, characterized by the two horizontal lines in each panel of Figure 5.5, is conditioned on the nature of the firm's marginal profit function due to the capital stock, $\pi_K(\cdot)$[8].

5.6 Conclusion

In this chapter, we consider a continuous-time optimal control model to determine the effect of tax magnitude uncertainty on a firm's capital stock and R&D spending in alternative, cost-reducing non-carbon energy input technology. First, we find that uncertainty in the magnitude of a future tax reduces near-term investment in non-

[8]We know that $\pi_K > 0$ and $\pi_{KK} < 0$, but we cannot determine the sign of π_{KKK} that characterizes the nature of π_K.

carbon fuel technology. This result is driven by the trajectory of capital accumulation in the first period which is everywhere increased with increasing marginal firm value function post the tax change. Marginal firm value, in turn, depends on the distribution of the future tax. We note that an increasing tax reduces firm value because the tax policy lowers the firm's cash flow. However, firm value increases in the capital stock, but at a decreasing rate, implying that there is diminishing marginal firm value to capital stock accumulation by a firm. In our analysis, the expected marginal firm value in the capital stock is concave in the tax implying that the marginal firm value decreases in risk. This in turn implies that optimal investment decreases in risk in the tax. On the other hand, we show that having a high capital stock at the time of the tax increase makes the firm robust to changes in the tax, since they will not be investing regardless of the level of the tax.

We contrast this with the result from Chapter 4 where uncertainty in the tax *increases* near-term investment in this technology. We argue that the possibility of investing in both periods puts a downward pressure on near term investment in this two-stage model. That is, there is an option value to waiting. Waiting for the tax to be resolved increases the value of information guiding the firm's investment decision. We hypothesize that investment lags or delays in observing the effect of investment may reverse this effect. This is open to future analysis.

We pay close attention to the impact of the cost of investment on the optimal investment decision. The differences in optimal investment in response to different capital stock levels also hinge on the definition that the cost of investment is a decreasing function of the capital stock—investment is very costly when the capital stock is low, and it costs less when the capital stock is high.

Second, we extend this analysis by investigating how investment is influenced by uncertainty in the timing of a future carbon tax. We find that uncertainty in the timing of a known future tax increase increases near-term investment. In this case, the effect of uncertainty on the shadow value of investment, and thus the firm's optimal investment, depends on the firm's belief about the probability of the timing of the known tax increase. Under uncertainty in the timing of the tax increase, the firm's optimal response is to increase its capital stock through higher investments. We hypothesize that there may be jumps to the saddlepath that lead to the steady state at the time of the tax change. The nearer the resolution of the tax increase, the higher the jump to the saddlepath. The saddlepath depends on how the firm's relative marginal profits change due to changes in the capital stock level. Future work will explore more exact characteristics of the optimal paths to a the steady state.

Simplifying assumptions in this analysis include the use of fixed prices. However, if prices are allowed to increase over time, we expect the results to become more resounding. An avenue for future work is to consider random or stochastic price paths in the prices. While this has the tendency to describe, more closely, a firm's behavior, we anticipate that the results will not be diametrically different. Also, in our model, we ascribed to the use of just one variant of a price-based policy—carbon tax—future work will look at other market-based policies like the current congressional consideration of a cap-and-price mechanism, subsidies and permit systems. Beyond this realm are emission standards and other command-and-control policies which are better suited for some other technologies. Intuitively, we conjecture that given the same framework of policy uncertainty, the results may be similar.

CHAPTER 6

CONCLUSION AND FUTURE WORK

This chapter summarizes the overall impacts of this dissertation with emphasis on the conclusions of the three major segments addressed. By extension, it also discusses the relevant contributions of this dissertation to the discussion on the impact of uncertainty on environmental policy and investment in technological change. We conclude this chapter by raising a number of related avenues for future work.

6.1 Summary of findings

In Chapter 3, we explored the possibility and implications of environmental technological advance that increases marginal abatement costs for higher levels of abatement. Through an illustrated example of a simplified electricity sector, we show the possibility of such innovation. With detailed consideration of the implications of such advance using the framework from Milliman and Prince (1989), we observed that policy implications can be substantively different when innovation increases marginal abatement costs. This suggests that the best framework for analyzing how policy instruments impact abatement technology is with a portfolio of technologies—as addressed in Chapter 4. Since different instruments have different incentives for different technologies, using the "wrong" instrument may promote the "wrong" technology. In fact, one interpretation of these results compared to Milliman and Prince (1989) is that taxes and auctioned permits provide more incentives for firms to choose technologies that lower marginal cost; while direct controls, subsidies, and free permits provide more incentives for firms to choose technologies with higher marginal cost.

