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Sigma exchange in the nuclear force and effective field theory

JF Donoghue
donoghue@physics.umass.edu

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John F. Donoghue,

Department of Physics
University of Massachusetts
Amherst, MA 01003, USA
and
Institut des Hautes Études Scientifiques
Bures sur Yvette, F-91440, France

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Abstract

In the phenomenological description of the nuclear interaction a crucial role is traditionally played by the exchange of a scalar I=0 meson, the sigma, of mass 500-600 MeV, which however is not seen clearly in the particle spectrum and which has a very ambiguous status in QCD. I show that a remarkably simple and reasonably controlled combination of ingredients can reproduce the features of this part of the nuclear force. The use of chiral perturbation theory calculations for two pion exchange supplemented by the Omnes function for pion rescattering suffices to reproduce the magnitude and shape of the exchange of a supposed σ particle, even though no such particle is present in this calculation. I also show how these ingredients can describe the contact interaction that enters more modern descriptions of the inter-nucleon interaction.

When describing QCD to non-physicists, we generally say that it is the theory that accounts for nuclear binding. However, in practice our understanding of the precise way that QCD leads to nuclear bound states is still not good. Nuclear binding is traditionally described by an internucleon potential which can be parameterized by the exchange of mesons[1, 2]. The most important exchange producing binding in the central potential is a scalar isoscalar meson, the sigma, of mass around 500-600 MeV. While other exchanges in the potential are correlated with clear resonances seen in the particle spectrum, the sigma is a puzzle. It is not seen in the usual way in the spectrum and, after 40 years of debate, does not have a clear interpretation in terms of the quarks and gluons of QCD[1]. It is unfortunate that the key ingredient in the signature effect of the strong interactions has such an ambiguous status.

The expectation is that the sigma represents, in some way, the exchange of two pions. The quantum numbers certainly are correct for this. Sophisticated attempts that construct the potential from scattering data (e.g. [5]) have two pions as the lightest intermediate state. However, while phenomenologically useful, these are not able to answer the question of the fundamental nature of the sigma effect. Modern descriptions of the internucleon interaction use chiral perturbation theory to calculate two pion exchange at low energy[7]. However, these do not produce the sigma effect because the chiral amplitudes grow monotonically with the energy. In this paper I add a simple and well-motivated addition to the chiral description, i.e. the Omnes function describing pion rescattering. We will see that this will produce an interaction remarkably close in structure to the exchange of a 600 MeV sigma meson. It is clear that there is no resonance in this description, yet the needed properties of sigma exchange are reproduced[2].

Recently, there have been successful applications of ideas of effective field theory in which the nuclear interaction is treated not by potentials but by contact interactions - delta function interactions[7, 8]. At low energy (recall that the energy typical of nuclear binding is 10 MeV/nucleon) the result of the exchange of a heavy particle can be described by a local interaction. Mathematically, this is consistent with the potential description because, as the mass m gets large, the Yukawa potential forms a representation of a delta function. Physically, this follows from the uncertainty principle, as the exchange of a heavy particle has a short range. Nonlocality, to the extent

[1] A discussion of the status of the sigma which is very much in the spirit of the present work can be found in [3]. A careful recent analysis of $\pi\pi$ scattering describing the sigma as a pole on second sheet, quite far from the real axis, is found in [4]

[2] Other attempts to describe the nuclear interaction without a sigma are seen in [6]

it is needed, can then be described by contact derivative interactions. This development greatly increases the generality of the description of nuclei, as it reduces multiple potentials with different functional forms to a small number of constants giving the strengths of the contact interactions. I will also show how the use of chiral perturbation theory plus the Omnes function can provide a reasonably controlled calculation of the I = 0 scalar contact interaction.

