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Essays on Financial Behavior and its Macroeconomic Causes and Implications

Soon Ryoo

University of Massachusetts - Amherst, sryoo@econs.umass.edu

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ESSAYS ON FINANCIAL BEHAVIOR AND ITS MACROECONOMIC CAUSES AND IMPLICATIONS

A Dissertation Presented
by
SOON RYOO

Submitted to the Graduate School of the University of Massachusetts Amherst in partial fulfillment of the requirements for the degree of
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Economics
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ESSAYS ON FINANCIAL BEHAVIOR AND ITS MACROECONOMIC CAUSES AND IMPLICATIONS

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SOON RYOO

Approved as to style and content by:

________________________________________
Peter Skott, Chair

________________________________________
James Crotty, Member

________________________________________
James Heintz, Member

________________________________________
Diane Flaherty, Department Chair
Economics
ACKNOWLEDGMENTS

In the years during my graduate program at University of Massachusetts Amherst, I benefited enormously from Peter Skott. He provided generous support for my research, valuable suggestions and constructive comments on every aspect of this dissertation. I was very fortunate to have him as my dissertation advisor. I am also grateful to James Crotty and James Heintz who read this dissertation and provided several constructive criticisms and helpful suggestions. Financial support from Political Economic Research Institute at University of Massachusetts at Amherst is gratefully acknowledged.

I would also like to thank my mother, two brothers and four sisters in Korea for their support and encouragement. Finally, I wish to extend my special thanks to my wife, Bong A, for her support, patience and, most importantly, taking good care of our adorable baby, Ian. This dissertation could not be completed without them.
This dissertation consists of three independent essays. The first essay, “Long Waves and Short Cycles in a Model of Endogenous Financial Fragility,” presents a stock flow consistent macroeconomic model in which financial fragility in firm and household sectors evolves endogenously through the interaction between real and financial sectors. Changes in firms’ and households’ financial practices produce long waves. The Hopf bifurcation theorem is applied to clarify the conditions for the existence of limit cycles, and simulations illustrate stable limit cycles. The long waves are characterized by periodic economic crises following long expansions. Short cycles, generated by the interaction between effective demand and labor market dynamics, fluctuate around the long waves.

The second essay, “Macroeconomic Implications of Financialization,” examines macroeconomic effects of changes in firms’ financial behavior (retention policy, eq-
uity financing, debt financing), and household saving and portfolio decisions using models that pay explicit attention to financial stock-flow relations. Unlike the first essay, the second essay focuses on the effects of financial change on steady growth path. The results are insensitive to the precise specification of household saving behavior but depend critically on the labor market assumptions (labor-constrained vs dual) and the specification of the investment function (Harrodian vs stagnationist).

The last essay, “Finance, Sectoral Structure and the Big Push,” studies the role of finance in the presence of investment complementarities using a big push model. Due to complementarities between different investment projects, simultaneous industrialization of many sectors (big push) may be needed for an underdeveloped economy to escape from an underdevelopment trap. Such simultaneous industrialization requires costly coordination by a third party, such as the government. Some recent papers show that private banks with significant market power may also solve the problem of coordination failure. We show that private coordination may not work since even large private banks may find it more profitable to finance firms in the traditional sector than in the modern sector.
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INTRODUCTION

Chapters in this dissertation represent an attempt to understand changes in financial behavior and their macroeconomic causes and implications from three different angles. Chapter 1 examines changes in financial behavior from a perspective of macroeconomic instability and cycles. Chapter 2 analyzes macroeconomic implications of financialization from a perspective of long-run steady growth. Chapter 3 studies the role of finance in the presence of investment complementarities from a viewpoint of development and industrialization.

According to Minsky’s financial instability hypothesis, a capitalist economy cannot lead to “a sustained, stable-price, full employment equilibrium” and serious business cycles are unavoidable due to the unstable nature of the interaction between investment and capitalist finance (Minsky, 1986, 173). An initially robust financial system is endogenously transformed into a fragile system as cash-flow relations change during tranquil years. During expansions, an investment boom generates a profit boom but this induces investors and bankers to adopt more speculative financial arrangements. While Minsky’s hypothesis has received growing attention in light of the current financial crisis, the logic behind it is intricate and not always easy to understand.¹

¹For instance, Foley (2001) states “When Minsky’s followers try to formulate his vision into mathematical models, they face a series of methodological riddles. It is not easy to formulate a single, generic, range of assets to represent the multifarious vehicles for the financial maneuvers that lie behind financial fragility. The model needs to be able to represent a shift in the average riskiness of position... It is not clear exactly where to locate a parameter to represent the financial boldness of investors. In Minsky’s discourse, the shift toward more exposed financial positions is not simply a psychological phenomenon based in the increasing optimism or level of denial of investors (though that is surely part of the process), but involves strong competitive pressures on individual investors to conform to group norms that are themselves shifting...”
Chapter 1 incorporates Minskian ideas into a macroeconomic model in order to clarify mechanisms of financial instability and cycles. Many attempts have been made to formalize Minsky’s ideas. Early contributions include Taylor and O’Connell (1985), Foley (1986), Semmler (1987), Jarsulic (1989, 1996), Delli Gatti and Gallegati (1990), Skott (1994), Dutt (1995), Keen (1995) and Flaschel, Franke and Semmler (1998, Ch.12), and recent studies include Setterfield (2004), Nasica and Raybaut (2005), Lima and Meirelles (2007), Fazzari et al. (2008), and Charles (2008). My own contributions to this literature are threefold:

First, the model in chapter 1 is stock-flow consistent. Financial stocks are explicitly introduced and their implications for income and financial flows are carefully modeled. In particular, unlike the previous studies listed above, capital gains from holding stocks are not assumed away and enter the definition of the rate of return on equity. The rate of return on equity defined in this way provides a basis of households’ portfolio decision. Firms’ and households’ financial decisions jointly determine stock prices and the rate of return on equity in equilibrium. Thus, stock markets receive a careful treatment in this model and play a central role in producing cycles.

Second, my model pays attention to both firms’ and households’ financial decisions. Minsky’s own account of financial instability tends to privilege the firm sector as a source of fragility. Most previous studies follow this tradition and tend to neglect the role of households’ financial decisions in creating instability and cycles. Some of the previous studies, including Taylor and O’Connell (1985), Delli Gatti and Gallegati (1990), and Flaschel, Franke and Semmler (1998, Ch.12), do not suffer from this kind of limitation but analyze households’ portfolio decision as well. However, their neglect of the role of capital gains in households’ portfolio decision makes it difficult to analyze the implication of households’ financial decisions and stock mar-

2Empirically, the movements of capital gains explain most of cyclical movements of the rate of return on equity.
ket behavior for instability and cycles. In contrast to these models, the model in chapter 1 analyzes both households’ and firms’ financial decisions. Capital gains and stock markets are considered explicitly in a stock-flow consistent framework and the interactions between households and firms turn out to be critical to the behavior of the system. Firms’ financial decision is captured by changes in the debt-capital ratio and households’ decision by changes in the equity-deposit ratio. Changes in those two variables represent endogenous changes in financial fragility, and interact with each other through goods and financial markets.

Lastly, existing Minskian models are not clear about the periodicity of cycles. My model in chapter 1 is explicit in this matter. It produces two distinct cycles: long waves and short cycles. Long waves are produced by the interaction between firms’ and households’ financial decisions, while short cycles are generated by the interaction between effective demand and labor market dynamics. In this framework, Minsky’s financial instability hypothesis is seen as a basis of long waves. To the best of my knowledge, my model is the first to integrate an analysis of Minskian long waves with that of short cycles.

Capitalist economies have been often characterized as increasing financialization and it has been growing concern for some economists. For example, Lazonick and O’Sullivan argue that the increasing dominance of shareholder value has forced changes in management strategy from ‘retain and invest’ to ‘downsize and distribute’ and this change in strategy is reflected in increases in dividend rate and corporate buybacks (Lazonick and O’Sullivan, 2000, p.18). Crotty (2005) has suggested that

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3Minsky’s two papers (Minsky, 1964, 1995) provide a strong support for this view. In these two papers, Minsky argues that there exists a mechanism in a capitalist economy that generates a ‘long swing’: the “mechanism which has generated the long swings centers around the cumulative changes in financial variables that take place over the long-swing expansions and contractions.” (Minsky, 1964). “The more severe depressions of history occur after a period of good economic performance, with only minor cycles disturbing a generally expanding economy.” (Minsky, 1995, p.85) During this long expansion, an initially robust financial structure is transformed to a fragile structure.
financialization weakens non-financial corporations and constrained the growth of aggregate demand and Dumenil and Levy (2001) has suggested that financialization tends to depress growth and employment while destabilizing economies.

Chapter 2 examines the macroeconomic effects on steady growth path of changes in firms’ financial behavior (retention rate, net equity issues, debt finance), shifts in the investment function, household saving/portfolio decisions, and interest rates. The analysis is conducted in fully developed macroeconomic models to avoid the danger of partial analysis. The chapter analyzes several models which are distinguished based on the assumptions on accumulation behavior (Harrodian vs. Kaleckian) and labor market (dual vs. mature).

Financial stocks are explicitly introduced in those models and households’ saving/portfolio behavior is characterized by desired stock-flow ratios (equity/income and deposit/income ratios in a two-asset context). In addition to these features, this chapter adopts a novel approach to the analysis of the long-term effects of financial change. The approach consists of two steps: the first step investigates the effects of financial change, assuming that the desired stock-flow ratios are exogenous; the second step examines the effects of induced changes in the stock-flow ratios.

Chapter 2 is an outcome of a joint work with Peter Skott and published in Cambridge Journal of Economics (Skott and Ryoo, 2008). While some of the models build on previous works by Skott, I carried most of the analysis in the paper. I would like to list some elements I contribute to this joint work:

First, the main focus of chapter 2 is on the comparative statics regarding the macroeconomic effects of some financial changes associated with financialization. As-

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4Dangers of a partial analysis are discussed in 2.2.2 below.
5For the advantage of this approach, see Ryoo and Skott (2008)
6Ryoo and Skott (2008) complements this work by providing an analysis of Kaleckian economy with labor constraint.
suming the desired stock-flow ratios are exogenous, the comparative statics are relatively simple and transparent. The question is whether the qualitative results carry over to the case with endogenously determined stock-flow ratios. I showed that using two influential works, Lavoie and Godley (2001-02) and Godley and Lavoie (2007), empirically plausible specifications yield small induced changes in the stock-flow ratios that do not overturn the qualitative results from the simple model with exogenous ratios. This approach turns out to be useful to evaluate the results of other models regarding the effects of financialization. For instance, van Treeck (2008) presents simulation results as an example of contractionary effects of increasing dividend or stock buyback. Ryoo and Skott (2008) show that these contractionary effects are based on implausible adjustments of financial stocks.

Second, the analysis of Lavoie and Godley (2001-02) and Godley and Lavoie (2007) leads to a conclusion that the qualitative effects of financialization are insensitive to the precise specification of household saving behavior. Lavoie and Godley (2001-02) assume that consumption depends on household income and capital gains. Godley and Lavoie (2007) assume that consumption is determined by household income and wealth. Our own specification in Skott and Ryoo (2008) directly relates consumption to stock-flow ratios. All of these specifications produce the same qualitative results regarding the effects of financialization. In addition, several sensitivity analyses were conducted to show that the qualitative results are robust to wide ranges of parameter values in each consumption function.

Third, the analysis in this chapter achieves a substantial degree of generality by focusing on both Harrodian and Kaleckian specifications of investment behavior. In particular, the chapter analyzes the Kaleckian framework in great depth (though our own preference lies with a Harrodian framework). The analysis is performed based on
a general form of Kaleckian accumulation function (see equation 2.4. below), which include most of existing Kaleckian specifications as its subcases.\(^7\)

Implications of finance for developing countries have been the subject of much empirical research in the mainstream literature on development. One of the important topics is whether financial development explains countries’ growth performance. Many studies along this line accept the view that “the role of financial markets and institutions arises to mitigate the effects of information and transaction costs that prevent direct pooling and investment of society’s savings” (Demirguc-Kunt, 2006, p.1) From this perspective, good financial markets, or more developed financial markets, are identified as “deep” markets which mean more liquid stock markets and larger banking sectors. In spite of econometric and measurement issues, it is argued, there exists a robust positive relationship between financial development in this sense and growth performance (Levine, 2003). However, a common feature of recent mainstream studies on finance and development is characterized by its neglect of structural problems. This literature presumes that underdeveloped countries’ financial structures gradually evolve toward a more advanced one with deeper and thicker financial markets. The implications of increasing returns such as multiple equilibria, poverty traps and uneven sectoral development, which were main topics of traditional development economics, receive a minor role or are simply ignored in recent trends.

Chapter 3 addresses the role of finance in the presence of investment complementarities. The key questions are whether a competitive or a monopolistic banking structure is conducive to solving a coordination failure caused by investment complementarities and how the sectoral structure of an economy interacts with the banking structure. Little is known about this in the existing literature, but Da Rin and Hell-

\(^7\)Skott and Ryoo (2008) considers only a dual economy version of Kaleckian model but Ryoo and Skott (2008) extends the analysis to include a Kaleckian model with labor constraint.
man (2002) recently provides, based on a big push model, a theoretical argument that banks with market power may be able to solve this kind of coordination problem if they have sufficient market power to make profits from costly coordination.

The analysis in chapter 3 suggests that the conditions under which banks with market power can solve the coordination problems are restrictive and depend critically on the sectoral structure of an economy. Banks with market power, let alone small competitive banks, may fail to provide a solution to coordination failures if they find it more profitable to finance firms in the traditional sector than in the modern sector. Thus, the analysis emphasizes the limitation of private coordination.
CHAPTER 1
LONG WAVES AND SHORT CYCLES IN A MODEL OF ENDGENEOUS FINANCIAL FRAGILITY

1.1 Introduction

Financial crisis hit the U.S and world economy in 2008. Giant financial institutions have collapsed. Stock markets have tumbled, and exchange rates are in turmoil. Governments and central banks around the world have responded by implementing bailout plans for troubled financial institutions and cutting interest rates to contain the financial panic, and expansionary fiscal packages are being pushed through to prop up aggregate demand. Hyman Minsky’s Financial Instability Hypothesis offers an interesting perspective on these developments, which came after a long period of financial deregulation, rapid securitization and the development of a range of new financial instruments and markets.\(^1\)

According to Minsky’s financial instability hypothesis, a capitalist economy cannot lead to a sustained full employment equilibrium and serious business cycles are unavoidable due to the unstable nature of the interaction between investment and finance (Minsky, 1986, 173). An initially robust financial system is endogenously turned into a fragile system as a prolonged period of good years induces firms and bankers to take riskier financial practices. During expansions, an investment boom generates a profit boom but this induces investors and banks to adopt more speculative financial arrangements. This is typically reflected in rising debt finance, which

\(^1\)Wray (2008), Kregel (2008), Cynamon and Fazzari (2008) and Crotty (2008), among others, provide perspectives on how shaky are the foundations of these ‘sophisticated’ developments in financial markets.
eventually turns out to be unsustainable because the rising debt changes cash flow relations (or income-payment commitment relations) and leads to various types of financial distress. Minsky suggests that this kind of endogenous change in financial fragility can generate debt-driven long expansions followed by deep depressions (Minsky 1964, 1995). In Minsky’s theory of long waves, short cycles fluctuate around the long waves produced by endogenous changes in financial structure. Thus, the distinction between short cycles and long waves is an important characteristic of Minsky’s cycle theory.

In spite of difficulties inherent in the formalization of Minsky’s theories, Minsky’s financial instability hypothesis has inspired a number of researchers to model the dynamic interaction between real and financial sectors. Taylor and O’Connell (1985), Foley (1986), Semmler (1987), Jarsulic (1989, 1996), Delli Gatti and Gallegati (1990), Skott (1994), Dutt (1995), Keen (1995) and Flaschel, Franke and Semmler (1998, Ch.12) are early contributions. Recent studies include Setterfield (2004), Nasica and Raybaut (2005), Lima and Meirelles (2007), Fazzari et al. (2008), and Charles (2008). While each of these studies captures a particular aspect of real-financial interactions, none of these tries to distinguish long waves from short cycles and the average periodicity in most models is ambiguous.

This chapter presents a stock-flow consistent model that produces long waves around which short cycles fluctuate. In the model, firms’ and households’ financial practices endogenously evolve through the interaction between real and financial sectors. The interaction between changes in firms’ and households’ financial practices, which are captured by the debt-capital ratio and the equity-deposit ratio, respectively, produce long waves. We prove the existence of limit cycles describing the long waves by using the Hopf bifurcation theorem and conduct simulation exercises to illustrate stable limit cycles. The resulting long waves are characterized by periodic economic crises following long expansions. Short cycles, generated by the interaction between
effective demand and labor market dynamics, fluctuate around the long waves. The main contribution of this paper is to provide a mechanism which explains both long waves and short business cycles in a unifying framework.

It is worth noting that long waves in this chapter emerge from the interaction between firms’ and households’ financial decisions. Minsky’s own accounts of cycles and crises tend to privilege firm and banking sectors as a source of instability and cycles, while they tend to ignore the role of households’ financial behavior in generating cycles. Our model of long waves pays close attention to both firms’ and households’ financial decisions. The model consists of two subsystems: firms’ debt dynamics and households’ portfolio dynamics. One interesting result of our analysis is that two stable subsystems can be combined to produce instability and cycles in the whole system (See section 1.4). Thus, the resulting instability and cycles are genuinely attributed to the interaction between sectors rather than characteristics of one particular sector. The stock-flow consistent approach provides a useful framework that enables us to keep track of the implications of one sector’s decisions for the other sector’s.

Short cycles as well as long waves in our model are the result of endogenous interactions among economic agents, not the result of exogenous shocks. Most of the mainstream literature on business cycles follows the Frisch tradition in which the source of cyclical behavior is exogenous shocks external to the models, and agents’ reactions to the shocks are important only as propagation mechanisms. Regarding this broad aspect, there is little difference between New Classical and New Keynes-

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2Minsky’s neglect of the household sector is explained by his observation that “[H]ousehold debt-financing of consumption is almost always hedge financing.” (1982, p. 32) This position, however, has been challenged by some Minskian explanations of the sub-prime mortgage crisis. (e.g. Wray(2008) and Kregel (2008))

sian theories. In contrast to those recent theories, the older tradition rooted in Harrod (1939, 1948), Kaldor (1940), Samuelson (1939) and Goodwin (1967) puts a great emphasis on the endogenous nature of cycles. The interaction between Keynesian multiplier and investment accelerator plays a central role in Harrod, Kaldor and Samuelson whereas the interaction between saving-constrained accumulation and class conflict in the labor market provides a basic mechanism in the Goodwin cycle. Skott (1989A) provides a synthesis of effective demand and labor market dynamics - two important elements in the older tradition - into a single framework. In the Skott model, the rate of growth of employment and production depends on both labor market conditions and effective demand. A limit cycle results from the interaction between destabilizing forces in the goods market and stabilizing forces in the labor market. The same mechanism as that in the Skott model is used to generate short cycles in this paper. However, unlike the Skott model, short cycles in this chapter oscillate around the long waves for which financial elements play a vital role.

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4 It is well known that exogenous stochastic productivity shocks are a main cause of New Classical real business cycles. While New Keynesian theories emphasize the effect of monetary shocks on cyclical behavior, the nature of exogenous shocks does not matter in the New Keynesian theories in the fundamental level, and New Keynesians accept the basic framework of real business cycle theory. For instance, Bernanke and Gertler (1989) say: “We present a formal analysis of the role of borrower’s balance sheets in the business cycle. Our vehicle is a modified real business cycle model, in which a characteristic of the investment technology is an asymmetry of information between the entrepreneurs who organize and manage physical investment and the savers from whom they borrow.” (p.14).

5 The Skott model has some weaknesses from a Minskian perspective. Skott assumes that in the context of two financial assets (equity and deposit), the ratio of the value of equity to net profit and the ratio of deposit to nominal income are exogenously given. The constancy of the equity-profit ratio implies that the price-earnings ratio is always constant, not only on steady state path but also during the course of cycles. More importantly, the assumption of constant deposit to nominal income ratio suggests that firms’ stock of debt passively moves in proportion to nominal income so that the debt-income ratio always remains constant on and off steady state path (In the Skott model, as in this paper, all deposits in banks are lent to firms. Thus, the deposit/income ratio is always equal to the debt/income ratio). Given this assumption, the debt-capital ratio (leverage ratio) will follow cyclical change in the utilization rate but the influence of endogenous changes in firms’ and bankers’ financial practices on the trajectories of real and financial variables are hardly conceivable in this framework.
The rest of this chapter is structured as follows. Section 1.2 presents some of
the stylized facts about financial changes in the firm and household sectors. Section
1.3 sets up our model. Section 1.4 analyzes how the interaction between firms’ and
households’ financial practices produces long waves. Section 1.5 briefly introduces a
model of short cycles into the current context. Section 1.6 combines our model of
long waves with the short-cycle model and provides simulation results. Section 1.7,
finally, offers some concluding remarks.

1.2 Stylized facts

This section provides a brief description of some financial changes in firm and
household sectors which happened in the U.S in the years since 1952.

In Figure 1.1(a), the ratio of debt to the replacement value of capital for non-
farm nonfinancial corporations is shown. This ratio has exhibited fluctuations but an
apparent strong upward trend. In 1952, the debt ratio barely exceeded 30% but in-
creased steadily in the 1950s and 1960s, reaching almost 50% in 1973 before dropping
back to 36% in 1981. The leverage ratio has since displayed a steep rise, with minor
downturns, up to now.

If the profit rate had increased along with the debt ratio, the rising debt ratio
could have been validated by firms’ increasing cash generating capability. This was,
however, not the case. Figure 1.1(b) displays the ratio of debt to profit before tax for
the same sector. The ratio had been below four until the mid 1960s and then began to
rise, reaching at six, the highest point since the World War II. It then fell, reflecting
a significant fall in the debt ratio, and hit the bottom in 1978. The early 1980s was
characterized by a sharp rise in the debt/profit ratio. It has shown fluctuations since
then. The pattern was similar to that of the debt-capital ratio in Figure 1.1(a) but
exhibited much more volatility.
Figure 1.1. U.S. Nonfarm Nonfinancial Corporations (1952-2007)

Notes: Federal Reserve Board, *Flow of Funds Accounts of the United States*. Author’s calculation. See Appendix A for details.

Figure 1.1(c) shows the ratio of real interest payment$^6$ to profit before tax. The trend shows a sharp contrast before and after 1980. Throughout the 1950s and the 1960s, it remained at very low levels below 15%. In the 1970s, this ratio even further decreased mainly due to the favorable effects on interest payment obligations of high inflation rates. Interest payments then dramatically increased above 60% of total investment.

$^6$The measures of real interest payments are calculated using bank prime loan rates. Thus the measures tend to hide the difference in interest rates associated with particular debt units depending on various maturity and riskiness.
profit in the early 1980s, reaching the historical high, 80%, in 1986. Since then, it has exhibited substantial fluctuations and the high peaks hit in 1989 and again in 2000.

Both Figure 1.1 (b) and 1.1 (c) indicate that firms’ profitability relative to their indebtedness and payment commitments was sound until the mid 1960s but the upward trend in the debt-capital ratio since the mid 1960s changed the tendency afterwards.

In the meantime, the importance of equity finance has been greatly reduced. Figure 1.1 (d) shows that in the 1950s-1970s a small positive fraction of investment was financed by new issues. Since 1980, however, the ratio has been negative in most years and fluctuated with much more volatility than in previous periods. The amount of corporations’ stock buybacks sometimes reaches more than 50% of their investment.

The household sector also has experienced significant changes since 1952. The ratio of consumption to disposable income had steadily fallen until the early 1980s with cyclical movements. The ratio then exhibited an upward trend until recent years (Figure 1.2(a)). Figure 1.2(b) shows, household indebtedness rose substantially throughout the whole period. In 1952, household credit market debt was simply 39% of their disposable personal income but it reached 136%. A large part of the upward movement is explained by the mortgage debt as shown in the figure. In Figure 1.2(c), the ratio of equity to deposit holding, indicating household portfolio, displays large swings. Throughout the 1950s and 1960s, the equity-deposit ratio remained at relatively high levels, on average, 1.45. This tendency was reversed in the 1970s and 1980s: the historical average of the equity-deposit ratio during the period marked merely 0.67. The ratio, however, began to rise steeply since the early 1990s. The ratio hit the historical high in 1999, followed by an astonishing collapse in 2000. Two major collapses in this ratio, 1973 and 2000, match those in the rate of return on equity shown in Figure 1.2(d).
Figure 1.2. U.S. Household and Nonprofit Organization (1952-2007)

Notes: Federal Reserve Board, *Flow of Funds Accounts of the United States*. Author’s calculation. See Appendix A for details.

1.3 Model

This section presents a model. Firms make decisions concerning pricing/output, accumulation and financing; households make consumption and portfolio decisions; banks accept deposits and make loans. It is assumed that there are only two types of financial assets - equity and bank deposits - and banks are the only financial institution. It is assumed that the available labor force grows at a constant rate.\(^7\)

---

\(^7\)We assume that there is no technical progress but the model can easily accommodate Harrod-neutral technical progress.
and long run growth is constrained by the availability of labor. Thus the economy is ‘mature’ in Kaldor’s (1966) terminology.

1.3.1 Firms

1.3.1.1 The finance constraint

Firms’ flow of funds account consists of sources and uses of funds. Firms have three sources of funds in our framework: profits, new issue of equity and debt finance. Using these funds, firms make investments in real capital, pay out dividends and make interest payments. Algebraically,

$$pI + Div + iM = Π + v\dot{N} + \dot{M}$$

(1.1)

where $I$, $Π$, $Div$, $M$, and $N$ are real gross investment, gross profits, dividends, bank loans and the number of shares, respectively. Bank loans carry the nominal interest rate ($i$). $p$ represents the price of investment goods as well as the general price of output in this one-sector model. All shares are assumed to have the same price $v$. A dot over a variable refers to a time derivative ($\dot{y} = dy/dt$).

I assume that firms’ dividend payout is determined as a constant fraction of net profits (= gross profits − depreciation − real interest payments). The dividend payout rate is denoted as $1 − s_f$ and, consequently, $s_f$ represents firms’ retention rate. Thus, we have

$$Div = (1 − s_f)(Π − δpK − rM)$$

(1.2)

where $K$ and $δ$ are real capital stock and the rate of depreciation of real capital. $r$ represents the real interest rate, $r = i − \hat{p}$ (A hat over a variable is used to denote a growth rate of the variable, for instance, $\dot{y} = (1/y)(dy/dt)$). Lavoie and Godley (2001-2002) and Dos Santos and Zezza (2007), among others, use the specification (1.2) regarding firms’ retention policy.
New equity issue can be represented by the growth of the number of shares ($\hat{N}$) or by the share of investment financed by new issues denoted as $x$. Skott (1981, 1988, 1989A) and Foley and Taylor (2004) use the former and Lavoie and Godley (2001-2002) the latter. Two measures, however, are related to each other in the following manner.\footnote{$\hat{N} = \dot{N}/N$. Given this notation, $v\hat{N} = vN\hat{N}$.}

\[ vN\hat{N} = xpI \quad (1.3) \]

Substituting (1.2) into (1.1), we get

\[ pI - \delta pK = s_f(\Pi - \delta pK - rM) + vN\hat{N} + M(\hat{M} - \dot{p}) \quad (1.4) \]

Scaling by the value of capital stock ($pK$), we finally have

\[ \hat{K} \equiv g = s_f(\pi u\sigma - \delta - rm) + x(g + \delta) + \dot{m} + gm \quad (1.5) \]

where $\pi$, $u$, and $m$ is the profit share ($\pi \equiv \frac{\Pi}{pY}$), the utilization rate ($u \equiv \frac{Y}{Y_F}$, $Y_F$ is full capacity output) and the leverage ratio ($m \equiv \frac{M}{pK}$). The technical output/capital ratio, $\sigma$ ($\equiv \frac{Y_F}{K}$), is assumed to be fixed. $\delta$ is the depreciation rate. Equation (1.5) has a straightforward interpretation: firms’ investment ($g$) is financed by three sources: retained earnings, $s_f(\pi u\sigma - \delta - rm)$, new equity issue, $x(g + \delta)$ and bank loans, $\dot{m} + gm$. Given this finance constraint, firms’ financial behavior is characterized by $s_f$, $x$ (or $\hat{N}$) and $m$ in steady state. Most theories treat the rates of firms’ retention and equity issue as parameters and debt finance as an accommodating variable (Skott 1988, 1989A, Lavoie and Godley 2001-2002 and Dos Santos and Zezza 2007). This chapter assumes that the retention rate ($s_f$) is exogenous as in the above literature but both the rate of equity issue ($x$ or $\hat{N}$) and the leverage ratio $m$ are endogenous. However, our way of treating equity finance and debt finance is not symmetric.
Debt finance evolves through endogenous changes in firms’ and banks’ financial practices which are directly influenced by the relationship between firms’ profitability and leverage ratio (see section 1.3.1.2 below). With debt finance determined in this way, equity finance \(x\) serves as a buffer in the sense that once the other sources of finance — the retention and debt finance policies — and investment plans are determined, equity issues fill the gap between the funds needed for the investment plans and the funds available from retained earnings and bank loans. In this regard, equity finance is seen as a pure residual of firms’ financing constraint. Formally, for a given set of parameters \(s_f, \sigma, \delta\) and \(r\), the trajectories of endogenous variables \(g, \pi, u, m\) and \(\dot{m}\) determine the required ratio of equity finance to gross investment:

\[
x = \frac{g - s_f(\pi u \sigma - \delta - rm) - \dot{m} - gm}{g + \delta}
\]

The treatment of equity finance as a residual may not be entirely satisfactory, especially from a point of view that emphasizes substantial difficulty involved in raising capital in equity markets compared to the other methods of finance. However, as Figure 1.1 (d) shows, the degree of flexibility in issuing equities was historically very large. This was even more prominent when the rate of net issue of equity was negative \((x < 0)\).

1.3.1.2 Endogenous changes in firms’ liability structure

Endogenous changes in firms’ liability structure, which are captured by changes in firms’ debt-capital ratio \((m)\), are central in this paper, and a Minskian perspective suggests that the debt-capital ratio evolves according to sustained changes in firms’ profitability relative to their payment obligations on debt. Changes in profitability that are perceived as highly temporary have only limited effects on desired leverage. I, therefore, distinguish cyclical movements in profitability from the trend in average

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profitability and assume that changes in liability structure are determined as the trend of profitability.\footnote{See section 1.4.1 for more discussion.}

The perception of strong profitability relative to payment commitments during good years, Minsky argues, induces bankers and businessmen to adopt riskier financial practices which typically results in increases in the leverage ratio. Following Minsky’s idea (Minsky, 1982, 1986), I assume that changes in the ratio of profit to debt service commitments drive changes in the debt structure. Formally,

\[
\dot{m} = \tau \left( \frac{\rho_T}{rm} \right); \quad \tau'(\cdot) > 0 \tag{1.7}
\]

where \(\rho_T\) represents the trend rate of profit\footnote{A definition of the trend rate of profit will be provided in section 1.4.} and \(\tau\) is an increasing function. The equation suggests the ratio of profit to debt service commitments drives changes in the debt structure. More specifically, during the period of tranquility when the level of profit is sufficiently high compared to interest payment obligations, firms’ and bankers’ optimism reinforced by their success tends to make them adopt riskier financial arrangements which involve higher leverage ratios. A high profit level compared to debt servicing is typically associated with a low probability of default which helps bankers maintain their optimism. Recent history characterized by strong profitability may motivate firms to implement riskier projects that require a large amount of debt.

The opposite is true when the ratio of profit to interest payments is low. Firms’ failure to repay debt obligations - defaults and bankruptcies in the firm sector - puts financial institutions linked to those firms in trouble as well. To the extent that financial institutions are intricately linked to each other through complex financial networks, a financial failure in one unit can easily produce that in another unit. When the profitability of the firm sector turns out to be too weak, situations can lead to
systemic crises. A collapse in the financial system may be unavoidable especially when financial networks are so complex that the level of systemic risk is high. A financial crisis forces firms and bankers to reduce their indebtedness and it is often manifested in a system-wide credit crunch. Thus, this situation may be represented by a sudden collapse of the debt-capital ratio.