However, for innovation to increase marginal cost, it must impact technologies associated with less than full abatement. By improving these technologies, the marginal cost of abatement must ultimately increase at some point as we move toward full abatement. In contrast, innovations that reduce the costs of full abatement—for example, lowering the costs of photovoltaic cells—will decrease the marginal costs of abatement. Also, the optimal level of abatement must be in the range where marginal costs have increased.

In Chapter 4, we considered a portfolio of R&D options in terms of reducing the effective cost of inputs. We distinguish between the R&D programs based on their influence on the price for inputs, which in turn drives the effective demand of inputs, and thus the optimal investment level in each of them. We find that optimal R&D investment is influenced by the relative cost of R&D programs into efficiency programs and the firm's flexibility in substituting between fossil and non-fossil energy inputs. In particular, overall investment appears to be highest when the carbon tax is high enough to provide incentives for using CCS, but not so high that firms start to substitute away from fossil fuel energy significantly. The effects of the elasticity of substitution between energy inputs is such that a high substitution elasticity between fossil and non-fossil energy leads to the optimal investments in CCS exceeding that of non-fossil at low tax levels, but at higher levels, the converse is true. The explanatory and logical interpretation is that investment in the CCS program increases in a carbon tax to offset the influence of the tax, but at high tax levels, these investments reduce since it is more economical to focus solely on non-fossil programs that are not influenced by the carbon tax. On the other hand, when short term elasticity of substitution between fossil and non-fossil energy inputs is low, the carbon tax does not significantly influence the investment level in non-fossil energy programs. Overall portfolio investment reduces in the tax when the firm's ability to substitute away from fossil related inputs is limited. Uncertainty in the tax decreases the overall optimal investment in the portfolio for a firm with enough flexibility in its use of fossil and non-fossil energy—as observed with non-fossil technology substituting for CCS at considerably high tax levels.

Thus, given the current uncertainty about a future tax, it looks like optimal R&D investments should be relatively small. However, it appears that non-fossil fuel program can be a hedge against uncertainty, and so more attention should be spent on this technology. In addition, this analysis provides insights to policymakers concerned about setting a carbon tax and crafting R&D policy because it is evident that reducing uncertainty will increase investment.

We considered a continuous-time optimal control model, in Chapter 5, to determine; (1) the effect of tax magnitude uncertainty on a firm's capital stock and R&D spending in alternative, cost-reducing non-fossil energy input technology, and (2) how investment is influenced by uncertainty in the timing of a future carbon tax. We find that uncertainty in the magnitude of a future tax reduces near-term investment in non-fossil fuel technology. This result is driven by the trajectory of capital accumulation in the first period which is everywhere increased with increasing marginal firm value function post the tax change. Marginal firm value, in turn, depends on the distribution of the future tax. We note that an increasing tax reduces firm value because the tax policy lowers the firm's cash flow. However, firm value increases in the capital stock, but at a decreasing rate, implying that there is diminishing marginal firm value to capital stock accumulation by a firm. In our analysis, the expected marginal firm value in the capital stock is concave in the tax implying that the marginal firm value decreases in risk. This in turn implies that optimal investment decreases in risk in the tax. On the other hand, we show that having a high capital stock at the time of the tax increase makes the firm robust to changes in the tax, since they will not be investing regardless of the level of the tax. We contrast this with the result from Chapter 4 where uncertainty in the expected tax increases near-term investment in this technology. We argue that the value of the option to invest in both periods puts a downward pressure on investment in this two-stage model allowing for near-term investment to be held back prior to the resolution of the future tax.

Our analysis indicates that uncertainty in the timing of a tax increases the optimal near term investment in this technology. Thus, overall uncertainty—about both the timing and the magnitude—has ambiguous effects on near term optimal investment. We will investigate this further in the future.

6.2 Contributions

One of the aims of this dissertation is to underscore the crucial relevance of technical change representation in the modeling of endogenous technological change in climate change literature. We show that this understanding has far reaching consequences on optimal policies for inducing technological change. In addition, and given our understanding of the different approaches to representing technological change, we accentuate the importance of a firm's investment in a portfolio of technologies. Although we use an emissions tax as a policy of choice, however, one caveat—which also simultaneously serves as an advantage for our portfolio analysis—is that it cannot differentiate between technologies. It is an advantage because using any other price-based control mechanism may not fully alter the allocation of private investments in the technologies.

