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Light baryon magnetic moments and $N \to \Delta\gamma$ transition in a Lorentz covariant chiral quark approach

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We calculate magnetic moments of light baryons and $N \to \Delta\gamma$ transition characteristics using a manifestly Lorentz covariant chiral quark approach for the study of baryons as bound states of constituent quarks dressed by a cloud of pseudoscalar mesons.

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I. INTRODUCTION

The study of the magnetic moments of light baryons and of the $N \to \Delta\gamma$ transition represents an old and important problem in hadron physics. Many theoretical approaches—lattice QCD, QCD sum rules, Chiral Perturbation Theory (ChPT), various quark and soliton methods, techniques based on the solution of Bethe-Salpeter and Faddeev field equations, etc. – have been applied in order to calculate these quantities.

It should be stressed that analysis of the $N \to \Delta\gamma$ transition is of particular interest because it allows one to probe the structure of both the nucleon and $\Delta(1232)$-isobar and can help to shed light on their possible deformation. This reaction represents a crucial test for the various theoretical approaches. For example, naive quark models based on SU(6) symmetry, which model the nucleon and its first resonance as a spherically symmetric 3q-configurations, fail to correctly describe the electric G_{E2} and Coulomb G_{C2} quadrupole form factors, which vanish in such models in contradistinction with experiment.

In Refs. [1, 2, 3] a model-independent analysis of the $N \to \Delta\gamma$ transition amplitude has been performed. Based on gauge and Lorentz covariance it was shown that the corresponding vertex function can be expressed in terms of three linearly independent form factors. All aspects of the reaction, such as helicity or multipole amplitudes, are expressible in terms of these form factors.

A comprehensive review of the role of nucleon resonances in nuclear structure has been presented in Ref. [4]. A didactic introduction to the $N - \Delta$ transition involving the main theoretical ideas and predictions of the constituent quark model (CQM) and its applications to the electromagnetic properties of nucleons and nuclei is given in Ref. [5]. This paper reviews the Isgur-Karl model and presents basic formulae for calculation of the baryon spectrum. Quarks are fundamental carriers of the baryon charge and coupling of the photon is introduced at the quark level. The model is used in the evaluation of the electromagnetic properties in terms of nucleon form factors, the Δ electromagnetic form factors, and excitations of the nucleon resonances. Further improvements are proposed: inclusion of relativistic effects, introduction of pion degrees of freedom, etc.

An effective Lagrangian incorporating chiral symmetry has been utilized in [6]. This Lagrangian includes at tree level the pseudo-vector Born terms, leading $t-$channel vector-meson exchanges, as well as $s-$ and $u-$channel Δ-isobar exchanges. The magnetic dipole ($M1$) and electric quadrupole ($E2$) amplitudes are expressed in terms of two independent gauge couplings at the $\gamma N\Delta$ vertex. The investigation of pion photoproduction from threshold through the $\Delta(1232)$ resonance region is accomplished using various unitarization methods, such that the errors obtained for both $E2$ and $M1$ multipoles reflect theoretical uncertainties as well as model dependence.

Ref. [7] analyzed the vector and axial form factors of the NN and $N\Delta$ systems as well as the πNN and $\pi N\Delta$ coupling constants (calculated defining two effective Lagrangians for the πNN and $\pi N\Delta$ interactions) within a constituent quark model. The main conclusion is that while the Goldberger-Treiman relation remains valid, the experimental couplings are found to be larger by 30% or so than those predicted by the model. Also, the use of a constituent quark model provides significant mass-dependent corrections to the naive predictions of SU(6) symmetry.

Complex form factors were calculated to order $O(\epsilon^3)$ in the "small scale expansion" formalism (inclusion of the Δ degrees of freedom consistent with chiral symmetry), within the framework of chiral effective theory [8]. It is shown that the low-q^2 dependence of the three transition multipoles – $M1(q^2)$, $E2(q^2)$ and $C2(q^2)$ – is governed by the πN and $\pi\Delta$ loop effects. The effective chiral lagrangian incorporates both the spontaneous and explicit breaking of chiral invariance. The way in which unknown low energy constants affect the ratios $\mathrm{EMR}(q^2) = E2(q^2)/M1(q^2)$ and $\mathrm{CMR}(q^2) = C2(q^2)/M1(q^2)$ is elucidated, and estimated values for the three individual couplings are obtained.

In Ref. [9] it was demonstrated that the $C2/M1$ ratio is related to the neutron elastic form factor ratio G_C^n/G_M^n not only at zero momentum transfer, but also for the entire range of momentum transfer where data is available. Relations are presented between the charge quadrupole transition form factor and elastic nucleon charge form factor on one side and the magnetic dipole transition form factor and elastic neutron magnetic form factor on the other. For example, at zero momentum transfer, the transition quadrupole moment and the neutron charge radius are related, leading the authors to the conclusion that the phenomena of deviation from the nucleon's spherical symmetry has its origin in a nonspherical cloud of quark-antiquark pairs in the nucleon. Performing an extrapolation of the $C2/M1$ result to $Q^2 \to \infty$ the ratio asymptotically approaches a small negative constant in qualitative agreement with perturbative QCD (pQCD).

Ref. [10] studied the chiral behavior (M_π dependence) of the $\gamma N\Delta$ EMR and CMR ratios using a relativistic effective chiral Lagrangian involving pion and nucleon fields supplemented by relativistic Δ-isobar fields. The calculation of observables in the pion electroproduction amplitude was performed to next-to-leading order (NLO) in the δ-expansion. The parameters entering the calculation of the various cross sections are the couplings g_M, g_E and g_C characterizing the individual $M1, E2$ and $C2$ transitions.

In Ref. [11] a theoretical framework using the light-cone sum rule approach has been suggested for the calculation of the $\gamma^* N \to \Delta$ transition. All three possibilities for virtual photon polarization were allowed, so the transition is described by three independent form factors. Since predictions are close to data in the region above $Q^2 \sim 2$ GeV^2,

the main conclusion on the result for the magnetic form factor is that the "soft" contribution is dominant at the experimentally accessible momentum transfers.

There are a number of interesting problems which we address in the present paper:

i) if one believes that both valence and sea-quark effects are important in the description of the electromagnetic properties of light baryons, then how large is the contribution of the meson-cloud;

ii) what is the physics required to correctly predict the $M1$ amplitude for the $N \to \Delta$ transition, which is considerably underestimated in constituent quark models;

iii) what input is needed in order to explain the experimental data for $E2/M1$ and $C2/M1$.

To possibly answer the above questions we use a Lorentz covariant chiral quark model recently developed in Ref. [12]. The approach is based on a non-linear chirally symmetric Lagrangian, which involves constituent quarks and the chiral (pseudoscalar meson) fields as the effective degrees of freedom. In a first step, this Lagrangian can be used to perform a dressing of the constituent quarks by a cloud of light pseudoscalar mesons and other heavy states using the calculational technique of infrared dimensional regularization (IDR) of loop diagrams. Then within a proper chiral expansion, we calculate the dressed transition operators which are relevant for the interaction of the quarks with external fields in the presence of a virtual meson cloud. In a following step, these dressed operators are used to calculate baryon matrix elements. Note, that a simpler and more phenomenological quark model which was based on the similar ideas of the dressing of the constituent quarks by a meson cloud has been developed in Refs. [13].

In the manuscript we proceed as follows. First, in Section II, we discuss basic notions of our approach. We derive the chiral Lagrangian motivated by baryon ChPT [14]-[21], and formulate it in terms of quark and mesonic degrees of freedom. Next, we use this Lagrangian to perform a dressing of the constituent quarks by a cloud of light pseudoscalar mesons and by other heavy states, using the calculational technique developed in Ref. [14]. We derive dressed transition operators within a proper chiral expansion, which are in turn relevant for the interaction of quarks with external fields in the presence of a virtual meson cloud. Then we discuss the calculation of matrix elements of dressed quark operators between baryons states using a specific constituent quark model [22]-[24] based on a specific hadronization ansatz of quarks in baryons. In Section III, we apply our approach to the study of magnetic moments of light baryons (nucleons and hyperons) and to the properties of the $N \to \Delta\gamma$ transition. In Section IV we present a short summary of our results.

II. APPROACH

A. Chiral Lagrangian

The chiral quark Lagrangian $\mathcal{L}_{qU}$ (up to order p^4), which dynamically generates the dressing of the constituent quarks by mesonic degrees of freedom, consists of two primary pieces $\mathcal{L}_q$ and $\mathcal{L}_U$:

$$\mathcal{L}_{qU} \,=\, \mathcal{L}_q + \mathcal{L}_U \,, \qquad \mathcal{L}_q \,=\, \mathcal{L}_q^{(1)} + \mathcal{L}_q^{(2)} + \mathcal{L}_q^{(3)} + \mathcal{L}_q^{(4)} + \cdots \,, \qquad \mathcal{L}_U \,=\, \mathcal{L}_U^{(2)} + \cdots \,. \tag{1}$$

The superscript (i) attached to $\mathcal{L}_{q(U)}^{(i)}$ denotes the low energy dimension of the Lagrangian:

$$\begin{aligned}
\mathcal{L}_U^{(2)} \;&=\; \frac{F^2}{4}\langle u_\mu u^\mu + \chi_+\rangle \,, \qquad \mathcal{L}_q^{(1)} \,=\, \bar{q}\left[i\not{D} - m + \frac{1}{2}\, g \not{u}\, \gamma^5\right] q \,, \\
\mathcal{L}_q^{(2)} \;&=\; -\frac{c_2}{4m^2}\,\langle u_\mu u_\nu\rangle\,(\bar{q}\, D^\mu\, D^\nu\, q \,+\, \text{h.c.}\,) \,+\, \frac{c_4}{4}\,\bar{q}\, i\, \sigma^{\mu\nu}\,[u_\mu, u_\nu]\, q \,+\, \frac{c_6}{8m}\,\bar{q}\,\sigma^{\mu\nu}\, F^+_{\mu\nu}\, q \,+\, \cdots, \\
\mathcal{L}_q^{(3)} \;&=\; \frac{i d_{10}}{2m}\,\bar{q}\,[D^\mu, F^+_{\mu\nu}]\, D^\nu\, q \,+\, \text{h.c.} \,+\cdots \,, \\
\mathcal{L}_q^{(4)} \;&=\; \frac{e_6}{2}\,\langle\chi_+\rangle\,\bar{q}\,\sigma^{\mu\nu}\, F^+_{\mu\nu}\, q \,+\, \frac{e_7}{4}\,\bar{q}\,\sigma^{\mu\nu}\,\{F^+_{\mu\nu}\hat{\chi}_+\}\, q \,+\, \frac{e_8}{2}\,\bar{q}\,\sigma^{\mu\nu}\,\langle F^+_{\mu\nu}\hat{\chi}_+\rangle\, q \,-\, \frac{e_{10}}{2}\,\bar{q}\,[D^\alpha, [D_\alpha, F^+_{\mu\nu}]]\sigma^{\mu\nu}\, q \,+\cdots,
\end{aligned} \tag{2}$$

where $\hat{\chi}_+ = \chi_+ - \frac{1}{3}\langle\chi_+\rangle$, the symbols $\langle\ \rangle$, $[\ ]$ and $\{\ \}$ occurring in Eq. (2) denotes the trace over flavor matrices, commutator and anticommutator, respectively. In Eq. (2) we display only the terms involved in the calculation of the dressed electromagnetic quark operator. Here, for simplicity, we drop the contribution of vector mesons. The detailed form of the chiral Lagrangian can be found in Ref. [12].

The couplings m and g denote the quark mass and axial charge in the chiral limit, c_i, d_i and e_i are the second-, third- and fourth-order low-energy coupling constants, respectively, which encode the contributions of heavy states. Parameter m is counted as as quantity of order $O(1)$ in the chiral expansion.

