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THREE ESSAYS
ON CONFLICT AND COOPERATION

A Dissertation Presented

by

SUNGHA HWANG

Submitted to the Graduate School of the
University of Massachusetts Amherst in partial fulfillment
of the requirements for the degree of

DOCTOR OF PHILOSOPHY

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Economics

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To my parents

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ABSTRACT

THREE ESSAYS
ON CONFLICT AND COOPERATION
MAY 2009

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Conflict theory has in recent years found important applications and made contributions in fields such as economics, political sciences and evolutionary biology. Economists have examined various aspects and implications of appropriation, a typical example of conflicting economic interests, in rent-seeking models. Political scientists, focusing on political turmoil such as war, civil war and demonstration, have scrutinized the effects of conflictual outcomes on political transitions and political systems. More importantly, early human lethal conflict is being recognized as a key factor in explaining human cooperation in evolutionary biology.

The first essay concerns the technical aspects of conflict theories. Two well-known forms of contest success functions predict contest outcomes from the difference between the resources of each side and from the ratio of resources. The

analytical properties of a given conflict model, such as the existence of equilibrium, can be drastically changed simply by altering the form of the contest success function. Despite this problem, there is no consensus about which form is analytically better or empirically more plausible. In this essay we propose an integrated form of contest success functions which has the ratio form and the difference form as limiting cases and study the analytical properties of this function. We also estimate different contest success functions to see which form is more empirically probable, using data from battles fought in seventeenth-century Europe and during World War II.

In the second essay we explore the application of conflict theory to the collective action problem in large groups. We examine critically the traditional understanding of the role of large groups in collective action using an idea initiated in evolutionary biology. Bingham uses Lanchester's square law to claim that the remote killing ability of humans and their precursors decreases the cost of punishment, when cheating behavior can be punished by other members. By modeling this technology and incorporating individual members' choice of behavior types, we show that as long as the defector is, even slightly, less collective than the punisher, the large group effect pervades. So we may conclude that the large group effect is quite robust, considering the fact that the defectors, because of their behavioral predisposition, would be reluctant to cooperate in any type of collective action.

In the final essay we address conflict and cooperation from a slightly different perspective: conflict and cooperation associated with class alliances and conflict in a society. Economic and political problems have been examined primarily within the context of a dyadic relationship, i.e. between two actors. However, when two different categories of groups are considered, subgroups within these groups may have both common interests and conflicts. Appropriative activity by a ruling class

of capitalists and landlords gives rise to class conflict between the ruling class and the ruled class. The struggle over the relative price between the goods of the urban manufacturing sector and the products of the agricultural sector can divide the ruling and ruled classes and unite the capitalists and the workers. Using coalitional game theory, we study the various conditions, such as the political strength of one class relative to that of other classes and the degree of economic conflict among classes, for coalition formation among these classes. We show that when the economic conflict over tariffs and the rate of appropriation escalates and one class is politically superior to others, the exclusion of that class might occur, so the originally strong class can end up being disadvantaged.

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CHAPTER 1

INTRODUCTION: OVERVIEW AND MAIN RESULTS

1.1 Overview

1.1.1 Conflict and Contests

Traditionally scholars treated conflict as a pathological state that needed special treatment. However in recent years, various theories of conflict have found important applications and made contributions in fields such as economics, political sciences, and evolutionary biology.

Among economists, research activities to combine the analysis of conflict with the traditional economic theory were pioneered by Jack Hirshleifer and Herschel Grossman (Hirshleifer, 1978, 1985, 1991; Grossman, 1994, 2001). Hirshleifer (1991) studies appropriative activities such as actual or threatened theft, robbery, or confiscation when the property rights are not well-defined. More concretely, Hirshleifer (1991) explores the trade-off between production and appropriation in a conflict model, in which two sides produce a common pool income and spend their resources to fight for a share of income. One of the important implications is that increased productive complementarity between the parties does not systematically favor peace. Moreover, the poorer side is motivated to fight more decisively, so conflict can be an income-equalizing process. Allowing for conflict and appropriation changes the standard findings from traditional economic theories.

Adopting similar approaches, Grossman examines topics covering conflict over claims to property, conflict between producers and predators, civil conflict, class conflict, and peace and war in interstate conflict. Effective property rights, which the conventional economic approaches treat as given, can be altered by appropriative activities. As a result, the allocation of resources and the distribution of gains does not depend solely on productive activities as the conventional economic theories presuppose. So we need to consider the theory of "endogenous determination of the equilibrium distribution of property"(Grossman, 1994, p.705). Focusing on the interplay between production and appropriation Grossman studies the effects of conflict in the allocation of scarce resources among alternative uses and the distribution of the resulting products.

Another way of understanding conflict is in terms of a contest, which is usually defined as "a game in which participants expend resources on arming so as to increase the probability of winning if conflict were to actually take place" (Garfinkel and Skaperdas, 2006, p.1). In a contest the side that spends the most obtains the prize; examples include auctions, athletic races, election campaignings, and wars (Hirshleifer and Riley, 1992; Konrad, 2007) The most significant fact about a contest is that its rewards is not proportional to marginal product, average product, or total product.

Conflict or theories of contest also have figured in rent-seeking literature. In a paper titled "The welfare costs of tariffs, monopolies, and theft", Gordon Tullock points out that interest group activities were somewhat similar to theft, as new policies transfer income and wealth from one group to another without any compensation (Tullock, 1967).

A government can create a monopolistic position for a firm or an industry by creating various entrance barriers and imposing tariffs, and the special surplus or profit engendered by this protection is called rents. The monopolistic rent can be regarded as a prize in a contest and each interested group can affect the government's decision by hiring lobbyists to acquire the prize. Krueger (1974) uses the term, "rent-seeking", to describe this kind of behavior as opposed to "profit-seeking". So we can view a game where groups spend their resources to obtain a monopolistic rent as an example of contests, and hence the activities of capturing the prize can be described by the technology of conflict. It is in this context that Tullock uses a well-known contest success functions, often called the Tullock form or ratio form, in his model of rent-seeking (Tullock, 1980). We provide a more precise description of contest success functions below.

The main point of Tullock and Krueger is that when we correctly account for resources spent in lobbying, bribing, and various rent-seeking activities, the welfare loss is much bigger than the case in which we only consider Harberger's "deadweightloss" triangle (Harberger, 1954). Since these important contributions, the arguments of rent-seeking bahavior were extended to cover a broad range of topics: privatization, development policy, foreign aid, the aging problem in a society, and tax avoidance (For example, see Buchanan, Tollison, and Tullock, 1980).

In a broader context, Schelling views conflict as a bargaining situation in which "one participant to gain his ends is dependent on the choices of decisions of others" (Schelling, 1980, p.5). More specifically Schelling considers "the strategy of conflict" such as surprise attacks in a situation of mutual distrust and threats like deterrence, when one's strategic behavior can influence another's expectations.

Inter-state wars or civil wars, both obvious examples of conflicts, have been important topics in political science and operational research. To political scientists the main question is why war takes place or recurs even if bargaining or negotiating based on unwasted resources by avoiding wars is possible. Fearon (1995), as one of the rationalist explanations of wars, asserts that rational leaders cannot achieve a mutually beneficial settlement because of the commitment problem and asymmetric information − for example, leaders' private information about their militaries capabilities or willingness to fight. The specific implications of conflict technology have been explored and studied by political scientists; for instance, Kalyvas, Balcelles, and Rohner (2008) studied the effect of the demise of cold war on the decline and transformation of civil wars by varying fighting effectiveness of adversaries.

The Correlates of War (COW) project is a preeminent project to collect data about international wars and civil wars, initiated by David Singer, a political scientist. A vast body of empirical research about the cause and the continuation of war has been conducted using this data (For example, see Collier and Hoeffler, 2001). Sambanis (2004) raises questions about the credibility of this data set because of coding problems, which critically depend on how civil wars are defined and measured.

Retired army colonel Dupuy has also constructed the HERO (Historical Evaluation and Research Organization) data set covering wars from 1600 through 1973, based on a vast literature including Bodart (1908). Dupuy scrutinizes 81 engagements between the Allied and German forces in 1943-44, and concludes that Germans consistently outperformed the Allies in their ability to wage combat by comparing the combat efficiencies of each force. Dupuy also emphasizes the importance of other qualitative aspects including morale and leadership. This point has been further developed by Biddle (2004) in his book *Military Power*. Biddle advances the arguments that the doctrine and tactics which armies use − "force employment" in his words − is a key determinant of the result of modern wars. Consequently, he also claims that supremacy in numbers sometimes can be almost irrelevant depending on the force enforcement of two sides (Biddle, 2004).

More significantly, evolutionary biologists or anthropologists turned to conflict theory in explaining the evolution of human cooperation. Since human cooperation is individually costly, but socially beneficial, the evolution of human cooperation poses similar questions to the public good problem. Boyd and Richerson (1992) argue that the enforcement of cooperation, and hence the evolution of it, would be possible by retribution - punishment that is directed solely at noncooperators by the punisher. Thus, retribution differs from the punishment strategy in a repeated prisoner's dilemma, which withholds future cooperation. Subsequently, many researchers adopt retribution as a key institution ensuring cooperation in human society (Bingham, 1999, 2000; Boyd, Gintis, Bowles, and Richerson, 2003).

Another instance in which conflict plays an important part in the exposition of human cooperation is group conflict and group selection. Group selection theory, ignored for a long time among biologists, has received renewed attention since Boyd and Richerson (1990). Boyd, Gintis, Bowles, and Richerson (2003) showed that intergroup competition and reproductive leveling might have allowed the proliferation of altruistic cooperation. Recently Choi and Bowles (2007) shows that altruism − the nature of cooperative types − and parochialism − hostility toward other groups − can coevolve by promoting group conflict; Bowles (2008) asserts that paradoxically the parochial conflict was the "midwife" of altruism.

1.1.2 Technology of Conflict

The technology of conflict − "a technology whereby some or all contenders incur costs in an attempt to weaken or disable competitors" − is often described by contest success functions (Hirshleifer, 1991, p.130). Contest success functions, which give the winning probabilities of each side according to resources devoted to conflict, have been used in most conflict models. Two major families of contest success functions have been adopted in conflict models (Tullock, 1980; Hirshleifer, 1989). The first is the difference form of contest success functions which predicts the outcome of contests based on the difference between contenders' forces or resources; the second one, called the Tullock form or the ratio form of contest success function, specifies contest power as a function of the ratio of forces or effort.

Comparing these two forms, Hirshleifer (1989) observes that in contests described by the ratio form, one side will certainly lose a whole prize if it does not exert any effort, while in a conflict model with the difference form a side has a positive probability of obtaining the prize even if it does not devote any effort. Because of these properties, he claims that the ratio form is applicable "when clashes take place under close to "idealized" conditions such as: an undifferentiated battlefield, full information, and unflagging weapon effectiveness". In contrast, the difference form applies "where there are sanctuaries and refuges, where information is imperfect, and where the victorious player is subject to fatigue and distraction" (Hirshleifer, 1989, p.104).

In an attempt to put the use of contest success functions on a "surer footing" and develop "a better understanding of any advantages or limitations" of various contest success functions, Skaperdas axiomatizes them (Skaperdas, 1996, p.284). He shows that when the contest success function satisfies (1) probability, (2) monotonicity, (3) anonymity, (4) consistency, and (5) independence from irrelevant alternatives, the functional form is given by an additive form. In addition, within the additive form, the difference form is shown to be a unique form which predicts winning probabilities based on the difference between resources; the ratio from is proven to be a unique form which depends on the ratio of resources.

Another derivation of contest success functions is the stochastic derivation, which originated from econometric studies of discrete choice. If resources or effort E_i is a proxy for the performance of contenders and the performance of each depends on the noisy term ϵ_i, where $i = 1, 2$ denotes each side, the winning probability for side 1 may be represented in $\Pr\{E_1 + \epsilon_1 > E_2 + \epsilon_2\}$. McFadden (1974) shows that when ϵ_i follows a type I extreme value distribution, $\Pr\{E_1+\epsilon_1 > E_2+\epsilon_2\}$ gives the difference form of contest success functions. Similar characterization for the ratio form is given by Jia (2008); when the performance depends on $E_i\epsilon_i$, the ratio form of contest success function corresponds to $\Pr\{E_1\epsilon_1 > E_2\epsilon_2\}$.

We note that when ϵ_i is an additive noise and follows a normal distribution, we might obtain a new contest success function, possibly called "probit "form, from $\Pr\{E_1 + \epsilon_1 > E_2 + \epsilon_2\}$. However this "probit" form has not been used much in the existing literature, because this form cannot be expressed explicitly in known functions.

A more direct description of conflict technology was given by Frederick Lanchester during the height of World War I (Lanchester, 1916). Lanchester originally proposed two types of differential equations which describe the attrition rates of engaging armies in warfare:

$$\text{Square Law: } \frac{dx}{dt} = -\kappa y,\ \frac{dy}{dt} = -\phi x$$

$$\text{Linear Law : } \frac{dx}{dt} = -\kappa xy,\ \frac{dy}{dt} = \phi yx$$

In the first equation fighters in armies can concentrate on each other, so the reduction in x, Δx, is proportional to the total number of fighters of opponent y. The situation is different if Y cannot concentrate; for example, fighters of X might be invisible to Y.as in guerrilla battles. In this case, which is described by the second equation, we expect that the reduction in x depends on x in addition to y since the larger X is, the more likely it will be that Y hits X's combatants.

By simple manipulations we see that in the first system of equations the outcome of battles depends on the square of the ratio of competing forces, hence its name, square law. In the second system of equations the outcome varies linearly with the ratio of forces, so this rule is called linear law.

Lanchester's model of attritions has been reinterpreted and extended to the conflict among social animals such as fire ants (Adams and Mesterton-Gibbons, 2003; Plowes and Adams, 2005) and early human lethal conflict (Bingham, 1999, 2000). Through examination of various paleontological and archaeological evidence, Bingham (1999, 2000) argues that early humans developed a remote killing competence, and so Lanchester's square law describes well the conflict situation between punishers and cheaters in non-kin cooperation.

Adams and Mesterton-Gibbons (2003) point out that (1) Lanchester's equations assume that the death rate of one side is not directly affected by the fighting ability of its own army, and (2) the square law and the linear law tend to be mixed. They suggest a kind of interpolation between the two types of Lanchester's equations:

$$\frac{dx}{dt} = -\frac{\kappa}{\phi^{\lambda-1}}x^{2-\theta}y,\ \frac{dy}{dt} = -\frac{\phi}{\kappa^{\lambda-1}}xy^{2-\theta}$$

for some constants λ, θ. We easily see that the specific choices of λ and θ would reproduce either square law version equations or linear law equations. For the empirical test of Lanchester's theory, Engel (1954) and Samz (1972) verify Lanchester's law using actual combat data of Iwo Jima during World War II.

Since Lanchester's equations predict the outcome of battles on the square of the ratio of contenders' forces, the ratio of combat power is a determinant of the outcome of the battle. Because of its deterministic nature the contest success function associated with Lanchester's theory may be regarded as a step function, which gives the side with larger forces 100% probability of winning (Hirshleifer, 1989). From a different perspective the defining feature of conflict, as opposed to other activities such as production and cooperation, is that one side's resources devoted to conflict weaken and reduce the adversary's resources. So we can regard Lanchester's equations as a prototype of modeling of attrition arising from various conflict situations (Hirshleifer, 1991).

As we have seen, to describe the technology two forms of contest success functions are frequently used. However, there is an issue of "appropriateness of a contest success function for any particular contest situation", and thus "finding ways to discriminate among functional forms empirically would be a complementary and welcome endeavor" as Skaperdas (1996) writes at the end of the paper (Skaperdas, 1996, p.638). Considering the fact that the two forms show different analytical properties in the same conflict models (Hirshleifer, 1989, 1991), to discriminate between and compare these two forms is crucial. This is the main topic of the first essay, which provides a plausible generalization of the two existing forms.

Lanchester's equations provide a direct description of the conflict process, in which the attrition of fighting ability of contending sides occurs. Researchers, especially in biology, have extended Lanchester's theory to explaining the behavior of animals including early humans. The second essay of the dissertation develops the idea that conflict between punishers and defectors in a public good game can be favorable for the support of cooperation in large groups. Particularly, we propose a formal model to appraise this argument and identify conditions for large group advantages. In the final essay we address conflict and cooperation from a slightly different perspective: conflict and cooperation associated with class alliances and oppositions in a society. This approach follows the view of conflict in terms of bargaining process (Schelling, 1980).

1.2 Main Results

The first essay concerns technical aspects of conflict theories. The main purpose of this essay is twofold: (1) to propose and derive a new family of contest success functions, which provides a generalization of the two existing forms and (2) to estimate the new contest success function using actual battle data. The new form of contest success functions generalizes the existing ones, having each as limiting cases. We provide two derivations: axiomatic derivation and probabilistic derivation.

The essence of axiomatic derivation is that the new form preserves and inherits one of the common properties of two contest success functions − a constant elasticity of augmentation, which we will define more precisely in the text. We argue that the degree of overvaluing or undervaluing of success probabilities can be measured by this elasticity. So in effect we show that the two forms of contest success functions belong to one family.

In addition we provide a probabilistic derivation of this form following McFadden (1974) and Jia (2008). Using a simple conflict model we study an equilibrium when the contest success function is given by this new form, and show that the solution corresponds to an interpolation between the ratio form and the difference form, as we expect.

Empirical studies of contest success functions are rare; Jia (2006) tries to estimate the ratio form and difference form of contest success functions using NBA data, but his work is incomplete. Dobson, Goddard, and Stahler (2008) use data from English soccer games to estimate Tullock's contest model. However, their main focus is to estimate equilibrium effort levels in Tullock's model, not to estimate contest success functions. We estimate various forms of contest success functions to see which form is more empirically probable using war data − battles in the seventeenth-century European wars and World War II. The advantage of using war data is that the data has a natural candidate for a variable that measures effort or resources, namely the number of combatants.

In the second essay we explore the application of conflict theory to the collective action problem in large groups. Since the publication of *Logic of Collective Action* by Mancur Olson, the theory of groups and the collective action in a group have been an intriguing research topic in social sciences. Collective action encompasses various activities ranging from the provision of public goods to political demonstrations. Through the prisoner's dilemma framework, an intrinsic logic of resolution of collective action problems provides solutions to the puzzle of evolution of human cooperation and vice versa.

We examine critically the traditional understanding of the role of large groups in collective action when members of the population punish defectors in the public good game. Particularly, we focus on describing conflict technology using Lanchester's equations. The idea of using Lanchester's equations in the collective action is not new; biologists have been applying Lanchester's law to collective action among animals. For example, Franks and Partridge (1993) use Lanchester's square law to explain why predatory army ants rely on large numbers of workers that are smaller than their prey.