Whether it is in climate damages, costs, or technological advance, uncertainty has proved difficult to model, especially, its resolution through time, i.e., knowledge updating. In summary, the contributions to literature include:

1. Sensitize the environmental economics arena that not all environmental technical change are the same. The impact of different representations of endogenous technological change has far-reaching consequences on the resolution of policies, and the inherent uncertainties that come in the climate change context.

2. Provide a theoretical template for analyzing investment in a portfolio of related technologies under an increasing and uncertain carbon tax scheme. This is an extension of related investment analyses which target specific technologies without provision for possible interactions with related technologies. This is because the future direction of energy systems is for investments to be targeted across related technologies.

3. Create a platform for the resolution of uncertainties using a continuous-time model that uses these uncertainties to determine near-term investment levels and capital stock accumulation.

6.3 Future Work

The treatment of how uncertainty in environmental policy influences investment decisions in technological progress is a continuing discussion in the economic arena of global climate change. While this dissertation has involved extensive and intensive examination of this interaction, there are still many avenues for future exploratory analysis. One such avenue is in the analysis of endogenous technological learning in bottom-up energy systems. Top-down modeling as addressed in this dissertation is only one window by which decision makers and policymakers get informed on the optimal profile of policies and investments. However, bottom-up energy systems seek to provide a more robust interpretation of *close-to-reality* data-driven studies. The formulation of the energy landscape problem in an endogenous model of technological learning is defined by the data regarding consumption patterns of energy use and conversion processes from the sectors of the world in a regional data. This dissertation is largely based on analyses that considers decision making at the firm level, however, under the assumption of a multi-regional world, expanded analysis of the interaction between regions on energy flow and technology transfer will lead to a more pragmatic hold on the direction of investments given the drive to achieve a pegged stabilization limit in the global carbon footprint.

Another prism for viewing future research work in this arena is through the evolution in the prices of energy inputs following a random walk or pattern. Injecting inherent uncertainties in the price paths of inputs is as important as the uncertainty in the guiding policy. To this end, other pertinent questions include; what is the impact of other policies on outcomes—especially non-market based policies? and how is the optimal investment policy in energy technologies affected when uncertainty in magnitude and timing are combined?

In summary, my research objective is to inform climate technology policy. Developing a good technology policy is difficult for a number of reasons including (1) modeling complexities concerning both economic and damage uncertainties as revealed in some parts of this dissertation; (2) lack of prior knowledge on the cost of R&D efforts; and (3) the probabilities of success in these efforts are unknown. My research focus will continue to explore how uncertainty influences optimal technology policy. For example, with uncertainties in prices, damages, technical success and policy, what is the socially optimal level of energy R&D investment? Although the climate change problem can only be ameliorated in the long-run, what are the short-run approaches that should be implemented toward achieving the long term result? Finally, given that different sectors and world regions have different contributions to this negative trend in climate, how should the mitigation efforts be distributed between them?

These points underscore the boundaries of future work. The discussion on the effect of uncertainties in the crafting of an optimal technology policy is dynamic and thus, this topic will always be open to future analysis.

APPENDIX A

MILIMAN AND PRINCE REVISITED

Relative ranking to promote innovation

Table A.1 shows the firm's initial incentive to innovate. Note that a negative cost in line 9 of Table A.1 implies a benefit to the firm from innovation; the more negative the cost, the higher the benefit. The ranking of the instruments with respect to the promotion of innovation remains unchanged from MP—direct controls under-perform the other instruments. The only significant difference is that under free permits, the firm accrues a transfer loss rather than a gain, since the firm chooses to emit more after technical change.

