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Deepankar Basu
University of Massachusetts Amherst, dbasu@econs.umass.edu

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DEPARTMENT OF ECONOMICS

Working Paper

Comparative Growth Dynamics in a Discrete-time Marxian Circuit of Capital Model

By

Deepankar Basu

Working Paper 2011-12

UNIVERSITY OF MASSACHUSETTS AMHERST

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Deepankar Basu*

June 13, 2011

Abstract

This paper develops a discrete-time formalization of the circuit of capital model presented by Marx in Volume II of *Capital* Marx (1993) as a tool for aggregate economic analysis of capitalist economies. The discrete-time formalization closely follows and extends the continuous-time formalization in Foley (1982, 1986a). The discrete-time model is used to address two important issues of interest to the heterodox economic tradition: profit-led versus wage-led growth, and the growth-reducing impact of non-production credit. First, it is demonstrated that both profit-led and wage-led growth regimes can be accommodated within the Marxian circuit of capital model. Second, it is demonstrated that the steady-state growth rate of a capitalist economy is negatively related to the share of consumption credit in total net credit, when the total credit is large to begin with.

JEL Codes: B51, O1.
Keywords: circuit of capital, economic growth, consumption credit, Marxian political economy.

1 Introduction

The determining motive of capitalist production is profit. In Volume I of *Capital*, Marx provided a consistent explanation of the phenomenon of profit, at the aggregate level, as arising from the exploitation of the working class by capitalists through the institution of wage labour. To establish this path-breaking result, Marx borrowed and further extended the labour theory of value of classical political economy.

*Department of Economics, 1012 Thompson Hall, University of Massachusetts, Amherst, MA 01003, email: dbasu@econs.umass.edu. I would like to thank Duncan Foley and participants in the 2011 URPE Summer School at the University of Massachusetts, Amherst for useful comments.

Marx demonstrated that the classical economists' category of value is nothing but objectified (socially necessary, abstract) labour, that value is expressed through the social device of money (so that money is intrinsic to commodity production) and that capital is self-valorizing value (Marx, 1992). Capital, self-expanding value, was represented by Marx as:

$$M - -C \cdots (P) \cdots C' - -M'. \tag{1}$$

In this well-known formula, M represents the sum of money that a capitalist enterprise commits to the process of production by purchasing commodities C. C is composed of two very different kinds of commodities, labour power and the means of production. These are brought together in the process of production, represented by P, which leads, after a period of time (the production time) to the emergence of finished products, C'. These commodities are then sold in the market for a sum of money $M' = M + \Delta M$, to not only recoup the original sum thrown into production but also a surplus ΔM, the proximate determinant of the whole process.

Marx's analysis in Volume I of *Capital* Marx (1992) demonstrated that the secret of surplus value that leads to the self-expansion of capital is the exploitation of labour by capital. In Volume II of capital Marx (1993), he highlighted the fact that the process of self-valorization of capital can only complete itself by traversing a circular movement, during which value assumes and discards three different forms. Marx called this circular movement of value the circuit of capital, where

> ... capital appears as a value that passes through a sequence of connected and mutually determined transformations, a series of metamorphoses that form so many phases or stages of a total process. Two of these phases belong to the circulation sphere, one to the sphere of production. In each of these phases the capital value is to be found in a different form, corresponding to a special and different function. Within this movement the value advanced not only maintain itself, but it grows, increases its magnitude. Finally, in the concluding stage, it returns to the same form in which it appeared at the outset of the total process. This total process is therefore a circuit (Marx, 1993, pp. 133).

The two phases of circulation that Marx refers to are M–C and C'–M' in (1), which together with the process of production, P, comprise the circuit of capital. Hence, the flow of value that comprises the circuit can be broken up into three distinct phases (or stages):

1. the flow of capital outlays to start the production process, M–C;
2. the flow of finished commodities emerging from the process of production impregnated with surplus value, (P)–C'; and
3. the flow of sales, C'–M', which sets the stage for another round of capital outlays and production.

As a representation of the flow of value through the capitalist economy, the circuit of capital model highlights two crucial aspects of the self-expanding flow of value. First, it pays close attention to the forms that value assumes and discards, at different stages of the circuit, as it attempts to expand itself quantitatively (a dialectical interaction of quality and quantity). Second, it highlights the crucial aspect of turnover, of how the different forms of value complete their own circuits at different speeds and how capital re-creates its own conditions of existence and growth.

Attending carefully to the issue of aggregation, the circuit of capital model in Volume II of *Capital*, can be extended to an ensemble of capital or even the whole capitalist economy. The analytical move to construct a circuit of capital model for the aggregate economy involves dealing with at least two conceptual issues. The first issue that one needs to deal with is the cross sectional heterogeneity across capitalist enterprises. At any point in time, different individual capitalist enterprises would be at different stages of the three phases of their individual circuit.

> In so far as each of these circuits is considered as a particular form of the movement in which different individual industrial capitals are involved, this difference also exists throughout simply at the individual level. In reality, however, each individual industrial capital is involved in all the three [phases] at the same time. The three circuits, the forms of reproduction of the three varieties of capital, are continuously executed alongside one another. One part of the capital value, for example, is transformed into money capital, while at the same time another part passes out of the production process into circulation as new commodity capital ... The reproduction of the capital in each of its forms and at each its stages is just as continuous as is the metamorphosis of these forms and their successive passage through the three stages. Here, therefore, the entire circuit is the real unity of its three forms (Marx, 1993, pp. 181).

The issue of cross sectional heterogeneity can be dealt with by aggregating across individual circuits of capitals at any point in time over the three phases to arrive at corresponding aggregate flows:

1. the aggregate flow of capital outlays
2. the aggregate flow of finished commodities
3. the aggregate flow of sales.

The second issue is more subtle but also more important. As stressed by Marx in chapter 12–14 of the second volume of *Capital* Marx (1993), the process of production and circulation takes finite amounts of time. The sum of the two is what Marx calls "turnover time":

> ...the movements of capital through the production sphere and the two phases of the circulation sphere are accomplished successively in time. The duration of its stay in the production sphere forms its production time, that in the circulation

> sphere its circulation time. The total amount of time it takes to describe its circuit is therefore equal to the sum of its production and circulation time (Marx, 1993, pp. 200).

Thus, for instance, a sum of money laid out as capital outlays will only emerge as finished products with a definite time lag; the finished products will be sold only with a definite time lag; and the sales revenue will be recommitted to production, once again, with a finite time lag. Thus, each of the three phases of the circuit come with its own time lag.

The time lags have two important implications. First, they establish definite relationships between each of the flows (involved in the three phases of the circuit) over time. Second, non-zero time lags imply that at any point in time, there will be a build-up of stocks of value, in three different forms (corresponding to the three flows), in the economy: productive capital, commercial capital and financial capital.

> It lies in the nature of the case, however, that the circuit itself determines that capital is tied up for certain intervals in the particular sections of the cycle. In each of its phases industrial capital is tied to a specific form, as money capital, productive capital or commodity capital. Only after it has fulfilled the function corresponding to the particular form it is in does it receive the form in which it can enter a new phase of transformation Marx (1993, pp. 133).

Aggregating across individual capitalist enterprises at any point in time, we can arrive at the corresponding aggregate stocks of value:

1. the aggregate stock of productive capital (inventories of unfinished products, raw materials and undepreciated fixed assets);

2. the aggregate stock of commercial (or commodity) capital (inventories of finished products awaiting sales); and

3. the aggregate stock of financial capital (money and financial assets).

The circuit of capital model can, therefore, be conveniently represented as a circular flow of expanding value with three nodes (representing stocks of value) connected by three flows. Figure 1, drawn from Foley (1986b), is a graphical representation of the circuit. It is important to note that each element of the circuit of capital corresponds to observable quantities in real capitalist economies. While the flows of value in the circuit are recorded in the profit-loss statements of capitalist enterprises, the stocks are recorded in their balance sheets. This implies that a circuit of capital model can be empirically operationalized and used to study tendencies in "actually existing capitalism".