In the context of human collective action problems, Bingham (1999, 2000) invoke Lanchester's square law to claim that the cost of punishment decreases exponentially as the number of punishers becomes larger. He argues that the remote killing ability of humans and their precursors − the special capacity of the human species to kill at a distance from its target − enables a large number of punishers to attack a single cheater simultaneously, and hence Lanchester's square law applies.

We observe that Bingham's point is valid in the situation where a large number of punishers face a single cheater, and that the Lanchester effect depends on the existence of a large number of punishers in his argument. Because the number of punishers is not always large in a large group, it is not clear whether the same argument can carry over to the collective action problem in large groups. In addition, even though the remote killing ability is developed, when the same number of punishers and cheaters engage in conflict, there is no reason to expect that only punishers can concentrate on attacking. Thus the remote killing competence is a necessary condition for the large group effect, but not sufficient; we need to find some conditions to ensure that a larger number of punishers confront less cheaters in fighting such as the collective punishment by punishers (Boehm, 1982).

We develop a simple model of $n-$prisoner dilemma with punishment, which combines the standard evolutionary model of three behavioral types − cooperator, punisher, and defector − with the Lanchester-type conflict between punishers and defectors. By modeling this technology and incorporating individual members' choice of behavioral types, we show that as long as the defector is, even slightly, less collective than the punisher, the large group effect pervades. So we may conclude that the large group effect is quite robust, considering the fact that the defectors, because of their behavioral predisposition, would be reluctant to cooperate in any kind of collective action.

Of course this argument does not assert that larger groups are always successful in collective action; larger groups may have other disadvantages − for instance, higher coordination costs and information costs. However by providing one instance of large-group advantages, we verify that the Olsonian view of collective action and group size does not always provide a correct answer. Moreover, an appropriately modified theory including coordination costs, we believe, would provide some keys to interesting questions of group and collective action, such as the determination of the optimal size of a group.

The final essay addresses class alliances and conflict in a society. Understanding interactions within groups and among groups in a society is the key to explaining important social changes, as Kindleberger (1951) suggests. Social scientists, such as Gerschenkron (1943), Moore (1965), Gourevitch (1977), Bowles (1984) and Luebbert (1991), have emphasized the role of class alliances and conflict, or more generally coalition formations among various classes in explaining institutional changes in the late nineteenth-century Europe.

In Germany, according to Moore (1965), the Junkers successfully formed an alliance with independent peasants and capitalists in big industries and repressed industrial workers. Moore's theory emphasizes the success or failure of compromise between the ruling classes and the role of peasants in political transitions. Furthermore, the working class in the nineteenth-century German society was stronger and better-organized than those in other countries. The question of coalition formation is more interesting if one asks how such a strong working class was excluded from the major coalition, the so-called *solidarity bloc*, and ended up being politically unsuccessful.

We consider four classes - the capitalists, the workers, the landlords and the peasants - which are commonly considered important actors in the economic or political arena. As Basu (1986) points, economic and political problems have been examined primarily within the context of a dyadic relationship, i.e. between two actors. However, when two different categories of groups are considered, subgroups within these groups may have both common interests and conflicts. Appropriative activity by a ruling class of capitalists and landlords gives rise to class conflict between the ruling class and the ruled class which consists of the peasants and the workers. The struggle over the relative price between the goods of urban manufacturing sector and the products of the agricultural sector can divide the ruling and ruled classes and unite the capitalists and the workers; opposition occurs between urban population and rural population.

Using coalitional game theory we study the various conditions for coalition formation among these classes, such as the relative political strength of each class and the degree of economic conflict among classes. We show that when economic conflict over tariffs and the rate of appropriation escalates and one class is politically superior to others, the exclusion of that class might occur. So the originally strong class can end up being disadvantaged. Though in general the initially more advantaged group is believed to remain more successful economically and politically, this need not be the case when the various common interests and conflicts cross and intertwine. The checks and balances among the four classes may lead to this paradoxical outcome.

1.3 Future Research Agendas

The derivation of contest success functions has been either axiomatic or probabilistic, and neither of these approaches is based on a concrete model of conflict processes or conflict situations. On the other hand, Lanchester's equations describe a concrete situation, a combat between two firing armies. We note that Lanchester's ordinary difference equations can be understood as approximations of underlying stochastic environments. This approximation is known to give a good prediction when the sizes of engaging armies are large and when the match is not evenly balanced(Kingman, 2002; Darling and Norris, 2008). In this sense Hirshleifer's assertion on the correspondence between the step-shaped contest success functions and Lanchester's equations can be misleading.

Stochastic Lanchester models have been used extensively in the operation research literature (See McNaught, 1999). A Markov chain obtained from this stochastic model has multiple absorbing states − the states in which one of the engaging sides has zero fighters. So we can calculate probabilities of the chain reaching one of these absorbing states, and by aggregating these probabilities we can obtain winning probabilities for each engaging side.

We may derive explicit formulas for these winning probabilities, and study the relationship between these formulas and the existing contest success functions. In addition we can explore the effect of combat effectiveness and the effect of splitting of initial forces on these probabilities, and hence on corresponding contest success functions.

Furthermore, we recall that conflict is the situation whereby one party devotes resources in order to reduce or offset opponents' resources: for example, eliminating the other side's combatants in battles or working hard to press the opposition and make tackles in a soccer game. In this interpretation the definition of conflicts covers strikes, promotional competitions, lawsuits, athletic competitions, and R&D races. Through promotional competitions, firms spend resources on advertising to affect their market share. If we interpret the winning probabilities as a share of income or profit, promotional competitions provide an example of contests. With this connection, the contest technology, or the technology of conflict in these various contexts may relate to the attrition process described by Lanchester's equation or its proper extensions.

Regarding collective action in large groups, the key source of large group advantage is Lanchester's square law. As we easily verify from the shape of the quadratic functions, this effect can be regarded as an acceleration effect, which was observed and studied by many economists. Schelling asserts this effect is ubiquitous in social behavior (Schelling, 1978, p.33), so we may observe a Lanchester-type effect in other various social interactions or contexts. If successful, we believe this research will contribute to a better understanding of the nature of conflict technologies, the connection between conflict situations and contest success functions, and the relationship between conflict technologies and other social phenomena.

CHAPTER 2

CONTEST SUCCESS FUNCTIONS: THEORY AND EVIDENCE

2.1 Introduction

Conflict theory has received growing attention in various disciplines. Economists have examined various aspects and implications of appropriation, a typical example of conflicting economic interests, in rent-seeking models (Tullock, 1980; Hirshleifer, 1989). Political scientists, focusing on political turmoil such as war, civil war and demonstration, have scrutinized the effects of conflictual outcomes on political transitions and political systems (Kalyvas, Balcelles, and Rohner, 2008). More importantly early human lethal conflict is being recognized as a key factor in explaining human cooperation in evolutionary biology (Bowles, 2008; Choi and Bowles, 2007; Garcia and Bergh, 2008).

In these studies the technology of conflict is usually described by a function called the contest success function. A contest is “a game in which participants expend resources on arming so as to increase their probability of winning if conflict were to actually take place” (Garfinkel and Skaperdas, 2006, p.1) and contest success functions show how probabilities of winning depend on the resources devoted to conflict. Two well-known forms of contest success functions predict contest outcomes from the difference between resources of each side and from the ratio of resources.

In spite of the frequent use of the two different forms of contest success functions, there is no agreement on which form better represents the technology of conflict. Jack Hirshleifer points out that the ratio form has the impractical implication that a side investing zero effort loses everything as long as the opponents spend a small amount of resources (Hirshleifer, 1989, 1991). However, since the difference form does not admit the existence of an interior pure strategy Nash equilibrium in widely used conflict models, the ratio form is more commonly used. One of the few attempts to empirically estimate these functions is Jia (2006), who estimates the ratio form and difference form of contest success functions using NBA data. However, the work is incomplete.

In this chapter we present an integrated form of contest success functions which has the ratio form and the difference form as limiting cases and study the analytical properties of this form. We also estimate different contest success functions using war data, which provides a natural candidate for a variable that measures effort or resources, namely the number of combatants.

To compare these two common functions we consider the following example. For concreteness we use the language of military combat, following (Hirshleifer, 1991). Suppose p is the winning probability of side 1 when two fighters of side 1 face one fighter of side 2, denoted by $(2, 1)$.We ask the following question: when a thousand and one fighters of side 1 contend with a thousand fighters of side 2, namely $(1001, 1000)$, should we still assign the same value of p to the winning probability of side 1? Similarly, if the number of fighters of side 1 and side 2 are 2000 and 1000 respectively, would p be the correct probability of side 1's winning in $(2000, 1000)$?

One may argue that because the importance of one more fighter becomes smaller as the total number of fighters grows, we should assign a probability less than p to $(1001, 1000)$. Regarding the case $(2000, 1000)$, one may think that the effectiveness of fighting ability may increase faster as fighter size increases, so side 1 can have a higher probability of winning in $(2000, 1000)$ (see Lanchester, 1916, for example).

The problem is that in analysis one necessarily chooses one specific form of contest success, equivalent to choosing one interpretation of these functions, even though we do not have a good answer to the above questions. The main purpose of this chapter is to define a new contest success function which provides more flexibility in specification than the existing forms. Section 2 provides the derivation, which closely resembles that of a CES production function (Arrow, Chenery, Minhas, and Solow, 1961). The probabilistic derivation, like McFadden (1974) and Jia (2008), is also provided. We examine the existence of pure strategy interior Nash equilibrium. In section 3 we present the empirical estimation of various contest success functions using battle data of seventeenth-century Europe and World War Two.

2.2 Integrated Form

2.2.1 Derivation

We present the difference form and the ratio form of contest success functions (Hirshleifer, 1989) . Denoting the resources or fighting effort devoted to a contest by side 1 and side 2 by x_1 and x_2 respectively and winning probabilities of side 1 and side 2 by $u(x_1, x_2)$ and $v(x_1, x_2)$, we have

$$\text{Difference}: u^d(x_1, x_2) = \frac{\exp(\kappa x_1)}{\exp(\kappa x_1) + \exp(\kappa x_2)} \quad \text{for } 0 \le x_1, x_2$$

$$v^d(x_1, x_2) = \frac{\exp(\kappa x_2)}{\exp(\kappa x_1) + \exp(\kappa x_2)} \quad \text{for } 0 \le x_1, x_2$$

$$\text{Ratio}: u^r(x_1, x_2) = \begin{cases} \frac{(x_1)^\kappa}{(x_1)^\kappa + (x_2)^\kappa} & \text{if } 0 < x_1 \text{ or } 0 < x_2 \\ \frac{1}{2} & \text{if } x_1 = 0 \text{ and } x_2 = 0 \end{cases}$$

$$v^r(x_1, x_2) = \begin{cases} \frac{(x_2)^\kappa}{(x_1)^\kappa + (x_2)^\kappa} & \text{if } 0 < x_1 \text{ or } 0 < x_2 \\ \frac{1}{2} & \text{if } x_1 = 0 \text{ and } x_2 = 0 \end{cases}$$

The superscript, d or r, indicates the difference or the ratio form. It is clear from the specifications that the difference form gives the probabilities of winning based on the difference in resources $x_1 - x_2$ since $u^d(x_1, x_2) = \frac{1}{1+\exp(-\kappa(x_1 - x_2))}$, while the winning probability in the ratio form depends only on the ratio, x_1/x_2, because $u^r(x_1, x_2) = \frac{1}{1+(x_2/x_1)^\kappa}$. We also note that the ratio form of contest success functions is not continuous at (0,0) which accounts for the impossibility of having (0,0) as Nash equilibrium in a conflict model. We will discuss this more precisely in section 3.2.2.

We note that in the example given in the introduction to this chapter, the ratio of the increase in fighters of side 1, necessary to keep the winning probability constant, to the corresponding increase in fighters of side 2 captures the degree of overvaluing (or undervaluing) of the winning probability. Specifically we compute

$$\text{case 1: ratio of fighters' increases} = \frac{1001 - 2}{1000 - 1} \approx 1 \tag{2.1}$$

$$\text{case 2: ratio of fighters' increases} = \frac{2000 - 2}{1000 - 1} \approx 2 \tag{2.2}$$

Motivated by this we define a new rate which can serve as a measure comparing two forms of contest success functions and call this the marginal rate of augmentation (MRA). This measure shows the quantity of additional resources side 1 needs to augment its existing resources to keep the winning probability a constant against an increase in other side's resources. More precisely if we consider the level set of side 1's contest success function $\bar{u} = u(x_1, x_2)$ and use the notation $x_2 = x_2(x_1)$ such that $\bar{u} = u(x_1, x_2(x_1))$, MRA is the slope of $x_2(x_1)$:

$$\text{MRA} := \frac{dx_2}{dx_1} = -\frac{u_{x_2}}{u_{x_1}} \tag{2.3}$$

So if MRA is high more resources should be devoted to obtain the same success probability. Using MRA we now define an elasticity of augmentation as follows:

$$\begin{aligned} &\text{elasticity of augmentation } (\rho) && (2.4)\\ &= \frac{\text{percentage increase in MRA}}{\text{percentage increase in relative size of contestants' resources}} && (2.5)\\ &= \frac{d\ln(-u_{x_2}/u_{x_1})}{d\ln(x_1/x_2)} \end{aligned}$$

where $u_{x_1} = \partial u/\partial x_1, u_{x_2} = \partial u/\partial x_2$

The elasticity of augmentation is a normalized percentage increase in MRA and since $u + v = 1$, we can also write $\rho = \frac{d\ln(v_{x2}/u_{x_1})}{d\ln(x_1/x_2)}$. When ρ is low we expect that side 1 would need to augment its resources by a smaller amount to keep up the same success probability. This may correspond to the situation described by the difference form. On the other hand a high ρ implies that side 1 should extend its resources by greater amounts to gain the same success probability. This situation is possibly captured by the ratio form. By simple calculation we verify that for the difference form the elasticity is 0 of augmentation whereas for the ratio form the elasticity is 1.

On the other hand the parameter κ in the difference form and ratio form is "mass effect parameter scaling the decisiveness of fighting effort disparities" (Hirshleifer, 1991) and measures the slope of the contest success function in even matching − the contest where $x_1 = x_2$. By computing the marginal winning probabilities in even matching for the difference form and the ratio form,

$$\frac{\partial u^d}{\partial x_1} = \kappa u^d(1-u^d), \quad \frac{\partial u^r}{\partial x_1} = \frac{\kappa}{x_1} u^r(1-u^r)$$

we see that $\partial u^d/\partial x_1 = \partial u^r/\partial x_1 = \frac{\kappa}{4}$ if $x_1 = x_2 = 1$. Since we would like to find an interpolation between the difference form and the ratio form with a constant elasticity of augmentation we require this probability to equal $\frac{\kappa}{4}$ at $x_1 = 1$ in the newly derived function. With these parameters, we have proposition 2.1:

Proposition 2.1 *Suppose we have the following equations:*

$$\frac{d\ln(-u_{x_2}/u_{x_1})}{d\ln(x_1/x_2)} = \rho \qquad \textit{for } x_1, x_2 \geq 0 \tag{2.6}$$

$$u_{x_1}(x_1, x_2) = \frac{\kappa}{4} \quad \textit{for } x_1 = x_2 = 1 \tag{2.7}$$

$$u(x_1, x_2; \rho) = \frac{f_\rho(x_1)}{f_\rho(x_1) + f_\rho(x_2)} \tag{2.8}$$

where $0 \leq \rho$, $\rho \neq 1$, $\kappa > 0$ *and* $f_\rho(0) > 0$, f_ρ *is increasing and differentiable for* $x_1, x_2 \geq 0$. *Then equation (2.9) is a unique solution satisfying (2.6), (2.7), and (2.8)*

$$u(x_1, x_2; \rho) = \frac{\exp\left(\kappa \frac{1}{1-\rho} x_1^{1-\rho}\right)}{\exp\left(\kappa \frac{1}{1-\rho} x_1^{1-\rho}\right) + \exp\left(\kappa \frac{1}{1-\rho} x_2^{1-\rho}\right)} \quad \textit{for } 0 \leq \rho < 1 \tag{2.9}$$

Moreover we have

$$u(x_1, x_2; 0) = u^d(x_1, x_2) \quad \textit{and} \quad u(x_1, x_2; \rho) \to u^r(x_1, x_2) \ \textit{as } \rho \to 1 \tag{2.10}$$

Proof. By rearranging (2.6) we obtain

$$c(x_1)^\rho u_{x_1} + (x_2)^\rho u_{x_2} = 0 \quad \text{for some } c \neq 0 \tag{2.11}$$

Using (2.8) and (2.11) we find

$$c(x_1)^\rho f_\rho'(x_1) f_\rho(x_2) - (x_2)^\rho f_\rho(x_1) f_\rho'(x_2) = 0 \ \text{ for } x_1, x_2 \geq 0 \tag{2.12}$$

By evaluating (2.12) at $x_1 = x_2 > 0$ we conclude $c = 1$. We set $x_2 = 1$ in (2.12) and find

$$\frac{f'_\rho(x_1)}{f_\rho(x_1)} = \frac{f'_\rho(1)}{f_\rho(1)} \frac{1}{x_1^\rho} \tag{2.13}$$

and using (2.7) we see that $f'_\rho(1)/f_\rho(1) = \kappa$. Then by solving (2.13) we obtain

$$u(x_1, x_2; \rho) = \frac{\exp(\kappa \frac{1}{1-\rho} x_1^{1-\rho})}{\exp(\kappa \frac{1}{1-\rho} x_1^{1-\rho}) + \exp(\kappa \frac{1}{1-\rho} x_2^{1-\rho})}$$

So we have $u(x_1, x_2; 0) = u^d(x_1, x_2)$ and the fact that $u(x_1, x_2; \rho) \to u^r(x_1, x_2)$ as $\rho \to 1$ follows from an application of L'Hopital's rule. ■

We call $u(x_1, x_2; \rho)$ in (2.9) an integrated form of contest success function and write $u_\rho(x_1, x_2) := u(x_1, x_2; \rho)$. According to proposition 2.1 an integrated contest success function equals the difference form when $\rho = 0$ and approaches the ratio form as $\rho \to 1$. Skaperdas (1996) shows that a function of the form (2.8) satisfies the desirable axioms of contest success function, where the desirable axioms include monotonicity, anonymity, and independence from irrelevant alternatives (see Skaperdas, 1996, pp.284-286, for the definitions of axioms). Hence by proposition 2.1 we also conclude that the integrated form is a unique function which satisfies the properties of (2.6, 2.7) and the desirable properties of a contest success function, which provides an axiomatic characterization of the new integrated form. Figure 1 depicts the level sets of the integrated form, difference form, and ratio form. As we expect the integrated form describes the intermediate levels of probabilities between the difference form and the ratio form.

