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Abstract

This paper examines the role of fiscal policy in the long run. We show that (i) dynamic inefficiency may be empirically relevant in a modified Diamond OLG model with imperfect competition, (ii) fiscal policy may be needed to avoid inefficiency (if investment adjusts passively to saving) and maintain full employment (if investment and saving decisions are taken separately), (iii) a simple and distributionally neutral tax scheme can maintain full employment in the face of variations in 'household confidence', and (iv) the debt ratio is inversely related to both the growth rate and government consumption.

JEL classification: E62, E22

Key words: Public debt, Keynesian OLG model, dynamic efficiency, confidence, sustainability.

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1 Introduction

Diamond (1965) provides a classic analysis of the usefulness of public debt in dynamically inefficient economies. This well-known result gains real-world significance insofar as actual economies become dynamically inefficient in the absence of public debt. The general consensus seems to be that this is not the case: empirically the rate of return on capital appears to exceed the rate of growth.

The empirical argument has two potential weaknesses. To ascertain the need for fiscal policy and public debt one would need to evaluate the rate of return in a state without public debt; it is not sufficient to show that dynamic efficiency may hold if the evidence applies to an economy with significant amounts of public debt. Moreover, the standard efficiency criterion – that the rate of return exceed the growth rate – is based on the identification of the rate of return with the marginal product of capital. This identification breaks down in our modified Diamond model with imperfect competition: with a markup on marginal cost, the rate of return exceeds the marginal product of capital.

In a second modification of the Diamond model, we introduce Keynesian concerns. Households save but investment decisions are made by firms, and what appears as a problem of dynamic inefficiency in a neoclassical version of the model turns into a problem of aggregate demand and unemployment in the Keynesian setting. We use a Leontief production function in most of the analysis. This specification brings out the key issues quite sharply. But the Leontief assumption can be relaxed – we do this in section 4 which allows for factor substitution – and the main argument is independent of the specification of the production function.

The motivation behind our analysis derives from the recent focus on public debt in policy debates. This focus is peculiar for many reasons. Most obviously, a recession with high unemployment would seem the wrong time to address public debt issues, whatever their long-term significance. From a theoretical perspective, moreover, the focus is surprising because dominant macroeconomic models imply that public debt is largely irrelevant. This irrelevance of debt in benchmark theoretical models may lie behind another curious feature of the debate: the emphasis on purely empirical studies. The empirical strand is represented most prominently by Reinhart and Rogoff's (2009) celebrated analysis of crisis episodes from eight centuries and a range of economies. In a subsequent paper, Reinhart and Rogoff (2010) suggest that debt-income levels above 90 percent tend to be associated with lower rates of economic growth. The impact of this claim – and the atheoretical approach, more generally – seems surprising in a profession which for many years has been guided by the Lucas critique; the interpretation of statistical correlations would seem quite daunting in this particular application.

The Reinhart and Rogoff claim has been challenged by other studies (e.g. Irons and Bivens (2010)) but the challenge has been largely empirical. Our analysis contributes a theoretical perspective on this (and other) policy issues: we find a relationship between debt and growth rates – a low growth rate generates a high steady-growth ratio of debt to income – but the causal link unambiguously runs from growth to debt.

OLG models with imperfect competition have been developed by, among others, d'Aspremont et al. (1995), Pagano (1990) and Jacobsen and Schultz (1994). Like our analysis in this paper, these models examine the potential usefulness of fiscal policy. But this similarity masks fundamental differences in the structure of the models and the nature of the fiscal effects. Assuming Cournot competition, d'Aspremont et al. show that fiscal policy can be used to influence the equilibrium markup which – along with an elastic labor supply – determines equilibrium employment; the model has no capital, no dynamic inefficiency, no Keynesian unemployment, and the government balances its budget in each period. The details are different in Pagano and Jacobsen and Schultz, but the fiscal effects run through changes in competition and market power in these papers too. By contrast, we take the markup as given, treat the labor supply as inelastic, and focus on questions of dynamic inefficiency, Keynesian unemployment, and the dynamics of public debt.

Public debt dynamics have been examined in an OLG setting by Chalk (2000). Assuming a constant primary deficit per worker, he finds that even if the economy is dynamically inefficient when public debt is zero, a constant primary deficit may be unsustainable. Moreover, in those cases where a primary deficit is sustainable, convergence is to a steady growth path that is dynamically inefficient. These results invite several questions. Why would a government want to pursue policies of this kind? Why focus on trajectories that keep a constant primary deficit? Economic analysis of monetary policy typically looks for optimal policies (or policy rules), given some welfare function and a model of how the economy operates. Our 'functional finance' approach may introduce market failures that are usually absent in contemporary analysis of monetary policy but the search for appropriate policies is in a similar spirit. The Keynesian literature on 'functional finance' has a long history, going back to Lerner (1943).[1] We know of no other studies, however, that use a formal OLG model to examine the long run implications of functional finance.

Fiscal policy, we argue, can be used to avoid dynamic inefficiency (when investment adjust passively to saving) and maintain full employment (when investment and saving decisions are separate). We first consider the requirements in steady growth and then examine the implications of shifts in 'household confidence' that lead to fluctuations in saving rates. A simple and distributionally neutral tax scheme can maintain full employment in the face of these shifts in confidence. Moreover, in the special case where households correctly anticipate future taxes, no variations in taxes will be needed: the tax policy effectively functions as an insurance scheme. Concerns over the sustainability of the public debt trajectory, finally, find no support. The required debt is inversely related to both the growth rate and government consumption, and fiscal policies based on functional finance may in some cases lead to high levels of public debt. But in this OLG setting no scenarios become explosive or otherwise unsustainable.

Section 2 gives a brief overview of the standard OLG model before presenting our modified version with imperfect competition and Keynesian features. Taxation and public debt are added in section 3, and we derive the full-employment requirements for fiscal policy with special attention to questions of intergenerational distribution. Section 4 extends the model by including monetary policy and a choice of technique. Section 5 discusses implications of the analysis, relating it to recent policy debates. Section 6 presents a few concluding remarks.

2 OLG models

2.1 Perfect competition and neoclassical production functions

Following Diamond (1965) all agents live for two periods: they work in the first period and live off their savings in the second.[2] The utility function for a young

[1] Pedersen (1937) articulated a similar principle of functional finance (Olesen (2001)). Recent contributions include Schlicht (2006), Godley and Lavoie (2007), Arestis and Sawyer (2010), Kregel (2010), and Ryoo and Skott (2012).

[2] Expositions of the model can be found in many textbooks, see e.g. Romer (2012, chapter 2).

agent in period t takes the standard CIES form

$$U = \frac{c_{1,t}^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho}\frac{c_{2,t+1}^{1-\theta} - 1}{1-\theta}; \qquad \theta \geq 0 \tag{1}$$

where $c_{1,t}$ and $c_{2,t+1}$ are the levels of consumption per capita when the agent is young and old, θ is the inverse of the intertemporal elasticity of substitution, and ρ is the discount rate. There is full employment and we take the labor supply to be inelastic.[3] Normalizing the supply of an individual worker to one, the budget constraint is given by

$$c_{1,t} + \frac{1}{1+r_{t+1}}c_{2,t+1} = w_t \tag{2}$$

where r_{t+1} is the rate of return on savings and w_t is the real wage.