In QCD, the low energy interactions of two nucleons obeys an unsubtracted dispersion relation in the different partial waves[1, 2, 5] Specializing to the scalar isoscalar channel, we can write the momentum space and coordinate space interaction as

$$
\begin{aligned}
V_S(q^2) &= \frac{2}{\pi}\int_{2m_\pi}^{\infty} d\mu\ \mu \frac{\rho_S(\mu)}{\mu^2+q^2} \\
V_S(r) &= \frac{1}{2\pi^2 r}\int_{2m_\pi}^{\infty} d\mu\ \mu\ e^{-\mu r}\ \rho_S(\mu) \qquad (1)
\end{aligned}
$$

Here ρ_S describes the physical intermediate states that occur at energy μ in the crossed channel. The threshold occurs at twice the pion mass and the lowest energy intermediate state is two pions.

At low energy these spectral functions can be rigorously calculated in chiral perturbation theory. The imaginary part of the Feynman diagrams describe the physical intermediate states and generate the spectral function ρ_S. For the diagrams of Figs 1 a,b,c these imaginary parts are[9, 10]

$$
\rho_S^{\mathrm{a,b}}(\mu) = \frac{3g_A^2}{64F_\pi^4}\left[4c_1 m_\pi^2 + c_3(\mu^2-2m_\pi^2)\right]\frac{(\mu^2-2m_\pi^2)}{\mu}\ \theta(\mu-2m_\pi) \qquad (2)
$$

$$
\begin{aligned}
\rho_S^{\mathrm{c}}(\mu) &= -\frac{3}{32\pi F_\pi^4}\sqrt{1-\frac{4m_\pi^2}{\mu^2}}\ \theta(\mu-2m_\pi) \qquad (3)\\
&\left(\left[4c_1 m_\pi^2 + \frac{c_2}{6}(\mu^2-4m_\pi^2) + c_3(\mu^2-2m_\pi^2)\right]^2 + \frac{c_2^2}{45}(\mu^2-4m_\pi^2)^2\right)
\end{aligned}
$$

Here c_1, c_2, c_3 are parameters that describe the $NN\pi\pi$ vertex - these have been measured in pion nucleon interactions[9, 10, 11]. I will address the box and crossed box diagrams below. These spectral functions are valid in the low energy regime only, and one observes that they grow monotonically with the energy.

However, there is another ingredient which necessarily enters. In the description of the $\pi\pi$ system, unitarity requires the inclusion of $\pi\pi$ rescattering.

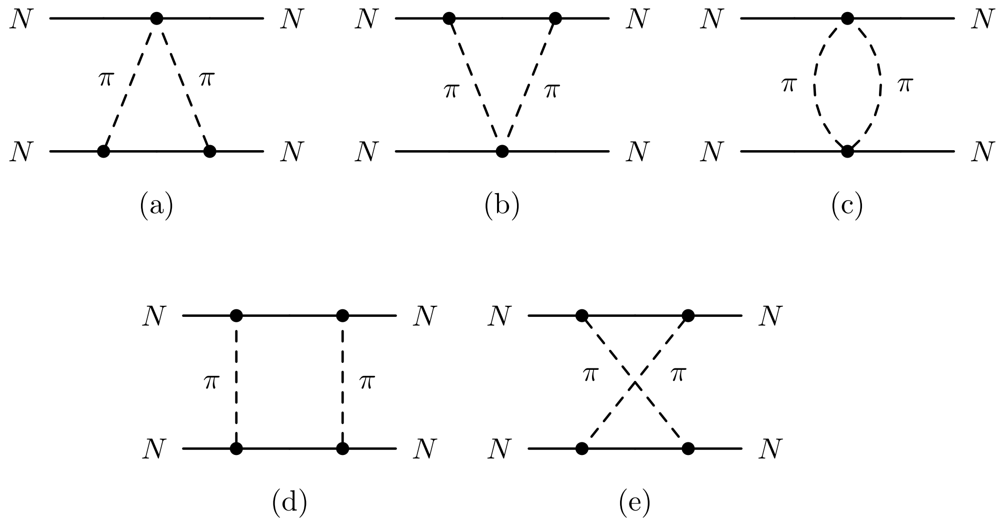

Figure 1: Two pion exchange diagrams which arise in chiral perturbation theory.