Here q is the quark field, and the octet of pseudoscalar fields

$$\phi = \sum_{i=1}^{8} \phi_i \lambda_i = \sqrt{2} \begin{pmatrix} \pi^0/\sqrt{2} + \eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2} + \eta/\sqrt{6} & K^0 \\ K^- & \bar{K}^0 & -2\eta/\sqrt{6} \end{pmatrix} \tag{3}$$

is contained in the SU(3) matrix $U = u^2 = \exp(i\phi/F)$ where F is the octet decay constant. We introduce the standard notations [14, 16, 18]

$$D_\mu = \partial_\mu + \Gamma_\mu, \quad \Gamma_\mu = \frac{1}{2}[u^\dagger, \partial_\mu u] - \frac{i}{2} u^\dagger R_\mu u - \frac{i}{2} u L_\mu u^\dagger, \tag{4}$$

$$u_\mu = i u^\dagger \nabla_\mu U u^\dagger, \quad \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \quad \chi = 2B\mathcal{M} + \cdots .$$

The fields R_μ and L_μ include external fields (electromagnetic A_μ, weak, etc.):

$$R_\mu \, = \, e\, Q\, A_\mu \, + \cdots , \qquad L_\mu \, = \, e\, Q\, A_\mu \, + \cdots$$

where $Q = \mathrm{diag}\{2/3, -1/3, -1/3\}$ is the quark charge matrix. The tensor $F^+_{\mu\nu}$ is defined as $F^+_{\mu\nu} \, = \, u^\dagger F_{\mu\nu} Q u + u F_{\mu\nu} Q u^\dagger$ where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the conventional photon field strength tensor. Here $\mathcal{M} = \mathrm{diag}\{\hat{m}, \hat{m}, \hat{m}_s\}$ is the mass matrix of current quarks (we work in the isospin symmetry limit with $\hat{m}_u = \hat{m}_d = \hat{m} = 7$ MeV and the mass of the strange quark $\hat{m}_s$ is related to the nonstrange one as $\hat{m}_s = 25\,\hat{m}$).

The quark vacuum condensate parameter is denoted by

$$B = -\frac{1}{F^2} \langle 0 | \bar{u} u | 0 \rangle = -\frac{1}{F^2} \langle 0 | \bar{d} d | 0 \rangle \, . \tag{5}$$

To distinguish between constituent and current quark masses we attach the symbol $\hat{}$ ("hat") when referring to the current quark masses. We rely on the standard picture of chiral symmetry breaking ($B \gg F$). In leading order of the chiral expansion the masses of pseudoscalar mesons are given by

$$M_\pi^2 = 2\hat{m} B, \qquad M_K^2 = (\hat{m} + \hat{m}_s) B, \qquad M_\eta^2 = \frac{2}{3}(\hat{m} + 2\hat{m}_s) B \, . \tag{6}$$

In the numerical analysis we will use: $M_\pi = 139.57$ MeV, $M_K = 493.677$ MeV (the charged pion and kaon masses), $M_\eta = 574.75$ MeV and the canonical set of differentiated decay constants: $F_\pi = 92.4$ MeV, $F_K/F_\pi = 1.22$ and $F_\eta/F_\pi = 1.3$ [25].

B. Dressing of quark operators

Any bare quark operator (both one- and two-body) can be dressed by a cloud of pseudoscalar mesons and heavy states in a straightforward manner by use of the effective chirally-invariant Lagrangian $\mathcal{L}_{qU}$. To illustrate the idea of such a dressing we consider the Fourier-transform of the electromagnetic quark operator:

$$J^{\rm bare}_{\mu,\,{\rm em}}(q) = \int d^4x \, e^{-iqx} \, j^{\rm bare}_{\mu,\,{\rm em}}(x) \, , \qquad j^{\rm bare}_{\mu,{\rm em}}(x) = \bar{q}(x) \, \gamma_\mu \, Q \, q(x) \, . \tag{7}$$

In Fig.1 we display the tree and loop diagrams which contribute to the dressed electromagnetic operator $J^{\rm dress}_{\mu,\,{\rm em}}$ up to fourth order. Note, here we restrict our consideration to the one-body quark operator. An extension of our method to two-body quark operators will be done in future.

The dressed quark operator $j^{\rm dress}_{\mu,\,{\rm em}}(x)$ and its Fourier transform $J^{\rm dress}_{\mu,\,{\rm em}}(q)$ have the following forms

$$j^{\rm dress}_{\mu,\,{\rm em}}(x) \; = \; \sum_{q=u,d,s} \left\{ f^q_D(-\partial^2) \, [\bar{q}(x) \gamma_\mu q(x)] \; + \; \frac{f^q_P(-\partial^2)}{2m_q} \, \partial^\nu \, [\bar{q}(x) \sigma_{\mu\nu} q(x)] \right\} \tag{8}$$

$$J^{\rm dress}_{\mu,\,{\rm em}}(q) \; = \; \int d^4x \, e^{-iqx} \, j^{\rm dress}_{\mu,\,{\rm em}}(x) = \int d^4x \, e^{-iqx} \sum_{q=u,d,s} \bar{q}(x) \left[\gamma_\mu \, f^q_D(q^2) \; + \; \frac{i}{2m_q} \, \sigma_{\mu\nu} \, q^\nu \, f^q_P(q^2) \right] q(x) \, ,$$

where m_q is the dressed constituent quark mass generated by the chiral Lagrangian (2) (see details in Ref. [12]); $f_D^u(q^2)$, $f_D^d(q^2)$, $f_D^s(q^2)$ and $f_P^u(q^2)$, $f_P^d(q^2)$, $f_P^s(q^2)$ are the Dirac and Pauli form factors of u, d and s quarks. Here we use the appropriate sub- and superscripts with a definite normalization of the set of $f_D^q(0) \equiv e_q$ (quark charges) due to charge conservation. Note, that the dressed quark operator satisfies current conservation: $\partial^\mu j^{\rm dress}_{\mu,\,{\rm em}}(x) = 0$. Evaluation of the diagrams in Fig.1 is based on the *infrared dimensional regularization* suggested in Ref. [14] to guarantee a straightforward connection between loop and chiral expansion in terms of quark masses and small external momenta. We relegate the discussion of the calculational technique to Ref. [12].

To calculate the electromagnetic transitions between baryons we project the dressed quark operator between the corresponding baryon states. The master formula is:

$$\langle B(p')|\, J^{\rm dress}_{\mu,\,{\rm em}}(q)\,|B(p)\rangle \;=\; (2\pi)^4\,\delta^4(p'-p-q)\,\bar u_B(p')\Bigg\{\gamma_\mu\,F_1^B(q^2)\;+\;\frac{i}{2\,m_B}\,\sigma_{\mu\nu}q^\nu\,F_2^B(q^2)\Bigg\}u_B(p)$$
$$=\;(2\pi)^4\,\delta^4(p'-p-q)\sum_{q=u,d,s}\Bigg\{f_D^q(q^2)\,\langle B(p')|\,j^{\rm bare}_{\mu,q}(0)\,|B(p)\rangle + i\,\frac{q^\nu}{2\,m_q}\,f_P^q(q^2)\,\langle B(p')|\,j^{\rm bare}_{\mu\nu,q}(0)\,|B(p)\rangle\Bigg\}\,, \qquad (9)$$

where $B(p)$ and $u_B(p)$ are the baryon state and spinor, respectively, normalized as

$$\langle B(p')|B(p)\rangle = 2E_B\,(2\pi)^3\,\delta^3(\vec p-\vec p\,')\,, \qquad \bar u_B(p)u_B(p) = 2m_B \qquad (10)$$

with $E_B = \sqrt{m_B^2+\vec p^{\,2}}$ being the baryon energy and m_B the baryon mass. In Eq.(9) we focus on the diagonal $\frac{1}{2}^+ \to \frac{1}{2}^+$ transitions (the extension to the nondiagonal transitions and transitions involving higher spin states like the $\Delta(1232)$ isobar is straightforward). Here $F_1^B(q^2)$ and $F_2^B(q^2)$ are the Dirac and Pauli baryon form factors. In Eq. (9) we express the matrix elements of the dressed quark operator in terms of the matrix elements of the bare operators. In our application we deal with the bare quark operators for vector $j^{\rm bare}_{\mu,q}(0)$ and tensor $j^{\rm bare}_{\mu\nu,q}(0)$ currents defined as

$$j^{\rm bare}_{\mu,q}(0) \;=\; \bar q(0)\,\gamma_\mu\,q(0)\,, \qquad\qquad j^{\rm bare}_{\mu\nu,q}(0) \;=\; \bar q(0)\,\sigma_{\mu\nu}\,q(0)\,. \qquad (11)$$

Eq. (9) contains our main result: we perform a model-independent factorization of the effects of hadronization and confinement contained in the matrix elements of the bare quark operators $j^{\rm bare}_{\mu,q}(0)$ and $j^{\rm bare}_{\mu\nu,q}(0)$ and the effects dictated by chiral symmetry (or chiral dynamics) which are encoded in the relativistic form factors $f_D^q(q^2)$ and $f_P^q(q^2)$. Due to this factorization the calculation of $f_D^q(q^2)$ and $f_P^q(q^2)$, on one side, and the matrix elements of $j^{\rm bare}_{\mu,q}(0)$ and $j^{\rm bare}_{\mu\nu,q}(0)$, on the other side, can be done independently. In particular, in a first step we derived a model-independent formalism based on the ChPT Lagrangian, which is formulated in terms of constituent quark degrees of freedom, for the calculation of $f_D^q(q^2)$ and $f_P^q(q^2)$. The calculation of the matrix elements of the bare quark operators can then be relegated to quark models based on specific assumptions about hadronization and confinement. The explicit forms of $f_D^q(q^2)$ and $f_P^q(q^2)$ are given in Appendix C of Ref. [12].

C. Matching to ChPT

The matrix elements of the bare quark operators should be calculated using specific model-dependent assumptions about hadronization and confinement. In Ref. [12] it was shown that in the case of nucleons the use of certain symmetry constraints leads to a set of relationships between the nucleon and corresponding quark form factors at zero momentum transfer. In general, due to Lorentz and gauge invariance, the matrix elements in Eq. (9) can be written as

$$\langle B(p')|\,j^{\rm bare}_{\mu,q}(0)\,|B(p)\rangle \;=\; \bar u_B(p')\Bigg\{\gamma_\mu\,F_1^{Bq}(q^2)\;+\;\frac{i}{2\,m_B}\,\sigma_{\mu\nu}\,q^\nu\,F_2^{Bq}(q^2)\Bigg\}u_B(p)\,, \qquad (12)$$
$$i\,\frac{q^\nu}{2\,m_q}\langle B(p')|\,j^{\rm bare}_{\mu\nu,q}(0)\,|B(p)\rangle \;=\; \bar u_B(p')\Bigg\{\gamma_\mu\,G_1^{Bq}(q^2)\;+\;\frac{i}{2\,m_B}\,\sigma_{\mu\nu}\,q^\nu\,G_2^{Bq}(q^2)\Bigg\}u_B(p)\,,$$

where $F^{Bq}_{1(2)}(q^2)$ and $G^{Bq}_{1(2)}(q^2)$ are the Pauli and Dirac form factors describing the distribution of quarks of flavor $q=u,d,s$ in the baryon B.

Let us briefly review the constraints on the nucleon form factors derived in Ref. [12]. The first set of relations arise from charge conservation and isospin invariance:

$$F_1^{pu}(0) = F_1^{nd}(0) = 2\,, \qquad F_1^{pd}(0) = F_1^{nu}(0) = 1\,, \qquad G_1^{Nq}(0) = 0\,, \qquad (13)$$
$$F_2^{pu}(0) = F_2^{nd}(0)\,, \qquad F_2^{pd}(0) = F_2^{nu}(0)\,, \qquad G_2^{pu}(0) = G_2^{nd}(0)\,, \qquad G_2^{pd}(0) = G_2^{nu}(0)\,.$$

Note, that the quantities $G_2^{Nq}(0)$ are related to the bare nucleon tensor charges $\delta_{Nq}^{\rm bare}$ [12]. The second set of constraints are the so-called *chiral symmetry constraints*. They are dictated by the infrared-singular structure of QCD and reproduce the leading nonanalytic (LNA) contributions to the magnetic moments and the charge and magnetic radii of nucleons [20, 26]:

$$\begin{aligned}
\mu_p &= -\frac{g_A^2}{8\pi}\,\frac{M_\pi}{F_\pi^2}\,\mathring{m}_N + \cdots\,, \\
\langle r^2\rangle_p^E &= -\frac{1+5g_A^2}{16\,\pi^2\,F_\pi^2}\ln\frac{M_\pi}{\mathring{m}_N} + \cdots\,, \\
\langle r^2\rangle_p^M &= \frac{g_A^2}{16\,\pi\,F_\pi^2\,\mu_p}\,\frac{\mathring{m}_N}{M_\pi} + \cdots\,,
\end{aligned} \tag{14}$$

where g_A and $\mathring{m}_N$ are the axial charge and the mass of the nucleon in the chiral limit. In order to fulfill the strictures of chiral symmetry (14) we demand the following identities involving the $F_2^{Nq}(0)$ and $G_2^{Nq}(0)$ form factors

$$1 + F_2^{pu}(0) - F_2^{pd}(0) = G_2^{pu}(0) - G_2^{pd}(0) = \left(\frac{g_A}{g}\right)^2 \frac{m_N}{\bar{m}}\,, \tag{15}$$

$$1 + F_2^{nd}(0) - F_2^{nu}(0) = G_2^{nd}(0) - G_2^{nu}(0) = \left(\frac{g_A}{g}\right)^2 \frac{m_N}{\bar{m}} \tag{16}$$

where $\bar{m} = m_u = m_d$ is the dressed nonstrange constituent quark mass in the isospin limit. In Ref. [12], the SU(6)-symmetry relations of the naive nonrelativistic quark model have been used for further constraints on $F_2^{Ni}(0)$ and $G_2^{Ni}(0)$. In this paper we go beyond the simple SU(6) picture, utilizing the relativistic constituent quark model [22]-[24] to calculate the bare baryonic matrix elements or to evaluate the contribution from the valence degrees of freedom.

