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Long distance chiral corrections in B meson amplitudes

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Abstract

We discuss the chiral corrections to f_B and B_B with particular emphasis on determining the portion of the correction that arises from long distance physics. For very small pion and kaon masses all of the usual corrections are truly long distance, while for larger masses the long distance portion decreases. These chiral corrections have been used to extrapolate lattice calculations towards the physical region of lighter masses. We show in particular that the chiral extrapolation is better behaved if only the long distance portion of the correction is used.

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1 Introduction

Lattice calculations of B meson properties are presently done with parameters such that the light quark masses are larger than their physical values. In order to make predictions that are relevant for phenomenology, these calculations are extrapolated down to lower quark masses. One of the extrapolation methods uses some results from chiral perturbation theory, and this appears to produce rather large effects due to the chiral corrections. A recent summary of the field [1] noted that this chiral extrapolation is the largest uncertainty (17%) at present in the calculation of the B meson decay constant f_B.

Chiral perturbation theory is an effective field theory involving pions, kaons and η mesons. These mesons are the lightest excitations in QCD and the effective field theory is designed to describe the effects of long range propagation of these light degrees of freedom. Even in loop diagrams there are long distance effects which are described well by the effective field theory. However, chiral perturbation theory is not a good model of physics at short distances and is not valid for large meson masses. If we consider mesons of variable mass, as the masses become heavier, less and less of the loop corrections are truly long distance.

The chiral corrections are sometimes used in ways that hide the separation of long distance and short distance physics. Consider for example the chiral correction to the B meson decay constant in dimensional regularization [2, 3, 5]

$$f_B = f_0 \left[1 - \left(\frac{1+3g^2}{16\pi^2 F_\pi^2} \right) \frac{3}{8} m_\pi^2 \ln \frac{m_\pi^2}{\mu^2} + ... \right] , \tag{1}$$

where g is the heavy meson coupling to pions. The ellipses denote the kaon and eta contributions as well as analytic terms in the masses that carry unknown coefficients which must be fit. We see that the corrections vanish for massless mesons and grow continuously with large meson masses[1]. This is the opposite of the behavior that one might expect, which would be to have larger chiral corrections when the pions are nearly massless. For very large masses of the "pions", physically we expect that the loop effects must decouple from the observables. The expression of Eq. 1 does not illustrate this decoupling. The key point is that as the mesons become heavier, most of the correction given in Eq. 1 comes from short distance physics, which is not

[1]Note that we keep the B meson mass unchanged, so that when we refer to large and small meson masses, we are always referring to the masses of the chiral particles - pions, kaons and etas - that occur in the loop diagrams.

a reliable part of the effective field theory. We will show this in more detail below. This behavior is not a problem in principle. The free coefficients in the chiral lagrangian allow one to compensate for the unwanted behavior and correctly match the short distance physics of QCD. However the reliance on Eq. 1 at large masses can have a deleterious effect on phenomenology in some applications.

The way that present lattice extrapolations of F_B are performed apply the chiral predictions outside their region of validity. An example is given in Fig. 1, describing the results of the JLQCD collaboration [4].

