Non-Computable Functions in Optimality Theory

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Non-Computable Functions in Optimality Theory

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Abstract

Is Optimality Theory a constraining theory? A formal analysis shows that it is, if two auxiliary assumptions are made: (1) that only markedness and faithfulness constraints are allowed, and (2) that input and output representations are made from the same elements. Such OT grammars turn out to be incapable of computing circular or infinite chain shifts. These theoretical predictions are borne out by a wide range of natural phonological processes including augmentation, alternations with zero, metathesis, and exchange rules. The results confirm, extend, and account for the observations of Anderson & Browne (1973) on exchange rules in phonology and morphology.
1. Introduction

"Phonological theory", say Prince and Smolensky (1993, herinafter P&S, p. 67), "contains two parts: a theory of substantive universals of phonological well-formedness, and a theory of formal universals of constraint interaction." The theory they espouse, Optimality Theory (hereinafter OT), claims to provide an exhaustive statement of how constraints interact, leaving the grammarian with just two tasks: to discover what the universal constraints are, and how they are ranked in particular languages.

In this view, languages differ only in how they rank the universal constraint set. Predictions about what will turn up in natural languages follow from P&S's "factorial typology" — one looks at the empirical consequences of all possible rankings of the universal constraint set. This naturally involves knowing what the constraints actually are.

There is another way to go about it, which this paper will illustrate. If we have reason to believe that every constraint (or at least every constraint relevant to a particular question) has such-and-such a property, it may follow that the constraint hierarchy as a whole is incapable of computing certain mappings from underlying to surface forms. A local property of each individual constraint can entail a global property of the whole grammar, thus allowing us to make predictions without exact knowledge of the constraints.

We will use this method to test a possibility that has haunted the OT literature for some time (Kirchner 1995), and which can be roughly stated as follows:

(1) There are only markedness and faithfulness constraints.

This will turn out to be an over-broad generalization. In order to prove our theorem we will be compelled to narrow it down to assert only that certain kinds of constraints must be either of the markedness or the faithfulness variety — again roughly speaking, those constraints which refer exclusively to segmental phonology or low-level prosodic features. As a necessary intermediate step, we will propose precise formal definitions of markedness and faithfulness.

It will be seen to follow that phonological processes involving only those elements cannot create certain kinds of exchange-rule, metathesis, or augmentation effects, while processes involving other elements (such as morphology) can do so. This extends and explains an observation originally due to Anderson & Browne (1973) to the effect that exchange-rule effects are always morphologically triggered.

The paper is organized as follows: Section 2 translates OT and our auxiliary hypotheses into formal terms so that we can prove theorems about them. The actual proofs are done in Section 3. Section 4 draws real-world conclusions and tests them against the empirical data. A final summing-up and discussion are to be found in Section 5.

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1 This paper owes a lot to the efforts of people besides the author. I would like to thank Isadora Cohen, Shu-Chen Susan Huang, André Isaak, Jo-Wang Lin, Mauricio Mixco, Willem de Reuse, Lisa Selkirk, Michael Walsh Dickey, and most particularly John McCarthy for consultation and critique. A special debt is owed the resourceful people at the UMass Interlibrary Loan.

The 1999 ROA archive version has been slightly edited from the original 1996 manuscript to remove minor typographical errors and snide comments. Correspondence is welcome; please address it to elliott@linguist.umass.edu or Department of Linguistics, University of Massachusetts, Amherst, MA 01003, USA.
2. Formal properties of Optimality Theory

In order to ask formally, "What can OT do?", we need precise formal definitions of "OT" and "do". We'll begin with the easier task, introducing the notion of computability by a grammar in Section 2.1 as a formalization of "do".

Formalization of "OT" will proceed by stages, starting in Section 2.2 with a broader class of formal grammars called constraint-hierarchy grammars which will turn out to be computationally uninteresting—for any given function of the relevant type, there exists a constraint-hierarchy grammar that computes it.

Section 2.3 introduces the auxiliary assumptions which define a computationally much more interesting model called here “classical OT”, in which underlying and surface representations are constructed in the same fashion (“homogeneity”), there is always a fully faithful candidate (“inclusivity”), and every constraint is either a markedness constraint or a faithfulness constraint (“conservativity”).

2.1. Phonological grammars

Phonological theories normally involve at least three levels of linguistic representation:

- The underlying (phonological) representation, assembled directly from information stored in the lexicon. This forms the input to the phonological component of the grammar. Underlying representations will be written between slashes: /kentapal/.

- The surface (phonological) representation, which is the output of the phonological component and input to the phonetic component, here written between square brackets: [kentapal].

- The phonetic representation, which is what the speaker actually does with their mouth, written here between double quotes: "kentapal".

Optimality Theory is a theory of constraint interaction, not of representations. We want our deductions about OT to hold even if the theory of representations changes. Hence, we have to treat input and output forms as unanalyzable atoms:

The term grammar will refer to a function G from a countable set A ("inputs") to subsets of another countable set B ("outputs"). A grammar G such that the inputs and outputs are precisely the underlying and surface phonological representations, respectively, of a natural language, and such that G performs the same mapping as a speaker's phonological competence, will be called a natural grammar.

In natural grammars, these subsets of B usually contain only one element: free phonological (as distinguished from phonetic) variation is less common, and will be ignored here. In order to simplify exposition, we shall incorrectly assume that all natural grammars have the property of exactness:

(2) **Defn** Let A and B be two countable sets, and suppose G: A → 2^B is such that ∀ a ∈ A, |G(a)| = 1. Then G is said to be exact.
Confining our interest to the class of exact grammars, we are now able to formulate the question, “what does a grammar do?”.

(3) **Defn** Suppose \( A \) and \( B \) are countable sets, and \( G : A \rightarrow 2^B \) is an exact grammar. We say that \( G \) computes the function \( f : A \rightarrow B \) if \( \forall a \in A, G(a) = \{f(a)\} \).

Our question now becomes: for a given class of exact grammars from \( A \rightarrow 2^B \), what functions can be computed by a member of that class?

### 2.2. Constraint-hierarchy grammars and Optimality Theory

Let \( A, B \) be countable sets.

(4) **Defn** A constraint over \( A \times B \) is any function \( C : A \times B \rightarrow \mathbb{N} \) such that the domain of \( C \) is all of \( A \times B \). If \( a \in A, b \in B \), we write \( C/a[/b] \) (rather than the more usual \( C(a,b) \) — as a reminder of which argument corresponds to the underlying representation, and which to the surface representation). Then \( a \) is called the input to \( C \), and \( b \) is called the candidate. The value of \( C/a[/b] \) is called the score awarded by \( C \) to the candidate \( b \) for the input \( a \).

(5) **Defn** A constraint hierarchy over \( A \times B \) is an ordered \( n \)-tuple \( C = (C_1, \ldots, C_n) \), where each \( C_i \) is a constraint over \( A \times B \).

(6) **Defn** A score vector (of length \( n \)) is an ordered \( n \)-tuple \( v = (r_1, \ldots, r_n) \), where \( r_i \in \mathbb{N} \).

(7) **Defn** Let \( v = (r_1, \ldots, r_n), v' = (r'_1, \ldots, r'_n) \) be score vectors. We say \( v < v' \) iff \( \exists k \) such that

(i) \( \forall i < k: r_i = r'_i \)

(ii) \( r_k < r'_k \)

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2 Intuitively, a constraint is a measure of how bad a given input-candidate pair is in a particular respect. Here we are allowing the badness score to be no smaller than zero, and to be arbitrarily large. We have to justify both of these assumptions.

In constructing a grammar to describe competence in a particular language, one might find it convenient, and linguistically "insightful", to hypothesize a constraint \( C \) which gives negative scores. If there is a number \( n \) which is the smallest score that \( C \) ever awards, then \( C \) is just a notational variant of an orthodox constraint \( C' \), where \( C'/a[/b] = C/a[/b] + n \). Replacing \( C \) with \( C' \) does not change the output of any constraint-hierarchy grammar containing \( C \). If allowing negative scores is to have any detectable effect, we must allow arbitrarily large negative scores. But then we are no longer guaranteed that Lemma (8) will hold; it is possible to construct sets of score vectors with no smallest element. The "detectable effect" is to make \( G \) possibly uncomputable. Hence, we can get along without negative-valued constraints.

To forbid constraints which give arbitrarily large positive scores would certainly simplify the definition of the \( \leq \)-relation on score vectors, since then, as P&S (p. 200) point out, there would be a simple order-preserving isomorphism between score vectors and the natural numbers. However, any such constraint-hierarchy grammar would just be a special case of the class of grammars described here, so the only effect on our arguments in this paper would be to make the proof of Lemma (8) trivial.

3 I'm ignoring the possibility that there might be infinitely many constraints, because it complicates matters and is not linguistically plausible.
If it is not true that $v < v'$, we say $v \geq v'$. If $v < v'$ or $v = v'$, we say $v \leq v'$. The $<$ relation is clearly transitive, antisymmetric, and irreflexive, so it is a total ordering.

(8) \textbf{Lemma} Let $V$ be a nonempty set of score vectors of length $n$. Then $V$ contains a minimal element; i.e., $\exists v_0 \in V$ such that $\forall v \in V$, we have $v_0 \leq v$.\footnote{This lemma is essential for the success of P\&S's H-eval; without it there is no guarantee that the algorithm will terminate.}

\textbf{Proof} Choose any $v^* \in V$, where $v^* = (r_1^*, \ldots, r_n^*)$. Let $L = \{v' = (r_1', \ldots, r_n') \mid r_i' \leq r_i^* \text{ for all } i \leq n\}$. Then $L$ is finite, since the $r_i'$ are bounded below by 0 and above by $r_i^*$. It follows that $L \cap V$ is finite, and therefore (since $<$ is a total ordering of $V$) contains a minimal element $v_0$. Then $v_0$ is also a minimal element of $V$: Let $v \in V$. If $v \in L$, then $v_0 < v$ by minimality of $v_0$ in $L$. If $v \notin L$, then $v^* < v$ by definition of $L$, so by transitivity $v_0 < v$. Hence $v_0$ is minimal in $V$.

(9) \textbf{Defn} Let $C = (C_1, \ldots, C_n)$ be a constraint hierarchy over $A \times B$, $a \in A$, $b \in B$. We define $C / a / [b]$ to be the score vector $(C_1/a/[b], \ldots, C_n/a/[b])$, and say that $C / a / [b]$ is the score awarded by $C$ to the candidate $b$ relative to the input $a$.

(10) \textbf{Defn} Let $G$ be a 3-tuple $(A \times B, \text{Gen}, C)$ such that

(i) $A \times B$ is a countable set;

(ii) $\text{Gen} : A \rightarrow 2^B$ is such that $\forall a \in A$, $\text{Gen}(s) \neq \emptyset$;

(iii) $C$ is a constraint hierarchy over $A \times B$.

For any $a \in A$ we define

$$G(s) = \{b_0 \in B \mid C / a / [b_0] \leq C / a / [b] \text{ for all } b \in \text{Gen}(a)\}$$

Then $G$ is said to be a constraint-hierarchy grammar. The element $a$ is called the input, the set $\text{Gen}(a)$ is the candidate set, and the set $G(b)$ is the output.

(11) \textbf{Lemma} If $G = (A \times B, \text{Gen}, C)$ is a constraint-hierarchy grammar, then $\forall a \in A$, $G(a) \neq \emptyset$.

\textbf{Proof} By (ii) of (9) above, $\text{Gen}(a) \neq \emptyset$. Hence we can apply Lemma () to the set $V = \{C / a / [b] \mid b \in \text{Gen}(a)\}$. This set contains a minimal element $v_0$, and $G(a)$ is simply the (obviously nonempty) subset of $S$ whose elements receive a score of $v_0$ relative to $a$. QED.

Constraint-hierarchy grammars are, as a class, uninteresting:

(12) \textbf{Lemma} Let $A, B$ be any countable sets, and let $f : A \rightarrow B$ be any function defined on all of $A$. Then there exists a constraint-hierarchy grammar $G = (A \times B, \text{Gen}, C)$ that computes $f$.

\textbf{Proof} There are at least two equally trivial ways to construct such a grammar.