D. Evaluation of the matrix elements of the valence quark operators

In this section we discuss the calculation of the baryonic matrix elements

$$\langle B(p')|\, j_{\mu,q}^{\rm bare}(0)\,|B(p)\rangle \qquad \text{and} \qquad \langle B(p')|\, j_{\mu\nu,q}^{\rm bare}(0)\,|B(p)\rangle \tag{17}$$

induced, respectively, by the bare quark operators:

$$j_{\mu,q}^{\rm bare}(0) \;=\; \bar{q}(0)\,\gamma_\mu\,q(0)\,, \qquad \text{and} \qquad j_{\mu\nu,q}^{\rm bare}(0) \;=\; \bar{q}(0)\,\sigma_{\mu\nu}\,q(0)\,. \tag{18}$$

We will consistently employ the relativistic three-quark model (RQM) [22]-[24] to compute such matrix elements (17). The RQM was previously successfully applied for the study of properties of baryons containing light and heavy quarks [23]-[24]. The main advantages of this approach are: Lorentz and gauge invariance, a small number of parameters, and modelling of effects of strong interactions at large (~ 1 fm) distances. Various properties of light and heavy baryons have been analyzed within this RQM [22]-[24], and a preliminary analysis of the electromagnetic properties of nucleons has been performed in Ref. [22] where the effects of valence quarks have been consistently taken into account. Here we extend this analysis to the case of hyperons as well as to the $N \to \Delta\gamma$ transitions and we include meson-cloud effects.

Let us begin by briefly reviewing the basic notions of the RQM approach [22]-[24]. The RQM is essentially based on an interaction Lagrangian describing the coupling between baryons and their constituent quarks. The coupling of a baryon $B(q_1q_2q_3)$ to its constituent quarks q_1, q_2 and q_3 is described by the Lagrangian

$$\mathcal{L}_{\rm int}^{\rm str}(x) = g_B\bar{B}(x)\int dx_1\int dx_2\int dx_3\, F_B(x,x_1,x_2,x_3)\, J_B(x_1,x_2,x_3) \;+\; \text{H.c.} \tag{19}$$

where $J_B(x_1,x_2,x_3)$ is the three-quark current with the quantum numbers of the relevant baryon B [27, 28]. One has

$$J_B(x_1,x_2,x_3) \;=\; \epsilon^{a_1a_2a_3}\,\Gamma_1\,q_1^{a_1}(x_1)\,q_2^{a_2}(x_2)C\,\Gamma_2\,q_3^{a_3}(x_3)\,, \tag{20}$$

where $\Gamma_{1,2}$ are Dirac structures, $C = \gamma^0\gamma^2$ is the charge conjugation matrix and $a_i, i = 1,2,3$ are color indices.

The function F_B is related to the scalar part of the Bethe-Salpeter amplitude and characterizes the finite size of the baryon. In the following we use a particular form for the vertex function [22]-[24]

$$F_B(x, x_1, x_2, x_3) \;=\; \delta^4(x - \sum_{i=1}^{3} w_i x_i)\; \Phi_B\Big(\sum_{i<j}(x_i - x_j)^2\Big) \tag{21}$$

where Φ_B is the correlation function of three constituent quarks with masses m_1, m_2, m_3. The variable w_i is defined by $w_i = m_i/(m_1 + m_2 + m_3)$ and therefore depends only on the relative Jacobi coordinates (ξ_1, ξ_2) as $\Phi_B(\xi_1^2 + \xi_2^2)$, where

$$\begin{aligned} x_1 &= x - \frac{\xi_1}{\sqrt{2}}\,(w_2 + w_3) + \frac{\xi_2}{\sqrt{6}}\,(w_2 - w_3)\,, \\ x_2 &= x + \frac{\xi_1}{\sqrt{2}}\,w_1 - \frac{\xi_2}{\sqrt{6}}\,(w_1 + 2w_3)\,, \\ x_3 &= x + \frac{\xi_1}{\sqrt{2}}\,w_1 + \frac{\xi_2}{\sqrt{6}}\,(w_1 + 2w_2)\,, \end{aligned} \tag{22}$$

and with $x = \sum\limits_{i=1}^{3} w_i x_i$ being the center of mass (CM) coordinate. Expressed in relative Jacobi coordinates and the center of mass coordinate, the Fourier transform of the vertex function reads [22]-[24]:

$$\Phi_B(\xi_1^2 + \xi_2^2) = \int \frac{d^4p_1}{(2\pi)^4} \int \frac{d^4p_2}{(2\pi)^4} e^{-ip_1\xi_1 - ip_2\xi_2}\, \widetilde{\Phi}_B(-p_1^2 - p_2^2)\,. \tag{23}$$

The choice of light baryon three-quark currents has been discussed in detail in Refs. [27, 28]. (For the discussion of the heavy baryon currents see Refs. [29] and [22]-[24]). When restricted to the unitary flavor SU(3) symmetry and the octet of light baryons, one can construct two linearly independent currents: vector and tensor. For the light baryon decuplet there exists only a single vector current. In Appendix A we list these three-quark currents for the baryon octet and for the $\Delta(1232)$-isobar. Note that the vector and tensor currents of the baryon octet [27, 28] are degenerate in the nonrelativistic limit. In the following we show that even in the relativistic case they give similar predictions for the magnetic moments of the nucleons, hyperons and for the $N \to \Delta\gamma$ transition. The quantities which *are* sensitive to the choice of the baryon octet currents are the $E2/M1$ and $C2/M1$ ratios, which are generated by relativistic effects and vanish in the nonrelativistic limit.

The baryon-quark coupling constants g_B are determined by the compositeness condition [22]-[24] (see also [30, 31]), which implies that the renormalization constant of the hadron wave function is set equal to zero:

$$Z_B = 1 - \Sigma'_B(m_B) = 0 \tag{24}$$

where $\Sigma'_B(m_B) = g_B^2 \Pi'_B(m_B)$ is the derivative of the baryon mass operator described by the diagram in Fig.2 and m_B is the baryon mass. To clarify the physical meaning of this condition, we first want to remind the reader that the renormalization constant $Z_B^{1/2}$ can also be interpreted as the matrix element between the physical and the corresponding bare state. For $Z_B = 0$ it then follows that the physical state does not contain the bare one and is described as a bound state. The interaction Lagrangian Eq. (19) and the corresponding free parts describe both the constituents (quarks) and the physical particles (hadrons) which are taken to be the bound states of the constituents. As a result of the interaction, the physical particle is dressed, *i.e.* its mass and its wave function have to be renormalized. The condition $Z_B = 0$ also effectively excludes the constituent degrees of freedom from the physical space and thereby guarantees that there is no double counting for the physical observable under consideration. In this picture the constituents exist in virtual states only. One of the corollaries of the compositeness condition is the absence of a direct interaction of the dressed charged particle with the electromagnetic field. Taking into account both the tree-level diagram and the diagrams with the self-energy and counter-terms insertions into the external legs (that is the tree-level diagram times $(Z_B - 1)$) one obtains a common factor Z_B which is equal to zero [31].

The quantities of interest, the matrix elements (17), are described by the triangle diagram in Fig.3a. In the case of the matrix element of the vector current we need to take into account two additional so-called "bubble" diagrams in Figs.3b and 3c to guarantee gauge invariance of matrix elements (see details in Refs. [22]-[24] and [32]). In particular, the "bubble" diagrams are generated by the non-local coupling of the baryon to the constituent quarks and the external gauge field which arises after gauging of the non-local strong interaction Lagrangian (19) containing the vertex function (21).

In the evaluation of the quark-loop diagrams we use the free fermion propagator for the constituent quark [22]-[24]:

$$i\,S_q(x-y) = \langle 0|T\,q(x)\,\bar{q}(y)|0\rangle \;=\; \int \frac{d^4k}{(2\pi)^4 i}\,e^{-ik(x-y)}\;\tilde{S}_q(k) \tag{25}$$

where

$$\tilde{S}_q(k) = \frac{1}{m_q - \not{k} - i\epsilon} \tag{26}$$

is the usual free fermion propagator in momentum space. We shall avoid the appearance of unphysical imaginary parts in Feynman diagrams by postulating the condition that the baryon mass must be less than the sum of the constituent quark masses $M_B < \sum_i m_{q_i}$.

In the next step we have to specify the vertex function $\tilde{\Phi}_B$, which characterizes the finite size of the baryons. In principle, its functional form can be calculated from the solutions of the Bethe-Salpeter equation for baryon bound states [33, 34]. In Refs. [35] it was found that, using various forms for the vertex function, the basic hadron observables are insensitive to the details of the functional form of the hadron-quark vertex form factor. We will use this observation as a guiding principle and choose a simple Gaussian form for the vertex function $\tilde{\Phi}_B$. Any choice for $\tilde{\Phi}_B$ is appropriate as long as it falls off sufficiently fast in the ultraviolet region of Euclidean space to render the Feynman diagrams ultraviolet finite. We employ the Gaussian form

$$\tilde{\Phi}_B(k_{1E}^2, k_{2E}^2) \doteq \exp(-[k_{1E}^2 + k_{2E}^2]/\Lambda_B^2)\,, \tag{27}$$

for the vertex function, where k_{1E} and k_{2E} are the Euclidean momenta. Here Λ_B is a size parameter, which parametrizes the distribution of quarks inside a given baryon. In previous papers we determined the following set of parameters for light baryons:

$$m_u = m_d = 420 \;\; \text{MeV}\,, \quad m_s = 570 \;\; \text{MeV}\,, \quad \Lambda_B = 1.25 \;\; \text{GeV}\,. \tag{28}$$

Note that the quoted value of the nonstrange constituent quark mass and the size parameter Λ_B have been obtained from the analysis of nucleon properties using the tensor $3q$ current, with the inclusion of only valence degrees of freedom. Below we intend to test the second choice – the vector current – and also include the meson-cloud contributions to the baryon properties.

E. $N \to \Delta\gamma$ transition

In this subsection we specify our approach for the case of the $N \to \Delta\gamma$ transition. In particular, we discuss in detail two important issues: i) projection of the dressed quark operator between nucleon and $\Delta(1232)$ states and ii) evaluation of the bare vector and tensor valence quark operators between the nucleon and the $\Delta(1232)$.