An elegant continuous-time formalization of Marx's analysis of the circuits of capital was developed in Foley (1982, 1986a). This model was empirically operationalized for the U.S. manufacturing sector in Alemi and Foley (1997) and has been used recently in dos Santos (2011) to study the impact of consumption credit on economic growth. Matthews (2000) develops an econometric model of the circuit of capital model. A different, but related,

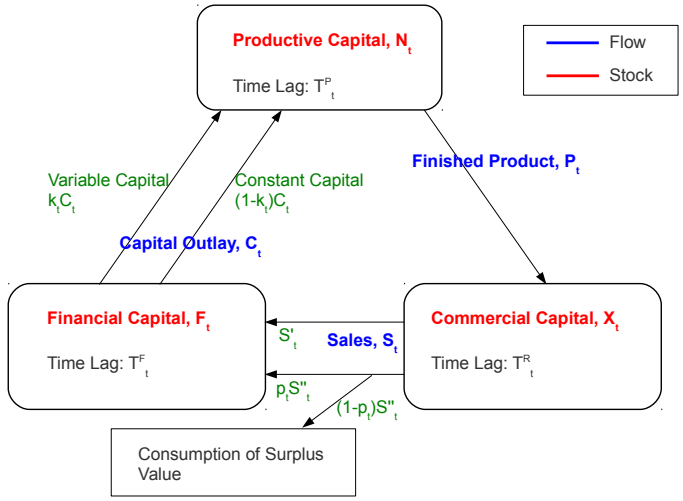

Figure 1: *The Circuit of Social Capital: A snapshot of the economy in period t, which shows the stocks of value at the beginning of the period, and the flows of value that occur within the period. (Source: Foley (1986b).)*

strand of the literature emerged from Kotz (1988, 1991), who used a circuit of capital model to analyze crisis tendencies witin capitalist economies. Loranger (1989) used the circuit of capital model to offer a new perspective on inflation.

This paper builds on and extends the approach in Foley (1982, 1986a) by developing a discrete-time version of the circuit of capital model. It is in this paper argued that the circuit of capital model offers a distinctive approach to analyzing macroeconomic behaviour of capitalist economies, which is different from both the neoclassical and Keynesian approaches. The neoclassical approach focuses on supply-side issues to the virtual neglect of demand-side factors; hence it is one-sided. The Keynesian approach restores the importance of demand-side issues within macroeconomics but, in turn, neglects the centrality of the profit-motive (the need for the generation and realization of surplus value) in driving the capitalist system. Hence, the Keynesian approach is one-sided too, because it overlooks the constraints that are imposed on the capitalist system due to the blind drive for surplus value even in the absence of aggregate demand problems. By transcending both kinds of one-sidedness, the Marxian circuit of capital model offers a distinctive framework for macroeconomic analysis of capitalist economies which accords centrality to the generation and circulation of surplus-value.

To demonstrate its strength and versatility, two important issues of interest to a broad range of heterodox economists are addressed within the Marxian circuit of capital model: wage-led versus profit-led growth regimes, and the impact of growing consumption credit on economic growth.

The rest of the paper is organized as follows. Section 2 offers a discussion of one of the crucial conceptual innovation of the circuit of capital model: the time lag structures. Section 3 sets up and solves the baseline model (i.e., model without aggregate demand considerations). Section 4 extends the baseline model by explicitly incorporating aggregate demand. Section 5 presents the main results of the paper, and the following section concludes the discussion.

2 Time Lag Structure

One of the main conceptual innovations of the circuit of capital model is the time lag structures that attach to each of the three phases of the circuit. To conceptualize the time lag structures rigorously and to develop the rest of the argument in this paper we will work in a discrete-time setting. The main reason to choose a discrete-time set-up over a continuous-time set-up, as in Foley (1982, 1986a), is that all economic variables are recorded only at discrete points in time. Since empirical operationalization of the Marxian circuit of capital model will need to work with discrete-time data, it seems analytically suitable to develop the model in a discrete-time set-up from the outset.

In a discrete-time setting, we can use a value emergence function (VEF) to capture the time lags involved in the different phases of the circuit of capital in the most general fashion. The VEF gives the time structure of emergence of value within the circuit of capital for value that has been injected into the circuit in some past time period. To fix ideas, suppose the process of injection of value occurred in period t'; then the most general form of a VEF would be the function $a_{t-t';t'}$, which represent the fraction of the injected value that emerges in the circuit $(t - t')$ periods later. Since all the value that was injected in period t' has to eventually emerge back into the circuit, we have

$$\sum_{t'=0}^{\infty} a_{t-t';t'} = 1. \tag{2}$$

There are four different ways to operationalize the VEF.

1. Fixed Time Lead (FTL): This is the simplest VEF, where value emerges after a fixed number of periods, T say, all at once;

2. Variable Time Lead (VTL): In this case we relax the assumption that the time structure of emergence is fixed across periods so that the value emerges, all at once as before, but after a variable number of periods, T_t say, i.e., the time of emergence varies across periods (hence the time subscript on T_t);

3. Finite Distributed Lead (FDL): In this case we relax the assumption that the value emerges all at once (which, for instance, is clearly relevant to the case of investment in long-lived fixed assets); hence value emerges gradually but over a finite number of future periods.

4. Infinite Distributed Lead (IDL): This generalizes FDL further by allowing the injected value to emerge gradually over all future periods.[1]

Returning to (2), it is obvious that for FTL, $a_{t'+T;t'} = 1$, and for VTL $a_{t'+T_{t'};t'} = 1$ so that the sum in (2) will have only one term (the only difference being the time dependence of $T_{t'}$ in the case of VTL). For FDL and IDL, on the other hand, the sum will have a finite and infinite number of terms, respectively.

We can now use the VEF to conceptualize the time lag structures involved in the circuit of capital. Let us start by looking back, from the vantage point of period t, and consider value injections (capital outlays, say) t' periods ago. Let us denote this value injection into the circuit in period $(t-t')$ by $C_{t-t'}$. Now consider the VEF for period $(t-t')$, $a_{t;t-t'}$. Recall that $a_{t;t-t'}$ gives the fraction of value committed in period $(t-t')$ that emerges back into the circuit in period t. Hence, the product of the two, $a_{t;t-t'}C_{t-t'}$, represents the quantum of value emerging in period t due to value committed in period $(t-t')$. In the most general case, i.e., using a IDL type of VEF, the flow of value emerging in period t will be the result of values injected into the circuit in *all* past periods (because value emerges, with an IDL type of VEF, over an infinite number of future periods). If we denote the flow of value emerging in period t as P_t (flow of finished products, say), then

$$P_t = \sum_{t'=-\infty}^{t} a_{t;t-t'}C_{t-t'}. \tag{3}$$

It can be seen immediately that with a FDL type of VEF, the sum in (3) will only have a finite number of terms. With a FTL type of VEF, on the other hand, the sum would have only one term, giving us $P_t = C_{t-T}$; and with a VTL, we would have $P_t = C_{t-T_t}$ (the only difference being the time dependence of T_t with a VTL)[2].

While the IDL provides the most general mathematical way to formulate a VEF, in the sense of imposing the least restrictions on the VEF, a FDL is conceptually sufficient for the analysis of value flows in capitalism. The only distinction between the IDL and the FDL is that the latter assumes the existence of a *finite number* of periods, no matter how large, at the end of which the relevant value emergence process has completed itself; the IDL assumes, on the contrary, that no such finite number exists.