(i) (Trivial Gen) For any $a \in A$, let $\text{Gen}(s) = A$. Let $C = (C_1)$, where for any $a \in A, b \in B$, $C_1$ is given by
C_1/a/[b] = 0 iff b = f(a)
C_1/a/[b] = 1 otherwise.

Then obviously \( \forall a \in A, G(a) = \{f(a)\} \).

(ii) (Trivial C) For any \( a \in A \), let \( \text{Gen}(a) = \{f(a)\} \). Let \( C = (C_1) \), where \( C_1 \) is the trivial constraint such that for any \( a \in A, b \in B, C_1/a/[b] = 0 \). Then obviously \( \forall a \in A, G(a) = \{f(a)\} \).

The claim that every natural-language grammar can be computed by some constraint-hierarchy grammar is therefore an empty one. We already know it to be true, since it rules nothing out. Definition (10) provides a framework or notational device, rather than a theory of language. If we want falsifiable predictions, we will have to constrain \( \text{Gen}() \) and \( C \) to prevent the tricks used in (12 i) and (12 ii). The next section of this paper investigates a widely-used model which does so constrain them, and which consequently cannot compute certain functions.

2.3. The “classical OT” model

The model we are going to look at is a variant of the standard version of Optimality Theory, as presented in, e.g., Prince & Smolensky (1993), and as commonly practiced in OT work, such as that collected in Beckman, Walsh Dickey, and Urbanczyk (1995) and the Rutgers Optimality Archive (http://ruccs.rutgers.edu/roa.html). This framework, which we will call "Classical OT", makes two assumptions which disallow the tricks used in (12).

One is that for any input \( /a/ \), \([a] \in \text{Gen}(/a/) \); that is, the candidate set always contains a fully faithful candidate. This forestalls (12 i), the trivial-\( C \) trick.

The other is that there are only two types of constraint: faithfulness constraints, which favor candidates that are like the input over those that differ from it, and markedness constraints, which favor candidates that lack some configuration over those that have (have) it. Intuitively, markedness constraints represent the tendency of a grammar to prefer certain surface forms over others, while faithfulness constraints represent the tendency to keep the output like the input. This disposes of (12 ii), the trivial-\( \text{Gen}() \) trick, since the single constraint needed to make the trick work will not necessarily belong to either of these categories.

The first of these assumptions is standard (see, e.g., P&S p. 80); the second, while implicit in much OT work (e.g., Kirchner 1995), has not to my knowledge been explicitly and seriously proposed — and with good reason, as we shall see.

It will be necessary to add one more hypothesis, homogeneity, which asserts that input and output representations are made out of the same structural elements. This is not a standard OT assumption, nor are we proposing it as an axiom about natural language. Rather, it will be used in the theoretical argument to prove a theorem, and in the empirical argument to single out the class of real-world processes to which we expect the theorem to apply.

In the following subsections (2.3.1 - 2.3.5), we formalize the classical OT assumptions and briefly discuss their interpretation. The next section, Section 3, will explore the computational power of classical OT grammars, and show how its hypotheses restrict the class of computable functions.
2.3.1. \textit{Gen()} and the fully faithful candidate

This is an uncontroversial assumption\textsuperscript{5}, whose formalization is repeated here in (13):

\begin{equation}
\text{Defn} \quad \text{If for some constraint-hierarchy grammar \((A \times B, Gen, C)\) we have} \quad \forall a \in A, [a] \in Gen (/a/), \text{then Gen() is said to be \textit{inclusive}.}
\end{equation}

Classical OT grammars have inclusive Gen() functions; P&S (p. 80) even propose a version in which Gen(/a/) = A for all \(a \in A\).

Inclusivity implies that \(A \subseteq B\). This will be take up again in when homogeneity is discussed in Section 2.3.4.

2.3.2. Markedness constraints

A markedness constraint is one that ignores the input and looks only at the candidate:

\begin{equation}
\text{Defn} \quad \text{If} \ C \text{ is a constraint over} \ A \times B \text{ such that} \quad \forall a, a' \in A, C/a/ = C/a'/, \text{ then} \ C \text{ is a} \textit{markedness constraint.}\textsuperscript{6}
\end{equation}

Markedness constraints are therefore those that penalize ill-formed surface structures. Examples include

(15) \textbf{Ex} The Onset Constraint (ONS). Syllables must have onsets (P&S, p. 16).

(16) \textbf{Ex} *PL/Lab. Don’t have [+labial] Place (P&S, p. 181).

(17) \textbf{Ex} OCP [Obligatory Contour Principle]. Adjacent identical elements are prohibited (adapted from Urbanczyk 1995, originally due to Leben and to Goldsmith).

(18) \textbf{Ex} CODA-COND. A [Place] node linked to a coda position must also be linked to something else. (Itô 1986).

Since the input is irrelevant to a markedness constraint, we can omit the first argument and write simply “C[/b]”.

2.3.3. Faithfulness constraints

A faithfulness constraint is one which always gives a perfect score to the fully faithful candidate.

\begin{equation}
\text{Defn} \quad \text{If} \ C \text{ is a constraint over} \ A \times B \text{ such that} \quad \forall a \in A, C/a/ [a] = 0, \text{ then} \ C \text{ is a} \textit{faithfulness constraint}.
\end{equation}

Familiar examples include\textsuperscript{7}

\textsuperscript{5} McCarthy & Prince (1995) represent the underlying form of the reduplicative morpheme as an abstract symbol RED, which cannot surface at all since it is not of the right representational type to appear in outputs.

\textsuperscript{6} The constraint C is of course really a two-place function; however, \(C/a/\) is a one-place function obtained by saturating the first argument of C.

\textsuperscript{7} Surprisingly, P&S’s constraints FILL and PARSE are not faithfulness constraints at all under this definition, but rather markedness constraints. The P&S Gen() function neither
(20) **Ex** DEP-IO. Every segment of the output has a correspondent in the input [discourages epenthesis] (McCarthy & Prince, 1995)

(21) **Ex** MAX-IO. Every segment of the input has a correspondent in the output [discourages deletion] (McCarthy & Prince 1995).

(22) **Ex** IDENT-IO(nas) If an input segment and an output segment are in correspondence, they have the same value of [nasal] (McCarthy & Prince, 1995).

### 2.3.4. Homogeneity of inputs and outputs

We have saved the most unintuitive assumption for last. It will be both convenient and crucial to assume further that $A = B$, that is, that the grammar is **homogeneous**. Convenient, because it allows us to state and prove theorems about constraint interaction much more simply; crucial, because (as we shall see later) it is precisely the homogeneous part of natural grammars that is well-behaved with respect to the theorems we shall derive.

(23) A grammar $G$: $A \rightarrow 2B$ is said to be **homogeneous** if $A = B$.

Once we start talking about formal grammars that mimic natural grammars, $A$ will become the set of possible underlying representations, and $B$ the set of possible surface representations. A real linguistic theory defines $A$ and $B$ by enumerating the formal elements used to build these representations (nodes, association lines, etc.) and the rules for putting them together. To claim that $A = B$ is to claim that, in the grammars of present interest, underlying and surface phonological representations are made of the same components, assembled in the same way.

Homogeneity is not a credible condition on OT grammars of natural languages. Right from the very start, OT grammar fragments have assumed the input to contain structures never found in outputs, and vice versa.

It is, for example, a common assumption that the input to the "phonological" component of the grammar is heavily annotated with nonphonological information — phonologically empty morphemes (e.g., McCarthy & Prince 1995), morphological constituency (e.g., Beckman 1995), Level 1 vs. Level 2 affix class (e.g., Benua 1996), part of speech and case (e.g., P&S’s Lardil), syntactic constituency (Truckenbrodt 1996), and much more. None of these annotations can be changed by Gen(), no output is ever unfaithful to them, and none are in the candidate output set.\(^8\)

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epenthesizes nor deletes segments; it only constructs a prosodic parse of the input; all candidates therefore have the same segments parsed in different ways. FILL penalizes candidates with empty prosodic structure; PARSE penalizes candidates with unparsed segments. Neither one needs to refer to the input in order to assign scores. The fully faithful candidate (which has no prosodic structure at all) will either trivially satisfy or trivially violate either one, depending on how the constraints are formulated, so they may or may not be faithfulness constraints as defined here.

\(^8\) True, it is conventional to represent morpheme boundaries in the candidates. This is just to make them easier to read. With the advent of Correspondence Theory (McCarthy & Prince 1995), there is no reason to mark boundaries in the candidates -- since Gen() can't change morphological affiliations, one can tell which morpheme a given surface segment is affiliated to by looking at its correspondent in the input. Furthermore, as John McCarthy (p.c., 1995) pointed out in reference to an earlier draft of this paper, leaving the boundary marks in the candidates leads to absurdity if, say, two segments separated only by a morpheme boundary are
There are also structures commonly supposed to be present in the output but not the input. Phonological phrase boundaries, for instance, seem to be determined entirely by the interaction of constraints (Truckenbrodt 1996). After all, where would the phrasing in the input come from? Not the lexicon, surely. And not the syntax alone either, since phrasing exhibits highly unsyntactic properties like rate- and length-sensitivity (Shih 1986, Du 1988). Moreover, if a phrase boundary somehow made it into the input, Truckenbrodt’s constraints would in effect erase it and overwrite it; the output would be faithful only by coincidence.

But most “phonological” representations are in fact present in both input and output: distinctive features, low-level prosodic constituency such as syllables or prosodic words, association lines. These are the elements that turn up in the lexicon, and that can be changed by Gen(). Homogeneous grammars can deal with much of the core business of phonology — assimilation, dissimilation, segmental inventories, phonotactics, syllable structure, phonologically conditioned allophony, anything resulting from the influence of sounds on other sounds.

In a natural grammar, those representational elements that are found in both inputs and outputs we may call homogeneous elements. Those constraints which refer only to homogeneous elements in both arguments we may call homogeneous constraints. Any natural grammar will have nonhomogeneous elements and nonhomogeneous constraints, but it will also have homogeneous elements and homogeneous constraints. This paper will argue that, if we confine our attention to phonological processes involving only homogeneous elements, we will need only conservative constraints to implement them. We will accordingly so confine our attention for the rest of the theoretical discussion (up to the end of Part 3).

2.3.5. Summary: Classical OT

The model of OT with which we are concerned here is thus the one satisfying the following postulate:

\[(24) \text{Defn} \quad \text{Let } G = (S \times S, \text{Gen}, C) \text{ be a homogenous constraint-hierarchy grammar. If every } C_i \text{ is either a markedness constraint or a faithfulness constraint, we say that } C \text{ is conservative. If, in addition, Gen()} \text{ is inclusive and } G \text{ is exact, then we say that } G \text{ is a classical OT grammar.}
\]

3. Computable functions in classical OT grammars

We are now in a position to show that classical OT is a constraining theory; there are functions it cannot compute, and hence phonological phenomena it predicts to be nonexistent. Our argument can be stated informally as follows:

metathesized. What happens to the plus juncture then?

This half of homogeneity is like "Consistency of Exponence" (McCarthy & Prince 1994), only a bit stronger, since it applies to more than just morphology.

9 The stock of available representational elements is here assumed to be universal; so is the status of each one as homogeneous or non-homogeneous. A homogeneous element is homogeneous in all languages -- Richness of the Base (P&S) makes it available in the input, and Gen() makes it available in the output. If that element never surfaces in a given language, we assume that some constraints conspire to stifle it.
The requirement in (24) of a conservative C and inclusive Gen() means that if the output is not identical to the input, it must be less marked than the input. To see why, notice that the inclusive-Gen() requirement means that the candidate set always contains the input, which scores perfectly on all faithfulness constraints. Since the output can’t do better on the faithfulness constraints, it must do better on the markedness constraints.

If, therefore, a classical OT grammar sends underlying /A/ to surface [B], then [B] must be less marked than the fully faithful candidate [A]. It follows that the grammar cannot also send underlying /B/ to surface [A], since this would entail that [A] is less marked than the fully faithful candidate [B], and hence that [A] is less marked than itself.

We will show more generally that a classical OT grammar cannot compute circular chain shifts — any function that sends /A1/ → [A2], /A2/ → [A3], ..., /An/ → [A1] — or infinite chain shifts — any function that sends /A1/ → [A2], /A2/ → [A3], .... Furthermore, this is an if-and-only-if theorem: any function that does not give rise to a circular or infinite chain shift can be computed by a classical OT grammar.