The projection of the dressed quark operator between nucleon and $\Delta(1232)$ states reads

$$\begin{aligned}\langle\Delta(p')|\,J^{\rm dress}_{\mu,\,{\rm em}}(q)\,|N(p)\rangle \;&=\; (2\pi)^4\,\delta^4(p'-p-q)\,\bar{u}^{\nu}_{\Delta}(p')\,\Lambda_{\mu\nu}(p,p')\,u_N(p)\\ =\;(2\pi)^4\,\delta^4(p'-p-q)\sum_{q=u,d}&\left\{f_D^q(q^2)\,\langle\Delta(p')|\,j^{\rm bare}_{\mu,q}(0)\,|N(p)\rangle + i\,\frac{q^{\nu}}{2\,m_q}\,f_P^q(q^2)\,\langle\Delta(p')|\,j^{\rm bare}_{\mu\nu,q}(0)\,|N(p)\rangle\right\},\end{aligned} \tag{29}$$

where $\Lambda_{\mu\nu}(p,p')$ is the $N \to \Delta\gamma$ vertex function, and $\bar{u}^{\nu}_{\Delta}(p')$ is the spin-$\frac{3}{2}$ Rarita-Schwinger spinor satisfying the supplementary conditions [36]:

$$\bar{u}^{\nu}_{\Delta}(p')\,\gamma_{\nu} = 0 \qquad \text{and} \qquad \bar{u}^{\nu}_{\Delta}(p')\,p'_{\nu} = 0\,. \tag{30}$$

The vertex function $\Lambda_{\mu\nu}(p,p')$ for on-shell nucleon and Δ-isobar states can be decomposed in terms of relativistic form factors $b_i(q^2)$ with $i = 1,2,3,4$:

$$\Lambda_{\mu\nu}(p,p') = [g_{\mu\nu}b_1(q^2) + p_\mu q_\nu b_2(q^2) + \gamma_\mu q_\nu b_3(q^2) + q_\mu q_\nu b_4(q^2)]\gamma^5 \tag{31}$$

Due to gauge invariance the fourth form-factor is a linear combination of the other three:

$$b_1(q^2) + b_2(q^2)p\cdot q + b_3(q^2)m_+ = -q^2 b_4(q^2)\,, \tag{32}$$

where $m_\Delta = 1232$ MeV is the mass of the Δ-isobar, $p \cdot q = (m_+ m_- - q^2)/2$ and $m_\pm = m_\Delta \pm m_N$. The $N\Delta$ vertex function $\Lambda_{\mu\nu}(p,p')$ can then be rewritten in a manifestly gauge-invariant form in terms of b_1, b_2 and b_3:

$$\Lambda_{\mu\nu}(p,p') \;=\; [g^\perp_{\mu\nu} b_1(q^2) + p^\perp_\mu q_\nu b_2(q^2) + \gamma^\perp_\mu q_\nu b_3(q^2)]\gamma^5 \tag{33}$$

or in terms of b_2, b_3 and b_4:

$$\Lambda_{\mu\nu}(p,p') \;=\; \Big[L^\perp_{2\,\mu\nu}\, b_2(q^2) \;+\; L^\perp_{3\,\mu\nu}\, b_3(q^2) \;+\; L^\perp_{4\,\mu\nu}\, b_4(q^2) \Big]\gamma^5 \tag{34}$$

where the superscript $\perp$ denotes the Lorentz-structures perpendicular to the photon momentum:

$$\begin{aligned} & g^\perp_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\,, \qquad p^\perp_\mu = p_\mu - q_\mu \frac{pq}{q^2}\,, \qquad \gamma^\perp_\mu = \gamma_\mu - q_\mu \frac{\not{q}}{q^2}\,, \\ & L^\perp_{2\,\mu\nu} \;=\; p^\perp_\mu q_\nu - g^\perp_{\mu\nu}\, pq\,, \qquad L^\perp_{3\,\mu\nu} \;=\; \gamma^\perp_\mu q_\nu - g^\perp_{\mu\nu}\, m_-\,, \qquad L^\perp_{4\,\mu\nu} \;=\; -g^\perp_{\mu\nu}\,, \end{aligned} \tag{35}$$

with

$$g^\perp_{\mu\nu}\, q^\mu = 0\,, \qquad p^\perp_\mu\, q^\mu = 0\,, \qquad \gamma^\perp_\mu\, q^\mu = 0\,. \tag{36}$$

It is easy to see that the gauge invariance of the $N \to \Delta\gamma$ matrix element is fulfilled: $q^\mu\, \Lambda_{\mu\nu}(p,p') = 0$. Alternative but equivalent sets of relativistic form factors defining the $N \to \Delta\gamma$ transition [1]-[11] are given in Appendix B.

Note that for the evaluation of the $N \to \Delta\gamma$ matrix element we use the *same* universal dressed electromagnetic quark operators including chiral corrections (8). For the calculation of the bare matrix elements $\langle \Delta(p')|\, j^{\rm bare}_{\mu,q}(0)\, |N(p)\rangle$ and $\langle \Delta(p')|\, j^{\rm bare}_{\mu\nu,q}(0)\, |N(p)\rangle$ we apply the same quark approach RQM [22]-[24] as for the case of the octet transitions. Again, for the case of the "vector" matrix element $\langle \Delta(p')|\, j^{\rm bare}_{\mu,q}(0)\, |N(p)\rangle$ we need to take into account the triangle diagram in Fig.3a as well as the two "bubble" diagrams shown in Figs.3b and 3c. For the case of the "tensor" matrix element $\langle \Delta(p')|\, j^{\rm bare}_{\mu\nu,q}(0)\, |N(p)\rangle$ we require the contribution of the triangle diagram only. In addition, due to the nondiagonality of the $N \to \Delta\gamma$ transition, we need to include the diagram in Fig.3d in the calculation of the "vector" matrix element in order to guarantee gauge invariance. This diagram describes the sub-process wherein the nucleon converts into the Δ-isobar via a quark loop followed by the interaction of the Δ with the external field. Note that the analogous diagram where the nucleon interacts with the external field and then converts into the Δ vanishes due to the Rarita-Schwinger conditions (30).

In analogy with the $\frac{1}{2}^+ \to \frac{1}{2}^+$ transitions [see Eq. (12)] we, for convenience, perform the expansion of the bare matrix elements describing $N \to \Delta$ transitions:

$$\langle \Delta(p')|\, j^{\rm bare}_{\mu,q}(0)\, |N(p)\rangle \;=\; \bar{u}^\nu_\Delta(p') \sum_{i=2}^{4} L^\perp_{i\,\mu\nu}\, b^V_i(q^2)\, \gamma^5\, u_N(p)\,, \tag{37}$$

$$i\, \frac{q^\nu}{2\, m_q}\, \langle \Delta(p')|\, j^{\rm bare}_{\mu\nu,q}(0)\, |N(p)\rangle \;=\; \bar{u}^\nu_\Delta(p') \sum_{i=2}^{4} L^\perp_{i\,\mu\nu}\, b^T_i(q^2)\, \gamma^5\, u_N(p)\,, \tag{38}$$

where the superscripts V and T denote the partial contributions of vector and tensor matrix element to the relativistic form factors b_i. Finally, the total results for the form factors b_i are:

$$\begin{aligned} & b_i(q^2) \;=\; b^{\rm bare}_i(q^2) \;+\; b^{\rm cloud}_i(q^2)\,, \\ & b^{\rm bare}_i(q^2) \;=\; \sum_{q=u,d} e_q\, b^V_i(q^2)\,, \qquad b^{\rm cloud}_i(q^2) \;=\; \sum_{q=u,d} \Big[(f^q_D(q^2) - e_q)\, b^V_i(q^2) \;+\; f^q_P(q^2)\, b^T_i(q^2) \Big]\,. \end{aligned} \tag{39}$$

where we have separated each form factor into its bare and meson cloud components.

III. PHYSICAL APPLICATIONS

In this section we consider the application of our technique to the problem of magnetic moments of light baryons and the static characteristics of the $N \to \Delta\gamma$ transition. We calculate the contributions of both valence and sea-quarks to these quantities using the approach discussed above. We remind the reader that such an analysis was performed in Ref. [12] using symmetry constraints in order to determine values of valence baryon form factors at zero recoil. In particular, exact values of the contributions of the valence degrees of freedom to the Pauli form factors were deduced using the requirements of SU(6) spin-flavor symmetry. In addition we considered a second possibility when we included SU(6) breaking corrections but without specific calculations. Here we precisely evaluate the valence quark effects (matrix elements of the bare quark operators (17)) using a Lorentz covariant framework, which helps to take relativistic effects into account. The proper inclusion is essential for a consistent calculation of the $N \to \Delta\gamma$ transition. In this paper we restrict our attention to the magnetic moments of the baryon octet and the multipoles of the $N \to \Delta\gamma$ transition.

A. Definition of baryon quantities

Below we give a set of definitions of baryon quantities which are the subject of the present calculations. First we recall the definition of the magnetic moments μ_B of the baryon octet in terms of the Dirac – F_1^B – and Pauli - F_2^B – form factors derived in Eq. (9):

$$\mu_B \;=\; [\, F_1^B(0) + F_2^B(0)\,]\;\frac{e}{2m_B}\,, \tag{40}$$

where we have set $\hbar = 1$. In terms of the nuclear magneton – $\mu_N = \frac{e\,\hbar}{2m_p}$ – the baryon magnetic moment is given by

$$\mu_B = \;[\, F_1^B(0) + F_2^B(0)\,]\;\frac{m_p}{m_B}\,, \tag{41}$$

where m_p is the proton mass. The off-diagonal $M1$ moment $\mu_{\Sigma\Lambda}$ defining the transition $\Sigma^0 \to \Lambda\gamma$ is given (again in units of nuclear magnetons)

$$\mu_{\Sigma\Lambda} = F_2^{\Sigma\Lambda}(0)\,\frac{2m_p}{m_\Sigma + m_\Lambda}\,, \tag{42}$$

where $F_2^{\Sigma\Lambda}(0)$ is the value of the corresponding Pauli form factor at zero recoil.

As noted above, in our formalism the magnetic moments of the octet baryon can be split into the contribution from valence quarks $\mu_B^{\rm bare}$ and from the meson cloud $\mu_B^{\rm cloud}$:

$$\mu_B \;=\; \mu_B^{\rm bare} \;+\; \mu_B^{\rm cloud} \tag{43}$$

where

$$\mu_B^{\rm bare} \;=\; \sum_{q=u,d,s} f_D^q(0)\,\left(F_1^{Bq}(0) + F_2^{Bq}(0)\right)\,, \tag{44}$$

$$\mu_B^{\rm cloud} \;=\; \sum_{q=u,d,s} f_P^q(0) G_2^{Bq}(0)\,. \tag{45}$$

Here the values of the meson-cloud Dirac form factors f_D^q at zero recoil coincide with the quark charges due to charge conservation: $f_D^q(0) \equiv e_q$. These meson-cloud form factors f_D^q and f_P^q have been calculated in Ref. [12], and the calculational method for the valence-quark form factors F_i^{Bq} and G_i^{Bq} has been discussed in detail in Refs. [22]-[24].

A complete description of the $N \to \Delta\gamma$ transition is then given in terms of the set of relativistic form factors $b_i(q^2)$ (expressions in terms of form factors from equivalent definitions can be found using the relations given in Appendix B and in Refs. [1]-[11]):

1) Magnetic form factor $G_{M1}(Q^2)$:

$$G_{M1}(Q^2) \;=\; \frac{1}{4}\left\{ b_3(Q^2)\frac{m_+(3m_\Delta + m_N) + Q^2}{m_\Delta} \;+\; b_2(Q^2)(m_+ m_- + Q^2) \;-\; 2b_4(Q^2)Q^2 \right\}. \tag{46}$$

2) Electric form factor $G_{E2}(Q^2)$:

$$G_{E2}(Q^2) \;=\; \frac{1}{4}\Bigg\{ b_3(Q^2)\frac{m_+m_- - Q^2}{m_\Delta} \;+\; b_2(Q^2)(m_+m_- + Q^2) \;-\; 2b_4(Q^2)Q^2 \Bigg\}. \tag{47}$$

3) Coulombic form factor $G_{C2}(Q^2)$:

$$G_{C2}(Q^2) \;=\; \frac{|\vec{q}\,|}{2}\Bigg\{ b_3(Q^2) \;+\; b_2(Q^2)E_N \;+\; b_4(Q^2)\omega \Bigg\}. \tag{48}$$

4) Helicity amplitudes $A_{3/2}(Q^2)$ and $A_{1/2}(Q^2)$:

$$A_{3/2}(Q^2) \;=\; -\sqrt{\frac{\pi\,\alpha\,\omega}{2m_N^2}}\,\left[G_{M1}(Q^2) \;+\; G_{E2}(Q^2)\right], \tag{49}$$

$$A_{1/2}(Q^2) \;=\; -\sqrt{\frac{\pi\,\alpha\,\omega}{6m_N^2}}\,\left[G_{M1}(Q^2) \;-\; 3\,G_{E2}(Q^2)\right]. \tag{50}$$