Is the FDL restrictive? In the case of circulating capital, it is immediately obvious that an FDL is appropriate. In the case of fixed capital too, an FDL is appropriate. This because the distinction between fixed and circulating capital is only a matter of degree; for the former, the turnover time is a finite multiple of the turnover time for the latter. "The part of the capital value that is fixed in the means of labour circulates, just like any other part ... the whole of the capital value is in constant circulation, and in this sense, therefore, all capital is circulating capital" (Marx, 1993, pp. 238). But then what is the distinction between fixed and circulating capital? According to Marx, the distinction resides in the different ways in which the two circulate.

[1] Foley (1982) uses a continuous-time analog of the IDL-type VEF.

[2] Note that, in a discrete-time setting, a VEF is a probability mass function over the positive integers; in a continuous-time setting, it is a probability density function over the positive real line.

> But the circulation of the part of the capital considered here [i.e., fixed capital] is a peculiar one. In the first place, it does not circulate in its use form. It is rather its value that circulates, and this does so gradually, bit by bit, in the degree to which it is transferred to the product that circulates as a commodity. A part of its value always remains fixed in it [i.e., in the means of labour that is the material expression of fixed capital] as long as it continues to function, and remains distinct from the commodities that it helps to produce. This peculiarity is what gives this part of constant capital the form of *fixed capital*. All other material components of the capital advanced in the production process, on the other hand, form, by contrast to it, circulating or *fluid capital* (Marx, 1993, pp. 238) (emphasis in original).

Thus, all capital is, from the perspective of circulation, circulating capital. But circulating capital transfers its value to the product in a finite number of periods; the value emergence process completes itself in a finite number of periods. Hence, an FDL-type VEF is conceptually sufficient to deal with the analysis of capitalism.[3]

But we can move one step further. Any FDL-type VEF is analytically equivalent to a VTL-type VEF. This is because it is always possible to represent the gradual emergence of value entailed by a FDL-type VEF with a all-at-once emergence entailed by a VTL, where the time lead in the VTL measures the "average" of the total (finite) number of periods for which a FDL-type VEF allows nonzero value emergence. Thus, if a general FDL-type VEF, $a_{t'+i;t'}$, entails complete value emergence over the M periods following the injection into the circuit, then we can always find a number M' such that

$$M' = \frac{1}{M}\sum_{i=1}^{M} i \times a_{t'+i;t'} \times \mathbb{1}_{\left[a_{t'+i;t'}\neq 0\right]}, \tag{4}$$

where $\mathbb{1}_{\left[a_{t'+i;t'}\neq 0\right]}$ represents an indicator function picking up the periods of nonzero value emergence between period $(t'+1)$ and $(t'+M)$. Thus, the FDL-type VEF, $a_{t'+i;t'}$ (with $i = 1, 2, \ldots, M$), is equivalent to the VTL with time lead M'. Therefore, a VTL-type VEF can be used to develop a Marxian circuit of capital model which, without sacrificing any of the analytical strengths of Foley (1982), comes with the tractability of Foley (1986b, chap. 5). The next section develops such a model.[4]

[3] A more sophisticated treatment of the depreciation of fixed capital might require some modification of FDL-type VEF; this is left for future research.

[4] The main difference of the model in this paper with the model in Foley (1986b, chap. 5) is that I use a VTL-type VEF while Foley (1986b, chap. 5) uses a FTL-type VEF. The VTL-type VEF allows the time lags to be endogenized but a FTL-type VEF does not. Hence, the former can be used to study out-of-steady-state behavior and the phenomenon of crisis, for which purpose the latter cannot be used.

3 The Model without Aggregate Demand

3.1 Basic Set-up

The circuit of capital model, as represented graphically in Figure 1, involves three flow variables: C_t, the flow of capital outlays; P_t, the flow of finished products; and S_t, the flow of sales. Assuming a VTL-type VEF underlying each of the three flows, we can posit T_t^P, T_t^R and T_t^F, to denote the production, realization and finance lags, respectively, in period t.[5]

The production lag of T_t^P periods mean that the flow of finished products in any period is equal to the flow of capital outlays T_t^P period ago:

$$P_t = C_{t-T_t^P}. \tag{5}$$

In a similar manner, the presence of the realization lag implies that the flow of sales in any period is equal to the flow of finished products T_t^R periods ago:

$$S_t = (1+q_t)P_{t-T_t^R}, \tag{6}$$

where q_t is the mark-up over cost in period t arising from the exploitation of labour by capital. The mark-up over cost is defined as the product of the rate of exploitation, e_t, and the share of capital outlays devoted to variable capital, k_t (see Foley, 1986b, chap. 2).

It is useful to break up the flow of sales into two parts

$$\begin{aligned} S_t &= (1+q_t)P_{t-T_t^R} \\ &= P_{t-T_t^R} + q_t P_{t-T_t^R} \\ &= S_t' + S_t'' \\ &= \frac{S_t}{1+q_t} + \frac{q_t S_t}{1+q_t} \end{aligned}$$

where S_t' is the flow of sales corresponding to the recovery of capital outlays, and S_t'' is the part of sales flow that corresponds to the realization of surplus value.

Capital outlays, in turn, are financed by the flow of past sales but only with a time lag, the finance lag, T_t^F; hence,

$$C_t = S'_{t-T_t^F} + p_t S''_{t-T_t^F}, \tag{7}$$

where T_t^F is the finance lag (the number of periods that is required for realized sales flows to be recommitted to production), and p_t is the fraction of surplus value that is recommitted to production, the rest being consumed by capitalist households, unproductive labour and the State.

Positive production, realization and finance lags imply that there will be build-up of stocks of value at any point in time. If N_t denotes the stock of productive capital in period

[5] Here, I will rapidly go over the basic concepts related to the circuit of capital; for a more detailed exposition, see Foley (1986a).

t, then the accumulation (or decumulation) of the stock of productive capital will be given by

$$\Delta N_{t+1} = N_{t+1} - N_t = C_t - P_t. \tag{8}$$

Similar, letting X_t denote the stock of commercial capital, we will have

$$\Delta X_{t+1} = X_{t+1} - X_t = P_t - \frac{S_t}{1+q_t} = P_t - S'_t. \tag{9}$$

If F_t denotes the stock of financial capital in period t, then

$$\Delta F_{t+1} = F_{t+1} - F_t = S'_t + p_t S''_t - C_t. \tag{10}$$

The basic circuit of capital model is defined by the six equations, (5) through (10), q_t, p_t and the three lags T_t^P, T_t^R, T_t^F being the parameters governing the behaviour of the system.

3.2 Baseline Solution: Expanded Reproduction

On a steady state growth path, all the flow and stock variables of the circuit of capital model grow at the same rate and the parameters are constant over time. Thus, on a steady state growth path

$$p_t = p; q_t = q$$

and the three time lags are constant across time periods:

$$T_t^P = T^P; T_t^R = T^R; T_t^F = T^F.$$

What determines the rate of profit? How fast does such a system expand? Is the rate of expansion related to the exploitation of labour and the periodicity of the flow of value? These questions were analyzed by Marx in Part Two of Volume II of *Capital* and are presented here as

Proposition 1. *On a steady state growth path with time-invariant parameters, the system represented by (5) through (10), grows at the rate g given by*

$$g = \frac{pq}{T^F + T^R + T^P},$$

and the aggregate rate of profit is given by

$$r = \frac{q}{T^F + T^R + T^P},$$

so that the Cambridge equation holds true: $g = p \times r$.

