Light Quark Masses and Mixing Angles

LIGHT QUARK MASSES AND MIXING ANGLES

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ABSTRACT

I review the present state of our knowledge about the masses and weak mixing elements of the u, d, s quarks. This is the written version of lectures given at the 1993 Theoretical Advanced Study Institute (TASI).

1 Introduction

The Standard Model is clearly one of the triumphs of modern science. However one of the less pleasant aspects of the theory is that it contains so many free parameters. Some of these parameters form the topic of these lectures, namely the masses m_u, m_d and m_s and the weak mixing elements V_{ud} and V_{us}. Within the model, all are products of the Higgs sector. They seem to be almost arbitrary numbers, but perhaps they are clues as to the structure beyond the Standard Model. Perhaps someday we will learn to decode these clues.

There is also a second topic hidden below the surface in these lectures, i.e., how to make reliable calculations at low energy. We will see that V_{ud} is known to 0.1%, V_{us} to 1% and at least one mass ratio to 10%. For the physics of hadrons these accuracies are remarkably good. [For example, $\alpha_s(M_z)$ is also only known to 10%]. The key is the use of symmetries as a dynamical tool. In particular, we will be using chiral perturbation theory. While we do not have the time for a full pedagogical presentation of this [1,2], we will see what it is and how it is used.

My approach here will reserve the heavy formalism as long as possible. I will treat quark masses crudely at first in order to get a basic feel for them with a minimum of formalism. Following that is the description of V_{ud}. Before proceeding on to describe the extraction of V_{us}, I will spend some time introducing chiral perturbation theory. Finally I return to quark masses and try to be as precise as possible.

2 Quark Masses I

Before turning to my real topic, we need to have a brief digression on 'constituent' vs. 'Lagrangian' or 'current' masses. The Lagrangian of QCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} + \bar{\psi}(iD - m)\psi \tag{1}$$

is a nonlinear field theory which contains small mass parameters m_u, m_d, m_s. Because these masses are small, the theory is almost chirally symmetric, as well as almost classically scale invariant. Masses also enter into the quark model of hadron structure, with

$$\mathcal{H}_{QM} = \sum_i \frac{p_i^2}{2M_i} + V(r_1 - r_2) \tag{2}$$

Given that this is also supposed to represent the strong interactions, it is remarkable how far this is from QCD. The mass parameters are large, $M \sim m_p/3$, and there is no trace of the symmetries of QCD. The large mass of the quark model has very little relation to the mass in the Lagrangian. The former is commonly referred to as a 'constituent' mass. Our topic here concerns only the mass parameters in the Lagrangian. In many ways these are defined by the symmetry properties and they are called 'current' (i.e., from divergences of Noether currents) or 'Lagrangian' masses.

Our first task is to learn to treat quark masses in the same way that we do coupling constants. Our mass parameters are not inertial masses of hadrons, and because of confinement one cannot find any poles in quark propagators. How then can we come up with a way to actually measure masses? The procedure is the same as with coupling constants. Observables depend on the masses, i.e.,

$$\begin{aligned} M &= M(m) \\ &= M_0 + am + bm^2 + \ldots \end{aligned} \tag{3}$$

We measure the quark mass m by its effect on observables. But we have a problem; we cannot reliably calculate observables at low energy, and so it is tough to learn how masses influence the observables. It is here that symmetry comes to the rescue. There will be exact relations between observables in the symmetry limit. Quark masses break the symmetry and disturb these

relations. That means that the deviations from the symmetry predictions are measures of quark mass. In the most basic of examples we will see that the pion and kaon masses start off as

$$\begin{aligned} m_\pi^2 &= 0 + (m_u + m_d)B_0 + \ldots \\ m_{K^+}^2 &= 0 + (m_u + m_s)B_0 + \ldots \end{aligned} \tag{4}$$

where B_0 is same constant. This lets us measure the ratio

$$\frac{m_u + m_d}{m_u + m_s} = \frac{m_\pi^2}{m_{K^+}^2} + \ldots \tag{5}$$

This is the general plan for measuring quark masses [3].

Once we are treating masses as coupling constants, we are led to the issue of renormalization. If the Lagrangian is written in terms of bare parameters, the interactions will induce mass shifts and we need to define renormalized masses. What then are the renormalization conditions and how are these connected to observables? I must admit that for the light quarks the answer to this question has not been completely satisfactorily found at present. In perturbation theory, of course, renormalization can be carried out. However we do not have a full connection between perturbation theory and low energy measurements. One key feature of perturbative renormalization in QCD is that the mass shift of a fermion is proportional to the mass of that fermion. In general then we will find

$$m_i^{(R)} = m_i^{(bare)} \left[Z_0 + Z_1 m_i + Z_1' \sum_{j \neq i} m_i + \ldots \right] \tag{6}$$

To first order (in m) we always have

$$m_i^{(R)} \alpha \, m_i^{(bare)} \tag{7}$$

so that ratios of the renormalized masses are equally ratios of the bare parameters. This nice feature can be preserved in mass independent perturbative renormalization schemes.

In perturbative theory one can also choose to define running masses, $m_i(q^2)$. In QCD, these get smaller as q^2 increases. For light quarks there is not much value for using these in the measurement of mass. We have our best information on ratios of masses, and in a mass independent renormalization

scheme, ratios are independent of the scale. Another point to be emphasized is that running masses for light quarks, despite getting large at low q^2, do not make a good model for constituent masses. This is because all of the running masses vanish at all q^2 in the chiral limit ($m_i^{(bare)} \to 0 \Rightarrow m_i(q^2) \to 0$), in contrast to constituent masses which approach a constant ($\approx 300MeV$) in this limit.

Non-perturbative effects can also induce mass shifts. One possible new form has been suggested by instanton calculations [4] with a mass shift

$$\delta m_u \ \alpha \ m_d m_s \tag{8}$$

We will see later that this in fact is consistent with the symmetries of QCD. It raises the question of what mass we are measuring in a given observable. However let us save these issues for later and now turn to the simple lowest order estimates of mass.

Consider first a world with massless u, d, s quarks. The quark helicity (L, R) is not changed by QCD interactions in this limit, and is unchanged under all Lorentz boosts. There are then two separate worlds, with left handed and right handed quarks being separately conserved. This implies an $SU(3)_L \times SU(3)_R$ symmetry. Any mass will break this symmetry because, at the very least, one can boost a massive left handed quark to a frame where it is right handed. However, if m is 'small' we are close to the symmetry limit. More precisely, in the massless limit, we have separate global invariance under

$$\begin{aligned} \psi_L \to \psi'_L = L\psi_L \\ \psi_R \to \psi'_R = R\psi_R \end{aligned} \tag{9}$$

with

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \tag{10}$$

and L in $SU(3)_L$, R in $SU(3)_R$. If there is a common mass $m_u = m_d = m_s$, this chiral symmetry is explicitly broken to $SU(3)_V$, and separate masses for u, d, s breaks even this latter symmetry.