5) Ratios EMR $= E2/M1 = -G_{E2}(Q^2)/G_{M1}(Q^2)$ and CMR $= C2/M1 = -G_{C2}(Q^2)/G_{M1}(Q^2)$:

$$\mathrm{EMR}(Q^2) \;=\; -\,\frac{G_{E2}(Q^2)}{G_{M1}(Q^2)} \qquad \text{and} \qquad \mathrm{CMR}(Q^2) \;=\; -\,\frac{G_{C2}(Q^2)}{G_{M1}(Q^2)}\,. \tag{51}$$

6) Transition dipole moment $\mu_{N\Delta}$:

$$\mu_{N\Delta} \;=\; \frac{2}{\sqrt{6}}\,G_{M1}(0)\,. \tag{52}$$

7) Transition quadrupole moment $Q_{N\Delta}$:

$$Q_{N\Delta} \;=\; -\,\frac{4\sqrt{6}}{m_+\,m_-}\,\frac{m_\Delta}{m_N}\,G_{E2}(0)\,. \tag{53}$$

8) $\Delta^+ \to p + \gamma$ decay width:

$$\Gamma(\Delta^+ \to p\gamma) = \frac{m_\Delta\,m_N}{8\,\pi}\left[1 - \frac{m_N^2}{m_\Delta^2}\right]^2 \Bigg\{\,|A_{1/2}(0)|^2 + |A_{3/2}(0)|^2\Bigg\}, \tag{54}$$

where $Q^2 = -q^2$ is an Euclidean momentum squared, $\alpha = 1/137$ is the fine structure coupling,

$$E_N \;=\; m_\Delta - \omega = \frac{m_\Delta^2 + m_N^2 + Q^2}{2m_\Delta} \qquad \text{and} \qquad \omega = \frac{m_\Delta^2 - m_N^2 - Q^2}{2m_\Delta} \tag{55}$$

are the nucleon and photon energies, and

$$|\vec{q}\,| \;=\; \frac{\lambda^{1/2}(m_\Delta^2, m_N^2, -Q^2)}{2m_\Delta} \tag{56}$$

is the 3-momentum of the virtual photon in the Δ-isobar rest frame. Here

$$\lambda(x,y,z) \;=\; x^2 + y^2 + z^2 - 2xy - 2xz - 2yz \tag{57}$$

is the Källen triangle function.

Note, that the form factors $G_{M1}(Q^2)$ and $G_{E2}(Q^2)$ can be written in a more compact form as combinations of two form factors $b_1(Q^2)$ and $b_3(Q^2)$ using the identity (39):

$$G_{M1}(Q^2) \;=\; \frac{1}{2}\Bigg\{ -b_1(Q^2) + b_3(Q^2)\frac{m_+^2 + Q^2}{2m_\Delta} \Bigg\}. \tag{58}$$

and

$$G_{E2}(Q^2) \;=\; \frac{1}{2}\left\{-b_1(Q^2) - b_3(Q^2)\frac{m_+^2+Q^2}{2m_\Delta}\right\}. \tag{59}$$

Therefore, the sum of $G_{M1}(Q^2)$ and $G_{E2}(Q^2)$ is defined by the $b_1(Q^2)$ form factor, while their difference involves only the form factor $b_3(Q^2)$:

$$\begin{aligned} G_{M1}(Q^2) + G_{E2}(Q^2) \;&=\; -b_1(Q^2)\,, \\ G_{M1}(Q^2) - G_{E2}(Q^2) \;&=\; b_3(Q^2)\,\frac{m_+^2+Q^2}{2m_\Delta}\,. \end{aligned} \tag{60}$$

B. Numerical results

As stressed above, for the octet states there exist two possible choices for the three-quark current: vector and tensor. A preliminary analysis (see also Ref. [22]) showed that these two types of currents give practically the same (or at least very similar) results in the case of the static properties of light baryons, *e.g.*, magnetic moments. This result is easily understood because the vector and tensor currents of the baryon octet become degenerate in the nonrelativistic limit. Also, the magnetic moments of light baryons are dominated by the nonrelativistic contributions, with relativistic corrections being of higher order and small. This explains why the simple nonrelativistic quark approaches work so well in the description of the magnetic moments of light baryons. Therefore, in order to distinguish between the two types of currents of the baryon octet we need to examine quantities which are dominated by relativistic effects. Two such quantities are the well known ratios $E2/M1$ and $C2/M1$ of the multipole amplitudes characterizing the $N \to \Delta\gamma$ transition. Here we find that the sole use of vector and tensor currents gives *opposite* results for the signs of these ratios. In particular, the use of the pure vector current for the proton gives reasonable results for $E2/M1$ and $C2/M1$ both with a *correct* (negative) sign, while the use of the pure tensor current yields ratios with *wrong* (positive) sign. Therefore, the study of the ratios $E2/M1$ and $C2/M1$ allows one to select the appropriate current for the description of the bound-state structure of the baryon octet (nucleons and hyperons). It is interesting to note that in the QCD sum rule method [27] dealing with current quarks the vector current structure is also preferred. This choice originally gave an explanation of the nucleon mass, while the use of the tensor current yields a suppression of the nucleon mass due to the "bad" chiral properties of this type of the three-quark current. We would like to stress, however, that this preference of the vector current for the description of the baryon octet in our approach and in QCD sum rules is apparently just coincidental because here we are dealing with constituent quarks instead of current quarks. Later on we will discuss why the tensor current fails for the ratios $E2/M1$ and $C2/M1$.

First we give a summary of the results obtained. In Table 1 we present our results for the magnetic moments of nucleons, hyperons and nondiagonal transitions $\Sigma^0 \to \Lambda\gamma$ and $N \to \Delta\gamma$ using the canonical set (Set I) [22]-[24] of parameters for the constituent quark masses $m_u = m_d = 420$ MeV and $m_s = 570$ MeV and the dimensional parameter $\Lambda_B = 1.25$ GeV characterizing the distribution of quarks in light baryons. For the octet baryon states we use the vector current. Another two solutions (Set II and Set III) corresponding to fixed values of the constituent quark masses, but with $\Lambda_B = 0.8$ GeV and $\Lambda_B = 0.75$ GeV, are also presented in Table 1. The reason for decreasing the value of the dimensional parameter Λ_B from 1.25 GeV to 0.75 GeV will be discussed below.

In Table 2 we show for comparison the results for the magnetic moments of baryons using a pure vector or tensor current for the octet states. The model parameters are fixed as $m_u = m_d = 420$ MeV, $m_s = 570$ MeV and $\Lambda_B = 0.8$ GeV. Here, for convenience, we restrict to the bare results (contributions of valence quarks only).

In Tables 3, 4, and 5 we give our results for observables of the $N \to \Delta\gamma$ transition such as the ratios EMR and CMR (at zero recoil and finite $Q^2 = 0.06$ GeV^2), the helicity amplitudes, the form factors G_{E2}, G_{M1} and G_{C2} at zero recoil, the dipole $\mu_{N\Delta}$ and quadrupole $Q_{N\Delta}$ moments, and the decay width. Again we give results for the three sets of parameters: Set I ($\Lambda_B = 1.25$ GeV), Set II ($\Lambda_B = 0.8$ GeV) and Set III ($\Lambda_B = 0.75$ GeV) while using the vector current for the octet baryons. In all cases, that is for Tables 1,3,4 and 5, we show the contributions both of the valence quarks ($3q$) and of the meson cloud. Note that the mesonic cloud contribution to the magnetic moments has been calculated [12] with the use of the chiral Lagrangian (2) and is expressed in terms of the following parameters: the constituent quark mass m, the axial quark charge g in the chiral limit, the low-energy coupling constants c_2, c_6, e_7, and e_8. As in Ref. [12] the parameters m and g are fixed input parameters. The value of the parameter c_2 has been deduced from the analysis of the nucleon mass, meson-nucleon sigma-terms and the q^2-dependence of electromagnetic nucleon form factors. The remaining parameters c_6, e_7 and e_8, controlling the size of the meson cloud contribution to the magnetic moments, are directly fitted to reproduce the experimental values of μ_p, μ_n and μ_Λ.

The set of values for c_6, e_7 and e_8 used here differs from the ones of Ref. [12], where instead the valence quark contributions have been fixed using gauge, isospin and chiral symetry constraints (as also discussed in Sec.IIc).

In addition, in Ref. [12] we additionally studied the implementation of the naive SU(6) valence quark model and corrections, expressed for example in the valence quark form factors of hyperons.

To state it clearly, in the present context we again use and test important symmetry constraints already derived in Ref. [12] and discussed in Sec.IIc. But now we calculate the contribution of the valence quarks using the relativistic quark model [22]-[24]. Most of the constraints (like gauge and isospin invariance constraints) are satisfied automatically. Fulfillment of the so-called chiral constraints

$$1 + F_2^{pu}(0) - F_2^{pd}(0) = G_2^{pu}(0) - G_2^{pd}(0) = \left(\frac{g_A}{g}\right)^2 \frac{m_N}{\bar{m}}\,, \tag{61}$$
$$1 + F_2^{nd}(0) - F_2^{nu}(0) = G_2^{nd}(0) - G_2^{nu}(0) = \left(\frac{g_A}{g}\right)^2 \frac{m_N}{\bar{m}}$$

is nontrivial. The direct calculation of the valence quark form factors $F_2^{Nq}(0)$ and $G_2^{Nq}(0)$ shows a slight violation of these identities (61). In particular, for the Set I, II and III we get, respectively:

$$1 + F_2^{pu}(0) - F_2^{pd}(0) \equiv 1 + F_2^{nd}(0) - F_2^{nu}(0) = 4.06\,, \tag{62}$$
$$G_2^{pu}(0) - G_2^{pd}(0) \equiv G_2^{nd}(0) - G_2^{nu}(0) = 3.51\,, \tag{63}$$

$$1 + F_2^{pu}(0) - F_2^{pd}(0) \equiv 1 + F_2^{nd}(0) - F_2^{nu}(0) = 4.25\,, \tag{64}$$
$$G_2^{pu}(0) - G_2^{pd}(0) \equiv G_2^{nd}(0) - G_2^{nu}(0) = 3.62\,, \tag{65}$$

and

$$1 + F_2^{pu}(0) - F_2^{pd}(0) \equiv 1 + F_2^{nd}(0) - F_2^{nu}(0) = 4.26\,, \tag{66}$$
$$G_2^{pu}(0) - G_2^{pd}(0) \equiv G_2^{nd}(0) - G_2^{nu}(0) = 3.63\,. \tag{67}$$

Due to the importance of satisfying these chiral constraints (they follow from the infrared singularities of the nucleon form factors and are model-independent identities), it is necessary to modify the vector $\bar{q}\gamma_\mu q$ or the tensor $\bar{q}\sigma_{\mu\nu}q$ currents used in the evaluation of the $F_{1(2)}^{Nq}$ or G_2^{Nq} form factors. In the following we argue that a modification of the G_2^{Nq} form factors only leads to a fulfullment of these constraints. From Eq. (44) one can see, that a modification of the F_1^{Nq} and F_2^{Nq} form factors leads to a modification of the bare contribution of the baryon magnetic moment since the quantity $f_D^q(0) \equiv e_q$ is fixed by charge conservation. Therefore, a modification of the F_1^{Nq} and F_2^{Nq} form factors is not possible since it would lead to a modification of physical quantities (*e.g.* the bare baryon magnetic moments). This is not the case for the G_2^{Nq} form factors. We can modify the G_2^{Nq} form factors ($G_2^{Nq} \to \tilde{G}_2^{Nq}$) to guarantee the fulfillment of the chiral constraints (61). Then we need to modify the form factors $f_P^q(0) \to \tilde{f}_P^q(0)$ to guarantee invariance of the meson-cloud contributions $\mu_N^{\rm cloud}$ (see Eq. (45):

$$\mu_N^{\rm cloud} \;\equiv\; \sum_{q=u,d} f_P^q(0) G_2^{Nq}(0) \;\equiv\; \sum_{q=u,d} \tilde{f}_P^q(0) \tilde{G}_2^{Nq}(0)\,. \tag{68}$$

On the other hand the modication $f_P^q(0) \to \tilde{f}_P^q(0)$ can be achieved by a redefinition of the low-energy constants c_6, e_7 and e_8, which are free parameters in the chiral Lagrangian (2).