Proof. Repeated substitutions into the equations relating the three basic flows comprising the circuit of capital, (5), (6) and (7), allow us to solve for the rate of steady state growth of the system. By definition, we have

$$\begin{aligned} C_t &= S'_{t-T^F} + pS''_{t-T^F} \\ &= \frac{S_{t-T^F}}{1+q} + \frac{pqS_{t-T^F}}{1+q} \\ &= \frac{(1+pq)}{1+q} S_{t-T^F} \\ &= \frac{(1+pq)}{1+q} \times (1+q)P_{t-T^F-T^R} \quad \text{(using (6))} \\ &= (1+pq)C_{t-T^F-T^R-T^P} \quad \text{(using (5))}. \end{aligned} \tag{11}$$

If the system grows at the rate g every period, then

$$C_t = C_0(1+g)^t;$$

hence, substitution in (11), gives

$$C_0(1+g)^t = (1+pq)C_0(1+g)^{t-T^F-T^R-T^P}.$$

Taking logarithms of both sides and simplifying gives us the characteristic equation of the system

$$1 = \frac{1+pq}{(1+g)^{T^P+T^R+T^F}}. \tag{12}$$

This gives us the growth rate of the system as

$$\ln(1+g) \approx g = \frac{\ln(1+pq)}{T^F+T^R+T^P} \approx \frac{pq}{T^F+T^R+T^P}. \tag{13}$$

An immediate corollary is that the system does not grow when all the surplus value is consumed (simple reproduction). According to the notation of the model, simple reproduction requires $p = 0$. But, if $p = 0$, this implies, by (11), that $g = 0$.

The second part of the proposition requires us to compute the rate of profit, and this, in turn, requires us to compute the *size* of the flows and stocks of value on the steady state growth path. Since the steady state growth rate of the system has been computed, the size of the flows and stocks in any given period is completely determined by its size in the initial period, i.e., period 0. But we can only compute the size of the stocks and flows relative to each other; hence, we will normalize the flow of capital outlays in period 0 to unity, i.e., $C_0 = 1$, and compute the other flows and stocks relative to C_0.

Since, according to (5), the flow of finished products (valued at cost) and the flow of capital outlays are related as

$$P_t = C_{t-T^P},$$

so, on the steady state growth path,

$$P_0(1+g)^t = C_0(1+g)^{t-T^P}.$$

Hence,

$$P_0 = \frac{C_0}{(1+g)^{T^P}} = \frac{1}{(1+g)^{T^P}}. \tag{14}$$

In a similar manner, using (6), we see that

$$S_0 = (1+q)\frac{C_0}{(1+g)^{T^P+T^R}} = \frac{1+q}{(1+g)^{T^P+T^R}}, \tag{15}$$

which implies that

$$S_0' = \frac{1}{(1+g)^{T^P+T^R}}, \tag{16}$$

and

$$S_0'' = \frac{q}{(1+g)^{T^P+T^R}}. \tag{17}$$

The size of the stocks of value in the initial period can be computed in a similar manner. According to (8), the stock of productive capital changes period t as:

$$\Delta N_{t+1} = N_{t+1} - N_t = C_t - P_t.$$

Hence

$$\begin{aligned} &N_t[(1+g)-1] = C_t - P_t \\ &N_0(1+g)^t g = C_0(1+g)^t - P_0(1+g)^t \\ &N_0 = \frac{1-P_0}{g}. \end{aligned} \tag{18}$$

Using (14) this becomes

$$N_0 = \frac{1}{g}\left\{1 - \frac{1}{(1+g)^{T^P}}\right\}. \tag{19}$$

In a similar manner, using (9), we can compute the size of the stock of commercial capital in the initial period as

$$X_0 = \frac{1}{g(1+g)^{T^P}}\left\{1 - \frac{1}{(1+g)^{T^R}}\right\}; \tag{20}$$

and using (10), we can compute the size of the stock of financial capital in the initial period as

$$F_0 = \frac{1}{g}\left\{\frac{1+pq}{(1+g)^{T^P+T^R}} - 1\right\}. \tag{21}$$

This allows us to compute the aggregate rate of profit, i.e., the rate of profit that the total social capital earns in a period

$$r_t = \frac{S''_t}{N_t + X_t + F_t}.$$

On a steady state growth path, the rate of profit is constant and is given by

$$r == \frac{S''_0}{N_0 + X_0 + F_0}.$$

Using the expression for S''_0 from (17), N_0 from (19), X_0 from (20) and F_0 from (21), we get the "Cambridge equation"

$$r = \frac{g}{p}.$$

Since $g = pq/(T^P + T^R + T^F)$, we get

$$r = \frac{q}{T^P + T^R + T^F}. \tag{22}$$

□

Working through the proof of this Proposition is instructive because it helps us master the techniques needed to solve the Marxian circuit of capital model. But even without going through the details of the proof, it easy to see that this result is crucial for the Marxian understanding of capitalism. It provides an intuitive explanation of the rate of profit in capitalism, arguably the most important variable governing the dynamics of a capitalist system.

The result shows clearly that the rate of profit rests on two factors, the rate at which surplus value is extracted from labour (captured by q), and the speed with which each atom of value traverses the circuit of capital (captured by what Marx termed the turnover time of capital, $T^F + T^R + T^P$). Social relations of production and class struggle impacts q through the rate of exploitation, e; technological factors relating to production impacts both q (through the proportion of capital outlays devoted to variable capital,k) and the production time lag T^P and technological factors relating to circulation impacts the time lags, T^R and T^F. Thus, this formulation makes it clear that the rate of expansion of the system is directly impacted by the rate of profit, and the rate of profit, in turn, is affected by both social and technological factors.

How does the rate of expansion of the system respond to changes in the distribution of aggregate income? Is the system a wage-led or profit-led growth regime? It can be seen

that the baseline Marxian circuit of model (without aggregate demand) is a pure profit-led regime. This can be immediately seen from Proposition 1: $\partial g/\partial q = p/(T^F + T^R + T^P) \geq 0$. Hence, a shift in the income distribution towards the capitalist class increases the growth rate of the system in the baseline model.[6] This is certainly not analytically satisfactory from a heterodox perspective; it seems intuitive that growth-impacts of income distribution in capitalist economies should allow for both wage-led and profit-led growth regimes (Bhaduri and Marglin, 1990; Foley and Michl, 1999; Taylor, 2006). We will see in a later section, and this is one of the main results of this paper, that this is indeed so: as soon as we incorporate aggregate demand in the circuit of capital model, we recover the possibility of both wage-led and profit-led growth.

4 The Model with Aggregate Demand

The results in Proposition 1 were derived under the admittedly unrealistic assumption that an adequate amount of aggregate demand was forthcoming in each period to realize all the value contained in the commodities offered for sale.[7] This is unrealistic because, under capitalism, there is no automatic mechanism to ensure that aggregate demand will equal aggregate supply (the total value contained in the commodities offered for sale). To explore the issue we need to explicitly account for the sources of demand.

4.1 The Realization Problem

What are the sources of demand in a closed capitalist economy?

> In so far as the capitalist simply personifies industrial capital, his own demand consists simply in the demand for means of production and labour-power ... In so far as the worker converts his wages almost wholly into means of subsistence, and by far the greater part into necessities, the capitalist's demand for labour power is indirectly also demand for the means of consumption that enter into the consumption of the working class Marx (1993, pp. 197).

In a capitalist economy closed to trade and *without the mechanism of credit* available to workers and capitalists, there are three sources of aggregate demand, all finally deriving from expenditures of capitalist enterprises: (a) the part of capital outlays that finances the purchase of the non-labour inputs to production (raw materials and long-lived fixed assets), (b) the consumption expenditure by worker households out of wage income, and (c) the consumption expenditure of capitalist households, unproductive labour and the State out of surplus value. Thus, if D_t denotes aggregate demand in period t, then

$$D_t = (1 - k_t)C_t + E_t^W + E_t^S \tag{23}$$

[6] An increase in the mark-up is a way to capture the shift of income towards the capitalist class.

[7] In this paper we will abstract from changes in the value of money.

where E_t^W denotes consumption expenditure out of wage, E_t^S denotes consumption expenditure out of surplus value, and C_t denotes capital outlays (as before).