However, while we see approximate $SU(3)_V$ multiplets in the spectrum of hadrons, we do not see even approximate multiplets for $SU(3)_L \times SU(3)_R$.

This is because the symmetry is hidden by the phenomena of dynamical symmetry breaking. This is characterized by a vacuum which is not invariant under the symmetry, and the appearance of Goldstone bosons. The π, K, η are the Goldstone bosons, and would be massless if the quarks were massless. This fact can be used to yield the best known measure of quark masses. For it, we need to use only first order perturbation theory, i.e., that the energy shift results from taking the matrix element of the perturbing Hamiltonian between unperturbed wavefunctions. The perturbation is

$$\mathcal{H}_m = m_u \bar{u}u + m_d \bar{d}d + m_d \bar{s}s \tag{11}$$

and the results are

$$\begin{aligned} < \pi \mid \mathcal{H}_m \mid \pi > &= m_\pi^2 = (m_u + m_d)B_0 \\ < K^+ \mid \mathcal{H}_m \mid K^+ > &= m_{K^+}^2 = (m_u + m_s)B_0 \\ < K^0 \mid \mathcal{H}_m \mid K^0 > &= m_{K^0}^2 = (m_d + m_s)B_0 \\ < \eta \mid \mathcal{H}_m \mid \eta > &= m_\eta^2 = \frac{1}{3}(4m_s + m_u + m_d)B_0 \end{aligned} \tag{12}$$

where B_0 is a constant (the reduced matrix element).

Defining

$$\hat{m} = \frac{m_u + m_d}{2} \tag{13}$$

we have

$$\begin{aligned} \frac{\hat{m}}{m_s} &= \frac{m_\pi^2}{2m_K^2 - m_\pi^2} \\ \frac{m_d - m_u}{m_s - \bar{m}} &= \frac{m_{K^0}^2 - m_{K^+}^2}{m_K^2 - m_\pi^2} \end{aligned} \tag{14}$$

valid to first order in the quark masses. Actually the second of these needs to be corrected for electromagnetic effects, which can also influence the $K^0 - K^+$ mass difference. Here we use Dashen's theorem [5], i.e., that to lowest order in chiral SU(3) [that is, with no quark mass effects], the kaon and pion electromagnetic splitting are the same

$$(m_{K^+}^2 - m_{K^0}^2)_{EM} = m_{\pi^+}^2 - m_{\pi^0}^2. \tag{15}$$

Subtracting off this contribution leads to

$$\frac{m_d - m_u}{m_s - \hat{m}} = \frac{m_{K^0}^2 - m_{K^+}^2 - m_{\pi^0}^2 + m_{\pi^+}^2}{m_K^2 - m_\pi^2} = 1/43 \tag{16}$$

or

$$\frac{m_d - m_u}{m_d + m_u} = \frac{m_{K^0}^2 - m_{K^+}^2 - m_{\pi^0}^2 + m_{\pi^+}^2}{m_\pi^2} = 0.28 \tag{17}$$

This is the estimate that most of the community is familiar with. However, the full story on quark masses is considerably more involved (or else my lectures would stop here).

Even at first order in the masses, there are other measures of quark mass ratios. Another interesting example is the decay $\eta \to 3\pi$, which is forbidden by isospin. The electromagnetic effect vanishes at lowest order in chiral SU(2) (Sutherland-Veltman theorem [6]) and all estimates beyond this order indicate that electromagnetism has a negligible effect. This leaves the isospin breaking $m_d - m_u$ as the feature which induces the decay. Soft pion theorems can relate the amplitude to

$$\langle \pi^0 \mid \mathcal{H}_m \mid \eta \rangle = \sqrt{\frac{1}{3}}(m_u - m_d)B_0 \tag{18}$$

or the result can be read off from the effective Lagrangian described later. One finds

$$\begin{aligned} \frac{m_d - m_u}{m_d + m_u} &= \frac{3\sqrt{3}\, F_\pi^2 A_0(\eta \to 3\pi^+\pi^-\pi^0)}{m_\pi^2} \\ &= 0.56 \end{aligned} \tag{19}$$

where A_0 is the amplitude in the center of the Dalitz plot and the error bars are purely experimental. This is considerably larger than the previous result, and would imply $m_u/m_d = 1/3.5$. However in this case we do know some of the higher order effects (described later) are sizeable, and will modify this result [7]. This result does indicate that first order measurements do not agree, and that we will need to confront the analyses at second order.

A third measurement of quark masses at first order involves $\psi' \to J/\psi\pi^0$ and $\psi' \to J/\psi\eta$. The former violates isospin and the second violates SU(3). Again an electromagnetism is estimated to play a very minor role, so that

these decays are driven by $m_d - m_u$ and $m_s - \hat{m}$ respectively. The analysis, using degenerate perturbative theory, yields the result

$$\begin{aligned} \frac{m_d - m_u}{m_s - \hat{m}} &= \left[\frac{16}{27}\frac{\Gamma\left(\psi' \to J/\psi + \pi^0\right)}{\Gamma\left(\psi' \to J/\psi + \eta\right)}\frac{\rho_\eta^3}{P_\pi^3}\right]^{1/3} \\ &= 0.033 \pm 0.004 \end{aligned} \tag{20}$$

This calculation uses only vectorial SU(3), not chiral SU(3). The result lies almost exactly halfway between the answer given by meson masses and by $\eta \to 3\pi$ (which yields 0.023 and 0.046 respectively). If we look at the spread around the central value, the first order values have a standard SU(3) breaking spread of $1 \pm 30\%$.

At this stage, one might ask about the absolute values of the masses. However for the light quarks there is no measurement of the light quark masses in the sense that I am using measurement. The basic problem is that the mass enters the theory multiplied by $\bar{\psi}\psi$, i.e., $\mathcal{H}_m = m\bar{\psi}\psi$. While their product is well defined, both m and $\bar{\psi}\psi$ are separately renormalization scheme dependent, and the measurements of the product do not measure m or $\bar{\psi}\psi$ separately. A very rough determination is as follows [9]. Since $m_{u,d} << m_s$, we have at first order

$$\begin{aligned} m_\Lambda - m_p &= <\Lambda \mid \mathcal{H}_m \mid \Lambda> - <P \mid \mathcal{H}_m \mid P> \\ &\approx <\Lambda \mid m_s\bar{s}s \mid \Lambda> - <P \mid m_s\bar{s}s \mid P> \\ &\equiv m_s Z \\ &\approx 180 MeV \end{aligned} \tag{21}$$

where

$$Z = <\Lambda \mid \bar{s}s \mid \Lambda> - <P \mid \bar{s}s \mid P> \tag{22}$$

Because $<\Lambda \mid \bar{s}\gamma_0 s \mid \Lambda> = 1$, we might expect $Z \sim O(1)$. [However, for the vacuum state $<0 \mid \bar{s}\gamma_0 s \mid 0> = 0$ but $<0 \mid \bar{s}s \mid 0>$ is quite large.] Explicit quark model calculation [1] yields $Z = 0.5 \to 0.75$, which seem reasonable, but not extremely solid. If we use these we get

$$m_s \sim 150 \to 300 MeV$$

$$
\begin{aligned}
m_\alpha &\sim 8 \to 16 MeV \\
m_u &\sim 3 \to 9 MeV
\end{aligned} \tag{23}
$$

However, since these and other estimates of light quark masses are based on models, not on measurements, we will not consider absolute values further.