The modification of the G_2^{Nq} form factors is achieved by appending the so-called "chiral" counterterm constructed with the use of nucleon fields. In particular, the tensor currents originally constructed in terms of quark fields and used for the calculation of the matrix element $\langle B(p')|\, j_{\mu\nu,q}^{\rm bare}(0)\,|B(p)\rangle$ should be modified by adding a term containing nucleon fields, viz.:

$$\bar{q}(x)\sigma_{\mu\nu}q(x) \;\to\; \bar{q}(x)\sigma_{\mu\nu}q(x) \;+\; \frac{\bar{m}}{m_N}\,\bar{N}(x)\sigma_{\mu\nu}\,\delta G^{Nq} N(x)\,, \tag{69}$$

where $\delta G^{Nu} = {\rm diag}\{\delta G^{pu}, \delta G^{nu}\}$ and $\delta G^{Nd} = {\rm diag}\{\delta G^{pd}, \delta G^{nd}\}$ are the diagonal 2×2 flavor matrices and $q = u$ or d.

These matrix elements are fixed to enhance the magnitudes of the form factors $G_2^{Nq}(0)$ and to satisfy the chiral constraints (15). The idea is to increase the combinations $G_2^{pu}(0) - G_2^{pd}(0)$ and $G_2^{nd}(0) - G_2^{nu}(0)$ from 3.51 (Set I),

3.62 (Set II) or 3.63 (Set III) to 4.06 (Set I), 4.25 (Set II) or 4.26 (Set III). Note that such modifications alter the normalization of the form factors G_2^{Nq} without additional change of these form factors at finite values of q^2. To fulfill the chiral constraints (61), we fix the constants δG^{Nq} in (69) as:

Set I

$$\delta G^{pu} \equiv \delta G^{nd} \, = \, -4\delta G^{pd} \equiv -4\delta G^{nu} = 0.440\,, \tag{70}$$

Set II

$$\delta G^{pu} \equiv \delta G^{nd} \, = \, -4\delta G^{pd} \equiv -4\delta G^{nu} = 0.504\,, \tag{71}$$

Set III

$$\delta G^{pu} \equiv \delta G^{nd} \, = \, -4\delta G^{pd} \equiv -4\delta G^{nu} = 0.504\,. \tag{72}$$

After introducing the counterterm (69) the form factors G_2^{Nq} are modified as

$$G_2^{Nq}(0) \to \tilde{G}_2^{Nq}(0) = G_2^{Nq}(0) + \delta G_2^{Nq}(0)\,. \tag{73}$$

As we stressed before we need to modify the form factors $f_P^q(0) \to \tilde{f}_P^q(0)$ to guarantee invariance of the meson-cloud contributions $\mu_N^{\rm cloud}$. As stated above, the result for $\tilde{f}_P^q(0)$ has no physical meaning: we merely need to redefine the low-energy couplings parametrizing this quantity.

Finally, to guarantee the invariance of the meson-cloud contributions to the magnetic moments of other baryons (including $N \to \Delta\gamma$ transition) we need to introduce the "chiral" counterterms by extending the tensor quark operator

$$\bar{q}(x)\sigma_{\mu\nu}q(x) \;\to\; \bar{q}(x)\sigma_{\mu\nu}q(x) \,+\, \frac{\bar{m}}{m_B}\sum_B \bar{B}(x)\sigma_{\mu\nu}\delta G^{Bq}B(x) + \cdots\,, \tag{74}$$

where δG^{Bq} is fixed from the condition

$$\mu_B^{\rm cloud} \;\equiv\; \sum_{q=u,d,s} f_P^q(0)G_2^{Bq}(0) \;\equiv\; \sum_{q=u,d,s} \tilde{f}_P^q(0)\tilde{G}_2^{Bq}(0)\,. \tag{75}$$

Note that we do not modify the form factors associated with the strange quark, that is $f_P^s \equiv \tilde{f}_P^s$ and $G_2^{Bs} \equiv \tilde{G}_2^{Bs}$, because we do not have special constraints on the strange quark contributions. In Eq. (74) we display for transparency only the diagonal operators in terms of baryon fields. The nondiagonal terms relevant for $\Sigma^0 \to \Lambda\gamma$ and $N \to \Delta\gamma$ transitions are omitted (symbol $\cdots$) and can be derived in analogy.

Now we discuss, how the parameters intering in the calculation of meson-cloud contributions are determined. With the use of the chiral constraint (61), the physical mass of the nucleon $m_N = m_p = 938.27$ MeV and its axial charge $g_A = 1.267$ [37] we fixed the quark axial charge as: $g = 0.94$ (Set I), $g = 0.92$ (Set II), $g = 0.92$ (Set III). For the parameter c_2 we use the value fixed in Ref. [12]: $c_2 = 2.502$ GeV^{-1}. The parameters c_6, e_7 and e_8 are fixed as:

Set I

$$\tilde{c}_6 = 0.163\,,\;\; \bar{e}_7 = -0.426\ {\rm GeV}^{-3}\,,\;\; \bar{e}_8 = -0.097\ {\rm GeV}^{-3}\,. \tag{76}$$

Set II

$$\tilde{c}_6 = 0.067\,,\;\; \bar{e}_7 = -0.318\ {\rm GeV}^{-3}\,,\;\; \bar{e}_8 = -0.076\ {\rm GeV}^{-3}\,. \tag{77}$$

Set III

$$\tilde{c}_6 = 0.067\,,\;\; \bar{e}_7 = -0.314\ {\rm GeV}^{-3}\,,\;\; \bar{e}_8 = -0.082\ {\rm GeV}^{-3}\,. \tag{78}$$

In the calculation of the q^2-dependence of the meson-cloud contribution following constants in the chiral Lagrangian (1) enter: c_4, d_{10} and e_{10}. Here for c_4, d_{10} and e_{10} we use the values fixed previously [12]:

$$c_4 = 1.693\ {\rm GeV}^{-1}\,,\;\; \bar{d}_{10} = 1.110\ {\rm GeV}^{-2}\,,\;\; \bar{e}_{10} = 0.039\ {\rm GeV}^{-3}\,. \tag{79}$$

In Eqs. (76)-(78) the constants $\bar{d}_{10}$, $\bar{e}_7$ and $\bar{e}_8$ and $\bar{e}_{10}$ refer to the renormalized coupling constants (see details in Ref. [12]) and $\tilde{c}_6 = c_6 - 16m(2\hat{m} + \hat{m}_s)$.

Now we return to discuss our results. As evident from Table 1 the magnetic moments of the baryon octet can be described with good accuracy for different values of the parameter Λ_B. There is only a weak dependence on the variation of this parameter from 0.8 GeV to 1.25 GeV. However, the dipole moment $\mu_{N\Delta}$ of the $N \to \Delta\gamma$ transition is quite sensitive to the variation of Λ_B. The reason for this stronger dependence is that $\mu_{N\Delta}$ is proportional to the combination of the form factors b_3 and b_2:

$$\mu_{N\Delta} \sim b_3(0)\frac{m_+(3m_\Delta + m_N)}{m_\Delta} + b_2(0)m_+m_- . \tag{80}$$

The main contribution to $\mu_{N\Delta}$ comes from the $b_3(0)$ form factor which has dimension $1/M$. If we restrict our attention to the leading contribution to $\mu_{N\Delta}$ coming from the $3q$ core then we immediately realize that b_3 scales as $1/\Lambda_B$. Hence we need to decrease the parameter Λ_B to get a reasonable description of $\mu_{N\Delta}$. With $\Lambda_B \simeq 0.75$ GeV one can fit the central value of $\mu_{N\Delta}$ precisely.

The next point of discussion is the sign of the EMR and CMR ratios. Again we restrict our attention to zero recoil – $Q^2 = 0$. The contribution of the b_4 form factor can essentially be neglected in our considerations since we find $b_4(0)/b_3(0) \simeq -1/10$ and $b_4(0)/b_2(0) \simeq 1/5$. If we temporarily also neglect the b_2 form factor in EMR and CMR then (this is a well-known result in the literature [1]-[11]) these ratios become degenerate and equal to

$$\text{EMR} \;=\; \text{CMR} \;=\; -\frac{m_-}{3m_\Delta + m_N} \simeq -6\% . \tag{81}$$

Therefore, to reproduce the *experimental* results for these quantities we require a contribution from the b_2 form factor. In both cases (for a vector and a tensor current) the value of the b_2 form factor is negative, but in the case of the tensor current it is twice as large than what is required phenomenologically. As a result, in the case of the tensor proton current the G_{E2} and G_{C2} form factors actually change sign from positive to negative, leading to positive ratios EMR and CMR. This is *not* the case for the vector current for the proton, and we therefore conclude that the vector current is strongly preferred in the calculations of the properties of $N \to \Delta\gamma$ transition. To further illustrate this issue, in Table 6 we demonstrate the sensitivity of the EMR and CMR ratios on the choice of the three-quark proton current (see discussion below).

Again, Table 2 shows that the pure vector and the pure tensor current used for the baryon octet give similar results for the bare magnetic moments of light baryons for the same set of model parameters: constituent quark masses and dimensional parameter Λ_B. In Table 2 we restrict ourselves to the specific choice of model parameters (Set II): $m_u = m_d = 420$ MeV, $m_s = 570$ MeV and $\Lambda_B = 0.8$ GeV. However, the *similarity* of results for the two respective octet currents is not very sensitive to a variation of the model parameters.

In Tables 3, 4 and 5 we present the detailed results for the properties of the $N \to \Delta\gamma$ transition for different values of the dimensional parameter $\Lambda_B = 1.25$, 0.8 and 0.75 GeV, respectively. For the EMR and CMR ratios we present our predictions at zero recoil ($Q^2 = 0$) and at the finite value $Q^2 = 0.06$ GeV^2 (recently the A1 Collaboration at Mainz [38] measured these quantities at this kinematic point). Our predictions are in good agreement with the experimental data of the LEGS Collaboration at Brookhaven [39] and of the GDH, A1 and A2 Collaborations at Mainz [38, 40]. The quantities which are sensitive to the choice of the dimensional parameter Λ_B are the magnetic, electric and Coulomb form factors and related quantities – helicity amplitudes, dipole and quadrupole moment, decay width. As we stressed before, the magnetic form factor G_{M1} and the dipole moment $\mu_{N\Delta}$ increase when the parameter Λ_B decreases. Other quantities mentioned above have the same tendency. Therefore, the best description of the data is achieved for values of $\Lambda_B = 0.8$ or 0.75 GeV. In Figs.4-10 we demonstrate the dependence of the G_{M1}, G_{E2}, G_{C2} form factors, the helicity amplitudes $A_{1/2}$ and $A_{3/2}$, and the ratios EMR and CMR as functions of Q^2 up to values of 0.2 GeV^2. Results are indicated for the parameter Set II with $\Lambda_B = 0.8$ GeV. In the figures the solid line corresponds to the total contribution while the dashed line marks the bare contribution or the one of the valence quarks.

Future refinement of the present work will involve tests of the functional form of the vertex functions entering into the strong interaction Lagrangian (19) as well as the form of the quark propagator modified in order to account for confinement. More precise data will also allow to study the possible mixture of vector and tensor currents.

In Table 6 we demonstrate the sensitivity of the EMR and CMR ratios at $Q^2 = 0$ on the choice of the proton three-quark current for typical values of the parameter $\Lambda_B = 0.75, 0.8$ and 1.25 GeV. The proton current is used in the form

$$J_p \;=\; (1-\beta)J_p^V \;+\; \beta J_p^T \tag{82}$$

where β is a tensor-vector mixing parameter. The limiting cases $\beta = 0$ and $\beta = 1$ correspond to the use of pure vector and pure tensor currents, respectively. We hope that forthcoming more precise experiments on the ratios EMR and CMR can yield a relatively precise limit on the value of the mixing parameter β.

IV. SUMMARY

In this paper we have calculated the magnetic moments of light baryons as well as the $N \to \Delta\gamma$ transition properties using a manifestly Lorentz covariant chiral quark approach to the study of baryons as bound states of constituent quarks dressed by a cloud of pseudoscalar mesons. Our main results are:

- The contribution of the meson cloud to the static properties of light baryons is up to 20%, which is consistent with the perturbative nature of their contribution and, together with the relativistic corrections, helps to explain how the 30% shortfall in the SU(6) prediction is ameliorated;

- We showed that the numerical value of the dipole magnetic moment $\mu_{N\Delta}$ is sensitive to the scale parameter Λ_B describing the distribution of quarks in the baryon. In particular, this quantity scales as $1/\Lambda_B$ and a reasonable description of data is achieved at $\Lambda_B \leq 0.8$ GeV due to the enhancement of the valence quark contribution;

- The multipole ratios EMR and CMR are sensitive to the choice of the proton current: vector J_p^V or tensor J_p^T (see Appendix A). The use of a pure vector current J_p^V gives a reasonable description of the data. The pure tensor current J_p^T gives results for EMR and CMR with the wrong (positive) sign. However, a small admixture of the tensor current is possible, and forthcoming experiments can give a strong restriction on the mixing parameter of such currents;

- We presented a detailed analysis of the light baryon observables all of which are in good agreement with experimental data.