1.3.1.3 Accumulation

In general, capital accumulation is affected by several factors including profitability, utilization, Tobin’s q, the level of internal cash flows, the real interest rates and the debt ratio, but there is no consensus among theorists concerning the sensitivity of firms’ accumulation behavior to changes in the various arguments. In particular, the long run sensitivity of the accumulation rate to changes in the utilization rate has been in debate among the structuralist/post-Keynesian economists.11 This chapter follows the Harrodian perspective in which capacity utilization has foremost importance in firms’ accumulation behavior (Harrod, 1939, 1948). A Harrodian perspective is characterized by the assumption that firms have a desired rate of utilization. In the short run, the actual rate of utilization may substantially deviate from the desired rate since firms’ demand expectations are not always met and capital stocks slowly adjust. If the actual rate exceeds the desired rate, firms will accelerate accumulation to increase their productive capacity and if the actual rate is smaller than the desired rate, they will slow down accumulation to reduce the undesired reserve of excess productive capacity. However, in the long run, it is not reasonable to assume that the actual rate can persistently deviate from the desired rate because capital stocks

flexibly adjust to maintain the desired rate. This perspective naturally distinguishes the short-run accumulation function from the long-run accumulation function.\textsuperscript{12}

A simple version of the long-run accumulation function can be written as

\[ u = u^* \]  \hspace{1cm} (1.8)

where \( u^* \) is an exogenously given desired rate of utilization. (1.8) represents the idea that in the long run, the utilization rate must be at what firms want it to be and capital accumulation is perfectly elastic so as to maintain the desired rate. The strict exogeneity of the desired rate in (1.8) may exaggerate reality but tries to capture mild variations of the utilization rate in the long-run. Taking an example of the U.S. economy from 1948 to 2008, Figure 1.3 (a) and 1.3 (b) plot the rate of capacity utilization for the industrial sector and the manufacturing sector, respectively. The Hodrick-Prescott filtered series (dotted lines) are added to capture the long-run variations in the utilization rate. The figures show that the degree of capacity utilization is subject to significant short-run variations but exhibits only mild variations around 80\% in the long-run.

In this chapter, I use the long run accumulation function (1.8) to analyze long waves: as long as we are interested in cycles over a fairly long period of time, the assumption that the actual utilization rate is on average at the desired rate is a reasonable approximation.

For the analysis of short cycles, however, the accumulation function (1.8) cannot be an appropriate specification because the deviation of the actual from the desired rate normally occurs in the short run. Thus, we will use the following specification (1.9) to describe accumulation behavior during a course of short cycles.

\textsuperscript{12}This Harrodian perspective is elaborated in Skott (1989, 2008A, 2008B) in greater detail.
The strong positive effect of utilization on accumulation in (1.9) embodies the Harrodian accelerator principle and the function $\phi$ is configured so that the desired rate of utilization is consistent with steady growth at a natural rate. We will use (1.9) in section 1.5 where short cycles are analyzed.\textsuperscript{13}

\section*{1.3.2 Banks}

In the model, banks’ active role in shaping firms’ financial structure is represented by equation (1.7) which reflects both firms’ and banks’ behavior. For a given profit-interest ratio, equation (1.7) determines the trajectory of the debt-capital ratio $m$.

At any moment, the amount of loans supplied to firms will be $M = mpK$. I assume

\textsuperscript{13}The specification (1.9) is clearly an oversimplification since it leaves out other determinants of investment. For instance, it does not capture the direct impact of financial variables such as cash flow and asset prices which are highly emphasized by Minsky (1975, 1982, 1986) and Tobin (1969), as well as current New Keynesian economics (Fazzari et al. (1988) and Bernanke, Gertler and Gilchrist (1996), among others). However, equation (1.9) can be easily extended to accommodate the effect of those variables without affecting major results of this study. In fact, the effect of cash flow and Tobin’s q on accumulation, it can be shown, reinforces the utilization effect on accumulation embodied in (1.9). The merit of simple specification in equation (1.9) is that it shows the underlying mechanisms in a transparent way.
that neither households nor firms hold cash, the loan and deposit rates are equal and there are no costs involved in banking. With these assumptions, the amount of loans to the firm sector must equal the total deposits of the household sector.

\[ M = M^H \]  

(1.10)

where \( M^H \) represents households’ deposit holdings.

Banks set the nominal interest rate \( i \), which is typically affected by inflation. To simplify the analysis, I assume that banks effectively control the real interest rate \( r \).

1.3.3 Households

Households receive wage income, dividends in return for their stock holdings and interest income. Thus, household real disposable income denoted as \( Y^H \) is given as:

\[ Y^H = W + Div + rM^H \]  

(1.11)

Households hold stocks and deposits in our two financial asset world and household wealth is denoted as \( NW^H \). Thus, we have

\[ NW^H = \frac{vN^H + M^H}{p} \]  

(1.12)

Based on their income and wealth, they make consumption and portfolio decisions. We adopt a conventional specification of consumption function.

\[ C = C(Y^H, NW^H); \quad C_{Y^H} > 0, \quad C_{NW^H} > 0 \]  

(1.13)

The life cycle hypothesis (Ando and Modigliani, 1963), among others, may justify this specification. Similar specifications have been used by Boyer (2000), Godley and Lavoie (2007) and Dos Santos and Zezza (2007).
For simplification, we assume that the function takes a linear form. We then have, after normalizing by capital stock and simple manipulations,

$$\frac{C}{K} = c_1[u\sigma - s_f(\pi u\sigma - \delta - r_m)] + c_2q \quad (1.14)$$

where $u\sigma - s_f(\pi u\sigma - \delta - r_m)$ is disposable income scaled by capital stock and Tobin’s q captures household wealth. $c_1$ and $c_2$ are household propensities to consume out of disposable income and wealth.

In addition to consumption/saving decisions, households make portfolio decisions. The long run evolution of household portfolio plays a pivotal role in generating long waves in this model. We denote the equity-deposit ratio as $\alpha$:

$$\alpha \equiv \frac{vN^H}{M^H} \quad (1.15)$$

We assume that the composition of households’ portfolio is affected by their views on stock market performance. Applying a Minskian hypothesis to household behavior, it is assumed that during good years, households tend to hold a greater proportion of financial assets in the form of riskier assets. In our two-asset framework, equity represents a risky asset and deposits a safe asset. Thus, a rise in fragility during good years is captured by a rise in $\alpha$. We introduce a new variable $z$ to represent the degree of households’ optimism about stock markets. We can normalize the variable $z$ so that $z = 0$ corresponds to the state where households’ perception of tranquility is neutral and there is no change in $\alpha$. Given this framework, the evolution of $\alpha$ is determined by an increasing function of $z$.

$$\dot{\alpha} = \zeta(z); \quad \zeta(0) = 0, \quad \zeta'(z) > 0 \quad (1.16)$$

The next question is what determines the degree of households’ optimism about stock markets, $z$. It is natural to assume that household portfolio decisions, the
division of their wealth into stocks and deposits, will be affected by the difference between the rates of return on stocks and deposits.

Our specification of the process in which households form their views on stock markets emphasizes historical elements in financial markets. Thus, the past trajectories of rates of return on assets as well as those of \( \alpha \) matter in the formation of \( z \). As a crude approximation of this perception formation process, the following exponential decay specification is introduced:

\[
z = \int_{-\infty}^{t} \exp \left[ -\lambda(t - \nu) \right] \kappa \left( r_e^\nu - r, \alpha_\nu \right) d\nu \tag{1.17}
\]

where \( r_e \) is the real rate of return on equity, \( \kappa_{re} \equiv \frac{\partial \kappa \left( r_e - r, \alpha \right)}{\partial r_e} > 0 \) and \( \kappa_\alpha \equiv \frac{\partial \kappa \left( r_e - r, \alpha \right)}{\partial \alpha} < 0 \). In expression (1.17), \( \kappa \left( r_e^\nu - r, \alpha_\nu \right) \) represents the information regarding the state of asset markets at time \( \nu \). The higher the rate of return on equity relative to the deposit rate of interest, the more optimistic households' view on stock markets becomes (\( \kappa_{re} > 0 \)). However, other things equal, a higher proportion of their financial wealth in the form of stock holdings (high \( \alpha \)) tempers the desire of further increases in equity holdings, i.e. \( \kappa_\alpha < 0 \).

Information on asset markets at different times enters in the formation of \( z \) with different weights. The term, \( \exp \left[ -\lambda(t - \nu) \right] \), represents these weights, implying that a more remote past receives a smaller weight in the formation of households' perception of tranquility. Thus, \( \lambda \) may be seen as the rate of loss of relevance or loss of memory of past events. The higher \( \lambda \), the more quickly eroded is the relevance of past events.

An alternative specification to (1.16) and (1.17) is possible. Consider the following specification.

\[
\dot{\alpha} = \zeta (\alpha^* - \alpha) \tag{1.16a}
\]

\[
\alpha^* = \int_{-\infty}^{t} \exp \left[ -\lambda(t - \nu) \right] \bar{\kappa} \left( r_e^\nu - r \right) d\nu \tag{1.17a}
\]

where \( \bar{\kappa} \) \( (\cdot) > 0 \) and \( \alpha^* \) is the desired equity-deposit ratio. (1.17a) tells us that households’ desired portfolio is determined by the trajectory of the difference between the rates of return on equity and
Differentiation of (1.17) with respect to \( t \) yields the following differential equation:

\[
\dot{z} = \kappa \left( r^e - r, \alpha \right) - \lambda z
\]  

(1.18)

Two dynamic equations (1.16) and (1.18), along with the equation describing the evolution of firms’ liability structure, (1.7), are essential building blocks for our model of long waves. To proceed, we need to see how the rate of return on equity, \( r^e \), is determined. \( r^e \) is defined as follows:

\[
r^e \equiv \frac{Div + \Gamma}{vN^H} = \frac{(1 - s_f)(\Pi - \delta pK - rM) + (\hat{v} - \hat{p})vN^H}{vN^H}
\]  

(1.19)

where \( \Gamma \) is capital gains adjusted for inflation \((\Gamma \equiv (\hat{v} - \hat{p})vN^H)\).

The rate of return on equity is determined by stock market equilibrium. Stock market equilibrium requires that the number of shares supplied by firms equals that of shares held by households, \( N = N^H \), which implies \( \dot{N} = N^H \) in terms of the change in the number of shares. The issue of shares is determined by firm’s financing needs. Firms issue new shares whenever retained earnings and bank loans fall short of the funds needed to carry their investment plans. Thus firms’ finance constraint (1.1) implies that:

---

deposit. This desired ratio may not be instantaneously attained so that the adjustment of the actual to the desired ratio takes time. (1.16a) represents this kind of lagged adjustment of the actual equity-deposit ratio toward the desired ratio. In spite of different interpretations, the two specifications, (1.16)-(1.17) and (1.16a)-(1.17a), are qualitatively similar. To see this, let \( z = \alpha^* - \alpha \). Then \( \dot{z} = \dot{\alpha}^* - \dot{\alpha} \). Differentiating (1.17a) with respect to \( t \), we have \( \dot{\alpha}^* = \dot{\kappa}(r^e) - \lambda \alpha^* = \dot{\kappa}(r^e) - \lambda(\alpha + z) \). Therefore, we can rewrite (1.16a) and (1.17a) to:

\[
\dot{\alpha} = \zeta(z) \quad (1.16b)
\]

\[
\dot{z} = \dot{\kappa}(r^e) - \lambda \alpha - \zeta(z) - \lambda z \quad (1.18a)
\]

One may want to compare (1.16b) and (1.18a) with (1.16) and (1.18).
\[
\dot{N} = \frac{1}{v} [pI + D iv + iM - \Pi - \dot{M}] \quad (1.20)
\]

Simple algebra shows that capital gains can be expressed as follows:

\[
\Gamma = (\dot{\alpha} - \dot{\hat{p}})v N^H = (\dot{\alpha} + \dot{\hat{m}} + \dot{\hat{K}})v N^H - v \dot{N}^H \quad (1.21)
\]

\((\dot{\alpha} + \dot{\hat{m}} + \dot{\hat{K}})v N^H\) represents the total increase in the real value of stock market wealth\(^{15}\) but some of the increase is attributed to the increase in the number of shares \((= v \dot{N}^H)\). To get the measure of capital gains, the latter should be deducted from the total increase.

Using \(N = N^H\), substituting (1.20) in (1.21) and plugging this result in (1.19), we get the new expression for \(r^e\):

\[
r^e = \frac{\Pi - iM + \dot{M} + (\dot{\alpha} + \dot{\hat{m}} + \dot{\hat{K}})v N^H - pI}{v N^H} \quad (1.22)
\]

Normalizing by \(pK\), we finally get the expression for \(r^e\) as a function of \(\pi, u, m, \dot{m}, \alpha\) and \(\dot{\alpha}\):

\[
r^e = \frac{\pi u \sigma - \delta - rm + (1 + \alpha)[\dot{\hat{m}} + m \phi(u)] + \dot{\alpha}m - \phi(u)}{\alpha m} \quad (1.23)
\]

\[
r^e = r^e(\pi, u, m, \alpha, \dot{m}, \dot{\alpha}) \quad (1.24)
\]

Substituting this expression in the dynamic equation (1.19), we have:

\[
\dot{z} = \kappa [r^e(\pi, u, m, \alpha, \dot{m}, \dot{\alpha}) - r, \alpha] - \lambda z \quad (1.25)
\]

(1.25) shows that households’ views of tranquility are affected by a number of variables and the relationship is complex. We consider several cases according to the property of (1.25) in section 1.4.

\(^{15}\)Note that \(\dot{\alpha} + \dot{\hat{m}} + \dot{\hat{K}} = \dot{\hat{v}} + \hat{N} - \hat{p}\).
1.3.4 Goods market equilibrium

The equilibrium condition for the goods market is:

\[
\frac{C}{K} + \frac{I}{K} = \frac{Y}{K}
\]  

(1.26)

The definition of \( q \) implies that \( q = (1+\alpha)m \). Using this, the equilibrium condition for the goods market can be written as:

\[
c_1[u\sigma - s_f(\pi u\sigma - \delta - rm)] + c_2(1 + \alpha)m + \phi(u) + \delta = u\sigma
\]

(1.27)

We take the profit share (\( \pi \)) as endogenous and the equilibrium value of \( \pi \) can be found for given \( u, m \) and \( \alpha \). Explicitly, we have:

\[
\pi = \frac{\phi(u) + \delta - (1 - c_1)u\sigma + c_2(1 + \alpha)m + c_1 s_f(\delta + rm)}{c_1 s_f u\sigma}
\]

\[
\equiv \pi(u, m, \alpha)
\]

(1.28)

(1.29)

As \( u, m \) and \( \alpha \) evolve over time, the profit share changes as well. The Harrodian investment function adopted in this paper emphasizes a high sensitivity of investment to changes in the utilization. Specifically, it assumes that investment rises much faster than saving as the utilization rate changes. Algebraically,

\[
\frac{\partial (I/K)}{\partial u} = \phi'(u) \gg (1 - c_1)\sigma + c_1 s_f \pi \sigma = \frac{\partial (S/K)}{\partial u}
\]

(1.30)

This Harrodian assumption has an implication for the effect of changes in utilization on profitability: utilization has a positive effect on the profit share and the magnitude will be quantitatively large. The partial derivative is given as
The large effect of changes in utilization on the profit share plays an important role in generating short cycles. (See section 1.5)

It is also readily seen that changes in the debt ratio and the equity-deposit ratio positively affect the profit share. Increases in the debt ratio or the equity-deposit ratio raise consumption demand though increases in disposable income or wealth, thereby increases the profit share.

\[
\frac{\partial \pi}{\partial m} = \frac{c_1 s_f r + c_2 (1 + \alpha)}{c_1 s_f u \sigma} > 0 \quad (1.32)
\]

\[
\frac{\partial \pi}{\partial \alpha} = \frac{c_2 m}{c_1 s_f u \sigma} > 0 \quad (1.33)
\]

The effects of changes in the state variables \((u, m\) and \(\alpha\)) on the current profit rate are straightforward. Since the current profit rate equals \(\pi u \sigma\), the positive effects of changes in \(u, m\) and \(\alpha\) on the profit share all carry over to those on the profit rate.

**1.4 Long waves**

This section shows how endogenous changes in firms’ and households’ financial practices generate long waves. Our model of long waves consists of two subsystems: one describes changes in firms’ liability structure and the other specifies changes in households’ portfolio composition. Section 1.4.1 analyzes the evolution of firms’ liability structure, assuming households’ portfolio composition is frozen. Section 1.4.2 examines households’ portfolio dynamics, given the assumption that firms’ liability structure does not change. Section 1.4.3 combines two subsystems and shows how long waves emerge from the interaction between two subsystems.
1.4.1 Long-run debt dynamics

This section analyzes the long-run evolution of firms’ debt structure. For convenience, we reproduce equation (1.7).

\[ \dot{m} = \tau \left( \frac{\rho_{T}}{r_m} \right) \quad \text{where} \quad \tau'(\cdot) > 0 \]  

(7)

Regarding the shape of \( \tau \) in (1.7), Minsky’s discussion suggests that the prosperity during tranquil years tends to induce firms and bankers to gradually raise the leverage ratio; the rise in the leverage ratio, however, cannot sustain because it worsens the profit/interest relation. Minsky points out that the financial system is prone to crises as the ratio of profit to interest traverses a critical level (Minsky, 1995). The resulting systemic crisis may prompt a rapid de-leveraging process. To capture this idea, we assume that \( \tau'(\cdot) \) takes relatively small positive values within a narrow bound when \( \frac{\rho_{T}}{r_m} \) is above a threshold level (good years), whereas it takes relatively large negative values when \( \frac{\rho_{T}}{r_m} \) is below the threshold level (bad years). When falling profit/interest ratio passes through the threshold level, \( \dot{m} \) sharply falls reflecting a rapid de-leveraging process. Thus, \( \tau'(\cdot) \) is likely to be very large when \( \frac{\rho_{T}}{r_m} = \tau^{-1}(0) \). Figure 1.4 reflects this assumption.

![Debt-Capital Ratio and Profit-Interest Ratio](image)

**Figure 1.4.** Debt-Capital Ratio and Profit-Interest Ratio
As briefly discussed in section 1.3.1.2, we use the trend rate of profit $\rho_T$ as a basis of the evolution of firms’ liability structure. Behind equation (1.7) is the idea that firms’ liability structure evolves endogenously over time and that the key determinant of the evolution is firms’ and banks’ perception of tranquility. The level of firms’ profit relative to payment commitments on liabilities is an indicator of firms’ performance and solvency status. Movements of the profit rate in general include both trend and cyclical components. It seems reasonable to assume that the long-run evolution of firms’ liability structure is primarily determined by the trend of the profit rate rather than the current profit rate.\(^{16}\)

The driving force of the short-run cyclical movements in the current profit rate is changes in capacity utilization while the desired rate, $u^*$, provides a good approximation of the long-run average of actual rates of utilization. Thus setting the utilization rate at the desired rate, the short-run cyclical component in the profit rate is effectively eliminated, and we have

$$\rho_T = \pi(u^*, m, \alpha)u^*\sigma$$

$$= \frac{n + \delta - (1 - c_1)u^*\sigma + c_2(1 + \alpha)m + c_1s_f(\delta + rm)}{c_1s_f}$$

(1.34)

The trend rate of profit defined as (26) depends positively on the debt-capital ratio $m$ and the equity-deposit ratio $\alpha$ ($\frac{\partial \rho_T}{\partial m} > 0$ and $\frac{\partial \rho_T}{\partial \alpha} > 0$). The profit-interest ratio, the key determinant of the liability structure, is written as

$$\frac{\rho_T}{rm} = \frac{n + \delta - (1 - c_1)u^*\sigma + c_2(1 + \alpha)m + c_1s_f(\delta + rm)}{c_1s_frm}$$

(1.35)

\(^{16}\)This perspective is in line with Minsky’s statement that “[T]he inherited debt reflects the history of the economy, which includes a period in the not too distant past in which the economy did not do well. Acceptable liability structures are based on some margin of safety so that expected cash flows, even in periods when the economy is not doing well, will cover contractual debt payments” (Minsky, 1982, 65).
(1.35) implies that for a given value of $\alpha$, the profit-interest ratio is uniquely determined by the debt-capital ratio $m$. Minsky’s implicit assumption that a rising debt ratio deteriorates the profit/commitment relation can be written as:

$$n + \delta - (1 - c_1)u^*\sigma + c_1 s_f \delta > 0$$

(1.36)

The average gross saving rate is typically greater than household marginal propensity to save out of disposable income, and this condition ensures that (1.36) will be met: if $\frac{S}{Y} = \frac{L}{Y} = \frac{n + \delta}{u^*\sigma} > (1 - c_1)$, then $n + \delta - (1 - c_1)u^*\sigma + c_1 s_f \delta > 0$. Thus, we assume that this condition is satisfied.$^{17}$

Using (1.7) and (1.35), $\dot{m}$ can be written as a function of $m$ and $\alpha$. 

$$\dot{m} = \tau \left( \frac{n + \delta - (1 - c_1)u^*\sigma + c_2(1 + \alpha)m + c_1 s_f (\delta + rm)}{c_1 s_f r m} \right) \equiv F(m, \alpha)$$

(1.37)

(1.37), along with the condition (1.36), implies that for any value of $\alpha$, (i) $F$ is decreasing in $m$, (ii) there exists a unique value of the debt ratio $m^*(\alpha)$ such that if $m = m^*(\alpha)$, $\dot{m} = 0$, and (iii) $m^*(\alpha)$ depends positively on $\alpha$, i.e. $m'^*(\alpha) > 0$. By setting $\dot{m}$ to zero and solving for $m$, we obtain the algebraic expression for $m^*(\alpha)$:

$$m^*(\alpha) \equiv \frac{n + \delta - (1 - c_1)u^*\sigma + c_1 s_f \delta}{\tau^{-1}(0) - 1} - \frac{c_2(1 + \alpha)}{c_1 s_f r - c_2(1 + \alpha)}$$

(1.38)

Using these properties (i), (ii) and (iii), Figure 1.5 illustrates the motion of the debt-capital ratio.

It is straightforward from Figure 1.5 that (assuming $\alpha$ is constant) our dynamic specification of Minsky’s financial instability hypothesis implies that firms’ debt structure monotonically converges to a stable fixed point $m^*(\alpha)$. The intuition is simple.

---

$^{17}$Otherwise, an increase in the debt ratio will raise the profit-interest ratio which leads to a self-repelling process of debt ratio without any ceiling.
When the actual debt ratio \((m)\) is lower than \(m^*(\alpha)\), the corresponding profit-interest ratio is greater than the threshold level at which the debt ratio does not change. This will induce firms to raise the debt ratio. The same kind of event will happen as long as \(m < m^*(\alpha)\): \(m\) will eventually converge to \(m^*(\alpha)\). The opposite will happen when the debt ratio is greater than the critical level \((m > m^*(\alpha))\).

It is worth noting that since the slope of the graph in Figure 1.5 is very steep at \(m = m^*(\alpha)\), the derivative of \(F(m, \alpha)\) with respect to \(m\) is strongly negative at \(m = m^*(\alpha)\), i.e. \(|F_m|\) is very large. In a limiting case where the de-leveraging process is instantaneous at \(m^*(\alpha)\), the graph in Figure 1.5 takes a step-like shape and \(F_m \rightarrow -\infty\).

Given assumption (1.36), a stable dynamics is inevitable in a one-dimensional continuous time framework. Moving from continuous to discrete time framework may change the picture so that firms’ debt dynamics alone can produce long-run cyclical movements. In this paper, however, I explore another avenue toward long waves by integrating firms’ debt dynamics into households’ portfolio dynamics.
1.4.2 Household portfolio dynamics

The other subsystem of our model of long waves, which describes households’ portfolio dynamics, consists of two dynamic equations:

\[ \dot{\alpha} = \zeta(z) \]  
\[ \dot{z} = \kappa (r^e - r, \alpha) - \lambda z \]

Analogously to the analysis of firms’ debt dynamics, we are interested in the long-run evolution of household portfolio decisions and, to simplify the analysis abstracts from the effect of short-run variations in capacity utilization. The rate of return on equity evaluated at \( u = u^* \) equals

\[ r^e|_{u=u^*} = \frac{\rho_T(m, \alpha) - \delta - rm + (1 + \alpha)[F(m, \alpha) + mn] + \zeta(z)m - n}{\alpha m} \]

Given this expression for \( r^e \), equation (40) becomes

\[ \dot{z} = \kappa (r^e|_{u=u^*} - r, \alpha) - \lambda z \equiv G(m, \alpha, z) \]

(1.37), (1.39), and (1.42) constitute a self-contained three-dimensional dynamical system. To better understand the mechanics of the three dimensional system, let us take a look at the subsystem (1.39) and (1.42), assuming that \( m \) is fixed. By differentiating (1.42) with respect to \( \alpha \) and \( z \), the effects of \( \alpha \) and \( z \) on \( \dot{z} \) are given by:

\[ G_\alpha = \kappa_{r^e} \frac{\partial r^e}{\partial \alpha} + \kappa_\alpha \geq 0 \]  
\[ G_z = \frac{\partial r^e}{\partial z} - \lambda = \kappa_{r^e} \frac{\zeta'}{\alpha} - \lambda \leq 0 \]

The effect of changes in \( \alpha \) on \( z \), \( G_\alpha \) in (1.43), is decomposed into two parts. First, changes in \( \alpha \) affect the rate of return on equity, which changes households’ views on
stock markets, $\kappa r_e \frac{\partial r^e}{\partial \alpha}$. The effect of an increase in $\alpha$ on $r^e$, $\frac{\partial r^e}{\partial \alpha}$, can be negative or positive in the steady state. Second, an increase in $\alpha$ mitigates the desire for further increases in equity holdings ($\kappa_\alpha < 0$). Thus, the overall effect depends on the precise magnitude of these two effects.

The effect of $z$ on $\dot{z}$ is also unclear. On the one hand, an increase in households’ optimism about stock markets accelerates stock holdings, which raises capital gains and the rate of return on equity. The increase in $r^e$ reinforces their optimism ($\kappa r_e \frac{\partial r^e}{\partial z} > 0$). On the other hand, the degree of optimism will erode at a speed of $\lambda$, holding $r^e$ and $\alpha$ constant. Thus, the net effect is ambiguous.

Let $J^H$ be the Jacobian matrix evaluated at the fixed point of (1.39) and (1.42). The ambiguity of the signs of $G_\alpha$ and $G_z$ yields four cases. Table 1.1 summarizes it.

**Table 1.1.** Classifying fixed points

<table>
<thead>
<tr>
<th>$G_\alpha$</th>
<th>$G_z &lt; 0$</th>
<th>$G_z &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_\alpha &lt; 0$</td>
<td>Case I Stable</td>
<td>Case II Unstable</td>
</tr>
<tr>
<td></td>
<td>$\text{Tr}(J^H) &lt; 0$ and $\text{Det}(J^H) &gt; 0$</td>
<td>$\text{Tr}(J^H) &gt; 0$ and $\text{Det}(J^H) &gt; 0$</td>
</tr>
<tr>
<td>$G_\alpha &gt; 0$</td>
<td>Case III Saddle</td>
<td>Case IV Saddle</td>
</tr>
<tr>
<td></td>
<td>$\text{Tr}(J^H) &lt; 0$ and $\text{Det}(J^H) &lt; 0$</td>
<td>$\text{Tr}(J^H) &gt; 0$ and $\text{Det}(J^H) &lt; 0$</td>
</tr>
</tbody>
</table>

A locally stable steady state in the subsystem is obtained when $G_z$ and $G_\alpha$ are both negative (Case I). In this case, $\lambda$ is large relative to $\kappa r_e \frac{\partial r^e}{\partial z}$, and $\kappa r_e \frac{\partial r^e}{\partial \alpha}$ is negative or, if positive, relatively small compared to the absolute value of $\kappa_\alpha$. Thus, to get a local stable steady state for households’ portfolio dynamics, the positive effect of changes in $\alpha$ and $z$ on $\dot{z}$ via the rate of return on equity needs to remain relatively small in the neighborhood of the steady state.

Moving from Case I, as $\lambda$ gets smaller than $\kappa r_e \frac{\partial r^e}{\partial z}$ ($G_z > 0$), keeping the condition $G_\alpha < 0$, the steady state becomes locally unstable, yielding Case II. In this case,
high optimism further boosts households’ optimistic views on stock markets, creating destabilizing forces. The locally unstable steady state, along with nonlinearities of (1.39) and (1.42), can produce limit cycles as long as $\lambda$ is not too small (See Figure 1.6). Thus, in this case, households’ portfolio dynamics alone can generate persistent long waves.

**Figure 1.6.** A Limit Cycle Generated by Household Portfolio Dynamics Alone (Case II)

If $G_{\alpha} > 0$, i.e. $\frac{\partial r_e}{\partial \alpha}$ is positive and its magnitude is large so that $\kappa_r \frac{\partial r_e}{\partial \alpha}$ is greater than $|\kappa_{\alpha}|$, then the fixed point of the households’ portfolio dynamics becomes saddle, regardless of the sign of $G_z$ (Case III and IV). In both Case III and IV, a high level of equity holdings creates increasing optimism ($G_{\alpha} > 0$), making the steady state a saddle point. However, Case IV is distinguished from Case III because it is an exceptional case: it turns out that the destabilizing force in Case IV is too strong to produce a limit cycle for the three dimensional full system ((1.37), (1.39), and (1.42)), whereas, in all other three cases I, II, and III, an appropriate choice of parameter values can produce a limit cycle for the full system. The next section analyzes the full system of long waves.
1.4.3 Full dynamics: long waves

We now put together firms’ debt and households’ portfolio dynamics and obtain the following three dimensional dynamical system:

\[
\dot{m} = F(m, \alpha) \tag{1.37}
\]

\[
\dot{\alpha} = \zeta(z) \tag{1.39}
\]

\[
\dot{z} = G(m, \alpha, z) \tag{1.42}
\]

Let us first consider the Jacobian matrix of the system evaluated at the steady state.

\[
J = \begin{bmatrix}
F_m & F_\alpha & 0 \\
0 & 0 & \zeta' \\
G_m & G_\alpha & G_z
\end{bmatrix}
= \begin{bmatrix}
- & + & 0 \\
0 & 0 & + \\
- & +/ & +/
\end{bmatrix}
\tag{1.45}
\]

\(G_\alpha\) and \(G_z\) are ambiguously signed but the partial derivative of \(G\) with respect to \(m\) is likely to be negative:

\[
G_m = \kappa_r \frac{\partial r^e}{\partial m} \tag{1.46}
\]

where

\[
\frac{\partial r^e}{\partial m} = \left[ \frac{\partial \rho_T}{\partial m} m - \rho_T \right] + (1 + \alpha)mF_m + n + \delta \tag{1.47}
\]

in the steady state. The sign of (1.47) may appear to be indeterminate: while \(\frac{\partial \rho_T}{\partial m} m - \rho_T\) is negative due to assumption (1.36) and \((1 + \alpha)mF_m\) is negative since \(F_m < 0\), \(n + \delta\) is positive. The discussion on the shape of \(\tau(\cdot)\) in section 1.4.1, however, suggests that \(F_m\) is large in magnitude at the steady growth path. Thus, at the steady state, the negative terms in the numerator in (1.47) dominate, and the rate of return on equity will decrease as firms’ indebtedness increases in the neighborhood of the steady state. Thus, we have \(G_m = \kappa_r \frac{\partial r^e}{\partial m} < 0\).

We are interested in the conditions under which the system exhibits limit cycle behavior. As 1.4.1 and 1.4.2 showed, the specification of firms’ financial decisions,
(1.37), leads to asymptotically stable dynamics, whereas households’ portfolio dynamics, (1.39) and (1.42), produces several cases presented in Table 1. Our analytic result suggests that if households’ portfolio dynamics is neither strongly stabilizing nor strongly destabilizing, our baseline system of (1.37), (1.39) and (1.42) tends to generate limit cycles. Our analysis of limit cycles is based on the Hopf bifurcation theorem. The Hopf bifurcation occurs if the nature of the system experiences the transition from stable fixed point to stable cycle as we gradually change a parameter value of a dynamical system (Medio, 1992, section 2.7). I will use \( \lambda \) as the parameter for the analysis of bifurcation. \( \lambda \) is particularly useful for the analysis not only because it has an obvious behavioral importance but also because it provides analytic tractability due to the fact that changes in \( \lambda \) do not affect steady state values. Proposition 1\(^{18}\) provides the main results of our analysis of long waves:

**Proposition 1** Consider the three dimensional system of (1.37), (1.39) and (1.42) and the Jacobian matrix (1.45) where the partial derivatives are taken at the steady state values. Let

\[
 b \equiv \frac{(|F_m|^2 - \zeta' G_\alpha) - \sqrt{(|F_m|^2 - \zeta' G_\alpha)^2 + 4\zeta' |F_m||G_m|F_\alpha}}{2|F_m|} < 0
\]

(I) **(Case I and Case II)** Suppose that \( G_\alpha < 0 \) and \( G_z < \min \left\{ |F_m|, \frac{\zeta' |G_m|}{|F_m|} \right\} \).\(^{19}\) Then a Hopf bifurcation occurs at \( \lambda = \lambda^* \equiv \kappa_r \frac{\partial \kappa_r}{\partial z} + |b| \). As \( \lambda \) falls passing through \( \lambda^* \), the system with a stable steady state loses its stability, giving rise to a limit cycle.