These statements are formalized and proven in Section 3.1 as the characterization theorem for classical OT grammars, defining precisely what functions classical OT can and cannot compute.

### 3.1. The characterization theorem

We begin by establishing that the output of a classical OT grammar is either the input, or something less marked than the input.

#### (25) Defn
Let C be a conservative constraint hierarchy. Define $C_M$ and $C_F$ to be the constraint hierarchies consisting respectively of the markedness constraints ($C_{M1}, ..., C_{Mp}$) and the faithfulness constraints ($C_{F1}, ..., C_{Fq}$) of C, such that the dominance relations in $C_M$ and $C_F$ are the same as those in C.

#### (26) Lemma
Let $G = (S, \text{Gen}, C)$ be a classical OT grammar, and suppose $a, b \in S$ are such that $a \neq b$ and $G(a) = \{b\}$. Then $C_M[a] > C_M[b]$.

**Proof** By definition of the function $G(s)$, we have $C/a/\{a\} > C/a/\{b\}$. By definition of faithfulness, though, we have $C_F/a/\{a\} = (0, 0, ..., 0) \preceq C_F/a/\{b\}$. If it were also true that $C_M[a] \leq C_M[b]$, then it would follow that $C/a/\{a\} \preceq C/a/\{b\}$, contradicting our hypothesis. Hence $C_M[a] > C_M[b]$. QED.

This sharply restricts the set of computable functions:

#### (27) Defn
Let $f:S\rightarrow S$ be any function. Let $f^0(s)$ represent $f(f(\ldots(s)\ldots))$, with f iterated n times. Let $f^0(s) = s$. Suppose that for any $s \in S$, there is a smallest number $\pi(s)$ such that $\pi(s)(s) = \pi(s)+1(s)$. Then we say that $f$ is eventually idempotent, and that $\pi(s)$ is the potential of $s$ under $f$.

#### (28) Thm (characterization theorem for classical OT grammars)
Let $f:S\rightarrow S$ be any function. Then f is computable by a classical OT grammar if and only if f is eventually idempotent.

**Proof** ($\Rightarrow$) Suppose f is computable by a classical OT grammar $G = (S, C)$. Let $a$ be any element of S, and consider the sequence $A = (\{a\}, G(a), G^2(a), G^3(a), \ldots)$, where again
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G^k(a) represents G iterated k times on a. (Each G^i(a) is guaranteed to exist by the homogeneity of G.) For convenience’s sake let us write A = ([a^0], [a^1], [a^2], ...). Let M = {CM(a^0), CM(a^1), ...}. This is a set of score vectors, so by Lemma (8) it has a minimal element CM(a^K) for some K. Then CM(a^K) ≤ CM(a^(K+1)). By Lemma (8), this means a^K = a^(K+1). But a^K = f^K(a), so f^K(a) = f^(K+1)(a), making f eventually idempotent. QED.

(⇐) Suppose f is eventually idempotent. We will construct a classical OT grammar G = (S, C) that computes f. Let C = (C_1, C_2), where for any a, b ∈ S,

C_1/a/[b] = 0 iff b ∈ {a, f(a)}
C_1/a/[b] = 1 otherwise
C_2[b] = π(b)

where π(b) is the potential of b under f, a number which is guaranteed to exist since f is eventually idempotent. Then G is a classical OT grammar, since C_1 is a faithfulness constraint and C_2 is a markedness constraint. For any input a ∈ S, the candidates will receive the following score vectors:

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Score Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>(0, π(a))</td>
</tr>
<tr>
<td>f(a)</td>
<td>(0, π(f(a)))</td>
</tr>
<tr>
<td>others</td>
<td>(1, unknown)</td>
</tr>
</tbody>
</table>

The set containing the candidate with the smallest score vector is G(a). Clearly, if f(a) = a, then G(a) = a = {f(a)}. If f(a) ≠ a, then π(f(a)) = π(a)-1, so (0, π(f(a))) < (0, π(a)), and again G(a) = {f(a)}. QED.

(29) Corollary Suppose G = (S, C) is an OT grammar. Then there exists a classical OT grammar G' = (S, C') such that G' computes the same function as G, and C' = (C'_1, C'_2).

Proof Done already in the proof of (28).

So everything that can be done in classical OT can be done with just one faithfulness constraint and one markedness constraint! This does not mean that classical OT is in any sense trivial. Quantifier order is important here. What Corollary (29) says is that for any eventually idempotent function, there exists a two-constraint classical OT grammar that computes it. What OT claims is that there exists a set of constraints such that for any phonological system found in nature, the constraints can be ranked to give a classical OT grammar that computes it.

Two constraints that would handle, say, Berber, could not be reranked to yield any other language, nor are they likely to be at all linguistically insightful or psychologically real. Any list of hypothetical constraints that might plausibly be the universal constraints will be much more interesting than that.

10 Not a unique one, though. In the construction of C'_2, we can replace "1" with "2" or "1776" or whatever we please, as long as it doesn’t equal 0.
3.2. Phonological processes incompatible with classical OT

Classical OT can compute all and only eventually idempotent functions; hence, a natural grammar that was not eventually idempotent would be a clear counterexample. What makes a function \( f: S \rightarrow S \) fail to be eventually idempotent? There are only two possibilities:

- **Infinite chain shift**: This occurs when \( S \) contains an element \( s \) such that \( s, f(s), f^2(s), ... \) are all distinct, as shown schematically in Figure (30).

- **Circular chain shift**: This occurs when \( S \) contains an element \( s \) such that for some \( k > 0 \), \( f^k(s) = s \), as shown schematically in Figure (31).

An eventually idempotent function \( f \) will therefore look like Figure (32). Elements having equal potential under \( f \) are shown at the same height from the bottom of the diagram; the right-hand column shows what that potential is.
The phonological interpretation is a bit tricky. Suppose there is a natural grammar $N$ containing a phonological process which causes underlying /b/ to emerge as [p] syllable-finally, and underlying /p/ to emerge as [b] syllable-finally. Suppose further that we know there exists an underlying form /rab/ (perhaps morphologically /ra/ + /b/). Then $N(rab) = [rap]$, so $N^2(rab) = N(rap) = [rab]$. Wouldn’t this be an instance of circular chain shift?

The answer is that we can’t tell, because we don’t really know that the phonological representations are /rab/ and [rab]. The output could differ in all sorts of inaudible ways from the input\(^{11}\).

On the one hand, maybe the input isn’t what we think it is. It could be festooned with features and prosodic structures that we never hear, because faithfulness to them is low-ranked and so they never surface\(^{12}\). Or there might be some predictable information that isn’t present in the underlying forms, but is added to the output by the phonology, such as allophonic variation.

\(^{11}\) We are only talking about \textit{homogeneous} elements here. It is also true by definition, though not germane, that inputs and outputs differ as to which of the nonhomogenous representational elements they have.

\(^{12}\) This idea -- specifying in the lexicon structures that never surface -- has been proposed as a
On the other hand, maybe the output isn’t what we think it is either. It is, after all, the output of the phonology, which is not directly observable, since between it and us stands the phonetics. The output could have picked up or lost any number of purely formal features that weren’t present in the input. If there were two adjacent segments in the input, each with a [+nasal] autosegment, the output could easily contain one doubly-linked [+nasal] and one unlinked one; we would never know the difference.

In other words, Theorem (28) is directly applicable only when we are sure we know what the underlying and surface forms actually look like, but sometimes we don’t.

Fortunately, there is a way around this problem. Consider the rap-rab example again:

\[(33) \text{ Input forms } /\text{rap } \& \ X_1/ \quad /\text{rab } \& \ X_2/ \]

\[(\text{Output forms } [\text{rap } \& \ Y_1] \quad [\text{rab } \& \ Y_2])\]

\[(\text{Phonetic forms } "\text{rap}" \quad "\text{rab}" )\]

where the Xs and Ys represent the inaudible structures accompanying the given forms, and the arrows show the action of N.

Our question is not really "Is there a classical OT grammar G which computes N ?", but rather "Is there a linguistically plausible classical OT grammar G which computes N ?", i.e., one in which each constraint makes linguistic sense. Suppose that, knowing what we do about the language in question and about linguistics in general, we are willing to make the following claim:

means of getting around a knotty paradox in tonology (e.g., John McCarthy, UMass/Amherst class lectures, Fall 1995). The tone-bearing unit in the Bantu languages seems to be the syllable, and tones can be lexically specified as associated with certain tone-bearing units. Hence a tone must be specified as associated with a certain syllable, meaning that the syllable in turn must also be lexically specified. But since syllabification in Bantu is predictable, to lexically specify syllabification would be redundant -- usually a sign that the grammarian is missing something important. The solution is to link a tone in the lexicon to a syllable node, which is linked to a mora node, which is linked to whatever segment is destined to be the nucleus of the syllable with that tone. Thus, the minimum of structure needed to locate the tone is supplied at the cost of lexicalizing a horribly wrong syllabification. This is not a very large cost, however, since faithfulness to the lexical syllabification is low-ranked, and a much better one will inevitably be imposed upon the output.
For any linguistically plausible conservative constraint hierarchy $\mathbf{C}$,

\[
\mathbf{C}^{F}_{/\text{rap} \& X_1 / [\text{rap} \& Y_1]} \leq \mathbf{C}^{F}_{/\text{rap} \& X_1 / [\text{rap} \& Y_2]}
\]

\[
\mathbf{C}^{F}_{/\text{rap} \& X_2 / [\text{rap} \& Y_2]} \leq \mathbf{C}^{F}_{/\text{rap} \& X_2 / [\text{rap} \& Y_1]}
\]

In other words, suppose we think that in any set of constraints that has any hope of being the sought-after universal constraint set, the output that we hear as "rap" is more faithful to underlying /rap/ than the output form heard as "rab" is, and that the output corresponding to "rab" is more faithful to underlying /rab/ than the output corresponding to "rap" is. Then Lemma (25) tells us that

\[
\mathbf{C}^{M}_{[\text{rap} \& Y_1]} > \mathbf{C}^{M}_{[\text{rap} \& Y_2]}
\]

\[
\mathbf{C}^{M}_{[\text{rap} \& Y_2]} > \mathbf{C}^{M}_{[\text{rap} \& Y_1]}
\]

which can't possibly be true. Therefore, no such $\mathbf{C}$ can exist.

We can relate this (rather uninsightfully) to the characterization theorem by noting that if we had a conservative constraint hierarchy $\mathbf{C}$ for the hypothetical language of the example, we could construct a grammar $G = (S, \mathbf{C})$ where $S = \{\text{rap} \& Y_1, \text{rab} \& Y_2\}$. Then $G(\text{rap} \& Y_1) = \{[\text{rab} \& Y_2]\}$, and $G(\text{rab} \& Y_2) = \{[\text{rap} \& Y_1]\}$. Then $G$ would not be eventually idempotent, so by the characterization theorem, such a $\mathbf{C}$ could not exist.

In general, the configurations that are ruled out are the following ones, in which the arrows indicate the action of $G$, and in which $[b_i]$ is assumed to be more faithful to /a_i/ than $G(a_i)$ is.

(36) **Infinite rotation**

\[
/\text{a}_0/ /\text{a}_1/ /\text{a}_2/ /\text{a}_3/ /\text{a}_4/ \ldots
\]

\[
[\text{b}_1] [\text{b}_2] [\text{b}_3] [\text{b}_4] \ldots
\]

(37) **Circular rotation**

\[
/\text{a}_0/ /\text{a}_1/ /\text{a}_2/ /\text{a}_3/ /\text{a}_4/ \ldots /\text{a}_n/
\]

\[
[\text{b}_0] [\text{b}_1] [\text{b}_2] [\text{b}_3] [\text{b}_4] \ldots [\text{b}_n]
\]
Stated formally,

\[ \textbf{Thm} \ (\text{rotation theorem}) \ \text{Let } S \text{ be a countable set, and let } N: S \rightarrow 2^S \text{ be exact. Suppose } A \text{ and } B \text{ are nonempty subsets of } S, \text{ and suppose there exist functions } f, g : A \rightarrow B \text{ such that} \]

(i) \ f \text{ and } g \text{ are bijections from } A \text{ to } B

(ii) \ \forall \ a \in A, \ f(a) \neq g(a)

(iii) \ \forall \ a \in A, \ f(a) = G(a)

(iv) \ \forall \ a \in A, \ CF/a/[g(a)] \leq CF/a/[f(a)]

Then there does not exist any classical OT grammar \( G = (S \times S, C) \) such that \( G(s) = N(s) \) for all \( s \in S \).

\[ \text{Proof} \] By contradiction. Suppose such a \( G \) exists; then \( C \) is conservative. By (i), \( h = f^{-1}g \) is a bijection from \( B \) to \( B \). Choose any \( b_0 \in B \), and let \( b_i = h^i(b_0) \). Let \( a_i = g^{-1}(b_i) \). Then \( f(a_i) = g(a_{i+1}) = b_{i+1} \).