Just like the recommittal of surplus value designated to production occurs with a time lag, the finance lag (T_t^F), consumption expenditure out of wages and surplus value will also occur with a time lag. If T_t^W denotes the time lag of expenditure out of wages, then

$$E_t^W = k_{t-T_t^W} C_{t-T_t^W};$$

similarly, if T_t^S denotes the time lag of expenditure out of surplus value, then

$$E_t^S = (1 - p_{t-T_t^S}) S''_{t-T_t^S}.$$

The three spending lags, T_t^F, T_t^W and T_t^S are crucial variables of the system. When the system is out of steady state, an increase in any of the spending lags implies that the amount of aggregate demand is falling relative to supply so that the ability of capitalist enterprises to sell finished products declines; when these fall, the opposite is implied. Thus, these spending lags can be used as aggregate demand parameters of the system Foley (1986a, pp. 24).

After incorporating the spending lags, the expression for aggregate demand becomes

$$D_t = (1 - k_t) C_t + k_{t-T_t^W} C_{t-T_t^W} + (1 - p_t) S''_{t-T_t^S}. \tag{24}$$

On a steady state growth path, the time lags of expenditure will be constant, i.e., $T_t^W = T^W$ and $T_t^S = T^S$, so that

$$D_t = (1 - k) C_t + k C_{t-T^W} + (1 - p) S''_{t-T^S}. \tag{25}$$

Hence, on a steady state path with growth rate g,

$$\begin{aligned}
D_0 &= (1 - k) + \frac{k}{(1+g)^{T^W}} + \frac{(1-p) S''_0}{(1+g)^{T^S}} \\
&= \left[1 - k \left\{1 - \frac{k}{(1+g)^{T^W}}\right\}\right] + \frac{q(1-p)}{(1+g)^{T^S + T^P + T^R}} \\
&= \frac{1}{(1+g)^{T^P + T^R}} \left[1 - k\{1 - \frac{1}{(1+g)^T W}\} \frac{1}{(1+g)^{T^F}}\right] \\
&\quad + \frac{1}{(1+g)^{T^P + T^R}} \times q \left[\left\{1 - k[1 - \frac{1}{(1+g)^{T^W}}]\right\} \frac{p}{(1+g)^{T^F}} + \frac{1-p}{(1+g)^{T^S}}\right] \\
&< \frac{1}{(1+g)^{T^P + T^R}} \times (1 + q) \\
&= S_0,
\end{aligned} \tag{26}$$

where the strict inequality holds when g, T^F, T^W, T^S are all strictly positive and the third line uses (12). This classic result about realization problems in capitalist economies, proved in Foley (1982) in a continuous-time setting and in a different, but related, framework in Kotz (1988), is summarized here as

Proposition 2. *In a capitalist economy where all capital outlays are financed from past sales revenue and there is no consumption credit in the economy there will always be insufficient aggregate demand relative to the flow of sales on any steady state growth path with a positive growth rate unless the three spending lags, T^F, T^W and T^S, are identically equal to zero.*

Since the spending lags cannot all be identically equal to zero, capitalist economies without the mechanism of credit will be perpetually plagued with the problem of insufficient aggregate demand. While this crucial insight about capitalist economies is usually attributed to Keynes (1936), it is interesting to note that Marx anticipated more or less the same result half a century ago:

> The capitalist casts less value into circulation in the form of money than he draws out of it, because he casts in more value in the form of commodities than he has extracted in the form of commodities. In so far as he functions merely as the personification of capital, as industrial capitalist, his supply of commodity-value is always greater than his demand for it...What is true for the individual capitalist, is true for the capitalist class (Marx, 1993, pp. 196-7).

It is intuitively clear that there are two ways to "solve" the realization problem. In a commodity money system, the production of the correct amount of gold (the money commodity) can cover the deficiency in aggregate demand (because the money commodity does not need to be sold to realize the value contained in it). In a non-commodity money system, new borrowing by households and capitalist enterprises can bridge the gap between aggregate supply and demand. Since, unlike Marx's times, we no longer operate in a commodity money system, we will only deal with the case of new borrowing.

4.2 Solution to the Realization Problem with Credit

To allow for positive amounts of *net credit* in the system in each period, let B_t denote the new borrowing by capitalists to finance capital outlays, i.e., production credit, so that

$$C_t = S'_{t-T_t^F} + p_t S''_{t-T_t^F} + B_t. \tag{27}$$

On a steady state growth path, the time lags are constant across periods, so that

$$C_t = S'_{t-T^F} + p_t S''_{t-T^F} + B_t. \tag{28}$$

On simplification, this gives,

$$1 = B_0 + \frac{(1+pq)S_0}{(1+q)(1+g)^{T^F}}, \tag{29}$$

where B_0 is the amount of production credit in the initial period as a proportion of capital outlays; we will assume that $0 \leq B_0 < 1$, where the strict inequality ensures that production

credit is never larger than capital outlays. Similarly, let B'_t denote new borrowing by households (worker, capitalist or the State) to finance consumption expenditure, i.e., consumption credit, so that aggregate demand becomes

$$D_t = (1-k_t)C_t + k_{t-T_t^W}C_{t-T_t^W} + (1-p_t)S''_{t-T_t^S} + B'_t. \tag{30}$$

If, on a steady state growth path, the realization problem is to be solved every period by new borrowing, i.e., if new borrowing is to cover the gap between sales and production, we must have

$$S_t = D_t \quad \text{which implies} \quad S_0 = D_0.$$

Using (29) and simplifying, this gives

$$S_0 = D_0 = \frac{1-k\left\{1-\frac{1}{(1+g)^{T^W}}\right\}+B'_0}{\left\{1-\frac{q(1-p)}{(1+q)(1+g)^{T^S}}\right\}},$$

where B'_0 denotes the amount of consumption credit in the initial period as a ratio of capital outlays. Substituting for S_0 in (29) gives us the characteristic equation of the system with positive net credit

$$1 = B_0 + \frac{(1+pq)}{(1+g)^{T^F}} \times \frac{1-k\left\{1-\frac{1}{(1+g)^{T^W}}\right\}+B'_0}{\left\{1-\frac{q(1-p)}{(1+q)(1+g)^{T^S}}\right\}}. \tag{31}$$

Re-arranging the characteristic equation, we see that the value of the steady state growth rate, g^*, solves

$$H(g;p,q,k,T^W,T^S,T^F,B_0,B'_0) = F(g;p,q,T^S,T^F) - G(g;p,q,k,T^W,B_0,B'_0) = 0, \tag{32}$$

where

$$F(g;p,q,T^S,T^F) = (1+g)^{T^F}(1+q) - \frac{q(1-p)}{(1+g)^{T^S-T^F}} \tag{33}$$

and

$$G(g;p,q,k,T^W,B_0,B'_0) = \frac{(1+pq)(1-k+B'_0)}{1-B_0} + \frac{k(1+pq)}{(1-B_0)(1+g)^{T^W}}. \tag{34}$$

It is obvious that (32) defines the steady state growth rate as an implicit function of the parameters of the system. Hence, we can use the Implicit Function Theorem (IFT) to work out the effects of changes in the parameters on the steady state growth rate, i.e., comparative dynamic results. But to apply the IFT we must prove two things: (a) that a solution to (32) exists, and (b) that the partial derivative of $H(g;.)$ with respect to g evaluated at the solution is nonzero.[8] Both these results are given as

[8]There are several versions of the IFT; for the results in this paper the version available as Theorem 15.2 in Simon and Blume (1994) is used.