---

\(^{18}\)The proof of Proposition I is found in Appendix B but the proof is concerned about only the existence of a limit cycle. The computation of the coefficient that shows whether the limit cycle is stable is very complicated and hard to interpret. Therefore, we extensively use simulation exercises to observe the stability of cycles.

\(^{19}\)Note that Case I automatically satisfies the second condition since \( G_z < 0 \) in Case I.
(II) (Case III) Suppose that \( G_z < 0 \) and \( 0 < G_\alpha < \min \left\{ \frac{|F_m|^2 G_\alpha}{c'}, \frac{F_m G_m}{|F_m|} \right\} \). Then a Hopf bifurcation occurs at \( \lambda = \lambda^* \equiv \kappa_r \frac{\partial \kappa}{\partial z} + |b| \). As \( \lambda \) falls passing through \( \lambda^* \), the system with a stable steady state loses its stability, giving rise to a limit cycle.

(III) (Case IV) Suppose that \( G_\alpha > 0 \) and \( G_z > 0 \). Then the steady state is unstable. There exists no limit cycle by way of the Hopf bifurcation.

![Figure 1.7. A Limit Cycle in the 3D System of Long Waves (Case I)](image)

Part (I) in the proposition suggests that the existence of a limit cycle requires at least three conditions: first, the mitigation effect of a high proportion of equity holdings on increasing optimism (\(|\kappa_\alpha|\)) is sufficiently large so that \( G_\alpha < 0 \); second, households’ optimistic or pessimistic view on stock markets is not excessively persistent (\( G_z < \min \left\{ |F_m|, \frac{\kappa_m G_m}{|F_m|} \right\} \)); third, the rate of loss of relevance of past events (\( \lambda \))

\[^{20}\text{Or the positive effect of changes in } \alpha \text{ on } \dot{z} \text{ via its effect on the rate of return on equity should not be too large.}\]
should not be too large (\(\lambda < \lambda^*\)).\(^{21}\) The second and third conditions imply that for the existence of a limit cycle, \(\lambda\) should be of appropriate magnitude:

\[
\kappa r^e \frac{\partial r^e}{\partial z} - \min \left\{ \frac{|F_m|}{|F_m|}, \frac{\zeta'|G_\alpha|}{|F_m|} \right\} < \lambda < \kappa r^e \frac{\partial r^e}{\partial z} + |b| \quad (1.48)
\]

All of these conditions imply that to get a limit cycle, households’ portfolio dynamics should be neither strongly stabilizing nor strongly destabilizing.

One interesting aspect of Part (I) in Proposition 1 is that the interaction between two stable subsystems - firms’ debt and households’ portfolio dynamics - can generate an unstable steady state and a limit cycle in the whole system (Case I). Thus, in this case, the source of the resulting long waves does not lie in a particular sector but purely in the interaction between both firm and household sectors. Figure 1.7 depicts the emergence of a limit cycle in this case in a three dimensional space. Figure 1.8 shows the trajectories of the debt-capital ratio and the equity-deposit ratio in this case. The

![Figure 1.8. Long Waves](image)

\(^{21}\)If \(\lambda\) exceeds \(\lambda^*\), then the system will be stabilized.
years and twenty seven years, respectively. This expansion is followed by a sharp fall in $m$ and $\alpha$, which have significant negative impacts on effective demand and trigger an abrupt downturn in the real sector (See section 1.5 below).

Part (I) also covers Case II where the subsystem of households’ portfolio dynamics is unstable. As shown in 1.4.2, in Case II, portfolio dynamics alone can create a limit cycle. Part (I) in the proposition suggests that the system can still have a limit cycle when the portfolio dynamics is combined with firms’ debt dynamics. Then what is the implication of introducing the debt dynamics into portfolio dynamics? The qualitative analysis does not tell much about the answer to this question. A numerical experiment, however, provides a case in which the amplitude and period of long waves get larger and the quantitative effect is very large as we move from the 2D subsystem of portfolio dynamics to the full 3D system (Figure 1.9).

\[\text{Figure 1.9. Comparison Between 2D and 3D Systems (Case II)}\]

\(^{22}\)The functions and parameter values for this simulation, which are also used for the simulation in section 1.6, are found in Appendix C. A sufficiently long period of time (from $t = 0$ to $t = 30000$) is taken in all simulation exercises in this paper.

\(^{23}\)In the simulation behind Figure 1.9, the exogenous debt-capital ratio in the subsystem of household portfolio dynamics is configured to the same value as the steady state level of $m$ in the full 3D system.
Part (II) in the proposition concerns Case III where the household portfolio subsystem yields a saddle point steady state. Thus, this part of Proposition 1 shows how stabilizing debt dynamics and households’ portfolio dynamics with the saddle property are combined to produce a limit cycle. Not surprisingly, not all saddle cases can generate a limit cycle. First, the destabilizing effect that makes the fixed point in the 2D household subsystem saddle – the magnitude of \( G_\alpha \) – should be mild: 
\[
G_\alpha < \min \left\{ \frac{|F_m||G_z|}{\zeta}, \frac{F_\alpha|G_m|}{|F_m|} \right\}.
\]
Second, \( G_z \) should be negative. If it is positive (\( G_z > 0 \)), the condition for the saddle point, \( G_\alpha > 0 \), eliminates the possibility of the emergence of a limit cycle by way of the Hopf bifurcation. Proposition 1-(III) makes this point. Intuitively, if both \( G_\alpha > 0 \) and \( G_z > 0 \) (Case IV), the portfolio dynamics in the household sector is excessively destabilizing in the sense that stabilizing forces in firms’ debt dynamics cannot contain such a strong destabilizing effect.

To understand the mechanism behind the long waves, it is illuminating to compare the full system with the subsystem of debt dynamics. As seen in section 1.4.1, with households’ portfolio composition (\( \alpha \)) fixed, the debt-capital ratio (\( m \)) monotonically converges to its steady state value \( m^*(\alpha) \) (See Figure 1.5). The main reason for this convergence is the inverse relation between \( m \) and \( \frac{\rho T}{r_m} \): a rising debt-capital ratio deteriorates firms’ profit-interest ratio. Thus, for any given \( \alpha \), \( m \) and \( \frac{\rho T}{r_m} \) move in the opposite direction. However, once households’ portfolio composition evolves endogenously, this kind of strict inverse relationship breaks down because changes in \( \alpha \) also affect \( \frac{\rho T}{r_m} \). Figure 1.10 illustrates this. In Figure 1.10, the horizontal dotted line represents the threshold level (= \( \tau^{-1}(0) \)) of the profit-interest ratio that makes \( \dot{m} \) zero. In the area above the horizontal line, the debt-capital ratio increases and in the area below the line, it decreases. With \( \alpha \) held fixed, the movement along the curve AB is not possible since for any given \( \alpha \), a rise in \( m \) is incompatible with a rise in \( \frac{\rho T}{r_m} \). However, increases in \( \alpha \) fueled by households’ optimism during an expansion have a positive effect on the profit-interest ratio by raising aggregate demand. Thus,
from A to B, the economy experiences increases in both $\alpha$ and $m$ (The positive effect of the rise in $\alpha$ on the profit-interest ratio dominates the negative effect of the rise in $m$ and consequently the profit-interest ratio also increases during this period). However, households’ optimistic views on stock markets eventually fade as both $m$ and $\alpha$ increase. As a result, the negative effect of a rise in the debt ratio starts to be dominant at some point and the profit-interest ratio begins falling (point B). Because the profit-interest ratio is still above the threshold level, the debt ratio still keeps increasing and the profit-interest ratio falls along the curve BC. When the profit-interest ratio passes through point C, the debt-capital ratio starts to fall. If $\alpha$ is fixed, this fall in the debt-capital ratio quickly restores profitability but during contractions, $\alpha$ falls as well and the profit-interest ratio tends to relatively slowly improve along the curve from C to A. When the economy reaches point A, a new cycle begins.

Figure 1.11 depicts the same story from a slightly different angle. The solid line plots a trajectory of the actual debt-capital ratio over time ($m$) and the dotted line
Figure 1.11. Actual and Desired Debt Ratios

A trajectory of the desired debt ratio ($m^*(\alpha)$). Figure 1.5 suggests that the actual debt ratio $m$ tends to gravitate toward the desired ratio $m^*(\alpha)$. However, when $\alpha$ changes, the desired ratio becomes a moving target of the actual ratio. From this perspective, a period of expansion (contraction) is the time when the actual ratio is below (above) the desired ratio, i.e. $m < m^*$ ($m > m^*$), and consequently the actual debt ratio is increasing (decreasing). In words, a stock market boom (rising $\alpha$) tends to raise the tolerable level of the debt-capital ratio which the actual ratio is chasing. When the relation between $m$ and $m^*$ is reversed, a long downturn begins (See point C in Figure 1.11).

1.5 Short cycles

The model of long waves in section 1.4.3 can be combined with a model of short cycles. To complete our model of short cycles, we need to introduce short-run accumulation function and firms’ pricing/output decisions.
In our analysis of long waves, the degree of capacity utilization is set at its long run average in order to remove its short-run impact on profitability and to focus on the trend profitability. However, when it comes to short cycles, the utilization rate can deviate from the desired rate due to falsified demand expectations and slow adjustment of capital stocks. As briefly discussed in section 1.3.1.3, we introduce the following accumulation function for the analysis of short cycles.

\[ g = \phi(u); \quad \phi'(u) > 0, \quad \phi(u^*) = n \]  
(1.9)

In 1.3.4, using this accumulation function (1.9) and the consumption function (1.14), I derived the profit share that ensures the goods market equilibrium. For convenience, I reproduce the expression for the equilibrium profit share.

\[ \pi(u, m, \alpha) = \frac{\phi(u) + \delta - (1 - c_1)u\sigma + c_2(1 + \alpha)m + c_1s_f(\delta + rm)}{c_1s_fu\sigma} \]  
(1.28)

The positive effect of \( u \) on \( \pi \) is obtained from the Harrodian assumption (1.30): investment rises faster in response to changes in \( u \) than saving. Note that both the debt-capital ratio and the equity-deposit ratio have an expansionary effect on the profit share.

Regarding firms’ pricing/output decisions, this paper adopts a Marshallian approach elaborated in Skott (1989A, 2008B). The Keynesian literature often assumes that prices are sticky while output adjusts instantaneously and costlessly to absorb demand shocks but the Marshallian approach assumes the opposite. Output does not adjust instantaneously due to a production lag and substantial adjustment costs. For instance, increases in production and employment require substantial search, hiring and training costs. Hiring or layout costs include not only explicit costs but also hidden costs such as a deterioration in industrial relations and morale. The approach
assumes the adjustment of prices is fast compared to slow output expansion. In this framework, fast adjustments in prices and the profit share establish product market equilibrium for a given level of output (and stocks of real capital and financial assets). In a continuous-time setting, sluggish output adjustment can be approximated by assuming that output is predetermined at each moment and that firms choose the rate of growth of output, rather than the level of output. Then output growth is determined by comparing the costs and benefits involved in the output adjustment which in turn are determined by the labor market conditions and the profit signal in the goods market, respectively. Thus we can formulate:

\[
\hat{Y} = h(\pi, e); \ h_\pi > 0, \ h_e < 0 \\
\text{(1.49)}
\]

where \(e\) is the employment rate. A higher profitability induces firms to expand output more rapidly whereas the tightened labor market gives firms negative incentives to expand production.\(^{24}\) Assuming a fixed-coefficient Leontief technology, \(Y = \min\{\sigma K, \nu L\}\), the employment rate can be expressed as:

\[
e = \frac{Y}{\nu} \\
\text{(1.50)}
\]

where \(\nu\) is constant labor productivity and \(\bar{L}\) is available labor force which exponentially grows at a constant natural rate \(n\). (1.50) implies:

\[
\hat{e} = \hat{Y} - n \\
\text{(1.51)}
\]

From the definition of \(u\), we have:

\[^{24}\text{For more details about the behavioral foundation of (1.49), see Skott (1989A, Ch.4).}\]
\[ \dot{u} = \dot{Y} - \dot{K} \]  

(1.52)

Putting together (1.9), (1.28), (1.49), (1.51) and (1.52), we get the following system of short cycles.

\[ \dot{u} = h(\pi(u, m, \alpha), e) - \phi(u) \]  

(1.53)

\[ \dot{e} = h(\pi(u, m, \alpha), e) - n \]  

(1.54)

When \( m \) and \( \alpha \) are fixed, the system of (1.53) and (1.54) exhibits essentially the same dynamic properties as Skott (1989A). As Skott shows, under plausible assumptions, the system of (1.53) and (1.54) guarantees the existence of a steady growth equilibrium and the steady state is locally asymptotically unstable unless the negative effect of employment on output expansion is implausibly large.\(^{25}\) Once the boundedness of the trajectories is proved, the system (1.53) and (1.54) will generate a limit cycle by way of the Poincare-Bendixson theorem (See Skott 1989A, Appendix 6C for the proof). The limit cycle is characterized by a clockwise movement in \( e - u \) space. The underlying mechanism is the interaction between destabilizing output dynamics and stabilizing labor market dynamics. Simply put, a high level of effective demand associated with a high level of output creates strong profitability which tends to stimulate output growth. This kind of positive feedback between demand and production tends to destabilize the system. Output growth, however, cannot last forever.

\(^{25}\)Local stability is determined by inspecting the trace and the determinant of the Jacobian matrix of (1.53) and (1.54), denoted as \( J(u, e) \). They are given

\[ J(u, e) = \begin{bmatrix} (h \pi \pi_u - \phi')u & h_e u \\ h \pi \pi_u e & h_e e \end{bmatrix}, \quad Tr(J) = (h \pi \pi_u - \phi')u + h_e e, \quad Det(J) = -\phi' h_e u e > 0 \]

The determinant is always positive, so the possibility of saddle point instability is excluded. If the sign of \( Tr \) is negative (positive), the equilibrium is locally asymptotically stable (unstable). The sign of the trace will be positive unless the negative employment effect is implausibly large (Skott, 1989A).
because it increases the employment rate and the tightened labor market negatively affects the condition of output growth.

1.6 Putting all together: long waves and short cycles

This section finally puts all elements together in order to integrate long waves with short cycles. Our full model of long waves and short cycles is a five dimensional dynamical system that consists of (1.37), (1.39), (1.42), (1.53), and (1.54). We have seen that (1.37), (1.39), and (1.42) provide a model of long waves, whereas (1.53) and (1.54) generate a mechanism of short cycles. By using (1.34) as our definition of trend profitability based on $u = u^*$, the system of long waves becomes independent of that of short cycles, while the latter depends on the former. This kind of unilateral dependence can be relaxed by adopting an alternative formulation of trend profitability. For instance, we can use a weighted moving average of current profit rates as a measure of the trend rate of profit (See Appendix D). Then the two systems become interdependent but as long as the alternatively defined trend rate of profit exhibits sufficiently smooth movements over time, the qualitative results based on (1.34) still remain valid. The rest of this section presents our simulation results based on (1.37), (1.39), (1.42), (1.53) and (1.54).26

As seen in section 1.5, if $m$ and $\alpha$ are fixed, (1.53) and (1.54) produce a limit cycle under plausible conditions. It can be shown that the resulting limit cycle exhibits a clockwise movement on the $e-u$ space, or alternatively, the $e-\pi$ space. Figure 1.12 (a) presents an example of the limit cycle on the $e-\pi$ space. The system of (1.37), (1.39) and (1.42), however, generates long waves of the debt-capital ratio ($m$) and the equity-deposit ratio ($\alpha$), which are represented in Figure 1.8. As $m$ and $\alpha$ change endogenously, the limit cycle in Figure 1.12 (a) breaks down and the clockwise move-

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26Parameter values and functions used for this simulation are available in Appendix C. The simulation is based on Case I in Table 1.
Figure 1.12. Profit-Employment Cycle

ment of $e$ and $\pi$ spirals up to the northeast or down to the southwest, depending on the direction of changes in $m$ and $\alpha$. Figure 1.12 (b) illustrates this. The upward spiral from A to B represents a long expansion driven by increases in the debt-capital ratio and the equity-deposit ratio, whereas the downward spiral from B to A an economic downturn prompted by sharp decreases in $m$ and $\alpha$.

Figure 1.13 (a) and (b) reproduce Figure 1.8 (a) and (b) for convenience. Figure 1.13 (c) displays Tobin’s $q = (1 + \alpha)m$. Corresponding changes in household wealth have immediate consequences for aggregate demand via the wealth effect. Figure 1.13 (d) shows fluctuations of net issues of equity as a share of gross investment. The pattern is a mirror image of changes in the debt-capital ratio. Increases in debt-capital ratio during a long expansion lead to a fall in the ratio of net equity issues.

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27 Figure 1.8 (b) shows the steady increase in the equity-deposit ratio during a long expansion. This implies that firms’ debt/equity ratio steadily falls during that period (Note that firms’ stock of debt is always equal to total household deposit in this model. Thus, firms’ debt/equity ratio is given by $1/\alpha$). Minsky often uses the debt/equity ratio to refer to the degree of indebtedness. The result in this paper, however, shows that rising indebtedness, measured by the debt-capital ratio ($m$), is consistent with falling debt-equity ratio ($1/\alpha$). Interestingly, Lavoie and Seccareccia (2001) question the empirical relevance of Minsky’s Financial Instability Hypothesis based on their finding that the debt-equity ratio is not procyclical. The result of this paper suggests that Minsky’s Instability Hypothesis does not necessarily imply the procyclical movement of debt-equity ratio.
which amounts to the substantial volume of stock buybacks (negative net issues of equity).28

During each long expansion, the profit share exhibits a strong upward movement with mild cyclical fluctuations around the trend (Figure 1.14(a)). The similar pattern characterizes the movements in the profit rates (Figure 1.14(b)). During crises, the rate of profit net of depreciation and interest payment ($\pi u \sigma - \delta - rm$) tumbles even

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28Increasing stock buybacks in corporate firms have been highlighted in the financialization literature (For an analysis of financialization and critical reviews on related studies, see Skott and Ryoo (2008)), where rises in stock buybacks are viewed as a consequence of shareholder value orientation in management and finance. The result of the analysis in this paper proposes an alternative view on this development: increasing stock buybacks may be seen as a consequence of increasing financial fragility embodied in firms’ decisions on the liability structure during long expansions.
to negative rates. A change in the debt structure has large impacts on the real sector performance through its effect on the profitability. This is prominently shown in the behavior of the employment rate (Figure 1.14(c)). Figure 1.14 (d) depicts a trajectory of the rate of return on equity. During long booms, the rate of return on equity is strong and sound on average but during crises, it suddenly drops to significantly negative rates.

Figure 1.15(b) shows the growth rate of output where the Hodrick-Prescott filtered trend is added.\textsuperscript{29} A financial sector induced crisis triggers a deep recession in the real

\textsuperscript{29}The filtered series is only for illustrative purpose since it simply smoothes the original series and it does not adequately capture asymmetric features and structural breaks in the original series.
sector which is reflected in the negative growth rates during periodic deep downturns. Capacity utilization and capital accumulation follow the pattern similar to that of output growth (Figure 1.15 (a) and (c)). Figure 1.15(d), finally, plot the ratio of consumption to disposable income. The series follows the basic long waves/short cycles pattern as shown in the profit share and the employment rate but the movement in the consumption/income ratio is noticeably smooth compared to other simulated series.\footnote{The long run behavior of consumption is closely related to movements in household net worth to income ratio: \( \frac{C}{Y_H} = c_1 \frac{Y^d}{Y_H} + c_2 \frac{NW^d}{Y_H} = c_1 + c_2 \frac{NW^d}{Y_H} \) where \( NW^d_H = \frac{(1+\alpha)m}{\sigma-s_f\sigma-s-d-r_m}. \)}
1.7 Conclusion

The U.S. economy is going through a deep recession triggered by the biggest financial crisis since the Great Depression. A Minskian perspective suggests that the explanation of this crisis should be found in endogenous changes in financial fragility.

This study has modeled a Minskian theory of long waves. The model clarifies the underlying mechanism of endogenous changes in financial fragility and the interaction between real and financial sectors. At a theoretical level, the study provides a promising way of integrating two types of instability principles: Minsky’s Financial Instability Hypothesis and Harrod’s Instability Principle. While both principles provide a source of cycles, they have distinct frequencies and amplitudes in this model. The Minskian instability hypothesis creates long waves and the Harrodian instability principle produces short cycles. The limit to the upward trend created by Minskian instability is imposed by financial crisis, while explosive trajectories implied by Harrodian instability are contained by stabilizing labor market dynamics.\(^{31}\) When two principles are combined into a coherent stock-flow consistent framework, the proposed pattern of long waves and short cycles emerges.

A purely mathematical model of this kind may clarify the logic of interactions but clearly has many limitations. The depth of the current crisis and the time needed to initiate a new cycle depend on institutional and policy dimensions. Minsky devotes a large part of his analysis to the institutional and historical developments of financial markets and policy responses. Thus, the patterns of long waves are heavily affected by these elements. The full account of long waves and crises is possible only when one takes a serious look at these dimensions.

\(^{31}\)The following quote from Minsky (1995, 84) is suggestive: “As reasonable values of the parameters of the endogenous interactions lead to an explosive endogenous process, and as explosive expansions and contractions rarely occur, then constraints by devices such as the relative inelasticity of finance or an inelastic labor supply need to be imposed and be effective in generating what actually happens.”
Disregarding the historical contingencies of actual movements, it may be useful to extend the model in a number of directions. First, it may be desirable to explicitly treat the banking sector as an active profit-seeking unit. Bankers’ perception of tranquility, possibly affected by their own profitability, may not always agree with those of the firm and household sectors.\textsuperscript{32} Next, this paper did not explore the implications of households’ indebtedness. Instead, it has focused on an increasing share of stocks (riskier asset) in households’ financial wealth as an indicator of increasing fragility in the household sector. It would be interesting to see the effect of the introduction of the evolution of household debt into the model.\textsuperscript{33} Third, the proposed model is inflation neutral in the sense that the decisions on real quantities such as investment, consumption and output expansion are made with no reference to inflation and the banking sector holds the real interest rate at a constant level. In some account of Minskyan ideas (e.g see Fazzari et al., 2008), changes in the inflation rate play an important role. Finally, the assumption of a closed economy in this paper is another major limitation. Unfettered international capital flows, in contrast to the belief of its proponents, have created growing instability and global imbalances (Blecker, 1999). Several authors suggest that Minsky’s theory can be extended to an international context (e.g. Wolfson, 2002), but few attempt has been made to formalize the ideas and to propose precise mechanisms behind them. Addressing these issues is left for future research.

\textsuperscript{32}Setterfield (2004) assumes that the private sector (the aggregate of firm and household sectors) and the banking sector have different fragility functions but does not try to justify the assumed shapes of those functions.

\textsuperscript{33}To introduce this aspect, the model may have to be extended to allow heterogeneity among households as long as the household sector as a whole is in a net credit position.
CHAPTER 2
MACROECONOMIC IMPLICATIONS OF FINANCIALIZATION

2.1 Introduction

Along with neoliberalism and globalization, financialization has become a buzzword in recent years. The precise definition is not always clear but in a broad sense the term refers to “the increasing role of financial motives, financial markets, financial actors and financial institutions in the operation of the domestic and international economies” (Epstein 2005a, p. 3). More specifically, financialization has been associated with a number of developments. These developments include shifts in central bank policy toward a near-exclusive focus on price stability, large increases in financial flows both internationally and in domestic financial markets, improved financing for households and elements of consumption/credit driven growth, changes in corporate governance and attempts to align managerial incentives with shareholder interests via stock option plans, and an increased influence of financial institutions and institutional investors. Financial pressures, it is argued, have induced changes in management strategy from “retain and invest” to “downsize and distribute” (Lazonick & O’Sullivan (2000, p. 18)) and have affected firms’ dividend, new issue and debt finance policies. In some accounts non-financial corporations have been “forced to

1Eatwell and Taylor (2000), Blecker (1999), Crotty (2005), Stockhammer (2004, 2006), Duménil and Lévy (2001), Boyer (2000), Aglietta and Breton (2001) and Froud et al (2000) are among the contributions to the growing literature on financialization. An International Working Group on Financialisation has also been set up with the aim of bringing together “an interdisciplinary network of researchers and practitioners interested in financialization and all the issues around relations between the capital market, firms and households.” (http://www.iwgf.org/Events.htm).
fund most of their capital investment externally in the neoliberal era” (Crotty 2005, p. 99).

These various changes associated with financialization may have implications for macroeconomic performance. Crotty (2005) has argued that financialization weakens non-financial corporations and constrains the growth of aggregate demand. In a similar vein, Duménil and Lévy (2001) suggest that financialization leads to instability and undermines growth and employment. Meanwhile, most of mainstream economics has been praising the potential benefits of financial liberalization, and some non-mainstream contributors have also seen financialization as a spur to growth. Thus, Boyer (2000) has suggested the potential for finance-led growth regimes as an alternative to the defunct Fordist regime.

Although most of the existing literature on financialization has been descriptive and empirical, more precise analytical treatments of some of the macroeconomic linkages have been presented by Boyer (2000), Aglietta and Breton (2001), Dutt (2005), Stockhammer (2004, 2006), and Hein and van Treeck (2007).

According to Aglietta and Breton “[g]rowing financial liberalization has profoundly changed the connections between finance and the rest of the economy” (2001, p. 434). Their analysis, however, is hard to follow, and the formal model does little to elucidate the mechanisms that could support the claims that are being made in the paper.² Boyer’s model of finance-led growth basically boils down to profit-led / exhilarationist regimes with a profit-wealth-consumption nexus as a driving force. Given the cen-

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²It is difficult, for instance, to justify their assumption of an exogenously given and constant (average) net rate of return (E(ρ)). The firm’s credit constraint, second, is peculiar, as is the assumption that a risk premium is added to the risk-free interest rate only if the quantity constraint is binding. This problem has implications for the analysis of the firm’s optimization problem. In this analysis, the crucial first order condition with respect to the debt ratio d overlooks the dependence of the interest rate r on the debt ratio. Intuitively, why would any firm ever want to choose d = d_max if by reducing its debt ratio marginally the interest rate on its debt drops by a finite amount? The calculation of solutions for r and d_max in the constrained regime is also wrong since it overlooks the fact that the default probability is itself a function of r (aside from this important point, the expressions for r and d_max also contain a minor error).
trality of this nexus, however, a more careful modeling of the stock-flow relations and of the effects of financialization on wage formation would have been desirable. Boyer, for instance, assumes an exogenously given, constant $q$–ratio. This constancy assumption with respect to a key financial variable seems particularly unsatisfactory in a model that addresses the effects of financialization. The mechanism through which an increase in the ‘profitability norm’ generates a decline in the wage bill (for given values of output and the capital stock) is also unclear, as is the determination of the “profitability norm”.³

The Stockhammer and Dutt papers do not suffer from weaknesses of the same kind. Stockhammer’s 2004 analysis, however, is partial and his 2006 model is rudimentary in its treatment of the financial system; Dutt’s analysis focuses exclusively on the relaxation of households’ credit constraint and considers neither capital gains nor firms’ financial decisions and balance sheets. Hein and van Treeck, finally, analyse the effects of changes in firms’ financial behavior in a Kaleckian model. They assume, however, that these changes have no effect on the debt-capital ratio, the equity-capital ratio and the accumulated earnings-capital ratio. Since changes in financial behavior will, in general, lead to movements in these ratios, their analysis appears to be confined to the very short run.

In this chapter we explore the macroeconomic implications of changes in firms’ financial decisions (retention rate, new equity issues, debt finance), ‘animal spirits’ (shifts in the investment function), household financial behavior (saving and portfolio decisions), and the level of interest rates. These changes are among the ones that have been highlighted by the financialization literature but clearly make up only a small subset of the issues that have been raised.

³Is this norm fixed without any feedback from actual profit rates? On p. 124 it is suggested that, as an extension, the norm could be determined “using an adaptive process taking into account the past record of the achieved rate of profit”, but this extension is not pursued in the paper and it would seem to undermine the exogenous ‘financialization’ argument.
Three further limitations should be emphasized at the outset: (i) we limit ourselves to a closed economy, (ii) the emphasis is on the medium- and long-run effects with little or no attention to questions of stability and short-run fluctuations, and (iii) we ignore fiscal policy altogether and our treatment of monetary policy is kept almost embarrassingly simple. We limit the analysis in this way partly to keep it tractable, but also because many of the arguments advanced by the financialization literature concern the medium- and long-run effects of the changes in financial behavior and appear to be unrelated to open-economy complications or government policy. Thus, our simplifications may be justified by the limited objective of our analysis: to examine the logic underlying some of the claims that have been made in the financialization literature.

The specification of expectations would be critical in a full dynamic analysis of the trajectory of the economy but, given our focus on the medium and long run, we simply assume that expectations are being met. If the economy follows a steady growth path, this assumption will be satisfied for any standard process of expectations formation. More generally, fluctuations around a steady growth path will be associated with an approximate consistency between average expectations and average outcomes. It should be noted, however, that financialization may affect the properties of cyclical fluctuations, leading perhaps to an increase in the amplitude of fluctuations, and our analysis is clearly incomplete since we ignore these effects on the higher moments of the variables. A more radical perspective, finally, may regard increasing financialization as merely a phase in a long cycle of endogenous changes in financial behavior and Minskian fragility. From this Minskian perspective our fo-
cuse in this chapter may be misleading and our neglect of the dynamic interactions underlying the observed changes in financial behavior represents a major limitation. 4

Two different settings are examined. The economy may be ‘mature’ in Kaldor’s (1966) terminology and have a growth rate that is constrained by the available labor force. Alternatively, in the ‘dual-economy’ setting, the labor supply to the modern / capitalist sector of the economy is perfectly elastic. Both of these settings are analyzed using two alternative models: one is derived from Skott (1981, 1988, 1989) and the other from Lavoie and Godley (2001-2002). Both of the models are in a broadly structuralist/ post Keynesian tradition and both pay explicit attention to balance sheets and financial stock-flow relations. The two models differ in a number of respects. Interestingly, however, the differences with respect to the specification of financing, saving and portfolio decisions have little effect on the qualitative results. By contrast, the effects of financialization depend critically on the labor market assumptions (labor-constrained vs dual) and the specification of the investment function (Harrodian vs Kaleckian).

The rest of this chapter is structured as follows. In section 2.2 we discuss some of the stylized facts relating to financialization and comment on the dangers of a purely partial analysis. Section 2.3 outlines our general framework, and Sections 2.4 and 2.5 consider the implications of changes in key financial variables in the context of the different models. Section 2.6, finally, discusses the main results and offers a few concluding comments and suggestions for further research.
Figure 2.1. The Retention Rate (1952-2005)

Notes: The retention rate adjusted for inflation = 1—{ Net Dividends ÷ (U.S. Internal Funds + Net Dividends + Inflation rate × Net Liabilities)}. The inflation rates are based on the CPI and Net Liabilities refer to nonfarm nonfinancial corporate net liabilities. U.S internal funds = Profit (before taxes and after net interest payments) − Taxes on corporate income − Net dividends + Consumption of fixed capital + capital consumption adjustment.

Sources: Federal Reserve Board, Flow of Funds Accounts of the United States, Table F.102 and Table B.102; Bureau of Labor Statistics, The Consumer Price Index. Authors’ calculation.