For a single elastic partial wave, unitarity of the S matrix and analyticity require a unique form of the solution, given originally by Omnes[12]. The amplitudes in the elastic region are described by a polynomial in the energy times the Omnes function

$$\Omega(\mu) = exp[\frac{\mu^2}{\pi} \int \frac{ds}{s} \ \frac{\delta(s)}{s-\mu^2}] \tag{4}$$

Here δ is the $\pi\pi$ scattering phase shift, in our case for the I=0, J=0 channel. Chiral perturbation theory is consistent with this order by order in the energy expansion. Following Ref. [13], it is known how to match this general description to the results of chiral perturbation theory by appropriately identifying the polynomial. The elastic region in this channel extends effectively up to energies of 1000 MeV.

In practice there has been good success at using the lowest order chiral amplitudes, supplemented by the Omnes function. An example close to the present problem is $\gamma\gamma \to \pi\pi$ in the S wave. Here a lowest order calculation supplemented with an Omnes function[14] yields results in close agreement with both experiment and with a two loop chiral calculation up to energies beyond 700 MeV[15].

I will adopt the Omnes solution matched to the leading order chiral result, and will explore possible modifications below. The description of the

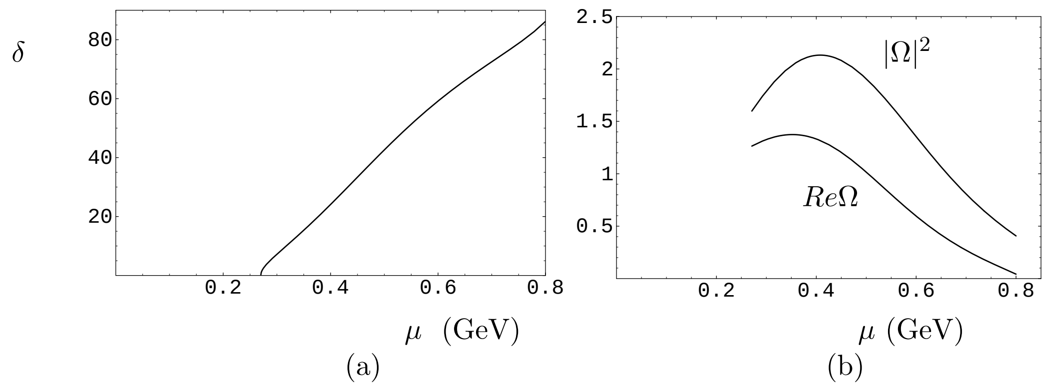

Figure 2: The left graph gives the pion phaseshifts that are the input into the Onmes function, while the right figure shows the real part and the absolute square of the Omnes function.

spectral function then becomes

$$\rho_S(\mu) = \rho_S^{a,b} Re\Omega(\mu) + \rho_S^c |\Omega(\mu)|^2 \tag{5}$$

The phase shifts can be analyzed in chiral perturbation theory in combination with experiment, with the definitive treatment of Colangelo et al (CGL)[16]. Their result for the I = 0, J=0 phase shift is shown in Fig 2, along with the resulting Omnes functions. Note that there is no sigma resonance in the phase shift near 300 - 600 MeV. A resonance in the elastic region is manifest by the phase shift passing through 90 degrees, which certainly does not happen near the sigma mass. (If one explores the complex plane there is a pole on the second sheet very far from the real axis[4].) In producing the Omnes function, I had to extend the phase shifts above the μ = 850 MeV endpoint of the CGL analysis in order that the principle value part of the Omnes function integral be well behaved near the upper end. As long as this extension is smooth it has little effect on this calculation.