By (ii), we have \( h(b_i) \neq b_i \) so \( b_i \neq b_{i+1} \). Therefore, by (iii), \( C/a_i/[b_i] > C/a_i/[b_{i+1}] \) (since \( [b_{i+1}] = G(a_i) \), the victorious candidate).

By (iv), \( CF/a_i/[b_i] \leq CF/a_i/[b_{i+1}] \). Hence, \( CM[b_i] > CM[b_{i+1}] \). Since \( i \) can become arbitrarily large, the set \( \{CM[b_i]\mid i = 0, 1, 2, \ldots\} \) has no least element. But Lemma (8) tells us that it must have a least element. \rightarrow\leftarrow \text{ Ergo, no such } G \text{ can exist. QED.} \]

(The "only if" half of the characterization theorem follows from this as a corollary by setting \( A = B = S \) and letting \( g \) be the identity function.)

Theorem (38), and the other theorems hitherto proven, are representationally neutral — i.e., independent of any theory of phonological representations. The hypothesized /\( a_i /\) could be a string of taxonomic phonemes, or a string of feature matrices, or a sophisticated autosegmental tier structure parsed into a prosodic tree — it doesn’t matter. As long as \( \text{Gen()} \) and \( C \) satisfy the hypotheses of conservativity, the conclusion of eventual idempotence follows.

**4. Empirical test of Classical OT**

What we have up to this point is a mathematical theorem which can be informally summarized as follows:

\[ \textbf{(39)} \] Suppose you have written an OT grammar which has the following properties:

(i) \ The inputs and candidates are made from the same kit of representational elements ("homogeneity").

(ii) \ There is always a fully faithful candidate ("inclusivity").

(iii) \ Every constraint is either a markedness constraint or a faithfulness constraint
("conservativity").

Then that grammar cannot compute circular or infinite rotations.\(^{13}\)

In order to translate this into a testable prediction about natural languages, we have to confront the problem, already discussed in § 2.3.4, that natural languages are far from homogeneous. Our solution is to use parsimony and assume that if a really existing phonological process can be described homogeneously, then it must be. Take for example a familiar phenomenon like codanausal place assimilation in Japanese (Itô 1986). Since the entire process takes place without reference to nonhomogeneous features like part of speech, we assume that in any plausible natural grammar of Japanese, none of the constraints active in this process make reference to nonhomogeneous elements. We can then excise that part of the grammar responsible for the process and apply the theorem to it in isolation.

Hypothesis (39 ii), inclusivity, is a theory-internal assumption which is taken to be true whenever (39 i) is true; its validity will not be questioned here.

Hypothesis (39 iii), conservativity, is a claim about natural languages, as discussed in §2.3. It is what is being tested. If this hypothesis is correct, then the only way for the forbidden rotations to arise is for Hypothesis (39 i), homogeneity, to be violated. The empirical test of conservativity is therefore:

\[(40)\] Suppose a process is found in a natural language which creates a circular or infinite rotation. Then the process can only be expressed with reference to nonhomogeneous representational elements.

In order to falsify this claim, we must seek something very specific: a process that satisfies the hypotheses of Theorem (38), but violates its conclusion. This process must relate homogeneous sets of underlying and surface representations, which means

- An SPE-style rule relating two intermediate levels of representation in a suspicious fashion (e.g., the famous SPE vowel-shift rule) is not relevant, since OT does not recognize any intermediate levels.

- A process which relies upon some representational feature found only in inputs or only in candidate outputs is not relevant, since Theorem (38) requires homogeneity. Processes triggered by, e.g., morphology or by phrase-level phonology therefore do not count.

The violation must occur synchronically and productively, so that we know it is occurring in the grammar rather than being a lexicalized historical relic.

We can imagine at least five different processes that could cause forbidden rotations. In order of increasing plausibility, they are:

- **Unconditional augmentation.** Regardless of how long the input is, the output is longer.

- **Symmetrical metathesis.** Underlying /ab/ surfaces as [ba], while underlying /ba/ becomes [ab].

\(^{13}\) An ordinary finite linear chain shift can be computed without any difficulties by a classical OT grammar, as pointed out by Kirchner (1995), who uses conservative constraints to implement chain shifts of lengths two and three in various languages.
• **Alternations with zero.** The language deletes and epenthesizes the same segment in the same environment, so that /abc/ → [ac], while /ac/ → [abc].

• **Segmental exchange rules.** These rules invert the value of a feature specification, changing, e.g., voiced to voiceless and vice versa.

• **Rules of paradigmatic replacement.** The language arbitrarily substitutes one segment or autosegment for another, turning /a/ into [b] and /b/ into [a].

We will show that the attested instances of these phenomena can be divided into four groups:

\[(41)\] Those that apply between intermediate levels of a serial derivation.

Those whose triggering environment refers to morphological information.

Those whose triggering environment refers to phrase-level prosodic information.

Those that were misreported.

In other words, it never happens that the hypotheses of Theorem (38) are fulfilled and its conclusions violated. For homogeneous processes, therefore, it seems that only markedness and faithfulness constraints are necessary.

This extends and generalizes an empirical observation about segmental exchange rules due to Anderson and Browne (1973). We state it in OT terms as follows:

\[(42)\] **Conservativity of phonology**

If a constraint refers only to homogeneous representational elements — those occurring both in inputs and in candidate outputs — then it is either a markedness constraint or a faithfulness constraint, in the sense of Definitions (14) and (19).

### 4.1. Unconditional augmentation

A natural grammar with a phonological requirement that the output be longer than the input would create an infinite chain shift, violating Theorem (28) (and hence Theorem (38)): /ba/ → [baba], /baba/ → [bababa], et cetera. This simply doesn’t happen, ever (John McCarthy, p.c. 1995). Augmentation is always to meet some static target condition — word minimality, say, or a syllable-weight requirement — that fixes a floor beneath which the input is not allowed to fall. These facts are eminently consistent with the conservativity of phonology.

### 4.2. Symmetrical metathesis

A metathesis rule that simply switched two segments around irrespective of their original order, such as that shown in (43), would exhibit circular rotation.

\[(43)\] \(V_1V_2 \rightarrow V_2V_1\)

(Provided, of course, that the language had two different vowel phonemes.) We would see effects like

\[(44)\] /boa/ → [bao]
Syncretic metathesis rules of any sort seem to be extremely rare. They have been claimed to exist for at least the following languages: Clallam, Classical Arabic, Hanunoo, Indonesian, Jamaican Creole, Kiliwa, Korean, Leti, Lithuanian, Tagalog, Tiberian Hebrew, Rotuman, Sierra Miwok, Yagua, and Zoque. (See Webb 1974 and Últan 1976 for comprehensive surveys of alleged metathesis rules; also Chomsky & Halle 1968, Thompson & Thompson 1969, Langdon 1976, Kenstowicz & Kisseberth 1979, Besnier 1987, Goldsmith 1990, Hume 1995 (citing van der Hulst and van Engelenhoven 1994) for examples and discussion of particular languages.) With one exception, the rules described do not meet our criteria — they are morphological, or nonproductive, or diachronic, or interdialectal, or they destroy their own triggering environment when they apply and hence cannot apply to their own output to create a circular chain shift.

The lone exception is Kiliwa, a Yuman language described by Mixco (1971; see also Langdon 1976) which at the time was spoken by a small community (Mixco knew of fourteen speakers) in Baja California. The language is probably still alive today, since there were at least five monolingual children in the language community at the time.

Kiliwa has three vowel phonemes, /i, u, a/, which are variously realized on the surface. There are two degrees of vowel length, contrastive only in stressed syllables, and three pitch accents: H (´), L (,), and HL (ˆ), also contrastive only in stressed syllables. Among the consonant phonemes is a glottal stop ///, which never surfaces intervocally.

Mixco (1971) gives the following rule affecting glottal stops:

$$\begin{align*}
\text{Mixco's Rule 16} & \quad /C_1C_2/ \rightarrow [C_2C_1] / \; \# \; X \; _{} \; V \\
\text{where } C_1C_2 = /?N/ \; \text{or} /N?/ \; \text{and } V \; \text{is stressed.} \\
\end{align*}$$

"A sequence of a glottal stop and a nasal when preceded by another segment in pretonic position undergo a reversal in sequential order."

Mixco (1971) has only the following examples:

$$\begin{align*}
(46) & \quad /N?/ \rightarrow [?N] \\
/h + ñ + ?u:/ & \rightarrow [h?ñôː] \quad \text{‘mother’} \\
p + m + ?t:/ & \rightarrow [p?mêː] \quad \text{‘you said’} \\
s + m + ?t:/ & \rightarrow [s?mêː] \quad \text{‘desiduative modal aux.’} \\
\end{align*}$$

$$\begin{align*}
(47) & \quad /?N/ \rightarrow [N?] \\
/? + ñ + ñÔː/ & \rightarrow [?’ñ?êː] \quad \text{‘I do’} \\
\end{align*}$$

Phonetically, the [?N] sequence is "very likely to be glottalized", and causes the following vowel (a stressed root vowel) to be uttered with creaky voice, while the [N?] sequence is "a real cluster" (M. Mixco, p.c. 1995).\(^{14}\)

\(^{14}\) In the following three cases from Mixco (1971), the environment exists for /N?/ → [?N] metathesis, but it doesn’t happen: /pæː + m + ?t:/ → [pæː m?êː] 'that’s the way to say it’ (p.
Mixco (1971) is a synchronic grammar of Kiliwa. Once he has finished stating his phonological rules, he stops giving the surface forms of utterances except in a few isolated spots; we see only the underlying morphophonemic representations. There are at least thirteen such cases cited where the environment for (45) exists. These cases are given in (48) and (49). Recent personal communication (M. Mixco, p.c., 1995) indicates that metathesis occurs only in the forms in (48), but not in those of (49)\(^{15}\).

(48) \(/?N/ \rightarrow [N?]\) (metathesis)

\[
\begin{align*}
/\mathbf{?}+?+\mathbf{n}i:+p+m+a/? & \quad \text{‘I+do+Declar.+Emph.’ (181)} \\
/t+p+?+\mathbf{n}i:/ & \quad \text{‘Progr. + I do’ (164)} \\
/\mathbf{m}a?+p \, m+?+\mathbf{n}i: \, s+\mathbf{u}/ & \quad \text{‘you’ll do it (you you+do Potential+be)’ (172)}
\end{align*}
\]

(49) \(/?N/ \rightarrow [?N]\) (no metathesis)

\[
\begin{align*}
/h+u?+\mathbf{m}áq+p/ & \quad \text{‘to turn back’ (page 183)} \\
/h+u?+\mathbf{m}l/ & \quad \text{‘to give name’ (183)} \\
/h+u?+\mathbf{m}á:/ & \quad \text{‘to help to eat’ (183)} \\
/k\mathbf{W}+?+\mathbf{m}á:/ & \quad \text{‘I’ve eaten’ (163)} \\
/s+?+\mathbf{n}i:y+m/ & \quad \text{‘to point that way’ (139)} \\
/k\mathbf{W}án \, h+?+\mathbf{n}i:/ & \quad \text{‘wife’s mother’ (121)} \\
/\mathbf{k}i/s+h+?+\mathbf{n}i:/ & \quad \text{‘wife’s father’ (121)} \\
/h+?+\mathbf{n}i:/ & \quad \text{‘husband’s mother’ (121)} \\
/\mathbf{n}âp+n+k\mathbf{W}+?+\mathbf{m}á:w/ & \quad \text{‘son’s child’ (119)} \\
/h+?+\mathbf{n}i:+\mathbf{u}/ & \quad \text{‘oh, mother!’ (113)}
\end{align*}
\]

The \(/?N/ \rightarrow [?N]\) half of the metathesis rule seems to apply only to forms of the verb ‘to do’ — a very restricted environment indeed. Mixco remarks (p.c., 1995) that the etymologically related verb ‘to have’ does not undergo metathesis:

(50) \(/?N/ \rightarrow [?N]\) (no metathesis)

\[
\begin{align*}
/\mathbf{n}i+:i:+y/ & \quad \text{Dative+Copula+Attributive} \\
/\mathbf{?}+\mathbf{n}i+:i:+y/ & \quad \text{1+Dat+Cop+Attr ‘I have’} \\
/\mathbf{?}+\mathbf{n}i+:i:+y+u/ & \quad \text{1+Dat+Cop+Attr+Pl:subj ‘We have’}
\end{align*}
\]

There are only two possibilities: either the metathetic surface forms are out-and-out lexicalized (and hence are not derived by the phonology at all), or else the metathesis is triggered specifically by the morpheme ‘to do’, and hence, relying on a non-homogeneous representational element, does not fall in the scope of the theorem.