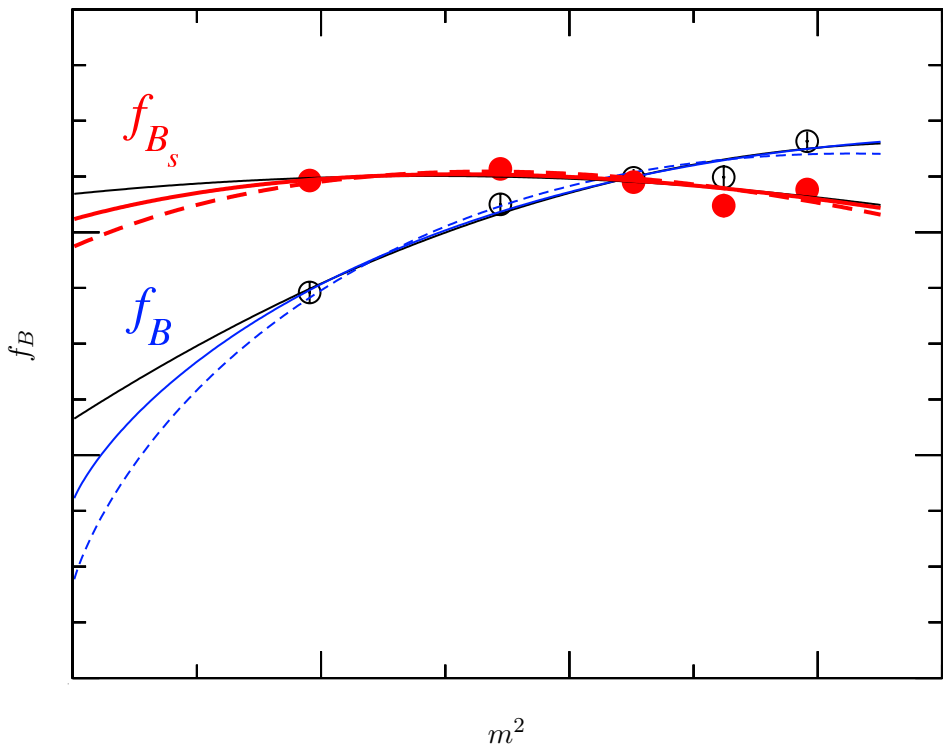

Figure 1: Lattice data points for f_B and f_{B_s} and fitted curves with quadratic fit (upper solid curve) and with chiral logs for $g = 0.27$ and $g = 0.59$ (dashed)

In order to address the issue of the chiral exptrapolation, the lattice data was fit with the function of Eq. 1 at large mass and the form is used to extrapolate the results to small values of the mass. The fact that there appears to be a large effect at $m = 0$ does not imply that the chiral cor-

rection is large here. Indeed, inspection of Eq. 1 shows that the chiral log correction vanishes at zero mass, so the chiral logarithm is not large at the physical masses. Rather, the big effect seen comes from using Eq. 1 at large masses. Since the chiral logs grow at large mass, and appear in this formula with a fixed coefficient, normalizing the function at large mass produces a sizeable difference when compared to smaller masses. Since chiral perturbation theory is not applicable at such large masses, this shift is not a valid consequence of chiral perturbation theory.

We will explore the long-distance/short-distance structure of the chiral corrections [7], and show that the undesirable effects described above come from short distance physics that chiral perturbation theory is not able to describe. The application of Eq. 1 at large masses then amounts to a bad model of the short distance physics. We will give formulas for the one loop corrections of Eq. 1 which *removes* the unwanted short-distance component. At small quark masses, our method is just a different regularization of the theory, and reproduces the usual chiral corrections. When applied at large quark masses, our formulas must also be considered as a model. However, it is a relatively innocuous model in that it makes no assumptions about short distance physics and and it produces a small correction since the loop effect decouples at large mass. When used to extrapolate the lattice results to the physical masses, our results lead to more reasonable estimates of the chiral corrections. Our methods are similar to some work on long distance regularization in baryon chiral perturbation theory [7] and on chiral extrapolations in other processes [8]. In particular, the JLQCD group has explored the use of the Adelaide-MIT approach [8]in the extrapolation of the pion decay constant[4]. Our work describes the rationale and benefits of a modified approach for the heavy-light system.

2 A study of the chiral corrections to f_B

The chiral corrections were initially calculated by Grinstein et al [2] (see also [3, 6, 5]). The methods are standard and we will not reproduce the details. However we note that, although there are various Feynman diagrams in the calculation, in the end the loop calculations involve only one loop integral,

$$\mathbf{I}(m) = i \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2 + i\epsilon)} \,. \tag{2}$$

The chiral expansion involves unknown parameters for the reduced decay constant at zero mass ($\bar{f}_0$) and for the slopes (α_1, α_2) parameterizing linear

dependence in the masses. The results are [2, 3, 5]

$$f_{B_{u,d}} = \frac{1}{\sqrt{m_B}}\,\bar{f}_0 \Bigg[1 + \alpha_1 m_\pi^2 + \alpha_2 (2m_K^2 + m_\pi^2) - \frac{1+3g^2}{4\,F_\phi^2}\left(\frac{3}{2}\,\mathbf{I}\,(m_\pi) + \mathbf{I}\,(m_K) + \frac{1}{6}\,\mathbf{I}\,(m_\eta)\right)\Bigg] \tag{3}$$