The maximization problem implies that

$$c_{1,t} = (1-s_t)w_t \tag{3}$$

where the young generation's saving rate s can be written

$$s_t = s(r_{t+1}) = \frac{(1+r_{t+1})^{(1-\theta)/\theta}}{(1+\rho)^{1/\theta} + (1+r_{t+1})^{(1-\theta)/\theta}} \tag{4}$$

Thus, total saving by the young (=the capital stock in the following period) is given by

$$K_{t+1} = S_t = s_t L_t w_t \tag{5}$$

where L_t is the number of (young) workers at time t. We assume that L_t grows at the constant rate $n \geq 0$,

$$L_{t+1} = (1+n)L_t \tag{6}$$

.

Using a CES production function without technical change, we have

$$Y_t = [\alpha K_t^{\gamma} + (1-\alpha)L_t^{\gamma}]^{1/\gamma} \tag{7}$$

[3] Lopez-Garcia (2008) provides an extension with an endogenous labor supply.

Assuming perfect competition,

$$
\begin{aligned}
w_t &= (1-\alpha)\left[\alpha\left(\frac{K_t}{L_t}\right)^{\gamma} + (1-\alpha)\right]^{(1-\gamma)/\gamma} &(8)\\
r_{t+1}+\delta &= \alpha\left[\alpha + (1-\alpha)\left(\frac{L_{t+1}}{K_{t+1}}\right)^{\gamma}\right]^{(1-\gamma)/\gamma} &(9)
\end{aligned}
$$

where δ is the rate of depreciation.

Substituting (4), (8) and (9) into (5) and dividing through by L_{t+1}, the dynamics of $k_t = K_t/L_t$ can be written

$$
\begin{aligned}
k_{t+1} &= \frac{\{1+\alpha[\alpha+(1-\alpha)k_{t+1}^{-\gamma}]^{(1-\gamma)/\gamma}-\delta\}^{(1-\theta)/\theta}}{(1+\rho)^{1/\theta}+\{1+\alpha[\alpha+(1-\alpha)k_{t+1}{}^{-\gamma}]^{(1-\gamma)/\gamma}-\delta\}^{(1-\theta)/\theta}}\\
&\quad\times\frac{(1-\alpha)[\alpha k_t{}^{\gamma}+(1-\alpha)]^{(1-\gamma)/\gamma}}{1+n} \qquad (10)
\end{aligned}
$$

In general this equation may have multiple steady-growth solutions with $k_{t+1} = k_t$, and (one or more of) the solutions may be dynamically inefficient.

2.1.1 Example: Cobb-Douglas production and logarithmic utility

Consider the case where $\theta \to 1$ (logarithmic utility) and $\gamma \to 0$ (Cobb-Douglas production function). In this case,

$$
\begin{aligned}
Y_t &= K_t^{\alpha} L_t^{1-\alpha}; \qquad 0<\alpha<1 &(11)\\
w_t &= (1-\alpha)K_t^{\alpha}L_t^{-\alpha} = (1-\alpha)k_t^{\alpha} &(12)
\end{aligned}
$$

and

$$s_t = \frac{1}{2+\rho} \tag{13}$$

Substituting (12) and (13) in (5) and dividing through by L_{t+1}, we get

$$k_{t+1} = \frac{(1-\alpha)}{(2+\rho)(1+n)}k_t^{\alpha} \tag{14}$$

This equation has a unique stationary solution

$$k^* = \left[\frac{1-\alpha}{(2+\rho)(1+n)}\right]^{1/(1-\alpha)} \tag{15}$$

and the (gross) marginal product of capital at $k_t = k^*$ is given by

$$r + \delta = \alpha \frac{(2+\rho)(1+n)}{1-\alpha} \tag{16}$$

The steady growth path is stable but, depending on the values of the parameters α and ρ, it may or may not be dynamically efficient (may or may not satisfy the condition $r > n$).

2.1.2 Example: Leontief production and logarithmic utility

If $\gamma \to -\infty$, the production function converges to the Leontief form,[4]

$$Y_t = \min\{\sigma K_t, \lambda L_t\} \tag{17}$$

In order for full-employment growth to be technically feasible, the capital stock must grow at least as fast as the labor force when all output is being invested. Algebraically,

$$\sigma K_t \geq Y_t \geq (n+\delta) K_t \tag{18}$$

or

$$\sigma \geq n + \delta \tag{19}$$

This technical feasibility condition is not sufficient, however. With a logarithmic utility function and a saving rate of $1/(2+\rho)$ out of wage income, the parameters need to satisfy the more restrictive condition

$$s_t Y_t = \frac{Y_t}{2+\rho} \geq K_{t+1} = (1+n)K_t \geq \frac{1+n}{\sigma} Y_t \tag{20}$$

or

$$\sigma \geq (1+n)(2+\rho) \tag{21}$$

We assume that this condition is met.

Turning to the determination of factor prices, perfect competition implies

[4] The general fixed coefficients form

$$Y_t = \min\{\sigma K_t, \lambda L_t\}$$

is the limiting case of the CES function

$$Y_t = [\alpha(\sigma K_t)^\gamma + (1-\alpha)(\lambda L_t)^\gamma]^{1/\gamma}$$

In equation (7) we assumed $\sigma = \lambda = 1$.

that

$$w_t = \begin{array}{lll} 0 & \text{if} & \lambda L_t > \sigma K_t \\ \lambda & \text{if} & \lambda L_t < \sigma K_t \end{array} \tag{22}$$

$$r_t + \delta = \begin{array}{lll} \sigma & \text{if} & \lambda L_t > \sigma K_t \\ 0 & \text{if} & \lambda L_t < \sigma K_t \end{array} \tag{23}$$

The economy has two (non-trivial) steady growth paths.[5] There is a full-utilization path with $\sigma K_t = \lambda L_t$ and[6]

$$w = (2+\rho)(1+n)\frac{\lambda}{\sigma} < \lambda \tag{24}$$

$$r + \delta = \sigma - (2+\rho)(1+n) > 0 \tag{25}$$

The steady growth path described by equations (24)-(25) is dynamically efficient: it follows from (19) and (23) that any reduction in the capital-labor ratio would produce a return to capital that exceeds the growth in the labor force ($\sigma - \delta > n$). The path is also unstable, however: a negative shock to w_t reduces S_t and implies that $K_t/L_t < \lambda/\sigma$ in the next period; as a result, w_t drops to 0, there is no saving, and the capital stock drops to 0.