3 The CKM Elements V_{ud}, V_{us}

The weak mixing elements V_{ud} and V_{us} are best measured in semileptonic decays, as nonleptonic transitions are not under theoretical control. The focus of theoretical analysis in the semileptonic decays is the quest for precision in handling the strong interactions. With V_{ud}, the main issues are the electroweak radiative correction and small effects due to isospin breaking. For V_{us}, the primary concern is SU(3) breaking in the current matrix elements.

The reference standard, to which the hadronic decays are compared, is $\mu^- \to e^- \bar{\nu}_e \nu_\mu$. With the Hamiltonian

$$
\mathcal{H}_w = \frac{G_\mu}{\sqrt{2}} \bar{\nu}_\mu \gamma_\mu (1+\gamma_5) \mu \bar{e} \gamma^\mu (1+\gamma_5) \nu_e \tag{24}
$$

and including the electroweak radiative correction, one has the rate

$$
\begin{aligned}
\Gamma(\mu \to e \nu \bar{\nu}) &= \frac{G_\mu^2 m_\mu^5}{192\pi^3} \left[1 - \frac{\alpha}{2\pi} \left(\pi^2 - \frac{25}{4} \right) - \frac{8 m_e^2}{m_\mu^2} + \frac{3}{5} \frac{m_\mu^2}{m_W^2} + \ldots \right] \\
&= \frac{G_\mu^2 m_\mu^5}{192\pi^3} \left[1 + (4203.85 - 187.12 + 1.05) \times 10^{-6} \right]
\end{aligned} \tag{25}
$$

where the corrections, in the order written, are due to photonic radiative effects, phase space, and the W propagator. The value of G_μ thus extracted is

$$
G_\mu = 1.16637(2) \times 10^{-5} GeV^{-2} \tag{26}
$$

For $\Delta S = 0$ beta decays we have

$$
\mathcal{H}_w = \frac{G_\beta}{\sqrt{2}} V_{ud} \bar{u} \gamma^u (1+\gamma_5) d \bar{e} \gamma_\mu (1+\gamma_5) \nu_e \tag{27}
$$

At tree level $G_\mu = G_\beta$, but at one loop this is no longer true as there is an important difference in the radiative correction. For the weak transition $1+3 \to 2+4$ some of the radiative corrections are shown in Fig. 1. Diagrams a, b are ultraviolet finite. This can be understood by noting that the calculation is the same as the vertex renormalization of a conserved current, which we know leads to no renormalization at $q^2 = 0$. Figures c, d are similar if we use the Fierz transformation

$$\bar{\psi}_2\gamma_\mu(1+\gamma_5)\psi_1\bar{\psi}_4\gamma^\mu(1+\gamma_5)\psi_3 = \bar{\psi}_4\gamma_\mu(1+\gamma_5)\psi_1\bar{\psi}_2(1+\gamma_5)\psi_1 \tag{28}$$

However diagrams e, f fall into a different class and are log divergent if we use the Fermi interaction with no propagator. The ultraviolet portion is then proportional to $(Q_1Q_3 + Q_2Q_4)$, i.e.,

$$M_{e,f}^{(u.v.)} = -M^{(0)}\frac{3\alpha}{2\pi}(Q_1Q_3 + Q_2Q_4)ln\Lambda/\mu_L \tag{29}$$

where Λ is a high energy cutoff and μ_l is a low energy scale. In muon decay $(1,2,3,4) = (\mu^-, \nu_\mu, \nu_e, e^-)$ so that

$$Q_\mu Q_{\nu_e} + Q_{\nu_\mu}Q_e = 0. \tag{30}$$

However in beta decay $(1,2,3,4) = (d, u, \nu_e, e^-)$ with

$$Q_d Q_{\nu_e} + Q_{u_\mu}Q_{e^-} = -\frac{2}{3}. \tag{31}$$

In a full treatment, including the W propagator and γ, Z loops one finds the cutoff $\Lambda = m_Z$, so that there is a universal 'model independent' correction [10] which can be absorbed in the definition of G_β

$$G_\beta = G_\mu\left(1 + \frac{\alpha}{\pi}ln\frac{M_Z}{\mu_L}\right) \tag{32}$$

To this also needs to be added smaller 'model dependent' low energy effects and coulomb corrections.

For $\Delta S = 0$ decays, the key to mastering the strong interactions is that the vector current is conserved (in the limit $m_u = m_d$), so that the matrix element is absolutely normalized. In contrast it is not possible to predict axial current matrix elements to high accuracy. In neutron beta decay, $n \to pe\nu$, where both vector and axial currents contribute, one needs to measure the

axial form factor g_A in order to be able to predict the rate and measure V_{ud}. This works, but at present the statistical accuracy is not the best. Pion beta decay, $\pi^{\pm} \to \pi^0 e^{\pm} \overline{\nu}$, only involves the vector current and would be the ideal channel to study, but there are not yet enough events. The most sensitive process is $0^+ \to 0^+$ nuclear beta decay between isospin partners [11]. This also only involves the vector current, and has very high statistics.

The superallowed $0^+ \to 0^+$ transitions have a single form factor

$$< N_2(I_z = 0) \mid V_\mu \mid N_1(I_z = 1) >= a(q^2)(p_1 + p_2). \tag{33}$$

with $a(0) = \sqrt{2}$. One calculates the half life $t_{1/2}$ times a kinematical phase space factor F, and adds hard and soft radiative corrections, Coulomb corrections to the wavefunction and finite size effect

$$Ft_{1/2} = \frac{2\pi^3 ln2}{G_\beta^2 m_e^5 \mid V_{ud} \mid^2 a^2(0)}[1 + \ldots] \tag{34}$$

Present efforts center on the nuclear wavefunction mismatch. When one plots the Ft values for different nuclei vs. Z, the result should be a constant value if all the nuclear effects have been taken into account completely. In practice there seems to be some indication for a slope to this line [12], indicating that some effect linear in Z is not fully accounted for. In the recent analysis of Ref. 11 this has been corrected for phenomenologically be extrapolating the Ft values to Z = 0, with the result

$$V_{ud} = 0.9751 \pm 0.0005 \tag{35}$$

One obtains a values for V_{ud} about 2σ lower if one simply averages the Ft measurements. Neutron and pion beta decays are consistent with Equation 35.

4 Effective Lagrangian Description

Before going on to the measurement of V_{us}, I need to describe the uses of effective Lagrangian techniques in chiral perturbation theory. In these notes, I will be somewhat brief as Andy Cohen covers effective field theory in these TASI lectures [2] and I have elsewhere [1] had the opportunity to present the subject in considerably greater depth.