2.2 Evidence

2.2.1 Some stylized facts

The stylized facts are largely well-known, and we confine ourselves to a brief description of some US data. The retention rate, first, has declined from around 85% in the 1970s to about 73% (Figure 2.1). It is worth pointing out that this change marks a return to retention rates that are at or below the levels of the 1950s. Looking

4 Minskian models of endogenous movements in financial fragility have been presented by, among others, Taylor and O’Connell (1986), Lavoie (1986/87), Delli Gati and Gallegati (1990), Semmler (1987), Skott (1994).
at the whole period since 1950, the aberration may have been the high retention rates of the 1970s and 1980s.

![Graph showing the rate of net issues of equities (1952-2005)](image)

**Figure 2.2.** The Rate of Net Issues of Equities (1952-2005)

*Notes:* Net issues of nonfinancial corporate equities divided by the market value of nonfinancial corporate equities outstanding

*Sources:* Federal Reserve Board, *Flow of Funds Accounts of the United States*, Table F.213 and Table B.102. Authors’ calculation.

The behavior of non-financial corporations with respect to new equity issues shows a clearer picture. Whether measured in terms of the value of new issues divided by the market value of outstanding equities (Figure 2.2) or, alternatively, by the share of new investment financed by new equity (Figure 2.3), there has been a significant decline in new issues. In the 1950s-1970s a small positive fraction of gross investment - on average about 5 percent - was financed by new issues. Since 1980, however, the rate of net issues has been negative in most years, and on average non-financial corporations have spent an amount equal to about 12 percent of their gross fixed investment to buy back equity.
Figure 2.3. The Ratio of Net Issues of Equities to Fixed Investment (1952-2005)

Notes: Net issues of nonfinancial corporate equities divided by nonfarm nonfinancial corporate (gross) fixed investment Quarterly data. Sources: Federal Reserve Board, Flow of Funds Accounts of the United States, Table F.213 and Table F.102. Authors’ calculation.

Debt finance has become increasingly important. As shown in Figure 2.4, the ratio of debt to the replacement value of capital has increased from a level just above 30 percent in the 1950s to about 60 percent. The ratio increased steadily in the 1950s and 1960s reaching about 50 percent in the early 1970s before dropping back to about 35 percent around 1980. Thus, the increase has been very steep over the last 25 years. It should be noted, however, that Figure 2.4 depicts gross debt. Insofar as non-financial firms hold increasing amounts of financial assets, the movements in net debt could be very different. Data issues make it difficult to get a clear picture of changes in net debt.

Real rates of interest have fluctuated substantially (Figure 2.5). The early 1980s saw historically very high interest rate, but rates gradually decrease in the late 1980s and early 1990s and, after another increase in the mid 1990s, are now at, and in some
Figure 2.4. The Ratio of Gross Debt to Capital: Nonfarm Nonfinancial Corporations (1952-2005)

Notes: Gross debt = commercial paper + municipal securities + corporate bonds + bank loans + other loans and advances + mortgages. Capital = replacement cost of structures + replacement cost of equipment and software.

Sources: Federal Reserve Board, Flow of Funds Accounts of the United States, Table B.102. Authors’ calculation.

cases below, their historical average. Thus, there is little support for common view that financialization has led to persistently high real rates of interest.

Turning now to household behavior, the well-known rise in the ratio of personal consumption to disposable personal income comes out clearly in Figure 2.6. The ratio of households’ net financial wealth to disposable income, however, has shown much more stability (Figure 2.7). The stock market boom of the 1990s shows up in this ratio, but the value of the ratio is now back at the level that characterized the “golden age” of the 1950s and 1960s. The effects of stock market fluctuations, finally, show up strongly in the ratio of capital gains to disposable income in Figure 2.8. The distribution of these gains has been very unequal, but as an average for the household
sector the capital gains (and losses) on financial assets have been very significant in some periods.

2.2.2 Dangers of a partial analysis

While the stylized facts of changes in financial variables are (relatively) clear, the interpretation and importance of these changes for the performance of the economy may not be obvious, and many of the arguments that have been advanced by the financialization literature have a partial flavor. As a case in point we may consider Stockhammer (2004). This paper, with its combination of theoretical argument and econometric work, presents a clear and interesting analysis. The partial nature of the analysis, however, is a limitation.
Financialization, Stockhammer argues, has generated a shift in firms’ behavior from growth objectives toward shareholder interests. He formalizes this argument by assuming that the representative firm faces a growth-profit tradeoff. Managers pick some point on this $g - r$ frontier, and an increased emphasis on shareholder interests (partly because of increased takeover threats and partly because of changes in managerial pay structures) moves the optimal position in the direction of higher profit rates and lower growth.

The macroeconomic implications of this microeconomic analysis are not as straightforward as they may seem. Stockhammer does not specify firms’ finance constraint or discuss firms’ financing decisions in any detail. Presumably, however, the movements along a $g - r$ frontier must be reflected - via the finance constraint - in changes in retention rates, external finance or the rate of new share issues. The changes in

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**Figure 2.6.** The Ratio of Personal Consumption Expenditures to Disposable Personal Income (1952-2006)

Figure 2.7. The Ratio of Households’ Net Financial Worth to Disposable Personal Income (1952-2006)

Notes: Net Financial Worth = Households’ Net Worth − Households’ Tangible Assets. In other words, the gap between two graphs shown in the figure represents households’ tangible assets divided by disposable person income

Sources: Federal Reserve Board, Flow of Funds Accounts of the United States, Table B.100. Authors’ calculation.

investment and firms’ financial decisions interact with household and government behavior, and these macroeconomic interactions - equilibrium conditions for financial and goods markets - are ignored in the analysis. Putting it differently, an individual firm may face a perceived $g - r$ tradeoff but this perceived tradeoff does not extend to the macroeconomic level: changes in accumulation and financial behavior affect aggregate demand and thereby the position of the $g - r$ frontier. Thus, the micro tradeoff may not be stable.

Stockhammer tests the theory by estimating an investment function that includes “rentiers’ share of the non-financial business sector” as an explanatory variable. It is unclear, however, how one should interpret the results. One might have thought, first,
Figure 2.8. The Ratio of Capital Gains on Financial Assets to Disposable Personal Income: Households and Nonprofit Organizations (1952-2005)

Notes: Capital Gains on Corporate Equities = (Holding gains on corporate equities − inflation rate using the CPI × corporate equities outstanding held by households and nonprofit organizations)/ disposable personal income. Capital Gains on Financial Assets = (Holding gains on all financial assets − inflation rate using the CPI × all financial assets held by households and nonprofit organizations)/ disposable personal income.

Sources: Federal Reserve Board, Flow of Funds Accounts of the United States, Table B.100 and Table R.100; Bureau of Labor Statistics, The Consumer Price Index. Authors’ calculation.

that a shift in firms’ accumulation behavior would imply changes in the parameters of the investment function. Stockhammer does not consider this possibility. Instead, he argues, the behavioral shift is captured by an increase in the “rentiers’ share”, and a negative coefficient on this variable is seen as lending support to the theoretical argument. Even assuming, however, that an increase in the “rentiers’ share” captures financial implications of a behavioral shift, a negative coefficient on this variable in the empirical work does not necessarily imply that the changes in financial behavior

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have had a negative effect on accumulation.\textsuperscript{5} Aggregate demand and thereby the values of other explanatory variables in the regression may have been affected by the changes in financial behavior, and these indirect effects need to be taken into account.

Unlike in the 2004 paper, the macroeconomic dimension of shareholder-induced shifts in firms’ investment behavior is analyzed by Stockhammer (2006) but this happens in a setting without differentiated financial assets and explicit stock-flow relations.

2.3 General framework

2.3.1 Firms, banks and households

This section presents our general framework. The framework leaves out open economy issues, there is no analysis of the short run and stability issues, and very limited attention to government policy. The purpose is to look at the interaction between firms and households across labor, goods and financial markets. Firms, it is assumed, make decisions concerning pricing / output, accumulation, and financing; households receive a return on their financial assets as well as wage income, and they make consumption and portfolio decisions; banks accept deposits and make loans. There are only two types of financial assets, equity and bank deposits, and banks are the only financial institution in the model.\textsuperscript{6}

\textsuperscript{5}In fact the coefficient on rentiers’ share is not negative in all specifications and it is insignificant in many. Moreover, there may be several explanations for a negative coefficient. Net financial income, first, is included in the gross profit share and for any given profit share, an increase in financial income implies a reduction in operating profits which presumably reduces the incentive to accumulate fixed capital. As noted by Stockhammer, second, an increase in gross financial income may mirror an increase in the cost of capital. Firms have both financial assets and liabilities and if the return on these move together, a rise in the cost of capital will be associated with an increase in gross financial income. This correlation becomes particularly important if the cost-of-capital variable that is included in the analysis provides a poor approximation to the actual cost of capital.

\textsuperscript{6}The liquid asset could also be interpreted as a short bond.
This framework - which generalizes the one in Skott (1988, 1989) - covers a number of special cases, including Harrodian specifications with or without labor constraints and Kaleckian models.

2.3.1.1 Firms

2.3.1.1.1 Finance constraint  Consider first the finance constraint facing a single firm. The firm invests in real capital and pays out dividends and interest on its debt (bank loans). These expenses have to be matched by income flows and the proceeds from new issues of equity and new debt. As argued by the financialization literature, the firm may hold equity in other firms and own other financial assets (bank deposits). Income flows therefore include both profits and the interest and dividend income from the firm’s current holdings of financial assets. Algebraically, the finance constraint can be written

\[ pI_j + Div^L_j + iM^L_j + vN^A_j + \dot{M}^A_j = \Pi_j + v\dot{N}_j + M^L_j + iM^A_j + Div^A_j \]

where \( I, \Pi, Div, M \) and \( N \) denote real investment, nominal profits, dividends, bank loans / deposits and the number of shares. Subscripts \( j \) indicate firm, and superscripts denote assets (\( A \)) and liabilities (\( L \)); thus \( M^A_j \) is firm \( j \)’s bank deposits and \( M^L_j \) the firm’s bank loans. Bank loans and deposits carry the same nominal interest rate (\( i \)), the price of investment goods (\( p \)) equals the general price of output in this one-sector model and, for simplicity, it is assumed that all shares have the same price, \( v \). A dot over a variable is used to denote a time derivative (\( \dot{x} = dx/dt \)).

If we aggregate across firms, the cross holdings of financial assets net out, and the aggregate finance constraint for the firm sector simplifies to:

\[ pI + Div + iM = \Pi + v\dot{N} + \dot{M} \]
where $I, \Pi, Div, M$ and $N$ without sub- and superscripts - denote aggregate investment and aggregate profit, net dividend payments from firms to other sectors, net debt to other sectors, and the aggregate number of shares held by other sectors.

We assume that dividends are given by

$$Div = (1 - s_f)(\Pi - rM) \quad (2.1)$$

where $r$ is the real rate of interest, $r = i - \hat{p}$, and $s_f$ is the retention rate out of profits net of interest payments. This specification is used by, among others, Lavoie and Godley (2001-02) and Dos Santos and Zezza (2007), but clearly, other specifications are possible. Skott (1989), for instance, assumes $Div = (1 - s_f)\Pi$, and another alternative would be to assume that dividends are set so as to leave sufficient retained earnings to cover some fraction of current investment. These specifications all imply that real dividend payments will be unaffected by a change in the rate of inflation, keeping constant the real rate of interest. This ‘inflation neutrality’ ceases to hold if the real rate of interest is replaced by the nominal rate in equation (2.1) since in this case an increase in inflation reduces the ratio $Div/\Pi$ of dividends to profits. As long as the inflation rate is constant, however, the switch to a nominal interest rate in equation (2.1) would not affect any of the qualitative results.

Using equation (2.1), the finance constraint can be rewritten

$$pI = s_f(\Pi - rM) + vN\hat{N} + M(\hat{M} - \hat{p}) \quad (2.2)$$

---

7Both Lavoie and Godley (2001-02) and Dos Santos and Zezza (2007) assume a constant price level, but Lavoie and Godley’s discussion on p. 300 of changes in interest rates indicates that they view the real interest rate is the relevant rate in the case of inflation.

8As shown in Figure 2.1, the ratio $(\Pi - rM - Div)/(\Pi - rM)$ increased in the high inflation years of the 1970s. Inflation effects of this kind may have contributed to this increase.
where a hat over a variable denotes the growth rate of the variable ($\dot{x} = \frac{\dot{x}}{x} = \frac{(dx/dt)}{x}$). The finance constraint (2.2) shows that, given the levels of investment and profits and the inherited debt, firms cannot choose the retention rate, the rate of new issues and the amount of new debt independently. One of these three variables will have to accommodate so as to ensure that the finance constraint is being met. In reality, of course, there may be dynamic feedback effects: an unexpected need for external finance in one period, for instance, may influence firm’s retention and/or new issue policies in subsequent periods.

Our purpose is to examine the comparative statics of changes in financial behavior and from this perspective it does not matter much which financial variable is designated as residual. In the analysis below we describe firms’ financial behavior in terms of their retention rate ($s_f$). New issue policies can be captured by the growth of the number of shares ($\dot{N}$) or by the share of investment that is being financed by new issues. Skott uses the former and Lavoie-Godley the latter parameterization, and we follow these different parameterizations in the respective versions of the model.  

2.3.1.1.2 Pricing / output: the growth function It is often assumed that firms set prices and that output adjusts instantaneously and costlessly to match demand. The empirical evidence in favour of significant price rigidity is quite weak, however, and output does not adjust instantaneously. Production is subject to a

---

9One could also, following Eichner (1976) and Wood (1975) - assume that firms set the shares of investment that are to be financed by the three different sources, with both $s_f$ and $\dot{N}$ varying in response to changes in accumulation. This case is considered in Skott (1989, chapter 7); it is also the approach used in Godley and Lavoie (2007).

10The study by Levy et al. (1997) of menu costs in five supermarkets, for instance, is often cited in support of menu costs and price stickiness (e.g. Romer 2001, pp. 315-316). This study found that on average 16 percent of all prices were changed each week. These frequent changes in prices were not costless but the finding that menu costs constitute a significant proportion of net profits is largely irrelevant for an evaluation of price flexibility. With prohibitively high menu costs, for instance, there would be no price changes and the share of menu cost in revenue would be zero; negligible menu costs on the other hand may allow firms to change prices frequently as part of their marketing strategies, and the observed share of menu costs in net profits could be very high in this case.
production lag, and increases in production and employment give rise to substantial search, hiring and training costs; firing or layoffs also involve costs, both explicit costs like redundancy payments and hidden costs in the form of deteriorating industrial relations and morale.

In a continuous-time setting one may approximate the effects of lags and adjustment costs by assuming that output is predetermined at each moment, that firms choose the rate of growth of output at each moment, rather than the level of output, and that this choice is made so as to balance the costs of changes against the benefits of moving toward a preferred level of output and employment. These costs and benefits are determined by demand signals from output markets and cost signals from input markets.

The demand signal can be captured by the prevailing profit share if prices are fully flexible. By assumption the level of output is predetermined, and with flexible prices a rise in demand leads to an increase in the price of output. Wage contracts are cast in terms of money wages and in the absence of perfect foresight or instantaneous feedbacks from output prices to money-wage rates, the real wage rate and the share of profits in income respond to unanticipated movements in prices: a positive demand shock generates a rise in the profit share.

The assumption of fully flexible prices is extreme, of course. Our reading of the evidence suggests that prices are less sticky than output, but in general there will be some stickiness in both prices and output, leaving changes in inventories and/or quantity rationing as accommodating variables. For the aggregate economy, however, quantity rationing is insignificant and movements in inventories tend to amplify fluctuations in other demand components - even in the short run - and thus do not obviate the need for price adjustments. For simplicity, we therefore disregard movements in inventories and assume that the demand signal is reflected in the profit share.
Turning to the signals from input markets, we leave out intermediate inputs and take labour to be the only input that is variable in the short run; changes in the capital stock take longer to implement and, partly because of that, firms typically maintain excess capital capacity. As far as production decisions are concerned, the labour market therefore provides the relevant signal, and we use the employment rate as the indicator of the state of the labour market. The rate of employment influences the costs of changing output through its effects on the availability of labour with the desired qualifications. High rates of employment increase the costs of recruitment, and since the quit rate tends to rise when labour markets are tight, the gross recruitment needs associated with any given rate of expansion increase when low unemployment makes it difficult to attract new workers. High employment and high turnover of the labour force, on the other hand, may allow firms to contract production and employment more rapidly without significant redundancy costs. These standard microeconomic effects may be reinforced by broader Marxian effects on the social relations of production. A high rate of employment may have a negative impact on firms’ growth plans because it strengthens workers vis-à-vis management and may lead to increased shop-floor militancy.

The analysis suggests that the rate of growth of production will be positively related to the profit share ($\pi$) and negatively related to the employment rate ($e$). Thus, the pricing / output decisions can be described by the following ‘growth function’\textsuperscript{11}

$$\dot{Y} = h(\pi, e); \; h_\pi > 0, \; h_e < 0.$$  

(2.3)

The case of unlimited labor supplies can be obtained by setting $h_e = 0$, and the growth function yields the standard Kaleckian assumption of a fixed profit share $\bar{\pi}$ (a fixed

\textsuperscript{11}Or ‘output expansion function’, using the terminology in Skott (1989, 1989a). The behavioral foundations of the function are discussed in greater detail in Skott (1989, chapter 4).
markup on wage cost) if we have both $h_\pi = \infty$ at $\pi = \bar{\pi}$ and $h_e = 0$. There is also an affinity between the growth function (2.3) and Robinson’s (1962, pp. 48-49) analysis of the rate of accumulation induced by a rate of profit. Since Robinson assumes that utilization is at an exogenously given normal level, the profit rate and the profit share move together, and a constant utilization rate implies that the accumulation rate is equal to the growth rate of output. Equation (2.3) generalizes the relation between growth and profits by allowing for the influence of labor market conditions. One may note, finally, that a static counterpart to equation (2.3) can be obtained by setting $\dot{Y} = 0$. The equation then defines the profit share as an increasing function of the employment rate. A short-run equilibrium relation of this kind could be derived from profit maximization if firms have monopsony power and the perceived elasticity of labor supply to the individual firm is inversely related to the aggregate rate of employment.

2.3.1.1.3 Accumulation With a fixed coefficient production function, a general specification of the investment function includes the rate of capital utilization, the profit share, and financial variables like the real rate of interest, the valuation ratio (Tobin’s q), and the ratios of debt and retained earnings to the value of the capital stock. Algebraically,

$$\frac{I}{K} = f(u, \pi, r, q, m, c)$$  \hspace{1cm} (2.4)

Comparing Robinson’s analysis to our ‘Harrodian - dual economy’ case below, the difference is that in Robinson’s model competition and pricing decisions keep utilization at the normal level while the profit share and the growth rate are determined by the equilibrium condition for the product market; the Harrodian - dual economy case assumes that the long-run properties of the accumulation function pins down utilization at the normal level, with the profit share and the growth rate determined by the ‘growth function’ in combination with saving behavior.

A positive relation between employment and the profit share could also arise from an inverse relation between the perceived demand elasticity and aggregate employment or as a result of a fixed markup on variable cost in a setting with overhead labor.
where \( u = \frac{Y}{K} \) is a measure of utilization, \( q \) is the valuation ratio \( (q = \frac{M + vN}{pK}) \), and \( m \) and \( c \) the ratios of debt and retained earnings to capital \( (m = \frac{M}{pK}, c = \frac{s_f(\Pi - rM)}{pK}) \).

There is no consensus in the structuralist / post Keynesian literature concerning the long-run sensitivity of the accumulation rate to changes in the various arguments.\(^{14}\) In the analysis below, we explore both Harrodian and Kaleckian specifications.

### 2.3.1.2 Banks

Banks give loans to firms and accept deposits from households. Neither firms nor households hold cash. When banks provide a loan to a firm, the money therefore returns to the bank immediately, either as deposits from households or because other firms use their increased revenues to reduce their debt. The loan and deposit rates are equal and there are no costs involved in banking. Thus, banks make neither profits nor losses,\(^ {15}\) and the firm sector has a net debt \( (M) \) that must equal the total deposits of the household sector \( (=\text{money demand}, M^H) \):

\[
M = M^H
\]

Banks determine the nominal interest rate. This nominal rate, however, will typically depend on inflation and to simplify the exposition, we treat the real rate of interest \( r \ (= i - \hat{p}) \) as the variable that is set by the banking system (and kept constant in steady growth).

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\(^{15}\)The share valuation of banks therefore is zero, and this simple version of the model does not capture the increasing share of the financial sector in GDP and of financial-sector profits in total profits.


2.3.1.3 Households

In analogy with firms, households face a budget (or finance) constraint. For the household sector as a whole it takes the form

\[ pC + v\dot{N}^H + \dot{M}^H = W + Div^H + iM^H \]  \hspace{1cm} (2.5)

where \( C \) is consumption, \( W \) wage income, \( N^H, M^H \) indicate household holdings of shares and deposits (money), and \( Div^H \) is dividend payments received by the household sector.

The steady-growth implications of household consumption and saving behavior can be described in terms of stock-flow ratios of assets to income. Specifically, let

\[ M^H = \beta(i, r, r_e, \pi, ...)pY \]  \hspace{1cm} (2.6)
\[ vN^H = \alpha(i, r, r_e, \pi, ...)pY \]  \hspace{1cm} (2.7)

where the stock-flow ratios \( \alpha \) and \( \beta \) may depend on a number of variables, including the real rates of return on deposits \((r)\) and equity \((r_e)\). Theories differ with respect to the determination of the (steady-growth) values of these stock-flow ratios, and in sections 2.4-2.5 we examine different specifications. Some theories are cast in terms of flow-flow relations (e.g. consumption as a function of distributed incomes and capital gains, as in the Lavoie-Godley model) but even when this is the case, the specification of the flow-flow relations have implications for the steady-growth values of the stock-flow ratios, and the implied stock-flow ratios provide a clearer picture of the mechanisms behind the effects of changes in financial behavior.

The relation between the stock-flow ratios and consumption is straightforward. Using the budget constraint (2.5) and the dividend equation (2.1), the stock-flow relations (2.6)-(2.7) imply the following consumption function:
\[
\frac{C}{K} = u[1 - s_f(\pi - r\beta) + \beta(\hat{p} - \hat{M}) - \alpha\hat{N}]
\] (2.8)

2.4 Harrodian accumulation

In this section we follow the Harrodian tradition and assume that the degree of excess capital capacity is at (or near) where firms want it to be. Firms will typically want a reserve of excess capacity, but if the degree of excess capacity persistently exceeds the desired reserve, they reduce their accumulation rate; conversely, if they find themselves with less than the desired excess capacity, they will gradually increase their rate of accumulation. Thus, a steady growth path with a constant accumulation rate requires the consistency of desired and actual degrees of excess capacity, that is,

\[
u = u^*\quad (2.9)
\]

where \(u\) is the output-capital ratio and \(u^*\) denotes the value of \(u\) when firms have the desired degree of excess capacity.\(^{16}\) Equation (2.9) expresses the steady-growth accumulation function. The equation need not be satisfied outside steady growth, but a simple Harrodian specification implies that if \(\hat{K}\) fluctuates within a relatively narrow band, the time-average of the output-capital ratio \(u\) must be approximately equal to \(u^*\) when the average is taken over a long period. To see this, consider a Harrodian investment function

\[
\frac{d}{dt}\hat{K} = \lambda(u - u^*); \quad \lambda > 0
\]

Integration implies that \(\bar{u} - u^* = \frac{\hat{K}_{t_1} - \hat{K}_{t_0}}{\lambda(t_1 - t_0)}\) where \(\bar{u}\) is the average output-capital ratio over the interval \([t_0, t_1]\). If \(\hat{K}_{t_1} - \hat{K}_{t_0}\) is bounded below some constant for all \((t_0, t_1)\),

\(^{16}\)The \(u = u^*\) condition is necessary but not sufficient. Firms must also make positive profits, cf. note 17 below.

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it follows that \( \bar{u} \) is close to \( u^* \) if the period is long (\( \bar{u} \) converges to \( u^* \) for \( t_1 - t_0 \) going to infinity).

### 2.4.1 A mature economy: labor-constrained steady growth

The growth rate in a mature economy is labor constrained and the employment rate is constant in steady growth. The growth rate therefore must be equal to the growth of the labor force and, for simplicity, we shall take this ‘natural rate of growth’ \( (n) \) to be an exogenously given constant. Thus, in steady growth

\[
\dot{Y} = n
\]  

Using (2.9) and (2.10) the equilibrium condition for the product market can now be written

\[
\frac{C}{K} + n = u^*
\]

or, using (2.6), (2.8), (2.9) and (2.10),

\[
[1 - s_f(\pi - r\beta) - \beta n - \alpha \hat{N}] = \frac{u^* - n}{u^*}
\]  

The effects of changes in firms’ financial behavior \( (s_f, \hat{N}) \), bank policy \( (r) \), or household saving and portfolio behavior can be derived from this equation. The qualitative results, however, depend on the properties of the \( \alpha \) and \( \beta \)–functions that describe household behavior.

#### 2.4.1.1 Inelastic stock-flow ratios

Assume first that \( \alpha \) and \( \beta \) are both independent of the various rates of return and other variables in the expressions (2.6)-(2.7). In this case with \( \alpha \) and \( \beta \) are
parameters rather than functions and the constancy of the term on the right hand side of equation (2.11) implies that

\[
\frac{\partial \pi}{\partial s_f} = -\frac{\pi - \beta r}{s_f} < 0 \quad (2.12)
\]

\[
\frac{\partial \pi}{\partial \hat{N}} = -\frac{\alpha}{s_f} < 0 \quad (2.13)
\]

\[
\frac{\partial \pi}{\partial r} = \beta > 0 \quad (2.14)
\]

\[
\frac{\partial \pi}{\partial \alpha} = \frac{\dot{N}}{s_f}
\]

\[
\frac{\partial \pi}{\partial \beta} = \frac{s_f r - n}{s_f}
\]

The signs of the effects of changes in \( s_f, \hat{N} \) and \( r \) are unambiguous. If firms raise the retention rate or increase the rate of new issues, this will depress profitability, while an increase in the real interest rate raises the profit share.\(^{17}\) The intuition is simple. An increase in \( s_f \) increases aggregate saving, given the share of profits, and to bring saving back into line with the steady-growth requirement, a reduction in the profit share is needed. An increase in the real interest rate \( (r) \) has the opposite effect since it reduces retained earnings and thus saving at any given share of profits. An increase in new issues \( (\hat{N}) \), like increases in the retention rate, raises aggregate saving but the mechanism may be a little less transparent. Saving goes up because the rise in \( \hat{N} \) induces households to raise their saving. Share prices adjust so as to maintain a constant ratio \( (= \alpha) \) of the value of shares to income. The growth of real income is given, and if the rate of new issues has gone up, this means that real share prices will increase at a lower rate. Capital gains therefore are smaller and as a result households choose to save a larger proportion of their wage, dividend and interest income.

\(^{17}\) A capitalist economy would not be viable if the steady growth path implied that profits fell short of real interest payments on the debt. Thus, the condition \( \pi - \beta r > 0 \) must hold, otherwise accumulation would collapse.
Financialization has been associated primarily with increased dividends (a decline in \( s_f \)), a decrease in the rate of new issues (\( \hat{N} \)) and an increase in the real rate of interest (although, as shown in section 2.2, the evidence for interest rates is questionable). Strikingly, in this model all of these changes unambiguously generate a rise in the steady-growth profit share and the steady-growth employment rate. The employment effect follows immediately from the growth function (2.3): whenever the profit share goes up, the employment rate must do the same in order to keep the growth rate unchanged.\(^{18}\)

So far we have taken the stock-flow ratios \( \alpha \) and \( \beta \) to be constant parameters. Even leaving aside the functional dependence of these ratios on, inter alia, the rates of return, financialization might generate a shift in the values of these parameters. Thus, it could be argued that financialization increases the availability of consumer credit and thereby tends to reduce the ratio \( \beta \). A reduction in \( \beta \) has two effects: it increases retained earnings (which tends to reduce consumption) but if the growth rate of income is positive it also reduces the amount of saving that households need to carry out in order to maintain the money-income ratio at the desired value. Depending on parameter values, the balance of these two effects can be positive or negative.\(^{19}\)

\(^{18}\)In this paper we do not consider nominal wage formation and inflation explicitly. The NAIRU literature is enormous; Skott has analysed reasons for the absence of a NAIRU in earlier work (Skott 1997, 1999, 2005).

\(^{19}\)Our results for changes in \( \beta \) are closely related to those of Dutt’s (2005) analysis of changes in consumer debt. Using a Kaleckian (stagnationist) model, Dutt shows that the short-run effect of an increase in households’ debt-income ratio (corresponding to a decrease in \( \beta \) in this model) is unambiguously positive. This short-run result is not surprising since the the transition to a higher debt ratio is associated with extra consumption. The long-run effects on growth are ambiguous, however. In the long run, the debt ratio has increased (\( \beta \) has decreased), and this increase in the debt ratio implies a shift of disposable income from low-saving workers to high-saving capitalists. This contractionary effect may or may not be offset by a positive effect. Consumer debt grows at the same rate as output (and the capital stock) and this expansionary effect - consumers being allowed to increase their debt when output grows - depends on the growth rate. Thus, in Dutt’s model, an increase in consumer debt will raise the growth rate if the initial growth rate is high while if output grows slowly, the increase in debt will reduce the growth rate. In this version of our model, the growth rate is exogenous but the analogous result in our model is that a decrease in \( \beta \) raises the profit share if the growth rate is high but reduces the profit share if the growth rate is small is low.
Changes in the $\alpha$ ratio are not usually seen as a key mechanism behind changes in economic performance.\textsuperscript{20} Moreover, in this model the effects of autonomous shifts in $\alpha$ depend on the values of $\hat{N}$. This result is quite intuitive. The value of the equity-income ratio ($\alpha$) simply does not affect saving if there are no new issues. Households can only save in the form of shares if other sectors (firms) are willing to sell shares. If that is not the case then an increase in the desire to own shares will simply generate higher share prices, and the desire will be met without any extra saving. With positive new issues, a higher valuation of shares (a higher $\alpha$) implies an increase in household saving; with negative new issues, on the other hand, a higher valuation of shares implies that households receive higher revenues from their net sale of shares, and their saving out of wages, dividends and interest income is reduced.

In addition to the changes in financial behavior, financialization may have been associated with a downward shift in the investment function. In this Harrodian setting, such a shift would be reflected in a rise in the desired output-capital ratio $u^*$. This kind of change has the consequences that one would expect. Equation (2.11) implies that a rise in $u^*$ leads to a decline in the profit share and, using the growth function (2.3), a fall a employment. Thus, according to this model the changes associated with neoliberalism and financialization have contradictory effects. The net effect may have been a deterioration of economic performance, but the negative impact comes from the shift in the investment function, rather than from the changes in financial behavior that have been highlighted in the literature.

How general are these conclusions? The assumption of exogenous $\alpha$— and $\beta$—ratios is clearly restrictive, but the qualitative results survive as long as $\alpha$ and $\beta$ are

\textsuperscript{20}One might consider the possibility that $u^*$ depend on the valuation ratio (Tobin’s $q$) and thereby on $\alpha$ and $\beta$. A high valuation ratio indicates a rate of profit that exceeds the cost of finance. The desired output-capital ratio may therefore be inversely related to the valuation rate. This expansionary impact of an increase and $\alpha$ and $\beta$ is considered by Skott (1988, 1989).
relatively insensitive to changes in the financial parameters \((s_f, \hat{N}, r)\) and the profit share \((\pi)\).

Differentiating equation (2.11) totally, we get

\[-s_f d\pi - \pi ds_f + (s_f r - n)d\beta + \beta(s_f dr + rds_f) - \alpha d\hat{N} - \hat{N} d\alpha = 0\]  

\[(2.15)\]

where

\[
d\alpha = \frac{\partial \alpha}{\partial s_f} ds_f + \frac{\partial \alpha}{\partial \hat{N}} d\hat{N} + \frac{\partial \alpha}{\partial r} dr + \frac{\partial \alpha}{\partial \pi} d\pi \]  

\[(2.16)\]

\[
d\beta = \frac{\partial \beta}{\partial s_f} ds_f + \frac{\partial \beta}{\partial \hat{N}} d\hat{N} + \frac{\partial \beta}{\partial r} dr + \frac{\partial \beta}{\partial \pi} d\pi \]  

\[(2.17)\]

\(s_f, \hat{N}, r\) and \(\pi\) may not influence \(\alpha\) and \(\beta\) directly but they will do so indirectly via their effects on the various rates of return. Thus, the partial \(\frac{\partial \alpha}{\partial s_f}\) includes the indirect effect on \(\alpha\) of changes in rates of return generated by the change in \(s_f\).