In addition to the efficient steady growth path and the trivial path with $K_t = 0$, there is a locally stable steady growth path with less than full utilization of capital. Starting from the efficient path, a positive shock to w raises saving and capital intensity increases in the next period to give $K_t/L_t > \lambda/\sigma$. The wage rate then rises to $w_t = \lambda$ in subsequent periods, and the economy will be following a steady growth path with excess capacity:

$$k = \frac{\lambda}{(2+\rho)(1+n)} > \frac{\lambda}{\sigma} \tag{26}$$

This steady-growth path clearly is dynamically inefficient; the net return on capital is negative along this path and we have $r_t = -\delta \leq n$. Consumption could be increased by reducing investment and eliminating the excess capacity.

2.2 Fixed coefficients and imperfect competition

Empirical arguments against the relevance of dynamic inefficiency have relied on the identification of the rate of return with the marginal product of capital. This

[5] In addition to these two paths there is a trivial steady-growth solution with $K_t = Y_t = 0$.
[6] The inequalities in (24)-(25) follow from condition (21).

identification becomes invalid under imperfect competition, and the argument loses its power.

Consider the Leontief case with logarithmic utility but assume that – unlike in section 2.1 – there is imperfect competition. For simplicity assume that the markup on variable cost is constant and that firms maintain some amount of excess capital capacity. Thus,

$$Y_t = \min\{\sigma K_t, \lambda L_t\} = \lambda L_t < \sigma K_t \tag{27}$$

$$\pi_t = \frac{Y_t - w_t L_t}{Y_t} = \frac{m}{1+m} = \bar{\pi} \tag{28}$$

where π_t is the profit share and $1+m$ is the markup factor. A constant markup is consistent with profit-maximization if firms' perceived demand function is isoelastic.

Using (5), (27) and (28), and dividing through by K_t, the growth rate of the capital stock ($\hat{K}_t$) is given by

$$\hat{K}_t = \frac{K_{t+1}}{K_t} - 1 = s(1-\bar{\pi})\frac{Y_t}{K_t} - 1 = \frac{(1-\bar{\pi})u_t\sigma}{2+\rho} - 1 \tag{29}$$

where $u_t = Y_t/(\sigma K_t) \leq 1$ is the utilization rate of capital and a hat over a variable is used to denote growth rates ($\hat{x}_t = (x_{t+1} - x_t)/x_t$).

The utilization and accumulation rates are constant in steady growth, and full employment requires that $\hat{K}_t = n$. Thus, for given values of $\bar{\pi}, \sigma$ and n, equation (29) determines the steady-growth solution for utilization, u^*.[7] There is an upper bound on utilization, $u^* \leq 1$, and full-employment growth becomes impossible if there are no solutions satisfying this restriction. The restriction can be written

$$\sigma > \frac{1}{1-\bar{\pi}}(1+n)(2+\rho) \tag{30}$$

Unlike in section 2.1, the share of wages is fixed at $1-\bar{\pi}$; this tightens the feasibility condition, relative to the perfect competition case in equation (21).

Assuming that (30) is met, adjustments in the utilization rate play the same role as movements along the production function in specifications with smooth substitution; these adjustments allow full employment growth. But the dynamic inefficiency problem is brought into stark focus by fixed coefficients: for utilization rates below one, the marginal product of capital is zero, even though

[7] With fixed coefficients and a given profit share, the solution is unique, even if $\theta \neq 1$. Thus, the analysis can be extended to cover a general CIES specification of the utility function.

capital gains a positive rate of return. More generally, having profit rates that exceed the rate of growth does not imply dynamic efficiency under imperfect competition.

2.3 Investment and aggregate demand

The presence of excess capacity serves to highlight another issue. So far investment has been determined passively by household saving, making dynamic inefficiency the only downside to high saving. The problem is transformed into one of aggregate demand if the level of investment is determined by profit-maximizing firms: some amount of excess capacity may be desired for a variety of reasons, including entry deterrence (Spence 1977), but profit maximizing firms will not maintain a constant rate of accumulation if they have persistent, unwanted excess capacity. Assuming for simplicity that there is a well-defined, constant, desired rate of utilization, $u_t = u^{**}$, a steady growth path must satisfy[8]

$$1 + \hat{K}^* = s^*(1 - \bar{\pi})u^{**}\sigma \tag{31}$$

$s^* = s(\bar{\pi}u^{**}\sigma)$.[9] Equation (31) describes the equilibrium condition for the product market along a steady growth path.

Instead of dynamic inefficiency we now have a Harrodian problem of discrepancies between natural and 'warranted' growth rates, $\hat{K}^* \gtreqless n$. A low saving rate implies that $\hat{K}^* < n$ and accumulation will be insufficient to keep up with the growth in the labor force; our feasibility assumption in the previous section essentially excludes this possibility.[10] More interesting for present purposes is the case of high saving rates and $\hat{K}^* > n$. In this situation labor constraints imply that in the long run output cannot grow at the rate determined by (31). Excess capacity must emerge and there will be downward pressure on investment. A reduction in investment, however, cannot restore steady growth at

[8]Imperfect competition is critical for the uniqueness of this 'warranted growth rate', to use Harrod's terminology. As argued in section 2.1, we have $u_t = u^{**} = 1$ under perfect competition, the profit share becomes indeterminate, and equation (8) with $\hat{K} = n$ can be used to determine π_t (for $u_t = u^{**}$), rather than u_t with $\pi_t = \bar{\pi}$.

[9]Steady growth paths with a (desired) utilization rate below unity ($u^{**} < 1$) are inefficient, strictly speaking. Inefficiencies of this kind cannot be addressed using fiscal policy and are beyond the scope of the present paper.

[10]If $u^{**} < 1$ the condition (30) needs to be strengthened slightly:

$$u^{**}\sigma > \frac{1}{1 - \bar{\pi}}(1 + n)(2 + \rho)$$

a lower rate: $\hat{K}^*$ is uniquely determined by (31). The dynamics depend on the full specification of investment behavior – not just the steady growth requirement that $u_t = u^{**}$ – but the likely result is downward instability and depression.[11] Whatever the details, if $\hat{K}^* \neq n$, there is no steady growth path with full employment and equilibrium in the product market.

3 Public debt

Dynamic inefficiency problems can be overcome by introducing a public sector and public debt. Diamond (1965) analyzed a neoclassical case with smooth substitution; we focus on the Keynesian case with imperfect competition and a Leontief production function.

3.1 Adding a public sector

Extending the model, we introduce a government that consumes (G_t), levies lumpsum taxes on the young and old generations (T_t^Y and T_t^O) and has debt (B_t).[12] Young households save in the form of fixed capital and government bonds. We assume that these assets are perfect substitutes and have the same rate of return, r_t.