The main idea is that if predictions follows from symmetry alone, then any general Lagrangian with the right symmetry will yield the correct predictions [13]. For physics of the light mesons, we seek then the most general Lagrangian with chiral SU(3) symmetry containing only the π, K, η fields. This can be accomplished with the 3×3 matrix representation

$$U = exp\left[i\frac{\vec{\lambda}\cdot\vec{\phi}}{F_\pi}\right] \quad , \tag{36}$$

with transformation

$$U \to LUR^\dagger \tag{37}$$

with L in $SU(3)_L$ and R in $SU(3)_3$. The only Lagrangian invariant under chiral SU(3) with 2 derivatives (there are none with zero derivatives) is

$$\mathcal{L}_{SYM} = \frac{F_\pi^2}{4}Tr(\partial_\mu U\partial^\mu U^\dagger) = \frac{1}{2}\partial_\mu\phi^A\partial^\mu\phi^A + \ldots \tag{38}$$

For QCD we also need some explicit chiral symmetry breaking, which at lowest order will be linear in the quark masses. It preserves parity and has the same chiral properties as $\bar{\psi}m\psi = \bar{\psi}_L m\psi_R + \bar{\psi}_R m\psi_L$. At lowest order the unique choice is

$$\mathcal{L}_{Breaking} = \frac{F_\pi^2 B_0}{2}Tr(mU + U^\dagger m) \tag{39}$$

where B_0 has been chosen to be the same constant as in Section II. The full lowest order Lagrangian

$$\mathcal{L} = \mathcal{L}_{sym} + \mathcal{L}_{Breaking} \tag{40}$$

when applied at tree level reproduces all of the lowest order predictions of chiral symmetry, such as the mass relations given previously.

What about effective Lagrangian with more derivatives or more powers of the quark masses? These may also have the correct chiral SU(3) properties. The key to practical applications is the energy expansion. Consider two possible chirally symmetric Lagrangians

$$\mathcal{L}^1 = aTr(\partial_\mu U\partial^\mu U^\dagger) + bTr(\partial_\mu U\partial_\nu U^\dagger\partial^\mu U\partial^\nu U^\dagger) \tag{41}$$

The Lagrangian has dimension $(mass)^4$, which implies that a has dimension $mass^2$ and $b/a \sim 1/mass^2$. When matrix elements are taken, derivatives turn into powers of momentum so that

$$M = aq^2 \left[1 + \frac{b}{a} q^2 \right] \tag{42}$$

If we define $b/a \equiv 1/\Lambda^2$, then for $q^2 \ll \Lambda^2$ there is little effect of the higher derivative terms. As q^2 increases, the four derivative term provides a correction to the lowest order result. In practice we most often find $\Lambda \sim m_\rho$, so that lowest order chiral predictions are modified as momenta approach m_ρ.

In constructing the effect of quark masses it is useful to consider an external field of the form [14]

$$\mathcal{L}_{QCD} = \bar{\psi} i \not{D} \psi - \frac{1}{2B_0} \left(\bar{\psi}_L \chi \psi_R + \bar{\psi}_R \chi^\dagger \psi_L \right) \tag{43}$$

such that QCD is obtained with $\chi = 2B_0 m$. However, if we allow a transformation rule

$$\chi \to L \chi R^\dagger \tag{44}$$

the Lagrangian will be chirally invariant. The effect of masses is then found by writing chirally invariant Lagrangians containing χ. We do this in Sec. 5.

Finally loop diagrams can, and must, be included. Divergences appear, but these just go into the renormalization of the parameters in the effective Lagrangian. Finite effects left over after renormalization account for the low energy propagation of the pions and kaons.

The application of effective Lagrangians to the chiral interactions of π, K, η is called Chiral Perturbation Theory. To next to leading order (i.e., $O(E^4)$) the instructions are:

1. Write the most general Lagrangians to $O(E^2)$ and $O(E^4)$; $\mathcal{L}_2$ contains two derivative or one power of the quark masses, and $\mathcal{L}_4$ has either 4 derivatives, 2 derivatives and one mass, or two powers of the mass.

2. Calculate all one loop diagram involving $\mathcal{L}_2$

3. Renormalize the parameters in the Lagrangian, determining the unknown parameters from experiment.

4. Find relations between different observables

These relations are the predictions of chiral symmetry.

5 V_{us} and SU(3) Breaking

One of the applications of chiral perturbation theory is in the determination of V_{us}. We will need to obtain the form factors in $\Delta S = 1$ processes such as $K \to \pi e \nu$ and $\Lambda \to p e \nu$. These are related by SU(3) to the $\Delta S = 0$ form factors which we have already discussed ($\pi^+ \to \pi^0 e \nu, n \to p e \nu$). However typical SU(3) breaking enters into other processes at the 30% level. We want to be more accurate than this.

A crucial ingredient here is the Ademollo Gatto theorem [15] which says that the vector form factors are modified from their SU(3) values only by terms second order in the SU(3) breaking mass difference $m_s - \hat{m}$. This again points to the value of using vector form factors in the extraction of V_{us}. The two possible sources of data are hyperon decays and $K \to \pi e \nu$.

Hyperon decays involve many modes and high statistics. The axial form factors cannot be predicted reliably from theory and must be measured. SU(3) parameterizes these form factors in terms of two reduced matrix elements, the neutron to proton axial coupling g_A and a D/F ratio. The vector form factors are predicted via SU(3) plus the Ademollo Gatto theorem. The history of our ability to treat these decays has undergone fluctuations. Before 1982, SU(3) fits worked well. In 1982, the data improved enough that SU(3) breaking at the 5% level was observed and caused troubles with fits based on SU(3) symmetry, invalidating any fits using SU(3) symmetry [16]. A few years later the quark model was used to provide an SU(3) breaking pattern that was consistent with the data, allowing a good fit and the extraction of V_{us} [17]. Unfortunately by 1990, the data was again better than theory, and the simple quark model pattern does not fit without modification [18]. Unless theory can recover once again, hyperon decays can not be analysed in any greater precision than this, because future increased statistics will only tell us more details about SU(3) breaking.

Kaon semileptonic decays involves only two modes (K^0 and K^+ decay). However the analysis is particularly strong since it can make use of a body of work on chiral perturbative theory. In addition these modes have very high statistics. For these reasons, kaon decay is the prime mode for measuring V_{us}.