The rate of return on equity, for instance, is given by

\[r_e = \frac{1}{\hat{N}} \left(1 - s_f\right)(\pi - r\beta + \hat{\nu} - \hat{\rho} = \frac{(1 - s_f)(\pi - r\beta)}{\alpha} + n - \hat{N}\]  

and an increase in \(s_f\) reduces \(r_e\)\(^{21}\)

Using (2.15)-(2.17) we get

\[
\frac{\partial \pi}{\partial s_f} = \frac{-\pi - \beta r - (s_f r - n)\frac{\partial \beta}{\partial s_f} + \hat{N} \frac{\partial \alpha}{\partial s_f}}{s_f - (s_f r - n)\frac{\partial \beta}{\partial \pi} + \hat{N} \frac{\partial \alpha}{\partial \pi}} \]  

\[(2.18)\]

\[
\frac{\partial \pi}{\partial \hat{N}} = \frac{-\alpha - (s_f r - n)\frac{\partial \beta}{\partial \hat{N}} + \hat{N} \frac{\partial \alpha}{\partial \hat{N}}}{s_f - (s_f r - n)\frac{\partial \beta}{\partial \pi} + \hat{N} \frac{\partial \alpha}{\partial \pi}} \]  

\[(2.19)\]

\[
\frac{\partial \pi}{\partial r} = \frac{s_f \beta + (s_f r - n)\frac{\partial \beta}{\partial r} - \hat{N} \frac{\partial \alpha}{\partial r}}{s_f - (s_f r - n)\frac{\partial \beta}{\partial \pi} + \hat{N} \frac{\partial \alpha}{\partial \pi}} \]  

\[(2.20)\]

The signs of the partials of the profit share with respect to these three financial parameters are the same as in (2.12)-(2.14) as long as

\(^{21}\)Mathematically, perverse results are possible in which a rise in \(s_f\) increases the return. This could happen, for instance, if there is a strong inverse relation between \(\alpha\) and \(r_e\). The conditions that would give these perverse results can be ruled out on economic grounds.
\[
\pi - \beta r > (sfr - n) \frac{\partial \beta}{\partial s_f} - \hat{N} \frac{\partial \alpha}{\partial s_f}
\]
\[
\alpha > (sfr - n) \frac{\partial \beta}{\partial \hat{N}} - \hat{N} \frac{\partial \alpha}{\partial \hat{N}}
\]
\[
sf\beta > -(sfr - n) \frac{\partial \beta}{\partial r} + \hat{N} \frac{\partial \alpha}{\partial r}
\]
\[
sf > (sfr - n) \frac{\partial \beta}{\partial \pi} - \hat{N} \frac{\partial \alpha}{\partial \pi}
\]

These ‘inelasticity conditions’ will automatically be satisfied if \((sfr - n) = \hat{N} = 0\), irrespective how sensitive are \(\alpha\) and \(\beta\) to variations in their arguments. Empirically, both \(sfr - n\) and \(\hat{N}\) are close to zero, having at times been positive and at times negative. In fact, setting \((sfr - n) = \hat{N} = 0\) is arguably a reasonable empirical benchmark. Thus, the qualitative results in (2.12)-(2.14) survive - at least as an outcome that holds for a range of empirically very plausible parameter values - in a more general model in which the stock-flow ratios are determined endogenously. It should be noted also that the different specifications used in Skott (1981, 1988, 1989) are special cases of the general model with endogenous \(\alpha\) and \(\beta\) ratios;\(^{22}\) all of these special cases satisfy the inelasticity conditions for any reasonable set of parameters, as does the flow-flow specification used by Lavoie and Godley (2001-2002) and the stock-flow specification in Godley and Lavoie (2007) (see below).

Overall, then, while the implications of assuming elastic stock-flow ratios are clear - the comparative statics will be reversed - inelastic ratios appear to be the more interesting and empirically relevant case.\(^{23}\)

\(^{22}\)Skott (1989), for instance, assumes that \(\beta\) is exogenous and that \(vN = \alpha(p, u, r, \beta)pY = (\pi - \frac{\delta}{u} - r\beta)pY\) where \(\delta\) is the rate of depreciation; thus, share valuation is proportional to profits net of depreciation and real interest payments.

\(^{23}\)We use the terms ‘inelastic’ and ‘elastic’ to denote the cases when the conditions hold and fail to hold, respectively. Intermediate cases in which some but not all of the conditions hold are clearly possible; in these cases only some of the signs of the partials in (2.12)-(2.14) will be preserved.
2.4.1.2 The Lavoie-Godley specification of consumption

In the Lavoie-Godley model, consumption is a function of distributed income and capital gains. Thus, the consumption function is specified as a flow-flow relation. Using our notation, a general version of their consumption function can be written as:

\[
\frac{C}{K} = \psi(y, \gamma), \quad \psi_y > 0, \quad \psi_\gamma > 0
\]  
(2.21)

where \( y \) is households’ distributed income and \( \gamma \) is capital gains, both variables as ratios of the capital stock \( y = [1 - s_f(\pi - r\beta) + \hat{\beta}]u \) and \( \gamma = \frac{vN(\hat{\beta} - \hat{\beta})}{pK} \). The proportion of the investment expenditure that is financed by equity issues is denoted as \( x \). Lavoie and Godley take this proportion as the parameter describing new issue policies (instead of \( \hat{N} \)). By definition

\[
\frac{vN}{pK} \hat{N} = x \frac{I}{K} = xg
\]

where \( g \) is the accumulation rate. Thus, the ratio of capital gains to capital can be written

\[
\gamma = \alpha u g - xg
\]

The equilibrium condition (2.11) is general and still holds in the Lavoie-Godley specification and - using the definition of \( x \) - the equation can be written

\[
u^* - n = u^* \left[ 1 - s_f(\pi - r\beta) - \beta n - \frac{xn}{u^*} \right] = \psi(y, \gamma)
\]  
(2.22)

The steady growth value of \( \beta \) (and \( \alpha \)) is affected by the consumption / saving function (2.21) and household portfolio decisions. In the Lavoie-Godley model these portfolio decisions are described by
\[
\frac{M}{M + vN} \equiv \frac{\beta}{\alpha + \beta} = z(r, r^e, y, q), \quad z_r > 0, z_{r^e} < 0, z_y > 0, z_q < 0
\]  
(2.23)

where \(r^e\) is the rate of return on equities \(r^e = \frac{(1-s_f)(\pi-\beta r)u + n(\alpha u-x)}{\alpha u}\) and \(q\) can be written as \((\alpha + \beta)u\).

For some functional forms of \(\psi\) in (2.22) and \(z\) in (2.23) it may be possible to obtain analytical expressions for \(\alpha\) and \(\beta\), as in our general representation for the stock-flow ratios, (2.6) and (2.7); other specifications - including the ones used by Lavoie and Godley - may preclude explicit analytical expressions but the stock-flow implications can still be evaluated numerically.

With the relevant definitions, (2.22) and (2.23) determine the equilibrium values of \(\pi\), \(\alpha\) and \(\beta\). Each exogenous variable \((s_f, x, r, \text{among others})\) affects the equilibrium stock-flow ratios \(\alpha\) and \(\beta\) as well as the profit share \(\pi\), and we get expressions that are analogous to (2.18)-(2.20):

\[
\begin{align*}
\frac{\partial \pi}{\partial s_f} &= \frac{\pi - \beta r - (s_f r - n) \frac{\partial \beta}{\partial s_f}}{s_f - (s_f r - n) \frac{\partial \beta}{\partial \pi}} \\
\frac{\partial \pi}{\partial x} &= \frac{n - (s_f r - n) u^* \frac{\partial \beta}{\partial \pi}}{s_f u^* - (s_f r - n) u^* \frac{\partial \beta}{\partial \pi}} \\
\frac{\partial \pi}{\partial r} &= \frac{s_f \beta + (s_f r - n) \frac{\partial \beta}{\partial \pi}}{s_f - (s_f r - n) \frac{\partial \beta}{\partial \pi}}
\end{align*}
\]

The total effect on the profit share of each parameter can be decomposed into the effect for a given \(\alpha\) and \(\beta\), and the derived effect via changes in \(\alpha\) and \(\beta\). The first effect is clear and straightforward as shown in section 2.4.1.1. Our main concern here is whether ‘the inelasticity conditions’ for stock-flow ratios hold in the Lavoie-Godley specification.
Table 2.1. Harrodian mature economy I

<table>
<thead>
<tr>
<th>Variable</th>
<th>Constant $\alpha$ and $\beta$ regime</th>
<th>Variable $\alpha$ and $\beta$ regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>The retention ratio</td>
<td>-0.238</td>
<td>-0.238</td>
</tr>
<tr>
<td>Equity issues</td>
<td>-0.386</td>
<td>-0.228</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>2.589</td>
<td>2.575</td>
</tr>
<tr>
<td>Utilization</td>
<td>-1.948</td>
<td>-1.634</td>
</tr>
<tr>
<td>Propensity to hold equity</td>
<td>-</td>
<td>0.0717</td>
</tr>
</tbody>
</table>

1. Numbers in the table show the partial derivatives of the profit share with respect to the parameters listed in the first column.

2. The structure and parameter values of the model are the same as in Lavoie and Godley (2001-2002) including the specification of consumption, but the closure of the model is different.

Using Lavoie and Godley’s values for the parameters,\textsuperscript{24} we find that in the Harrodian mature economy, the indirect effects via changes in $\alpha$ and $\beta$ are quite small, with the direct effects corresponding to constant stock-flow ratios explaining most of the total effects. Table 2.1 shows the numerical results. The numbers in Table 2.1 indicate the derivatives of the profit share with respect to each exogenous parameter, evaluated at the equilibrium associated with Lavoie and Godley’s original values of parameters. A thorough examination of whether our ‘inelasticity conditions’ are robust with respect to reasonable variations in all parameter values has been left for future research; preliminary results, however, show robustness as we vary the parameters of the consumption function.\textsuperscript{25}

Lavoie and Godley have changed their consumption function in recent work. Godley and Lavoie (2007) use a stock-flow specification with consumption as a linear

\textsuperscript{24}Lavoie and Godley (2001-2002) did not report parameter values but have provided the values in private correspondence. These parameter values and our procedure of decomposition are given in Appendix E.

\textsuperscript{25}Below we report some of the sensitivity results for the ‘Kaleckian - dual economy’ case, which is the case that is closest to Lavoie and Godley’s own model.
Table 2.2. Harrodian mature economy II

<table>
<thead>
<tr>
<th>Variable</th>
<th>Constant $\alpha$ and $\beta$ regime</th>
<th>Variable $\alpha$ and $\beta$ regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>The retention ratio</td>
<td>-0.296</td>
<td>-0.296</td>
</tr>
<tr>
<td>Equity issues</td>
<td>-0.342</td>
<td>-0.261</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>1.023</td>
<td>1.005</td>
</tr>
<tr>
<td>Utilization</td>
<td>-1.102</td>
<td>-0.564</td>
</tr>
<tr>
<td>Propensity to hold equity</td>
<td>-</td>
<td>0.091</td>
</tr>
</tbody>
</table>

1. Numbers in the table show the partial derivatives of the profit share with respect to the parameters listed in the first column.

2. The structure and parameter values of the model are the same as in Lavoie and Godley (2001-2002) except the specification of consumption and the closure of the model. The alternative specification of consumption is given by the one in Godley and Lavoie (2007).

function of income and wealth. This specification is closer in spirit to our analysis in section 2.3.1.3 and, using their new specification and parameter values, our inelasticity conditions are still satisfied; Table 2.2 lists the derivatives of the profit share for this case.$^{26}$

2.4.2 Dual economies: endogenous growth

We now turn to the case of dual economies, that is, economies in which the labor force does not constrain the rate of growth. This case may correspond to economies with large amounts of hidden unemployment in backward, non-capitalist sectors, or it could depict the case where the labor supply to the capitalist sector is perfectly elastic for other reasons (immigration, women’s participation rate, endogenous fertility, or technical progress).

$^{26}$The numerical results in Table 2.2, 2.4, 2.8, 2.9, and 2.10 are based on $\frac{C}{K} = c_1 \{u - s_f(\pi u - rm)\} + c_2 q$ where $c_1 = 0.75$ and $c_2 = 0.064$, which is equivalent to the one in Godley and Lavoie (2007) if there is no consumer loan, no bank profit and no inflation.
The growth function can be simplified in a dual economy of this kind. The employment rate no longer serves as a relevant signal and therefore drops out of the growth function. Hence,
\[ \hat{Y} = h(\pi); h' > 0 \]

In steady growth we still have \( g = \hat{Y} = \hat{K} \) and \( u = u^* \), and the equilibrium condition for the product market takes the form

\[ [1 - s_f(\pi - r\beta) - \beta h(\pi) - \alpha \hat{N}] = 1 - \frac{1}{u^*} h(\pi) \]  \( (2.24) \)

2.4.2.1 Inelastic stock-flow ratios

In the case with exogenous \( \alpha \) and \( \beta \) ratios, equation (2.24) gives the following comparative statics:

\[
\begin{align*}
\frac{\partial \pi}{\partial s_f} &= -\frac{\pi - r\beta}{s_f + (\beta - \frac{1}{u^*})h'(\pi)} \\
\frac{\partial \pi}{\partial \hat{N}} &= s_f + (\beta - \frac{1}{u^*})h'(\pi) \\
\frac{\partial \pi}{\partial \alpha} &= s_f + (\beta - \frac{1}{u^*})h'(\pi) \\
\frac{\partial \pi}{\partial \beta} &= s_f + (\beta - \frac{1}{u^*})h'(\pi) \\
\frac{\partial \pi}{\partial r} &= \frac{\beta s_f}{s_f + (\beta - \frac{1}{u^*})h'(\pi)} \\
\frac{\partial \pi}{\partial \alpha} &= -\frac{s}{s_f - n} \\
\frac{\partial \pi}{\partial \beta} &= s_f + (\beta - \frac{1}{u^*})h'(\pi)
\end{align*}
\]

The signs of these partials depend on the magnitude of \( h'(\pi) \). The expression \( \beta - \frac{1}{u^*} = \frac{M - pK}{pY} \) is negative for any empirically reasonable specification, and it follows that compared to the labor constrained case, the comparative statics are unchanged if \( h' \) is ‘small’ but reversed if \( h' \) is ‘large’. The standard Kaleckian formulation with \( \pi = \bar{\pi} \) corresponds to the limiting case with \( h' \to \infty \). This may be an extreme case, but in the absence of labor constraints one would expect a high sensitivity of growth.
to variations in profitability. Thus, the large-$h'$ case with the reversal of comparative statics for the profit share seems the most reasonable.

Changes in the profit share influence the growth rate in a dual economy, rather than the employment rate as in the labor-constrained economy. Expressions for the growth rate effects are readily obtained in the limiting case with a constant markup formulation ($h'(\pi) \to \infty$ at $\pi = \bar{\pi}$). In this limiting case equation (2.24) can be rewritten

$$[1 - s_f(\bar{\pi} - r\beta) - \beta g - \alpha \hat{N}] = 1 - \frac{1}{u^*} g$$

and

$$\frac{\partial g}{\partial s_f} = \frac{(\bar{\pi} - r\beta)u^*}{1 - \beta u^*} > 0$$
$$\frac{\partial g}{\partial \hat{N}} = \frac{\alpha u^*}{1 - \beta u^*} > 0$$
$$\frac{\partial g}{\partial r} = -\frac{s_f \beta u^*}{1 - \beta u^*} < 0$$
$$\frac{\partial g}{\partial \alpha} = \hat{N} u^*$$
$$\frac{\partial g}{\partial \beta} = \frac{(g - s_f r)u^*}{1 - \beta u^*}$$

The signs of the effects of changes in $s_f, \hat{N}$ and $r$ are clear. If firms raise the retention rate or increase the rate of new issues, this will increase the rate of capital accumulation, while an increase in the real interest rate slows down accumulation. The intuition is simple. Since $u^*$ and $\bar{\pi}$ are unaffected by changes in $s_f, \hat{N}$ and $r$, the effects on accumulation of changes in $s_f, \hat{N}$ and $r$ derive exclusively from their direct impacts on saving and the amount of available finance. Given that $u = u^*$ and $\pi = \bar{\pi}$, an increase in $s_f$ or $\hat{N}$ must increase the amount of financial resources available to
firms - raising the rate of capital accumulation - while a rise in $r$ has the opposite effect on accumulation since it reduces the amount of retained earnings.\footnote{It is easy to understand these comparative statics by looking at the closed-form solution for the rate of capital accumulation, i.e. $g = \frac{b-n(s-r)-\alpha N}{1-\beta u^*}$.}

### 2.4.2.2 The Lavoie-Godley specification of consumption

As we have seen in section 2.4.1.2, households’ consumption/saving and portfolio decisions in Lavoie-Godley (2001-2002) implicitly define the stock-flow ratios, $\alpha$ and $\beta$, as functions of a number of variables, and the accumulation rate becomes an additional influence on $\alpha$ and $\beta$ in the dual economy. Analogously to the analysis in section 2.4.1.2, we obtain the following comparative statics.

\[ \frac{\partial g}{\partial s_f} = \bar{\pi} - r\beta - (s_f r - g) \frac{\partial \beta}{\partial s_f} + \frac{1}{u^*} (1 - \beta u^* - x) + (s_f r - g) \frac{\partial \beta}{\partial g} \]

\[ \frac{\partial g}{\partial x} = \frac{1}{u^*} g - (s_f r - g) \frac{\partial \beta}{\partial x} + \frac{1}{u^*} (1 - \beta u^* - x) + (s_f r - g) \frac{\partial \beta}{\partial g} \]

\[ \frac{\partial g}{\partial r} = -\frac{s_f \beta + (s_f r - g) \frac{\partial \beta}{\partial r}}{\frac{1}{u^*} (1 - \beta u^* - x) + (s_f r - g) \frac{\partial \beta}{\partial g}} \]

We follow a decomposition procedure that is similar to the one in 2.4.1.1 in order to check if the inelasticity conditions for the stock-flow ratios hold in Harrodian dual economies. Table 2.3 reports the numerical results based on Lavoie and Godley’s parameter values.

The signs of the derivatives of $g$ with respect to the parameters are the same in the variable $\alpha$ and $\beta$ regime as in the constant $\alpha$ and $\beta$ regime, that is, our ‘inelasticity conditions’ hold in Harrodian dual economies with a Lavoie-Godley specification of consumption and portfolio behavior. However, the absolute values of the derivatives in the case of constant $\alpha$ and $\beta$ are much greater than those in the case of variable
Table 2.3. Harrodian dual economy I

<table>
<thead>
<tr>
<th></th>
<th>Constant $\alpha$ and $\beta$ regime</th>
<th>Variable $\alpha$ and $\beta$ regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>The retention ratio</td>
<td>0.073</td>
<td>0.037</td>
</tr>
<tr>
<td>Equity issues</td>
<td>0.118</td>
<td>0.021</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>-0.790</td>
<td>-0.396</td>
</tr>
<tr>
<td>Utilization</td>
<td>0.595</td>
<td>0.348</td>
</tr>
<tr>
<td>Profit share</td>
<td>0.305</td>
<td>0.154</td>
</tr>
<tr>
<td>Propensity to hold equity</td>
<td>-</td>
<td>-0.011</td>
</tr>
</tbody>
</table>

1. Numbers in the table show the partial derivatives of the growth rate with respect to the parameters listed in the first column.

2. The structure and parameter values of the model are the same as in Lavoie and Godley (2001–2002) including the specification of consumption, but the closure of the model is different.

$\alpha$ and $\beta$. Thus, the adjustment of $\alpha$ and $\beta$ caused by changes in the parameters produce significant and partially offsetting effects on accumulation.

The implications of the alternative specification of the consumption function in Godley-Lavoie (2007) are given in Table 2.4. The inelasticity conditions are satisfied and the effects of the changes in $\alpha$ and $\beta$ are more modest in this case.

2.5 A Kaleckian model

Our Kaleckian model differs from Harrodian models with respect to the specification of accumulation. Unlike in the Harrodian framework, the utilization rate $u$ becomes an accommodating variable, and a shift in aggregate demand may generate a permanent change in utilization.\(^{28}\) The profit share, by contrast, is treated as exogenous, $\pi = \bar{\pi}$, and the labor supply is taken be perfectly elastic (that is, the

\(^{28}\)A steady growth path for the Kaleckian model may have utilization at the normal or desired level, despite the accommodating changes in utilization. This equalization of actual and desired utilization rates can be achieved if the desired utilization rate itself adjusts to the actual rate (Lavoie 1995, Dutt 1997).
Table 2.4. Harrodian dual economy II

<table>
<thead>
<tr>
<th>Variable</th>
<th>Constant α and β regime</th>
<th>Variable α and β regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>The retention ratio</td>
<td>0.101</td>
<td>0.081</td>
</tr>
<tr>
<td>Equity issues</td>
<td>0.117</td>
<td>0.071</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>-0.349</td>
<td>-0.273</td>
</tr>
<tr>
<td>Utilization</td>
<td>0.376</td>
<td>0.329</td>
</tr>
<tr>
<td>Profit share</td>
<td>0.341</td>
<td>0.272</td>
</tr>
<tr>
<td>Propensity to hold equity</td>
<td>-</td>
<td>-0.025</td>
</tr>
</tbody>
</table>

1. Numbers in the table show the partial derivatives of the growth rate with respect to the parameters listed in the first column.

2. The structure and parameter values of the model are the same as in Lavoie and Godley (2001-2002) except the specification of consumption and the closure of the model. The alternative specification of consumption is given by the one in Godley and Lavoie (2007).

model describes the dual-economy case). The Kaleckian model, finally, often imposes ‘stagnationist’ assumptions which ensure that an increase in the profit share will reduce utilization; most of our results for the comparative statics of changes in financial behavior do not depend on these additional assumptions.

2.5.1 Inelastic stock-flow ratios

By using the definition of α and β, Tobin’s q, the debt-capital ratio, and the ratio of retained earnings to capital can be written as:

\[
q = (\alpha + \beta)u \\
m = \beta u \\
c = s_f(\pi - r\beta)u
\]
Thus, for given values of $\bar{\pi}, \alpha$ and $\beta$, the accumulation function (2.4) becomes a function of utilization only:

$$\frac{I}{K} = f(u, \bar{\pi}, r, q, m, c) = f(u, \bar{\pi}, r, (\alpha + \beta)u, \beta u, s_f(\bar{\pi} - r\beta)u) \equiv \phi(u; \alpha, \beta, r, \bar{\pi}, s_f)$$

From (2.8) and the product market equilibrium condition, we now have

$$\phi(u; \alpha, \beta, r, \bar{\pi}, s_f) = \left[ s_f(\bar{\pi} - r\beta) + \beta(\phi(u) + \alpha \dot{N})u \right]$$  \hspace{1cm} (2.25)

We may interpret the terms in the bracket on the right hand side of (2.25) as the average saving rate. Following the Kaleckian tradition, we assume that the traditional Keynesian short-run stability assumption holds in the long run, too, that is, we assume that saving is more responsive than investment to changes in the utilization rate. If the $\alpha$– and $\beta$–ratios are exogenous, we then have

$$s_f(\bar{\pi} - r\beta) + \beta(\phi'(u) + g) + \alpha \dot{N} - \phi' > 0$$  \hspace{1cm} (2.26)

and - assuming positive autonomous investment, $\phi(0; \alpha, \beta, r, \bar{\pi}, s_f) > 0$ - it can be shown that there is a unique positive solution for $u$ in the interval $(0, \frac{1}{\beta})$.

For empirically reasonable magnitudes of the negative effect on capital accumulation of the debt-capital ratio, accumulation is increasing in the utilization rate, i.e. $\phi'(u) > 0$, and we have the following comparative statics for the utilization rate:

\footnote{The mathematical condition for $\phi'(u) > 0$ is $\beta|f_m| < f_u + f_q(\alpha + \beta) + f_s s_f(\bar{\pi} - r\beta)$.}
\[
\frac{\partial u}{\partial \pi} = \frac{(1 - \beta u)f_c + us_f\{(1 - \beta u)f_c - 1\}}{sf(\bar{\pi} - r\beta) + \beta(\phi' u + g) + \alpha\hat{N} - \phi'} < 0 \quad (2.27)
\]

\[
\frac{\partial u}{\partial s_f} = \frac{sf(\bar{\pi} - r\beta) + \beta(\phi' u + g) + \alpha\hat{N} - \phi'}{(\bar{\pi} - r\beta)u\{(1 - \beta u)f_c - 1\}} < 0 \quad (2.28)
\]

\[
\frac{\partial u}{\partial \hat{N}} = -\frac{sf(\bar{\pi} - r\beta) + \beta(\phi' u + g) + \alpha\hat{N} - \phi'}{\alpha u} < 0
\]

\[
\frac{\partial u}{\partial r} = \frac{sf(\bar{\pi} - r\beta) + \beta(\phi' u + g) + \alpha\hat{N} - \phi'}{(1 - \beta u)(f_r - f_c s_f \beta u) + s_f \beta u}
\]

\[
\frac{du}{d\alpha} = \frac{f_q(1 - \beta u)u - \hat{N}u}{sf(\bar{\pi} - r\beta) + \beta(\phi' u + g) + \alpha\hat{N} - \phi'}
\]

\[
\frac{du}{d\beta} = \frac{(sf_r - g)u + (1 - \beta u)(f_q + f_m - f_c s_f r)u}{sf(\bar{\pi} - r\beta) + \beta(\phi' u + g) + \alpha\hat{N} - \phi'}
\]

The stagnationist case is obtained if an increase in the profit share generates a decline in utilization. Comparing (2.27) and (2.28) it is readily seen that in this stagnationist case an increase in the retention rate must depress the rate of utilization: it follows from (2.26) and (2.27) that \((1 - \beta u)f_c < 1\) is a necessary condition for \(\frac{\partial u}{\partial \pi} < 0\). However, the determinate sign of the partial derivative of the utilization rate with respect to the retention ratio can also be justified directly by the empirically mild assumption that \((1 - \beta u)f_c < 1\). Given this assumption, an increase in \(s_f\) lowers the utilization rate since, for a given \(u\), saving rises more sharply than investment, and the utilization rate must decrease in order to restore the product market equilibrium. Analogously - and independently of whether \(\frac{\partial u}{\partial \pi} < 0\) - the average saving rate rises as \(\hat{N}\) increases since more household income goes to purchasing equities rather than buying consumer goods. This depresses the level of effective demand and results in a lower rate of utilization.

The increase in the real interest rate has a negative impact on both saving and investment. It lowers the amount of corporate saving, and the decrease in retained earnings depresses accumulation for a given rate of utilization. Saving falls more

\textsuperscript{30}It is difficult to see how an increase in retained earnings - keeping constant \(u, \pi, r, q, m\) - can lead to a more than one-for-one increase in investment, that is, one would expect \(f_c \leq 1\).
sharply than investment if the direct negative impact on investment of changes in $r$ is not too large, i.e. $(1 - \beta u)(f_r - f_c s f \beta u) + s f \beta u > 0$. Under this assumption, to restore the product market equilibrium, a higher utilization rate is required. However, if $(1 - \beta u)(f_r - f_c s f \beta u) + s f \beta u < 0$, the higher real interest rate requires a lower utilization rate for the product market equilibrium. The effects of changes in $\alpha$ and $\beta$, again, are ambiguous.

The effects on accumulation of changes in the financial variables are given by:

$$\frac{\partial g}{\partial \pi} = f_\pi + s f u f_c + \phi' \frac{\partial u}{\partial \pi}$$

$$\frac{\partial g}{\partial s f} = f_c (\bar{\pi} - r \beta) u + \phi' \frac{\partial u}{\partial s f}$$

$$\frac{\partial g}{\partial \hat{N}} = \phi' \frac{\partial u}{\partial \hat{N}} < 0$$

$$\frac{\partial g}{\partial r} = f_r - f_c s f \beta u + \phi' \frac{\partial u}{\partial r}$$

The result for $\frac{\partial g}{\partial \pi}$ in equation (2.29) is parallel to Marglin and Bhaduri’s (1990) analysis of wage and profit led growth in a stagnationist regime. The direct and positive effect on accumulation of an increase in the profit share may or may not be dominated by the effect of a decline in utilization. A rise in the retention rate - equation (2.30) - also produces conflicting effects on accumulation. The first term in (2.30), $f_c (\bar{\pi} - r \beta) u$, captures a direct positive impact on accumulation from an increase in the amount of internal funds, but an increase in the retention rate also has a negative effect on accumulation by lowering the utilization rate (the second term in (2.30), $\phi' \frac{\partial u}{\partial s f}$, is negative). Which effect dominates is an empirical matter but - using the expressions for $\frac{\partial u}{\partial \pi}$ and $\frac{\partial u}{\partial s f}$ - it follows that in this model $\frac{\partial g}{\partial s f} > 0$ is a sufficient condition for growth to be profit led.\(^ {31}\)

\(^{31}\) We have
The effect on capital accumulation of an increase in the rate of equity issues is more clear-cut. An increase in $\hat{N}$ leads to a lower rate of utilization, and the lower utilization rate depresses capital accumulation.

Real interest rates have ambiguous effects. The direct effect on accumulation of a rise in the real rate of interest is negative but the derived effect on accumulation via changes in the utilization may be positive: $f_r - f_s f_r \beta u$ in (2.32) is negative, but the sign of $\phi' \frac{\partial u}{\partial r}$ in (2.32) can be positive or negative, leaving unclear the sign of the total effect. The ambiguity that characterizes the effects of changes in $\alpha$ and $\beta$ on utilization also carry over to the effects on the growth rate.

Financialization, finally, may have been associated with a downward shift in the accumulation function, $f$ (or $\phi$). A downward shift of this kind leads to a lower utilization rate, and this fall in utilization exacerbates the decline in accumulation.

Strikingly, the comparative static results for a Kaleckian dual economy resemble those for the mature Harrodian economy. A fall in the rate of new equity issues is expansionary in both models. In the Kaleckian model it leads to a higher utilization rate and a higher accumulation rate; in the Harrodian model profits and employment both increase. A decrease in the retention rate, moreover, may (but need not) increase both the utilization rate and the capital accumulation rate in the Kaleckian model and it raises profits and employment in the Harrodian case.

\[
\frac{\partial g}{\partial \pi} = f_\pi + s_f f_e + \phi' \frac{\partial u}{\partial \pi} f_\pi \\
= f_\pi + s_f f_e + \phi' \frac{(1 - \beta u) f_\pi + u s_f (1 - \beta u) f_e - 1}{s_f (\bar{\pi} - r \beta) + \beta (\phi' u + g) + \alpha N - \phi'} \\
= f_\pi + \phi' \frac{1 - \beta u}{s_f (\bar{\pi} - r \beta) + \beta (\phi' u + g) + \alpha N - \phi'} \\
+ \frac{s_f}{\bar{\pi} - r \beta} \frac{(1 - \beta u) f_\pi + s_f (\bar{\pi} - r \beta) u (1 - \beta u) f_e - 1}{s_f (\bar{\pi} - r \beta) + \beta (\phi' u + g) + \alpha N - \phi'} \\
= f_\pi + \phi' \frac{1 - \beta u}{s_f (\bar{\pi} - r \beta) + \beta (\phi' u + g) + \alpha N - \phi'} + \frac{s_f}{\bar{\pi} - r \beta} \frac{\partial g}{\partial s_f}
\]
2.5.2 The Lavoie-Godley specification of consumption and accumulation

In Lavoie and Godley (2001-2002), the accumulation function is given by

\[ g = \gamma_0 + \gamma_1 s_f (\bar{\pi} u - r m) - \gamma_2 r m + \gamma_3 q + \gamma_4 u \]

where \( \gamma_0, \gamma_1, \gamma_2, \gamma_3, \) and \( \gamma_4 \) are positive constants. Using the definitions of \( q, m, \alpha \) and \( \beta \), this accumulation function can be rewritten:

\[ g = \gamma_0 + [\gamma_1 s_f (\bar{\pi} - r \beta) - \gamma_2 r \beta + \gamma_3 (\alpha + \beta) + \gamma_4] u \quad (2.33) \]

If the \( \alpha \) and \( \beta \) ratios are constant, we have a special linear version of our function \( \phi(u) \) in the previous section, and the sensitivity of investment to the utilization rate depends on the various parameters, including \( \alpha \) and \( \beta \). The Lavoie-Godley specification of consumption and portfolio behavior, however, implies that the \( \alpha \) and \( \beta \) ratios are endogenous and that the response of investment to changes in \( u \) will be affected by the endogenous adjustment of the stock-flow ratios \( \alpha \) and \( \beta \).