Next we consider the probabilistic derivation of the integrated form. We write $X \sim F(s)$ to indicate that the distribution of a random variable, X, is $F(s)$, and recall X follows Gumbel type (type I) extreme value distribution if $X \sim \exp(-e^{-\kappa s})$

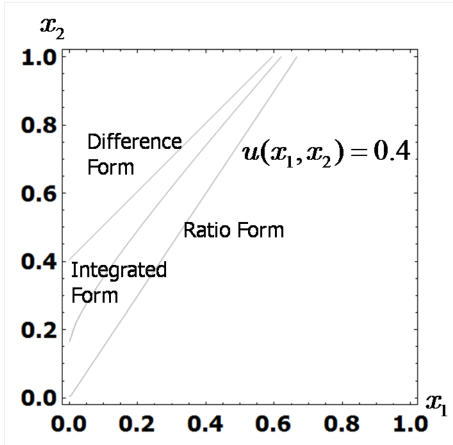

Figure 1. Comparison of contest success functions.

Each line shows the combinations of x_1 and x_2 small at which side 1's winning probability is 0.4. We use the values, $\kappa = 1, \rho = 0.3$.

and Fréchet type (type II) extreme value distribution if $X \sim \exp(-s^{-\kappa})$, where κ is a positive constant. We suppose the result of a contest depends on performance, h_i , and performance is in turn determined by x_i and a random factor is ϵ_i: i.e. $h_i = h_i(x_i, \epsilon_i)$ (Garfinkel and Skaperdas, 2006; Jia, 2008). If the specification of performance is in additive form, $h_i^d = x_i + \epsilon_i$ and ϵ_i follows a type III extreme value distribution, the difference form of the contest success function equals $\Pr\{h_1^d > h_2^d\}$ (McFadden, 1974). A similar derivation for the ratio form was not available until Jia (2008).

Jia (2008) shows that when ϵ_i follows a type I extreme value distribution and the specification of performance is in multiplicative form, $h_i^r = x_i\epsilon_i$,the ratio form of contest success function can be derived from $\Pr\{h_1^r > h_2^r\}$. Now we suppose that $h_i(\rho) = \frac{x_i^{1-\rho}-1}{1-\rho} + \frac{\epsilon_i^{1-\rho}-1}{1-\rho}$. Then it is easy to see that $\Pr\{h_1(0) > h_2(0)\} = \Pr\{h_1^d > h_2^d\}$ and $\Pr\{h_1(1) > h_2(1)\} = \Pr\{h_1^r > h_2^r\}$, where we use notations: $h_i(1) = \lim_{\rho\to 1} h_i(\rho)$.

Since we have $\Pr\{x_1 + \epsilon_1 < x_2 + \epsilon_2\} = \Pr\{\epsilon_1 - \epsilon_2 < x_2 - x_1\}$ for given x_1 and x_2, McFadden (1974)'s results is obtained by showing that $\epsilon_1 - \epsilon_2 \sim \Lambda(s)$ for $\epsilon_i \sim$

$\exp(-e^{-\kappa s})$, where $\Lambda(s) = \frac{1}{1+e^{-s}}$. Similarly because $\Pr\{x_1\epsilon_1 < x_2\epsilon_2\} = \Pr\{\log \epsilon_1 - \log \epsilon_2 < \log x_2 - \log x_1\}$ holds, Jia's derivation is equivalent to showing that $\log \epsilon_1 - \log \epsilon_2 \sim \Lambda(s)$ for $\epsilon_i \sim \exp(-s^{-\kappa})$. Proposition 2.2 provides the generalization of these derivations. In the proposition we use the following definition of the rational power of real numbers, $s^{\frac{n}{m}}$, for m, n natural numbers

$$s^{\frac{n}{m}} := \begin{cases} (\sqrt[m]{s})^n & \text{if } s \geq 0 \\ -(\sqrt[m]{-s})^n & \text{if } s < 0 \end{cases} \tag{2.14}$$

where $\sqrt[m]{s}$, for $s > 0$, denotes a unique positive real number y such that $y^m = s$.

Proposition 2.2 *Suppose that $\rho = 1 - \frac{n}{m}$, m, n are natural numbers such that $m > n$ and $\epsilon_1, \epsilon_2 \sim F(s)$ i.i.d and $F(s) = \exp\left(-e^{-\kappa \frac{1}{1-\rho} s^{1-\rho}}\right)$ for $-\infty < s < \infty$ and $h_i(\rho) := \frac{x_i^{1-\rho}-1}{1-\rho} + \frac{\epsilon_i^{1-\rho}-1}{1-\rho}$. Then*

$$\Pr\{h_1(\rho) > h_2(\rho)\} = \frac{\exp\left(\kappa \frac{1}{1-\rho} x_1^{1-\rho}\right)}{\exp\left(\kappa \frac{1}{1-\rho} x_1^{1-\rho}\right) + \exp\left(\kappa \frac{1}{1-\rho} x_2^{1-\rho}\right)}$$

Proof. From (2.14) we see that $\frac{1}{1-\rho} s^{1-\rho}$ is continuous, increasing and $\frac{1}{1-\rho} s^{1-\rho} \to -\infty$ as $s \to -\infty$ and $\frac{1}{1-\rho} s^{1-\rho} \to \infty$ as $s \to \infty$. Since $\Pr\{h_1(\rho) > h_2(\rho)\} = \Pr\{\frac{1}{1-\rho} x_1^{1-\rho} - \frac{1}{1-\rho} x_2^{1-\rho} > \frac{1}{1-\rho} \epsilon_2^{1-\rho} - \frac{1}{1-\rho} \epsilon_1^{1-\rho}\}$,in the view of McFadden (1974) (or lemma in the appendix) it is enough to show that $\frac{1}{1-\rho} \epsilon_1^{1-\rho} \sim G(s)$, $G(s) = \exp(-e^{-\kappa t})$. Again from (2.14) and the definition of $F(s)$ we have

$$\begin{aligned} \Pr\{\frac{1}{1-\rho} \epsilon_1^{1-\rho} < s\} &= \Pr\{\epsilon_1 < \left(\frac{n}{m} s\right)^{\frac{m}{n}}\} \\ &= \exp(-e^{-\kappa s}) \end{aligned}$$

■

We note that the distribution function $F(s)$ does not possess a continuous density since $F(s)$ is not differentiable at 0. Moreover if $\epsilon_1, \epsilon_2 \sim F_i(s)$ independently and $F_i(s) = \exp\left(-\gamma_i e^{-\kappa \frac{1}{1-\rho} s^{1-\rho}}\right)$, from Lemma 1 in the appendix we have

$$\Pr\{h_1 > h_2\} = \frac{\gamma_1 \exp\left(\kappa \frac{1}{1-\rho} x_1^{1-\rho}\right)}{\gamma_1 \exp\left(\kappa \frac{1}{1-\rho} x_1^{1-\rho}\right) + \gamma_2 \exp\left(\kappa \frac{1}{1-\rho} x_2^{1-\rho}\right)} \tag{2.15}$$

and as $\rho \to \infty$, (2.15) approaches a generalized ratio form (see Jia, 2008) . Thus γ_1 represents the relative fighting effectiveness of side 1 against side 2 (see Dupuy, 1987; Kalyvas, Balcelles, and Rohner, 2008).

2.2.2 *Existence of Pure-Strategy Nash Equilibrium*

Despite the fact that both the ratio form and the difference form have their respective analytical advantages, the ratio form of contest success function is more commonly used since this form admits an interior Pure-strategy Nash equilibrium for the frequently-used conflict model (Garfinkel and Skaperdas, 2006). We study the condition of ρ for an integrated form which allows the pure strategy Nash equilibrium using a simple conflict model in this section (Hirshleifer, 1989; Garfinkel and Skaperdas, 2006). Assume side 1 and side 2 have resources x_1 and x_2, where $x_1, x_2 \in [0, \bar{x}]$ and $\bar{x} \geq 1$, and they compete for a prize of the value $2\bar{x}$, the sum of total available resources. The costs of competing are resources devoted to the contest, so we write the expected payoffs for side 1 and side 2:

$$\pi_1(x_1, x_2) = 2\bar{x}\ u_\rho\,(x_1, x_2) - x_1 \tag{2.16}$$

$$\pi_2(x_1, x_2) = 2\bar{x}\ v_\rho(x_1, x_2) - x_2 \tag{2.17}$$

and suppose $\rho \in [0, 1)$.In the model with u_ρ being replaced by the ratio form u^r in (2.16) and (2.17) , $(x_1, x_0) = (0, 0)$ cannot be a Nash equilibrium since an arbitrary small increase in resources from 0 will raise the winning probability from 0.5 to 1 and hence the marginal winning probability at 0 is infinity (Hirshleifer, 1989). This is one of the main reasons why Hirshliefer criticizes the ratio form: peace is never observed as an equilibrium outcome.

We look for a symmetric interior pure-strategy Nash equilibrium. Denoting such an equilibrium by (x_1^*, x_2^*), we find the first order condition for the interior best response of side 1, x_1^{BR} given x_2.

$$2\kappa\bar{x}\frac{1}{(x_1^{BR})^\rho}u(x_1^{BR}, x_2)(1 - u(x_1^{BR}, x_2)) - 1 = 0 \tag{2.18}$$

At a symmetric equilibrium, $u(x_1^{BR}, x_2) = \frac{1}{2}$. Had we used the difference form instead of the integrated form or set $\rho = 0$, the left hand side of (2.18) would have not depended on x_1. Because of this an interior symmetric equilibrium generally fails to exist in the difference form, and this accounts for the more popular use of the ratio form in the conflict model.

In the integrated form, if a symmetric equilibrium (x_1^*, x_2^*) exists, from (2.18)

$$x_1^* = x_2^* = \left(\frac{\kappa\bar{x}}{2}\right)^{\frac{1}{\rho}} \tag{2.19}$$

To simply the analysis we assume $\kappa\bar{x} < 2$ and $\pi_*''(t) < 0$ for $t \in [0, \bar{x}]$ where $\pi_*(t) := 2\bar{x}\ u\,(t, x_2^*) - t$. The first assumption, $\kappa\bar{x} < 2$, guarantees $x_1^* = \left(\frac{\kappa\bar{x}}{2}\right)^{\frac{1}{\rho}} < \frac{\kappa\bar{x}}{2} < \bar{x}$ and $\pi_*''(t) < 0$ for $t \in [0, \bar{x}]$ ensures that $\pi_*(t)$ achieves the global maximum at x_1^*. With these two assumptions $x_1^* = x_2^* = \left(\frac{\kappa\bar{x}}{2}\right)^{\frac{1}{\rho}}$ is indeed a unique symmetric Nash equilibrium and we note that the solution $x_1^* = x_2^* = \left(\frac{\kappa\bar{x}}{2}\right)^{\frac{1}{\rho}}$ is a generalization of the solution in the case of the ratio form (For example, see equation (10) in Garfinkel and Skaperdas, 2006).

Moreover we verify that $\lim_{\rho\to 0} x_1^* = 0$; in the limiting case approaching the difference form, an interior pure strategy Nash equilibrium converges to 0 and this shows one instance where there is no interior pure strategy Nash equilibrium in the difference form. So we conclude that under reasonable conditions the integrated

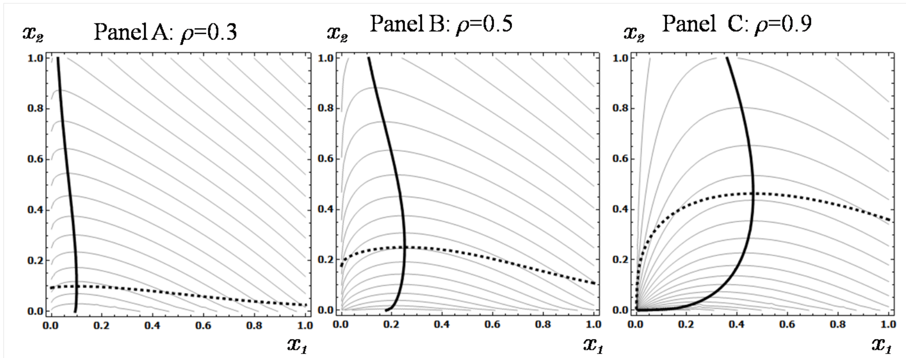

Figure 2. Existence of an interior pure strategy Nash equilibrium.
We draw side 1's best response (thick line), side 2's best response (thick dashed line), side 1's indifference curves (thin line) in each panel. We use values, $\kappa = 1, \bar{x} = 1$. From the shape of indifference curves we see that $\pi''_*(t) < 0$ for all t.

form of the contest success function allows an interior pure strategy Nash equilibria for all $\rho > 0$. In figure 2 we present a numerical example of this analysis.

2.3 Empirical Evidence

Since we do not have an *a priori* answer as to which form of contest success function is more plausible, we conduct an empirical analysis. As arms race and wars are the most important and obvious examples of conflictual contest (for various forms of conflict, see Konrad (2007)), we believe that the estimation of our contest success function using war data would provide meaningful estimates of the parameters, ρ and κ.

2.3.1 Estimation Method

We use battle data from the seventeenth century European wars in Bodart (1908, pp. 49-177) and from World War II in Dupuy (1987, pp. 293-295). Military combat, a violent, planned form of physical interaction between two hostile opponents, has a natural hierarchy: war, campaign, battle, engagement, action, and duel (Dupuy, 1987). Among these we consider two levels of military combat: war and battle. A war is an armed conflict or a state of belligerence usually lasting for months or years while a battle involves combat between two armies with specific missions, normally lasting one or two days. The seventeenth century European wars data covers 315 battles with each battle corresponding to one observation in our data. Each observation has a record of the winner, the loser, and the total numbers of personnel in winning and losing armies.

We note that data on each battle gives two pieces of information: the winning probability of the winner and the losing probability of the loser. Because of this difficulty in interpretation we consider two constructions of the data set from the original battle data. In the first construction − the case presented in the text − we associate each battle data with either a winning event or a losing event depending on a random draw from a fair coin. Alternatively we expand the original battle data by associating both winning and losing events to each battle, hence obtaining a new data set with 630 observations. In this case, presented in the appendix, we correct the standard errors by clustering battles. We provide descriptive statistics for European battles in the appendix.

Denoting the indicator of winning by y_i we use the following econometric model:

$$\begin{aligned} & y_i && \sim && \text{Bernoulli}(\pi_i) && (2.20) \\ \textbf{(D)} \ & \pi_i && = && F(\kappa(x_{1i} - x_{2i}) + \beta_1 + \beta_2 D_{army\ i} + \beta_3 D_{war\ i}) \\ \textbf{(R)} \ & \pi_i && = && F(\kappa(\ln x_{1i} - \ln x_{2i}) + \beta_1 + \beta_2 D_{army\ i} + \beta_3 D_{war\ i}) \\ \textbf{(I)} \ & \pi_i && = && F(\eta(x_{1i}^{1-\rho} - x_{2i}^{1-\rho}) + \beta_1 + \beta_2 D_{army\ i} + \beta_3 D_{war\ i}) \end{aligned}$$

where $F(s) = \frac{1}{1+e^{-s}}$, D_{army}, D_{war} are dummy variables indicating the identity of armies and the kind of wars (see appendix).

The specifications of models in equation (2.20) are the direct consequence of proposition 2.2 and dummy variables control for combat effectiveness (or ineffectiveness) due to the identity of armies or the specificity of the war. Indeed using $F(s)$ and $\eta = \frac{\kappa}{1-\rho}$ we can write model (**I**) as

$$\pi_i = \frac{\exp^{\beta_1+\beta_2 D_{army\ i}+\beta_3 D_{war\ i}} \exp(\kappa \frac{1}{1-\rho} x_{1i}^{1-\rho})}{\exp^{\beta_1+\beta_2 D_{army\ i}+\beta_3 D_{war\ i}} \exp(\kappa \frac{1}{1-\rho} x_{1i}^{1-\rho}) + \exp(\frac{1}{1-\rho} x_{2i}^{1-\rho})}$$

so we can regard the part $\exp^{\beta_1+\beta_2 D_{army\ i}+\beta_3 D_{war\ i}}$ as normalized γ_i, $\frac{\gamma_1}{\gamma_2}$, in (2.15). We also note that model (**D**) is a standard logit regression and model (**R**) is a logit regression with the data log-transformed. So model (**D**), (**R**), and (**I**) estimate the difference form, the ratio form, and the integrated form, respectively. We estimate each parameter using the maximum likelihood method, which is the standard method in estimating logit models. We could not estimate the difference form and the integrated form in the case of World War II data since the data provides only the ratio of combat powers.

2.3.2 *Estimation Results*

In table 1 we note that all coefficients of κ in the difference and the ratio forms are estimated to be positive, which shows that one side's winning probability is an increasing function of that side's own effort. For the integrated form we can recover implied $\kappa = 2.28009$, using the relation $\kappa = (1-\rho)\eta$. To compare κ's in each model we compute one side's marginal winning probability at even matching when the number of combatants is half of its total available resources ; i.e. if we denote the total available resources by $\bar{x}$, this marginal winning probability is $\frac{1}{4}\frac{\kappa}{(\bar{x}/2)^{\rho}}$. Since the mean number of combatants in the seventeenth century European war data is 21,035 (see appendix), we can use this number as a proxy for $\frac{\bar{x}}{2}$. We may interpret these numbers as follows: in response to an increase of 10,000 in

	17C European War			World War II
	Difference	Ratio	Integrated	Ratio
κ	1.98×10^{-5}	0.70377		3.41982
	(9.32×10^{-6})	(0.120365)		(0.6776)
η			-18.19199	
			(6.4571)	
ρ			1.125335	
			(0.21571)	
Number of Observations	315	315	315	188
Percentage of Correctly Predicted	65.40	67.62	67.30	84.04
Log-likelihood Value	-200.8481	-188.23251	-188.0628	-70.6855

Table 1. Estimation of contest success functions.
All estimates are significant at 99% level. We use dummy variables of armies and wars in European war estimation and dummy variables of armies indicating either Allied forces or German forces in World War II data. Standard errors are corrected for heteroskedasticity.

the number of combatants (from 21,035 original combatants), the difference form, the ratio form, and integrated form predict increases in the winning probabilities by 4.95%, 8.36%, 7.78% respectively. In the case of World War II, using the fact that the average strength of battles is around 14,000 (Dupuy, 1987, pp. 169) we compute a marginal winning probability $\times 10^4$ as 2.4472 (or 244.72%) which is a much larger number than those of the European wars. This fact suggests that the contest success function for World War II is more non-linear than the one for

	D	R	I
marginal winning probability $\times 10^4$	0.0495	0.08364	0.077827

Table 2. Estimates of the mass effect parameter

the seventeenth century European wars and "[T]he tremendous advantage of being even just a little stronger than one's opponent", which is pointed out as one of the stylized facts of warfare by Hirshleifer (1991, p 131), only appears in World War II data.