The saving equation (5) now takes the form

$$K_{t+1} + B_{t+1} = S_t \tag{32}$$

while the public sector budget constraint is given by

$$G_t + (1 + r_t)B_t = B_{t+1} + T_t^Y + T_t^O \tag{33}$$

Returning to a general CIES specification of the utility function, the young generation in period t maximizes (1) subject to a modified constraint,

$$c_{1,t} + \frac{1}{1 + r_{t+1}} c_{2,t+1} = w_t - \tau_t - \frac{1+n}{1 + r_{t+1}} \gamma_{t+1} \tag{34}$$

[11] A simple Harrodian specification assumes that firms respond to deviations of u from u^{**} by gradually changing their accumulation rate:

$$\hat{K}_{t+1} - \hat{K}_t = \phi(u_t - u^{**}); \qquad \phi' > 0$$

This specification leads to instability of the warranted path.

[12] Since the labor supply is taken to be inelastic, it does not matter whether the taxes on the young are lumpsum or based on wage income.

where $\tau_t \equiv T_t^Y/L_t$ and $\gamma_t = T_t^O/L_t$. This gives the following solution for saving

$$S_t = \left[s_t(w_t - \tau_t) + (1 - s_t)\frac{1+n}{1+r_{t+1}}\gamma_{t+1} \right] L_t \tag{35}$$

where s_t is given by (4). Alternatively, saving can be written

$$S_t = \tilde{s}_t(w_t - \tau_t)L_t \tag{36}$$

where the young generation's saving rate out of the current disposable income ($\tilde{s}_t$) is given by

$$\tilde{s}_t = \frac{s_t(w_t - \tau_t) + (1 - s_t)\frac{1+n}{1+r_{t+1}}\gamma_{t+1}}{w_t - \tau_t} \tag{37}$$

Using (36)-(37) and dividing through by L_t, (32)-(33) can be rewritten,

$$(1+n)(k_{t+1} + b_{t+1}) = s_t(w_t - \tau_t) + (1 - s_t)\frac{1+n}{1+r_{t+1}}\gamma_{t+1} \tag{38}$$

$$= \tilde{s}_t(w_t - \tau_t) \tag{39}$$

$$g_t + (1 + r_t)b_t = (1+n)b_{t+1} + \tau_t + \gamma_t \tag{40}$$

where $g_t \equiv G_t/L_t, b_t \equiv B_t/L_t, k_t = K_t/L_t$.

3.2 Steady growth: the Leontief case with imperfect competition

By definition $w_t = (1 - \pi_t)\lambda$ and $r_t = \pi_t u_t \sigma$, and steady growth requires that $b_t = b, \pi_t = \bar{\pi}, u_t = u^{**}$. Substituting these conditions into (38), using (40), and rearranging, we get

$$b = \frac{s^*(1-\bar{\pi})\lambda - (1+n)k^*}{1+n+s^*(r^*-n)} + \frac{1}{1+r^*}\gamma - \frac{s^*}{1+n+s^*(r^*-n)}g \tag{41}$$

where $r^* = \bar{\pi}\sigma u^{**}$ and $k^* = 1/(\sigma u^{**})$, and where $s^* = s(r^*)$ is determined by (4).

The required debt (b) depends *inversely* on public consumption (g) and *directly* on the level of taxes on the old generation (γ).[13] For any given γ, an increase in g implies that consumption has to contract in order to maintain

[13] An inverse relation between debt and government consumption is obtained in a non-OLG setting by Schlicht (2006) and Ryoo and Skott (2012).

equilibrium in the product market. This is achieved by increasing taxes on the young. As a result the desired saving decreases and this, in turn, reduces the need for government debt as an outlet for saving. Analogously, with a given value of g, an increase in γ must be accompanied by a reduction in τ in order to maintain the level of consumption and equilibrium in the goods market; the disposable income of the young increases, and the amount of public debt must also increase to meet the rise in saving.

The relation between public debt and economic growth has received attention recently (e.g. Reinhart and Rogoff 2010). It is interesting to note therefore that the required debt is inversely related to the growth rate n for empirically relevant values of the parameters; the partial $\partial b/\partial n$ is negative if $k+(1-s^*)(b-\gamma/(1+r^*)) > 0$. This result is quite intuitive. The reason for the debt is that the young generation wants to save 'too much'. But the threshold defining 'too much' depends on the growth rate: a higher growth rate implies that more fixed capital will be needed to employ the future generation and, consequently, that the required amount of public debt will be lower.

Equations (38) and (40) can also be used to derive the steady-growth solution for the tax rate τ :

$$\tau = \frac{(r^*-n)[s^*(1-\bar{\pi})\lambda-(1+n)k^*]}{1+n+s^*(r^*-n)} - \frac{1+n}{1+r^*}\gamma + \frac{1+n}{1+n+s^*(r^*-n)}g \qquad (42)$$

Thus, an increase in g raises τ (but reduces b) while an increase in γ reduces τ (but raises b). It follows that shifts in g or γ produce a negative correlation the steady-growth values of τ and b. Other parameter shifts yield a positive correlation; an increase in the wage share, for instance, will raise both b and τ if $r^* > n$.

3.3 Fluctuations in 'confidence'

The analysis can be extended to cover the effects of fluctuations in saving rates. Assume that the saving rate fluctuates across generations. These variations could be the result of variations in the discount rate across generations, but variations in 'confidence' or expected returns can do the trick too.[14] Thus, assume (in line with standard assumptions) that $\theta > 1$ and note that the saving rate at period t depends on the young generation's expected rate of return in the

[14] Deleveraging and asymmetries between debtors and creditors can also lead to changes in saving rates in a more general setting (Eggertsson and Krugman (2012)). Our simple OLG structure with saving by the young and dissaving by the old precludes these effects.

next period. Abandoning the assumption of perfect foresight and using r^e_{t+1} to denote the rate of return in period $t+1$ that is expected by the young generation in period t, we can re-write equations (37) as

$$\tilde{s}_t = \frac{s_t(w_t - \tau_t) + (1 - s_t)\frac{1+n}{1+r^e_{t+1}}\gamma_{t+1}}{w_t - \tau_t} \tag{43}$$

where

$$s_t = \frac{(1 + r^e_{t+1})^{(1-\theta)/\theta}}{(1 + \rho)^{1/\theta} + (1 + r^e_{t+1})^{(1-\theta)/\theta}} \tag{44}$$

From (43) and (44) it follows that $\tilde{s}_t$ is decreasing in r^e_{t+1} when $\theta > 1$. A pessimistic outlook leads to a high saving rate (and if the capital stock were to increase as a result, the realized return would in fact be reduced, thus partially vindicating the pessimistic expectations). Our primary concern in this paper, however, is not the sources of fluctuations in saving. To simplify the analysis, we therefore consider the effects of exogenous fluctuations in the young generation's saving rate out of current disposable income ($\tilde{s}_t$).

3.3.1 Distributionally neutral intervention

The effects of fluctuations in the saving rate on employment (in the Keynesian setting) or dynamic efficiency (in the non-Keynesian setting) can be offset by a distributionally neutral policy intervention: institute a transfer to those young generations that are unduly pessimistic (tend to consume too little) and finance the transfer by taxing the same generation when it gets old. The transfer increases the saving of the young generation as well as its consumption. But the additional saving will be more than fully absorbed by the issue of government bonds, thus leading to a decline in the amount of saving that has to be matched by fixed investment. Conversely, an overly optimistic generation can be taxed in the first period and compensated by a transfer in the second.