In order to be convinced that the theory of $K \to \pi e \nu$ is under control, we have to turn to internal consistency checks. The analysis is due to Gasser and Leutwyler [19]. There are two form factors, f_+ and f_-

$$\begin{aligned} < \pi^- \mid \bar{s}\gamma_\mu u \mid K^0 > &= f_+^{K^0\pi^-}(k+p)_\mu + f_-^{K^0\pi^-}(k-p)_\mu \\ < \pi^0 \mid \bar{s}\gamma_\mu u \mid K^+ > &= \frac{1}{\sqrt{2}}\left[f_+^{K^+\pi^0}(k+p)_\mu + f_-^{K^+\pi^0}(k-p)_\mu\right]. \end{aligned} \quad (45)$$

If one includes the next-to-leading order Lagrangian, as well as one loop diagrams, one obtains lengthy expressions for the form factors. Among the highlights of the results are

1. The Ademollo Gatto theorem has a correction due to isospin breaking

$$\begin{aligned} \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} &= 1 + \frac{3}{4}\frac{m_d - m_\mu}{m_s - \hat{m}} + l_{K\pi} \\ &= 1.029 \pm 0.010(Data) \end{aligned} \quad (46)$$

   where $l_{K\pi} = 0.004$ arise from loop diagrams. This value is consistent, because of the large uncertainty, with all of our previous estimates of the quark mass ratio.

2. The form factors are related to the chiral constant L_9 determined in the pion form factor,

$$\begin{aligned} \frac{f_-(0)}{f_+(0)} &= -\left[1 - \frac{F_K}{F_\pi} + \frac{2L_9}{F_\pi^2}\left(m_K^2 - m_\pi^2\right)\right] \\ &= -0.13\, theory \\ &= -0.20 \pm 0.08\, data. \end{aligned} \quad (47)$$

3. The slopes of the form factors are predicted in agreement with the data (although the data presently have a few internal inconsistencies).

Given that the theory appears to be under control Leutwyler and Roos [20] have extracted

$$V_{us} = 0.2196 \pm 0.0023 \tag{48}$$

(a 10% measurement). This value is consistent with the results of hyperon decay, and implies the check of the unitarity of the KM matrix

$$\mid V_{ud} \mid^2 + \mid V_{us} \mid^2 + \mid V_{ub} \mid^2 = 0.9990 \pm 0.0022 \tag{49}$$

with $\mid V_{ub} \mid^2 \leq 10^{-5}$.

6 Quark Masses Beyond Leading Order

Now we turn to the most difficult issue in these lectures; the analysis of quark masses at second order. There are several motivations for pursuing such an analysis. First of all, we have seen how the lowest order predictions lead to some discrepancies. In addition, there is the strong CP problem [21], where the effect of CP violation by the θ term of QCD would vanish if $m_u \to 0$. We then must question how well we know that $m_u \neq 0$. This solution to the strong CP problem is not natural, in the technical sense, within the Standard Model, but perhaps might be possible within an extension of our present theory. Finally there are several subtle issues which arise at second order in the mass, most notably the reparameterization ambiguity described below.

The mass sector of the theory is described by

$$\mathcal{L} = \ldots + \frac{F^2}{4} Tr\left(\chi^\dagger U + U^\dagger \chi\right) \tag{50}$$

at lowest order [recall Equation 43], and at higher order by

$$\begin{aligned} \mathcal{L}_4 \ldots \quad &+ \quad L_6 \left[Tr\left(\chi^\dagger U + U^\dagger \chi\right)\right]^2 + L_7 \left[TR\left(\chi^\dagger U - U^\dagger \chi\right)\right]^2 \\ &+ \quad L_8 Tr\left(\chi U^\dagger \chi U^\dagger + \chi^\dagger U \chi^\dagger U\right) \end{aligned} \tag{51}$$

where $L_{6,7,8}$ are dimensionless unknown reduced matrix elements in the basis of Gasser and Leutwyler [14].

The π, K, η masses can be analysed to second order in the quark masses [14].

$$
\begin{aligned}
F_\pi^2 m_\pi^2 &= 2\hat{m}F^2B_0\left[1+\frac{32L_6B_0}{F^2}\left(m_u+m_d+m_s\right)+\frac{32L_8B_0\hat{m}}{F^2}\right. \\
&\quad \left.-3\mu_\pi-2\mu_K-\frac{1}{3}\mu_\eta\right] \\
F_{K^+}^2 m_{K^+}^2 &= \left(m_s+m_u\right)F^2B_0\left[1+\frac{32L_6B_0}{F^2}\left(m_u+m_d+m_s\right)\right. \\
&\quad \left.+\frac{16L_8B_0}{F^2}\left(m_u+m_s\right)-\frac{3}{2}\mu_\pi-3\mu_K-\frac{5}{6}\mu_\eta\right] \\
F_{K^0}^2 m_{K^0}^2 &= \left(m_s+m_d\right)F^2B_0\left[1+\frac{32L_6B_0}{F^2}\left(m_u+m_d+m_s\right)\right. \\
&\quad \left.+\frac{16L_8B_0}{F^2}\left(m_s+m_d\right)-\frac{3}{2}\mu_\pi-3\mu_K-\frac{5}{6}\mu_\eta\right] \\
F_\eta^2 m_\eta^2 &= \frac{4}{3}F_K^2m_K^2-\frac{1}{3}F_\pi^2m_\pi^2-\frac{64}{3}\left(2L_7+L_8\right)B_0\left(m_s-\hat{m}\right)^2 \\
&\quad +\left(2\mu_\pi-\frac{4}{3}\mu_K-\frac{2}{3}\mu_\eta\right)\left(m_K^2-m_\pi^2\right)
\end{aligned} \tag{52}
$$

where

$$
\eta_i=\frac{m_i^2}{32\pi^2F_\pi^2}lnm_i^2/\eta^2 \tag{53}
$$

There are two main results of this analysis. One is that the deviation of the Gell Mann Okubo formula measures a useful combination of chiral coefficients

$$
\begin{aligned}
\delta_{GMO} &= \frac{4F_K^2m_K^2-3F_\eta^2m_\eta^2-F_\pi^2m_\pi^2}{4\left(F_K^2m_K^2-F_\pi^2m_\pi^2\right)} \\
&= \frac{16}{F_\pi^2}\left(2L_7+L_8\right)\left(m_\pi^2-m_K^2\right)-\frac{3}{2}\mu_\pi+\mu_K+\frac{1}{2}\mu_\eta \\
&= -0.06(Data)
\end{aligned} \tag{54}
$$

which yields

$$
2L_7+L_8=0.2\times10^{-3} \tag{55}
$$

at the chiral scale $\mu = m_\eta^2$. The other prediction is more important, producing a ratio of quark masses which are free from unknown parameters

$$\begin{aligned} \frac{m_d - m_u}{m_s - \hat{m}} \frac{2\hat{m}}{m_s + \hat{m}} &= \frac{m_\pi^2}{m_K^2} \frac{(m_{K^0}^2 - m_{K^+}^2)_{QM}}{m_K^2 - m_\pi^2} \\ \left(m_{K^0}^2 - m_{K^+}^2\right)_{QM} &\equiv \left(m_{K^+}^2 - m_{K^+}^2\right)_{expt} - \left(m_{K^0}^2 - m_{K^+}^2\right)_{EM} \end{aligned} \tag{56}$$

The only flaw in this wonderful relation is that we do not know $(m_{K^0}^2 - m_{K^+}^2)_{EM}$ to the order that we are working. Recall that Dashen's theorem was only valid to zeroth order in the quark mass. The next order results have not been fully explored in chiral perturbation theory.