Which form of contest success functions better describes battle? As we see in Table 1 and the Appendix, our tentative conclusion would be that a contest success function close to the ratio form would best describe the winning probabilities of battle. Of course the peculiarity of 17th century European wars or other possible data problems may have hindered the correct estimation of our model. This problem, if it exists, can be corrected by extending data sets to cover other kinds of wars or other forms of conflicts.

2.4 Discussion

In the chapter we have proposed an integrated contest success function which has the difference form and the ratio form as limiting cases. Also we have derived this new form from a constant elasticity of augmentation and provided a probabilistic derivation. These results give a generalization of the existing results. In addition we have shown that the integrated form has desirable analytical properties which admit an interior pure strategy Nash equilibrium. Regarding the question of a plausible specification of contest success functions, a tentative conclusion is that the seventeenth century European wars better fit the ratio form of contest success functions.

Another way of interpreting the integrated contest success function is the transformation of variables. Since we do not know the exact unit of measure for fighting effort or resources in various conflict situations, we may interpret the problem of choosing contest success functions as a transformation of observables into variables with correct measurement. In this interpretation, as we have seen in the text, the difference form with the log transformation corresponds to the ratio form. More generally, an integrated form of contest success functions arises from a transformation $X_i = \frac{x_i - 1}{1-\rho}$ and the difference form.

CHAPTER 3

LARGER GROUPS MAY ALLEVIATE THE COLLECTIVE ACTION PROBLEM

3.1 Introduction

Provisions of public projects in a community, revolutionary activities to overthrow corrupt and inefficient governments, and more generally various actions that members of a group take to achieve a common goal have been examined extensively by social scientists under the name of collective action. Group size has played an important role in explaining collective action. A standard argument put forward by Olson (1965) asserts that a larger group faces more difficulties in achieving a common goal compared to a smaller group, because of an aggravated free-rider problem: "unless the number of individuals in a group is quite small ... rational, self-interested individuals will not act to achieve their common or group interests (Olson, 1965, p.2)"

Since then, studies of the relationship between group size and the provision of collective goods have been conducted by various researchers (Chamberlin, 1974; Marwell and Pamela, 1993; McGuire, 1974; Oliver and Marwell, 1988; Sandler, 1992; Agrawal and Goyal, 2001; Esteban and Ray, 1999). Chamberlin (1974) emphasizes the distinction between goods with perfect nonrivalness ("inclusive" goods in Olson's terms) and goods with rivalness of consumption ("exclusive" goods) among nonexcludable goods. With this distinction he argues that Olson's claim that the larger group would provide fewer public goods only holds for goods with perfect rivalness. In the case of goods with non-rivalness, Chamberlin shows that as group size increases, the amount of total contribution, in absolute terms, would increase. This view that the Olson thesis holds when the collective good is private but may be reversed when the good is purely public, initiated by Chamberlin and substantiated by others (Chamberlin, 1974; Marwell and Pamela, 1993; McGuire, 1974; Oliver and Marwell, 1988; Sandler, 1992) is described as "common wisdom" by Esteban and Ray (2001)

Esteban and Ray (2001), criticizing the notion that this common wisdom only applies to the extreme case of "purely public" (perfect nonrivalness), examine group size effect using a model with explicit intergroup conflicts. In particular they show that under plausible assumptions about costs, the winning probabilities of a large group is greater than that of a smaller group even if the prize is purely private. However, the context in which they examine collection action − competition between several groups − may be more relevant in some instances, but it is neither a general situation nor the situation that Olson considers. Also, since the notion that a group provides less amounts of collective goods in absolute terms does not necessarily coincide with the fact that a group provides a suboptimal amount of collective goods, the "common wisdom" view is not a complete antithesis of Olson's arguments.

A variety of empirical or experimental studies have also examined the group size hypothesis (Isaac and Walker, 1988; Bagnoli and McKee, 1991; Isaac, Walker, and Williams, 1994; Hann and Koorema, 2002; Carpenter, 2007) and many of them find that "the size of a group is positively related to its level of collective action" (Marwell and Pamela, 1993, p.38). Hann and Koorema (2002) use data from their study of a candy bar honor system in 166 firms in the Netherlands and find evidence that free riding decreases with group size. Carpenter (2007) tests the group size hypothesis when punishment is allowed and finds that "large groups contribute at rates no lower than small groups because punishment does not fall appreciably in large group" (Carpenter, 2007, p.31) In sum, even though various empirical and experimental studies suggest that large groups may perform better, few theoretical works provide the logic and reasoning of how larger groups can overcome their aggravated free-rider problem. So the question of the relationship between group size and group's performance in collective action still remains unanswered.

An interesting connection between group size and collective action has been raised in evolutionary biology literature in the context of the use of punishment to explain the evolution of human cooperation (Bingham, 1999, 2000). Paul Bingham uses Lanchester's square law to claim that humans' ability to kill from a distance decreases the cost of punishment. The crux of Bingham's argument is that as the remote killing ability develops a large number of punishers can attack a single "cheater" simultaneously, and hence the cost of punishment decreases exponentially:

> When a large number of individuals - say n - simultaneously attack a single target, the risk to each is reduced by a factor of n because the target is incapacitated about n times faster. Moreover, during this n-fold shorter conflict, the risk to each attacker is further reduced by a second factor of n because the risk of return fire from the target is distributed across n attackers. Thus, the total risk to n remote attackers

is reduced by n^2 (Bingham, 2000, p.249).

Although Bingham's point is valid in the situation where the large number of punishers face a single cheater, the Lanchester effect − the effect derived from Lanchester's square law and explained in the quote − depends on the existence of large number of punishers. Because the number of punishers is not always large in a large group, it is not clear whether the same argument can carry over to the collective action problem with punishment in the large group. In addition, even though the remote killing ability is developed, when the same number of punishers and cheaters engage in conflicts, there is no reason to expect the cost of punishment to decrease exponentially. Thus the remote killing competence is a necessary condition for the large group effect, but not sufficient; we need to impose some conditions to ensure that a larger number of punishers confront less cheaters in fighting such as the collective punishment by punishers (Boehm, 1982).

To explore these questions we develop a simple model of $n-$prisoner dilemma with punishment, which combines the standard evolutionary model of three behavioral types − cooperator, punisher, and defector − with the Lanchester-type conflict between punishers and defectors (Bowles and Choi, 2002; Bowles and Gintis, 2004; Bingham, 1999, 2000; Panchanathan and Boyd, 2004; Sethi and Somanathan, 2006). In the model we introduce a parameter to capture the degree of collectiveness among punishers who engage the defectors, and study the conditions for large-group advantage. We find that group size has a positive effect in supporting higher levels of cooperation, so that collective action is more likely to be successful, as we expect from Bingham's argument. More interestingly we show that as long as the defector type is, even slightly, less collective than the punisher type, the large group effect pervades. Thus, we may conclude that the large group effect is quite robust, considering that the defector type, because of its behavioral disposition, would be reluctant to engage in any type of collective action.

Interestingly Esteban and Ray (2001) emphasize the role of "divide-and-conquer" which captures exactly the notion that the defector is less collective than the punisher in the punishment process: "Political entities have applied this rule [divide-and-conquer] with surprising universality, but if smaller groups are more potent, the division of one's opponent into a number of smaller units would entail more effective opposition (Esteban and Ray, 2001, p.664)"

The paper is organized as follows. In section 2 we briefly review the existing research on group size and collective action. Section 3 reminds readers of Lanchester's equations and Lanchester's square law. We present a model and results as well as numerical simulations of the model in the section 4. In section 5 we discuss implications and extensions of the model.

3.2 Lanchester's Law and an Illustrating Example

We imagine that x combatants of army A engage army B with y combatants. Lanchester's equations (Lanchester, 1916, p.20) read

$$\frac{dx}{dt} = -\kappa y, \quad \frac{dy}{dt} = -\phi x \tag{3.1}$$

where κ and ϕ denote the fighting effectiveness of each army. Equation (3.1) is derived from the assumption that the number of persons knocked out per unit time is directly proportional to the numerical size of the opposing force; during each per unit of time, Δt, the opposing force of magnitude y concentrates on the elimination of Δx, so $\Delta x = y\Delta t$. Engel (1954) and Samz (1972) verify the

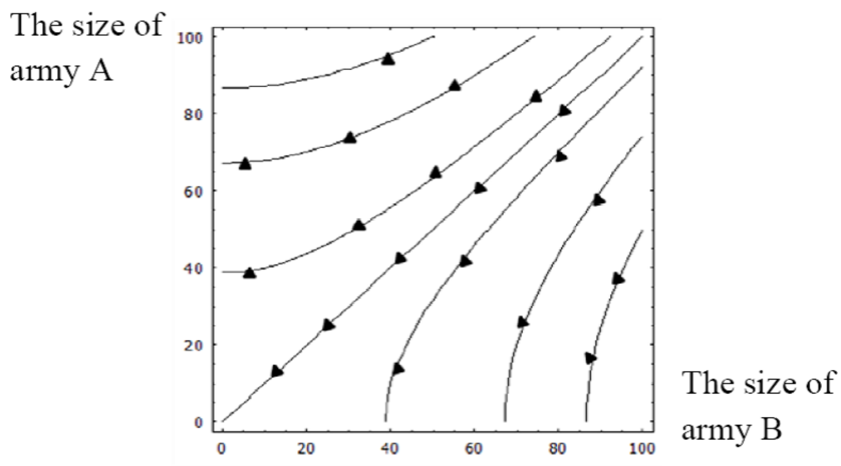

Figure 3. Solution curves for Lanchester's equation.
Each line corresponds to each solution of Lanchester's equation. We set $\phi = \kappa = 1$

validity of Lanchester's equation in an actual combat situation where U.S. forces captured the island of Iwo Jima during World War II. This system (3.1) is called a Hamiltonian system and the equation for solution orbits is given by

$$H(x, y) = \frac{\phi}{2}x^2 - \frac{\kappa}{2}y^2, \tag{3.2}$$

so if we evaluate $H(x, y)$ at the solutions of ODE the value of (3.2) only depends on the initial values, $H(x^*, y^*) = \frac{\phi}{2}x_0^2 - \frac{\kappa}{2}y_0^2$. In this way we obtain the phase diagram (see figure 3) which describes the time path of each solution.

We adopt the criterion that one army wins the battle if the other army vanishes first; i.e. army A wins the battle if and only if $\phi x_0^2 > \kappa y_0^2$ where x_0 and y_0 are initial values for x and y. Similarly, if army B divides evenly and, accordingly, army A engages twice with half of original army B $y_0/2$, army A wins both battles if and only if $\phi x_0^2 - \kappa \left(\frac{y_0}{2}\right)^2 > \kappa \left(\frac{y_0}{2}\right)^2$, where the left hand side represents the square of remaining combatants in army A after the first engagement. In general when army B is divided by n we obtain the following rule:

$$\phi x_0^2 > \kappa \frac{y_0^2}{n} \tag{3.3}$$

Now consider a population which consists of defectors, punishers, and possibly cooperators. For concreteness, suppose that 50% of the population are defectors. We assume that the defectors behave individually (divided by n), so $y_0 = n$ for simplicity. A punishment process – where the punishers eliminate defectors and defectors counteract – is described by Lanchester's equations with $\phi = \kappa$. Then equation (3.3) is reduced to $x_0 > \sqrt{y_0}$, where x_0 and y_0 denotes the initial number of punishers and defectors respectively. First consider the case in which population size is 20. Since 50% of the population are defectors, or equivalently 10 are defectors in the population, we need 3.1328 punishers – ignoring the integer problem – to eliminate defectors. However, if the size of the population is 200, only 10 punishers is enough for 100 defectors. In other words, when group size is 20, 15% of the population must be punishers in order to get rid of the defectors, whereas in a group of size 200 5% of the population will be enough to do this. In short the technology of punishment exhibits increasing returns to group size.

3.3 Model and Analysis

3.3.1 Model

Consider a population of size n playing a public good game. We suppose that each member in a population – identified with a player in a public good game – can choose to be one of three types: cooperator, punisher, and defector. Punishers (P) contribute to the public project and punish defectors, defectors (D) do not contribute to the public good, and finally cooperators (C) do not punish, but only contribute to the public project. A member chooses types taking account of the effect of this choice on costs he incurs because of ensuing conflicts, which we call a punishment process. In the punishment process punishers and defectors have a series of engagements described by Lanchester's equations following (Bingham, 1999, 2000). With the notation $E_i = 1$ if i contributes and $E_i = 0$ otherwise, member i's expected payoffs of each type or evaluation of utility upon adopting each type reads

$$\pi(P) = \frac{b}{n} \sum E_j + \frac{1}{n} - c - d \Pr(I_P) \tag{3.4}$$

$$\pi(D) = \frac{b}{n} \sum_{j \neq i} E_j - s \Pr(I_D) \tag{3.5}$$

$$\pi(C) = \frac{b}{n} \sum E_j + \frac{1}{n} - c \tag{3.6}$$

where b denotes a benefit from the public project, $\frac{1}{n}$ is the marginal contribution of contributing type, and c is the cost of contribution. We suppose that $c < b < nc$, so in the absence of punishment all members' contributing is socially optimal while none of them have enough material motivation to do so when n is sufficiently large. Term $\Pr(I_P)$,which we will specify precisely later, represents the probability with which punisher i would be injured or hurt during the punishment process and in which case he pays the cost d. Similarly defector i needs to pay s with the probability $\Pr(I_D)$, the probability of defector's being injured, and we assume that $s > c$ so the cost that the defector pays in case of being injured $-$ for example a cost for the recovery of injury or a forgone income from the exclusion of productive activities because of injury $-$ is greater than per-period contribution cost. We note when $d = s = 0$ no punishment takes place and payoffs replicate the n-prisoner dilemma.

Though we use the language of public goods problems such as cooperator, punisher and defector, we observer that this setting can be readily extended to the sit-

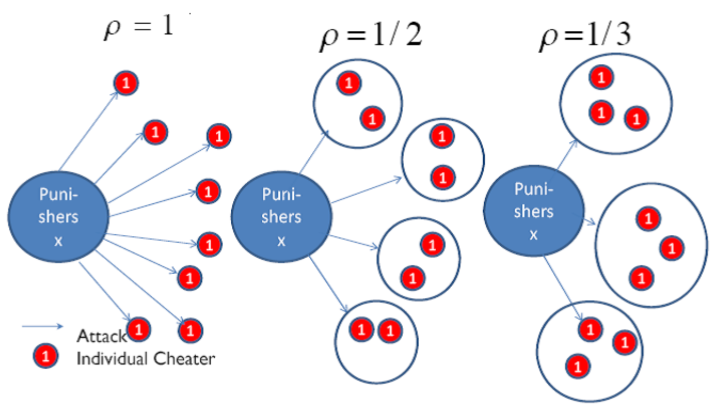

Figure 4. Defectors' tendency to act collectively in the punishment. $1/\rho$ represents the number of defectors who act together in the punishment process. When $\rho = 1$ all defectors behave individually in the punishing process and as $\rho \to 0$ cheaters behave more collectively.

uation of political collective actions (Tullock, 1971; DeNardo, 1985; Epstein, 2002). In the context of revolutionary activities to overthrow a corrupt and oppressive government, this setting models an individual's choice from among three activities: join the revolutionaries (punisher), join the force of repression (defector), or remain inactive (cooperator). With these name changes (3.4)∼(3.6) reproduce a similar specification of expected payoffs that Tullock (1971) used in his study of "paradox of revolution".

We proceed to specify terms $\Pr(I_P)$ and $\Pr(I_C)$ using Lanchester's theory. First we introduce a parameter ρ to describe the degree of collectiveness of the defectors in the punishment process:

$$\rho = \frac{1}{\text{the number of defectors who counteract together}}$$

Figure 4 illustrates this parameter schematically.

For instance, when the number of punishers and the defectors are x and $y = 2021$ and $\rho = 1/1000$, from a similar calculation to that presented in section 3 the condition for the punishers to defeat "the army of" defectors is as follows:

$$x^2 \geq 2 \times (1000)^2 + 21^2 = [0.001 \times 2021] \times (\frac{1}{0.001})^2 + (2021 - 2000)^2$$

where $[x]$ denotes the integer part of x. The first term, $2 \times (1000)^2$, indicates that the army of punishers faces 1000 defectors twice and then competes with the remaining 21 defectors. Thus in general we have the following condition for the punishers to annihilate all defectors from the population:

$$\phi x^2 \geq \kappa[\rho y] \left(\frac{1}{\rho}\right)^2 + \kappa(y - [\rho y]\frac{1}{\rho})^2 \tag{3.7}$$

where $\rho > 0$. Since a type is more likely to be injured or knocked out if the result of the punishment process is close to the defeat of that type, we suppose that $\Pr(I_D)$ and $\Pr(I_P)$ monotonically depends on (3.7). In particular we suppose that

$$\begin{aligned}\Pr(I_D) &= F(\phi x^2 - \kappa[\rho y] \left(\frac{1}{\rho}\right)^2 - \kappa(y - [\rho y]\frac{1}{\rho})^2) \\ \Pr(I_P) &= 1 - F(\phi x^2 - \kappa[\rho y] \left(\frac{1}{\rho}\right)^2 - \kappa(y - [\rho y]\frac{1}{\rho})^2)\end{aligned}$$

where $F(t)$ increasing, $\lim_{t\to\infty} F(t) = 1$, $\lim_{t\to-\infty} F(t) = 0$. For example, we have $F(t) = \mathbf{1}_{[0,\infty)}(t)$, or $F(t) = 1$ if $t \geq 0$, $= \exp(\beta t)$ if $t < 0$.

3.3.2 Static Analysis

Using the model developed we ask two questions: 1. How does an increase in group size change π_P, π_D,and π_C given a generation? 2. How does an increase in group size affect the long-run proportion of each type in the population when individuals update their types? The first question addresses the static characterization of the model, while the second one refers to the dynamic properties of the system induced by the model. These two are closely related as the standard result in game theory suggests – for instance the strict Nash equilibrium in the underlying game is an evolutionary stable strategy, hence the asymptotically stable state in the replicator dynamics (See Weibull, 1995). Concerning the first question we have proposition 3.1.

Proposition 3.1 *Suppose that* $\alpha = \frac{x}{n}, \beta = \frac{y}{n}$, *and* $s > c$. *Then for all* $\rho > 0$, $\alpha > 0, \beta > 0$, $\lim_{n\to\infty}(\pi_P - \pi_D) > 0$ *and* $\lim_{n\to\infty}(\pi_C - \pi_D) > 0$

Proof. From the definition of $\Pr(I_D)$ we have

$$\begin{aligned}\Pr(I_D) &= F(\phi\alpha^2 n^2 - \kappa[\rho\beta n]\left(\frac{1}{\rho}\right)^2 - \kappa(n\beta - [\rho n\beta]\frac{1}{\rho})^2) \\ &\to 1 \text{ as } n \to \infty\end{aligned}$$

Then $\pi_C - \pi_D \geq \pi_P - \pi_D \to -c + s > 0$. ■

As we see in the proof we have $\pi_C - \pi_D \geq \pi_P - \pi_D$,thus whenever $\pi_P > \pi_D$ playing D is strictly dominated by both strategies C and P. Figure 5 below characterizes the combinations of population proportion $(\alpha, \beta, 1 - \alpha - \beta)$ which support $\pi_P - \pi_D > 0$.