With these ingredients, we can display the result for the scalar interaction. In Fig. 3, I show the result for ρ, along with the individual contributions of the diagrams of Fig 1. If we had a pure sigma exchange this would be delta function at the mass of the σ, or a Breit-Wigner shape corresponding to a narrow resonance. One could be forgiven for seeing this result as a very broad resonance, even though no resonance exists in the formalism. The coordinate space potential is shown in Fig. 3. Also shown for comparison is the potential of an infinitely narrow 600 MeV scalar with a normalization chosen to match. In practice these are hard to differentiate

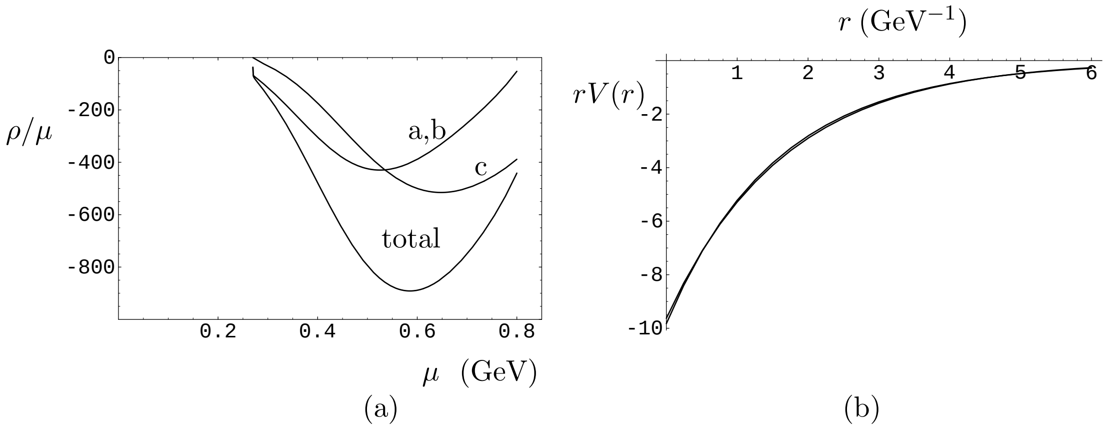

Figure 3: The left figure shows the our results for the spectral function $\rho(\mu)/\mu$ as well as the individual components of diagram 1 a,b,c. The right figure shows the coordinate space potential $rV(r)$. There are actually two curves in the figure on the right. One is the result of this calculation and the second is that of a narrow 600 MeV sigma, with normalization chosen to match. The curves cannot be distinguished.

because the curves are nearly identical. The simple description of Eq. 5 reproduces closely the spatial variation of the sigma potential. The strength of the interaction will be addressed below.

One can address the robustness of this result by considering possible higher order modifications of the basic representation. The $NN\pi\pi$ interaction has been described by the lowest order chiral Lagrangian. There are also energy dependent modifications to these low order results. In particular, we expect that there might be form factors depending on the energy. To probe this effect, let as modify the $\pi\pi$ interaction by a form factor.

$$c_3 \to \frac{c_3}{(\mu^2 + m^2)^n} \tag{6}$$

The choice of $n = 1$ and $m = 800$ is shown in Fig. 4. While the relative contribution of the two diagrams change (since the latter has the form factor squared) and the magnitude is different, the energy variation and spatial variation are remarkably similar to the original case. Use of a dipole form factor does not change this conclusion.

We can best address the strength of the interaction by describing the magnitude of the contact interaction G_s. For a narrow sigma this would

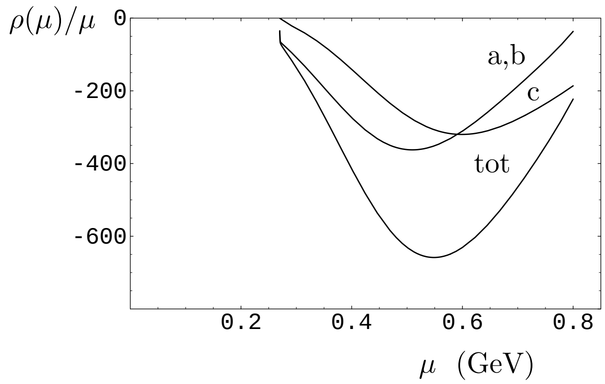

Figure 4: The spectral function $\rho(\mu)/\mu$ in the presence of higher order energy dependent modifications as described in the text