Lemma 1. *There exists a unique nonnegative value g^* that solves the system represented in (32), i.e,*

$$H(g^*; p, q, k, T^W, T^S, T^F, B_0, B_0') = 0.$$

Moreover,

$$\frac{\partial H}{\partial g}(g^*; p, q, k, T^W, T^S, T^F, B_0, B_0') < 0.$$

Proof. Let x denote the vector of parameters, i.e.,

$$x = (p, q, k, T^W, T^S, T^F, B_0, B_0').$$

Note that, as long as $0 \leq B_0 < 1$ and $0 \leq B_0' < 1$, $H(0; x) > 0$. This is because

$$1 + pq = F(0; x) \leq G(0; x) = (1 + pq) \times \frac{1 + B_0'}{1 - B_0}.$$

Moreover $H(g; x)$ is a monotonically decreasing function of g. This can be seen from the fact that

$$\frac{-k(1 + pq)T^W}{(1 - B_0)(1 + g)^{T^W+1}} = \frac{\partial G}{\partial g}(g; x) < 0,$$

and

$$\begin{aligned}\frac{\partial F}{\partial g}(g; x) &= \frac{T^F(1 + q)}{(1 + g)^{T^F+1}} - \frac{q(1 - p)(T^F - T^S)}{(1 + g)^{T^F+T^S+1}} \\ &= \frac{1}{(1 + g)^{T^F+1}} \left\{ T^F + q \times T^F \left[1 - \frac{1 - p}{(1 + g)^{T^S}} \right] + \frac{q(1 - p)T^S}{(1 + g)^{T^S}} \right\} > 0,\end{aligned}$$

so that

$$\frac{\partial H}{\partial g}(g; x) = \frac{\partial G}{\partial g}(g; x) - \frac{\partial F}{\partial g}(g; x) < 0.$$

Hence, there exists a nonnegative and unique value of g which solves (32). The solution can also be represented graphically as Figure 2, where the intersection of the two curves give the steady state growth rate, g^*, and changes in the parameters shift the curves to produce new steady-state growth rates.

The second part of the Proposition follows immediately. Since $\frac{\partial H}{\partial g}(g; x)$, holds for an arbitrary g, it must also hold for $g = g^*$. Hence,

$$\frac{\partial H}{\partial g}(g^*; p, q, k, T^W, T^S, T^F, B_0, B_0') < 0.$$

□

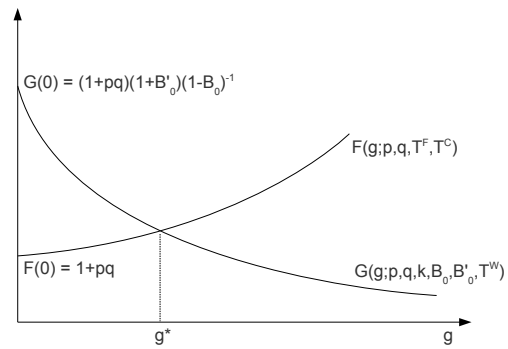

Figure 2: *Steady State Growth Rate of the System*

Graphical representation of the steady state solution by Figure 2 immediately allows us to derive some interesting results about comparative steady-state growth paths; these results are summarized as

Proposition 3. *Let g^* represent the unique, nonnegative solution of (32). The following comparative dynamics results hold true:*

1. *If B_0 (new borrowing to finance capital outlays) increases ceteris paribus then the steady state growth rate of the system, g^*, increases.*

2. *If B_0' (new borrowing to consumption expenditures) increases ceteris paribus then the steady state growth rate of the system, g^*, increases.*

3. *If either of the spending lags, T^W, T^S, T^F, increase ceteris paribus then the steady state growth rate of the system, g^*, falls.*

4. *If the proportion of capital outlays devoted to variable capital, k, increases ceteris paribus then the steady state growth rate of the system, g^*, falls.*

Proof. When B_0 or B_0' increases, one can immediately see, using (34), that the G-curve in Figure 2 shifts upwards. Hence, the steady state growth rate g^*, which corresponds to the intersection of the G and F curves, increase. Thus, the level of credit (new borrowing) in the economy is positively related to the steady state growth rate.

When T^F (finance lag) or T^S (spending lag for expenditure out of surplus value) increase, one can see, using (33), that the F-curve in Figure 2 shifts upwards. On the other hand, when T^W (spending lag for expenditure out of wages) increases, it follows from (34) that

the G-curve shifts downward. Hence, in all the three cases the value of g^* falls. Thus, lengthening of the spending lags reduces the rate of growth of the economy.

When k, the proportion of capital outlays devoted to variable capital, increases the G-curve shifts down, as can be seen using (34). Hence the intersection of the F and G curves shifts to the left, which implies that the steady state growth rate g^* falls. □

These comparative dynamics results can be re-stated more concretely with reference to two imaginary capitalist economies. First, between two identical capitalist economies, the one with higher amounts of credit will be on a higher steady-state growth path; this is simply because net credit solves the problem of insufficient aggregate demand by reducing the realization lag.

Second, the economy where a higher share of capital outlays is devoted to purchasing the non-labour inputs to production will have a lower rate of growth. This is because variable capital is the source of surplus value, the source of expansion of the system. Hence, a lower share devoted to variable capital will proportionately reduce the source of surplus value, and by implication, the source of expansion of the system.

Third, the economy with lower time lags will have a higher rate of profit and expansion. This is because lower time lags allows each atom of value to traverse the circuit in lesser time and thereby self-valorize itself in lesser time. Hence, the system grows faster per unit of time.

4.3 Maximal Growth Rate of the System

Since Proposition 3 shows that net positive credit always increases the steady state growth rate of the system, a question immediately arises: is there an upper limit to how fast the system can expand? Intuitively, it is clear that net positive credit increases the rate of growth of the system by reducing the length of the realization lag (the length of time that is required for finished products to be sold and the value, including surplus-value, to be converted into its money form). But the realization lag has a zero lower bound: finished products cannot be physically sold before they emerge from the process of production. Hence, the zero lower bound of the realization lag defines the maximal growth rate of the system.

To address this question in formal terms, we must endogenize the realization lag, T_t^R. To proceed, note that all the flow of finished products that emerge from the production process between periods $(t - T_t^R)$ and $(t + 1 - T_{t+1}^R)$ has to be realized by the flow of sales in period t. Hence,

$$P_{t-T_t^R} \times \left[(t + 1 - T_{t+1}^R) - (t - T_t^R)\right] \approx \frac{S_t}{1 + q_t};$$

hence,

$$\Delta T_{t+1}^R = T_{t+1}^R - T_t^R = 1 - \frac{S_t}{P_{t-T_t^R}(1 + q_t)}. \tag{35}$$

On a steady state growth path, $\Delta T^R_{t+1} = 0$; hence

$$S_t = (1+q)P_{t-T^R},$$

which implies

$$S_0 = (1+q)\frac{P_0}{(1+g)^{T^R}}.$$

Hence,

$$T^R = \frac{\ln([1+q]P_0/S_0)}{\ln(1+g)}.$$

The zero lower-bound of the realization lag, $T^R \geq 0$, implies $(1+q)P_0 \geq S_0$. Using the expression for S_0 and P_0 from the previous section, we have

$$\frac{1}{(1+g)^{T^P}} = \frac{1 - k\left\{1 - \frac{1}{(1+g)^{T^W}}\right\} + B'_0}{1 + q\left\{1 - \frac{(1-p)}{(1+g)^{T^S}}\right\}}. \tag{36}$$

The value of the growth rate, g^m, that solves (36) is the "maximal" growth rate of the system. What is the importance of this result? It shows that even in the *absence of aggregate demand problems*, a capitalist economy is limited by internal factors as to how fast it can expand. Thus, as Foley (1986a) points out, the Marxian circuit of capital model encompasses both Keynes (1936) and von Neumann (1945).

Note that when $T^W = T^S = B'_0 = 0$, (36) reduces to

$$1 + pq = (1+g^m)^{T^P},$$

the expression for the maximal growth rate in Foley (1986a).

4.4 Out of Steady State Behaviour

It is intuitively clear that spending lags and credit impact on the rate of steady state growth through their effect on the realization lag. To see this more concretely, we will study the out-of-steady-state behaviour of the system. In other words, we will endogenize the parameters of the system, i.e., allow them to vary across time and study the out-of-steady-state behaviour of the realization lag to changes in the spending lags and net credit.