In each panel of figure 5 the shaded regions show the population state where playing defect is strictly dominated by punishers, and hence by cooperators. This shows the significance of the size of the group. On the other hand when x, y belong to the unshaded regions, we have $\Pr(I_P) \approx 1$, $\Pr(I_D) \approx 0$, so $\pi_D > \pi_P$ and $\pi_D > \pi_C$ for large n; playing defect is individually rational which replicates the conventional argument for large groups. Because of these payoff structures we may regard the shaded regions, in a suitable dynamic process, as basins of attraction for some equilibrium consisting of punishers and cooperators; white regions corresponds to the basins of attraction for an all-defectors equilibrium. When the size of the group increases, the shaded region enlarges; being defector becomes less favorable. The

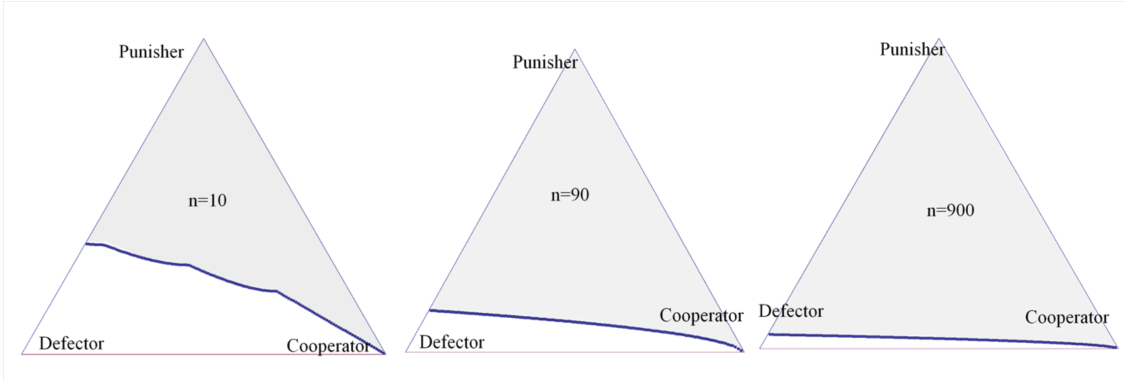

Figure 5. The fractions of population which supports punishment. Each point in the triangle uniquely corresponds to one population state, composed of fractions of each type, through the Bary centric coordinate. For example, the point located on the left bottom vertex corresponds to a population state in which all individuals choose the defector type. The points in the shaded area are population states which ensure $\Pr(I_P) = 1$, so $\pi_P - \pi_D > 0$. When $\Pr(I_P) = 1$, $\pi_P - \pi_D > 0$ so in the shaded region playing defect is strictly dominated. $\rho = 0.5, \kappa = \phi, F(t) = 1_{[0,\infty)}$ are used.

static analysis of the payoff structures and the basins of attraction strongly suggests that cooperation would be supported in the long run.

3.3.3 *Dynamic Analysis*

We consider a state space $\Xi_n = \{(x, y, z) \in \mathbb{R}^3 | \, x+y+z = n\,\}$, which is a subset of a simplex in $\mathbb{R}^3$. Assuming that the state at the end of period of t is (x, y, z) we can write $\pi_P(x, y) := \pi_P$, $\pi_D(x, y) := \pi_D$, and $\pi_C(x, y) := \pi_C$ to emphasize the dependence of payoffs, in particular $\Pr(I_P)$ and $\Pr(I_D)$, on x, y. During the period $t + 1$

D1 A proportion of individuals is drawn from the population at random.

D2 With probability $(1 - \epsilon)$ for $\epsilon \in (0, 1)$, the individuals draw choose types according to the following switching rule:

$$\text{type } i \text{ switch to type } j \text{ if } j \in \arg\max_k \pi_k(x, y)$$

whenever there is a tie between target strategies, an individual is assumed to choose one randomly from these strategies.

D3 With probability ϵ, individuals choose types randomly and the system moves into the next period.

D2 is called a best response update (Young, 1998; Kandori, Mailath, and Rob, 1993) and the specification of stochastic dynamics follows Young (1998) except D1; instead of drawing one individual as Young (1998) does, we draw a given proportion of individuals. If we draw one individual at each period, the convergence speed of the system to an equilibrium will slow down as n increases, so some positive level of punishing and cooperating behaviors may persist due simply to the sampling method. Since we wish to control this artifact and single out the large group effect from the irrelevant aspects of the modeling, we allow a proportion of individuals to update their strategies. In step 3, D3, we allow for the possibility of idiosyncratic behavior such as mistakes by individuals in choosing their best response strategies following the standard evolutionary model (Young, 1998).

Since the independent randomness, which arises each period both by D1 and D3, accumulates in the state of the system through time, the system follows a Markov process and the standard limit theorem for the finite state space applies. In particular D3 makes the chain irreducible and aperiodic so we have a unique invariant distribution μ. Since we are interested in the long run equilibrium value of population fractions, playing cooperators, punishers, and defectors, we estimate $\lim_{t\to\infty} E(X_t)$, $\lim_{t\to\infty} E(Y_t)$,and $\lim_{t\to\infty} E(Z_t)$ using a Monte Carlo simulation (Madras, 2002). As we do not know an invariant distribution we take the all-defectors state as an initial state. The state is least likely to support a high level of cooperation in the system in the long run.

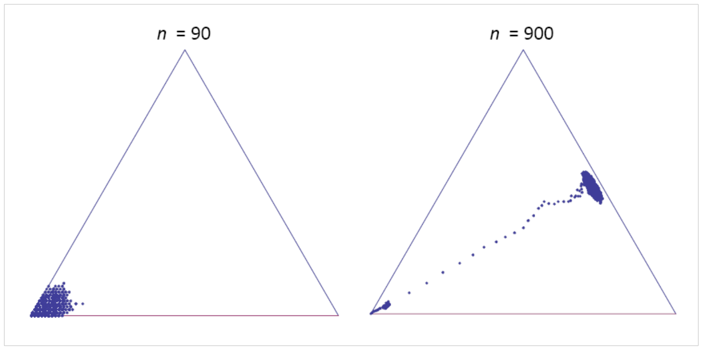

Figure 6. The fractions of population in each period.
Each point is each state in the simulation. The initial values of states are taken as $\alpha = 0, \beta = 1, \gamma = 0$. 10% of population are drawn at each period. Other parameters used are $\kappa = 1, \phi = 1, \rho = 0.5, \varepsilon = 0.1, b = 10, c = 2, d = 3, s = 3, T = 10000$.

Figure 6 depicts trajectories of the states of the system. In the first panel, where the size of population is relatively small, the population state starting from all defectors stays close to the all-defectors equilibrium. This may capture the situation in which all individuals are trapped in the basin of attraction of the all-defectors equilibrium in figure 5. The second panel shows the case where, as the basin of attraction for the all-defectors equilibrium shrinks, individuals in the population manage to escape and hence the higher level of cooperation is supported. The estimates in table 3 below show the large group effect more precisely.

To estimate the mean fractions and construct the confidence intervals, we follow the batch means method (See Madras, 2002) and choose 25 batches. Also to

	$n = 90$			$n = 900$		
	Punisher	Defector	Cooperator	Punisher	Defector	Cooperator
fraction	**0.03362**	**0.93242**	**0.03395**	**0.48231**	**0.033359**	**0.484325**
95% Conf.	[0.031855	[0.929807	[0.032531	[0.480872	[0.03275	[0.48274
Interval	0.0353861]	0.935051]	0.035368]	0.48376]	0.033960]	0.48591]

Table 3. Estimates of mean fractions of population
Estimates of mean fractions of population and 95% confidence interval.The parameters used are the same as figure 2

avoid the initialization bias the first five batches have been dropped and Table 3 confirms the large group effect. The main reason is already explained: as the size of population grows, it becomes easier to prevent the proliferation of defectors with a smaller fraction of punishers.

3.4 Discussion

We note two features of the model. When $\rho \to 0$, the above argument fails to hold in the limit. This is because if $\rho \to 0$, meaning the defectors behave as collectively as the punishers do, the punisher cannot exploit Lanchester's law. However proposition 3.1 does hold for all $\rho > 0$. As long as ρ is strictly greater than 0 (even if it is very close to 0) or the defector tends to behave less collectively, the punisher will always enjoy large-group advantages. In addition the result does not depend on the magnitude of d. This means that, however high the cost of punishment is, an increase in group size is always in favor of being a punisher or a cooperator, as the cost of punishment decreases to 0 as n increases. This fact suggests that the second-order free rider problem can be reduced by the size of the group (Panchanathan and Boyd, 2004).

In sum we show that if the punishment process is well described by Lanchester's equations, larger groups may favor cooperation. Of course we do not assert that larger groups always perform better than smaller groups; we provide one possible mechanism through which group size can enhance performance in collective action. We ignore the information costs and coordination problems that larger groups may suffer. An appropriately modified theory, we believe, would provide some keys to interesting questions of group size and collective action such as the determination of the optimal size of groups.

CHAPTER 4

CLASS ALLIANCES AND CONFLICTS: AN EXPLANATION OF POLITICAL TRANSITIONS

4.1 Introduction

Since Karl Marx defined class in terms of ownership and control of the means of production, many scholars have adopted class as a unit of analysis. Notably Bowles (1984) studies how the economic relations among various classes can give rise to class alliances and class conflicts.

> Similarly, if renters exchange some of their r-good [agricultural good] income for c-goods [manufacture good], they have a common interest with landlords in the relative price of r-goods. Not surprisingly, when tariff debates have dominated political discourse and organization, as in Germany before World War I, renter-landlord alliances have been common (Bowles, 1984, p.113).

Social scientists, such as Gerschenkron (1943), Moore (1965), Kindleberger (1951), Gourevitch (1977) and Luebbert (1991), have emphasized the role of class alliances and conflicts, or more generally coalition formation among various classes in explaining important institutional changes in the late nineteenth-century Europe. According to Moore (1965), in Germany, the Junkers successfully formed an alliance with independent peasants and capitalists in big industries and repressed industrial workers. Moore's theory emphasizes the success or failure of compromise between the ruling classes and the role of the peasant in political transitions. Class alliances or conflict between different classes can delay democratization or precipitate it.

The working class in nineteenth-century German society was stronger and better-organized than those in other countries, as has been suggested by Nolan (1986):

> Although Britain experienced the first industrial revolution and France developed the first significant socialist associations, Germany produced the largest and best-organized workers' movement in the late nineteenth century. By the mid-1890s, German social democracy had successfully built a mass party and a centralized trade union movement in spite of – or it could be argued, because of - its espousal of deterministic Marxism, its practice of ambivalent parliamentarianism and its isolation from the state and much of society (Nolan, 1986, p.352).

The question of coalition formation is more interesting if one asks how such a strong working class was excluded from the major coalition, the so-called *solidarity bloc*, and ended up being politically unsuccessful. In terms of institutional changes, Acemoglu and Robinson raise a similar question: "Why in the nineteenth century, [did] Germany, the country with the most developed socialist party at that time, institute the welfare state without franchise extension, while Britain and France extended the franchise?" (Acemoglu and Robinson, 2000, p.1176).

While the literature, by emphasizing the role of class alliances and conflicts, provides comprehensive historical description of coalition formation among classes and its implications for the ensuing political transitions, few of these studies provide a formal model to explain the coalition formation of classes. This paper proposes a simple model of coalition formation among four players who have common interests and conflicts with each other.

As Basu (1986) points out, economic and political problems have been examined primarily within the context of a dyadic relationship, i.e. between two actors. However, as history of coalition formation in the nineteenth century European countries shows, interests and conflict among four classes can be intertwined; the higher rate of overall exploitation would be of common interest to the capitalist class and the landed class, whereas these two classes may have a conflict over the relative price of a industrial product to an agricultural good. The working class and the peasant class can have similar relationships. We model this tetradic relationship of common interests and conflicts using the coalition game.

By explicitly introducing a parameter that reflects changes in economic conflict − such as dramatic changes in trade conditions induced by sudden inflow of cheap agricultural products from America in the nineteenth century − we study the conditions for forming various coalitions. Specifically we show that when the economic conflicts over tariffs and the rate of appropriation escalates and one class is politically superior to others, the exclusion of that class might occur, so the originally strong class can end up being disadvantaged. Though in general the initially more advantaged group is believed to remain more successful economically and politically, this need not be the case when the various common interests and conflicts cross and intertwine. The checks and balances among the four classes may lead to this paradoxical outcome.

A similar result is known as the "segregation of major player" in cooperative game theory (Von Neumann and Morgenstein, 1944) and the "paradox of voting power" in the voting literature (Deegan and Packel, 1982), but none of these theories addresses the conflictual aspect of players. In contrast to these models we show that the interaction between cooperation and conflict among players in the formation of coalitions yields this paradox. We empirically examine the coalition formation in nineteenth century Germany and England and assert that the historical formation of coalitions is congruent with the predictions of the model. Provided that the solidarity bloc is anti-democratic, through coalition formation this result also shows a possible link between trade policy and democracy, which is a central theme of Geschenkron's *Bread and Democracy* (Gerschenkron, 1943). Section two proposes a model and in section three we explain the method of finding equilibrium coalition structure and present the main results. Historical reviews follows in section four and section five summarizes the chapter.

4.2 Model

When two different categories of groups are considered, subgroups within these groups may have both common interests and conflicts. For example, the divisions of a society into ruling class versus ruled class, and into urban population versus rural population, gives rise to relationships among four subgroups, in which each has both a common interest and conflict with another. Concretely we consider the capitalist class, the working class, the landlord class, and the peasant class having common interests and conflicts described by figure 7.

In a society consisting of four classes – a capitalist class, a landlord class, a working class, and a peasant class – we suppose that there are two modes of

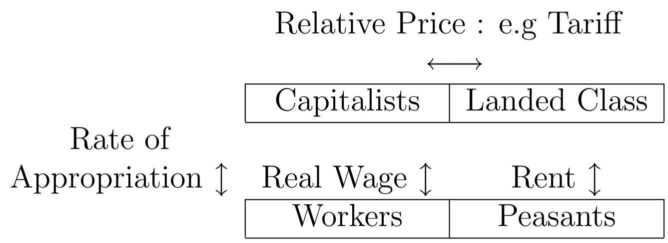

Figure 7. Class structure

production, or two distinct production sectors: the manufacturing sector and the agricultural sector. In the manufacturing sector the capitalists hire the workers, while the landlords hire the peasants in the agricultural sector and the two sectors trade their outputs. We suppose that the wage norm and the relative price between the outputs of the two sectors are the major determinants of the payoffs of each class. The wage norm ω is a proxy for the (inverse) rate of appropriation (or exploitation) and the relative price refers to the ratio of the price of agricultural products to the price of the industrial goods. The reason for workers and peasants to have a common economic interest in raising ω is that the labor market for agriculture and manufacturing are related through the pool of the unemployed. By the working of the pool of the unemployed or "reserve army of labor", the wage norm ω determines the overall rate of exploitation in a society and the workers and the peasants have a common interest in raising ω. Similarly τ, measuring the relative expensiveness of the agricultural products, yields the conflicting interests between the manufacturing sector and the agricultural sector. We summarize the sources of common interests and conflicts in table 4.

We reasonably believe that these kinds of relationships is common in social life. For example, consider females versus males and Republicans versus Democrats; a female Republican may share similar political opinions with a male Republican, but they may conflict on gender issues. Similarly, a female Republican may have

	Common Interest	Conflict
Capitalist and Landlord	Low ω	τ
Capitalist and Worker	Low τ	ω
Worker and Peasant	High ω	τ
Landlord and Peasant	High τ	ω

Table 4. Class conflicts and interests.

common interests in gender issues with a female Democrat, but they may have conflicting views on political agendas (See Lee and Roemer, 2006). Whenever two different categories of groups intersect, we can observe a relationship similar to the one in figure 7.

We use the following payoff structure to capture the conflicts and the common interests among four classes, focusing on two levels of τ and ω; $\omega_H, \omega_L, \tau_H, \tau_L$. Here the subscript H (L) indicates "High" ("Low").

First consider the case of $\kappa = 0$ as a benchmarking case. In this case we assign 1 to the most preferred outcome and -1 to the least preferred one as payoffs of each class, so the capitalist obtains 1 unit of payoff at (τ_L,ω_L) since the pairing of low relative price and low wage norm, (τ_L, ω_L), is the most preferred outcome. When the outcome is a mix of the preferred outcome and less preferred outcome, for example (τ_H, ω_L) and (τ_L, ω_H) for the capitalist, then we simply set the payoff equal

	τ_H,ω_H	τ_H,ω_L	τ_L,ω_H	τ_L,ω_L
Capitalist	-1	κ	$-\kappa$	1
Landlord	$-\kappa$	1	-1	κ
Worker	κ	-1	1	$-\kappa$
Peasant	1	$-\kappa$	κ	-1

Table 5. Payoffs for four classes. $-1 \leq \kappa \leq 1$

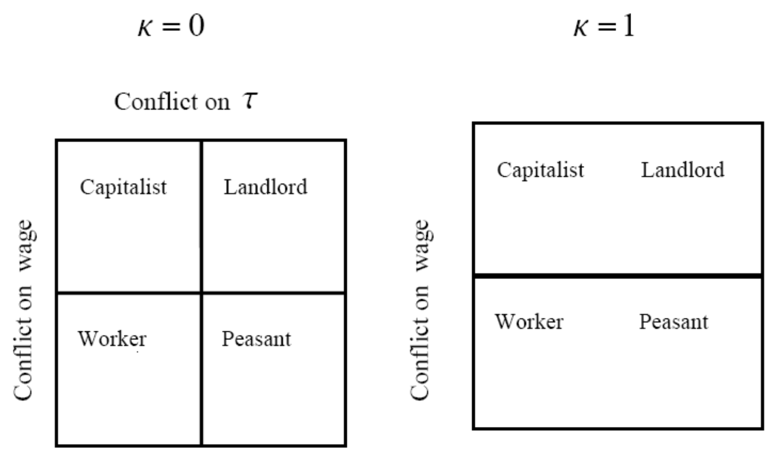

Figure 8. The changes in payoffs for various κ

to 0; i.e. the payoff advantage from the preferred choice exactly offsets the payoff disadvantage from the less preferred outcome. One may argue that the working of "reserve army" equalizes the worker's wage and the peasant wage, hence κ in (τ_H, ω_H) and (τ_H, ω_L) for workers and peasants would be 1. However, since we wish to study the class conflict between the working class and the peasants during the situation like tariff debates we restrict ourselves to payoffs in table 5.