The aim is to achieve $u_t = u^{**}$ and $k_t = k^* = 1/(\sigma u^{**})$ so as to maintain full employment and avoid inefficiency. Using these targets and substituting (39) into (40), we get

$$\tau_t = \frac{(1 + n)k^* + g + (1 + r^*)b_t - \gamma_t - \tilde{s}_t(1 - \bar{\pi})\lambda}{1 - \tilde{s}_t} \tag{45}$$

The tax on the old generation (γ_t) and the public debt (b_t) appear on the right hand side of equation (45), and these variables still need to be determined.

Distributional neutrality means that a generation should not be favored (or punished) because of its degree of confidence. If b^* and γ^* denote the steady-growth values of b and γ along the optimal path when there are no variations in confidence, this requirement can be stated formally as

$$(1+r^*)(k^*+b_{t+1}) - \gamma_{t+1} = (1+r^*)(k^*+b^*) - \gamma^* \tag{46}$$

The expression on the left hand side of equation (46) gives the income available to an old generation in period $t+1$. Neutrality requires that this income be equal to the level that characterizes the steady growth path. The stabilization of output and the consumption of the old generation at their steady-growth values implies that the consumption of the young will also be at its steady-growth value.

Using (45) and (46), the equation for the tax on the young at time t can now be written

$$\tau_t = \frac{(1+n)k^* + g + (1+r^*)b^* - \gamma^* - \tilde{s}_t(1-\bar{\pi})\lambda}{1-\tilde{s}_t} \tag{47}$$

Hence,

$$\frac{\partial \tau_t}{\partial \tilde{s}_t} = -\frac{(1-\bar{\pi})\lambda - \tau_t}{1-\tilde{s}_t} \tag{48}$$

As long as the disposable income of the young is positive ($(1-\bar{\pi})\lambda - \tau_t > 0$), we have $\frac{\partial \tau_t}{\partial \tilde{s}_t} < 0$.

3.3.2 Tax expectations

The above analysis uses systematic variations in τ_t and γ_{t+1} to get distributional neutrality across generations. We have taken the saving rate $\tilde{s}$ as exogenous, however. This combination of assumptions may seem unreasonable since in general the saving rate depends on the tax structure. The private sector's anticipation of future taxes does not, however, negate the possibility of distributionally neutral stabilization.

Taxes can be used as an insurance mechanism when future taxes are anticipated. Consider the extreme case where taxes are perfectly foreseen. Formally, let γ_{t+1} be determined by

$$(1+n)\gamma_{t+1} = (1+n)\gamma^* + \beta(1+r_{t+1})(w_t - \tau_t - c_{1,t}) \tag{49}$$

$$\beta = \frac{r_{t+1} - r^*}{1+r_{t+1}} \tag{50}$$

The tax scheme in (49)-(50) combines lumpsum taxes (γ^*) with a proportional tax on capital income (β). The proportional tax rate is conditional on the realized rate of return, and the conditionality ensures that the private after-tax rate of return always will be r^*. Thus, using (49)-(50), the household budget constraint (34) can be rewritten

$$\begin{aligned} c_{2,t+1} &= (w_t - \tau_t - c_{1,t})(1 + r_{t+1}) - (r_{t+1} - r^*)(w_t - \tau_t - c_{1,t}) - (1+n)\gamma^* \\ &= (1 + r^*)(w_t - \tau_t - c_{1,t}) - (1+n)\gamma^* \end{aligned} \tag{51}$$

The budget constraint becomes independent of 'confidence'. Consequently, the tax rate on the young should be set at the steady-growth level, $\tau_t = \tau^*$, and no variations are required. What happens is that the conditional tax scheme provides insurance and effectively guarantees that the rate of return will be at the steady-growth value.[15]

The assumptions underlying this example may be implausible. The tax scheme may not be 'credible' and the perceived budget constraint could differ from (51), even if policy makers were to follow (49)-(50). Or putting it differently, a combination of confidence effects and perfect anticipation of future taxes – as in the above example – may seem even more questionable than the more common Ricardian assumptions of prefect foresight with respect to both taxes and future returns. In response to this objection, one can take one of two routes: assume that there are no variations in confidence or, alternatively accept that variations in confidence do occur and that households do not fully anticipate future taxation. The first route we will leave to others; the second can be approached by examining – as in equation (46) – how variations in the saving rate can be neutralized by taxation.

3.3.3 Non-neutral intervention

The analysis may be subject to another objection. Sections 3.2.1-3.2.2 assume that the private sector is subject to swings in confidence but that the government correctly anticipates the rate of return on capital and has the ability to

[15] Intervention is also desirable in the case where the fluctuations in consumption derive from intergenerational differences in discount rates. In this case, the tax regime does not serve as an insurance scheme. Instead, the 'distortionary effects' of taxes on capital income can be used to stabilize the capital stock and avoid dynamic inefficiencies, even if private agents have perfect foresight. It should be noted that intra-generational discount rates have no direct bearing on the appropriate intergenerational discount rate and therefore cannot determine the socially optimal trajectory for k_t.

implement fairly sophisticated tax schemes. Intergenerational neutrality and perfect government foresight are not required, however, for the stabilization of k_t at k^*.

Without foresight, policy makers may not be able to implement a neutral insurance scheme but that does not preclude the stabilization of k at k^*. To see this, consider the simple case in which capital income is taxed at a constant (time-invariant) rate (β),

$$\gamma_t = \beta(k_t + b_t)(1 + r_t) \tag{52}$$

Unlike in section 3.2.2, β need not satisfy (50) and – returning to the case without private sector anticipation of future taxes – we take the variations in the young's saving rate out of current disposable income ($\tilde{s}_t$) to be exogenous.[16] By assumption the profit share is constant ($\pi_t = \bar{\pi}$) and the policy is designed to keep $k_t = k^*$ (and thus $r_t = r^*$).

Combining these assumptions and (52) with equations (39) and (40), the debt dynamics can be written

$$b_{t+1} = A_t - B_t b_t \tag{53}$$

where

$$\begin{aligned} A_t &= \frac{\tilde{s}_t[(1-\bar{\pi})\lambda - g + \beta(1+r^*)k^*] - k^*(1+n)}{(1+n)(1-\tilde{s}_t)} && (54) \\ B_t &= (1-\beta)\frac{1+r^*}{1+n}\frac{\tilde{s}_t}{1-\tilde{s}_t} && (55) \end{aligned}$$

For a constant value of $\tilde{s}$ and a sufficiently large tax rate β, the difference equation (53) has a unique, stable stationary point,

$$b^{**} = \frac{A}{1+B} \tag{56}$$

Random fluctuations in $\tilde{s}_t$ generate fluctuations in A_t and B_t (and thereby in b_t, also in the long run).[17] But if the ratio $A_t/(1 + B_t)$ is bounded then so are

[16] With the proportional tax on capital income the relation (36) between $\tilde{s}_t$ and s_t implies – using (38) – that

$$\tilde{s_t} = \frac{s_t}{1-(1-s_t)\beta}$$

Hence, the argument could also be phrased in terms of exogenous movements in s.