To proceed, note that we can use a logic similar to that used in (35) to derive how spending lags change over time. Thus, the lag for spending out of wage income varies across time as

$$\Delta T^W_{t+1} = T^W_{t+1} - T^W_t = 1 - \frac{E^W_t}{W_{t-T^W_t}} = 1 - \frac{E^W_t}{k_{t-T^W_t} C_{t-T^W_t}}.$$

Similarly, the spending lag for expenditure out of surplus value varies as

$$\Delta T^S_{t+1} = T^S_{t+1} - T^S_t = 1 - \frac{E^S_t}{(1 - p_{t-T^S_t})S''_{t-T^S_t}},$$

and the finance lag changes as

$$\Delta T^F_{t+1} = T^F_{t+1} - T^F_t = 1 - \frac{C_t - B_t}{S'_{t-T^F_t} + p_{t-T^S_t} S''_{t-T^S_t}}.$$

On an arbitrary time path, not necessarily a steady-state path, consistent with the basic circuit of capital model, the period by period solution to the realization problem implies

$$S_t = D_t = E^W_t + E^S_t + (1 - k_t)C_t + B'_t.$$

Using the expressions for changes in the spending lags, this becomes

$$\begin{aligned} S_t = {} & (1 - \Delta T^W_{t+1})k_{t-T^W_t} C_{t-T^W_t} + (1 - \Delta T^S_t)(1 - p_{t-T^S_t})S''_{t-T^S_t} \\ & + (1 - k_t)(1 - \Delta T^F_{t+1})(S'_{t-T^F_t} + p_{t-T^S_t} S''_{t-T^S_t}) + (1 - k_t)B_t + B'_t. \end{aligned} \tag{37}$$

Since, the change in the realization lag is given by

$$\Delta T^R_{t+1} = 1 - \frac{S_t}{P_{t-T^R_t}(1 + q_t)},$$

substituting (37) gives

$$\begin{aligned} \Delta T^R_{t+1} = 1 - {} & \frac{1}{P_{t-T^R_t}(1 + q_t)}[(1 - \Delta T^W_{t+1})k_{t-T^W_t} C_{t-T^W_t} + (1 - \Delta T^S_t)(1 - p_{t-T^S_t})S''_{t-T^S_t} \\ & + (1 - k_t)(1 - \Delta T^F_{t+1})(S'_{t-T^F_t} + p_{t-T^S_t} S''_{t-T^S_t}) + (1 - k_t)B_t + B'_t]. \end{aligned} \tag{38}$$

This shows immediately that changes in the spending lags are positively related to changes in the realization lag, i.e.,

$$\frac{\partial \Delta T^R_{t+1}}{\partial \Delta T^W_{t+1}} > 0, \quad \frac{\partial \Delta T^R_{t+1}}{\partial \Delta T^S_{t+1}} > 0, \quad \frac{\partial \Delta T^R_{t+1}}{\partial \Delta T^F_{t+1}} > 0,$$

and that net credit is negatively related to the changes in the realization lag

$$\frac{\partial \Delta T^R_{t+1}}{\partial B_t} < 0, \quad \frac{\partial \Delta T^R_{t+1}}{\partial B'_t} < 0.$$

Hence, decreases in the spending lags decrease the realization lag and increase the rate of growth of the system on an arbitrary path; similarly, increases in net credit reduces the realization lag and increases the rate of expansion of the system.

5 Main Results

The discrete-time Marxian circuit of capital that has been developed in the previous sections will be used in this section to address two important issues of interest to a broad range of heterodox economists: (a) growth impacts of changes in the class distribution of income, and (b) growth impacts of non-production credit.

5.1 Wage-led versus Profit-led Growth Regimes

Analyzing the impact of changes in the distribution of income between social classes on the rate of growth of the capitalist system has been one of the critical features that distinguish the heterodox tradition in macroeconomics from the mainstream, neoclassical, one (Foley and Taylor, 2006). The basic heterodox idea that distinguishes it from the neoclassical tradition is to see wage income as playing a dual role in capitalist economies. One the one hand it appears as a cost to capitalist enterprises and impacts the rate of profit, investment decisions and thereby aggregate demand (profit effect); on the other hand, it furnishes the revenue to finance consumption expenditures by worker households, thereby functioning as a crucial component of aggregate demand (wage effect). Hence, a shift of income towards the worker class has an ambiguous effect on the overall expansion of the system, depending on whether the wage effect is stronger than the profit effect.[9]

In the Marxian circuit of capital model, this issue can be addressed by analyzing the effect of changes in the mark-up over costs, q, which can be understood as a proxy for the share of total income accruing to the non-working class. Since the mark-up is $q = ek$, an increase in the rate of exploitation e increases q and captures the shift in income away from productive workers, assuming that the proportion of capital outlays devoted to variable capital, k, remains unchanged. Increases in q, therefore, can capture the increasing share of income appropriated by the capitalist class. How does this impact the growth rate of the system? We have already seen that the baseline Marxian circuit of capital model (without taking account explicitly of aggregate demand) is a pure profit-led growth regime. This result changes as soon as we bring aggregate demand into the picture.

To see this we can once again use the IFT on (32) to find the impact of changes in q on the steady state growth rate of the system.[10] Let x denote the vector of parameters as before, i.e., $x = (p, q, k, T^W, T^S, T^F, B_0, B_0')$, g stand for a generic growth rate and g^* denote the steady state growth rate of the system; then, using the IFT on (32) we have

$$\frac{\partial g}{\partial q}(g^*; x) = \frac{\frac{\partial H}{\partial q}(g^*; x)}{[-\frac{\partial H}{\partial g}(g^*; x)]}.$$

[9] There is a huge, and growing, body of literature devoted to the issue of wage-led versus profit-led growth regimes; see Bhaduri and Marglin (1990), Foley and Michl (1999, chap. 9), Taylor (2006), Bhaduri (2008) and the references therein for a relatively comprehensive list of contributions to this emerging area of research.

[10] Note that Lemma 1 allows us to use the IFT.

Since, by Proposition 1, $\frac{\partial H}{\partial g}(g^*;x) < 0$, we have

$$\text{sgn}(\frac{\partial g}{\partial q}(g^*;x)) = \text{sgn}(\frac{\partial H}{\partial q}(g^*;x)).$$

Using (34) and (33), we have

$$\frac{\partial g}{\partial q}(g^*;x) = \frac{1}{(1-B_0)}\left\{1 - k\left[1 - \frac{1}{(1+g)^{T^W}}\right] + B_0'\right\} - (1+g)^{T^F}\left\{1 - \frac{1-p}{(1+g)^{T^S}}\right\}. \quad (39)$$

This allows us to provide sufficient conditions for a wage-led and a profit-led growth regime as

Proposition 4. *Suppose g^* denotes the steady state growth rate of the system represented by (32). If the finance lag is large enough, then the system is wage-led, i.e.,*

$$\textit{if} \quad T^F > \frac{\ln(1+B_0') - \ln(1-B_0)}{\ln(1+g^*)} \approx \frac{B_0 + B_0'}{g^*} \quad \textit{then} \quad \frac{\partial g}{\partial q}(g^*;x) < 0.$$

If the finance lag is small enough, then the system is profit-led, i.e.,

$$\textit{if} \quad T^F < \frac{\ln(1+B_0'-k') - \ln(1-B_0) + \ln(p/p')}{\ln(1+g^*)} \quad \textit{then} \quad \frac{\partial g}{\partial q}(g^*;x) > 0,$$

where

$$p' = 1 - \frac{1-p}{(1+g)^{T^S}} \quad \textit{and} \quad k' = k\left\{1 - \frac{1-p}{(1+g)^{T^W}}\right\}.$$

Proof. The proof follows immediately from (39). □

This proposition has at least two important implications. First, it demonstrates that the Marxian circuit of capital model is not a pure profit-led growth regime once aggregate demand has been explicitly modeled within the system. Since Volume II of *Capital* deals explicitly with issues of aggregate demand and its relation to problems of realization, the common Keynesian assertion that Marxian economics lacks a proper appreciation of demand factors is erroneous. By demonstrating that the Marxian circuit of capital model allows for a wage-led growth regime, Proposition 4 reinforces the Marxian case.