The parameter κ captures the importance of the variables τ and ω in conflicts and common interests among the four classes. We may describe the conflict over τ (ω) as the vertical (horizontal) conflict. When $\kappa = 0$, the contrast in economic interests, depending on τ and ω, is most pronounced. On the other hand, if $\kappa = 1$, the relative price does not affect the payoffs of the classes and similarly when $\kappa = -1$, the wage norm is not involved in the determination of payoffs.

Now suppose that each class engages in the formation of coalitions which can determine the levels of τ and ω. Each class possesses the political resources which can be used to implement its preferred proposal, such as high relative price or low wage norm, in the coalition. To represent these resource constraints we suppose that each class possesses a certain proportion of total resources: α_i for class i's political resources, $\alpha_i \in [0,1]$, and $\sum \alpha_i = 1$. This introduction of political resources resembles the distinction between *de jure* political power and *de facto* political power in Acemoglu, Johnson, and Robinson (2005).

> Here *de jure* political power refers to power that originates from the *political institutions* in society...There is more to political power than political institutions, however. A group of individuals, even if they are not allocated power by political institutions, for example as specified in the constitution, may nonetheless possess political power. Namely, they can revolt, use arms, hire mercenaries, co-opt the military...We refer to this type of political power as *de facto* political power. (Acemoglu, Johnson, and Robinson, 2005, p.4)

The definition of political resources corresponds to *de facto* political power. Even though the working class and the peasant class do not have *de jure* political power, they may have *de facto* political power. When classes form a coalition, they use their *de facto* political power to achieve a goal. The coalition with aggregate political resources greater than a certain number, $\bar{\alpha}$, can be a majority coalition. We suppose $\bar{\alpha} \in [1/2, 1]$. If a coalition forms a majority this coalition can choose one of the proposals which maximizes its payoff and the proposals corresponds to four alternatives: (τ_H, ω_H), (τ_H, ω_L),(τ_L, ω_H),(τ_L, ω_L).

The worth of the coalition is, by definition, the maximum amount of the payoffs that the coalition can achieve so we can write the worth of this game as

$$v(S) = \begin{cases} \max\limits_{(\tau,\omega)} \sum_{i \in S} \pi_i(\tau,\omega) & if \ \ \sum_{i \in S} \alpha_i > \bar{\alpha} \\ 0 & \text{otherwise} \end{cases} \tag{4.1}$$

Once a coalition is formed, the coalition will choose (τ, ω) which maximizes the sum of the players in the coalition, while the other coalition or players which fail to be a majority simply cannot achieve anything. The specification of the worth, or the characteristic function, completes the presentation of our model. The next step is to find stable coalition structures and to do this we follows the three steps pioneered by Hart and Kurz (1983): 1. Find the coalition structure values, 2. Derive the normal form of the game, 3. Identify the stable coalition structures.

4.3 Methods and Results

4.3.1 Methods

Hart and Kurz (1983) explain the coalition values as follows:

> Our theory combines two kinds of game theoretic concepts: value and stability. The basic idea is, first, to evaluate the players' prospects in the various coalition structures, and then, based on these "values", to find which ones are stable. We call this value "coalition structure value“, or " *CS-value*" for short. (p.1047, Hart and Kurz, 1983)

A coalition structure $\mathcal{B}$ is a finite partition $\mathcal{B} = \{B_1, B_2, ..., B_m\}$ of the set of all players N ;i.e. $\bigcup_{k=1}^{m} B_k = N$ and $B_k \bigcap B_l = \phi$ for all $k,\ l \in M = \{1, ...m\},\ k \neq l$. For each coalition structure $\mathcal{B}$, a coalition value for each player $i \in N$ is defined as follows (Owen, 1977):

$$\phi_i(v, \mathcal{B}) = \sum_{\substack{H \subset M \\ j \notin H}} \sum_{\substack{S \subset B_j \\ i \notin S}} \frac{h!(m-h-1)!s!(b_j - s - 1)!}{m!b_j!} [v(Q \cup S \cup i) - v(Q \cup S)]$$

for $B_j \in \mathcal{B}$ and $i \in B_j$ where h, s and b_j are the cardinalities of H, S and B, and $Q = \bigcup_{k \in H} B_k$, $M = \{1, ...m\}$ such that $\mathcal{B} = \{B_1, B_2, ..., B_m\}$.

The "value" corresponds to the evaluation, or expected payoff, to the players given the coalitional structure. The value summarizes the complex possibilities facing each player in a game in characteristic function form (Roth, 1988, p.4). With the value attached to the coalition structure, each player is able to compare his "prospects" in the various coalition structures and to decide whether or not to change his coalition. A stable coalition structure is defined as a coalition structure where no defection by any group of players or individual player is desirable.

Another alternative value is the Aumann-Dreze value (Aumann and Dreze, 1974) which is based on the interpretation that the considered coalition would actually form. Consequently in the Aumann-Dreze case side payments among coalitions are not allowed and the interactions between coalitions are neglected, whereas side payments among coalitions are possible in the case of the coalition structure value and the bargaining between the coalitions is an important determinant of the values. Since we wish to study the interaction among various coalitions as well as the interaction within a coalition, we adopt the coalitional structure value for our analysis.

In the second step where we specify the normal form of game we have two alternative models, $\gamma-$ model and $\delta-$model:

Model γ (Hart and Kurz, 1983): The game $\Gamma \equiv \Gamma_{v,N}$ consists of:

(M1) the set of player is N

(M2) For each $i \in N$, the set of $\Sigma^i$of *strategies* of i consists of all coalitions S that contain i, namely, $\Sigma^i = \{S \subset N | i \in S\}$

(M3) For each n-tuple of strategies $\sigma = (S^1, S^2, ..., S^n) \in \Sigma^1 \times \Sigma^2 \times ... \times \Sigma^n$ where $n = |N|$and each $i \in N$, the payoff to i is $\phi^i(v, \mathcal{B}_\sigma^{(\gamma)})$, where

$$T_\sigma^i = \begin{cases} S^i, & \text{if } S^j = S^i \text{ for all } j \in S^i \\ \{i\}, & \text{otherwise} \end{cases}$$

$$and\ \mathcal{B}_\sigma^{(\gamma)} = \{T_\sigma^i | i \in N\}$$

Model δ: The game $\Delta \equiv \Delta_{v,N}$ is given by (M1), (M2), and (M4) :

(M4) For each n-tuple of strategies $\sigma = (S^1, S^2, ..., S^n) \in \Sigma^1 \times \Sigma^2 \times ... \times \Sigma^n$ and each $i \in N$, the payoff to i is $\phi^i(v, \mathcal{B}_\sigma)$, where

$$\mathcal{B}_\sigma^{(\delta)} = \{T \subset N | i, j \in T \text{ if and only } if\ S^i = S^j\}$$

The above definitions of the normal form game specify which coalition will arise as the result of the strategies of individuals. For example, consider a coalition structure [12|34567], which indicates that the coalition, $\{1, 2\}$, and the coalition $\{3, 4, 5, 6\}$ form. If 3 leaves his coalition, his previous coalition can either fall apart or stick together. So the resulting coalition following 3's departure would be either [12|3|4|5|6|7] or [12|3|4567] depending on the interpretation of the coalition. The first case – the "fall apart" scenario – is based on the view that a coalition results from a unanimous agreement and model γ describes this type of coalition formation. The second case – "stick together" – corresponds to the situation in which in a large coalition a small number of players leaving a coalition should not influence the other coalition members' agreement to act together. This case is specified by model δ.

Having specified the normal forms of the game, we need to identify the stable coalition structure in the final step. The stable coalition structure is defined to be a equilibrium outcome, so we need a proper equilibrium concept which addresses the coalitional deviations. For this purpose we use the concept of the strong equilibrium, first defined Aumann (1967) and subsequently adopted by Hart and Kurz (1983):

Definition 4.1 (Hart and Kurz, 1983) *The coalition structure $\mathcal{B}$ is γ-stable (δ-stable) in the game (v, N) if $\sigma_{\mathcal{B}}$ is a strong equilibrium in $\Gamma_{v,N}(\Delta_{v,N},$ respectively); i.e., if there exists no nonempty $T \subset N$ and no $\hat{\sigma}^i \in \Sigma^i$ for all $i \in T$, such that $\phi^i(v, \hat{\mathcal{B}}) > \phi^i(v, \mathcal{B})$ for all $i \in T$, where $\hat{\mathcal{B}}$ corresponds to $\left((\hat{\sigma}^i)_{i\in T}, (\sigma^j_{\mathcal{B}})_{j\in N\setminus T}\right)$ by (M3) (or (M4), respectively) (Hart and Kurz, 1983)*

4.3.2 Results

First we note that when the wage norm ω is the only factor in economic interests among classes and the ruling class has dominant political resources, we expect that the society would be characterized by a horizontal conflict and the corresponding coalition structure, $[CL|WP]$, would arise. In fact this is verified using the model as follows: (see the appendix for the proof.)

- If $\kappa = 1$, $\alpha_C = \alpha_L > \alpha_P = \alpha_W$, $\alpha_C + \alpha_L > \bar{\alpha}$, $[CL|WP]$ is $\gamma-$ and $\delta-$stable
- If $\kappa = -1, \alpha_C = \alpha_W > \alpha_L = \alpha_P$, $\alpha_C + \alpha_W > \bar{\alpha}$, $[CW|LP]$ is $\gamma-$and $\delta-$stable

Next we examine the effect of an imbalance in political resources in favor of one class, namely the working class, on the stability of the coalition structures. To do this we set $\alpha_C = \alpha_L = \alpha_P$ and conduct the comparative statistics comparing $\alpha_C < \alpha_W < \bar{\alpha}$ and $\alpha_W < \alpha_C < \bar{\alpha}$. We exclude the trivial case of $\alpha_W > \bar{\alpha}$ where the working class can form a majority coalition by standing alone and hence the coalition formation is unnecessary.

Proposition 4.2 *Suppose $\alpha^* = (1 - \bar{\alpha})/2$ and $\alpha_C = \alpha_L = \alpha_P$. We have the following stable coalition structure.*

- *If $0 \leq \kappa < 1$, $\alpha_C < \alpha^* < \alpha_W < \bar{\alpha}$, $[CLP|W]$ is a unique $\gamma-$and $\delta-$ stable coalition.*

- *If* $-1 < \kappa < 0$, $\alpha_C < \alpha^* < \alpha_W < \bar{\alpha}$, $[CLP|W]$ *is a* $\gamma -$ *stable, but not* $\delta -$ *stable coalition.*

- *If* $0 \leq \kappa < 1$, $\alpha_W < \alpha^* < \alpha_C < \bar{\alpha}$, $[CWP|L]$ *is a unique* $\gamma-$*and* $\delta -$ *stable coalition.*

- *If* $-1 < \kappa < 0$, $\alpha_W < \alpha^* < \alpha_C < \bar{\alpha}$, $[CWP|L]$ *is a* $\gamma -$ *stable, but not* $\delta -$ *stable coalition.*

Proportion 4.2 asserts that when economic interests and conflict over the relative price and the wage norm contradict each other, $(-1 < \kappa < 1)$, and when the working class has the greatest political resources among the four classes, $(\alpha_C < \alpha^* < \alpha_W)$, a coalition among C, L, P is likely to form against W. From (4.1) we see $v(WP) = 1+\kappa, v(WC) = 1-\kappa$, $v(WL) = 0$ since workers share common interests with peasants and capitalists, and have conflict with landlords. On the contrary when $\alpha_W < \alpha^* < \alpha_C$, the exclusion of L may arise. Even though the capitalists and the peasants can form a majority coalition because of $\alpha_C = \alpha_P = \alpha_W$, we note that the capitalists and the peasants have no common interest, but conflict over wage norm and relative price. So in this case only L can form a profitable coalition of size two; $v(LC) = 1+\kappa, v(LP) = 1-\kappa, v(LW) = 0$. We see that L and C plays the same role as W and P in the case $\alpha_C < \alpha^* < \alpha_W$ respectively, so we expect $[CWP|L]$ is a stable coalition. If we recall that $\delta-$model is more relevant to coalition formation in large groups, we may conclude that the model suggests strongly $[CLP|W]$ is stable when $\alpha_C < \alpha^* < \alpha_W < \bar{\alpha}$ while $[CWP|L]$ is stable for $\alpha_W < \alpha^* < \alpha_C < \bar{\alpha}$. In the next section we argue that the historical experiences in the nineteenth-century Germany and England square with the results of the model.

4.4 Historical Relevance

We review the existing literature on coalition formation in nineteenth century Germany and England and argue that the model identifies the historically plausible coalition structure as a stable coalition structure. In addition we attempt to use coalition formation to explain political transitions in these periods. We imagine that four classes engage in the formation of coalitions due to a certain exogenous change. The model suggests that if the political power of the working class is strong, the coalition $\{CLP\}$ would form, and conversely when the working class is weak, the coalition $\{CWP\}$ would arise.

From the payoff structure given in Table 2, we verify that the coalition of capitalists, landlords, and peasants will choose the outcome (τ_H, ω_L) since (τ_H, ω_L) maximizes the sum of payoffs of the classes in the coalition. If we interpret the relative price τ as the tariff level on the agricultural output, the coalition $\{CLP\}$ can be called the coalition of protective trade policy. The interpretation of the level of the relative price τ as the trade policy has a caveat. If some country adopts a high tariff on the industrial sector and a high tariff on the agricultural sector, this country's trade policy is protective with τ still being unchanged. However the protection of agricultural products was the main concern of protectionist policy in late nineteenth century European countries, and thus we may use the high τ as a representation of protectionist policy.

Since the coalition chooses the most preferred outcome of landlord class, (τ_H, ω_L), the coalition represents the interests of the agricultural sector. Similarly the coalition in the second case $\{CWP\}$ adopts the outcome (τ_L, ω_H), so $\{CWP\}$ is the coalition of free trade policy representing the economic interests of the manufacturing sector. Table 6 summarizes the results of the model in the historical contexts.

Strong Working Class	$\{CLP\}$ Coalition Protective Trade Policy Agricultural Based Coalition
Weak Working Class	$\{CWP\}$ Coalition Free Trade Policy Manufacture Based Coalition

Table 6. The summary of results of the model

4.4.1 Evidence on Coalition Formation

In the late 19th century cheap wheat from America was exported to most European countries. For example the price of wheat fell from \$1.70 to \$0.66 a bushel in England between 1873 and 1894 (Kindleberger, 1951). Reactions to this agricultural crisis varied from country to country. Some countries such as England took no action; other countries such as Germany adopted a protective trade policy. The German working class at that time is characterized as well-organized and powerful compared to those in other countries and the socialist party in England was weak and less developed (Nolan, 1986; Gerschenkron, 1943; Gourevitch, 1977). Accordingly we may construe the case, $\alpha_C < \alpha^* < \alpha_W$, in proposition 4.2 above as representative of the German situation and the case, $\alpha_W < \alpha^* < \alpha_C$, as representative of the English one. Moore explains how $\{CLP\}$ coalition arose in the following passage:

> The Junkers managed to draw the independent peasants under their wing and to form an alliance with sections of big industry that were happy to receive their assistance in order to keep the industrial workers in their place with a combination of repression and paternalism. (Moore, 1965, p.115)

The formation of a coalition $\{CLP\}$ means the exclusion of the working class from the major coalition; in *Bread and Democracy*, Gerschenkron provides the possible cause of this exclusion:

> An outstanding feature of that period was the rapid growth of the Social Democratic Party. In 1890 the antisocialist law was allowed to expire. The most impressive socialist victory was only a matter of years. At that time the prediction made by August Bebel, the leader of the Social Democratic Party, to the effect that socialist majority in the German Reichstag would be attained within the lifetime of his generation was widely believed. Such a contingency was regarded by most German farmers as a very real menace to their economic existence. It threatened socialization of the soil and transformation of the free peasants, working on the land of their fathers, into hired laborers of the socialist state. (Gerschenkron, 1943, p.28)

Because of the strong power of the working class, the peasants regarded the strengthening of working class as a menace and were willing to join the coalition of landlords. Similar responses from the capitalist class in the Germany can be found in the following passage.

> ...out of fear of the working class the greater part of the German bourgeoisie became reconciled to their "junior partner" status of the traditional ruling class....The bourgeois intelligentsia, heretofore the chief carrier of the political aspirations of the bourgeoisie, split deeply. (Rittberger, 1973, p.291)

Evidently the trade policy adopted in German was protective:

It was thus under Bismarck's aegis that the so-called protectionist "solidarity bloc" between industry and agriculture was created, celebrating its first success in the promulgation of the tariff of 1879, by which a number of industrial products and grain production were placed under protection. (Gerschenkron, 1943, p.44)

In addition the short-lived Caprivi's free trade policy (1890-1894) can be explained by the path of the coalition formation process (described in the appendix). From the coalition structure $[CWP|L]$ capitalist, peasants, and landlords have incentives to deviate and form a new coalition. Since $\{CWP\}$ represents the coalition of the free trade policy, the deviation route $[CWP|L] \to [CLP|W]$ may explain why the Caprivi's policy did not last very long.

Regarding the class coalition in England, Moore (1965) provides the following accounts of the coalition structure:

One of the reasons why such a scene seems incongruous in England of the nineteenth century is that, unlike the Junkers, the gentry and nobility of England had no great need to rely on political levels to prop up a tottering economic position (Moore, 1965, p.35)

The above passage suggests that the landlords did not need to form a coalition with other classes and $\{CWP\}$ coalition would arise in England. Moreover $\{CWP\}$ coalition is characterized by the coalition representing industrialist's interest:

In regard to agricultural problems, the Conservative governments of 1874-1879 took only small palliative measures; the Liberals from 1880 onward either let markets take their course or actively attacked agrarian interests. By and large agriculture was allowed to shift for itself, that

> is, to commit decorous suicide with the help of a few rhetorical tears (Moore, 1965, p.38)

In addition the trade policy against the nineteenth century crisis adopted by Britain was free trade policy (Moore, Kindleberg, Gourevitch). This fact also corroborates our contention that $\{CWP\}$ coalition is one of the most plausible class coalitions in England in the nineteenth century.

4.4.2 Relationship between the Class Coalitions and Political Transitions in the Nineteenth Century

Having identified the stable coalition structure, we proceed to consider the political ramifications of class alliances and conflicts. Clearly it is a difficult task to find the relationships between class coalitions and political transitions in general. However in nineteenth century England and Germany we may find this connection:

> The whole coalition of Junker, peasant, and industrial interest around a program of imperialism and reaction had disastrous results for German democracy. In England of the late nineteenth century, this combination failed to put in an appearance. (Moore, 1965, p.38)

What underlies Moore's argument is that the Junkers, the representative of agriculture, were against democratization. The rising bourgeois and urban sector are described as politically advanced and oriented towards democracy, whereas the peasant or the agricultural sector is regressive and anti-democratic. A similar response from the working class is founded in Gerschenkron (1943) .