and

$$f_{B_s} = \frac{1}{\sqrt{m_B}}\,\bar{f}_0 \Bigg[1 + \alpha_1 (2m_K^2 - m_\pi^2) + \alpha_2 (2m_K^2 + m_\pi^2) - \frac{1+3g^2}{4\,F_\phi^2}\left(2\,\mathbf{I}\,(m_K) + \frac{2}{3}\,\mathbf{I}\,(m_\eta)\right)\Bigg], \tag{4}$$

where g is the coupling of heavy mesons to pions[2] and F_ϕ is the pseudo-goldstone meson decay constant in the chiral limit[3]. Of course, the integral still needs to be regularized. In dimensional regularization, one absorbs the $1/(d-4)$ divergences into the slopes and finds the residual integral

$$\mathbf{I}^{d.r.}\,(m) = \frac{1}{16\pi^2}\left[m^2 + m^2 \ln \frac{m^2}{\mu^2} \right], \tag{5}$$

where μ is the arbitrary mass parameter that enters in dimensional regularization. The physical results do not depend on μ as it can be absorbed into a shift in the unknown slope coefficients.

Let us explore the loop integral and study the long-distance part. In order to do this, we use a cut-off defined in the rest frame of the B meson in order to remove the short-distance component. Specifically, we use a dipole cutoff yielding

$$\mathbf{I}\,(m,\Lambda) = i\Lambda^4 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2 + i\epsilon)\,(k^2 - \Lambda^2 + i\epsilon)^2}\,. \tag{6}$$

In related contexts, other forms of cut-offs have been studied [7, 8] - qualitatively similar results are found with other forms, although the parameter Λ will have different meanings in each case. We employ a finite value for

[2] In our numerical work, we will use $g = 0.59$.

[3] We use the normalization such that $F_\pi = 0.0924$ GeV.

the cut-off of order the size of the B meson. The integral may be calculated and has the form

$$\mathbf{I}\left(m,\Lambda\right)=\frac{\Lambda^{4}}{16\pi^{2}}\left[-\frac{1}{m^{2}-\Lambda^{2}}+\frac{m^{2}}{\left(m^{2}-\Lambda^{2}\right)^{2}}\ln\frac{m^{2}}{\Lambda^{2}}\right]. \tag{7}$$

More illuminatingly, this result is shown in Fig. 2. In this figure we compare the dimensionally regularized result to the long-distance portion, defined by Eq. 7.

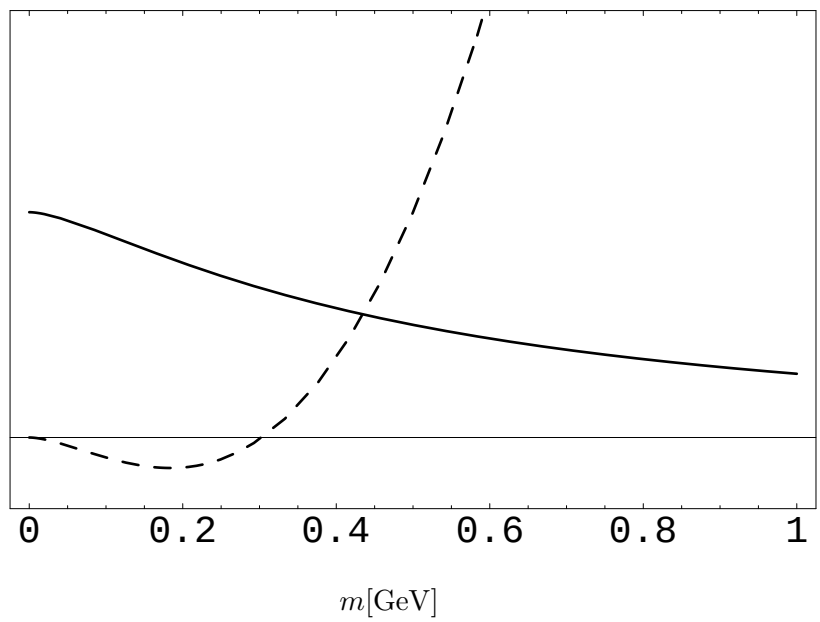

Figure 2: Integrals $\mathbf{I}\left(m,\Lambda\right)$ with $\Lambda=500$ MeV and $\mathbf{I}^{d.r.}\left(m\right)$ with $\mu=500$ MeV (dashed)