---

\(^{129}\) /? + m + ? + ā:/ → [ʔ?m?ā:] ‘Nom.+Stat.+‘say, be(cop.)’” (p. 130), /? + ŋ + ?i: + y/ → [ʔ?ŋ?:e:y] ‘belongs to me’ (p. 62). Mixco (1971) says that /? + ŋ + ?i: + y/ is a “lexicalized exception”; he has no comment on the other two examples. There is also the puzzling /?m?á: + sī/ → [ʔ?m?á:sī] ‘someone’ (p. 131). If we follow Rule (45) to the letter, we expect /?m?á:/ → “[ʔ?m?á:]” metathesis of two segments in a root. The rule was probably meant to require that the metathetic segments be separated by a morpheme boundary. This would also rule out metathesis in /nāy ?n?i:y i+k\mathbf{W}+mā:t+t/ ‘my(masc.) dead child’ (p. 122)

\(^{15}\) Incompatible computer character sets make Mixco’s vowel transcriptions illegible; this is why surface forms are not given in (49) and (50). However, it is crystal-clear what happens to the nasal and glottal stop.
4.3. Alternations with zero

A language which epenthesized and deleted the same segment in the same environment would generate a circular chain shift, as shown hypothetically in (51):

(51)

a. \( \emptyset \rightarrow e \rightarrow \emptyset \)

b. \( /bile/ \rightarrow [bil] \rightarrow /bil/ \)

There are languages which epenthesize and delete the same segment, but invariably in disjoint environments.

Eastern Massachusetts English (McCarthy 1993) epenthesizes [r] between a prosodic-word-final short vowel and a following vowel, but deletes it between a short vowel and a consonant in the next word. Sawai\(^{17}\) (Whisler 1992) requires the intonational phrase to end with a CV syllable, and will append an [e] if necessary to meet this target. At the same time, speakers delete [e] from a root-final closed syllable when it is followed by certain suffixes\(^{17}\). In Mohawk (Postal 1979, Hopkins 1987), vowel hiatus is resolved by deleting the vowel which is “weaker” (lower) on the language-specific scale \([i] > [o] > [\ddot{a}] > [e] > [i] > [a]\); the three weakest vowels \([e \ i \ a]\) are epenthesized in various consonantal environments to break up consonant clusters or provide syllable nuclei. The dialect of Bedouin Hijazi Arabic described by Al-Mozainy (1981) deletes underlying short high vowels from open syllables, but epenthesizes [i] in the environment \(C\_C\) to create a closed syllable word-finally. The high back unrounded [i] of Korean is reportedly deleted from verb stems or affixes to remedy vowel hiatus, and epenthesized in loan words to break up consonant clusters (Ahn 1991).

In no case\(^{18}\) does the insertion and deletion of the same segment give rise to a rotation forbidden by classical OT, again lending support to the contention that phonology is conservative.

4.4. Segmental exchange rules

A forbidden rotation could also be created by a segmental exchange rule, also known as a polarity rule, flip-flop rule, or alpha-switching rule. Such a rule inverts distinctive-feature values in such a way as to change Segment A into Segment B, and vice-versa, creating the circular rotation shown in the hypothetical example below:

\(^{16}\) An Austronesian language spoken by ten to twelve thousand people in Maluko Province, Indonesia.

\(^{17}\) It is not clear from Whisler’s description whether the deletion is part of the the realization of the morphemes in question, or whether it is a phonological consequence of, e.g., stress shifting.

\(^{18}\) A possible counterexample from Georgian is described by Fähnrich (1987): [v] is “usually” deleted before [o] and [u] (p. 25). Vowel hiatus, however, is “sometimes” resolved by epenthesis, “chiefly” of [v] and [b] (p. 27). Our immediate doubts about the regularity of this phenomenon are only deepened when he later informs us that verbs whose stems end in [-ev] exhibit many “peculiarities” in the passive conjugation, among them that they lose the [v] before the suffix [-od] (p. 82). If the deletion rule were regular, this would not be “peculiar”.
The term "polarity" was coined early in this century by the Africanist Carl Meinhof, who thought it a common organizing principle not only of phonology but of morphology as well (Meinhof 1912). The debate over the existence or nonexistence of phonological exchange rules dates from the publication of The Sound Pattern of English in 1968, in which they are used to account for the synchronic reflex of the Great English Vowel Shift. For discussion of the theoretical implications within rule-based phonology, see, e.g., Chomsky (1967), Chomsky & Halle (1968), McCawley (1974), Anderson & Browne (1973), Anderson (1974, 1975).

Chomsky & Halle's English data is not relevant here. The rotation relation they propose in their Vowel Shift Rule holds only between intermediate levels of representation; the rotated vowels undergo diphthongization and other mutations which destroy the triggering environment of the polarity rules (Chomsky & Halle 1968, p. 188). The same is true of their rules of Rounding Adjustment (op. cit., p. 217) and Backness Readjustment (op. cit., p. 209) (which in any case applies only "in certain irregular forms" such as mouse~mice, break~broke), and of Bever's (1967, pp. 85ff.) Menomini vowel-glide coalescence rule. The debate, however, sent many linguists off on a hunt for examples from other languages: Menomini (Bever 1967, Chomsky & Halle 1968), Czech (Wolfe 1970, Anderson & Browne 1973), Old Prussian (Wolfe 1970, Anderson & Browne 1973), Tiberian Hebrew (Malone 1972), Icelandic (Anderson 1972), Luo (Gregersen 1972, Anderson & Browne 1973), Dinka (Anderson & Browne 1973), and Flemish Brussels Dutch (Zonneveld 1976).

In their comprehensive review of the subject, Anderson & Browne (1973) conclude that "the class of segmental exchange rules is limited to the domain of morpholexical processes"; i.e., the triggering environment for the exchange refers at least in part to morphology; McCawley (cited in Zonneveld 1976) writes that "the more plausible examples that I know of of exchange rules … share an important characteristic: their environment is not phonological but morphological." A striking example is the use of voicing inversion in the inflectional morphology of the Luo noun (Gregersen 1972, Okoth-Okombo 1982).

Zonneveld (1976) presents data which is supposed to reflect a phonologically conditioned exchange rule in the Dutch dialect spoken in Flemish Brussels in the early 1930s (van Loey 1933, Mazereel 1931). Rules of Precluster Shortening (53) and Auxiliary Verb Reduction (54) result in a regular and productive alternation of /V:/ with [V].

\[(53) \quad /V/ \rightarrow [-long] / \_C1C2^{19}\]

\[(54) \quad /V/ \rightarrow [-long] / \_[\_\_\_\_\_\_\_\_, -stress, Aux]\]

For most vowels, that is the extent of the alternation. However, the back vowels exhibit a change of vowel quality as well: [u:]~[o], [o:]~[u]. Zonneveld's data (taken from van Loey 1933 and Mazereel 1931) is here cited in full:

\[(55) \quad [u:] \sim [o]

\[
[m\text{uko}] \quad \text{'to make'} \quad \quad [\text{vomakt}] \quad \text{'made'}
\]

\[^{19} \text{There are some constraints on the consonants: } C_2 \text{ must be an obstruent, and } C_2 \text{ has to be a plosive if } C_1 \text{ is a liquid (Zonneveld 1976).}\]
Non-computable functions in Optimality Theory

<table>
<thead>
<tr>
<th></th>
<th>'be ashamed'</th>
<th>'ashamed'</th>
</tr>
</thead>
<tbody>
<tr>
<td>[səˈmɑː]</td>
<td></td>
<td>[sə&amp;mɑː]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>'Fleming'</th>
<th>'Flemish'</th>
</tr>
</thead>
<tbody>
<tr>
<td>[vləˈmɪŋk]</td>
<td></td>
<td>[vləms]</td>
</tr>
</tbody>
</table>

(56) \[\alpha \sim [u]\]

<table>
<thead>
<tr>
<th></th>
<th>'foot'</th>
<th>'foot [diminutive]'</th>
</tr>
</thead>
<tbody>
<tr>
<td>[vɔt]</td>
<td></td>
<td>[vutsʰa]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>'to call'</th>
<th>'[s/he] calls'</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ro:pa]</td>
<td></td>
<td>[rupt]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>'must [stressed]'</th>
<th>'must [unstressed]'</th>
</tr>
</thead>
<tbody>
<tr>
<td>[moxtə]</td>
<td></td>
<td>[muto]</td>
</tr>
</tbody>
</table>

We seem to have here a perfect configuration for the rotation theorem (38). For any phonotactically permissible X and Y, we expect \([XuY]\) to be more faithful to \(/Xu:Y/\) than \([XoY]\) is, and we expect \([XoY]\) to be more faithful to \(/Xo:Y/\) than \([XuY]\) is.

(57)

The hypotheses of Theorem (38) are satisfied, hence, Flemish Brussels Dutch cannot be modelled by a classical OT grammar. Since the conditioning environment is purely phonological (the change of vowel quality is triggered by shortening), we conclude that phonology is not necessarily conservative.

Or do we?

The forms given in (55-56) do not come from contemporary informants; they were culled from a 1931 book (Mazereel) and a 1933 paper (van Loey); the latter simply repeats some of the former’s data without adding anything new. The three linguists’ descriptions of the same data are shown below.

(58) Flemish Brussels Dutch vowel shortening: back vowels

<table>
<thead>
<tr>
<th></th>
<th>Mazereel (1931)</th>
<th>Van Loey (1933)</th>
<th>Zonneveld (1975)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[oː] ~ [u, ə]</td>
<td>[oː] ~ [u]</td>
<td>[oː] ~ [u]</td>
<td></td>
</tr>
<tr>
<td>[uː] ~ [u, ə]</td>
<td>[uː] ~ [ə]</td>
<td>[uː] ~ [ə]</td>
<td></td>
</tr>
</tbody>
</table>
In (59), we have the entirety of Mazereel's data on shortening\textsuperscript{20}, from which van Loey's and Zonneveld's is drawn.

(59) $[u:] \sim [\circ]$

\begin{tabular}{lll}
[muka], [moka] & 'to make' & [vomokt] & 'made' \\
[struct], [stro:t] & 'street' & [strotsl\d] & 'street [diminutive]' \\
[rup] & 'turnip' & [ropk\a] & 'raapken' \\
[dup\d] & 'baptize' & [dups\l] & 'baptism' \\
[s\xomta] & 'be ashamed' & [s\xomta] & 'ashamed' \\
[v\xomn\ld] & 'Fleming' & [\v\lms] & 'Flemish' \\
\end{tabular}

$[\circ] \sim [u]$

\begin{tabular}{lll}
[vot] & 'foot' & [vutsl\a] & 'foot [diminutive]' \\
[moxt\a] & 'must [stressed]' & [mut\a] & 'must [unstressed]' \\
[don] & 'to do [stressed]' & [dun] & 'to do [unstressed]' \\
[rop\a] & 'to call' & [rupt] & '[s/he] calls' \\
\end{tabular}

There is clearly disagreement about what the vowel alternation really is. Where Zonneveld has $[\circ] \sim [u]$ and $[u:] \sim [\circ]$, Mazereel has $[\circ] \sim [u]$ and $[u:] \sim [\circ]$.