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APPENDIX A: THREE-QUARK BARYON CURRENTS

Here we specify the baryonic currents [27, 28]. The three-quark currents of the baryon octet are:

I. Vector currents

$$
\begin{aligned}
J_p^V &= \varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 d^{a_1}u^{a_2}C\gamma_\mu u^{a_3}\,,\\
J_n^V &= -\varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 u^{a_1}d^{a_2}C\gamma_\mu d^{a_3}\,,\\
J_{\Sigma^+}^V &= -\varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 s^{a_1}u^{a_2}C\gamma_\mu u^{a_3}\,,\\
J_{\Sigma^0}^V &= \sqrt{2}\,\varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 s^{a_1}u^{a_2}C\gamma_\mu d^{a_3}\,,\\
J_{\Sigma^-}^V &= \varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 s^{a_1}d^{a_2}C\gamma_\mu d^{a_3}\,,\\
J_{\Xi^-}^V &= \varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 d^{a_1}s^{a_2}C\gamma_\mu s^{a_3}\,,\\
J_{\Xi^0}^V &= \varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5 u^{a_1}s^{a_2}C\gamma_\mu s^{a_3}\,,\\
J_{\Lambda^0}^V &= \sqrt{\frac{2}{3}}\,\varepsilon^{a_1a_2a_3}\gamma^\mu\gamma^5(u^{a_1}d^{a_2}C\gamma_\mu s^{a_3}-d^{a_1}u^{a_2}C\gamma_\mu s^{a_3})\,.
\end{aligned}
\tag{A1}
$$

II. Tensor currents

$$
\begin{aligned}
J_p^T &= \varepsilon^{a_1a_2a_3}\sigma^{\mu\nu}\gamma^5 d^{a_1}u^{a_2}C\sigma_{\mu\nu} u^{a_3}\,,\\
J_n^T &= -\varepsilon^{a_1a_2a_3}\sigma^{\mu\nu}\gamma^5 u^{a_1}d^{a_2}C\sigma_{\mu\nu} d^{a_3}\,,\\
J_{\Sigma^+}^T &= -\varepsilon^{a_1a_2a_3}\sigma^{\mu\nu}\gamma^5 s^{a_1}u^{a_2}C\sigma_{\mu\nu} u^{a_3}\,,\\
J_{\Sigma^0}^T &= \sqrt{2}\,\varepsilon^{a_1a_2a_3}\sigma^{\mu\nu}\gamma^5 s^{a_1}u^{a_2}C\sigma_{\mu\nu} d^{a_3}\,,\\
J_{\Sigma^-}^T &= \varepsilon^{a_1a_2a_3}\sigma^{\mu\nu}\gamma^5 s^{a_1}d^{a_2}C\sigma_{\mu\nu} d^{a_3}\,,\\
J_{\Xi^-}^T &= \varepsilon^{a_1a_2a_3}\sigma^{\mu\nu}\gamma^5 d^{a_1}s^{a_2}C\sigma_{\mu\nu} s^{a_3}\,,\\
J_{\Xi^0}^T &= \varepsilon^{a_1a_2a_3}\sigma^{\mu\nu}\gamma^5 u^{a_1}s^{a_2}C\sigma_{\mu\nu} s^{a_3}\,,\\
J_{\Lambda^0}^T &= \sqrt{\frac{2}{3}}\,\varepsilon^{a_1a_2a_3}\sigma^{\mu\nu}\gamma^5(u^{a_1}d^{a_2}C\sigma_{\mu\nu}s^{a_3}-d^{a_1}u^{a_2}C\sigma_{\mu\nu}s^{a_3})\,.
\end{aligned}
\tag{A2}
$$

The three-quark (vector) currents of the Δ-isobar are:

$$
\begin{aligned}
J_{\Delta^{++}}^\mu &= \varepsilon^{a_1a_2a_3}u^{a_1}u^{a_2}C\gamma^\mu u^{a_3}\,,\\
J_{\Delta^+}^\mu &= \frac{1}{\sqrt{3}}\varepsilon^{a_1a_2a_3}(d^{a_1}u^{a_2}C\gamma^\mu u^{a_3}+2u^{a_1}u^{a_2}C\gamma^\mu d^{a_3})\,,\\
J_{\Delta^0}^\mu &= \frac{1}{\sqrt{3}}\varepsilon^{a_1a_2a_3}(u^{a_1}d^{a_2}C\gamma^\mu d^{a_3}+2d^{a_1}d^{a_2}C\gamma^\mu u^{a_3})\,,\\
J_{\Delta^-}^\mu &= \varepsilon^{a_1a_2a_3}d^{a_1}d^{a_2}C\gamma^\mu d^{a_3}\,.
\end{aligned}
\tag{A3}
$$

In the case of the Δ^+ and Δ^0 states it is also useful to proceed with the currents which contain two identical quarks (two "up" or two "down" quarks) contracted together as a diquark subsystem:

$$
\begin{aligned}
J_{\Delta^+}^\mu &= \frac{1}{\sqrt{3}}\varepsilon^{a_1a_2a_3}(2d^{a_1}u^{a_2}C\gamma^\mu u^{a_3}-i\gamma_\nu d^{a_1}u^{a_2}C\sigma^{\mu\nu}u^{a_3})\,,\\
J_{\Delta^0}^\mu &= \frac{1}{\sqrt{3}}\varepsilon^{a_1a_2a_3}(2u^{a_1}d^{a_2}C\gamma^\mu d^{a_3}-i\gamma_\nu u^{a_1}d^{a_2}C\sigma^{\mu\nu}d^{a_3})\,.
\end{aligned}
\tag{A4}
$$

Eqs. (A4) are derived from Eqs. (A3) using Fierz transformations.

APPENDIX B: SETS OF THE RELATIVISTIC FORM FACTORS FOR THE $N \to \Delta\gamma$ TRANSITION

In the literature one can find several equivalent decompositions of the vertex function $\Lambda_{\mu\nu}(p,p')$ describing the $N \to \Delta\gamma$ transition [1]-[11]

$$
\begin{aligned}
\Lambda^{(1)}_{\mu\nu}(p,p') &= [(\gamma_\mu q_\nu - g_{\mu\nu}\not{q})G_1(q^2) + (p'_\mu q_\nu - g_{\mu\nu}p'q)G_2(q^2) + (q_\mu q_\nu - g_{\mu\nu}q^2)G_3(q^2)]\gamma^5\,, \\
\Lambda^{(2)}_{\mu\nu}(p,p') &= [(g_{\mu\nu}q^2 + q_\mu p_\nu)a_1(q^2) + a_2(g_{\mu\nu}m_+m_- + P_\mu p_\nu)a_2(q^2) + (g_{\mu\nu}m_+ + \gamma_\mu p_\nu)a_3(q^2)]\gamma^5\,, \qquad \text{(B1)} \\
\Lambda^{(3)}_{\mu\nu}(p,p') &= -\frac{1}{2m_N}[(g_{\mu\nu}\not{q} - \gamma_\mu q_\nu)c_1(q^2) + (g_{\mu\nu}q^2 - q_\mu q_\nu)\frac{c_2(q^2)}{2m_N} + (g_{\mu\nu}pq - p_\mu q_\nu)\frac{c_3(q^2)}{2m_N} + g_{\mu\nu}c_4(q^2)]\gamma^5\,.
\end{aligned}
$$

where $P = p + p'$ and $m_\pm = m_\Delta \pm m_N$. The sets of the relativistic form factors G_i, a_i, b_i [see Eq. (31)] and c_i are related to each other as:

$$
\begin{aligned}
&G_1 = -a_3 = b_3 = \frac{c_1}{2m_N}\,, \\
&G_2 = -2a_2 = b_2 = \frac{c_3}{4m_N^2}\,, \\
&G_3 = -a_1 + a_2 = -b_2 + b_4 = \frac{c_2 - c_3}{4m_N^2}\,, \\
&c_4 \equiv 0\,. \qquad \text{(B2)}
\end{aligned}
$$

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Table 1. Magnetic moments of light baryons (in units of the nuclear magneton μ_N). Results are calculated for the case of a purely vector current.

	Set I ($\Lambda_B = 1.25$ GeV)			Set II ($\Lambda_B = 0.8$ GeV)			Set III ($\Lambda_B = 0.75$ GeV)			Experiment [37, 39]
	Bare (3q)	Meson cloud	Total	Bare (3q)	Meson cloud	Total	Bare (3q)	Meson cloud	Total	
μ_p	2.530	0.263	2.793	2.614	0.179	2.793	2.621	0.172	2.793	2.793
μ_n	-1.530	-0.383	-1.913	-1.634	-0.279	-1.913	-1.643	-0.270	-1.913	-1.913
μ_Λ	-0.575	-0.038	-0.613	-0.579	-0.034	-0.613	-0.578	-0.035	-0.613	-0.613 ± 0.004
μ_{Σ^+}	2.336	0.196	2.532	2.423	0.148	2.571	2.430	0.130	2.560	2.458 ± 0.010
μ_{Σ^-}	-0.942	-0.327	-1.269	-0.960	-0.223	-1.183	-0.962	-0.235	-1.197	-1.160 ± 0.025
μ_{Ξ^0}	-1.240	-0.096	-1.336	-1.303	-0.082	-1.385	-1.310	-0.076	-1.386	-1.250 ± 0.014
μ_{Ξ^-}	-0.599	0.033	-0.566	-0.567	0.012	-0.555	-0.562	0.014	-0.548	-0.6507 ± 0.003
$\lvert\mu_{\Sigma^0\Lambda}\rvert$	1.273	0.293	1.566	1.372	0.245	1.617	1.385	0.222	1.607	1.61 ± 0.08
$\mu_{N\Delta}$	2.357	0.439	2.796	2.984	0.354	3.338	3.102	0.356	3.458	3.642 ± 0.019 ± 0.085

Table 2. Sensitivity of the bare contributions to the light baryon magnetic moments on the choice of the octet baryon $3q$-current (in units of the nuclear magneton μ_N). The scale parameter is chosen to be $\Lambda_B = 0.8$ GeV.