The long distance component is seen to have several reassuring features in the cut-off regularization. It is largest when the meson is massless, as one would expect. It is small when the mass is big and exhibits decoupling, vanishing as the mass goes to infinity. It smoothly interpolates between these limits. When comparing it to the dimensionally regularized result, one sees a shift in the intercept at zero mass - this is not surprising because the regularization corresponds to removing the value when $m=0$. One also notices that, aside from this shift, both forms have the same logarithmic behavior near $m=0$. The small curvature noted at the smallest mass values

is the nonlinear behavior due to the chiral log factor $m^2 \ln m^2$. Without this term the result would be able to be Taylor expanded about $m = 0$, with the first term being a linear slope in m^2 - the nonlinear behavior is the result of the logarithm.

We also see that the chiral log by itself grows large quickly and has a large curvature at large masses in dimensional regularization. This effect is not mirrored in the long-distance component, so that it is clear that this behavior comes from the short-distance portion of the integral. This is not surprising. In dimensional regularization, there is no scale within the integration aside from the particle's mass, so that the the whole integral scales with $k \sim m$. These short distance effects are ones which are not reliable calculated by the effective field theory.

These results suggest that we should consider an extrapolation that only includes the long distance loop effects. The short distance effects are provided by the lattice simulation[4]. The truly long distance effects are supplied by chiral perturbation theory. We will use the long distance parts of the loops in performing the matching of the two regions. In our approach this matching is described by the parameter Λ specifying the separation of long and short distances. The residual dependence on this parameter, within some range, is a reflection of the present uncertainty in the matching procedure.

3 Long distance regularization of the chiral calculation

At small quark masses, the cut-off treatment of the integral can be promoted to a regularization of chiral perturbation theory. This has been studied in the context of baryon chiral perturbation theory in Ref. [7], where it was called long distance regularization. The use of a cut-off is clearly more painful calculationally than the usual dimensional regularization, but when the masses are small it reproduces the usual one-loop chiral expansion for matrix elements such as we are studying.

In order to regularize the calculation using the cut-off, the divergent pieces are separated in the Feynman integral. The result is

$$\mathbf{I}\,(m,\Lambda) = \frac{1}{16\pi^2}\left[\Lambda^2 - m^2\,\ln\frac{\Lambda^2}{\mu^2}\right] + \mathbf{I}^{\mathbf{ren}}(m,\Lambda)\,, \tag{8}$$

[4]The "smooth matching" procedure of Ref. [5] is another attempt to apply the chiral results only in their region of validity.

where $\mathbf{I^{ren}}(m,\Lambda)$ is finite in the limit $\Lambda \to \infty$. This residual integral has the form

$$\mathbf{I^{ren}}\,(m,\Lambda) = \mathbf{I^{d.r.}}\,(m) \;+\; \frac{1}{16\pi^2}\left[-\frac{m^4}{m^2-\Lambda^2} - \frac{m^4(m^2-2\Lambda^2)}{(m^2-\Lambda^2)^2}\,\ln\frac{m^2}{\Lambda^2}\right]. \tag{9}$$

We see that there are potentially divergent contributions proportional to Λ^2 and $\ln\Lambda^2$. However, these have exactly the right structure to be absorbed into the chiral parameters. In particular, the renormalization is

$$\begin{aligned}
\bar{f}_0^{ren} &= \bar{f}_0 - \frac{8}{3}\,\bar{f}_0\,\frac{1+3g^2}{64\pi^2\,F_\phi^2}\,\Lambda^2 \\
\alpha_1^{ren} &= \alpha_1 + \frac{5}{6}\;\frac{1+3g^2}{64\pi^2\,F_\phi^2}\,\ln\frac{\Lambda^2}{\mu^2} \\
\alpha_2^{ren} &= \alpha_2 + \frac{11}{18}\;\frac{1+3g^2}{64\pi^2\,F_\phi^2}\,\ln\frac{\Lambda^2}{\mu^2}.
\end{aligned} \tag{10}$$