Table A.1. Firm incentives ranking to promote innovation for discoveries

Pre-Innovation Costs						
		Direct Control	Emission Subsidy	Free Permit	Auctioned Permit	Emission Taxes
1	Direct Cost	$e^m ae^*$	$e^m ae^*$	$e^m ae^*$	$e^m ae^*$	$e^m ae^*$
2	Transfer Loss	–	–	–	$e^* aH0$	$e^* aH0$
3	Transfer Gain	–	$e^m aT^* e^*$	–	–	–
4	Total (1+2-3)	$e^m ae^*$	$-e^m aT^*$	$e^m ae^*$	$e^m aH0$	$e^m aH0$
Post-Innovation Costs						
5	Direct Cost	$e^m ce^*$	$e^m fe'$	$e^m fe'$	$e^m fe'$	$e^m fe'$
6	Transfer Loss	–	–	–	$e' fH0$	$e' fH0$
7	Transfer Gain	–	$e^m T^* fe'$	$-e^* afe'$	–	–
8	Total (5+6-7)	$e^m ce^*$	$-e^m T^* f$	$e^m fae^*$	$e^m fH0$	$e^m fH0$
9	Difference(8-4)	cax $-e^m x$	xfa $-e^m x$	xfa $-e^m x$	xfa $-e^m x$	xfa $-e^m x$
	Ranking	5	1	1	1	1

Innovator and non-innovator ranking to promote diffusion for non-patented discoveries

Table A.2 shows the incentives to promote diffusion for both the innovator and the non-innovator. Auctioned permits had the highest diffusion incentives in MP; here they have the lowest.When diffusion shifts MC to MC' for all firms, the auctioned

Table A.2. Firm incentives ranking to promote diffusion for non-patented discoveries

Innovator Costs						
		Direct Control	Emission Subsidy	Free Permit	Auctioned Permit	Emission Taxes
1	Pre-diffusion Cost (Line 8 Table A.1)	$e^m ce^*$	$-e^m T^* f$	$e^m fae^*$	$e^m fH0$	$e^m fH0$
2	Post-diffusion Cost	$e^m ce^*$	$-e^m T^* f$	$e^m ce^*$	$e^m cJ0$	$e^m fH0$
3	Cost (2-1)	–	–	caf	$cfHJ$	–
4	Ranking	1	1	4	5	1
Non-Innovator Costs						
5	Pre-diffusion Cost (Line 4 Table A.1)	$e^m ae^*$	$e^m aT^*$	$e^m ae^*$	$e^m aH0$	$e^m aH0$
6	Cost (2-5)	cax $-e^m x$	xfa $-e^m x$	cax $-e^m x$	$cJHax$ $-e^m x$	xfa $-e^m x$
	Ranking	3	1	3	5	1

permits increase in price, in contrast to MP where they decrease in price. This is because technical change increases the marginal cost of abatement. The other instruments remain in the same order as MP. Note, however, that none of the instruments offer a positive incentive for the innovator to promote diffusion—at best the innovator loses nothing. Non-innovators profit unambiguously from diffusion under all instruments except auctioned permits. It is possible that the increase in the price of auctioned permits may outweigh the benefit of lowered abatement costs. This would not necessarily prevent diffusion, however—any individual firm, taking the auctioned price as given, would benefit from adopting the new technology.

Industry-wide relative ranking to promote optimal agency response for non-patented discoveries

Table A.3 shows that, as noted in MP, the results for optimal agency response are exactly the opposite here as in MP. Given higher marginal costs, the optimal agency response is to lower the emissions limit (from e^* to e^{**}) or increase the tax/subsidy (from T^* to T^{**}). Thus, unsurprisingly, the industry has an incentive to support optimal agency response in every case except emissions taxes. In MP, when technical change decreased marginal costs, the optimal agency response is to increase the limit or decrease the tax/subsidy, thus the opposite results.

Table A.3. Firm incentives ranking to promote optimal agency response for non-patented discoveries

Pre-Control Costs						
		Direct Control	Emission Subsidy	Free Permit	Auctioned Permit	Emission Taxes
1	Direct Cost	$e^m ce^*$	$e^m fe'$	$e^m ce^*$	$e^m ce^*$	$e^m fe'$
2	Transfer Loss	–	–	–	$e^* cJ0$	$e' fH0$
3	Transfer Gain	–	$e^m T^* fe'$	–	–	–
4	Total (1+2-3)	$e^m ce^*$	$-e^m T^* f$	$e^m ce^*$	$e^m cJ0$	$e^m fH0$
Post Innovation Costs						
5	Direct Cost	$e^m de^{**}$	$e^m de^{**}$	$e^m de^{**}$	$e^m de^{**}$	$e^m de^{**}$
6	Transfer Loss	–	–	–	$e^{**} dI0$	$e^{**} dI0$
7	Transfer Gain	–	$e^m T^{**} de^{**}$	–	–	–
8	Total (5+6-7)	$e^m de^{**}$	$-e^m T^{**} d$	$e^m de^{**}$	$e^m dI0$	$e^m dI0$
9	Difference(8-4)	$-e^* cde^{**}$	$-fdT^* T^{**}$	$-e^* cde^{**}$	$-dcJI$	$fdIH$
	Ranking	$1-4$	$1-4$	$1-4$	$1-4$	5
	Cont. Adjust St.	favor	favor	favor	favor	oppose