The consumption function and households’ portfolio choice have been described already in section 2.4.1.2. For convenience we reproduce the key equations (2.22)-(2.23) here:

\[ u - g = u \left[ 1 - s_f (\bar{\pi} - r \beta) - \beta g - \frac{x g}{u} \right] = \psi(y, \gamma) \quad (2.34) \]

\[ \frac{\beta}{\alpha + \beta} = z(r, r^e, y, q) \quad (2.35) \]

where \( \gamma = \alpha u g - x g, y = [1 - s_f (\bar{\pi} - r \beta) + \hat{p}\beta] u, r^e = \frac{(1-s_f)(\bar{\pi} - r \beta)u + \gamma (\alpha u - x)}{\alpha u}, \) and \( q = (\alpha + \beta)u. \) Unlike in section 2.4.1.2, \( g \) and \( u \) are endogenously determined while \( \pi \) is a parameter.

The system (2.33)-(2.35) determines four endogenous variables, \( g, u, \alpha \) and \( \beta \) ((2.34) contains two equations). This system is equivalent to the steady-growth sys-
tem in Lavoie and Godley (2000-2001).\textsuperscript{32} It can be compared to one in which accumulation is described by (2.33), but in which $\alpha$ and $\beta$ are assumed constant (that is, in which we drop (2.35) and the last equation in (2.34)).

**Table 2.5.** Kaleckian dual economy I

<table>
<thead>
<tr>
<th>Regimes</th>
<th>Utilization</th>
<th>Accumulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant $\alpha$ and $\beta$ regime</td>
<td>Variable $\alpha$ and $\beta$ regime</td>
</tr>
<tr>
<td>The retention ratio</td>
<td>-0.162</td>
<td>-0.186</td>
</tr>
<tr>
<td>Equity issues</td>
<td>-0.342</td>
<td>-0.352</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>1.055</td>
<td>0.197</td>
</tr>
<tr>
<td>Profit share</td>
<td>-0.680</td>
<td>-0.780</td>
</tr>
<tr>
<td>Propensity to hold equity</td>
<td>-</td>
<td>0.296</td>
</tr>
</tbody>
</table>

1. Numbers in the table show the partial derivatives of the utilization rate and the growth rate share with respect to the parameters listed in the first column.

2. The structure and parameter values of the model are the same as in Lavoie and Godley (2001-2002) including the specification of consumption.

**Table 2.6.** Effects of changes in financial variables on stock-flow ratios in Kaleckian dual economy I

<table>
<thead>
<tr>
<th></th>
<th>$s_f$</th>
<th>$x$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>1.951</td>
<td>2.040</td>
<td>1.814</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2.426</td>
<td>2.537</td>
<td>2.226</td>
</tr>
</tbody>
</table>

Notes: 0.75($s_f$), 0.05($x$), 0.0275($r$), and the values for the parameters other than $s_f$, $x$, and $r$ are the same as those used in Lavoie and Godley (2001-2002).

Analytical solutions are hard to obtain, but using the original parameter values in Lavoie and Godley (2001-2002) our inelasticity conditions for stock-flow ratios survive in this Kaleckian Lavoie-Godley system. Table 2.5 describes the numerical results.\textsuperscript{32}

\textsuperscript{32}The only small difference between Lavoie and Godley steady-state system and ours lies in the lag structure of variables. In our analysis, we make all level variables in each equation contemporaneous.
Qualitatively, the macroeconomic effects of financialization on the steady state are the same in the fixed $\alpha, \beta$ system and the Lavoie-Godley model. In both models, the effects of an increase in the retention rate are negative for both utilization and accumulation. Thus, given the parameter configuration, the direct positive impact of a rise in $s_f$ on accumulation is dominated by its negative utilization effect on accumulation. A rise in the share of investment financed by new issues has a contractionary effect on both utilization and accumulation. An increase in the real interest rate on the utilization rate has a positive effect on the utilization rate, but this positive utilization effect is offset by the negative effect of the higher interest rate on accumulation: accumulation slows down in the face of the higher real rate of interest.

The similarity between the systems with constant and endogenous $\alpha$ and $\beta$ ratios is not just qualitative. The derivatives of $u$ and $g$ with respect to the various parameters are also similar in magnitude. Thus, the effects on $u$ and $g$ of induced adjustments of $\alpha$ and $\beta$ are quantitatively small.\(^{33}\) This result is not surprising since, as indicated by Table 2.6, the values of the $\alpha$ and $\beta$ ratios appear to be rather insensitive to variations in the financial parameters (the parameter changes in the table are very substantial).

The sensitivity of the qualitative results to variations in parameters of the consumption function is shown in Table 2.7. The effect of changes in the real interest rate could not be signed unambiguously for the case with a constant $\alpha$ and $\beta$, and it is therefore not surprising that the effect of changes in $r$ on utilization may depend on the precise parameters. The effects that could be signed with a constant $\alpha$ and $\beta$ are robust: the direction of the effects is preserved in the variable $\alpha, \beta$ case for all meaningful combinations of the consumption parameters. The violations in the top left corner of Table 2.7 arise when, as a result of low consumption, the model gener-

\(^{33}\)There is one possible exception: the quantitative effect of the real interest rate on utilization differs substantially in the two systems. Our numerical exercises, however, show that the difference tends to decrease if we consider non-marginal, discrete changes in the interest rate.
Table 2.7. Sensitivity analysis in Kaleckian dual economy I

<table>
<thead>
<tr>
<th>$a_2$</th>
<th>$a_1$</th>
<th>0.45</th>
<th>0.50</th>
<th>0.55</th>
<th>0.60</th>
<th>0.65</th>
<th>0.70</th>
<th>0.75</th>
<th>0.80</th>
<th>0.85</th>
<th>0.90</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
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<td>▽</td>
<td>○</td>
<td>○</td>
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<tr>
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<tr>
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<td>▽</td>
<td>○</td>
<td>○</td>
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<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>○</td>
<td>◊</td>
<td>◊</td>
<td>◊</td>
<td>◊</td>
<td>◊</td>
<td>×</td>
</tr>
</tbody>
</table>

1. The table shows that our stock-flow inelasticity conditions hold for a wide range of values of the consumption parameters $a_1$ and $a_2$ in the original Lavoie and Godley model (2001-2002).

2. Cases indicated by “▽”: The conditions for the stock-flow inelasticity hold for the partial derivatives of $u$ and $g$ with respect to changes in $s_f$, $x$ and $r$. $u_{s_f} < 0$, $u_x < 0$, $u_r > 0$, $g_{s_f} < 0$, $g_x < 0$, and $g_r < 0$.

3. Cases indicated by “◊”: The results are the same as cases with “▽” except that $u_r > 0$ in the case of fixed $\alpha$ and $\beta$ but $u_r < 0$ in the case of variable $\alpha$ and $\beta$.

4. Cases indicated by “×”: The results are the same as cases with “▽” except that $u_{s_f} > 0$ and $g_{s_f} > 0$ for both fixed and variable $\alpha$ and $\beta$. However, in these cases, $\pi - r\beta < 0$.

5. Two cases indicated by “×”, no economically significant solution is obtained.

ates an outcome with low utilization, high indebtedness, and an inability of firms to cover the real interest payments on their loans ($\pi - r\beta < 0$).

Turning, finally, to the alternative specification in Godley and Lavoie (2007), a similar picture emerges. Table 2.8 compares the effects of parameter changes using this specification to the constant $\alpha, \beta$ case. Table 2.9 illustrates the sensitivity of $\alpha$ and $\beta$ to variations in $s_f$, $x$ and $r$, and Table 2.10 indicates the sensitivity of the inelasticity conditions to variations in the consumption parameters. All the results are in line with what we observed for the 2001-02 specification.
Table 2.8. Kaleckian dual economy II

<table>
<thead>
<tr>
<th>Regimes</th>
<th>Utilization</th>
<th>Accumulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant α and β regime</td>
<td>Variable α and β regime</td>
</tr>
<tr>
<td>The retention rate</td>
<td>-0.470</td>
<td>-0.487</td>
</tr>
<tr>
<td>Equity issues</td>
<td>-0.806</td>
<td>-0.742</td>
</tr>
<tr>
<td>Real interest rate</td>
<td>0.587</td>
<td>0.154</td>
</tr>
<tr>
<td>Profit share</td>
<td>-1.592</td>
<td>-1.64</td>
</tr>
<tr>
<td>Propensity to hold equity</td>
<td>-</td>
<td>0.391</td>
</tr>
</tbody>
</table>

1. Numbers in the table show the partial derivatives of the utilization rate and the growth rate with respect to the parameters listed in the first column.

2. The structure and parameter values of the model are the same as in Lavoie and Godley (2001-2002) except the specification of consumption. The alternative specification of consumption is given by the one in Godley and Lavoie (2007).

2.6 Conclusion

Financialization is a short-hand expression for a number of developments over the last 30 years. The term is convenient but these developments may not have the coherence and unity suggested by the term and they may not signal the transition to some new ‘regime’.

This chapter is an attempt to show how the macroeconomic effects of some of the observed changes in financial behavior can be analyzed using existing theoretical frameworks. The models in sections 2.4-2.5 differ along three dimensions: (i) the role of labor constraints (mature vs dual economies), (ii) accumulation regimes (Harrodian vs Kaleckian specifications), and (iii) the specification of household behavior (elastic vs inelastic stock-flow ratios). All three dimensions are important when it comes to evaluating the effects of the behavioral changes that have been associated with financialization.

Looking first at the third dimension, the comparative statics in the elastic stock-flow case are reversed compared to the case with inelastic stock-flow ratios. Phrased
Table 2.9. Effects of changes in financial variables on stock-flow ratios in Kaleckian dual economy II

<table>
<thead>
<tr>
<th></th>
<th>$s_f$</th>
<th>0.55</th>
<th>0.75</th>
<th>0.95</th>
<th>$x$</th>
<th>-0.05</th>
<th>0.05</th>
<th>0.15</th>
<th>$r$</th>
<th>0.01</th>
<th>0.0275</th>
<th>0.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td></td>
<td>0.804</td>
<td>0.824</td>
<td>0.830</td>
<td>0.855</td>
<td>0.824</td>
<td>0.777</td>
<td>0.799</td>
<td>0.824</td>
<td>0.861</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td></td>
<td>0.994</td>
<td>1.023</td>
<td>1.035</td>
<td>1.058</td>
<td>1.023</td>
<td>0.967</td>
<td>0.978</td>
<td>1.023</td>
<td>1.089</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: 0.75($s_f$), 0.05($x$), 0.0275($r$), and the values for the parameters other than $s_f$, $x$, and $r$ are the same as those used in Lavoie and Godley (2001-2002). The structure and parameter values of the model are the same as in Lavoie and Godley (2001-2002) except the specification of consumption. The alternative specification of consumption is given by the one in Godley and Lavoie (2007).

in this way, however, the result is not interesting since reversal of the results formed the basis for the definition of elastic stock-flow ratios. More interesting is the finding that all our specifications fall into the category of inelastic stock-flow ratios. We may not be able to conclude from this that all reasonable specifications are inelastic. We have shown, however, that a range of empirically plausible specifications will be stock-flow inelastic; it is striking, in particular, that models like that of Lavoie-Godley (2001-02) which have been built up from flow-flow relations also generate stock-flow ratios that are inelastic.

Assuming inelastic stock-flow ratio, some of the main results for the other two dimensions are summarized in Table 2.11. Consider a change in new issue policies. A decrease in new issues will be expansionary in the mature Harrodian economy as well in the Kaleckian dual economy. Expansionary means different things in the two regimes: the growth rate is exogenously given in the mature economy and expansionary refers to an increase in the rate of employment; in the dual economy, on the other hand, the labor supply is infinitely elastic (and the rate of employment ill-defined), and an expansionary effect is one that raises the growth rate.

34The effects of changes in retention rates are a little less clear in that - essentially for Marglin-Bhaduri reasons - the growth effects are ambiguous in the Kaleckian dual-economy case.
Table 2.10. Sensitivity Analysis in Kaleckian dual economy II

<table>
<thead>
<tr>
<th>$c_2$</th>
<th>$c_1$</th>
<th>0.45</th>
<th>0.55</th>
<th>0.65</th>
<th>0.75</th>
<th>0.85</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.003</td>
<td>▽</td>
<td>▽</td>
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<td>〇</td>
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<td>〇</td>
<td>〇</td>
<td>〇</td>
<td>〇</td>
</tr>
</tbody>
</table>

1. The table below shows that our stock-flow inelasticity conditions hold for a wide range of values of $c_1$ and $c_2$ when we modify the specification of the consumption function in Lavoie and Godley (2001-2002) keeping intact other parts of the model. The alternative specification of consumption is given by the one in Godley and Lavoie (2007).

2. Cases indicated by “〇”: The conditions for the stock-flow inelasticity hold for the partial derivatives of $u$ and $g$ with respect to changes in $s_f$, $x$ and $r$: $u_{s_f} < 0$, $u_x < 0$, $u_r > 0$, $g_{s_f} < 0$, $g_{x} < 0$, and $g_{r} < 0$.

3. Cases indicated by “◊”: The results are the same as cases with “〇” except that $u_r > 0$ in the case of fixed $\alpha$ and $\beta$ but $u_r < 0$ in the case of variable $\alpha$ and $\beta$.

4. Cases indicated by “▽”: The results are the same as cases with “〇” except that $u_{s_f} > 0$ and $g_{s_f} > 0$ for both fixed and variable $\alpha$ and $\beta$. However, in these cases, $\pi - r_\beta < 0$.

The Harrodian dual economy produces the opposite result: a decrease in new issues reduces the growth rate.\(^{35}\) Intuitively, the growth rate (along the steady growth path) is constrained by saving in the Harrodian dual economy, and a decrease in new issues reduces saving and thereby the growth rate. This argument is a straightforward generalization of what happens in the textbook version of Harrod’s model. In a mature

\(^{35}\)The Harrodian dual economy could be split into two cases, depending on the sensitivity of the growth function with respect to changes in the profit share. We focus on the high-sensitivity case, cf. section 2.4.1.2.
Table 2.11. The effects of a decrease in the retention ratio or the rate of net issues of equities in different regimes

<table>
<thead>
<tr>
<th></th>
<th>Mature Economies</th>
<th>Dual Economies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harrodian</td>
<td>Profit share: Increase</td>
<td>Growth: Decrease</td>
</tr>
<tr>
<td></td>
<td>Employment: Increase</td>
<td></td>
</tr>
<tr>
<td>Kaleckian</td>
<td></td>
<td>Utilization: Increase</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Growth: Ambiguous when $s_f \downarrow$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Growth: Increase when $\dot{N} \downarrow$</td>
</tr>
</tbody>
</table>

economy, by contrast, the growth rate of output and the rate of accumulation will adjust to the natural rate. A decrease in new issues tends to reduce saving, and an increase in profits is needed to compensate for this reduction and maintain the rate of accumulation at the natural rate. An increase in profitability, in turn, must be offset by a rise in employment in order to keep the growth rate of output at the natural rate. Basically, moving from a mature to an dual-economy setting turns an expansionary change into a contractionary change.

Moving from a Harrodian to a Kaleckian economy also tends to reverse the comparative statics. This, again, generalizes results that are well-known from comparisons of the textbook Harrod model with standard stagnationist formulations (e.g. Rowthorn (1981) and Dutt (1984)). The only difference is that here we have expanded the models to include financial factors that are usually left out.

These comparisons between mature and dual-economy versions of the Harrodian model and between Harrodian and Kaleckian versions of the dual economy provide some intuition for the similarity between the mature Harrodian economy and the Kaleckian dual economy: these latter economies differ in two dimensions and the two reversals of the comparative statics offset each other.

Of course, the dependence of the comparative statics on the specification of the model is not surprising. One contribution in this study, however, is to clarify the conditions under which the different results obtain. Moreover, most studies of ad-
vanced capitalist economies by heterodox economists seem to be informed by either a Harrodian mature-economy perspective or by the Kaleckian dual-economy framework (our own preference lies with the former, but the majority view probably favors the latter). The two perspectives are quite different, but our results in this paper show that when it comes to an evaluation of the effects of the changes in financial behavior over the last 30 years, the qualitative conclusions are rather similar. A downward shift of the accumulation function will - not surprisingly - be contractionary in both frameworks, but contrary to the fears among some heterodox economists, key developments associated with the process of financialization have expansionary effects: decreases in retained earnings, a decline in new issues of equity and increased reliance on external finance tend to be expansionary in both frameworks.

Financialization involves broader issues that go beyond the questions discussed in this chapter. One set of issues concerns international capital flows and the constraints implied by these flows on the policy options of nation states. Leaving aside the international dimension, issues of power provide an another example. It is often claimed that financialization is associated with the increased power of financial institutions. Auerbach (1988), however, presents the case for an alternative view:

The present relationships between banks and firms, far from signalling the growing dominance of financial institutions represent a precisely contrary development. They result from the efforts of financial institutions to accommodate themselves to a far more insecure environment, one made insecure by the activities of financial institutions in competition with each other and by the ever more stringent demands made upon them by their clients, especially their business customers. (p. 198)

Disregarding power issues, an increase in competition and insecurity may have implications for financial stability as well as for the time horizons used by both firms and financial institutions. A relatively recent but now largely forgotten literature questioned the relative merits of competitive, market-based Anglo-Saxon financial systems compared to German-Japanese systems. The latter, it was argued, might help to al-
leviate a short-termist bias (e.g. Cosh et al 1990). More generally, a competitive financial system would not necessarily - even if it were fully 'efficient' - produce good macroeconomic results if the investment in physical and/or human capital gives rise to significant externalities (as suggested by traditional development theory, post Keynesians like Kaldor, and recent endogenous growth theory). In the case of positive externalities, ‘artificially low’ interest rates may be desirable (Auerbach and Skott 1992).36

One may note, finally, that concerns over the excesses and questionable benefits of the financial system have been voiced before and that even the extent of resources that are put into the financial system may cause concern. Thus, Tobin (1984; reprinted 1987) confessed

> to an uneasy Physiocratic suspicion, perhaps unbecoming in an academic, that we are throwing more and more of our resources, including the cream of our youth, into financial activities remote from the production of goods and services, into activities that generate high private rewards disproportionately to their social productivity. (1987, p. 294)

Tobin’s conclusion was motivated in part by the fact that 16 out of an elite group of 46 executives whose earnings exceeded one million dollars in 1983 were officers of financial companies. He also noted that graduates from the School of Organization and Management at Yale who took jobs in finance had starting salaries four times the poverty threshold for four-person families, and observed that the average holding period for shares was only 19 months and that the Department of Finance categories of Finance and Insurance generate 4.5-5 per cent of GNP (1987, p. 282).

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36The relatively strong German and Japanese economic performance during the Golden Age could be explained, of course, by other factors, unrelated to the financial systems. Likewise, the relatively poor performance by the two economies in the more recent years may not reflect a need for reforms of the financial and/or labor market systems, as claimed by OECD and other international organisations. See Nakatani and Skott (2007) for discussion of the Japanese case.
numbers seem almost quaint by today’s standards, and developments over the last 20 years can only reinforce one’s Physiocratic suspicions.

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37In 2005, among CEO’s in the top 189 efficient firms classified by Forbes, 164 earned more than $2 million (or approximately $1 million in 1983 dollars) and 46 of them belonged to financial companies (diversified financials, banking, and insurance). The average compensation of those 46 CEO’s in financial companies was $9.6 million or about 170 times the median U.S. family income in 2004 (see The State of Working America 2006/2007 published by Economic Policy Institute). In 2004, the average holding period for shares had dropped to 12.1 months (NYSE Historical Statistics, http://www.nysedata.com). Finance and Insurance, as categorized by the Department of Commerce, accounted for 5.5% of employee compensation, about 5% of the employed labor force, 7.5% of after-tax corporate profits, and about 3% of personal consumption in 1983; in 2005 those corresponding figures were 7.6%, 4.3%, 11.1% and 5.9% respectively (calculated from U.S. Department of Commerce, Bureau of Economic Analysis, National Income and Product Account).
CHAPTER 3
FINANCE, SECTORAL STRUCTURE AND THE BIG PUSH

3.1 Introduction

Complementarities among different investment projects may cause coordination failures where no investment project can break even if it is implemented alone, while simultaneous implementation of the investment projects (big push) makes individual projects profitable. The idea of big push dates back to Baran (1957), Myrdal (1957), Nurkse (1958), and Rosenstein-Rodan (1943). Due to the presence of pervasive complementarities, uncoordinated investment decisions may lead to an underdevelopment trap where no investment project is actually made and, in order to avoid this underdevelopment trap, coordinated implementation of investment projects may be needed.

Murphy et al. (1989) formulates this big push idea. Several contributions have since been made in the big push literature (i.e. see, among others, Ciccone and Matsuyama, 1996; Rodriguez-Clare, 1996; Rodrick, 1996; Skott and Ros, 1997). While existing studies have focused on various mechanisms that generate big push results, less attention has been paid to the role of the financial sector in addressing the problem of coordination failure generated by investment complementarities.¹ Da Rin and

¹There have been a number of empirical studies in the literature regarding finance and development. See Demirguc-Kunt (2006) and Levine (2003) for survey. One important topic in this literature is whether financial development explains countries’ growth performance. Many studies along this line accept the view that “the role of financial markets and institutions arise to mitigate the effects of information and transaction costs that prevent direct pooling and investment of society’s savings” (Demirguc-Kunt, 2006, p.1) From this perspective, good financial markets are identified as “deep” markets which mean more liquid stock markets and larger banking sectors. The role of financial institutions in coordinating investment projects in the presence of investment
Hellman (2002) is an important exception. They investigate the implications of introducing banks with market power into the Murphy-Schleifer-Vishny type model, and show that private banks can act as catalysts for industrialization and solve the coordination failure if the banks have sufficient monopoly power to secure profits from costly coordination and are sufficiently large to mobilize a critical mass of firms. Thus, Da Rin and Hellman suggest that banks’ profit maximizing behavior is consistent with mobilizing the critical mass of complementary investments.

It is difficult to deny the argument that the existence of banks with sufficient size and market power can solve coordination failures caused by investment complementarities. In principle, a complete monopolist can internalize gains generated by complementary investment projects and inefficiency caused by externality can be eliminated by the monopolist. An agent with a ‘sufficient’ degree of market power may be able to mimic the solution by the monopolist. However, more important questions are how the sufficient level of market power that can solve a coordination failure is determined, and what kinds of factors affect the determination of the critical level of market power.\(^2\) This chapter focuses on the effect of sectoral structure on the determination of the critical level of market power. In the Da Rin and Hellman model, the decision-making by the bank with market power is on whether to finance firms investing in the modern sector characterized by investment complementarities.

\(^2\)There is some empirical support for a positive relationship between bank concentration and the growth of particular types of firms. For example, Petersen and Rajan (1995) argues that younger firms have easier access to credit if banks have market power because banks with market power expect themselves to be able to extract future profits with those younger firms. This argument is based on the assumption that that new entrants in an industry are characterized by more innovative technologies, which would confer banks with market power increasing opportunities of sharing profits with the firms. Their argument is based on the existence of relationship banking. Cetorelli and Gambira (2001) suggest that higher bank concentration is conducive to the growth of industries where young firms heavily rely on external finance. However, in general, the empirical findings regarding the relationship between concentration and the growth of firms are mixed. More importantly, the implications of investment complementarities for the role of banks with market power have received little attention in these studies.
or invest at a risk-free interest rate. In our model, we explicitly introduce the traditional sector in which investment projects are strategically substitutable. The bank with market power considers financing investment projects in the traditional sector as well as those in the modern sector, and its profit maximization determines optimal credit allocation over the sectors. The characteristics of traditional and modern sectors interact with the bank’s decision on loan allocation. In particular, I explore a possibility that the existence of strong traditional sectors is an obstacle to private coordination by the bank with market power. The bank with market power may fail to provide a solution to the problem of coordination failure if they find it more profitable to finance firms in the traditional sector than in the modern sector. This will be the case if the economy has relatively strong traditional sectors which provide ample profit opportunities for banks with market power.

The role of market power in promoting or discouraging the industrialization process has been analyzed by de Fontenay (2004) but with a focus on industrial firms. de Fontenay suggests that firms with market power have a dual role in industrialization process: firms with market power may encourage investment in complementary industries but other firms may be discouraged from investing by the risk of hold-up by the firm with market power. de Fontenay maintains that whether the positive or negative impact will be dominant depends on the market structure and institutions by which firms with market power are organized. The analysis in de Fontenay (2004) makes a point similar to ours: the introduction of market power into big push models may not provide a solution to coordination failures and the role of market power depends on the underlying structural aspects of an economy.

The rest of this chapter is structured as follows. In section 3.2 we develop a model which pays explicit attention to sectoral structure. Section 3.3 discusses the intersectoral and intrasectoral relations that characterize the real sector of an economy. Section 3.4 studies multiple equilibria under a competitive banking system. Section
3.5 investigates the conditions under which banks with market power induce or fail to
induce industrialized equilibrium. In section 6, we discuss some implications of our
results.

3.2 Model

We consider an economy in which a final good is produced by two intermediate
good sectors. The model shares some common features found in big push models
(incl. Ciccone and Matsuyama, 1996; Murphy et al., 1989; Rodriguez-Clare, 1996).
Banking structure and firms’ belief structure are introduced along the line of Da Rin
and Hellman (2002).

3.2.1 Production processes

The economy has a final good Z which is produced by two goods X and Y under
a perfectly competitive condition. The production process is characterized by the
following CES production function:

\[
Z = \left[ \alpha X^{1-\frac{1}{\epsilon}} + \beta Y^{1-\frac{1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \quad (\epsilon > 1) \tag{3.1}
\]

where \( \epsilon \) is the elasticity of substitution of X and Y. The price of Z, \( P_Z \), is normalized
at one.

Good X is produced by a perfectly competitive firm using a finite number of
intermediate goods \( x(i) \)'s indexed by \( i = 1, 2, ..., n \). The intermediate goods, \( x(i) \)'s,
are produced by monopolistically competitive firms, and the elasticity of substitution
among \( x(i) \)'s in producing X is denoted as \( \sigma \). Thus, the production technology for
good X is represented by

\[
X = \left[ \sum_{i=1}^{n} x(i)^{1-\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (\sigma > 1) \tag{3.2}
\]
We will call the set of monopolistically competitive firms producing $x(i)$’s “sector I”. $n$ is the number of firms which actually invest in sector I. In our model, $n$ is endogenously determined by firms’ entry decisions.

Similarly, good $Y$ is produced by perfectly competitive firms each of which uses as inputs a variety of intermediate goods $y(j)$’s indexed by $j = 1, 2, ...$. Formally,

$$Y = \left[ \sum_{j=1}^{m} y(j)^{1-\frac{1}{\tau}} \right]^\frac{1}{\tau-1}, \ (\tau > 1) \quad (3.3)$$

The set of monopolistically competitive firms producing $y(i)$’s will be called “sector II”. $m$ is the number of firms which actually invest in sector II.

Given that there are two sectors I and II, one may want to call $\epsilon$ the inter-sectoral elasticity of substitution and $\sigma$ and $\tau$ the intra-sectoral elasticity of substitution. As we will see, the relative magnitude of $\epsilon$, $\sigma$ and $\tau$ will produce several sectoral structures (The meaning of a ‘sectoral structure’ used in this paper becomes clear in section 3.3).

Turning to the intermediate goods $x(i)$ and $y(j)$, production of $x(i)$ units of each variety in sector I requires $a \cdot x(i)$ units of labor and a start-up cost $K_I$. Both $a$ and $K_I$ are assumed to be exogenously given. Similarly, production of $y(j)$ units of each variety requires $b \cdot y(j)$ units of labor and a start-up cost $K_{II}$ where $b$ and $K_{II}$ are constant. Since the effect of the difference between $K_I$ and $K_{II}$ is not our primary focus, we will assume that $K_I = K_{II} \equiv K$. The presence of the start-up cost implies that there is an increasing returns to scale.

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3The Dixit-Stiglitz model of monopolistic competition (Dixit and Stiglitz (1977) and Ethier (1982)) is “grossly unrealistic” (Fujita, Krugman, and Venables, 2000). For our purposes, however, it provides a tractable and useful way of representing coordination problems in the entry decisions.

4A general discussion on the implications of different fixed cost assumptions for industrialization theories is found in Gans (1997).
The crucial assumption in this model is that the start-up cost must be financed by borrowing from banks. Since bank financing of firms in intermediate good sectors is essential in this model, banks’ decisions to extend loans and set interest rates play an important role in initiating production processes in the intermediate good sectors which in turn affects equilibrium of the model.

Each firm $i$ is charged the interest rate $r_i$ and each firm $j$ is charged $r_j$. Banks set these interest rates to maximize their profit. Let us denote the price of $x(i)$, the price of $y(j)$, and the wage rate as $p_x(i)$, $p_y(j)$, and $w$, respectively. Profits earned by firm $i$ in sector I and firm $j$ in sector II then can be written as:

$$\pi_I(i) = p_x(i) \cdot x(i) - w \cdot a \cdot x(i) - (1 + r_i)K$$

$$\pi_{II}(j) = p_y(j) \cdot y(j) - w \cdot b \cdot y(j) - (1 + r_j)K$$

Thus, each firm’s profit in intermediate good sectors depends on the price of the intermediate good it produces, the demand for the good, the wage rate, the level of fixed cost and the interest rate charged by banks.

### 3.2.2 Banking sector

There are two types of banks in the model. On the one hand, there are a number of competitive fringe banks each of which can finance up to $K$; in other words, each fringe bank cannot finance more than one firm. Financing $K$ incurs some costs of $\rho_cK$ to each fringe bank. On the other hand, there is a big bank — a bank with market power — which can finance up to $A$ firms where $A > 1$. The big bank’s cost of financing $K$ is given $\rho_mK$ where $\rho_m < \rho_c$. This implies that the big bank has a cost advantage over competitive fringe banks. Da Rin and Hellman (2002) sees this cost advantage as coming from the big bank’s “superior skills, a lower cost of capital, or some regulatory advantages.” (p.376)
Each fringe bank will offer a firm a particular rate of interest. The big bank will offer \( A \) firms interest rates. Firms that receive offers from banks will decide whether to accept the offer. Whether a firm \( i \) or \( j \) accepts a particular loan offer depends on the profitability of its investment project: profits given by (3.4) and (3.5) must be nonnegative in order for firms to accept the offers.

It is assumed that Bertrand competition prevails among fringe banks. Thus, fringe banks’ behavior is characterized by marginal cost pricing \((r = \rho_c)\) and in equilibrium each fringe bank will make zero profit. In contrast, the big bank has market power and sets its interest rates. There are two constraints on the exercise of this market power. On the one hand, because there are fringe banks, interest rates cannot exceed \( \rho_c \). We may call this the Bertrand constraint. On the other hand, the lending capacity of the big bank is fixed: it can finance only up to \( A \) firms. We may call this the capacity constraint. If the big bank finances \( n \) firms in sector I and \( m \) firms in sector II, then the capacity constraint is given by \( n + m \leq A \). Each of two constraints may or may not be binding in equilibrium.

Due to its market power and its capability to make positive profit, the big bank has a strategic position which may affect the model’s outcome. In the later part of this paper, we look for some conditions under which the model generates multiple equilibria: one equilibrium is characterized by the full development of sector I and the other by no development of sector I due to a coordination failure. Under those conditions, firms in sector I become strategically complementary (defined in 3.3.2) and the entry decisions of firms into sector I suffer from a coordination problem. A crucial question is under what circumstances the market power of the big bank can solve this coordination problem. The answer, it turns out, depends critically on the sectoral structure of an economy as well as the degree of market power.
3.2.3 Model closure

Full employment is assumed. Given a fixed size of labor force, $L$, the labor market equilibrium condition is

$$\sum_{i=1}^{n} a \cdot x(i) + \sum_{j=1}^{m} b \cdot y(j) = L$$ (3.6)

For expositional convenience, we normalize the units of intermediate goods so that

$$\left(\frac{\sigma}{\sigma-1}\right) a = 1$$ and \( \left(\frac{\tau}{\tau-1}\right) b = 1.5 $$

3.3 Sectoral structure

3.3.1 Profit functions

Banks offer interest rates to firms and firms which receive banks’ offers decide whether to accept those offers by considering their profits net of interest payments. A firm will accept an offer only if its investment project is expected to be profitable.