have the value g_σ^2/m_σ^2. There is some uncertainty in the appropriate value of G_S, because the preferred fit magnitudes depend somewhat on the calculational scheme used. However, phenomenological studies of the nuclei tend to require $G_S = 300 - 450$ MeV^{-2} . The contact interaction is given by the strength of the momentum space potential of Eq. 1 evaluated at $q^2 = 0$. This is just integral under the integrand shown in Fig. 3. The result depend most sensitively on the parameter c_3, which is not perfectly known. The phenomenological extraction of c_3 from πN data has a large error bar, $c_3 = -4.7^{+1.2}_{-1.0}$ GeV^{-2} [9, 11]. However, when using an Omnes representation, it is likely that this constraint is on the product $c_3\Omega(2m_\pi)$, in which case the value would be $c_3 = -3.7^{+1.0}_{-0.8}$ GeV^{-2}. (The other parameter choices used were $c_1 = -0.64$ GeV^{-2} and $c_2 = 3.3$ GeV^{-2}, although these have only a small impact on the results.) The result for G_S as a function of c_3 is shown in Fig 4. There is good agreement for the required range of magnitudes of G_S for the allowed values of c_3[7]. Here the use of a form factor does make a difference. With the form factor described above the value of G_S is 40% smaller than without it for a given value of c_3. However at present understanding this difference may be accounted for by adjusting the value of c_3. These uncertainties in the appropriate values of c_3 and G_S keep us from using the magnitude as a precise test of the method.

In an effective field theory treatment, one keeps pion exchange as an explicit degree of freedom while treating the shorter range interactions as contact terms. In such a treatment, we should treat the box and crossed

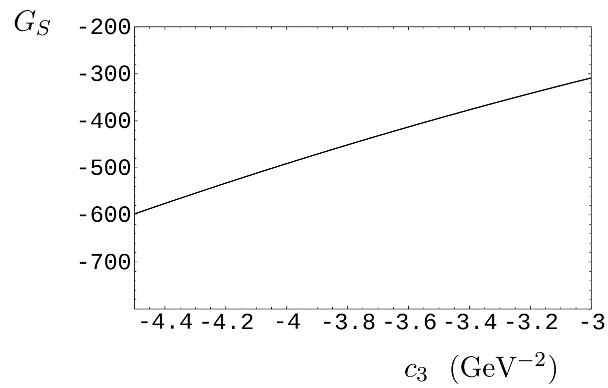

Figure 5: The strength of the scalar interaction as a function of the parameter c_3.

box diagrams of Fig 1 dynamically, and they should therefore not be included into the contact interaction. For this reason I did not include these diagrams in the calculation of the integrand $\rho_S(\mu)/\mu$ whose integral gives the strength of the contact interaction. However, it should be admitted that the different approximations schemes that are used to calculate nuclear properties treat the iteration of the one-pion exchange interaction in quite different ways. Use of time-dependent perturbation theory, field theoretic methods, mean-field methods, Bethe-Salpeter approximations, etc all capture different amounts of the box and crossed box diagrams. This is related to the serious ambiguity of what is meant by a potential in field theory[17], and this likely accounts for the range of fit values of G_S. It seems that for the scalar central potential the iteration of the one pion interaction is a numerically small compared to the irreducible two pion/sigma contribution, for example see [9] and Fig 3.15 of Ref. [2]. Hopefully this ambiguity will be cleared up as effective field theory techniques are extended to heavier nuclei[18].

Chiral perturbation theory plus the Omnes function give a quite simple description of the scalar central potential, with a result very similar to the exchange of a fictitious sigma particle. However fortunately there is no need in such a description to postulate such a scalar particle. This description appears to be robust, being qualitatively unchanged by the addition of higher order interactions. Besides elucidating a long standing puzzle, these results are useful because we have a reasonably solid control over all the ingredients, the chiral amplitudes and the $\pi\pi$ phase shifts. Questions such as the quark mass dependence of nuclear bending may now be addressed in a reasonable fashion[19]. The connection of the nuclear interaction to QCD becomes more under control.

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