After renormalization, we can express the chiral amplitudes in terms of these parameters plus the logarithmic contribution in the residual integral $\mathbf{I^{ren}}\,(m,\Lambda)$, providing the renormalized observables

$$\begin{aligned}
f_{B_{u,d}} =& \frac{1}{\sqrt{m_B}}\,\bar{f}_0^{ren}\Bigg[1 + \alpha_1^{ren} m_\pi^2 + \alpha_2^{ren}(2m_K^2 + m_\pi^2) \\
&- \frac{1+3g^2}{4\,F_\phi^2}\left(\frac{3}{2}\,\mathbf{I^{ren}}\,(m_\pi,\Lambda) + \mathbf{I^{ren}}\,(m_K,\Lambda) + \frac{1}{6}\,\mathbf{I^{ren}}\,(m_\eta,\Lambda)\right)\Bigg]
\end{aligned} \tag{11}$$

and

$$\begin{aligned}
f_{B_s} = \frac{1}{\sqrt{m_B}}\,\bar{f}_0^{ren}\Bigg[&1 + \alpha_1^{ren}(2m_K^2 - m_\pi^2) + \alpha_2^{ren}(2m_K^2 + m_\pi^2) \\
&- \frac{1+3g^2}{4\,F_\phi^2}\left(2\,\mathbf{I^{ren}}\,(m_K,\Lambda) + \frac{2}{3}\,\mathbf{I^{ren}}\,(m_\eta,\Lambda)\right)\Bigg].
\end{aligned} \tag{12}$$

Since at small mass, the residual integral $\mathbf{I^{ren}}\,(m,\Lambda)$ tends to $\mathbf{I^{d.r.}}\,(m)$, the usual chiral expansion is recovered at $m^2 << \Lambda^2$. At small mass, the cut-off is just another way to regularize the calculation.

4 The chiral extrapolation of f_B

If we are going to use any meson loop calculation at larger masses in order to match to the lattice, then all treatments are model dependent. We have argued above that the use of chiral logs at these scales amounts to a bad model because it builds in very large and spurious short distance effects. Our calculation above removes the short distance effects in the one loop diagrams. This is then a reasonable formalism to apply to the lattice calculation. The lattice calculation supplies the correct short distance physics, described there through terms analytic in m^2 (linear behavior, quadratic...). In addition, at smaller masses, our formulas naturally include the chiral logarithms in the regions where they should be valid. This motivates us to use the long-distance loop calculation in the chiral extrapolation for B meson properties.

Let us first fit our expression to a caricature of the lattice data by matching the data at two points. Such a linear extrapolation is appropriate for one loop since we have only the constants and linear counterterms in the one loop expression. This fit is demonstrated in Fig. 3, for various values of Λ. We see that the extrapolation is smoother and that there is no large curvature induced at large mass.

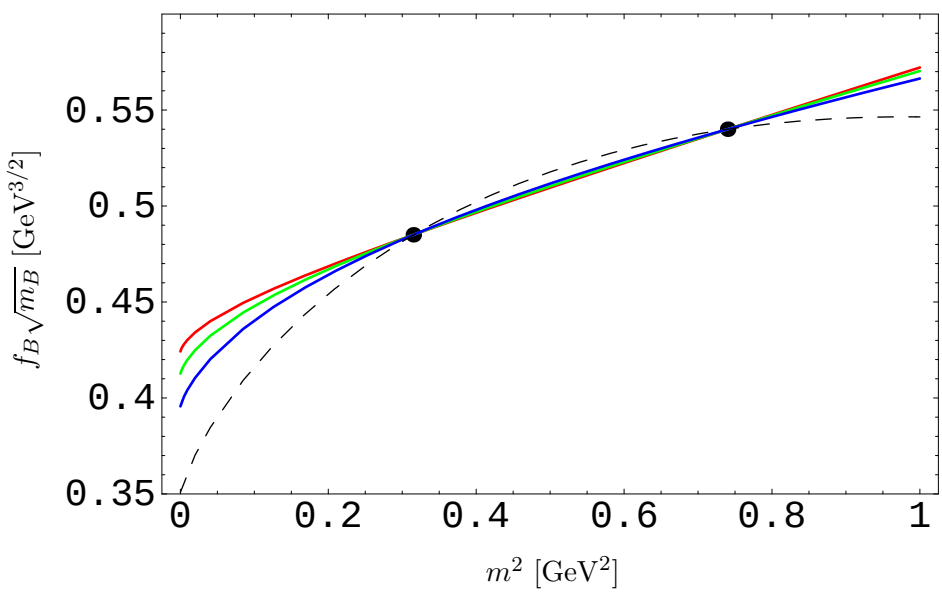

Figure 3: $f_B\sqrt{m_B}$ as a function of m^2 fitted to the Lattice data points for $\Lambda = 400, 600, 1000$ MeV and for the result from dimensional regularization (dashed)