> When Bulow entered the chancellery Social Democracy was a giant party, efficiently organized and working in close collaboration with powerful trade unions. In a few years it was to become the strongest party

> in the German Reichstag, efficiently organized and working in close collaboration with powerful trade unions. It was rapidly outstripping its revolutionary past and was transforming itself into a purely democratic, in fact the most democratic, party in the country – a party which had for its principal goal the democratization of Germany. (Gerschenkron, 1943, p.65)

So we may infer that Germany had late democratization, since there was a class alliance among bourgeois, Junkers, and peasants under the dominance of the Junkers. On the other hand in England the landed class was excluded from the major coalition, and early democratization was possible. Therefore the model suggests that strong working class power led to late democratization while weak working class power brought early democratization. Gourevitch summarizes this in the following:

> In Great Britain in the 1880s the regime was solid enough without any new sources of support but, as on the Continent, the decision on tariffs reinforced existing tendencies. With the reconfirmation of the Corn Law Repeal, the position of agriculture and landed interests crumbled. After 1880, the absolute number of people in farming declined sharply. While the Junkers were successfully preserving many of their privileges, the British aristocracy lost most of those which remained. The County Councils Act of 1888 (which ended Justice of the Peace control of local life), the secret ballot, reform of the House of Lords, educational reorganization, and reform of the status of the church can all be lined to the waning influence of agriculture; so can British reluctance to join the Common Market fifty years later. (Gourevitch, 1977, p.311)

4.5 Summary

When economic conflict over tariffs and wage norms escalates, the existence of a strong working class may lead to late democratization while the presence of a weak working class would provide favorable conditions for the democratization; the exclusion of the working class may delay the democratization. In the context of class alliances this means that there is a non-monotonic relationship between political power and economic performance. The class with strongest political power can be excluded from the formation of a major coalition and therefore perform worst in the bargaining process.

APPENDIX A

CONTEST SUCCESS FUNCTIONS: LEMMA AND TABLES

First we prove the lemma used in the text.

Lemma A.1 *Suppose that* $\epsilon_i \sim \exp(-\gamma_i e^{-\kappa s})$ *where* $\gamma_i > 0, \kappa > 0$ *and* $-\infty < s < \infty$ *for* $i = 1, 2$. *Then* $\epsilon_1 - \epsilon_2 \sim \frac{\gamma_1}{\gamma_1 + \gamma_2 e^{-\kappa s}}$.

Proof. It is easy to check $\Lambda(s) := \frac{\gamma_1}{\gamma_1+\gamma_2 e^{-\kappa s}}$ is a distribution function. From the definition of ϵ_i, we have $\Pr\{\epsilon_2 \in ds\} = \exp(-\gamma_2 e^{-\kappa s})\gamma_2 \exp(-\kappa s)\kappa ds$. Hence from the definition of conditional probability and the independence between ϵ_1 and ϵ_2, we have

$$\begin{aligned}
\Pr\{\epsilon_1 < \epsilon_2 + x\} &= \int_{-\infty}^{\infty} \Pr\{\epsilon_1 < s + x\} \Pr\{\epsilon_2 \in ds\} \\
&= \int_{-\infty}^{\infty} \exp(-\gamma_1 e^{-\kappa s - \kappa x}) \exp(-\gamma_2 e^{-\kappa s})\gamma_2 \exp(-\kappa s)\kappa ds \\
&= \int_{0}^{\infty} \exp(-t(\gamma_1 e^{-\kappa x} + \gamma_2))\gamma_2 dt \\
&= \frac{\gamma_2}{\gamma_1 e^{-\kappa x} + \gamma_2} \qquad \text{(A.1)}
\end{aligned}$$

We use the change of variable, $t = \exp(-\kappa s)$, in the third line and the asserted claim follows from (A.1). ∎

We provide tables containing descriptive statistics and alternative estimations.

War	Number of Battles
War of the Spanish Succession	108
Thirty Years' War	64
Austro-Turkish War	34
Great Northern War	29
Dutch War	19
War of the League of Augsburg	18
Other wars	43

Table 7. 17th century European wars.

Other wars include wars with less than ten battles. These are the Turkish War with Venice and Austria, English Civil War, Hungarian-Turkish War, Polish-Turkish War, Second English Civil War, The Fronde, War of the Quadruple Alliance, Polish-Swedish War, Spanish-Portuguese War, Swedish-Danish War, The First Northern War, War of Devolution, Chamber of Reunion, English Scottish War, Franco-Spanish War, Moldavian Campaign, Monmoth's Rebellion, Polish Insurgency, and Turkish-Ventian War. The classification of war is based on Dupuy and Dupuy. (1986) and Palmer and Colton (1984).

Army	Number of Battles
French	162
Imperial	173
Swedish	63
Spanish	99
Turkish	50
English	50
Dutch	64
Russian	21
Others	142

Table 8. Major armies in 17th century European wars.

More than two armies allied in some battles.

Statistics	Number of Personnel
Obs	630
Mean	21035
Max	260000
Min	1000
Standard Dev	24047
Median	15000

Table 9. Descriptive statistics for number of personnel involved in 17th century European war.

Statistics	Combat Power Ratio
Obs	188
Mean	1.4332
Max	7.54
Min	0.1326
Standard Dev	1.30212
Median	1

Table 10. Descriptive statistics for combat power ratio in World War II data.

	17C European War		
	Difference	Ratio	Integrated
κ	2.24×10^{-5}	0.803	
	(9.30×10^{-6})	(0.133)	
η			−20.478
			(5.159)
ρ			1.117
			(0.214)
Number of Observations	630	630	630
Percentage of Correctly Predicted	70.63	73.97	73.73
Log-likelihood Value	−382.443	−354.87	−354.552

Table 11. Alternative data Set.

Each battle represents both winning event and losing event. The standard errors are corrected for heteroskedasticity and clustering.

	Model 1	Model 2	Model 3	Model 4
η	−2505.666	−19.123	−20.099	−49.866
	(226.199)	(7.736)	(19.510)	(93.992)
Implied κ	0.749194	2.52424	3.739	13.9193
ρ	1.0003	1.132	1.186	1.279
	(0.0000299)	(0.204)	(0.242)	(0.340)
Number of Observation	308	315	315	184
Log-likelihood Value	−182.557	−185.612	−192.636	−95.279

Table 12. Alternative estimation

Model 1: Excludes observations with armies of size greater than 100,000; Model 2: Some observations indicate that the battle took place in the garrison. We use the dummy variable when the observation has this indication.; Model 3: Exclude dummy variables for wars; Model 4: Includes only battles among eight major armies: French, Imperial, Swedish, Spanish, Turkish, English, Dutch, Russian

APPENDIX B

CLASS ALLIANCES AND CONFLICTS: PROOF OF PROPOSITION 4.2

We first prove the proposition 4.2, and then provide the proof of the benchmarking case in the end.

Case 1: $\alpha_C = \alpha_L = \alpha_P < \alpha^* < \alpha_W < \bar{\alpha}$

First we consider the case: $\alpha_C = \alpha_L = \alpha_P < \alpha^* < \alpha_W < \bar{\alpha}$. This corresponds to the case where the political power of the working class is strong. For example, $(\alpha_C, \alpha_L, \alpha_W, \alpha_P) = (0.23,\ 0.23,\ 0.\,31,\ 0.23)$ for $\bar{\alpha} = 0.5$. (Recall the assumption of $\alpha_C = \alpha_L = \alpha_P$ and $\alpha_C + \alpha_L + \alpha_P = 1$.) Only the working class is powerful enough to makes majority coalitions of size 2. All other classes need either working class or two more other classes to form a majority coalition. Precisely the majority coalitions are $\{CW\}$,$\{LW\}$, $\{WP\}$, $\{CLW\}$, $\{CLP\}$, $\{LWP\}$, $\{CWP\}$, {CWLP} since $\alpha_i + \alpha_j < \bar{\alpha}$ for all $i, j \neq W$, $\alpha_i + \alpha_W > \bar{\alpha}$ for all $i \neq W$. So all the coalitions of workers are majority coalitions. Using (4.1) and table 5, we can find the worth of the game:

$$v(S) = \begin{cases} 1+\kappa & S = \{WP\} \\ 1-\kappa & S = \{CW\} \\ 1 & S = \{CLW\}, \{CLP\}, \{LWP\}, \{CWP\} \\ 0 & \text{otherwise} \end{cases} \quad \text{(B.1)}$$

This game is similar to apex games except that $v(LW) = 0$ and $v(CLWP) = 0$ (Hart and Kurz, 1984). However the differences turn out to be important since in game (B.1) is a *unique* stable equilibrium. In the context of class alliance, since landlords and workers do not have any common interest, the coalition between L and W generates nothing: $v(LW) = 0$. This makes the game asymmetric. The same is true for the grand coalition as can be verified from the table 13. The worth of the grand coalition is always 0 regardless of the levels of τ and ω. Because of this feature of the grand coalition, this game is not super-additive. The CS value for the game (B.1) is given in table 13.

Now we set $\kappa = 0$. The CS values for $[CLWP]$ and $[C|L|W|P]$ are the same as the Shapley value: (0, -1/6, 1/6, 0). This means that when the coalition structure is $[CLWP]$, capitalists, landlords, workers and peasants expect 0, -1/6, 1/6, and 0 as their payoffs, respectively. Imagine that the players enter a room in a random manner and they are allowed to form a coalition inside the room. C can be a "positive" pivot player, a player who contributes a positive worth as she enter the room or joins the existing coalition, or a "negative" pivot player, a player who contributes a negative worth. C is a "positive" pivot player when the orders of entering is $\{WCPL\}$, $\{WCLP\}$, $\{LWCP\}$, $\{WLCP\}$, $\{PLCW\}$, $\{LPCW\}$ (for example, since $v(WC) - v(W) = 1$ for $\{WCPL\}$). So C will expect 1/4, since out of total possibilities (4!) the number of orders where C become a "positive" pivot player is 6.

	C	L	W	P
$CLWP$	0	$\frac{1}{6}(\kappa-1)$	$\frac{1}{6}(1-\kappa)$	0
$C\|LWP$	$-\frac{1}{2}$	$\frac{1}{12}(3\kappa-1)$	$\frac{1}{12}(5-3\kappa)$	$\frac{1}{6}$
$C\|L\|WP$	$\frac{1}{6}(\kappa-2)$	$\frac{1}{6}(\kappa-2)$	$\frac{1}{12}(5-3\kappa)$	$\frac{1}{12}(3-\kappa)$
$C\|LW\|P$	$-\frac{1}{6}$	$\frac{1}{6}\kappa$	$\frac{1}{6}(2-\kappa)$	$-\frac{1}{6}$
$C\|LP\|W$	$-\frac{1}{6}\kappa$	$\frac{1}{12}(3\kappa-1)$	$-\frac{1}{6}\kappa$	$\frac{1}{12}(\kappa+1)$
$C\|L\|W\|P$	0	$\frac{1}{6}(\kappa-1)$	$\frac{1}{6}(1-\kappa)$	0
$CL\|WP$	$\frac{1}{4}(\kappa-1)$	$\frac{1}{4}(\kappa-1)$	$\frac{1}{4}(1-\kappa)$	$\frac{1}{4}(1-\kappa)$
$CW\|LP$	$\frac{1}{4}(1-\kappa)$	$\frac{1}{4}(\kappa-1)$	$\frac{1}{4}(1-\kappa)$	$\frac{1}{4}(\kappa-1)$
$CL\|W\|P$	$\frac{1}{12}(\kappa+1)$	$\frac{1}{12}(3\kappa-1)$	$-\frac{1}{6}\kappa$	$-\frac{1}{6}\kappa$
$CW\|L\|P$	$\frac{1}{12}(3-\kappa)$	$\frac{1}{6}(\kappa-2)$	$\frac{1}{12}(5-3\kappa)$	$\frac{1}{6}(\kappa-2)$
$CP\|LW$	0	0	0	0
$CP\|L\|W$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{6}$
$CWP\|L$	$\frac{1}{12}(\kappa+1)$	$-\frac{1}{2}$	$\frac{1}{6}(2-\kappa)$	$\frac{1}{12}(\kappa+1)$
$CLW\|P$	$\frac{1}{6}$	$\frac{1}{12}(3\kappa-1)$	$\frac{1}{12}(5-3\kappa)$	$-\frac{1}{2}$
$CLP\|W$	$\frac{1}{12}(3-\kappa)$	$\frac{1}{6}\kappa$	$-\frac{1}{2}$	$\frac{1}{12}(3-\kappa)$

Table 13. CS values for $-1 < \kappa < 1$

On the other hand, when the sequences of entering is $[PWLC]$, $[WPLC]$, $[LPWC]$, $[PLWC]$, $[WLPC]$, $[LWPC]$, C becomes a "negative" pivot player, who contributes -1. This is because the joining of C into the existing coalition drops the worth of that coalition from 1 to 0. This feature highlights the nature of class conflicts: a grand coalition would not do any good. Since C can be a "negative" pivot player with the probability of 1/4, C expects -1/4 from being a negative pivot. As the expected payoff from the positive pivot is 1/4 and the one from the negative pivot is -1/4, the capitalist expects 0 payoff in total when the coalition structure is $[CLWP]$. A similar explanation is applied to the payoff of peasants.

The payoff -1/6 of L comes from the fact that L cannot be a pivot player when L is the second players in the random order. In a random order such as $[OLOO]$,

L cannot be a positive player unlike the capitalist or the peasant. This is because L does not have a common interest with W: $v(LW) = 0$. Under the condition that L is less powerful, L can contribute a positive worth in the coalition of size 2 only with more powerful class, W. However, since L does not have a common interest with W, this is impossible unlike C and P. This is why L expects -1/6 in the coalition structure $[CLWP]$.

For the case of $[CW|L|P]$, CS values are given by $(C, L, W, P) =$ (1/4, -1/3, 5/12, -1/3). These numbers show well how the CS value can characterize interactions among coalitions and within coalition. First, in this coalition structure we think that $\{CW\}$ forms one representative agent, A and A ,L, P engages in bargaining. The worth for this new bargaining is $v(A) = v(AL) = v(AP) = 1$, $v(L) = v(P) = v(LP) = v(ALP) = 0$. Again by using a similar argument we find the Shapley value for this new game: $(A, L, P) =$ (2/3, -1/3, -1/3). This is the bargaining process among the coalitions.

Secondly, the bargaining within coalition occurs. Since we have only one coalition $\{CW\}$, we consider the bargaining within $\{CW\}$ only. To determine the bargaining within $\{CW\}$, we need to consider what is the "disagreement point". Because the player has a option to get out of the coalition, the natural "disagreement point" would be the expected payoff that the player will get when she gets out of the coalition. Therefore, C has 0 as its "fallback" payoff and W has 1/6 as its "fallback" since $[C|L|W|P] =$ (0, -1/6, 1/6, 0). Using this we obtain a new game which occurs within the coalition $\{CW\}$: $v(C) = 0$, $v(W) = 1/6$, $v(CW) = 2/3$ (since $v(A) = 2/3$ and $(A, LP) =$(2/3, -1/3, -1/3)). Again we find the Shapley values for this case and this turns out to be $(C, W) =$ (1/4, 5/12). Combining the first result and the second result, we find CS values for $[CW|L|P]$ as (C, L, W, P) = (1/4, -1/3, 5/12, -1/3). Other lines of CS values can be interpreted similarly.

Now we can show the following claims.

Claim 1. When $0 \leq \kappa < 1$, $[CLP|W]$ is γ- and δ- stable.

CS values for $[CLP|W]$ is $(\frac{3-\kappa}{12}, \frac{\kappa}{6}, -\frac{1}{2}, \frac{3-\kappa}{12})$. We note that $\frac{3-\kappa}{12}$, $\frac{\kappa}{6}$, $\frac{3-\kappa}{12}$ are the maximum values for capitalists, landlords, and peasants over all coalition structures, respectively. Therefore from the definition of the strong equilibrium, it follows that $[CLP|W]$ is γ- and δ- stable; workers cannot form a deviating coalition since all other classes achieve the maximum values at $[CLP|W]$.

Claim 2. When $-1 < \kappa < 0$, $[CLP|W]$ is γ- and δ- stable.

Since the only possible deviation for this case is from $[CLP|W]$ to $[CW|LP]$ ($\frac{3-\kappa}{12} < \frac{1-\kappa}{4}$ for C and $-\frac{1}{2} < \frac{1-\kappa}{4}$ for L) and this deviation corresponds to δ-model, we see that $[CLP|W]$ is γ-stable, but not δ-stable.

Claim 3. When $0 \leq \kappa < 1$, $[CLP|W]$ is a unique γ- and δ- equilibrium.

For this claim we classify the possible coalitional deviation as follows.

i. The exclusion of a working class

The coalition structure with a coalition size greater than or equal to 3 can deviate by excluding workers.

$[CLW|P] \rightarrow [CLP|W]$, $[CWP|L] \rightarrow [CLP|W]$, $[C|LWP] \rightarrow [CLP|W]$, $[CLWP]$ $\rightarrow [CLP|W]$

This follows from the fact that the maximum values for C, L, P do not occurs at these coalition structures with a coalition of size greater than 3 except $[CLP|W]$; C, L and P can form a profitable deviational coalition. Because $[C|L|W|P]$ has the same CS values as the grand coalition, we find the following deviation: $[C|L|W|P]$ $\rightarrow [CLP|W]$

This phenomenon is similar to the exclusion of the major player in the apex game (Hart and Kurz, 1984). The exclusion of the major player, the working class, was first established by von Neumann and Morgenstein who call this "the segregation of the major player" (Von Neumann and Morgenstein, 1944). When a certain class is strong enough, the other classes have a strong incentive to form a coalition against this class.

ii. The exclusion of capitalists, landlords, or peasants

$[CW|LP] \rightarrow [C|LWP]$, $[CL|WP] \rightarrow [CLW|P]$, $[CP|LW] \rightarrow [CWP|L]$

The possibility of these deviations can be verified using CS values of the table 13. The idea behind these deviations is that the working class always has an incentive to exclude the partner class in its coalition (e.g. C in $[CW|LP]$) and form a new coalition with the other remaining two classes. The vertical class conflict (the conflict between CW vs LP) and the horizontal class conflict (the conflict between CL vs WP) can be resolved by the "ternary" coalition led by the working class.

iii. The formation of opposing coalitions

$[CW|L|P] \rightarrow [CW|LP]$, $[C|LW|P] \rightarrow [CP|LW]$, $[C|L|WP] \rightarrow [CL|WP]$,
$[C|LP|W] \rightarrow [CW|LP]$,$[C|L|WP] \rightarrow [CL|WP]$, $[CL|W|P] \rightarrow [CL|WP]$,
$[CP|L|W] \rightarrow [CP|LW]$

These deviations are symbolically characterized as follows:$[OO|\square|\square] \rightarrow [OO|\square\square]$. If two of classes already form a coalition, the remaining two players always have incentives to form a opposing coalition against the existing coalition. Each class can get more benefit from forming a counter-coalition rather than remaining as a stand-alone class. These deviations show the routes along which the class conflict can be exacerbated by the formation of an opposing coalition.