[17] Other non-neutral schemes could be used, including one with a balanced government budget at all times. Continuous full employment ($k_t = k^*$) could be maintained without

the fluctuations in b_t.

4 Monetary policy and the choice of technique

As shown in section 2.1, a Leontief production function does not exclude market clearing and dynamic efficiency under perfect competition. Moreover, even if a smooth production function may allow long-run changes in capital intensity, it seems doubtful that variations in capital intensity can maintain full employment in response to swings in 'confidence', as exemplified by decreasing consumption rates following the collapse of a housing bubble. Thus, the fixed coefficient assumption which we used throughout section 3 can be defended on grounds of realism and relevance as well as analytical simplicity.

The Leontief assumption can also be justified along lines that are consistent with Lerner's analysis of functional finance. Monetary policy, Lerner argued, should be used to set interest rates at levels that induce an optimal amount of investment. In a long-term context, this criterion translates into interest rates that produce an optimal capital-output ratio. Thus, the fixed coefficients of the Leontief production function can be seen as the outcome of a profit maximizing choice of technique for a given cost of external finance (a given real rate of interest) and a given (profit maximizing) markup on marginal cost (Skott 1989, chapter 5).

Assume, for simplicity, that the smooth long-run production function is Cobb-Douglas,

$$Y_t = K_t^{\alpha} L_t^{1-\alpha} \tag{57}$$

Let W_t and P_t be the money wage rate and the price of capital goods, and let i and δ denote the cost of finance (the real rate of interest) and the rate of depreciation. Cost minimization requires

$$\begin{aligned} \min_{L_t, K_t} \quad & W_t L_t + (i+\delta) P_t K_t \\ s.t. \quad & (u^{**} K_t)^{\alpha} L_t^{1-\alpha} = Y_t \end{aligned} \tag{58}$$

where u^{**} is the expected average (= desired) utilization rate of the capital stock.

government deficits by taxing the pessimistic young and transferring the tax revenue to the currently old generation.

From the constraint in (58), $K_t = Y_t \lambda_t^{\frac{1-\alpha}{\alpha}} / u^{**}$ where $\lambda_t = Y_t / L_t$, and substituting this expression into the objective function, the minimization programme can be rephrased as

$$\min_{\lambda_t} \quad W_t \lambda_t^{-1} + (i+\delta) P_t \frac{1}{u^{**}} \lambda_t^{(1-\alpha)/\alpha} \tag{59}$$

The first order condition yields

$$\lambda_t = \left(\frac{u^{**}}{i+\delta} \frac{\alpha}{1-\alpha} \frac{W_t}{P_t} \right)^{\alpha} \tag{60}$$

The price of capital goods, P_t, may be exogenous to the individual firm but in a one-good model this price must be equal to the general price level in equilibrium. A simple mark-up rule (as in section 3) implies

$$P_t = (1+m) W_t \frac{1}{\lambda_t} \tag{61}$$

Combining equations (60)-(61) we have

$$\lambda_t = \left[\left(\frac{\alpha}{1-\alpha} \right) \left(\frac{u^{**}}{i+\delta} \right) \left(\frac{1}{1+m} \right) \right]^{\alpha/(1-\alpha)} \tag{62}$$

Thus, the choice of technique is fully determined by i, u^{**} and m. Cost minimization produces one relation between λ_t and W_t/P_t; pricing decisions give another relation. In equilibrium these two relations – equations (60) and (61) – must be mutually consistent. This consistency requirement fixes the real wage and the coefficients of the Leontief production function.

Assuming, for simplicity, that firms raise finance by issuing (short) bonds, the model now contains two financial assets, and it is reasonable to add another two, cash and equity. In advanced economies households typically do not own physical capital directly; ownership takes the form of equity shares. Thus, household portfolios consist of four financial assets, money, equity and two types of bonds. If we disregard risk, private agents will only want to hold non-interest bearing cash for transactions purposes; abstracting from this transactions demand cash holdings must be zero in equilibrium. absence of risk also implies that equity and the two bonds become perfect substitutes. These three assets must carry the same rate of return, and the household portfolio choice leaves the composition of the portfolio undetermined. The rate of return is determined by monetary policy: the central bank determines the return on bonds through its open market operations,[18] and the equality between the bond rate and the rate of return on equity is ensured via the valuation of shares. Appendix A provides the details.

A particular 'zero-debt natural rate of interest' could be defined as the rate which yields a choice of technique such that no public debt is required along a full employment growth trajectory. In the Cobb-Douglas case, for instance, the required capital-labor ratio is given by equation (15) . By definition $k_t = \lambda_t^{1/\alpha}/u^{**}$ and, using (62), the solution for i becomes

$$i = (u^{**})^{\alpha} \frac{\alpha}{(1-\alpha)^2} \frac{(2+\rho)(1+n)}{1+m} - \delta \tag{63}$$

The zero-debt natural rate of interest may imply dynamic inefficiency; in fact, the rate may be negative. In the latter case, full-employment growth with zero public debt will require a positive inflation rate; the economy suffers from a 'structural liquidity trap' (Nakatani and Skott 2007, Skott 2001). Putting it differently, the interest rate that is consistent with full-employment growth – the natural rate of interest – depends on fiscal policy and the debt ratio. Functional finance can be interpreted as an attempt to (i) identify the optimal capital intensity, (ii) set the interest rate at the associated level through an appropriate monetary policy, and (iii) use fiscal policy to make the natural interest rate equal to this optimally chosen rate.

5 Discussion

5.1 Public debt, interest rates and economic growth

An exogenous rise in debt will be associated with a fall in the capital stock and an increase in the return on capital in standard OLG models. A functional finance approach to fiscal policy makes this result irrelevant. Fiscal policy and the level of debt adjust endogenously: debt is allowed to increase if an increase is necessary to maintain both full employment and the optimal capital intensity.

[18] Since the private sector holds no cash when the interest rate is positive, the equilibrium net position of the central bank must also be zero for any positive interest rate.

Bank loans to firms and household bank deposits could be used instead of or in addition to corporate bonds. Assuming, for simplicity, that there are no costs in banking, that there is free entry and banks make no profits, and that households hold money only in the form of bank deposits, then total bank loans must equal total deposits and the same interest rate will apply to deposits and loans.

A perfectly executed fiscal policy of this kind would show fluctuations in debt (as in section 3.3) with a constant rate of interest.