There is a residual dependence of the extrapolated value on the parameter Λ. This is shown in Fig. 4. In the range $\Lambda = 400$ MeV $\rightarrow 1000$ MeV, this amounts to a 5% uncertainty in the extrapolated value. The formula used in previous extrapolations corresponds to $\Lambda \rightarrow \infty$. It is clear that the loop contributions that arise beyond the scale of $\Lambda = 1000$ MeV are of too short distance to be physically relevant for the effective field theory - there is no reliable chiral physics beyond this scale.

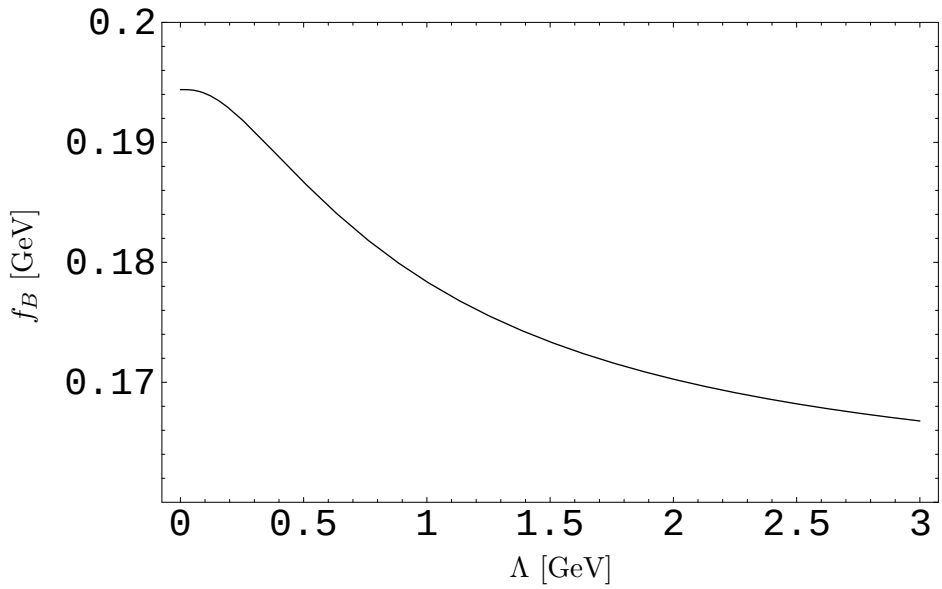

Figure 4: f_B at the physical pion mass as a function of Λ

This extrapolation can be systematically improved. Most favorably would be the situation in which the lattice data can be calculated at smaller mass squared - eventually no extrapolation would be needed. Even if the improved data goes only part of the distance to the physical masses, it would remove some of the model dependence of the result. The extrapolation needed would be smaller and the residual Λ dependence would be smaller. Another way that improvement possibly may be made is with increased precision even at larger masses. As shown by Eq. 9 above, the extrapolations for different Λ values differ only at order m^4/Λ^2. If one includes an extra $\mathcal{O}(m^4)$ in the one loop chiral calculation, fitting to a quadratic expression, then the extrapolations will be in closer agreement at this chiral order. Note however that the low mass region is still being extrapolated by a one-loop chiral formula - this procedure is not equivalent to a two-loop result in chiral

perturbation theory.

As the lattice data reaches higher precision, it may be that the range of Λ for which a good fit is obtained may shrink. While we are treating Λ as a regularization parameter, it is meant as a rough parameterization of a physical effect - the transition from long-distance to short-distance in the loop calculation. Therefore when using a fit to a given order in the chiral expansion, the lattice data may only be describable with Λ within some range near the scale of this physical effect. Indeed, already the present data is a poor fit for $\Lambda \to \infty$. Of course if one allows arbitrary orders in the chiral expansion, with free parameters at each order, it is always possible to correct the loop effect for any incorrect short distance behavior by adjusting the parameters. However, when using the one loop integral with precise data it may not be possible to obtain good fits for large values of Λ without introducing *several* new parameters at higher orders in the masses. In contrast, simpler fits with fewer parameters may be obtained with Λ within some optimal range.