Gasser and Leutwyler have also analysed $\eta \to 3\pi$ to second order [7]. The result can be expressed in parameter free form as

$$\frac{m_d - m_u}{m_s - \hat{m}} \frac{2\hat{m}}{m_s + \hat{m}} = \frac{3\sqrt{3} F_\pi^2 Re A_{\eta \to 3\pi}(0)}{[1 + \Delta_{\eta 3\pi}] (m_K^2 - m_\pi^2)} \frac{m_\pi^2}{m_K^2} = 2.35 \times 10^{-3} \tag{57}$$

where $\Delta_{\eta 3\pi} = 0.5$. Recall that this ratio was 1.7×10^{-3} from meson masses and 3.5×10^{-3} from $\eta \to 3\pi$, both at lowest order. The effects of the $O(E^4)$ analysis has been to produce a compromise value for the ratio.

One of the advances of the past year is that it is now reasonable to expect consistency between the analysis of the kaon mass difference and that of $\eta \to 3\pi$. The agreement of Eq. 54 and Eq. 55 would require $(\Delta m_K^2)_{QM} = 7.0 MeV$. However Dashen's theorem implies $(\Delta m_K^2)_{EM} = 5.3 MeV$. Are there significant violations of Dashen's theorem? Recent analyses suggest that there are [22]. I am, of course, most partial to the work which I participated in. We used a series of powerful constraints on the $\gamma\pi \to \gamma\pi$ and $\gamma K \to \gamma K$ amplitudes which serve to predict the electromagnetic mass difference when the photons are contracted into a propagator. These constraints include 1) data on $\gamma\gamma \to \pi\pi$, 2) low energy chiral constraints, 3) the dispersion theory of $\gamma\gamma \to \pi\pi$, 4) soft pion theorems and, 5) the generalized Weinberg sum rules. These features are compatible with a vector dominance model which yields

$$\frac{(\Delta m_K^2)_{EM}}{(\Delta m_\pi^2)_{EM}} = 1.8 \tag{58}$$

whereas Dashen's theorem says that the ratio should be unity. The difference has a rather simple origin: it is due to factors of m_K^2 in propagators instead of m_π^2. The larger electromagnetic contribution brings the kaon mass difference and $\eta \to 3\pi$ into considerably better agreement (10 %).

The most interesting feature of the study of quark masses beyond leading order is the reparameterization invariance, first made explicit by Kaplan and Manohar [23]. The crude statement is that when using SU(3) symmetry one obtains the same physics using either the masses (m_u, m_d, m_s) or the set

$$\begin{aligned} m_u^{(\lambda)} &= m_u + \bar{\lambda} m_d m_s \\ m_d^{(\lambda)} &= m_d + \bar{\lambda} m_u m_s \\ m_s^{(\lambda)} &= m_s + \bar{\lambda} m_u m_d \end{aligned} \tag{59}$$

for an $\bar{\lambda}$! The reason is that m_i and $m_i^{(\lambda)}$ both have the same chiral SU(3) properties. This can be seen using the Cayley-Hamilton theorem for a 3×3 matrix A

$$detA = A^3 - A^2 TrA - \frac{A}{2}\left[Tr(A^2) - (Tr(A))^2\right] \tag{60}$$

If we apply this to the matrix χ [recall $\chi = 2B_0 m$ for pure QCD] defining

$$\chi^{(\lambda)} = \chi + \lambda[det\chi^\dagger]\chi\frac{1}{\chi^\dagger\chi} \tag{61}$$

we have

$$\begin{aligned} \left[det\chi^\dagger\right]\chi\,\frac{1}{\chi^\dagger\chi} &= \left[detU\chi^\dagger\right]\chi\,\frac{1}{\chi^\dagger\chi} \\ &= U\chi^\dagger U\chi^\dagger U - U\chi^\dagger U Tr\left(U\chi^\dagger\right) \\ &- \frac{U}{2}\left[Tr\left(U\chi^\dagger U\chi^\dagger\right) - \left(TrU\chi^\dagger\right)^2\right] \end{aligned} \tag{62}$$

and

$$Tr(\chi^\lambda U^\dagger) = Tr(\chi U^\dagger) - \frac{\lambda}{2}\left[Tr(\chi^\dagger U\chi^\dagger U) - \left(Tr(\chi^\dagger U)\right)^2\right] \tag{63}$$

In an effective Lagrangian the use of $\chi^{(\lambda)}$ instead of χ leads to a Lagrangian of the same general form since

$$\begin{aligned} Tr(\chi^{(\lambda)}U^\dagger + U\chi^{(\lambda)\dagger}) &= Tr(\chi U^\dagger + U\chi^\dagger) \\ &+ \frac{\lambda}{2}\left[Tr(\chi U^\dagger + U\chi^\dagger)\right]^2 \\ &+ \frac{\lambda}{2}\left[Tr(\chi U^\dagger - U\chi^\dagger)\right]^2 \\ &- \lambda Tr(\chi U^\dagger \chi U^\dagger + \chi^\dagger U \chi^\dagger U) \end{aligned} \tag{64}$$

The last three terms lead to a modification of the chiral coefficients which we called L_6, L_7, L_8 previously. However the total effective Lagrangian has the same form. Use of $\chi^{(\lambda)}$ and one set of L_6, L_7, L_8 is equivalent to the use of χ and a different set of L_6, L_7, L_8. This property of χ is the same as that of the masses, when we use $\chi = 2B_0 m$, and

$$\chi^{(\lambda)} \equiv 2B_0 m^\lambda = 2B_0\left[m + (2B_0\lambda)m_u m_d m_s \frac{1}{m}\right] \tag{65}$$

and identify $\bar{\lambda} = 2B_0\lambda$. The precise statement of the reparameterization ambiguity is then that, using either SU(3) or chiral SU(3) any physics described by (m_u, m_d, m_s) and (L_6, L_7, L_8) can be equally well described by

$$\begin{aligned} m_u^{(\lambda)} &= m_u + \bar{\lambda} m_d m_s \quad & L_6^{(\lambda)} &= L_6 - \tilde{\lambda} \\ m_d^{(\lambda)} &= m_d + \bar{\lambda} m_u m_s \quad & L_7^{(\lambda)} &= L_7 - \tilde{\lambda} \\ m_s^{(\lambda)} &= m_s + \bar{\lambda} m_u m_d \quad & L_8^{(\lambda)} &= L_8 - 2\tilde{\lambda} \end{aligned} \tag{66}$$

with $\bar{\lambda} = 2B_0\lambda; \tilde{\lambda} = F_\pi^2\lambda/16$, for any reasonable λ.