Of course, fiscal policy may not always be conducted following the principles of functional finance – the current obsession with austerity testifies to that – but the result carries important implications for empirical evaluation: observed correlations between interest rates and debt depend on the interaction between policy regimes and variations in private sector behavior. Without knowledge of the sources of changes in the public debt, there is no way to predict the empirical correlation between debt and interest rates. Thus, it is not surprising that the results of empirical studies are weak and inconclusive.[19]

The potential OLG link between high debt, low levels of capital (and income) and high interest rates represents a level, rather than a growth effect of debt. The transition to a new level involves temporary changes in the growth rate, and the possibility of a debt - growth link has received great attention following the publication of Reinhart and Rogoff (2010) and Kumar and Woo (2010).[20] The theoretical story behind this link is a little unclear and theoretical ambiguities accentuate the difficulty of interpreting empirical results.[21]

Accepting, for the sake of the argument, that a negative correlation can be found between debt and economic growth, a key question concerns causation. This question has two parts. The first part asks whether actual past episodes of rising debt did in fact cause a decline a growth (as opposed to a reverse causal link between the two variables or an explanation in which a third factor accounts for the changes in both debt and growth). This part of the question has been addressed by Irons and Bivens (2010) who argue that empirically causation has run from growth to debt. Their analysis considers short and medium term effects of a slowdown in growth on deficits and debt. But as shown in section 3.2, functional finance may produce a long-run causal link between growth rates

[19] In the words of Engen and Hubbard (2005, p.83), there is "little empirical consensus about the magnitude of the effect ... some economists believe there is a significant, large, positive effect of government debt on interest rates, others interpret the evidence as suggesting that there is no effect on interest rates". Bohn (2010, p.14) makes a similar statement about the difficulty of finding significant interest effects of debt. He goes on to suggest that a "leading explanation is Ricardian neutrality". There is no need for Ricardian neutrality to explain the results, however; our OLG model does not display neutrality.

[20] A debt - capital link would seem to imply a deficit - growth link, however; not a debt - growth nexus.

[21] Kumar and Woo mention a number of possible channels, including the effect of higher interest rates on capital accumulation, and the potential effects of debt induced increases in "uncertainty about prospects and policies". As discussed above, the evidence on a debt - interest rate link is tenuous, at best. The latter effect seems to be a close cousin of what Krugman has been referring to as the 'confidence fairy', and it is hard to see how contractionary fiscal policies will enhance confidence in a recession.

and debt: a reduction in the long-run rate of growth will tend to produce an increase in the long-run debt ratio.

The second part of the question is more radical. One may ask whether it is at all meaningful to look for a general answer to a reduced-form question about the growth effects of public debt. A particular example of this general problem is sometimes used to suggest the seriousness of the current debt problem. According to Rogoff and Reinhart (2010, p. 6),

> . . .war debts are arguably less problematic for future growth and inflation than large debts that are accumulated in peace time. Postwar growth tends to be high as war-time allocation of man-power and resources funnels to the civilian economy. Moreover, high wartime government spending, typically the cause of the debt buildup, comes to a natural close as peace returns. In contrast, a peacetime debt explosion often reflects unstable underlying political economy dynamics that can persist for very long periods.

As pointed out by Michl (2011):

> To a Keynesian, the quote above would very sensibly read "high recession-time government spending, typically the cause of the debt buildup, comes to a natural close as growth returns." (In fact, Keynes (1972, p. 144) once aptly described government borrowing as "nature's remedy" for preventing a recession from deteriorating into a total collapse in production.) As for the rest of the quote, who would deny that "unstable political dynamics" can be an obstacle to growth?

The general point is simple. A fiscal expansion is intrinsically neither good nor bad. Appropriate fiscal policy – and appropriate movements in debt – depend on the economic circumstances. Reduced-form correlations – whether in a simple bivariate analysis or from growth regressions that include other controls – depend on the underlying sources of the movements in debt. A reckless fiscal expansion can cause overheating, inflation and macroeconomic instability. But these effects of bad policy say little about the growth effects of fiscal expansion in a deep recession.

5.2 Public debt and intergenerational distribution

Claims that high public debt hurts future generations have figured prominently in popular debates and also appear in the academic literature. Having found that public debt has at most small effects on interest rates, Engen and Hubbard (2005), go on to caution that public deficits and debt still matter because large levels of government debt "can represent a large transfer of wealth to finance current generations' consumption from future generations which much eventually pay down federal debt to a sustainable level." (p. 132)

The *possibility* that fiscal policy can hurt future generations is not controversial; inappropriate fiscal policy can have negative effects for future as well as for current generations. But our analysis of an OLG model without bequests – the setting that is most favorable to the case for adverse future effects of public debt – shows that debt need not be a burden on future generations. On the contrary, it can serve to remove dynamic inefficiencies and maintain full employment. Moreover, fluctuations in 'confidence' can be addressed through policies that are neutral in the effects on the intergenerational distribution. Even when a policy is not fully neutral in this sense, future generations may be better off than without the policy. With a fixed tax rate on capital income, for instance, the required variations in the tax on the young generation will have distributional effects: a pessimistic generation will be favored by a reduction in its taxes (section 3.3.3). This result does not imply that future generations would be better off without the reduction, however. In the absence of fiscal expansion, a lack of demand would affect utilization, reduce investment and the future capital stock, and jeopardize employment.

5.3 Austerity and long term consolidation

Austerity programs often include reductions in entitlement programs like social security or medicare. These reductions can clearly be distributionally regressive. Surprisingly, perhaps, they may also aggravate the 'debt problem': the changes correspond to a rise in the tax on the older generation, and as shown in section 3.2, this increases the debt that is required to maintain dynamic efficiency and full employment. A reduction in government consumption (g), likewise, requires an increase in the long-run debt. The general point, once again, is that the desirable level of public debt depends on a range of behavioral and policy variables.

These results suggest that from a functional finance perspective Krugman's powerful critiques of austerity may be insufficiently radical. His insistence that the slump is not the time to cut the debt is fully in line with functional finance[22] but Krugman also suggests that the US has long run budget problems that must be addressed once we are out of recession.[23] The nature of the long run debt problem is not made clear however. This is not to say that there can be no adverse consequences of high public debt. But these consequences have to be clearly specified and balanced against the benefits.

6 Conclusion

Are the current debt levels and fiscal deficits sustainable? It is not always clear what is meant by sustainability, but the question may be whether the fiscal requirements for full employment growth will generate an ever-increasing debt-GDP ratio.[24] The analysis in this paper says no. It suggests that fiscal policy and public debt may be needed to avoid dynamic inefficiency and maintain full employment, and that this fiscal policy need not and in our OLG model do not lead to any kind of unsustainability.[25] We have shown, moreover, that this fiscal policy need not produce adverse effects on interest rates or economic growth. Nor does it necessarily affect the inter-generational distribution: fluctuations in household 'confidence' can be addressed through distributionally neutral policies.

The model is abstract and has obvious limitations. Most prominently, perhaps, we have assumed a closed economy. Open (and local) economies are in a very different position than sovereign countries that control their own currency; this paper says nothing about the open-economy issues.[26] A second obvious

[22] Jayadev and Konczal (2010) and Guajardo et al. (2011) challenge the Alesina and Ardagna (2010) claim of expansionary austerity.