Innovator gains from the entire process of technological change for non-patented innovation

Table A.4 illustrates that for non-patented innovations that increase marginal abatement cost, direct controls, emission subsidies, and free permits guarantee positive gains, while for auctioned permits and emission taxes there is no clear-cut gain.

This result is in contrast to the result in MP, where auctioned permits and taxes resulted in gains, and direct controls, subsidies, and free permits were ambiguous. The reason for the difference is that for direct controls and free permits, technological change decreases the direct cost of abatement and reduces the stringency of the policy. For subsidies, costs are lower and the subsidies are greater. Taxes and auctioned permits, on the other hand, could lead to a loss if the transfer loss due to higher tax/price outweighs the savings in abatement cost. Note that if the marginal damages are constant then there is a clear gain for taxes and auctioned permits as well – it requires steeply sloped marginal damages to get a loss. However, all these calculations are net of the cost of technical change.More generally, this result, like

Table A.4. Innovator gains from the entire process of technological change for non-patented discoveries

		Direct Control	Emission Subsidy	Free Permit	Auctioned Permit	Emission Taxes
1	Abate. cost (Pre-Inno.)	$e^m ae^*$	$-e^m T^* a$	$e^m ae^*$	$e^m aH0$	$e^m aH0$
2	Abate. cost (Post-Inno.)	$e^m de^{**}$	$-e^m T^{**} d$	$e^m de^{**}$	$e^m dI0$	$e^m dI0$
3	Change in abate. cost	dxk $-kae^*e^{**}$ $-e^m x$	axf $-fdT^*T^{**}$ $-e^m x$	dxk $-kae^*e^{**}$ $-e^m x$	$kdIH$ $-e^m x$	$kdIH$ $-e^m x$
4	Reduced abate. cost	gain	gain	gain	uncer-tain	uncer-tain

the result in Milliman and Prince (1989) is heavily influenced by optimal agency response. If we only look at the combined incentives to innovate to promote diffusion, it can be shown that taxes and subsidies provide the greatest incentive, followed by free permits and direct controls, with auctioned permits last. In fact, it cannot be guaranteed that auctioned permits will lead to a gain after diffusion, because the loss from diffusion is potentially large. Taken altogether the dominant choice is emission subsidies: they tie for first in all permutations. Emission taxes, however, are not far behind, especially if the marginal damages are almost flat. In MP, auctioned permits are the dominant choice, but again, emissions taxes are not far behind. Thus, as long as marginal damages are not too steep, emission taxes may be the most robust instrument for promoting a variety of technologies.

APPENDIX B

OPTIMAL DEMAND FOR CARBON AND NON-CARBON ENERGY

First solving the energy sub-problem (let e represent the total energy demand) by considering the cost minimization problem

$$\min P_c e_c + P_{nc} e_{nc} \tag{B.1}$$

$$\text{s.t. } e_c^\gamma + e_{nc}^\gamma = e^\gamma \tag{B.2}$$

Taking first order conditions, we get

$$\begin{aligned} e_c &= P_c^{\frac{1}{\gamma-1}} \left[P_c^{\frac{\gamma}{\gamma-1}} + P_{nc}^{\frac{\gamma}{\gamma-1}} \right]^{-\frac{1}{\gamma}} e & \text{(B.3)} \\ e_{nc} &= P_{nc}^{\frac{1}{\gamma-1}} \left[P_c^{\frac{\gamma}{\gamma-1}} + P_{nc}^{\frac{\gamma}{\gamma-1}} \right]^{-\frac{1}{\gamma}} e & \text{(B.4)} \end{aligned}$$