Zonneveld's $[\circ]$ is "the usual" mid back vowel, and $[\circ:]$ presumably that times two; his $[u]$ is "the usual" high back vowel, and $[u:]$ twice that.

Mazereel's $[\circ]$ is tense mid back rounded\textsuperscript{21}; there is no $[\circ:]$. It is clearly distinct from his $[\circ]$, which is a lax mid back rounded vowel\textsuperscript{22} whose long counterpart has the same quality: $[\circ:]$. His $[u]$ is tense high back rounded\textsuperscript{23}; "$d[le] [u:]$, he says, "is de lange [u]" (Mazereel 1931, p. 16).

Lest there should be any doubt that $[\circ]$ and $[\circ:]$ are phonetically different, van Loey (1933, p. 314) illustrates the difference for his Dutch-speaking readers with a minimal pair showing their phonemic difference in Modern Dutch: $[\bom]$ 'bomb' versus $[\bom]$ 'tree'.

Thus, while the $[\circ] \sim [u]$ half of the alternation is confirmed, the original sources contradict the claim that $[u:]$ alternates with $[\circ]$. If anything, it alternates with $[\circ:]$. That's not all. A linguist who has done field work on the dialect has this to say:

"[I]t is the case that long /o:/ becomes /u/ in vowel shortening contexts, but it is not the case that /u:/ becomes /O/ (open o) in the Brussels dialect. This is because /u:/ does not exist in the Brussels dialect; what Mazereel, and other people as well represent as /u:/ is not [u:], but a mid back hyperrounded vowel, most likely with retracted tongue root as well. . . . [I]t clearly is not an [u:]." (W. de Reuse, electronic p.c., 1995. Quoted by permission.)

\textsuperscript{20} Mazereel (1931), pp 26-27, 44-45.
\textsuperscript{21} "De [\circ] is geronde wijde middelachterklinker (mid back wide round)." (Mazereel 1931, p. 15. English gloss in original.)
\textsuperscript{22} "De [o] is een korte nauwe geronde middelachterklinker (mid back narrow round)." (Mazereel 1931, p. 15)
\textsuperscript{23} "De [u] is de geronde bovenachterklinker (high back wide round)." (Mazereel 1931, p. 16)
The actual alternation, then, is between [o:] and [u] on the one hand, and two completely different vowels on the other — quite a long way removed from circular rotation.\textsuperscript{24}

Anderson and Browne’s generalization therefore stands: if an exchange rule creates a circular rotation, then it is conditioned at least in part by something other than just the phonological representation — or, in OT terms, the constraints that cause it to happen are sensitive to something beyond just phonology. This again is consistent with our claim that phonology is conservative.

4.5. Paradigm replacement: Tone sandhi in Taiwanese\textsuperscript{25}

One final possibility must be considered. There exists a well-known, well-documented, and dramatic instance of circular rotation which does not fit into any of the aforementioned categories: the tone-sandhi alternations of Taiwanese. In addition to other alternations, in certain environments an underlying high level tone becomes mid level, a mid level tone becomes low falling, a low falling tone becomes high falling, and a high falling tone becomes high level, producing a circular rotation of length four. A similar circle of length two is found elsewhere in the tone-sandhi system.

Previous analyses have taken one of three lines on this phenomenon. One school of thought holds tonemes to be distinctive-feature matrices, and treats the sandhi mechanism as an exchange rule operating on tonal features (Wang 1967, Cheng 1968, 1973, Shih 1986). Another views the tones as complexes of autosegments, and the sandhi process as an autosegmental rule affecting autosegments and association lines; the circular effect is achieved by having several different phonological representations for each phonetic tone (Yip 1980, Wright 1983, Du 1988). Finally, Schuh (1978) argues that, whatever the representations may be, the tone-sandhi process treats each toneme as an atomic unit, substituting one for another according to an arbitrary pattern of ”paradigm replacement”. (For reasons to be explained below, I agree with him.) All researchers agree that something odd is going on, and all solutions have a contrived look to them.

The discussion to come will illuminate several aspects of the rotation theorem. In Subsection 4.5.1, we take a first look at the bare facts of the Taiwanese tone-sandhi alternation. A review of previous analyses illustrates the importance of the rotation theorem’s representation-neutrality, and the problems caused by our ignorance of the mapping between phonological and phonetic representations. We will conclude that the Taiwanese facts do indeed point to a circular rotation between the underlying and surface phonological representations — i.e., a violation of the rotation theorem. Our hypothesis that phonology is conservative predicts

\textsuperscript{24} It might not even be the business of the phonology at all. Van Loey cites these forms because he is trying to prove that certain vowels were present in an earlier stage of the Brussels Dutch dialect; part of his evidence for this is that they are still present as “shortened sounds” in various contemporary words, among them those of (59). His discussion suggests (though not conclusively) that they were isolated lexical relics: “Een bewijs dat het Brussels eenmaal de klanken a) /ɔ:/, /u:/, /ɔ:/; b) /i:/, /u:/, /y:/; c) /e:/, /e:/, /o:/; d) /ɔ/ heft gekend, is niet alleen het feit, dat ze (of sommige daarvan) rondom Brussel nog gehoord worden . . . , maar ook dat ze nog anwezig zijn als verkorte klanken in [24 examples].” (p. 312)

\textsuperscript{25} The dialects in question belong to the Southern Min dialect group spoken on Taiwan and Hainan Island, and in parts of the Chinese provinces of Fujian and Guangdong adjoining the Formosa Strait (Chen 1987). The mainland dialects are variously termed Amoy, Amoy Hokkien, Hagu, Hoklo, or Xiamen; those spoken on Taiwan are called Taiwanese.
then that the triggering environment will turn out to refer to some non-homogeneous representational element. In Subsection 4.5.2., we inspect the triggering environment and conclude that this is in fact the case.

This much is sufficient to support our claim of phonological conservativity, and readers who are interested only in that may go directly to Section 5, skipping Subsection 4.5.3, which defends Schuh (1978)'s characterization of Taiwanese tone sandhi as "paradigm replacement".

4.5.1. The general tone-sandhi alternation


4.5.1.1. Tone sandhi in the Tainan dialect (Cheng 1968, 1973)

The tone-bearing unit is the syllable, of which we must distinguish three kinds: those ending in an oral stop /p t k/, those ending in a glottal stop /ʔ/, and all others, which end with vowels or nasals. The first two types, which I will call "oral-stopped" and "glottal-stopped", are known collectively as "checked syllables", the last as "free syllables".

Most syllables carry a member of the phonemic tone inventory. In certain syllables (decidedly a minority in actual speech), tone contrasts are neutralized; these syllables are said to bear the neutral tone and are perceived as having especially weak stress (Cheng 1968; for phonetic details see Du 1988).

When a word is produced in isolation, the last syllable which does not bear neutral tone appears with its citation tone. There are seven citation tones in Taiwanese — five that are found on free syllables, and two that are found on checked syllables. Phonetically, they are approximately high level [55], mid level [33], low falling [31], high falling [53], and low rising [23] on the free syllables; high falling [54] and mid falling [32] on checked syllables.

Under other circumstances, a syllable will appear with a tone other than its citation tone. The most common of these circumstances obtains, roughly speaking, when the syllable is not the final syllable of a phrase. In this case it bears its elsewhere tone. The elsewhere tones are the same as the citation tones, minus the low rising [23] tone. The correspondences between citation and elsewhere tones are shown in (60) below. "E1" denotes the citation environment, "E2" the elsewhere environment. The alternating pairs are given their traditional identifying numbers.

---

26 The neutral tone is phonetically a mid-falling or low-falling tone. The F0 contour of a neutral-toned syllable has nothing to do with its underlying lexical tone, nor with the phonetic tone of the preceding syllable. See the experimental results of Du (1988: 103-109).

27 Class 6 is missing. Classes 1 and 5 are respectively the high and low reflexes of the Middle Chinese ping2 (Level) tone; 3 and 7, of Middle Chinese qu4 (Departing), and 4 and 8, of Middle Chinese ru4. (Entering). Class 2 is the high reflex of the Middle Chinese shang4 (Falling) tone; the low reflex, which would be Class 6, has merged with Class 7 (Cheng 1973, 1987: 77).
Taiwanese general tone sandhi (adapted from Cheng 1968, 1973)

Free syllables (CV, CVN)

<table>
<thead>
<tr>
<th>Class</th>
<th>2</th>
<th>3</th>
<th>7</th>
<th>5</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>[53]</td>
<td>[31]</td>
<td>[33]</td>
<td>[23]</td>
<td>[55]</td>
</tr>
<tr>
<td>E2</td>
<td>[55]</td>
<td>[52]</td>
<td>[31]</td>
<td>[33]</td>
<td></td>
</tr>
</tbody>
</table>

Oral-stopped syllables (CV{p t k})

<table>
<thead>
<tr>
<th>Class</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>[32]</td>
<td>[54]</td>
</tr>
<tr>
<td>E2</td>
<td>[54]</td>
<td>[32]</td>
</tr>
</tbody>
</table>

Glottal-stopped syllables (CV/)

<table>
<thead>
<tr>
<th>Class</th>
<th>4</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>[32]</td>
<td>[54]</td>
</tr>
<tr>
<td>E2</td>
<td>[53]</td>
<td>[31]</td>
</tr>
</tbody>
</table>

Glottal-stopped syllables occur only in E1; they lose their glottal stops and become free in E2. Oral-stopped syllables have the same segments in both environments.

Two contrasts in E1 — that between tone classes 5 and 1 and that between free and glottal-stopped syllables — are neutralized in E2. For these reasons, it is usual to assume the E1 forms to be underlying and the E2 forms to be derived. If we do so, we can represent the E1–E2 sandhi alternation in the following perspicuous way, adapted from Wang (1967):

Taiwanese E1-E2 tone sandhi circles

This pattern seems to be a spectacular violation (in fact, two spectacular violations) of the conclusion to the rotation theorem.

4.5.1.2. Phonetic reality of the circular rotation

Before proceeding further, we ought to stop and review phonetic data that has become available in the almost thirty years after Cheng’s first article (1968), in order to assure ourselves that the circular alternations really are there. For instance, is the derived (E2) high
level tone really the same as the underlying (E1) tone? If not, the circle is broken, and the problematic phenomenon evanesces.

Ideally, we would like instrumental data showing that the underlying and derived tones have the same acoustic properties, and perceptual data showing that Taiwanese speakers cannot tell the difference.

There are several acoustic studies, but only that of H.-B. Lin (1988: 30-56) has directly compared the E1 and E2 tones, and then only the free-syllable tones, and without controlling for segmental or intonational context (the E2 tones were realized on [si], the E1 tones on utterance-final [do].) However, the copious data of two excellent though under-cited studies, those of Chang (1988) and Du (1988), yields average F0 and amplitude curves for all underlying and derived tones.

Only one relevant perceptual study, again that of H.-B. Lin (1988: 112-134), is available to me, and I am reluctant to conclude anything from it28.

There is little doubt about the length-4 circle in the free syllables. All of the Taiwanese authors are in agreement about their perceptions, and they are backed up by instrumental data. Both environments have a high level, high falling, low falling, and mid level tone. The absolute pitch of each derived tone is very close to that of its underived counterpart, and, aside from some variations in the slope of the falling tones, both sets have the same contours (Cheng 1968, Chang 1988, H.-B. Lin 1988: 47-53, Du 1988: 110-115)29. The match is especially close in the well-controlled experiment of Chang (1988), whose subjects produced the tokens in a frame sentence with phonetic material on either side, rather than in word lists. Unless and until perceptual evidence to the contrary is found, we ought to agree with the native speakers that the tones are phonetically the same.