Given the interest rates and firms’ expectations of $m$ and $n$, we can derive profit functions for individual producers in sector I and sector II. The determination of those interest rates itself will be discussed later.

**Proposition 2** In the model described in section 3.2, intermediate good producers have the following profit functions for given $m$, $n$ and interest rates:

\[
\pi_I(n, m, r_i) = \frac{\alpha^\epsilon \left[\alpha^\epsilon n^{\frac{1}{\sigma-1}} + \beta^\epsilon m^{\frac{1}{\tau-1}}\right]^\frac{1}{\epsilon}}{a\alpha^\epsilon n^{\frac{1}{\sigma-1}} + b\beta^\epsilon m^{\frac{1}{\tau-1}}} (1 - a) L - (1 + r_i) K \quad (3.7)
\]

\[
\pi_{II}(n, m, r_j) = \frac{\beta^\epsilon \left[\alpha^\epsilon n^{\frac{1}{\sigma-1}} + \beta^\epsilon m^{\frac{1}{\tau-1}}\right]^\frac{1}{\epsilon}}{a\alpha^\epsilon n^{\frac{1}{\sigma-1}} + b\beta^\epsilon m^{\frac{1}{\tau-1}}} (1 - b) L - (1 + r_j) K \quad (3.8)
\]

**Proof.** See Appendix F. ■

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5This implies that $b > a$ ($b < a$) whenever $\tau > \sigma$ ($\tau < \sigma$).
The relationship between individual firm’s profit and other firms’ investment decisions is determined by intersectoral (\(\epsilon\)) and intrasectoral (\(\sigma\) and \(\tau\)) elasticity of substitution. In the following section, we classify different cases of the profit functions according to the interfirm and intersectoral relations.

3.3.2 Strategic complementarity and substitutability

In order to characterize the properties of profit functions of intermediate good producers, we will use the following terminology:

- Firms in a sector are strategically complementary if each firm’s profit is increasing in the number of firms investing in that sector.

- Firms in a sector are strategically substitutable if each firm’s profit is decreasing in the number of firms investing in that sector.

- Sector X is strategically complementary to Sector Y if each firm’s profit in Sector Y is increasing in the number of firms investing in Sector X.

- Sector X is strategically substitutable to Sector Y if each firm’s profit in Sector Y is decreasing in the number of firms investing in Sector X.

Figure 3.1. Profit Functions of Intermediate Good Producers
Note that the first two statements define *intrasectoral* relations within a sector while the last two statements define *intersectoral* relations. For instance, firms in sector I are strategically complementary (substitutable) if \( \frac{\partial \pi_I}{\partial n} > 0 \) (\( \frac{\partial \pi_I}{\partial n} < 0 \)).

We define a sectoral structure as a particular combination of intersectoral and intrasectoral relations. Several sectoral structures can obtain depending on parameters, \( \epsilon, \sigma \) and \( \tau \). Below we focus on a particular case in which multiple equilibria naturally emerge: firms in sector I are strategically complementary among themselves (\( \frac{\partial \pi_I}{\partial n} > 0 \) for all \( n \) and \( m \)), firms in sector II are strategically substitutable (\( \frac{\partial \pi_{II}}{\partial m} < 0 \) for all \( n \) and \( m \)), and sector I and II are strategically substitutable to each other (\( \frac{\partial \pi_{II}}{\partial m} < 0 \) and \( \frac{\partial \pi_I}{\partial n} < 0 \) for all \( n \) and \( m \)). This case is illustrated by Figure 3.1 (a) and (b). In Figure 3.1(a), firm’s profit in sector I is increasing in \( n \) and the function shifts down as \( m \) increases. In Figure 3.1(b), firm’s profit in sector II is decreasing in \( m \) and the function shifts down as \( n \) increases. Proposition 3\(^7\) specifies the restrictions on the parameter values required to obtain this type of sectoral structure.

**Proposition 3** Given the profit functions (3.7) and (3.8), for any \( n \) and \( m \),

(i) Firms in sector I are strategically complementarity (\( \frac{\partial \pi_I}{\partial n} > 0 \)) if and only if \( \sigma < 2 \) and \( \sigma < \epsilon \).

(ii) Firms in sector II are strategically substitutable (\( \frac{\partial \pi_{II}}{\partial m} < 0 \)) if and only if \( \tau > 2 \) and \( \tau > \epsilon \).

(iii) Sector I and II are strategically substitutable for each other (\( \frac{\partial \pi_{II}}{\partial m} < 0 \) and \( \frac{\partial \pi_{II}}{\partial n} < 0 \)) if \( \epsilon > \frac{b}{a} + 1 \).

---

6 These inequalities may hold only locally: the signs of relevant partial derivatives depends on the level of \( n \) and \( m \). If there is no dependency of the signs of those partials on \( n \) and \( m \), strategic complementarity or substitutability may be said to hold globally.

7 This proposition is obtained by examining the partial derivatives of (3.7) and (3.8) with respect to \( n \) and \( m \) but the procedure is rather tedious.
Proposition 3 (i) tells us that a sufficiently low intra-sectoral elasticity of substitution in sector I ($\sigma$) compared to the inter-sectoral elasticity $\epsilon$ makes the profit level of intermediate good producers in sector I increase in $n$. When $\sigma < 2$ and $\sigma < \epsilon$, firms’ investment projects in sector I become global strategic complements.\(^8\) For expositional convenience, we will call sector I, the sector characterized as strategic complementarities, ‘the modern sector.’

Proposition 3 (ii) shows that a sufficiently high elasticity of substitution in sector II makes profits made by intermediate good producers in sector II decrease in $m$. We will call sector II ‘traditional sector.’ It is worth noting that our ‘traditional’ sector is ‘traditional’ not because the sector has decreasing or constant returns to scale technology rather than increasing returns but because it is characterized by strategically substitutable firms. In fact, in our model, firms in our ‘traditional sector’ (sector II) have increasing returns to scale technology due to the existence of fixed start-up cost (See equation (3.5)).\(^9\)

Proposition 3 (iii) gives us a condition under which the modern sector (sector I) and the traditional sector (sector II) are strategic substitutes for each other, i.e. individual firm’s profit function in one sector is decreasing in the number of firms

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\(^8\) $\sigma < 2$ is not necessary for local complementarity but for global complementarity. While the assumption for global complementarity ($\sigma < 2$) is not essential for our analysis, it greatly simplifies our analysis without losing any insight. If we have only $\sigma < \epsilon$ not $\sigma < 2$, the profit function (3.7) is initially increasing in $n$ but beyond some point decreasing in $n$ for any given $m$. In this case, we cannot have an outcome where $n$ is increasing without bound. However, it is still possible to obtain two equilibria in which the values of $n$ are finite. In contrast, $\sigma < \epsilon$ is essential to obtain local complementarity. Thus, even when goods in sector I are highly substitutable (a high value of $\sigma$ which exceeds 2), investment projects in sector I can be strategically complementary for some ranges of $n$ and $m$ as long as $\sigma < \epsilon$. This aspect is closely related to the notion of Hicks-Allen complements. See Matsuyama (1995).

\(^9\) The characteristics of individual producers’ technology are not necessarily seen as the ultimate determinant of making an economy ‘traditional’ or ‘less developed.’ Even old large plantation methods of production may exhibit increasing returns to scale for a certain range of production scale. For instance, there is no unanimous consensus regarding the degree of scale economies of large-scale slave plantations in the antebellum southern economy in the U.S. See Ransom and Sutch (2001, pp.73-78).
actually investing in the other sector. This requires a sufficiently large intersectoral
elasticity of substitution ($\epsilon > \frac{b}{a} + 1$).

Due to the parameter restrictions in Proposition 3, the economy has potential
to exhibit multiple equilibria. The following two sections provide an analysis of the
effects of banking structure on the determination of equilibrium.

### 3.4 Multiple equilibria in a competitive banking system

Let us assume that there is no bank with market power and all banks, which can
finance at most one firm, are subject to Bertrand competition. Then banks will adopt
marginal cost pricing, i.e. $r_i = r_j = \rho_c$ for all $i$ and $j$. The solution to the model
without bank with market power is simply the same as the solution to the model
where each intermediate good producer faces $(1 + \rho_c)K$ of fixed start-up cost with
interest payments.

Consider firms’ entry decisions in sector I and II. Firms’ expected profits are
determined by their expectations on the number of firms investing in sectors and
banks’ interest rate policies. Firms are assumed to share a common belief. We will
focus on two particular types of beliefs, the optimistic and the pessimistic belief
following Da Rin and Hellman (2002). The optimistic belief is defined as the one in
which firms believe that the largest $n$ in the equilibrium set and the corresponding
value of $m$ will be realized. The pessimistic belief is defined as the one in which firms
believe that the smallest $n$ in the equilibrium set and the corresponding value of $m$
will be realized.

Whenever a firm’s investment project is expected to generate nonnegative profit,
it will decide to invest. The following assumptions will be imposed to make our
problems non-trivial.
\[
\bar{m} \equiv \left[ \frac{\beta^{\frac{1}{\tau}} L}{(\tau - 1)(1 + \rho_c)K} \right]^{\frac{1}{\tau - 1}} \geq 1 \tag{3.9}
\]
\[
\bar{n} \equiv \left[ \frac{(\sigma - 1)(1 + \rho_c)K}{\alpha^{\frac{1}{\tau}} L} \right]^{\frac{1}{\tau - 2}} > 1 \tag{3.10}
\]

It can be shown that the first inequality ensures the positiveness of the equilibrium \(m\) when pessimistic beliefs prevail. Without the second inequality, \(n\) always diverges to \(\infty\) and \(m\) converges to 0 and, therefore, the case with multiple equilibria is eliminated. As long as \(\bar{n} > 1\), a single investment alone in sector I will not be profitable for any nonnegative \(m\). Industrialization requires mobilizing a minimum number \(\bar{n}\) of investment projects in sector I.

Suppose that every firm believes that the modern sector (sector I) will expand without bound. This implies that they believe that the traditional sector (sector II) will eventually vanish because due to the strategic substitutability between those two sectors the indefinite expansion of the modern sector eventually makes any investment project in the traditional sector unprofitable. This belief system is self-fulfilling. Because for any \(n > \bar{n}\) and \(m = 0\), which is implied by the optimistic belief, any individual investment project in sector I is profitable and \(n\) will grow without bound. At a sufficiently large value of \(n\), any investment project in sector II becomes unprofitable given that every firm in sector II believes nobody else will make in the sector. Thus, firms’ optimistic belief leads to industrialization equilibrium where the modern sector will be fully developed.

Now assume that every firm believes the modern sector will never develop \((n = 0)\). This belief implies that they must believe that \(\bar{m}\) number of firms will enter the traditional sector in equilibrium. This belief system again generates a self-fulfilling equilibrium. The modern sector will never develop under this pessimistic belief.

We have shown that the model generates multiple equilibria in which one equilibrium represents industrialization with a fully developed modern sector and the other
an underdevelopment trap with no development of the modern sector. Note that the introduction of banks without market power does not change the basic feature of the model. Actual equilibrium depends on the belief structure only. In section 3.5, we will examine the implication of introducing a bank with market power into our model.

3.5 Market power and industrialization

We consider decision-making by a bank with market power and its effect on the outcome of the model.

It is easy to see that under the optimistic beliefs, the existence of the big bank will make little difference. To see this, let us assume that every firm which considers investing in the modern sector believes that the modern sector will expand without bound and the traditional sector will eventually vanish. Then for any finite interest rate every firm will invest in the modern sector because the belief that \( n \to \infty \) and \( m \to 0 \) implies that the expected profit made by a firm in sector I eventually becomes positive due to intersectoral and intrasectoral relations specified in the model (see Figure 3.1 (a) and equation (3.7)). In addition, no firm will invest in the traditional sector because the profit in the sector becomes negative as \( n \to \infty \) and \( m \to 0 \) for a finite interest rate (see Figure 3.1 (b) and equation (3.8)). The big bank’s profit maximization implies that both the capacity constraint and the Bertrand constraint must be binding. Otherwise, the big bank could increase its profit by slightly increasing interest rates or the amount of loans. Thus, in equilibrium, the big bank will end up with financing \( A \) firms in the modern sector at \( \rho_c \), the maximum permissible rate under the Bertrand constraint. Other firms will be financed by fringe firms at the same rate of interest. The only difference between this case and the case without the big bank is that the big bank will make positive profits, \((\rho_c - \rho_m)KA\).
The rest of this section will focus on the case of pessimistic beliefs. The main purpose is to clarify the relation between the sectoral structure and the minimum degree of market power required to solve coordination failures.

### 3.5.1 Bank decision problem

In this section, we are interested in the relationship between the size of the bank’s lending capacity \( A \) (in short, the bank size below) and the sectoral structure of an economy. For the rest of the paper, we replace the profit functions (3.7) and (3.8) of intermediate good producers by their linear versions (3.11) and (3.12) below where all parameters are positive constants.\(^{10}\)

\[
\pi_I(n, m, r_i) = \phi_0 + \phi_1 n - \phi_2 m - (1 + r_i)K
\]

\[
\pi_{II}(n, m, r_j) = \gamma_0 - \gamma_1 n - \gamma_2 m - (1 + r_j)K
\]

The discussion on the sectoral structure in section 3.3 allows us to give natural interpretations to parameters in (3.11) and (3.12). Following our assumptions in section 3.3, firms in the modern sector (sector I) are strategically complementary \( (\frac{\partial \pi_I}{\partial n} = \phi_1 > 0) \), firms in the traditional sector (sector II) are strategically substitutable \( (\frac{\partial \pi_{II}}{\partial m} = -\gamma_2 < 0) \).

The cross derivatives of (3.11) and (3.12) which describe intersectoral relations are both negative — sector I and II are strategically substitutable to each other — and their magnitudes are given by \( \phi_2 \) and \( \gamma_1 \). Constant terms in (3.11) and (3.12) \( (\phi_0 \) and \( \gamma_0 \)) are the shifting parameters which determine the ‘height’ of the profit functions for any given \( n \) and \( m \). The levels of \( \phi_0 \) and \( \gamma_0 \) may reflect government subsidy policies, distribution of foreign aids over sectors, and the degree of abundance of sector-specific resources or technology.

\(^{10}\)By this linear specification, we do not lose main qualitative features of the model in section 3.2 and 3.3.
The bank with market power has two alternatives regarding optimal loan allocation over sectors. In the first alternative, the bank decides to induce an industrialized equilibrium. In order to do this, the bank has to find a way of mobilizing the firms in the critical mass of industrialization. If it succeeds in mobilizing the critical mass, the modern sector would fully develop on its own while the traditional sector would vanish. We expect there are some costs involved in mobilizing the critical mass and solving coordination failures. These costs will be reflected in the bank’s profit calculation.

In the second alternative, the bank decides not to mobilize the critical mass of industrialization. In this case, the bank will extend loans to firms in the traditional sector and may extend loans to a limited number of firms in the modern sector.\footnote{The number of firms in the modern sector to which the bank would grant loans must be smaller than that of firms in the critical mass because otherwise the modern sector would fully develop on its own which contradicts the scenario in this second alternative.}

In each alternative, the bank decides on interest rates and allocate loans so as to maximize its profits and, by comparing the profits from each alternative, the bank will decide whether to induce an industrial equilibrium or an underdevelopment trap. Let us analyze two cases one by one.

\subsection*{3.5.2 Profits from inducing industrialization}

The critical mass of industrialization $n^*_c$ is defined as the value of $n$ which satisfies
\[
\pi_1(n, m^*, \rho_c) = \phi_0 + \phi_1 n - \phi_2 m^* - (1 + \rho_c)K = 0,
\]
where $m^*$ is firms’ common belief on the number of firms investing in sector II. We then have:
\[
n^*_c = \frac{\phi_2 m^* + (1 + \rho_c)K - \phi_0}{\phi_1}
\]

Let us assume that $n^*_c > 1$ for all $m^* \geq 0$. Without this assumption, the model does not have a coordination problem since any single investment project in sector I

\begin{equation}
(3.13)
\end{equation}
will be profitable no matter how many other firms invest in the sector. In order for the big bank to mobilize firms in the critical mass, it has to subsidize those firms by managing interest rates. If firms’ expectations on \( m \) is \( m^* \), the big bank will charge the interest rates by the following rule in order to make investment projects of the firms in the critical mass break even:

\[
\rho_i^* = \frac{\phi_0 + \phi_1 i - \phi_2 m^*}{K} - 1 \quad \text{for all } i \in [1, n^*_c]
\]  

(3.14) is obtained by solving \( \pi_I(i, m^*, r_i) = 0 \) for \( r_i \). The big bank cannot charge firm \( i \) in the critical mass any interest rate above the level specified in (3.14) because if so, it will fail to induce the firm to invest, thereby failing to mobilize the critical mass. Moreover, the bank has no reason to charge firm \( i \) any other rate lower than the rate specified in (3.14) because it has the profit maximizing objective. Thus, the interest rates given by (3.14) are the maximum permissible rates on the loans extended to the firms in the critical mass. It is worth noting that (3.14), together with our assumption that \( n^*_c > 1 \) for all \( m^* \geq 0 \), implies that the rates charged to firms in the critical mass \( (\rho_i^*) \) must be strictly lower than the competitive rate \( (\rho_c) \), in other words, \( \rho_c > \rho_i^* \) for all \( i \in [1, n^*_c] \).

While the big bank charges the rates dictated by (3.14) to firms in the critical mass, the bank will charge \( \rho_c \) for other firms outside the critical mass, i.e. \( i \in (n^*_c, A] \) because it is subject to the Bertrand competition. It is trivial to see the capacity

---

12In order for firms’ beliefs to be rational, \( m^* \) must be zero in the case where the bank induces an industrialization equilibrium.

13In Da Rin and Hellman (2002), the condition dictating this type of interest rate policy is called ‘elimination constraint.’ Unlike their model, the interest rate policy given by (3.14) in our model depends on sector I firms’ expectations on \( m \). Keeping others constant, a rise in \( m^* \) requires a lower interest rate \( (\rho_i^*) \). Intuition is simple. Since we assume that two sectors are strategically substitutable, if firms in sector I expect sector II to expand, they expect their profits to be negatively affected, thus a lower interest rate is required to make them break even.

14For any \( i \in [1, n^*_c] \), if \( r_i > \rho_i^* \), then \( \pi_I(i, m^* r_i) < 0 \)
constraint will be binding and no credit will be granted to firms in sector II, i.e. 
\( n = A \) and \( m = 0 \).

Now we can derive the bank’s profits denoted as \( \Pi_H \) earned from inducing industrialization:

\[
\Pi_H = \sum_{i=1}^{n^*_c} (\rho^*_i - \rho_m)K + (A - n^*_c)(\rho_c - \rho_m)K = \sum_{i=1}^{n^*_c} (\rho^*_i - \rho_c)K + (\rho_c - \rho_m)KA
\]

(3.15)

(3.16)

(3.17)

Two remarks follow from observing (3.17). First, the bank’s profit from inducing industrialization is increasing in its size \( A \):

\[
\frac{d\Pi_H}{dA} = \rho_c - \rho_m \]

Second, since we assume that \( n^*_c > 1 \), \(-\frac{1}{2}\phi_1 n^*_c(n^*_c - 1)\) in (3.20) is always negative. Thus, the bank’s profit in this case must be strictly lower than \((\rho_c - \rho_m)KA\), i.e. \( \Pi_H < (\rho_c - \rho_m)KA \).

The analysis in this section can be summarized by Proposition 4:

**Proposition 4** Suppose that pessimistic beliefs prevail, \( n^*_c > 1 \) for all \( m^* \geq 0 \), and the bank with market power decides to induce industrialization. Then:

(i) Let \( \rho^*_i \) be such that \( \pi_1(i, m^*, \rho^*_i) = 0 \). The bank with market power will charge \( \rho^*_i \) to firm \( i \) in the critical mass \( (i \in [1, n^*_c]) \) and \( \rho_c \) to firms outside the critical mass \( (i \in [n^*_c, A]) \). The firms in sector I not financed by the big bank will be financed by fringe banks at \( \rho_c \).

(ii) \( \rho^*_i < \rho_c \) for all \( (i \in [1, n^*_c]) \).

(iii) The bank’s profit is given by \( \Pi_H = -\frac{1}{2}\phi_1 n^*_c(n^*_c - 1) + (\rho_c - \rho_m)KA \) and \( \Pi_H < (\rho_c - \rho_m)KA \).
3.5.3 Profits from inducing an underdevelopment trap

Now suppose that the bank with market power considers inducing an underdevelopment trap. We want to find the bank’s optimal interest rates and loan allocation. Let us define $m_L$ such that:

$$m_L \equiv \frac{\gamma_0 - (1 + \rho_c)K}{\gamma_2}$$ (3.18)

$m_L$ is the equilibrium number of firms investing in the traditional sector if there is no bank with market power and every competitive bank will charge $\rho_c$ subject to Bertrand competition.

Depending on whether $m_L > A$ or $m_L \leq A$, we have two different cases.

3.5.3.1 Strong traditional sector: $m_L > A$

Let us start with the case in which $m_L > A$. In this case, the bank size is less than the equilibrium number of firms investing in the traditional sector which would occur if there is no bank with market power. This condition may hold for the country which has large profitable traditional sectors due to high $\gamma_0$ or low $\gamma_2$.

Suppose that the bank with market power decides to induce an underdevelopment trap. Then it will use its entire resources to finance firms in the traditional sector at $\rho_c$ under the Bertrand constraint. There is no reason why the big bank extends loans to firms in sector I because the bank should provide loans to firms in sector I at a rate strictly lower than $\rho_c$. Instead, the big bank can provide its entire loans to firms in sector II at $\rho_c$. The loan offers from competitive fringe banks at $\rho_c$ will be accepted by firms which could not obtain finance from the big bank. In equilibrium exactly $m_L$ firms in the traditional sector will make break-even since every bank will charge $\rho_c$.\(^{15}\) Thus, in equilibrium, $A$ firms in sector II will be financed by the big bank and

\(^{15}\)Given that firms believe the modern sector would not develop at all (they should believe this in order for their beliefs to be rational), the profit of a representative firm in sector II is given by
\( m_L - A \) in the same sector by fringe banks. Let us denote as \( \Pi_L \) the level of profit in the underdevelopment trap. As long as \( m_L > A \), \( \Pi_L = (\rho_c - \rho_m)KA \). From this and Proposition 4 (iii), we have:

\[
\Pi_H < \Pi_L \text{ if } m_L > A.
\]

In other words, as long as \( m_L > A \), the bank with market power will induce an underdevelopment trap because it is more profitable to induce the underdevelopment trap than an industrialization equilibrium. From (3.18), \( m_L \) is positively related to \( \gamma_0 \) and negatively to \( \rho_c, K \) and \( \gamma_2 \). Thus, the higher \( \gamma_0 \) and the lower \( \rho_c, K \) and \( \gamma_2 \), the more likely the condition \( m_L > A \) be met for a given size of \( A \). The analysis in this section suggests that under the condition \( m_L > A \), banks with market power may fail to be a catalyst for industrialization since their profit maximization motives guide them to manage the traditional sector rather than stimulate the modern sector. Proposition 5 follows:

**Proposition 5** Suppose that \( m_L > A \) and pessimistic beliefs dominate. Then:

(i) In an underdevelopment equilibrium, \( m_L \) firms will enter sector II where \( m_L \equiv (\gamma_0 - (1 + \rho_c)K)/\gamma_2 \).

(ii) If the bank with market power chooses an underdevelopment trap, it will charge \( \rho_c \) to \( A \) firms in sector II. \( m_L - A \) firms in sector II will be financed by competitive fringe banks.

(iii) If the bank with market power chooses an underdevelopment trap, it will make a profit of \( \Pi_L = (\rho_c - \rho_m)KA \) which is always greater than the profit if it would

\[\pi_{II}(0, m, \rho_c) = \gamma_0 - \gamma_2 m - (1 + \rho_c)K. \text{ Firms will keep entering sector II up to the point where } \pi_{II}(0, m, \rho_c) = 0 \text{ which gives us the equilibrium level of } m.\]
make in the case of industrialization. Thus, the bank’s final decision is, indeed, 
to induce an underdevelopment trap as long as \( m_L > A \).

### 3.5.3.2 Weak traditional sector: \( m_L \leq A \)

Let us look at the other case in which \( m_L \leq A \). This case represents an economy 
with a relative weak traditional sector. This may be due to low \( \gamma_0 \) or high \( \gamma_2 \). The 
bank’s profit maximization problem is given by:

\[
\begin{align*}
\max_{m,n,r,r_i} & \quad (r - \rho_m)Km + \sum_{i=1}^{n}(r_i - \rho_m)K \\
\text{s.t.} & \quad r \leq \rho_c \\
& \quad m + n \leq A \\
& \quad \pi_{II}(n,m,r) = 0 \\
& \quad \pi_I(n+1,m,\rho_c) \leq 0 \\
& \quad \pi_I(i,m,r_i) = 0
\end{align*}
\]

(3.19)\text{ (3.20)\text{ (3.21)\text{ (3.22)\text{ (3.23)\text{ (3.24)\text{ }}\

- \( r \) represents the uniform rate of interest the bank charges on the loans granted 
to firms in the traditional sector.\(^{16}\) \( r_i \)'s are the rates of interest on the loans to 
firm \( i \) in the modern sector. (3.19) is the bank’s objective function showing its total 
profit. (3.20) and (3.21) are the Bertrand constraint and the loan capacity constraint, 
respectively. (3.22) describes the equilibrium relation between \( m, n \) and \( r \) which is 
implied by the zero profit condition for a representative firm in the traditional sector. 
(3.23) ensures that the economy should not industrialize: if this condition does not 
hold, the modern sector may fully develop on its own. (3.24) shows the interest rate 
discriminating policy set by the big bank. This policy specifies the rate of interest for

---

\(^{16}\)The bank does not have any incentive to interest-rate-discriminate firms in the traditional sector 
while it does have in the modern sector characterized by strategic complementarities among firms.
each firm \(i\) in the critical mass which makes each firm \(i\) break even at a given level of \(m\). Based on the analysis of this maximization problem, proposition 6 follows and the proof is given in Appendix G.

**Proposition 6** Suppose that \(m_L \leq A\) and pessimistic beliefs dominate. Then the bank with market power has a profit maximization problem given by (3.19)-(3.24).

(i) The profit maximization problem has a global maximum.

Let us write the maximized profit as a function of \(A\) and denote this as \(\Pi_L(A)\).

Then, \(\Pi_L(A)\) has the following properties (ii)-(v).

(ii) \(\Pi_L(A)\) is bounded from above. In other words, the capacity constraint (21) will eventually become slack.

(iii) \(\Pi_L(A)\) is nondecreasing in \(A\) and continuous.

(iv) \(\Pi_L(m_L) > \Pi_H(m_L)\).

(v) As long as the capacity constraint (3.21) is binding, \(\frac{d\Pi_L(A)}{dA} < (\rho_c - \rho_m)K\).

**Proof.** See Appendix G. ■

Proposition 6 along with proposition 4 and 5 will be used to construct the profit curves of the big bank in 3.5.4 and to see the bank’s final decision on sectoral credit allocation.

**3.5.4 Bank’s final decision on sectoral allocation**

The bank with market power will make its final decision on whether to induce industrialization or underdevelopment by comparing profits \(\Pi_H\) and \(\Pi_L\). Based on proposition 4(iii), 5(iii) and 6(ii)-6(v), we can construct profit curves of the big bank as shown in Figure 3.2. As the bank size \((A)\) increases, the bank profit from promoting industrialization \((\Pi_H)\) is unboundedly increasing starting from a negative level of
profit, $-\frac{1}{2} \phi_1 n^*_c (n^*_c - 1)$, while the profit from inducing an underdevelopment trap ($\Pi_L$) is nondecreasing starting from origin and eventually bounded since the expansion of the modern sector has a limit in this regime and as a result benefits from investment complementarities in the modern sector are limited.

![Figure 3.2. Bank’s Decision](image)

**Figure 3.2. Bank’s Decision**

Proposition 4, 5 and 6 imply that there exists a unique $A^*$ such that $\Pi_L(A^*) = \Pi_H(A^*)$. $A^*$ is the minimum bank size required for the bank to induce an industrialization equilibrium. For a range of the bank size $A < A^*$, the bank with market power decides not to promote industrialization. This case does not exclude a limited expansion of the modern sector, but the bank will not push forward the process of industrialization to the extent that the modern sector can develop on its own in this regime because it undermines its own profitability. Only when $A$ exceeds the critical value $A^*$, the bank has an incentive to promote industrialization. Two remarks are in order.

First, the minimum degree of market power required to induce the big bank to promote industrialization ($A^*$) depends on the position of $\Pi_H$ and $\Pi_L$, which in turn
is affected by the sectoral structure. Any change in a parameter which will shift up the bank profit in an underdevelopment case will raise $A^*$. In Figure 3.3, an upward shift in $\Pi_L$ increases the critical level from $A^*$ to $A^{**}$. Changes in parameters which are favorable for firms’ profits in sector II (i.e. a rise in $\gamma_0$ or a fall in $\gamma_1$ or $\gamma_2$) tend to shift up the bank’s profit function in an underdevelopment trap case and raise the critical value of $A$.

![Figure 3.3. The Effect of an Increase in Profitability in the Traditional Sector](image)

Second, it is worth noting that the critical value $A^*$ is greater than $m_L$. We may interpret $m_L$ as a status quo size of the traditional sector. Thus, the existence of a strong traditional sector may be captured by a high value of $m_L$ which makes the condition $m_L > A$ more likely to hold. As long as $m_L > A$, the bank with market power will induce an underdevelopment trap because the traditional sector provides profit opportunities better than the modern sector (Proposition 5 (iii)).
3.6 Implications

The analysis in the previous section suggests that the conditions under which the bank with market power can play a catalytic role for industrialization depend critically on the sectoral structure of an economy. A degree of market power sufficient to solve a coordination problem for one economy may not be sufficient to solve it for another economy. Banks with market power may fail to solve coordination problem when they have strong profit opportunities in traditional sectors. Thus, the conditions under which the bank’s profit maximization motive is compatible with the goal of promoting industrialization may not be met under some circumstances. In this regard, Epstein’s institutional study (2005b) is suggestive. Epstein emphasizes that central banks not only in the developing countries but also in the now developed countries have engaged in sectoral policies in order to support economic sectors. Historical cases supporting this arguments includes some ‘private central banks’ such as some continental European central banks in the nineteenth century, the Bank of France, the Bank of the Netherlands, and the Bank of Italy. After discussing these examples, Epstein adds a caveat:

“One should not overestimate the extent to which these central banks were agents of development in the sense of having a developmental vision and intent. These central banks were private, not public. As a result, their interest was in making a profit. At times, this concern even conflicted with their activities as central banks. Still, however imperfectly, these central banks helped mobilize and allocate finance to industry and to government in the service of economic development, sometimes directed by a developmental vision from the state.” (Epstein, 2005b, p.10)

Epstein’s study suggests that there can be conflicts between private profit maximizing motives and a national interest of industrialization. Despite these conflicts,
some optimistic perspective may suggest that spontaneous coordination among private agents can achieve a Pareto-optimal outcome and avoid a Pareto inferior outcome if they recognize that they will get huge gains from spontaneously coordinating themselves. However, proponents of a big push have given doubts to this possibility,\textsuperscript{17} and the analysis in this chapter suggests that the presence of profit opportunities in the traditional sector can be an obstacle to that kind of spontaneous coordination. This may be why government sponsored development banks, rather than private agents, have played a crucial role in promoting industrialization or sectoral development in many developing countries (Chang, 2002).\textsuperscript{18}

\textsuperscript{17}See Rosenstein-Rodan (1943) and Matsuyama (1995).

\textsuperscript{18}Armendariz de Aghion (1999) emphasizes the role of development banking from another perspective such as cofinancing arrangements and coownership with private financial institutions.
APPENDIX A

DATA SOURCES AND CALCULATIONS FOR FIGURES IN CHAPTER 1

□ Figure 1.1 Nonfarm Nonfinancial Corporations (1952-2007)

(a) The Ratio of Debt to Capital Debt (credit market instruments) = commercial paper + municipal securities + corporate bonds + bank loans + other loans and advances + mortgages. Capital = replacement cost of structures + replacement cost of equipment and software. Sources: Federal Reserve Board, Flow of Funds Accounts of the United States, Table B.102. Author’s calculation.