Now consider the original problem

$$\min wx + P_c e_c + P_{nc} e_{nc} \tag{B.5}$$

$$\text{s.t. } f(x, e_c, e_{nc}) = y \tag{B.6}$$

Rewriting (B.5) by substituting (B.3) and (B.4), we have

$$\min wx + P_c^{\frac{\gamma}{\gamma-1}} \left[P_c^{\frac{\gamma}{\gamma-1}} + P_{nc}^{\frac{\gamma}{\gamma-1}} \right]^{-\frac{1}{\gamma}} e + P_{nc}^{\frac{\gamma}{\gamma-1}} \left[P_c^{\frac{\gamma}{\gamma-1}} + P_{nc}^{\frac{\gamma}{\gamma-1}} \right]^{-\frac{1}{\gamma}} e \tag{B.7}$$

$$= \min wx + \left[P_c^{\frac{\gamma}{\gamma-1}} + P_{nc}^{\frac{\gamma}{\gamma-1}}\right]\left[P_c^{\frac{\gamma}{\gamma-1}} + P_{nc}^{\frac{\gamma}{\gamma-1}}\right]^{-\frac{1}{\gamma}} e \tag{B.8}$$

$$= \min wx + \left[P_c^{\frac{\gamma}{\gamma-1}} + P_{nc}^{\frac{\gamma}{\gamma-1}}\right]^{\frac{\gamma-1}{\gamma}} e \tag{B.9}$$

Now solving the above problem for x and e, using $\left[P_c^{\frac{\gamma}{\gamma-1}} + P_{nc}^{\frac{\gamma}{\gamma-1}}\right]^{\frac{\gamma-1}{\gamma}}$ for the price of e, we form the optimization problem as

$$\min wx + \left[P_c^{\frac{\gamma}{\gamma-1}} + P_{nc}^{\frac{\gamma}{\gamma-1}}\right]^{\frac{\gamma-1}{\gamma}} e \tag{B.10}$$

$$\text{s.t. } y = [x^\rho + (e_c^\gamma + e_{nc}^\gamma)^{\frac{\rho}{\gamma}}]^{\frac{1}{\rho}} \tag{B.11}$$

substituting and taking FOCs, we have

$$x = w^{\frac{1}{\rho-1}}\left[w^{\frac{\rho}{\rho-1}} + \left(P_c^{\frac{\gamma}{\gamma-1}} + P_{nc}^{\frac{\gamma}{\gamma-1}}\right)^{\frac{\gamma-1}{\gamma}\cdot\frac{\rho}{\rho-1}}\right]^{-\frac{1}{\rho}} y \tag{B.12}$$

$$= w^{\frac{1}{\rho-1}}\left[w^{\frac{\rho}{\rho-1}} + \left(P_c^{\frac{\gamma}{\gamma-1}} + P_{nc}^{\frac{\gamma}{\gamma-1}}\right)^{\frac{\rho(\gamma-1)}{\gamma(\rho-1)}}\right]^{-\frac{1}{\rho}} y \tag{B.13}$$

$$e = \left(P_c^{\frac{\gamma}{\gamma-1}} + P_{nc}^{\frac{\gamma}{\gamma-1}}\right)^{\frac{\gamma-1}{\gamma(\rho-1)}}\left[w^{\frac{\rho}{\rho-1}} + \left(P_c^{\frac{\gamma}{\gamma-1}} + P_{nc}^{\frac{\gamma}{\gamma-1}}\right)^{\frac{\rho(\gamma-1)}{\gamma(\rho-1)}}\right]^{-\frac{1}{\rho}} y \tag{B.14}$$

substituting (B.14) into (B.3) and (B.4), and letting $\bar{P} = P_c^{\frac{\gamma}{\gamma-1}} + P_{nc}^{\frac{\gamma}{\gamma-1}}$, we have

$$e_c = P_c^{\frac{1}{\gamma-1}}\bar{P}^{-\frac{1}{\gamma}}\bar{P}^{\frac{\gamma-1}{\gamma(\rho-1)}}\left[w^{\frac{\rho}{\rho-1}} + \bar{P}^{\frac{\rho(\gamma-1)}{\gamma(\rho-1)}}\right]^{-\frac{1}{\rho}} y \tag{B.15}$$

$$= P_c^{\frac{1}{\gamma-1}}\bar{P}^{\frac{\gamma-\rho}{\gamma(\rho-1)}}\left[w^{\frac{\rho}{\rho-1}} + \bar{P}^{\frac{\rho(\gamma-1)}{\gamma(\rho-1)}}\right]^{-\frac{1}{\rho}} y \tag{B.16}$$