Dialect variation in the checked-tone syllables, however, is enormous. At least three different patterns are described. In the Tainan dialect of Cheng (1968), the citation tones [32] and [4] alternate respectively with the elsewhere tones [4] and [32], from which (to judge from the rather sketchy numerical data) they are not phonetically distinguishable. The mainland Zhangzhou speakers of Chang (1988) have citation tones [42] and [5], which alternate respectively with [5] and a short rising-falling tone [232]. Finally, in the Taizhong and Gaoliong dialects of central and southern coastal Taiwan, two checked tones, [33] and [32] can occur before the neutral tone; they alternate respectively with [21] and [54] in E2, and are neutralized to [32] when produced in isolation (Du 1988: 110-115). None of these descriptions tallies exactly with that of Chiang (1967), whose speakers (we are told only that they are from 'southern Taiwan') had the citation tones [54] and [32]. Since the phonetically best-documented cases lack the circular rotation, we will say no more about the checked syllables.

28 In this study, listeners often misidentified a word in the E2 context as if it had been in the E1 context. For instance, having heard [si55] in the E2 context, they frequently (38% of responses) reported it correctly, using a character which has citation form [si53], but almost as frequently (34% of responses) reported it incorrectly, using a different character which has citation form [si55]. If the E1 and E2 tones were very different, such confusion ought not to have occurred. However, in later, similar experiments with the same subjects, the effect disappeared completely (H.-B. Lin 1988: 124-134).

29 Chang’s underlying and derived low falling tones are no more different than the underlying and derived rising tones of Mandarin in the study of Chuang, Hiki, Sone, & Nimura (1971), though they are far from the near-perfect identity reported for Mandarin rising tones by Peng (1996). Mandarin speakers cannot tell the difference (Wang & Li 1967, Peng 1996).
4.5.1.3. Phonological representations

The long-syllable circle may be there phonetically, but is it there phonologically? The rotation theorem only applies to circular rotations taking place between the input to and output from the phonology. If similar-sounding tones actually have different phonological representations, then the circle is illusory.

This is not an idle question. Several authors have tried to finesse the problem by positing for the E1 and E2 tones different phonological representations that sound the same (Yip 1980, Wright 1983, Du 1988). Take Yip’s account, for example, which uses a register feature and two tonal autosegments:

(62) Taiwanese general tone sandhi, free syllables (Yip 1980: 217)

\[\begin{array}{c|c|c|c|c}
  & & & & \\
  & E1 & Class & E2 \\
  & [23] & 5 & [33] & \\
  & \underline{\text{L}} & \underline{\text{H}} & \underline{\text{H}} & \\
  & [55] & 1 & [33] & \\
  & \underline{\text{H}} & \underline{\text{H}} & \underline{\text{H}} & \\
  & [33] & 7 & [21] & \\
  & \underline{\text{L}} & \underline{\text{L}} & \underline{\text{H}} & \\
  & [21] & 3 & [53] & \\
  & \underline{\text{L}} & \underline{\text{L}} & \underline{\text{H}} & \\
  & [53] & 2 & [55] & \\
  & \underline{\text{H}} & \underline{\text{L}} & \underline{\text{H}} & \\
\end{array}\]

In order to apply the rotation theorem to the last four tone pairs in (62), we would have to convince ourselves that the E2 tones are more faithful to the homophonous E1 tones than they are to the E1 tones from which they were respectively derived — and this we cannot do. Take the [33] tone derived from [55], for instance. It has no features in common with the homophonous underlying [33] tone, but shares two H autosegmental features with the [55] from which it sprung. Any plausible set of faithfulness constraints will have to declare it more faithful to E1’s [55] than to E1’s [33]. The hypotheses of the rotation theorem do not obtain, so it is perfectly possible for a classical OT grammar, using these representations, to model the grammar of Taiwanese.
Yip’s feature system avoids circular rotation at the price of having three different representations for [33], two for [55], and two for [21]. There are no phonetic grounds for believing that these are different tones; for instance, the [33] derived from [53] and the [33] derived from [13] have been found by Chang (1988) to be absolutely identical acoustically in both F0 and duration (see Li (1983) for numerous examples of surface homophony). Nor can we still maintain that the low falling tone is phonologically a low tone. It is common to analyze the Mandarin 3rd tone, which in most contexts is phonetically low falling, as underlyingly low level (e.g., Yip 1980, Wright 1983, Shih 1986). However, the more recent phonetic evidence reviewed here clearly shows that the Taiwanese low falling tone is a [31], and falls much more steeply than the Mandarin [21] (Chuang, Hiki, Sone, & Nimura 1971, Peng 1996).

Wright’s wholly autosegmental analysis takes the E2 tones as underlying and derives the E1 tones from them (Wright 1983). There are three phonological representations for phonetic [33] — a different one for each of Classes 1, 5, and 7. Underlying and derived [21] are also represented differently. This approach relies on floating tonal autosegments that in E2 remain floating and are lost, but in E1 displace the lexically pre-linked tones. Six of the seven underlying tones are so represented. As a consequence, the surface representations of Taiwanese utterances are packed with unassociated tonal autosegments, none of which have any perceptible effect (e.g., downstep, or influence on a following neutral-toned syllable).

From this standpoint, Du (1988)’s split-register system is an improvement, the only redundancy being that underlying [33] has a very different representation from the two derived [33]s. It also represents the low falling tone as low and falling. But, adequate as this may be for Taiwanese, it suffers from the same problem that motivated Yip’s choice of representational framework in the first place: namely, that it allows languages to have far more contour tones than they actually are observed to have (Yip 1980: 23-24).

There is probably no way to empirically confirm or refute proposals which, like Yip’s, Wright’s, and Du’s, depend on phonetic neutralization of a phonological contrast between an E1 form and an E2 form. We would have to either find a phonological process that treats a tone in the E1 environment differently from the homophonous tone in E2, or show that every process targeting both treats them exactly alike. Since it is unlikely that any process affects both environments (as we shall see in the next subsection, when we learn what they are), data cannot settle the question. Theory must. It is “an appeal from the stronger to the weaker authority”.

There are three theoretical reasons for not using phonetic neutralization to rescue the hypothesized conservativity of phonology from the hostile Taiwanese data.

- **The representations are implausible.** (See above).
- **The alternation would still be too bizarre.** Quite independently of the circular-rotation problem, we will have no easy time coming up with a plausible set of markedness constraints to drive the sandhi transitions. Suppose Du’s representations are correct, and the apparent circle is a linear chain shift that begins with one mid level tone, which becomes a low falling tone, which becomes a high falling tone, which becomes a high level tone, which finally becomes another, phonetically indistinguishable, mid level tone:

\[
[33]_a \rightarrow [31] \rightarrow [53] \rightarrow [55] \rightarrow [33]_b
\]

---

30 Ample independent motivation is given for the phonetic identity of two of the representations for [33], but the others are all ad hoc devices for the Taiwanese problem.
Each tone in the chain is more marked than the tones to its right; that, in classical OT, is the only reason they change at all. One immediately wants to know why [33]\text{a} is more marked — much more marked — than [33]\text{b}, why [31] is less marked than [33]\text{a} but more marked than [53], etc. There is no good answer, as we will discuss in detail in Section 4.5.3.

- We don’t need it. In view of the foregoing, we would adopt the phonetic-neutralization theory only in order to rid ourselves of the circular rotation. In the next subsection we will review evidence that Taiwanese general tone sandhi is conditioned by non-homogeneous representational factors and hence is at liberty to have circular rotations.

If not the Yip (1980), Wright (1983), or Du (1988) theories, what representation shall we adopt? That of Wang (1967)? Or of Woo (1969)? Or those of Yip, Wright, or Du, with the redundancies removed? Thanks to the representation-neutrality of the rotation theorem, it doesn’t matter. We have decided to assign the same representations (whatever they may be) to the underlying tones, the trivially derived E1 tones, and the homophonous E2 tones; this immediately tells us which candidates are faithful to which inputs. The rotation theorem can now be applied, predicting a partly nonphonological conditioning environment for general tone sandhi. For familiarity’s sake, we will keep using the Chao letters [55], [31], etc. to stand for these unspecified representations.

4.5.2. The conditioning environment of general tone sandhi

Since the general tone-sandhi alternation is not conservative, our hypothesis of phonological conservativity predicts that the conditioning environment will make reference to non-homogeneous factors. In the remainder of this section, we will look at evidence that this is in fact true.

Many authors have written on the E1 and E2 environments, including Cheng (1968, 1973), Shih (1986), Chen (1987), Hsiao (1991), and J.-W. Lin (1994a, b). Once again, valuable data and discussion not available elsewhere are to be found in Du (1988). These writers do not disagree on the facts; the environment conditioning the alternation seems to be much less vulnerable to dialectal variation than the alternation itself. However, the facts themselves are quite confusing and difficult to interpret, partly because we often aren’t sure what the syntactic structure of a sentence is, and partly because the phenomenon inhabits a very thinly populated twilight zone between syntax, morphology, and phonology.

The E2 environment is easily disposed of: Any full-toned syllable that is not in E1 is in E2. Those in E1 keep their citation tones; the rest surface with their elsewhere tones, as shown in (63) (adapted from Cheng 1968, 1973):

\[
\begin{array}{cccccccc}
\text{titsin} & \text{ku} & \text{i-tsiN} & \text{tai-lam} & \text{si} & \text{ts‘it} & \text{e} & \text{hu-sia} \\
/33 & 55 & 53 & 53-23 & 23-23 & 33 & 32 & 23 & 53-23/ \\
\text{at very long ago Tainan be a prefecture} \\
\text{capital} \\
\end{array}
\]

‘Very long ago Tainan was a prefectural capital’
These examples illustrate the two ways in which a syllable may find itself in E1: it can be final in certain phrasal constituents (usually called "tone groups"), or it can immediately precede a neutral-toned syllable.

(65) **E1 (citation environment)**

A syllable is in E1 iff one of the following holds:

(a) It is the last syllable of its tone group.

(b) It is immediately followed by a neutral-toned syllable.

The environment in which the neutral tone is licensed is precisely E1, so that the Taiwanese tone group has the tonal pattern

\[(E ... E \ C n ... n ) \text{TG}\]

where E represents an elsewhere tone, C a citation tone, and n a neutral tone (Cheng 1968, 1973, Du 1988). The E2 environment is boldfaced, the E1 environment plain.

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31 The two occurrences of \[\text{an-ne}\] are different lexical items, distinguished by different underlying tones.

32 There are two further environments in which a syllable will appear with its E1 tone. One is the quotation environment. A word or phrase that is being used as its own name, rather than as the name of its lexically specified denotation, has E1 tone on the final syllable even if it completely isolated ("How do you pronounce this character?" "____") or is an adjunct of the sort that doesn’t qualify as a tone group:

The E1 tone is also found foot-finally in versification and certain formulaic phrases (Boyce 1980, Chen 1980, Hsiao 1991). Interestingly, verse can be read with the E1 tones in E1, just as in normal speech; with the E1 tones foot-finally; or with the E1 tones in both environments (Hsiao 1991: 164-168). This is probably not due to the existence of two wholly independent tone-sandhi processes in Taiwanese, since “both” rules map E1 tones to E2 tones in exactly the same way, and the odds are pretty long against that happening by chance (say, one in \(3129 = 55 + 22\)). Perhaps in verse recitation each metrical foot is produced in the quotation environment as a separate utterance. If the segmentation into feet is done directly from the underlying phonological representation, the E1 tones appear at the ends of the two feet of which each line consists. If the line is first subjected to general tone sandhi and then versified, the E1 tones will appear in E1 and foot-finally. If the poem is read as prose, it is treated like ordinary speech and only undergoes general tone sandhi, giving E1 tones only in E1.
We will next look at current ideas of how Taiwanese speakers decide where to put tone-group boundaries. Then we will argue that, under current theories of the prosodic hierarchy at least, the Taiwanese tone group cannot be a phonological phrase, but must make reference to syntactic constituency and relations. This will suffice to establish that it is, as we predict, not wholly conditioned by homogeneous representational elements.

Tone-group boundaries — usually written "#" in the theoretical literature — invariably coincide with the right-hand edge of a syntactic maximal projection. However, not every ]XP is also a #:

    [ 52 21 21 55-23 # 55 33 # 21 52 33 #]

That old lady talks speech really slowly
'That old lady talks really slowly'

The example is from Du (1988: 129); only surface tones are given. The AP [lau] 'old' is not followed by a #, and [lau] surfaces with its E2 tone; all other XPs are followed by #, placing their final syllable in E1.