(b) The Ratio of Debt to Profit Debt = credit market instruments. Profit measures are before tax and after depreciation. Sources: Flow of Funds Accounts of the United States, Table B.102 and Table F.102; Author’s calculation.

(c) The Ratio of Interest Payment to Profit Interest Payment = (nominal bank prime rate − CPI inflation) × credit market instruments. Profit measures are before tax and after depreciation. Sources: Flow of Funds Accounts of the United States, Table B.102 and Table F.102; Federal Reserve Board, Federal Reserve Statistical Release; Bureau of Labor Statistics, The Consumer Price Index. Author’s calculation.

(d) The Ratio of Net Issues of Equities to Fixed Investment Net issues of nonfinancial corporate equities divided by nonfarm nonfinancial corporate (gross) fixed investment. Sources: Flow of Funds Accounts of the United States, Table F.102. Author’s calculation.
Figure 1.2 Households (1952-2007)

(a) The Ratio of Personal Consumption Expenditures to Disposable Personal Income  
    Sources: Flow of Funds Accounts of the United States, Table F.6 and F.100. Author’s calculation.

(b) Debt-Income Ratio  
    Household debt and income refer to credit market instruments and disposable personal income in the accounts, respectively. Sources: Flow of Funds Accounts of the United States, Table B.100. Author’s calculation.

(c) The Ratio of Equity Holding to Deposit Holding: Households and Nonprofit Organizations (1952-2007)  
    Sources: Flow of Funds Accounts of the United States, Table B.100. Author’s calculation.

Figure 1.2: Nonfarm Nonfinancial Corporations (1952-2007)

(d) The Rate of Return on Equity  
    The rate of return on equity = (net dividends + capital gains) ÷ market value of equity outstanding. Capital gains = Δ market value of equity − net new equity issues. CPI inflation rates are used to obtain the real rates. Sources: Flow of Funds Accounts of the United States, Table B.102 and Table F.102; Bureau of Labor Statistics, The Consumer Price Index. Author’s calculation.
APPENDIX B

PROOF OF PROPOSITION 1

To prove the existence of a limit cycle for the system of (1.37), (1.39), and (1.42), we need to show that the Jacobian matrix (1.45) evaluated at \((m(\lambda), \alpha(\lambda), z(\lambda), \lambda)\), where \((m(\lambda), \alpha(\lambda), z(\lambda))\) is a fixed point of the system,\(^1\) should have the following properties:

- The Jacobian matrix has a pair of complex conjugate eigenvalues \(\beta(\lambda) \pm \theta(\lambda)i\)
  such that \(\beta(\lambda^*) = 0\), \(\theta(\lambda^*) \neq 0\), and no other eigenvalues with zero real part exist at \((m(\lambda^*), \alpha(\lambda^*), z(\lambda^*), \lambda^*)\)

where \(\lambda^*\) is a Hopf bifurcation point.

To apply the above condition for the Hopf bifurcation to the current context, I will use the fact that the Jacobian matrix will have a negative real root and a pair of pure imaginary roots if and only if:

\[
\text{(R1)} \quad \text{Tr}(J) = F_m + G_z < 0
\]
\[
\text{(R2)} \quad J_1 + J_2 + J_3 = F_m G_z - \zeta' \cdot G_\alpha > 0
\]
\[
\text{(R3)} \quad \text{Det}(J) = -\zeta' \cdot (F_m G_\alpha - F_\alpha G_m) < 0
\]
\[
\text{(R4)} \quad -\text{Tr}(J)(J_1 + J_2 + J_3) + \text{Det}(J) = -(F_m + G_z)(F_m G_z - \zeta' \cdot G_\alpha) - \zeta' \cdot (F_m G_\alpha - F_\alpha G_m) = 0
\]

Let us denote the eigenvalues of the Jacobian matrix as \(\mu(\lambda)\) and \(\beta(\lambda) \pm \theta(\lambda)i\).

\(^1\)Note that in our case the fixed point is independent of the value of \(\lambda\).
Proof of (I)

Suppose that $G_\alpha < 0$. Then (R3) is always met. In order to satisfy (R1) and (R2), we should have $G_z < \min \left\{ |F_m|, \frac{\zeta' G_\alpha}{|F_m|} \right\}$. (R4) is quadratic in $G_z$. (R4) can be rewritten as:

$$a_1 G_z^2 + a_2 G_z + a_3 = 0 \quad (B.1)$$

where

$$a_1 \equiv -F_m > 0$$
$$a_2 \equiv -(F_m^2 - \zeta' G_\alpha) \leq 0$$
$$a_3 \equiv \zeta' F_\alpha G_m < 0$$

Solving (B.1) for $G_z$, we obtain one negative and one positive real roots. Let us select the negative root\(^2\), which is given as:

$$b \equiv \left( \frac{|F_m|^2 - \zeta' G_\alpha}{2|F_m|} - \sqrt{(|F_m|^2 - \zeta' G_\alpha)^2 + 4\zeta'|F_m||G_m|F_\delta} \right) < 0 \quad (B.2)$$

Since $G_z = \kappa \frac{\partial e}{\partial z} - \lambda$, the value of $\lambda$ that satisfies (R4) is: $\lambda = \kappa \frac{\partial e}{\partial z} + |b|$. Let $\lambda^* \equiv \kappa \frac{\partial e}{\partial z} + |b|$. We have shown that if $G_z < \min \left\{ |F_m|, \frac{\zeta' G_\alpha}{|F_m|} \right\}$ and $\lambda = \lambda^*$, then the Jacobian matrix has a negative real root and a pair of imaginary roots: $\mu(\lambda^*) < 0$, $\beta(\lambda^*) = 0$, and $\theta(\lambda^*) \neq 0$. To prove $\lambda^*$ is indeed the bifurcation point, we still need to show that $\beta'(\lambda^*) \neq 0$. To prove $\beta'(\lambda^*) \neq 0$, let us use the following fact:

$$\mu(\lambda) + 2\beta(\lambda) = F_m + G_z$$
$$2\mu(\lambda)\beta(\lambda) + \beta(\lambda)^2 + \theta(\lambda)^2 = F_m G_z - \zeta' G_\alpha$$
$$\mu(\lambda)[\beta(\lambda)^2 + \theta(\lambda)^2] = -\zeta' \cdot (F_m G_\alpha - F_\alpha G_m)$$

\(^2\)It can be shown that the positive root is irrelevant for the analysis.
Totally differentiating both sides with respect to $\lambda$, we get

\[
\begin{bmatrix}
1 & 2 & 0 \\
2\beta(\lambda) & 2[\mu(\lambda) + \beta(\lambda)] & 2\theta(\lambda) \\
[\beta(\lambda)^2 + \theta(\lambda)^2] & 2\mu(\lambda)\beta(\lambda) & 2\mu(\lambda)\theta(\lambda)
\end{bmatrix}
\begin{bmatrix}
\mu'(\lambda) \\
\beta'(\lambda) \\
\theta'(\lambda)
\end{bmatrix}
= \begin{bmatrix}
-1 \\
|F_m| \\
0
\end{bmatrix}
\] (B.3)

The right hand side of (B.3) is obtained using the fact that $\frac{\partial G_z}{\partial \alpha} = -1$ and $\lambda$ does not affect all other partial derivatives than $G_z$. Evaluating (B.3) at $\lambda = \lambda^*$, we have:

\[
\begin{bmatrix}
1 & 2 & 0 \\
0 & 2\mu(\lambda^*) & 2\theta(\lambda^*) \\
\theta(\lambda^*)^2 & 0 & 2\mu(\lambda^*)\theta(\lambda^*)
\end{bmatrix}
\begin{bmatrix}
\mu'(\lambda^*) \\
\beta'(\lambda^*) \\
\theta'(\lambda^*)
\end{bmatrix}
= \begin{bmatrix}
-1 \\
|F_m| \\
0
\end{bmatrix}
\]

Solving this for $\beta'(\lambda^*)$, we finally get:

\[
\beta'(\lambda^*) = \frac{2\mu(\lambda^*)\theta(\lambda^*)|F_m| - 2\theta(\lambda^*)^3}{4\mu(\lambda^*)^2\theta(\lambda^*) + 4\theta(\lambda^*)^3} < 0 \quad \text{since} \quad \mu(\lambda^*) < 0
\]

Thus, $\beta'(\lambda^*)$ is strictly negative.

**Proof of (II)**

Suppose that $G_\alpha > 0$ and $G_z < 0$. Then (R1) is always satisfied. To meet (R2) and (R3), we need $G_\alpha < \min\left\{\frac{|F_m||G_z|}{\zeta}, \frac{F_m|G_m|}{|F_m|}\right\}$. The rest of the proof is essentially the same as that of (I).

**Proof of (III)**

Routh-Hurwitz necessary and sufficient conditions for the local stability of a three dimensional system are (R1), (R2) and (R3) with replacing the equality in (R4) by the inequality: $-\text{Tr}(J)(J_1 + J_2 + J_3) + \text{Det}(J) > 0$. Suppose that $G_\alpha > 0$ and $G_z > 0$. Then (R2) is always violated and the fixed point is unstable. At the same time, since (R2) is not met, it is impossible to get a limit cycle a la the Hopf bifurcation.
APPENDIX C

FUNCTIONS AND PARAMETER VALUES FOR THE SIMULATION IN CHAPTER 1

Investment function is linear in the utilization rate and consumption function in household disposable income and wealth.

\[ g = \gamma_0 + \gamma_1 u \]  
\[ \frac{I}{K} = g + \delta \]  
\[ \frac{C}{K} = c_1[u\sigma - s_f(\pi u\sigma - \delta - rm)] + c_2q \]

The condition for the goods market equilibrium \( Y = C + I \) gives us the equilibrium profit share:

\[ \pi = \phi(u) + \delta - \frac{(1 - c_1)u\sigma + c_2(1 + \alpha)m + c_1 s_f(\delta + rm)}{c_1 s_f u \sigma} \]

\[ \equiv \pi(u, m, \alpha) \]

The growth function is assumed to take the following nonlinear form.

\[ \hat{Y} = h(\pi, e) = h_0 + \frac{h_1}{1 + \exp[-h_2(\pi + h_3 \ln(h_4 - e) + h_5)]]} \]

Given this specification and chosen parameter values, the response of output growth to changes in the profit share is small at high and low values of the profit share but large at intermediate values.
Nonlinearity plays an important role in firms’ debt dynamics and households’ portfolio dynamics (C.4 and C.5). Function $\tau$ and $\zeta$ is very steep at $\frac{\rho_T}{\tau m} = \tau^{-1}(0)$ and $z = 0$, respectively.

$$\dot{m} = \tau \left( \frac{\rho_T}{\tau m} \right) = \tau_0 + \frac{\tau_1 - \tau_0}{1 + \exp[-\tau_2 (\frac{\rho_T}{\tau m} - \tau_3)]}$$  
(C.4)

where $\rho_T = \pi(u^*, m, \alpha)u^*\sigma$ and $u^* = \frac{1}{\gamma_1}(n - \gamma_0)$

$$\dot{\alpha} = \zeta(z) = \zeta_0 + \frac{\zeta_1 - \zeta_0}{1 + \exp[-\zeta_2(z - \zeta_3)]}$$  
(C.5)

$$\dot{z} = \kappa\left( r^e|_{u=u^*} - r, \alpha \right) - \lambda z = \kappa_0 + \kappa_1\left( r^e|_{u=u^*} - r \right) - \kappa_2\alpha - \lambda z$$  
(C.6)

where $r^e|_{u=u^*} = \frac{\rho_T - \delta - \tau m (1 + \alpha)(\dot{m} + mn) + \dot{\alpha}m - n}{\alpha m}$

\begin{table}
\centering
\begin{tabular}{cccccccccc}
\hline
\hline
\gamma_0 & \gamma_1 & h_0 & h_1 & h_2 & h_3 & h_4 & h_5 & \sigma \\
\hline
-0.93 & 1.2 & -0.02 & 0.07 & 60 & 0.4 & 1.1 & 0.423 & 0.5 \\
\hline
n & $\delta$ & $s_f$ & r & $c_1$ & $c_2$ & $\kappa_0$ & $\kappa_1$ & $\kappa_2$ \\
0.03 & 0.09 & 0.7 & 0.04 & 0.65 & 0.04 & 0.066 & 0.007 & 0.06 \\
\hline
\lambda & \zeta_0 & \zeta_1 & \zeta_2 & \zeta_3 & \tau_0 & \tau_1 & \tau_2 & \tau_3 \\
0.0105 & -0.24 & 0.03 & 30 & -0.7 & -0.15 & 0.011 & 20 & 10.6 \\
\hline
\end{tabular}
\caption{Parameter values}
\end{table}
A weighed moving average specification may provide an alternative measure of the trend rate of profit:

\[ \rho_T = \int_{-\infty}^{t} \eta \exp\left[-\eta(t - \nu)\right] \rho_{\nu} d\nu \quad \text{where} \quad \mu > 0 \quad (D.1) \]

where \( \rho_{\nu} \) is the current rate of profit at each moment of time \( \nu \in (-\infty, t] \) and \( \eta \exp[-\eta(t - \nu)] \) represents the weight attached to \( \rho_{\nu} \) in the calculation of the trend rate of profit at time \( t \), which exponentially decreases as \( \nu \) gets further back to the past.

This specification implies that the trend profit rate is constantly updated based on the following averaging process.

\[ \dot{\rho}_T = \eta(\rho - \rho_T) \]

where \( \rho = \pi(u, m, \alpha)u\sigma \). Note that the expression for the current profit rate \( \rho \) includes capacity utilization \( (u) \) as well as the debt ratio \( (m) \) and the equity-deposit ratio \( (\alpha) \). Thus, the system of short cycles and that of long waves become interdependent.

The two specifications, (1.34) and (D.1), produce qualitatively similar results. The basic idea behind both specifications is to smooth actual profitability and get a measure of the long-run trend of profitability and one would expect the two specifications to produce qualitatively similar results. Simulations confirm that this is indeed the case. Analytically, the specification (1.34) is more tractable and the analysis in Chapter 1 has been based on (1.34).
APPENDIX E

PROCEDURE FOR NUMERICAL RESULTS IN 2.4.1.2, 2.4.2.2 AND 2.5.2.

In 2.4.1.2, 2.4.2.2, and 2.5.2 we used numerical methods to examine the comparative statics of two models: one with constant stock-flow ratios, $\alpha$ and $\beta$, and the other with Lavoie-Godley specifications on consumption and portfolio choice and induced variations in the stock-flows ratios. The analysis was conducted in the context of Harrodian mature economies (2.4.1.2), Harrodian dual economies (2.4.2.2), and Kaleckian dual economies (2.5.2) and the results summarized in Table 2.1 to 2.10. In this appendix, we present the procedure that was used to find the values of the derivatives of the endogenous variables with respect to financial and other parameters. We do this in the context of Kaleckian dual economies (see Table 2.5 in 2.5.2) where the model with variable stock-flow ratios is the same as the one in Lavoie and Godley (2001-2002). The procedure in the other cases is similar and, in fact, less complicated. Note that Table 2.1, 2.3, 2.5, 2.6 and 2.7 are produced based on the specification of consumption in Lavoie and Godley (2001-2002) - the flow-flow specification represented by (E.3) below- while the other tables (Table 2.2, 2.4, 2.8, 2.9 and 2.10) are produced based on that in Godley and Lavoie (2007). In the latter, the consumption function (E.3) is replaced by $\frac{C}{K} = c_1\{u - s_f(\pi u - rm)\} + c_2q$ where $c_1 = 0.75$ and $c_2 = 0.064$, keeping intact other equations and parameter values.

The Kaleckian dual economies with the variable $\alpha$ and $\beta$ – Lavoie and Godley (2001-2002)
\[ g = s_f(\pi u - rm) + mg + xg \quad (E.1) \]
\[ g = \gamma_0 + \gamma_1 s_f(\pi u - rm) - \gamma_2 rm + \gamma_3 q + \gamma_4 u \quad (E.2) \]
\[ u - g = a_1\{u - s_f(\pi u - rm)\} + \frac{a_1}{a_2} \gamma \quad (E.3) \]
\[ m = (1 - \lambda_0 + \lambda_1 r - \lambda_2 r_e) q + \lambda_3\{u - s_f(\pi u - rm)\} \quad (E.4) \]

where \( r^e = \frac{(1-s_f)(\pi u-rm)+\gamma}{q-m} \) and \( \gamma = g(q-m) - xg. \)

The symbols used here are the same as the ones in the main text of this paper. The inflation rate is assumed to be zero. (E.1) describes firms’ finance constraint, (E.2) is the investment function, (E.3) describes the equilibrium condition for the product market where the right-hand side specifies households consumption behavior as a function of household distributed income and capital gains. (E.4) shows households’ demand for money (portfolio choice). The following values are used by Lavoie and Godley (2001-2002).

\[
\begin{align*}
\gamma_0 &= 0.0075 & \gamma_1 &= 0.5 & \gamma_2 &= 0.5 & \gamma_3 &= 0.02 & \gamma_4 &= 0.125 \\
s_f &= 0.75 & x &= 0.05 & \pi &= 0.2498 \\
a_1 &= 0.8 & a_2 &= 4.5 \\
\lambda_0 &= 0.45 & \lambda_1 &= 0.2 & \lambda_2 &= 0.0133 & \lambda_3 &= 0.0001 & r &= 0.0275
\end{align*}
\]

Given these parameter, (E.1)-(E.4) determine the steady-state values of \( u, g, q \) and \( m \). The system has multiple solutions due to nonlinearities of some equations. The number of solutions is six but five of them can be discarded on economic grounds since at least one of the variables — including \( r^e \) — is negative. The positive numerical solution is:

\[
\begin{align*}
u^* &= 0.188 & g^* &= 0.0545 & q^* &= 0.8789 & m^* &= 0.487 \quad (E.5)
\end{align*}
\]
The partial derivatives of the solutions for $u$ and $g$ with respect to $s_f, x, r, \pi,$ and $\lambda_0$ are evaluated at $(u^*, g^*, q^*, m^*)$. The obtained values were reported in the third and fifth columns of Table 2.5.

Using the definitions of $\alpha$ and $\beta$, we obtain the following equilibrium values for $\alpha$ and $\beta$:

$$\alpha^* = \frac{q^* - m^*}{u^*} = 2.07936 \quad \beta^* = \frac{m^*}{u^*} = 2.58914$$

Using these steady-state values of stock-flow ratios, we can transform the variable $\alpha$ and $\beta$ regime to the constant $\alpha$ and $\beta$ regime by dropping the consumption and portfolio choice functions.

**Constant $\alpha$ and $\beta$ regime**

$$g = s_f(\pi u - rm) + mg + xg$$

$$g = \gamma_0 + \gamma_1 s_f(\pi u - rm) - \gamma_2 rm + \gamma_3 q + \gamma_4 u$$

$$q = (\alpha^* + \beta^*)u$$

$$m = \beta^* u$$

By construction, the above four equations must yield the same steady state values as in (E.5). Then, the partial derivatives of the solutions for $u$ and $g$ with respect to $s_f, x, r, \pi,$ and $\lambda_0$, again, are evaluated at $(u^*, g^*, q^*, m^*)$. The second and forth columns of Table 2.5 report these values.
APPENDIX F

PROOF OF PROPOSITION 2

**Proof.** The final good producer solves the following cost minimization problem where $P_X$ and $P_Y$ are price indices for good $X$ and $Y$:

\[
\text{Minimize } \begin{array}{c}
\underset{X,Y}{\text{subject to}} \\
P_X X + P_Y Y
\end{array}
\]  

(F.1)

subject to  

\[ Z = \left[ \alpha X^{1-\frac{1}{\epsilon}} + \beta Y^{1-\frac{1}{\epsilon}} \right]^{\frac{1}{1-\epsilon}} \]  

(F.2)

The first-order conditions give us the solution to the minimization problem:

\[ X = \alpha^\epsilon P_X^{-\epsilon} Z \left[ \alpha^\epsilon P_X^{1-\epsilon} + \beta^\epsilon P_Y^{1-\epsilon} \right]^{-\frac{1}{\epsilon}} \]  

(F.3)

\[ Y = \beta^\epsilon P_Y^{-\epsilon} Z \left[ \alpha^\epsilon P_X^{1-\epsilon} + \beta^\epsilon P_Y^{1-\epsilon} \right]^{-\frac{1}{\epsilon}} \]  

(F.4)

From the zero profit condition for the final good sector, we have:

\[ P_Z = \left[ \alpha^\epsilon P_X^{1-\epsilon} + \beta^\epsilon P_Y^{1-\epsilon} \right]^{-\frac{1}{\epsilon}} \]  

(F.5)

$P_Z$ is the final good price, which is normalized to unity ($P_Z = 1$).
A representative firm that produces good $X$ faces the following cost minimization problem:

$$\text{Minimize } \sum_{i=1}^{n} p_x(i)x(i)$$

subject to $X = \left[ \sum_{i=1}^{n} x(i)^{1-\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$ (F.6)

The first order conditions, along with (F.6), are given by:

$$p_x(i) - \lambda \left( \frac{\sigma}{\sigma - 1} \right) \left[ \sum_{i=1}^{n} x(i)^{1-\frac{1}{\sigma}} \right]^{\frac{1}{\sigma-1}} x(i)^{-\frac{1}{\sigma}} = 0 \text{ for all } i \quad \text{(F.7)}$$

The zero profit condition can be written:

$$P_X X - \sum_{i=1}^{n} p_x(i)x(i) = 0 \quad \text{(F.8)}$$

Similarly, a representative firm that produces good $Y$ has the following cost minimization problem:

$$\text{Minimize } \sum_{j=1}^{m} p_y(j)y(j)$$

subject to $Y = \left[ \sum_{j=1}^{m} y(j)^{1-\frac{1}{\tau}} \right]^{\frac{\tau}{\tau-1}}$ (F.9)

The first order conditions and the zero profit condition are:

$$p_y(j) - \mu \left( \frac{\tau}{\tau - 1} \right) \left[ \sum_{j=1}^{m} y(j)^{1-\frac{1}{\tau}} \right]^{\frac{1}{\tau-1}} y(j)^{-\frac{1}{\tau}} = 0 \text{ for all } j \quad \text{(F.10)}$$

$$P_Y Y - \sum_{j=1}^{m} p_y(j)y(j) = 0 \quad \text{(F.11)}$$

(F.6), (F.7), (F.9) and (F.10) give each intermediate good producer a conditional factor demand curve. The elasticity of demand for each $i$- and $j$-producer is $\sigma$ and $\tau$, respectively.
respectively.\textsuperscript{1} Thus, the profit maximization behavior of each individual intermediate good producer implies the following mark-up pricing rule:

\[ p_x(i) = \frac{\sigma}{\sigma - 1}aw \quad \text{for all } i \]

\[ p_y(j) = \frac{\tau}{\tau - 1}bw \quad \text{for all } j \]

By normalizing \((\frac{\sigma}{\sigma - 1}) a = (\frac{\tau}{\tau - 1}) b = 1\), we have:

\[ p_x(i) = p_y(j) = w \quad \text{for all } i, j \quad (F.12) \]

Furthermore, by symmetricity, we know that each producer produces the same quantity of output:

\[ x(i) = x \quad \text{for all } i \quad (F.13) \]

\[ y(j) = y \quad \text{for all } j \quad (F.14) \]

It follows from (F.6), (F.9), (F.13) and (F.14) that:

\[ x = n^{\frac{\sigma}{\sigma - 1}}X \quad (F.15) \]

\[ y = m^{\frac{\tau}{\tau - 1}}Y \quad (F.16) \]

(F.8), (F.12) and (F.15) are combined to give us the expression for the price index for good \(X\) in terms of the wage rate and the number of \(i\)-producers.

\[ P_X = p_x n^{\frac{1}{\sigma}} = wn^{\frac{1}{\sigma}} \quad (F.17) \]

\textsuperscript{1}This follows the conventional assumption in the literature that intermediate good producers do not engage in strategic interactions among them.
Similarly, using (F.11), (F.12) and (F.16), we have the expression for the price index for good $Y$.

$$P_Y = p_y m^{\frac{\sigma-1}{\tau}} = w m^{\frac{\sigma-1}{\tau}}$$  \hspace{1cm} (F.18)

The zero profit condition for the final good sector, (F.5), together with (F.17) and (F.18), yields the expression for the equilibrium wage rate for given $m$ and $n$.

$$w(m, n) = \left[ \alpha^\epsilon n^{\frac{\sigma-1}{\tau}} + \beta^\epsilon m^{\frac{\sigma-1}{\tau}} \right]^{\frac{1}{\epsilon-1}}$$  \hspace{1cm} (F.19)

Next, we need to find the expressions for $x$ and $y$ for given $m$ and $n$. Note that from (F.3), (F.4), (F.17) and (F.18), the relative factor demand from the final good sector is given by:

$$\frac{X}{Y} = \left( \frac{\beta P_X}{\alpha P_Y} \right)^{-\epsilon} = \left( \frac{\beta n^{1/(1-\sigma)}}{\alpha m^{1/(1-\tau)}} \right)^{-\epsilon}$$  \hspace{1cm} (F.20)

In symmetric equilibrium, the labor market clearing condition is written as:

$$a nx + b my = L$$  \hspace{1cm} (F.21)

Then, (F.15), (F.16), (F.20) and (F.21) can determine $x$ and $y$ for given $m$ and $n$.

$$x(m, n) = \frac{\alpha^\epsilon n^{\frac{\sigma-1}{\tau}} L}{a\alpha^\epsilon n^{\frac{\sigma-1}{\tau}} + b\beta^\epsilon m^{\frac{\sigma-1}{\tau}}}$$  \hspace{1cm} (F.22)

$$y(m, n) = \frac{\beta^\epsilon m^{\frac{\sigma-1}{\tau}} L}{a\alpha^\epsilon n^{\frac{\sigma-1}{\tau}} + b\beta^\epsilon m^{\frac{\sigma-1}{\tau}}}$$  \hspace{1cm} (F.23)

It is straightforward to see that (F.12), (F.19), (F.22), (F.23), (3.4) and (3.5) imply the profit functions (3.7) and (3.8).

$$\pi_1(n, m, r_i) = \frac{\alpha^\epsilon \left[ \alpha^\epsilon n^{\frac{\sigma-1}{\tau}} + \beta^\epsilon m^{\frac{\sigma-1}{\tau}} \right]^{\frac{1}{\epsilon-1}} n^{\frac{\sigma-1}{\tau}}}{a\alpha^\epsilon n^{\frac{\sigma-1}{\tau}} + b\beta^\epsilon m^{\frac{\sigma-1}{\tau}}}(1-a)L - (1+r_i)K$$  \hspace{1cm} (3.7)
\[ \pi_{\Pi}(n, m, r_j) = \frac{\beta^\epsilon \left[ \alpha^\epsilon n^{\frac{\sigma - 1}{\tau - 1}} + \beta^\epsilon m^{\frac{\tau - 1}{\tau - 1}} \right]^{\frac{1}{\tau - 1}} m^{\frac{\tau - 1}{\tau - 1}}}{a \alpha^\epsilon n^{\frac{\sigma - 1}{\tau - 1}} + b \beta^\epsilon m^{\frac{\tau - 1}{\tau - 1}}} (1 - b) L - (1 + r_j) K \] (3.8)
APPENDIX G

PROOF OF PROPOSITION 6

Proof. Using firms’ profit functions (3.11) and (3.12), the maximization problem (3.19)-(3.24) can be transformed to the following problem:

\[
\max_{m,n} [\gamma_0 - \gamma_1 n - \gamma_2 m - (1 + \rho_m)K]m + \sum_{i=1}^{n} [\phi_0 + \phi_1 i - \phi_2 m - (1 + \rho_m)K] \quad (G.1)
\]

subject to

\[
\frac{\gamma_1}{\gamma_2} n + m \geq m_L \quad (G.2)
\]

\[
m + n \leq A \quad (G.3)
\]

\[
\phi_0 + \phi_1 (n + 1) - \phi_2 m - (1 + \rho_c)K \leq 0 \quad (G.4)
\]

(G.2) rewrites the Bertrand constraint (3.20) in terms of \(n\) and \(m\). (G.4) is the deindustrialization constraint which is equivalent to (3.24).

(i) It is easy to show that (G.2), (G.3) and (G.4), along with nonnegative constraints \((m \geq 0 \text{ and } n \geq 0)\), forms a nonempty compact set under the assumptions \(A \geq m_L \text{ and } n^*_c > 1\). Since the objective function (G.1) is continuous, the maximization problem has a global maximum in the constraint set.

(ii) From the objective function (G.1), we have:

\[
\frac{\partial \Pi_L}{\partial m}(m,n) = \gamma_0 - 2\gamma_2 m - (\gamma_1 + \phi_2)n - (1 + \rho_m)K
\]

\[
\frac{\partial \Pi_L}{\partial n}(m,n) = \phi_0 + (1/2)\phi_1 - (\gamma_1 + \phi_2)m + \phi_1 n - (1 + \rho_m)K
\]

Let us consider a set \(S_0\) such that \(S_0 \equiv \{(m,n) \in \mathbb{R}_+^2 | \frac{\partial \Pi_L}{\partial m}(m,n) < 0 \text{ and } \frac{\partial \Pi_L}{\partial n}(m,n) < 0\}. Then construct another set \(S(A)\) such that \(S(A) \equiv \{(m,n) \in \mathbb{R}_+^2 | m + n = 150\} \).
A and $\phi_0 + \phi_1(n + 1) - \phi_2m - (1 + \rho_c)K \leq 0 \}$. It is easy to see $S(A)$ is a subset of the constraint set for the maximization problem. Furthermore, $S(A)$ is contained in $S_0$ for a sufficiently large $A$. We want to show that the maximum solution to the problem cannot belong to $S(A)$ for a sufficiently large $A$. To show this, we can choose a sufficiently large value $A'$ so that $S(A') \subset S(A)$ for any $A > A'$ because if $(n(A), m(A)) \in S(A)$, then $(n(A), m(A)) \in S_0$, and by slightly decreasing the value of $n(A)$ to $n(A) - \epsilon$, the bank can raise its profit due to the fact that $\frac{\partial \Pi_L}{\partial n} < 0$ for any $(m, n) \in S_0$. This, however, contradicts the assumption that $(n(A), m(A))$ is a maximum. This proves that the capacity constraint $m + n \leq A$ should be eventually slack as $A$ becomes very large.

(iii) Since the objective function is continuous and the compact constraint set continuously expands as $A$ grows, $\Pi_L(A)$ is nondecreasing in $A$ and continuous.

(iv) Proposition 4.3 (iii) yields that $\Pi_L(m_L) > \Pi_H(m_L)$.

(v) If the capacity constraint (G.2) is not binding, $\frac{d\Pi_L(A)}{dA} = 0$. If only the capacity constraint (G.2) is binding so that there exists an interior solution, we must have $\frac{\partial \Pi_L}{\partial m} = \frac{\partial \Pi_L}{\partial m}$ and $\frac{dn}{dA} + \frac{dm}{dA} = 1$ at the optimum. Then,

$$
\frac{d\Pi_L(A)}{dA} = \frac{\partial \Pi_L}{\partial m} \frac{dm}{dA} + \frac{\partial \Pi_L}{\partial n} \frac{dn}{dA} = \frac{\partial \Pi_L}{\partial m} \\
= \left[\gamma_0 - \gamma_1 n - \gamma_2 m - (1 + \rho_m)K\right] - \gamma_2 m - \phi_2 n \\
= (r - \rho_m)K - \gamma_2 m - \phi_2 n
$$

If another constraint as well as the capacity constraint is binding, we then have:

$$
\frac{d\Pi_L(A)}{dA} \leq (r - \rho_m)K - \gamma_2 m - \phi_2 n
$$
Thus, for all cases, we obtain:

\[
\frac{d\Pi_L(A)}{dA} \leq (r - \rho_m)K - \gamma_2m - \phi_2n
\]

\[
= (\rho_c - \rho_m)K - (\rho_c - r)K - \gamma_2m - \phi_2n
\]

From the Bertrand constraint (3.20), \( r \leq \rho_c \), we have

\[
\frac{d\Pi_L(A)}{dA} < (\rho_c - \rho_m)K
\]