$$e_{nc} = P_{nc}^{\frac{1}{\gamma-1}} \bar{P}^{\frac{\gamma-\rho}{\gamma(\rho-1)}} \left[w^{\frac{\rho}{\rho-1}} + \bar{P}^{\frac{\rho(\gamma-1)}{\gamma(\rho-1)}} \right]^{-\frac{1}{\rho}} y \tag{B.17}$$

hence, the cost function is

$$c(y) = y \left[w^{\frac{\rho}{\rho-1}} + \bar{P}^{\frac{\gamma-1}{\gamma} \cdot \frac{\rho}{\rho-1}} \right]^{\frac{\rho-1}{\rho}} \tag{B.18}$$

$$= y \left[w^{\frac{\rho}{\rho-1}} + \bar{P}^{\frac{\rho(\gamma-1)}{\gamma(\rho-1)}} \right]^{\frac{\rho-1}{\rho}} \tag{B.19}$$

Now, considering the monopolist's profit maximization problem

$$\max \; yP(y) - c(y) \tag{B.20}$$

where

$$P(y) = Ay^{-\frac{1}{b}} \tag{B.21}$$

hence, by substituting (B.19) and (B.21) into (B.20), taking FOC and solving for y

$$\frac{b-1}{b} Ay^{-\frac{1}{b}} = \left[w^{\frac{\rho}{\rho-1}} + \bar{P}^{\frac{\rho(\gamma-1)}{\gamma(\rho-1)}} \right]^{\frac{\rho-1}{\rho}} \tag{B.22}$$

$$y = \left(\frac{b}{b-1} \frac{1}{A} \right)^{-b} \left[w^{\frac{\rho}{\rho-1}} + \bar{P}^{\frac{\rho(\gamma-1)}{\gamma(\rho-1)}} \right]^{\frac{\rho-1}{\rho}(-b)} \tag{B.23}$$

$$= \left(\frac{b}{b-1} \frac{1}{A} \right)^{-b} \left[w^{\frac{\rho}{\rho-1}} + \bar{P}^{\frac{\rho(\gamma-1)}{\gamma(\rho-1)}} \right]^{\frac{b(1-\rho)}{\rho}} \tag{B.24}$$

substituting for (B.24) in (B.16) and (B.17), we have

$$e_c^* = P_c^{\frac{1}{\gamma-1}} \bar{P}^{\frac{\gamma-\rho}{\gamma(\rho-1)}} \left[w^{\frac{\rho}{\rho-1}} + \bar{P}^{\frac{\rho(\gamma-1)}{\gamma(\rho-1)}} \right]^{-\frac{1}{\rho}} \times \tag{B.25}$$

$$\left(\frac{b}{b-1} \frac{1}{A} \right)^{-b} \left[w^{\frac{\rho}{\rho-1}} + \bar{P}^{\frac{\rho(\gamma-1)}{\gamma(\rho-1)}} \right]^{\frac{b(1-\rho)}{\rho}} \tag{B.26}$$

$$= P_c^{\frac{1}{\gamma-1}} \bar{P}^{\frac{\gamma-\rho}{\gamma(\rho-1)}} \left[w^{\frac{\rho}{\rho-1}} + \bar{P}^{\frac{\rho(\gamma-1)}{\gamma(\rho-1)}} \right]^{-\frac{1}{\rho}+\frac{b(1-\rho)}{\rho}} \left(\frac{b}{b-1} \frac{1}{A} \right)^{-b} \tag{B.27}$$

$$= P_c^{\frac{1}{\gamma-1}} \bar{P}^{\frac{\gamma-\rho}{\gamma(\rho-1)}} \left[w^{\frac{\rho}{\rho-1}} + \bar{P}^{\frac{\rho(\gamma-1)}{\gamma(\rho-1)}} \right]^{\frac{b(1-\rho)-1}{\rho}} \left(\frac{b}{b-1} \frac{1}{A} \right)^{-b} \tag{B.28}$$

Similarly,

$$e_{nc}^{*} = P_{nc}^{\frac{1}{\gamma-1}} \bar{P}^{\frac{\gamma-\rho}{\gamma(\rho-1)}} \left[w^{\frac{\rho}{\rho-1}} + \bar{P}^{\frac{\rho(\gamma-1)}{\gamma(\rho-1)}} \right]^{\frac{b(1-\rho)-1}{\rho}} \left(\frac{b}{b-1} \frac{1}{A} \right)^{-b} \tag{B.29}$$

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