Second, it delivers the fairly intuitive result that the size of the finance lag, T^F, determines whether the system is profit-led or wage-led as far as growth is concerned. If, to start with, the finance lag is large relative to total new borrowing in the system normalized by the steady state growth rate, then a shift of income away from wages would be tantamount to shifting income towards economic agents who wait for a relative long period before converting realized sales revenues into new capital outlays. Hence, the level of aggregate demand would fall and the speed with which value traverses the circuit go down. This would lead to a fall in the rate of profit and the steady state growth rate. If, on the other hand, the finance lag is relatively small the opposite happens: the rate of profit and the rate of growth increases when income is shifted away from workers and towards capitalists.

5.2 Growth Impacts of Rising Consumption Credit

The consolidation of the neoliberal regime in the U.S. and elsewhere, since the early 1980s, went hand in hand with the growing dominance of finance over the economy.[11] One peculiar form of this dominance has been the explosion of debt levels, relative to aggregate income flows, in these advanced capitalist countries. A large part of the new borrowing has been incurred by working-class households (a component of consumption credit), as opposed to capitalist enterprises (production credit). Though the growth-reducing impact of financialization has been recently analyzed (Onaran et al., 2011), very few studies have been devoted to understanding the impact of consumption credit on growth.

An exception is dos Santos (2011), which has used a continuous-time Marxian circuit of capital model to demonstrate that the "maximal" rate of growth of a capitalist economy is negatively impacted by the growth of consumption credit.[12] This paper strengthens the result in dos Santos (2011) further by demonstrating that growth-reducing impact of consumption credit affects the actual growth rate too. This is important because the result about maximal growth rates does not imply the same about actual growth rates: an economy might have a lower maximal rate of growth compared to another at the same time as having a higher actual rate of growth.

While, according to Proposition 3, the growth of any kind of credit increases the growth rate of the economy, increases in the *share* of consumption credit *ceteris paribus* has an adverse impact on the steady state growth rate. To see this formally let us re-write the quantity of new consumption credit in the initial period as

$$B_0' = \lambda Z_0,$$

and the quantity of production credit as

$$B_0 = (1-\lambda)Z_0,$$

where Z_0 is the total amount of new borrowing in the economy in the initial period and $0 \leq \lambda \leq 1$ is the share of consumption credit in the total amount of new borrowing. To analyze the impact of changes in the share of consumption credit, we need to re-write (32) using λ and Z_0 in place of B_0' and B_0:

$$H(g; p, q, k, T^W, T^S, T^F, Z_0, \lambda) = F(g; p, q, T^S, T^F) - G(g; p, q, k, T^W, Z_0, \lambda) = 0. \quad (40)$$

The F-curve does not depend on the quantity of new borrowing; hence it remains the same as before

$$F(g; p, q, T^S, T^F) = (1+g)^{T^F}(1+q) - \frac{q(1-p)}{(1+g)^{T^S - T^F}}, \quad (41)$$

[11] For a detailed analysis of the rise and consolidation of neoliberalism, see Duménil and Lévy (2004).

[12] Recall that the maximal growth rate of the system is the rate at which it grows when the realization lag is zero. Since the realization lag is bounded below by zero, the maximal rate of growth defines the upper bound of the rate of expansion of the system.

but the G-curve changes to

$$G(g; p, q, k, T^W, Z_0, \lambda) = \frac{(1+pq)}{1-(1-\lambda)Z_0}\left\{1 - k + \lambda Z_0 + \frac{k}{(1+g)^{T^W}}\right\}. \tag{42}$$

Using the IFT on (40), we have

$$\frac{\partial g}{\partial \lambda}(g^*; x) = \frac{\frac{\partial H}{\partial \lambda}(g^*; x)}{[-\frac{\partial H}{\partial g}(g^*; x)]}.$$

Since $\frac{\partial H}{\partial g}(g^*; x) < 0$, the sign of $\frac{\partial g}{\partial \lambda}(g^*; x)$ is the same as the sign of $\frac{\partial H}{\partial \lambda}(g^*; x)$. But

$$\frac{\partial H}{\partial \lambda}(g^*; x) = \frac{(1+pq)Z_0}{[1-(1-\lambda)Z_0]^2} \times \left\{k - \frac{k}{(1+g^*)^{T^W}} - Z_0\right\}, \tag{43}$$

which gives us one of the key results of this paper: when the economy is operating with high levels of total credit, an increase in the share of consumption credit *ceteris paribus* will depress the steady state growth rate. We can state this result more formally as

Proposition 5. *Let Z_0 denote the level of total net credit in the economy, and λ denote the share of consumption credit, with $(1-\lambda)$ denoting the share of production credit. If*

$$Z_0 > k\left\{1 - \frac{1}{(1+g)^{T^W}}\right\}$$

then

$$\frac{\partial g}{d\lambda}(g^*; x) < 0.$$

Proof. The proof follows immediately from (43). □

What does this result imply? If we compare two identical capitalist economies (having the same amounts of total net credit), one with a higher share of consumption credit than the other, then Proposition 5 shows that the economy with a higher share of consumption credit can be expected to be on a lower steady state growth path than the other economy.

To understand the logic of this result, it is important to distinguish between consumption and production credit. By definition, only production credit finances capital outlays. Hence, it is only production credit that creates the flow of value for the creation of more surplus value. Hence, only production credit has the capacity to increase the size of value flowing through the circuit, and thereby expand the size of the system. Thus, while consumption credit solves the realization problem by reducing the realization lag, it cannot increase the size of the system by facilitating the generation of more surplus value (which production credit does). Since, when comparing steady state growth paths, the time lags are assumed constant, a higher share of consumption credit will reduce the rate of growth of the system.

An interesting corollary of Proposition 5 arises if the time lag for spending out of wages is "small": increases in the share of consumption credit is *always* growth-reducing. To see this note that if $T^W = 0$, then by Proposition 5 it follows that $\frac{\partial g^*}{d\lambda}(g^*; x) < 0$ for any positive amount of total net credit $Z_0 > 0$. Thus, if workers spend their wage income immediately, i.e., in the period in which they earn it, then an increase in the share of consumption credit will take the economy to a lower steady state growth path as long as there was positive net credit to begin with.

6 Conclusion

Marx's analysis of the circuits of capital in Volume II of *Capital* offers a unique framework for macroeconomic analysis of capitalist economies. This paper has developed a discrete-time version of Duncan Foley's formalization of the Marxian circuit of capital model (Foley, 1982, 1986a). Using this model two issues of wide interest among heterodox economists was addressed.

First, it was demonstrated that the Marxian circuit of capital model allows both wage-led and profit-led growth regimes. The crucial parameter of the system that determines whether a capitalist economy will be wage-led versus profit-led is the length of the finance lag (the period of time that elapses between realization of value, and surplus-value, through sale and its recommittal into production). When the finance lag is large, the economy is more likely to be wage-led; when the finance lag is small, the economy is more likely to be a profit-led growth regime.

Second, it was demonstrated that non-production credit has a negative impact on the rate of growth of the system. Between tow identical capitalist economies, the one with a higher proportion of non-production credit in total net credit will have lower steady state growth rate. This finding is relevant to understanding the slowdown, and wider growth problems, of capitalist economies saddled with high levels of household debt.

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