Since the deviation paths in the above satisfies both model γ and δ, there is no γ- and δ- stable coalition structure except $[CLP|W]$. Therefore proposition 4.2) follows from claim 1 and claim 2. Moreover since the all γ- and δ- stable coalition structures are α- and β- core states, $[CLP|W]$ is also α- and β- core states.(see Aumann, 1967; Hart and Kurz, 1983, for the definitions of core states and the relation among stability concepts) The existence and uniqueness of the coalition structure are strong results. There are some games which do not possess a stable coalition structure. Also considering the fact that many coalitional games have multiple stable coalition structures, the uniqueness result is critical.

Here we propose one likely path of coalition formation process:

$[C|L|W|P] \rightarrow [CW|L|P] \rightarrow [CW|LP] \rightarrow [CWP|L] \rightarrow [CLP|W]$

Starting from an initial coalition $[C|L|W|P]$, a coalition between C and W can arise. This leads to the coalition $[CW|L|P]$. Afterward the confrontation among classes increase by the coalition of L and P. This class conflict may be resolved by P's deserting L or the exclusion of L. Finally L can successfully induce C and P into the exclusion of W.

Case 2: $\alpha_C = \alpha_L = \alpha_P < \alpha^* < \alpha_W < \bar{\alpha}$

Next we consider the second case: $0 < \alpha_W < \alpha^*$. one example of this case is $(\alpha_C, \alpha_L, \alpha_W, \alpha_P) = (0.27,\ 0.27,\ 0.19,\ 0.27)$ for $\bar{\alpha} = 1/2$. Since $\alpha_i + \alpha_W < \bar{\alpha}$ for all $i \neq W$ and $\alpha_i + \alpha_j > \bar{\alpha}$ for all $i, j \neq W$ (because $\bar{\alpha}$> 1/2), majority coalitions are $\{CL\}$, $\{CP\}$, $\{LP\}$, $\{CLW\}$, $\{CLP\}$, $\{LWP\}$, $\{CWP\}$, $\{CWLP\}$

Therefore the following game is obtained.

$$v(S) = \begin{cases} 1+\kappa & S = \{CL\} \\ 1-\kappa & S = \{LP\} \\ 1 & S = \{CLW\}, \{CLP\}, \{LWP\}, \{CWP\} \\ 0 & \text{otherwise} \end{cases} \tag{B.2}$$

By the examination of equation $(B.1)$ for case 1 and equation $(B.2)$, we can see that the only differences are that L and W change the roles, and C and P change the roles. Specifically we define $\phi : \{C, L, W, P\} \rightarrow \{C, L, W, P\}$ by $\phi(C) = P$, $\phi(L) = W$, $\phi(W) = L, \phi(P) = C$. Then the game $(B.2)$ is obtained by applying

	C	L	W	P
$CLWP$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$
$C\|LWP$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0
$C\|L\|WP$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$
$C\|LW\|P$	$-\frac{1}{6}$	$\frac{1}{3}$	0	$-\frac{1}{6}$
$C\|LP\|W$	$-\frac{1}{6}$	$\frac{1}{3}$	$-\frac{1}{6}$	0
$C\|L\|W\|P$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$
$CL\|WP$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$CW\|LP$	0	0	0	0
$CL\|W\|P$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$CW\|L\|P$	$\frac{1}{3}$	$-\frac{1}{6}$	0	$-\frac{1}{6}$
$CP\|LW$	0	0	0	0
$CP\|L\|W$	$\frac{1}{3}$	$-\frac{1}{6}$	$-\frac{1}{6}$	0
$CWP\|L$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0
$CLW\|P$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{6}$	$-\frac{1}{2}$
$CLP\|W$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{6}$

Table 14. CS values for $\kappa = 1$

ϕ to $(B.1)$ Therefore the results follow. Finally, we check the benchmarking case. When $\kappa = 1$, $\alpha_C = \alpha_L > \alpha_W = \alpha_P$, $\alpha_C + \alpha_L > \bar{\alpha}$,we have

$$v(S) = \begin{cases} 2 & \text{if } \quad S = \{CL\} \\ 1 & \text{if } \quad S = \{CLW\}, \{CLP\}, \{LWP\}, \{CWP\} \\ 0 & \text{otherwise} \end{cases}$$

In this case the CS values are given by

so we see that $[CL|WP]$ is $\gamma - stable$ and $\delta - stable$. The second case follows from the symmetry among C, L, W, and P.

BIBLIOGRAPHY

Acemoglu, D., S. Johnson, and J. A. Robinson (2005): "Institutitons as the Fundamental Cause of Long-Run Growth," in *Handbook of Economic Growth*, ed. by P. Aghion, and S. Durlauf. North Holland.

Acemoglu, D., and J. A. Robinson (2000): "Why Did the West Extend the Franchise: Democracy, Inequality, and Growth in Historical Perspective," *Quartely Journal of Economics*, 115, 1167–1199.

Adams, E. S., and M. Mesterton-Gibbons (2003): "Lanchester's Attrition Models and Fights Among Social Animals," *Behavioral Ecology*, 14(5), 719–723.

Agrawal, A., and S. Goyal (2001): "Group Size and Collective Action: Third Party Monitoring in Common-Pool Resources," *Comparative Political Studies*, 34, 63–93.

Arrow, K., H. B. Chenery, B. S. Minhas, and R. M. Solow (1961): "Capital-Labor Substitution and Economic Efficiency," *Reveiw of Economics and Statistics*, 43(3), 225–250.

Aumann, R., and J. Dreze (1974): "Cooperative Games with Coalition Structures," *International Journal of Game Theory*, 3, 217–237.

Aumann, R. J. (1967): "A Survey of Cooperative Games Without Side Payments," in *Essays in Mathematical Economics*, ed. by M. Shubik, pp. 3–27.

Bagnoli, M., and M. McKee (1991): "Voluntary Contribution Games: Efficient Private Provision of Public Goods," *Economic Inquiry*, 29, 351–366.

Basu, K. (1986): "One Kind of Power," *Oxford Economic Papers*, 38(2), 259–282.

Biddle, S. D. (2004): *Military Power: Explaining Victory and Defeat in Modern Battle*. Princeton Univ. Press, Princeton.

Bingham, P. (1999): "Human Uniqueness: A General Theory," *Quarterly Review of Biology*, 74(2), 133–169.

——— (2000): "Human Evolution and Human History: A Complete History," *Evolutionary Anthropology*, 9(6), 248–257.

Bodart, G. (1908): *Militar-Historishes Kriegs-Lexikon*. C.W. Stern, Wien and Leipzip.

Boehm, C. (1982): "The Evolutionary Development of Morality as an Effect of Dominance Behavior and Conflict Interference," *Journal of Social and Biological Structures*, 5, 413–421.

Bowles, S. (1984): "Class Alliances and Surplus Labor Time," in *Economic Structure and Performance: Essay in Honor of Hollis B. Chenery*, ed. by M. Syrquin, L. Taylor, and L. Westpah, pp. 103–114. Academic Press.

Bowles, S. (2008): "Conflict: Altruism's Midwife," *Nature*, 456, 326–327.

Bowles, S., and J.-K. Choi (2002): "The First Property Rights Revolution," *Santa Fe Institute Working Paper*, 02-11-061.

Bowles, S., and H. Gintis (2004): "The Evolution of Strong Reciprocity: Cooperation in Heterogeneous Populations," *Theoretical Population Biology*, 65, 17–28.

Boyd, R., H. Gintis, S. Bowles, and P. Richerson (2003): "The Evolution of Altruistic Punishment," *Proceedings of the National Academy of Science*, 100(6), 3531–35.

Boyd, R., and P. Richerson (1990): "Culture and Cooperation," in *Beyond Self Interest*, ed. by J. Mansbridge, pp. 111–127. University of Chicago Press, Chicago.

——— (1992): "Punishment Allows the Evolution of Cooperation (or Anything Else) in Sizable Groups," *Ethlogy and Sociobiology*, 13(3), 171–195.

Buchanan, J. M., R. D. Tollison, and G. Tullock (eds.) (1980): *Toward a Theory of the Rent-Seeking Society*. Texas A and M University Press, College Station.

Carpenter, J. P. (2007): "Punishing Free-Riders: How Group Size Affects Mutual Monitoring and the Provision of Public Good," *Games and Economics Behavior*, 60, 31–51.

Chamberlin, J. (1974): "Provision of Collective Goods as a Function of Group Size," *American Political Science Review*, 68, 707–716.

Choi, J.-K., and S. Bowles (2007): "The Coevolution of Parochial Altruism and War," *Science*, 319, 636–640.

Collier, P., and A. Hoeffler (2001): "Greed and Grievance in Civil War," *Policy Research Paper*, 2355, World Bank.

Darling, R., and J. Norris (2008): "Differential Equation Approximations for Markov Chains," *Probability Surveys*, 5, 37–79.

Deegan, J., and E. W. Packel (1982): "To the (Minimal Winning) Victors Go the (Equally Divided) Spoils: A New Index of Power for Simple N-Person Games," in *Political and Related Models*, ed. by S. Brams, W. Lucas, and P. Sraffin, pp. 238–255. New York.

DeNardo, J. (1985): *Power in Numbers: The Political Srategy of Protest and Rebellion*. Princeton Univ. Press, Princeton.

Dobson, S., J. Goddard, and F. Stahler (2008): "Effort Levels in Contests: An Emprical Application of the Tullock Model," mimeo.

Dupuy, R. E., and T. N. Dupuy. (1986): *The Encyclopedia of Military History from 3500 B.C. To the Present*. Harper and Row, New York, 2nd Edition.

Dupuy, T. N. (1987): *Understanding War*. Paragon House, New York.

Engel, J. H. (1954): "A Verification of Lanchester's Law," *Journal of the Operational Research*, 2, 163–171.

Epstein, J. M. (2002): "Modeling Civil Violence:An Agent-based Computational Approach," *Proceedings of the National Academy of Sciences*, 99(3), 7243–7250.

Esteban, J., and D. Ray (1999): "Conflict and Distribution," *Journal of Economic Theory*, 87, 379–415.

Esteban, J., and D. Ray (2001): "Collective Action and the Group Size Paradox," *American Political Science Review*, 95(3), 663–672.

Fearon, J. D. (1995): "Rationalist Explanations for War," *International Organization*, 49, 379–414.

Franks, N. R., and L. W. Partridge (1993): "Lanchester Battles and the Evolution of Combat in Ants," *Animal Behavior*, 45, 197–199.

Garcia, J., and J. C. V. D. Bergh (2008): "Evolution of Parochial Altruism by Multiselection," *Journal of Theoretical Biology*, forthcoming.

Garfinkel, M., and S. Skaperdas (2006): "Economics of Conflict: An Overview," in *Handbook of Defense Economics*, ed. by T. Sandler, and K. Hartley, chap. 3.

Gerschenkron, A. (1943): *Bread and Democracy.* University of Chicago Press, Berkeley and Los Angeles.

Gourevitch, P. (1977): "International Trade, Domestic Coalitions, and Liberty: Comparative Responses to the Crisis of 1873-1896," *Journal of Interdisciplinary History*, 8(2), 281–312.

Grossman, H. (1994): "Production, Appropriation, and Land Reform," *American Economic Review*, 84(3), 705–712.

——— (2001): "The Creation of Effective Property Rights," *American Economic Review*, 91(2), 347–352.

Hann, M., and P. Koorema (2002): "Free Riding and Provision of Candy Bars," *Journal of Public Economics*, 83, 277–291.

Harberger, A. C. (1954): "Monopoly and Resouce Allocation," *American Economic Review*, 44, 77–87.

Hart, S., and M. Kurz (1983): "Endogenous Formation of Coalitions," *Econometrica*, 51(4), 1047–1064.

——— (1984): "Stable Coalition Structure," in *Coalitions and Collective Action*, ed. by M. Holler, pp. 236–258. Physica-Verlag.

Hirshleifer, J. (1978): "Competition, Cooperation, and Conflict in Economics and Biology," *American Economic Review*, 68(2), 238–243.

——— (1985): "The Expanding Domain of Economics," *American Economic Review*, 75(6), 53–68.

——— (1989): "Conflict and Rent-Seeking Success Functions: Ratio vs Difference Models of Relative Success," *Public Choice*, 63(3), 101–112.

——— (1991): "The Technology of Conflict as an Economic Activity," *American Economic Review*, 81(2), 130–134.

Hirshleifer, J., and J. G. Riley (1992): *The Analytics of Uncertainty and Information*. Cambridge Univ. Press, Cambridge.

Isaac, R. M., and J. M. Walker (1988): "Group Size Effects in Public Goods Provision: the Volutary Contributions Mechanism," *Quartely Journal of Economics*, 103, 179–199.

Isaac, R. M., J. M. Walker, and A. Williams (1994): "Group Size and the Volutary Provision of Public Goods - Experimental Evidence Utilizing Large Group," *Journal of Public Economics*, 54, 1–36.

Jia, H. (2006): "An Empirical Study of Contest Success Functions : Evidence from the NBA," *mimeo.*

——— (2008): "A Stochastic Derivation of the Ratio Form of Contest Success Functions," *Public Choice*, 135, 125–130.

Kalyvas, S. N., L. Balcelles, and D. Rohner (2008): "International System and Technologies of Rebellion: Howe the Cold War Shaped Internal Conflict," *mimeo.*

Kandori, M., G. J. Mailath, and R. Rob (1993): "Learning, Mutation, and Long Run Equilibria in Games," *Econometrica*, 61(1), 29–56.

Kindleberger, C. (1951): "Group Behavior and International Trade," *Journal of Political Economy*, 59(1).

Kingman, J. (2002): "Stochastic Aspects of Lanchester's Theory of Warfare," *Journal of Applied Probability*, 39, 455–465.

Konrad, K. A. (2007): "Strategy in Contests - An Introduction," .

Krueger, A. O. (1974): "The Political Economy of the Rent-Seeking Society," *American Economic Review*, 64, 291–303.

Lanchester, F. (1916): *Aircraft in Warfare*. D. Appleton and Company, New York.

Lee, W., and J. Roemer (2006): "Racism and Redistribution in the United States: A Solution to the Problem of Amerian Exceptionalism," *Journal of Public Economics*, 90(6-7), 1027–1052.

Luebbert, G. M. (1991): *Liberalism, Fascism, or Social Democracy*. Oxford Univ. Press, New York.

Madras, N. (2002): *Lectures on Monte Carlo Methods.* American Mathematical Society.

Marwell, G., and O. Pamela (1993): *The Critical Mass in Collective Action: A Micro-Social Theory.* Cambridge Univ. Press, Cambridge.

McFadden, D. L. (1974): "Conditional Logit Analysis of Qualitative Choice Behavior," in *Frontiers in Econometrics*, ed. by P. Zarembka, pp. 105–142. Academic.

McGuire, M. (1974): "Group Size, Group Homogeneity and the Aggregate Provision of a Pure Public Good under Cournot Behavior," *Public Choice*, 18, 107–126.

McNaught, K. R. (1999): "The Effects of Splitting Exponential Stochastic Lanchester Battles," *Journal of the Operational Research Society*, 50(3), 244–254.

Moore, B. (1965): *Social Origins of Dictatorship and Democracy.* Beacon, Boston.

Nolan, M. (1986): "Economic Crisis, State Policy, and Working-Class Formation in Germany, 1870-1900," in *Working-Class Formation: Nineteenth-Century Patterns in Western Europea and the United States*, ed. by K. I., and A. Zolberg. Princeton.

Oliver, P., and G. Marwell (1988): "The Paradox of Group Size in Collective Action," *American Sociological Review*, 53, 1–8.

Olson, M. (1965): *The Logic of Collective Action.* Harvard Univerity Press, Cambridge, MA.

Owen, G. (1977): "Value of Games with a Priori Unions," in *Essays in Mathematical Economics and Game Theory*, ed. by R. Hein, and O. Moeschlin, pp. 76–88. New York.

Palmer, R. R., and J. Colton (1984): *A History of the Modern World*. Random House, New York, 6 edn.

Panchanathan, K., and R. Boyd (2004): "Indirect Reciprocity can Stabilize Cooperation without the Second-order Free Rider Problem," *Nature*, 432, 499–501.

Plowes, N. J. R., and E. S. Adams (2005): "An Empirical Test of Lanchester's Square Law: Mortality During Battles of the Fire Ant Solenopsis Invicta," *Proceedings of the Royal Society B*, 272, 1809–1814.

Rittberger, V. (1973): *Evolution and International Organization: Toward a New Level of Sociopolitical Integration*. Martinus Nijhoff, The Hague.

Roth, A. (1988): "Introduction to the Shapley Value," in *The Shapley Value: Essays in Honor of Lloyd S. Shapley*, ed. by A. Roth. Cambridge Univ. Press, New York.

Sambanis, N. (2004): "What is Civil War?," *Journal of Conflict Resolution*, 48(6), 814–858.

Samz, R. (1972): "Some Comments on Engel's "Verification of Lancehser's Law"," *Operational Research*, 20(1), 49–52.

Sandler, T. (1992): *Collective Action: Theory and Applications*. Univeristy of Michigan Press, Ann Arbor.

Schelling, T. (1978): *Micro Motives and Macro Behavior*. Norton, New York.

Schelling, T. C. (1980): *The Strategy of Conflict*. Havard Univ. Press, Cambridge, MA.

Sethi, R., and E. Somanathan (2006): "A Simple Model of Collective Action," *Economic Development of Cultural Change*, 54(3), 725–747.

Skaperdas, S. (1996): "Contest Success Functions," *Economic Theory*, 7, 283–290.

Tullock, G. (1967): "The Welfare Cost of Traffis, Monopolies, and Theft," *Western Economic Journal*, 5, 224–232.

——— (1971): "The Paradox of Revolution," *Public Choice*, 11, 89–99.

——— (1980): "Efficient Rent Seeking," in *Toward a Theory of Rent Seeking Society*, ed. by J. M. Buchanan, R. D. Tollison, and G. Tullock, pp. 3–15. Texas A and M University Press.

Von Neumann, J., and O. Morgenstein (1944): *Theory of Games and Economic Behavior*. Princeton Univ. Press, Princeton.

Weibull, J. (1995): *Evolutionary Game Theory*. MIT Press, Cambridge, MA.

Young, P. (1998): *Individual Strategy and Social Structure: An Evolutionary Theory of Institutions*. Princeton Univ. Press, Princeton.