Let us see examples of how this works. For the ratio of quark masses measured above, we have

$$\begin{aligned} \frac{m_d^{(\lambda)} - m_u^{(\lambda)}}{m_s^{(\lambda)} - \hat{m}^{(\lambda)}} \frac{2\hat{m}^{(\lambda)}}{m_s^{(\lambda)} - \hat{m}^{(\lambda)}} &= \frac{(m_d - m_u)(1 - \bar{\lambda} m_s)}{(m_s - \hat{m})(1 - \bar{\lambda}\hat{m})} \frac{2\hat{m}(1 + \bar{\lambda} m_s)}{(m_s + \hat{m})(1 + \bar{\lambda}\hat{m})} \\ &= \frac{m_d - m_u}{m_s - \hat{m}} \frac{2\hat{m}}{m_s + \hat{m}} + \mathcal{O}(m^2) \end{aligned} \tag{67}$$

i.e., the ratio is invariant. Similarly the combination

$$2L_7^{(\lambda)} + L_8^{(\lambda)} = 2(L_7 - \tilde{\lambda}) + (L_8 + 2\tilde{\lambda}) = 2L_7 + L_8 \tag{68}$$

is invariant. Finally

$$\begin{aligned} m_\pi^2 &= 2B_0\hat{m}^{(\lambda)}\left[1 + \frac{32B_0}{F_\pi^2}\left(\hat{m}L_8^{(\lambda)} + (2\hat{m} + m_s)L_6^{(\lambda)}\right) + \ldots\right] \\ &= 2B_0\hat{m}(1 + \bar{\lambda}m_s)]\left[1 - \bar{\lambda}m_s + \frac{32B_0}{F_\pi^2}\left(\hat{m}L_8 + (2\hat{m} + m_s)L_6\right) + \ldots\right] \\ &= 2B_0\hat{m}\left[1 + \frac{32B_0}{F_\pi^2}\left(\hat{m}L_8 + (2\hat{m} + m_s)L_6\right) + \ldots\right] \\ &\quad + \mathcal{O}(m^3) \end{aligned} \tag{69}$$

is also unchanged in form under the reparameterization.

Physical quantities are invariant under the reparameterization transformation. Quark mass ratios (or the $L_i{}'s$) are not invariant and hence can not be uniquely measured by any analysis using SU(3) or chiral SU(3). This conclusion is general and extends to other systems, such as baryons or heavy mesons, when analysed to second order (or beyond). The best that we can do is to measure a one parameter family of masses.

There is a weak restriction on the transformation in that we can't choose λ so large as to destroy the energy expansion. The typical sizes of the chiral coefficients are of order a few times 10^{-3}. We should not allow any $\tilde{\lambda}$ that makes L_6, L_7, L_8 unnaturally large. In practice this does not happen for the mass range that we are most interested in.

A conventional choice for masses and chiral parameters is

$$\begin{aligned} \frac{m_u}{m_s} = \frac{1}{34} \quad &; \quad \frac{m_d}{m_s} = \frac{1}{19} \\ L_7 = -0.4 \times 10^{-3} \quad &; \quad L_8 = 1.1 \times 10^{-3} \end{aligned} \tag{70}$$

A second set which is equally consistent is one with $m_u = 0$

$$\begin{aligned} \frac{m_u}{m_s} = 0 \quad &; \quad \frac{m_d}{m_s} = \frac{1}{26} \\ L_7 = 0.2 \times 10^{-3} \quad &; \quad L_8 = -0.1 \times 10^{-3} \end{aligned} \tag{71}$$

obtained by a reparameterization transformation. A third compatible set is

$$\frac{m_u}{m_s} = \frac{1}{22} \quad ; \quad \frac{m_d}{m_s} = \frac{1}{16}$$
$$L_7 = -0.8 \times 10^{-3} \quad ; \quad L_8 = 1.9 \times 10^{-3} \tag{72}$$

In all cases L_7 and L_8 are natural in size. (Nothing is known about the magnitude of L_6). Note that since m_u is the smallest mass, it changes the most. This is to be expected since we have

$$\Delta m_u \sim m_d m_s \sim m_d \frac{m_K^2}{\Lambda^2} \sim \frac{1}{3} m_d \sim m_u \tag{73}$$

so that the change in m_u is of the same order as m_u itself.

The reparameterization transformation is an invariance of SU(3) effective Lagrangians, not of the fundamental QCD Lagrangian. However, there may be physics in QCD which generates effects like this [24]. Let us consider the allowed forms of radiative corrections to the masses in various limits.

1. If $m_u = m_d = m_s = 0$, we have an exact $SU(3)_L \times SU(3)_R$ chiral symmetry. There are no modifications to masses due to radiative corrections, as the quarks are protected by the chiral symmetry from picking up a mass.

2. If $m_u = m_d = 0$ and $m_s \neq 0$, there is an exact chiral SU(2) symmetry which protects m_u and m_d from any quantum shifts. Likewise m_u and m_s would be protected in an $m_u = m_s = 0, m_d \neq 0$ world.

3. Now consider $m_u = 0$, but $m_d \neq 0$ and $m_s \neq 0$. Now there is no symmetry protection at all, because chiral SU(2) is broken and axial U(1) is not a quantum symmetry. There can be radiative corrections to m_u. However, since the corrections must vanish as $m_d \to 0$ or as $m_s \to 0$, it must have the form

$$m_u = c m_d m_s \tag{74}$$

There is in the literature an interesting example of just such a renormalization, where instantons lead to this form of radiative correction, with the overall coefficient depending on the cutoff in instanton sizes [4]. We don't

need to take the details of this calculation too seriously, but we must acknowledge that this form of radiative correction can occur in QCD. It is always associated with the $U(1)_A$ anomaly. By permutation symmetry, if $m_u \neq 0$ we would have $\Delta m_d = c m_u m_s, \Delta m_s = c m_u m_d$.

These radiative corrections can produce different definitions of quark masses. For example in a mass independent renormalization scheme, one has

$$\left[m_i^{(r)}\right]_1 = Z m_i \tag{75}$$

with a common factor of Z. In a second renormalization scheme one might include the low energy effects (such as the instantons) which induce the radiative corrections of the preceding paragraph. The two schemes would be related by a finite renormalization

$$\left[m_u^{(r)}\right]_2 = \left[Z' m_u^{(r)} + \bar{\lambda} m_d^{(r)} m_s^{(r)}\right]_1 \tag{76}$$

for some $\bar{\lambda}$. For consistency, the various other parameters in the theory would also have to be related

$$[L_7]_2 = \left[L_7 - \tilde{\lambda}\right]_2 \tag{77}$$

such that observables are unchanged. From this point of view, there is the possibility of a renormalization scheme ambiguity in QCD which mirrors the reparameterization invariance.