Our procedure might be criticized as being a model, due to the choice of a separation function and a separation scale. However, at large masses, the dimensional regularization result is really more of a model as it introduces large and unphysical short distance physics. Our procedure is the "anti-model" because it removes most of that physics. The residual dependence on Λ comes from the ambiguity concerning how much of the short distance physics to remove. The value of Λ from the lattice results, introduced through the dipole cut-off, parametrizes the short distance physics. However, this dependence can itself be adjusted by using the coefficients of the chiral lagrangian. Despite the decoupling of the loop at large mass, we retain all of the correct chiral behavior in the limit of small quark mass.

5 Application to B_B

All of the preceding formalism can also be applied to the chiral extrapolation of the B_B parameter for $B-\bar{B}$ mixing. We have reproduced the calculations of Ref. [2, 3] using throughout the method of long distance regularization. As above, only the integral $\mathbf{I^{ren}}$ is needed in the final answer. The chiral

formulas after renormalization of the parameters are

$$B_{B_d} = B_0^{ren} \left[1 + \beta_1^{ren}\, m_\pi^2 + \beta_2^{ren}\, (2m_k^2 + m_\pi^2) \right.$$
$$\left. - \frac{1-3g^2}{4\,F_\phi^2} \left(\mathbf{I^{ren}}\,(m_\pi, \Lambda) + \frac{1}{3}\,\mathbf{I^{ren}}\,(m_\eta, \Lambda) \right) \right] \tag{13}$$

$$B_{B_s} = B_0^{ren} \left[1 + \beta_1^{ren}\, (2m_K^2 - m_\pi^2) + \beta_2^{ren}\, (2m_k^2 + m_\pi^2) \right.$$
$$\left. - \frac{1-3g^2}{3\,F_\phi^2}\,\mathbf{I^{ren}}\,(m_\eta, \Lambda) \right] \tag{14}$$

in the same notation as before. Here the new chiral constants B_0, β_1, β_2 describe the intercept and slope of the chiral expansion. At small masses the usual dimensional regularization results of Ref. [2, 3] are recovered in the limit of small m/Λ, as is seen using Eq. 9.

The chiral corrections for B_B are proportional to $1-3g^2$, while in the case of f_B the corrections contain the factor $1+3g^2$. This modification makes an important change in the result. For the coupling $g = 0.59$ that is favored by recent measurements [9] and supported by recent lattice calculations and theoretical predictions [9], the factor $1-3g^2$ almost vanishes. In this case, the one loop chiral corrections are tiny whether one employs the standard scheme or our long-distance regularization methods. (See also [10] for a discussion of this effect). For this reason, we do not display the numerical effect of the chiral extrapolation of B_B. Use of a significantly smaller value of the coupling g would lead to measurable effect in the B_B extrapolation.

6 Conclusions

We have presented a method for the extrapolation of lattice data to smaller quark masses. This includes the chiral logarithm in the region where it is valid. It has the advantage that it removes the large and unphysical short distance effects that caused problems in previous methods. There is still some residual model dependence that is visible in the variation of the results on Λ. However the extrapolations are better behaved than previous ones. The residual uncertainty in a linear extrapolation (i.e. with a slope proportional to m^2 and no chiral logarithm) for f_B is about 5% when the cut-off is constrained to the range 400 MeV–1000 MeV. For B_B the uncertainty

in the chiral extrapolation is negligible for $g = 0.59$. We would recommend that our method only be applied for values in this range.

The chiral corrections have the effect of producing a slight decrease in the extrapolated values of f_B and B_B when compared to an extrapolation which does not include chiral effects. This is the effect of the non-analytic behavior of the chiral logarithm at long distance. Our estimates suggest that the decrease due to the chiral log puts the chirally corrected result at 0.945 ± 0.025 of the uncorrected extrapolation for f_B. We hope that our method will be applied in future extrapolations of lattice data.

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