To the question of which XPs do not create tone groups, two detailed answers have been given. Chen (1987) characterizes the exceptional XPs as adjuncts which c-command their heads; J.-W. Lin (1994a), relying on a more sophisticated syntactic analysis, as XPs that are lexically governed. For more detailed discussion and examples, the reader is referred to the cited authors. For the present, the important point is that the tone group looks very much like an end-based prosodic domain in the sense of Selkirk (1986) — i.e., a phonological phrase. As J.-W. Lin (1994a) points out, 'O'odham (formerly Papago) has exactly the same tone-group formation rule as Taiwanese; hence, the OT analysis of the former by Truckenbrodt (1996) should carry over at once to the latter.

At this point we have accomplished our goal: we have shown that Taiwanese tone sandhi is conditioned by a nonhomogeneous factor, namely phrase-level prosody. We should mention, though, some evidence that the tone group is not in fact a phonological phrase — that the sandhi process makes direct reference to the syntax, as argued by Chen (1990) and Hayes (1990).

First, and most importantly, is the fact pointed out by Chen (1987), that tone groups do not fit inside intonational phrases. The intonational phrase, recognizable in Taiwanese by lengthening and pause at its right-hand margin, is thought to be the largest prosodic domain below the level of the utterance. The intonational-phrase boundary is, cross-linguistically, a very strong one, yet it fails to separate tone groups (examples from Chen 1987):

33 My syntactic bracketing differs slightly from Du's. She analyzes [kong55 ue33] 'talk' as a VP, which would make [kau52 ban 33] 'really slow' an adjunct of the sentence rather than of the VP, contrary to its function. But [kong55 ue33] is a verb-object construction literally meaning 'speak speech' (it is written with the same characters as Mandarin [shuo44 hua52]). We can take [ue33] 'speech', the direct object, to be an NP, and adjoin the AdvP [u21 kau53 ban33] 'really slowly' to the VP. This, as we will see, predicts the observed tone-group boundaries.

34 The definition of government comes from Hale & Selkirk (1987): "A governs B iff m-commands B and every barrier for B dominates A. A m-commands B iff A does not dominate B and every maximal projection M that dominates A dominates B. Barriers are defined roughly as follows: no maximal projection may appear between B and the maximal projection of A that dominates B."
Rather than admit overlapping prosodic constituents into the grammar, Hayes (1990) argues that the tone-sandhi rule must make direct reference to syntactic representations. The literature does contain another well-known case in which a language, Kimatuumbi, seems to have two different kinds of phonological phrase (Odden 1987, 1990). They do not, however, overlap; every right boundary of one type is a right boundary of the other. The recent reanalysis of Truckenbrodt (1996) views one as properly contained in the other.

Secondly, the tone group does not create a phrasal prominence. Du (1988) has amassed considerable experimental evidence, both instrumental and perceptual, that there is no syllable of a tone group that regularly receives stress. Intensity does not depend on E1 or E2, and Taiwanese listeners identify the highest-pitched syllable of a phrase as the “loudest”, regardless of where it occurs. Peng (1994), to be sure, found that a phrase-final (but utterance-medial) syllable can be considerably longer (up to 40% for some tones, almost no change for others) than a phrase-medial syllable, but Du’s listeners did not perceive the lengthening as prominence.

Thirdly, the size of the tone group is unlimited. If a syntactic constituent is very long, the corresponding tone group will also be very long (example from Chen 1987):

(69) yi kioN-kioN kio gua ke k'ua/ pua/ tiam-tsing ku ts'eq #
/55 23-23 31 53 55 32 32 53-55 53 32/
[33 33-33 53 55 33 53 53 55-33 55 32]
he by force cause I more read half hour long book
'He insisted that I read for another half hour.'

There is no tendency to break long domains into shorter ones³⁵. Tone groups are as long as the syntax needs them to be. This is not what we expect of a prosodic constituent (Selkirk 1993).

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³⁵ Well, hardly ever. In slow speech, coordinated APs, AdvPs, or quantifier phrases ("He is a steady, dependable, and honest worker", etc.), which we expect to have a single tone-group boundary, tend to break up into more tone groups when the number of coordinates exceeds two. The same is true of coordinate NPs adjoined to another N as modifiers ("umbrella export factory"). Monosyllabic APs or AdvPs modifying the same argument tend to be separated from each other by tone-group boundaries; oddly, this is obligatory in fast speech but optional at normal or slow rates. (Du 1988: 124-135).
4.5.3. The arbitrary nature of Taiwanese tone sandhi

The worldview of Optimality Theory — not just classical OT, but the entire school of thought — is built around explanation. There are reasons for things to happen, and reasons for things not to happen; what actually does happen emerges from the simple and regular interaction of these reasons incarnate as constraints. As a result, OT deals very poorly with anything arbitrary. There is, I submit, absolutely no reason for Taiwanese to have the tone alternations that it does. Every attempt to make sense out of them has failed.

We have already seen how an autosegmental approach leads to implausible underlying representations (Yip 1980, Wright 1983, Du 1988).

Another approach exploits coincidental properties of a certain featural representation to express the tone-sandhi rule as an exchange rule mapping the citation forms to the elsewhere forms (Wang 1967; similar but different ideas used by Cheng 1968, 1973, Woo 1969 require disjunctive rule ordering (see Chomsky 1967)):

(70) Taiwanese tone sandhi rule (adapted from Wang 1967)

\[ \begin{align*}
\alpha \text{ high} & \quad \beta \text{ high} \\
\beta \text{ falling} & \Rightarrow \gamma \text{ falling} \\
\gamma \text{ rising} & \quad \gamma \text{ rising}
\end{align*} \]

This produces the following long-syllable tone-sandhi circle:

(71) \[ [+h, - f, - r] \rightarrow [- h, + f, - r] \rightarrow [+ h, + f, - r] \]

Schuh (1978: 249) asks: "[T]his is a formal tour de force. But what does it mean linguistically?" Does a native speaker’s natural grammar really conceive of tone sandhi as a process permuting register and contour variables? Such a priori and esthetic considerations aside, though, there is a neighboring dialect of Taiwanese that is a serious problem for this approach.

According to Hsieh (1976), there is a "Coastal" dialect of Taiwanese which has the same tonal inventory as the standard dialect, and almost the same sandhi rules — except that the E1 [23] tone becomes [21] rather than [33] in E2, and — most importantly — the E1 [53] tone becomes [33] rather than [55] in E2. Thus, the free syllables in this dialect exhibit a cycle of length three: [53] \rightarrow [33] \rightarrow [21] \rightarrow [53]. The difference between the Coastal and standard dialects is small; we would want the difference between our analyses of the two to be small also. Hence, we expect the Coastal tone-sandhi rule to be expressible with an exchange rule of a similar sort, using the same features.

However, this cannot be done. If an SPE-style exchange rule contains a minus sign, the circle it generates will have a length divisible by two. Hence the Coastal exchange rule does not contain a minus sign. Therefore, if its input contains \( m \) features marked with "+" and \( n \) features marked with "-", so will its output — the rule cannot invert signs, but only move them around from feature to feature. But the three members of the Coastal circle do not all have the same number of "+" features and "-" features — [53] has two "+" features, [21] has one, and [33] has none. Ergo, no such rule can capture the sandhi alternation.

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36 But no one else.
The problem will persist no matter what features are used. In retrospect, we see that the exchange-rule analysis of the standard dialect was ill-chosen. The outlook is equally bad for any constraint-based analysis that merely emulates the action of an exchange rule in OT terms.

It looks as if the Taiwanese general tone-sandhi alternation is, synchronically speaking, completely arbitrary and idiosyncratic, not derivable from simple, logical, plausibly innate constraints, even if they are allowed to be non-conservative. It is not the kind of phenomenon OT was intended to explain, but a case of out-and-out paradigm replacement peculiar to Taiwanese.

5. Retrospect and prospect

In this paper we have developed and illustrated a novel means of getting predictions out of OT with only the vaguest idea of what the constraints actually are. A local property of the individual constraints can lead to a global property of every grammar that uses those constraints, which manifests itself in the data as a language universal.

Moreover, the particular local property we chose to illustrate this method (conservativity) led to a predicted language universal (the lack of circular and infinite chain shifts) which was in fact borne out by the data. The universal was remarkable enough to have attracted attention before; the OT framework provides a very natural explanation of it in terms of the basic concepts of markedness and faithfulness, which do not exist in serial-derivational theories. The motto of classical OT, embodied in its postulate of conservativity, is "if it ain't broke, don't fix it" — the phonology only changes things to make them "better", with "better" being defined as a condition on surface representations. This is why rotations are impossible within that framework — there is no way for A to be "better" than B and B to simultaneously be "better" than A. We have shown that this is the entire empirical content of classical OT, in the absence of information about the individual constraints.

So far, so good. But as usual, in trying to settle one problem we have stirred up two or more previously quiescent others. To prove this I will touch upon two of the most disturbing here, and suggest at least the outlines of a campaign against them.

One is the puzzling importance of representational homogeneity. In order to prove the characterization theorem, we had to be able to feed the grammar with its own output, so we insisted that input and output representations be built in the same way. And it turned out that real-world processes sensitive only to homogeneous representational elements behaved as predicted, while those sensitive to nonhomogeneous elements sometimes misbehaved.

This is not what one would expect at first blush. There is a very natural way to integrate nonhomogeneous constraints into the system of markedness and faithfulness: Morphological and syntactic information is only present in inputs; we expect it to figure in faithfulness constraints but not markedness constraints, producing effects like those of "positional faithfulness" discussed in Beckman (forthcoming) — faithfulness to root-initial syllables.

37 The Party teaches us that all progress arises out of contradiction, so it’s actually for the best.
38 Under our current working definitions, since morphological information is absent from the candidates, no candidate is fully faithful to the input, and hence no faithfulness constraints exist in a complete natural grammar. What we mean here is of course a "relativized faithfulness" — faithfulness to the input in all those features to which one can be faithful, i.e., the homogeneous ones. The morphological annotations do not count one way or the other.
differential faithfulness to roots and affixes, etc. Phrase-level phonological information is only present in outputs; we expect it to figure in markedness constraints but not faithfulness constraints, producing effects like phrase-final lengthening. Hence we expect no circular chain shifts at all. Yet the grammar seems to allow, for no immediately obvious reason, nonhomogeneous constraints a latitude forbidden to homogeneous ones.

Our second problem is the idiosyncratic nature of the circular chain shifts. When they do arise, they have a language-particular look about them, in either the triggering environment or the alternation itself. It is very difficult to believe that anything like the phenomena of Kiliwa ‘do’ or Taiwanese tone sandhi (see Section 4.5.3) could be attributed to the interaction of innate constraints — the very bedrock of the OT outlook.

The two problems may be linked. It may be that innateness and conservativity go hand-in-hand, and that the correct generalization is rather that constraints involving solely innate representational elements are themselves innate and conservative. If a language has an alternation that cannot be computed by these constraints, the learner can detect this by the failure of the learning algorithm to converge; the deficiency would then be suppleted by adding an undominated ad hoc constraint which simply insisted that a particular lexical item, in a particular environment, must surface in a particular way — making of each nonconservativity a lexical exception. Some such mechanism, is necessary in any case to deal with processes (such as Germanic ablaut, the dramatic vowel-coalescence phenomena of Yu’pik (Reed et al. 1977), or the mutations caused by Kwakiutl glottalizing suffixes (Boas 1947, Bach n.d.)) which are triggered by specific lexical items and have phonologically unwieldy effects that are probably most economically described as paradigm replacements.

If innateness and conservativity are a package deal, we would expect that constraints referring to presumably innate morphological or syntactic categories (such as N versus V, or content-word versus function-word, or root versus affix) would be conservative, while those referring to presumably language-particular ones (such as first versus second conjugation, or specific lexical items) would not be. Thus unfaithfulness to the underlying segmentation might be compelled by a particular affix, but not by the mere fact of being a root.

These questions are still wide open, though. We must be content for the nonce with having identified them.
Bibliography


Bach, Emmon. (no date). Building words in Haisla. MS, UMass/Amherst.


Li, Xin-Tai. 1983. *Xia4men2 fang1yan2 yin1ping2 he2 yang2ping2 de bian4dia04*. *Fang1yan2* 2:120-122.