[23]

> Yes, the United States has a long-run budget problem. Dealing with that problem is going to require, first of all, sharply bending the curve on Medicare costs; without that, nothing works. And second, it's going to require some combination of spending cuts and revenue increases, amounting to at least 3 percent of GDP and probably more, on a permanent basis. (Krugman, http://krugman.blogs.nytimes.com/2010/07/21/notes-on-rogoff-wonkish/)

[24] A convergent debt-income ratio is closely related to the "S2 indicator" of sustainability; see, e.g., Andersen (2012).

[25] As shown by Ryoo and Skott (2012), functional finance can produce unstable debt-income dynamics in settings with intra-generational heterogeneity. These unstable scenarios are closely linked to (intra-generational) distribution effects and can be avoided by changes in the structure of taxation.

[26] Chalk (2000, p. 319) argues that some OECD countries "have seen an explosion in their

limitation is the neglect of heterogeneity within generations and questions of intra-generational distribution. Public debt may have regressive distributional effects if taxes on wage income are used to finance interest payments to the rich. The incentive effects of taxes, third, have been ignored throughout. A higher level of debt need not be associated with higher tax rates but even if it is, the scale of government consumption and the form of taxation may be more important than the level of debt for the public sector's incentive effects.[27] Fourth, the model only indirectly addresses inflationary concerns. Engen and Hubbard (2005) suggest that "federal government debt may also pose the temptation to monetize the debt, causing inflation". They point out, however, that "this concern has not been a problem in the United States over the past fifty years" (p. 98); Reinhart and Rogoff (2010) also find no evidence for a link between debt and inflation in advanced economies. The inflation fear essentially boils down to a concern that policy may not in the future be governed by a functional finance criterion: "eliminate both unemployment and inflation" (Lerner 1943, p. 41). Fifth, the size of the public debt influences the effectiveness of monetary policy. A contractionary monetary policy raises interest rates and generates an automatic fiscal expansion unless it is matched by an increase in tax rates. Thus, monetary policy is blunted when debt is high and this may complicate short-run economic policy. The simple OLG structure, sixth, may be appealing for an analysis of public debt, but it has peculiar properties that find no support in data. The model implies that the saving rate is inversely related to the profit share: only the young save, and the young get their income as wage income. Empirically, however, saving rates are higher out of profits than wages. Thus, the saving assumptions that are at the center of the analysis in OLG models can be questioned.[28]

Our analysis, finally, has taken as given the level of government spending. Public investment in environmental areas, infrastructure, education and health clearly contribute to future welfare, and many items of public 'consumption' and social spending can have a high future payoff – even in narrow economic terms

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indebtedness to such an extent that the solvency of the public sector is brought into question". Solvency questions of this kind may be relevant for countries with debt in foreign currency. But it is unclear how a sovereign state could ever become insolvent if its debt obligations are denominated in a currency that it can print at will.

[27] The importance of incentives for growth are disputed. Fast growth during the 'golden age' was associated with high marginal tax rates in the US, and the Nordic welfare states show a very respectable performance, including low unemployment and labor force participation rates that exceed those of the US.

[28] We abandon these assumptions in Ryoo and Skott (2012) which builds on the 'stock-flow consistent' framework from Skott (1989) and Skott and Ryoo (2008).

– from lower crime and higher future earnings. The benefits and distributional effects of public spending could justifiably be ignored in the debt discussion if government investment (and other spending) were already at an agreed-upon optimal level. A good deal of the debate over public debt, however, may reflect underlying controversies over the desirable level of public spending. These issues are beyond the scope of this paper.

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Appendix A:

Using M and V to denote the amount of corporate bonds and the value of corporate equity and assuming a constant price of output (normalized to one), Tobin's (average) q is given by

$$q = \frac{M+V}{K} \tag{64}$$

Assume for simplicity that firms distribute all profits and finance investment exclusively through corporate bonds. Thus,

$$K_{t+1} = M_{t+1} \tag{65}$$

By assumption the rate of return on investment in corporate assets must be equal to the interest rate i. Hence,[29]

$$(1+i)(M_{t+1}+V_{t+1}) = Y_{t+1} - w_{t+1}L_{t+1} + (1-\delta)K_{t+1} + V_{t+2} \tag{66}$$

or

$$\begin{aligned} (1+i)q_{t+1} &= (1+i)\frac{M_{t+1}+V_{t+1}}{K_{t+1}} = \frac{Y_{t+1} - w_{t+1}L_{t+1} + (1-\delta)K_{t+1} + V_{t+2}}{K_{t+1}} \\ &= (1+R) + \frac{V_{t+2}}{K_{t+1}} = (1+R) + \frac{(M_{t+2}+V_{t+2}) - M_{t+2}}{K_{t+2}}\frac{K_{t+2}}{K_{t+1}} \\ &= 1 + R + (q_{t+2}-1)(1+n) \end{aligned} \tag{67}$$

where R is the rate of return on capital and – using (62) – $R = R(i)$.[30] In steady growth we have $q_t = q$ and (67) reduces to

$$(1+i)q = 1 + R(i) + (q-1)(1+n) \tag{68}$$

or

$$q = \frac{R(i)-n}{i-n} \tag{69}$$

[29] M_{t+1} and V_{t+1} denote the debt and equity valuation at the beginning of period $t+1$.

[30] Using (62) and $\sigma = \lambda^{\frac{\alpha-1}{\alpha}}$, we have

$$R = \bar{\pi}u^{**}\sigma - \delta = \bar{\pi}u^{**}\lambda^{\frac{\alpha-1}{\alpha}} - \delta = \left(\frac{\bar{\pi}}{1-\bar{\pi}}\right)\left(\frac{1-\alpha}{\alpha}\right)(i+\delta) - \delta \equiv R(i)$$

We have $R(i) \geq i$ and the rate of interest must exceed the growth rate to avoid dynamic inefficiency; with this restriction, equation (69) implies $q \geq 1$ and $\partial q/\partial i < 0$.

Household wealth takes the form of financial assets, and the saving equation (32) can be written

$$B_{t+1} + M_{t+1} + V_{t+1} = S_t \tag{70}$$

Dividing through by L_t and using (36), we get the following modified version of (38)

$$(1+n)(b_{t+1} + q_{t+1}k_{t+1}) = \tilde{s}_t(w_t - \tau_t) \tag{71}$$

Equation (40), with r_t replaced by i, still holds and and the steady growth solution for b becomes

$$b = \frac{s^*(1-\bar{\pi})(k^*u^{**})^\alpha - (1+n)qk^*}{1+n+s^*(i-n)} + \frac{1}{1+i}\gamma - \frac{s^*}{1+n+s^*(i-n)}g \tag{72}$$

where s^*, k^* and q are determined by the interest rate i; s^* via equation (4), k^* via the choice of technique and q via (69).