An caveat to the above argument involves the $U(1)_A$ dependence. In the presence of a non- zero vacuum angle θ in QCD the mass shift due to the instanton effect is actually [4]

$$\Delta m_u = c m_d m_s e^{i\theta} \tag{78}$$

The various masses of different renormalization schemes have different θ dependence, and can in principle be differentiated by their behavior under $U(1)_A$ transformations. This can also be seen in the transformation of the χ and $\chi^{(\lambda)}$ under $U(1)_A, L = e^{i\alpha}, R = e^{-i\alpha}$, in that

$$\begin{aligned} \chi &\rightarrow e^{2i\alpha}\chi \\ \chi^{\lambda} &\rightarrow e^{2i\alpha}\left[\chi + \lambda e^{-6i\alpha}\left[det\chi^{+}\right]\chi\frac{1}{\chi^{+}\chi}\right] \end{aligned} \tag{79}$$

so that m and $m^{(\lambda)}$ are not equivalent in their $U(1)_A$ properties. Of course, $U(1)_A$ is not a symmetry, but there are a set of anomalous Ward identities [25] which can in principle probe the $U(1)_A$ behavior. In practice, none of the measurements discussed above involve $U(1)_A$.

There is an example which shows how the $U(1)_A$ properties can measure masses independent of the reparameterization [24] transformation. Briefly summarized one adds the $\theta F\tilde{F}$ term to the QCD Lagrangian but with θ treated as an external source so that functional derivatives with respect to $\theta(x)$ yield matrix elements of $F\tilde{F}$. The $U(1)_A$ properties determine how $\theta(x)$ enters the effective Lagrangian, and these matrix elements are calculated to $O(E^4)$. The example shown was

$$\frac{<0 \mid F\tilde{F} \mid \pi^0>}{<0 \mid F\tilde{F} \mid \eta>} = \frac{3\sqrt{3}}{4}\left[\frac{m_d - m_u}{m_s - \hat{m}}\right]\frac{F_\eta}{F_\pi} \left[1 - \frac{32B_0}{F_\pi^2}(m_s - \hat{m})(L_7 + L_8) + \ldots\right] \tag{80}$$

This matrix element is not reparameterization invariant so that, if it could be measured, it could be used to disentangled the individual mass ratios.

What can be done in such a situation? There is at present no completely satisfactory solution. However, some possible directions have been at least partially explored. One possibility is to choose a definition of mass which is automatically reparameterization invariant. For example we can simply define invariant masses m_i^* by [24]

$$\begin{aligned} F_\pi^2 m_\pi^2 &\equiv F_0^2 B_0 \left[m_u^* + m_d^*\right] \\ F_{K^+}^2 m_{K^+}^2 &\equiv F_0^2 B_0 \left[m_s^* + m_u^*\right] + \delta_{GMO} F_K^2 \left(m_K^2 - m_\pi^2\right) \\ F_{K^0}^2 m_{K^0}^2 &\equiv F_0^2 B_0 \left[m_s^* + m_d^*\right] + \delta_{GMO} F_K^2 \left(m_K^2 - m_\pi^2\right) \\ F_\eta^2 m_\eta^2 &\equiv F_0^2 B_0 \left[\frac{4}{3} m_s^* + \frac{2}{3}\hat{m}^*\right] \end{aligned} \tag{81}$$

This results in

$$m_u^* = m_u \left[1 + \frac{32B_0}{F_\pi^2}\left(L_6(m_u + m_d + m_s) + L_8 m_u\right) - 3\mu_\pi - 2\mu_K\right.$$

$$
\begin{aligned}
& + \frac{1}{2}\left(\frac{m_d - m_u}{m_s - \hat{m}}\right)(\mu_\eta - \mu_\pi)\Big] \\
& + 32\,\frac{L_7 B_0}{F_\pi^2}(m_u - m_d)(m_u - m_s) \\
\mu_i^2 \;=\; & \frac{m_i^2}{32\pi^2\,F_\pi^2}\,ln\,m_i^2/\mu^2 \qquad (82)
\end{aligned}
$$

with m_d^* similar with $(m_u, m_d, m_s) \to (m_d, m_u, m_s)$, and m_s^* likewise with $(m_u, m_d, m_s) \to (m_s, m_d, m_u)$, plus some rearrangement of the chiral logs [24]. Each of these invariant masses m_i^* is also invariant under changes in the scale μ which enters when using dimensional regularization. Many ratios of the m_i^* are physical and can be evaluated

$$
\frac{m_d^*}{m_s^*} = \frac{1}{22}\ ;\ \frac{m_u^*}{m_d^*} = 0.2 \qquad (83)
$$

and are fine measures of the breaking of chiral SU(3) and SU(2) symmetry. They do not address the $U(1)_A$ properties and cannot answer the question of whether strong CP violation disappears due to the $m_u = 0$ option.

A second possible direction is to try to use a model to calculate one of the chiral coefficients. Leutwyler has given a sum rule for L_7 and saturated it with an η' pole [26]. This is reasonable, but it is a model, and it is being applied in a sector where we have no previous experience to see if resonance saturation works in the presence of the reparameterization transformation. As with many models, one often finds other contributions which upset the original conclusion – as has been suggested for the $\pi'(1300)$ intermediate state in the sum rule [27].

Ultimately the most promising way would be to find a way to measure observables connected to $U(1)_A$ anomalous Ward identities. Wyler and I proposed to use $\psi' \to J/\psi\pi^0$ and $\psi' \to J/\psi\eta$ to do this [24, 28]. These unlikely reactions were chosen because an analysis by Voloshin and Zakharov [29] claimed that by using a QCD multipole [30] expansion, these decays were mediated by the local operator $F\tilde{F}$, such that a ratio of the decay rates can be converted into the ratio of Eq.(80). This yielded a set of ratios with $m_u \neq 0$. However, the Voloshin Zakharov analysis has been criticized by Luty and Sundrum [31], and unless I am missing something it seems to me that the criticism is justified. I have some hopes of getting around this problem in the future, but it is otherwise difficult to measure masses in $U(1)_A$ processes.

7 Where do we stand?

We have been using symmetries to measure masses and mixing angles. The results

$$\begin{aligned} V_{ud} &= 0.9751 \pm 0.0005 \\ V_{us} &= 0.220 \pm 0.002 \\ \mid V_{ud} \mid^2 + \mid V_{us} \mid^2 &= 0.999 \pm 0.002 \end{aligned} \tag{84}$$

are gratifyingly precise. For the expert the interest lies in the error bars, which are dominated by nuclear uncertainties in the case of V_{ud}, and SU(3) breaking for V_{us}.

In the case of light quark masses, we have one firm ratio

$$\frac{m_d - m_u}{m_s - \hat{m}} \frac{2\hat{m}}{m_s + \hat{m}} = 2.3 \times 10^{-3}, \tag{85}$$

accurate to about 10%. We cannot at present measure a second ratio when we work beyond leading order, due to the reparameterization transformation. We are left instead with a one parameter family of mass ratios. The up quark mass has the widest range, presently including $m_u = 0$. Somewhat better known is $m_d/m_s \sim .05(1 \pm 0.3)$. More precise statements than this are model dependent. In order to do better in the measurement process, we need to find a way to exploit axial $U(1)$ anomalous Ward identities.

[1] I am of course pleased to recommend a recent book with a more detailed treatment: J.F. Donoghue, E. Golowich and B.R. Holstein, "Dynamics of the Standard Model", (Cambridge University Press, Cambridge, U.K., 1992.

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