	Vector current	Tensor current	Experiment [37, 39]
μ_p	2.614	2.804	2.793
μ_n	-1.634	-1.814	-1.913
μ_Λ	-0.579	-0.594	-0.613 ± 0.004
μ_{Σ^+}	2.423	2.509	2.458 ± 0.010
μ_{Σ^-}	-0.960	-0.973	-1.160 ± 0.025
μ_{Ξ^0}	-1.303	-1.385	-1.250 ± 0.014
μ_{Ξ^-}	-0.567	-0.560	-0.6507 ± 0.003
$\lvert\mu_{\Sigma^0\Lambda}\rvert$	1.372	1.398	1.61 ± 0.08
$\mu_{N\Delta}$	2.984	2.740	3.642 ± 0.019 ± 0.085

Table 3. Results for the N $\rightarrow \Delta\gamma$ transition (Set I: $\Lambda_B = 1.25$ GeV)

	Bare (3q)	Meson cloud	Total	Experiment [37, 38, 39]
EMR (%) at $Q^2 = 0$	-3.22	0.29	-2.93	-2.5 ± 0.5; -3.07 ± 0.26 ± 0.24
EMR (%) at $Q^2 = 0.06$ GeV^2	-3.14	0.42	-2.72	-2.28 ± 0.29 ± 0.20
CMR (%) at $Q^2 = 0$	-3.69	0.34	-3.35	
CMR (%) at $Q^2 = 0.06$ GeV^2	-4.75	0.44	-4.31	-4.81 ± 0.27 ± 0.26
$A_{1/2}(0)$ in 10^{-3} GeV$^{-1/2}$	-87.4	-11.8	-99.2	-135 ± 6
$A_{3/2}(0)$ in 10^{-3} GeV$^{-1/2}$	-173.0	-20.9	-193.9	-250 ± 8
$G_{E2}(0)$	0.093	0.002	0.095	0.137 ± 0.012 ± 0.043
$G_{M1}(0)$	2.887	0.359	3.246	4.460 ± 0.023 ± 0.104
$G_{C2}(0)$	0.107	0.008	0.115	
$Q_{N\Delta}$ (fm^2)	-0.073	-0.001	-0.074	-0.108 ± 0.009 ± 0.034
$\mu_{N\Delta}$	2.357	0.439	2.796	3.642 ± 0.019 ± 0.085
$\Gamma_{\Delta\rightarrow\gamma}$ (MeV)	0.30	0.09	0.39	0.58 - 0.67

Table 4. Results for the N $\rightarrow \Delta\gamma$ transition (Set II: $\Lambda_B = 0.8$ GeV)

	Bare (3q)	Meson cloud	Total	Experiment [37, 38, 39]
EMR (%) at $Q^2 = 0$	-3.41	0.31	-3.10	-2.5 ± 0.5; -3.07 ± 0.26 ± 0.24
EMR (%) at $Q^2 = 0.06$ GeV^2	-3.34	0.33	-3.01	-2.28 ± 0.29 ± 0.20
CMR (%) at $Q^2 = 0$	-3.95	0.26	-3.69	
CMR (%) at $Q^2 = 0.06$ GeV^2	-5.13	0.35	-4.78	-4.81 ± 0.27 ± 0.26
$A_{1/2}(0)$ in (10^{-3} GeV$^{-1/2}$)	-110.0	-14.3	-124.3	-135 ± 6
$A_{3/2}(0)$ in (10^{-3} GeV$^{-1/2}$)	-219.4	-25.3	-244.7	-250 ± 8
$G_{E2}(0)$	0.125	0.002	0.127	0.137 ± 0.012 ± 0.043
$G_{M1}(0)$	3.655	0.434	4.089	4.460 ± 0.023 ± 0.104
$G_{C2}(0)$	0.144	0.007	0.151	
$Q_{N\Delta}$ (fm^2)	-0.098	-0.001	-0.099	-0.108 ± 0.009 ± 0.034
$\mu_{N\Delta}$	2.984	0.354	3.338	3.642 ± 0.019 ± 0.085
$\Gamma_{\Delta\rightarrow N\gamma}$ (MeV)	0.49	0.12	0.61	0.58 - 0.67

Table 5. Results for the N $\rightarrow \Delta\gamma$ transition (Set III: $\Lambda_B = 0.75$ GeV)

	Bare (3q)	Meson cloud	Total	Experiment [37, 38, 39]
EMR (%) at $Q^2 = 0$	-3.43	0.30	-3.13	-2.5 ± 0.5; -3.07 ± 0.26 ± 0.24
EMR (%) at $Q^2 = 0.06$ GeV^2	-3.35	0.30	-3.05	-2.28 ± 0.29 ± 0.20
CMR (%) at $Q^2 = 0$	-3.98	0.25	-3.73	
CMR (%) at $Q^2 = 0.06$ GeV^2	-5.17	0.33	-4.84	-4.81 ± 0.27 ± 0.26
$A_{1/2}(0)$ in (10^{-3} GeV$^{-1/2}$)	-114.3	-14.3	-128.6	-135 ± 6
$A_{3/2}(0)$ in (10^{-3} GeV$^{-1/2}$)	-228.1	-25.4	-253.5	-250 ± 8
$G_{E2}(0)$	0.130	0.002	0.132	0.137 ± 0.012 ± 0.043
$G_{M1}(0)$	3.800	0.435	4.235	4.460 ± 0.023 ± 0.104
$G_{C2}(0)$	0.151	0.007	0.158	
$Q_{N\Delta}$ (fm^2)	-0.102	-0.002	-0.104	-0.108 ± 0.009 ± 0.034
$\mu_{N\Delta}$	3.102	0.356	3.458	3.642 ± 0.019 ± 0.085
$\Gamma_{\Delta\rightarrow N\gamma}$ (MeV)	0.53	0.13	0.66	0.58 - 0.67

Table 6. Sensitivity of the EMR and CMR ratios to the choice of the proton $3q$-current

	Mixing parameter β													
	0	0.025	0.05	0.075	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.5	0.75	1
Set I ($\Lambda_B = 1.25$ GeV)														
EMR (%)	-2.93	-2.54	-2.28	-2.04	-1.80	-1.35	-1.08	-0.55	-0.19	0.15	0.47	1.05	2.29	3.19
CMR (%)	-3.35	-3.03	-2.72	-2.42	-2.13	-1.59	-1.08	-0.61	-0.17	0.25	0.63	1.35	2.81	3.95
Set II ($\Lambda_B = 0.8$ GeV)														
EMR (%)	-3.10	-2.83	-2.56	-2.30	-2.06	-1.60	-1.17	-0.77	-0.40	-0.05	0.28	0.87	2.09	3.03
CMR (%)	-3.69	-3.35	-3.03	-2.64	-2.35	-1.80	-1.29	-0.81	-0.37	0.04	0.43	1.14	2.58	3.69
Set III ($\Lambda_B = 0.75$ GeV)														
EMR (%)	-3.13	-2.84	-2.58	-2.33	-2.07	-1.62	-1.19	-0.79	-0.41	-0.07	0.26	0.85	2.10	3.00
CMR (%)	-3.73	-3.39	-3.06	-2.75	-2.44	-1.87	-1.34	-0.85	-0.40	0.03	0.43	1.16	2.65	3.80

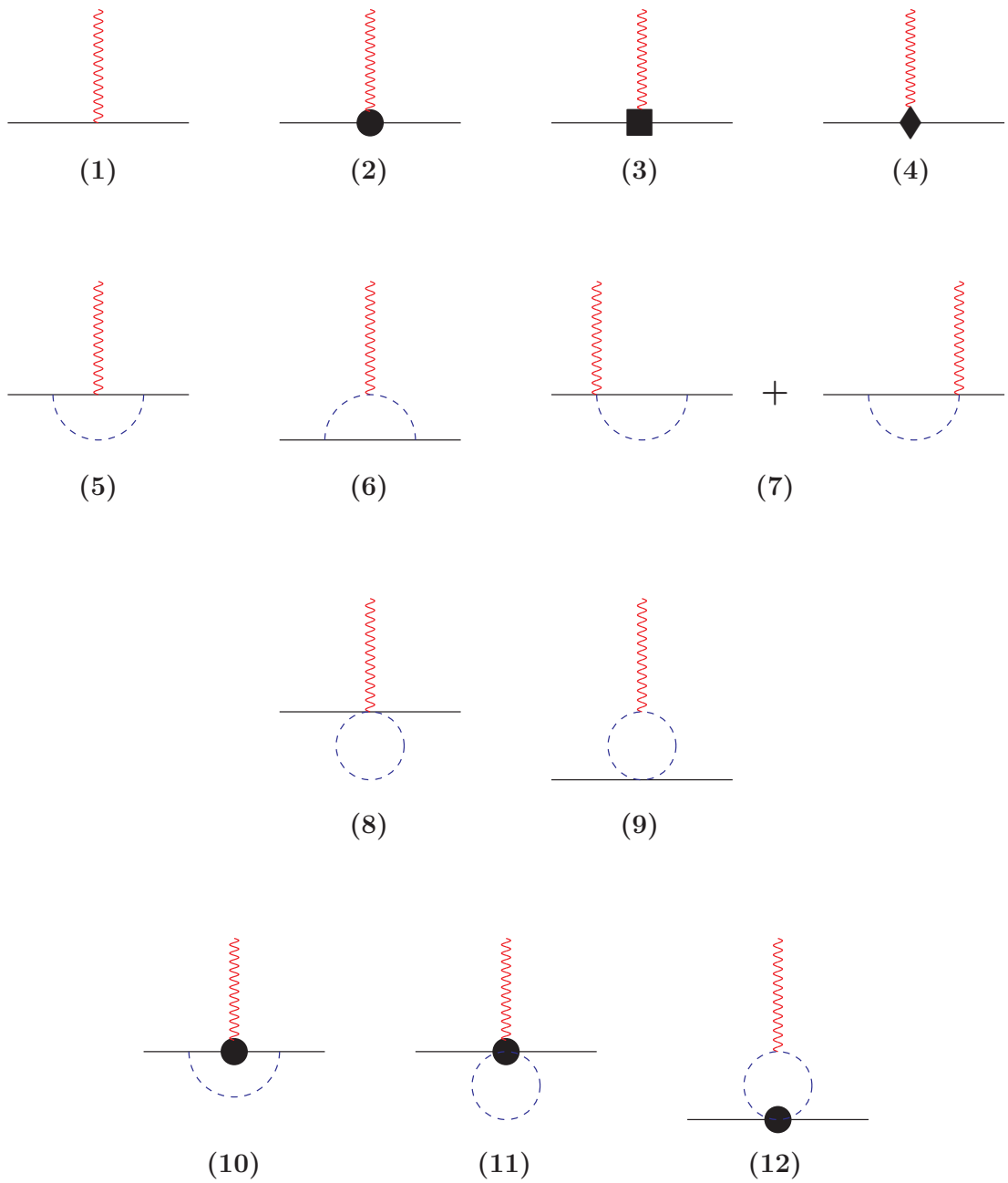

Fig. 1. *Diagrams including pseudoscalar meson contributions to the EM quark transition operator up to fourth order. Solid, dashed and wiggly lines refer to quarks, pseudoscalar mesons and the electromagnetic field, respectively. Vertices denoted by a black filled circle, box and diamond correspond to insertions from the second, third and fourth order chiral Lagrangian.*

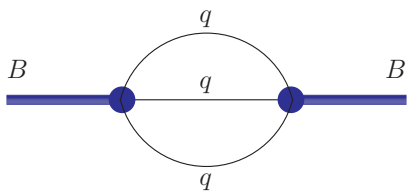

Fig.2 Baryon mass operator

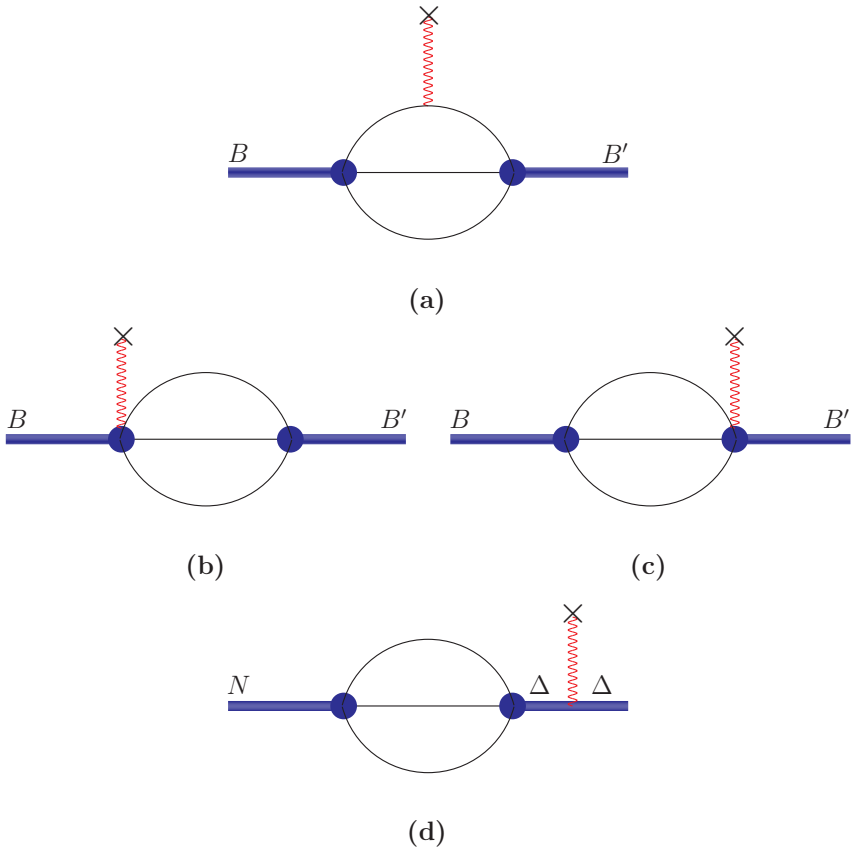

Fig.3 Diagrams contributing to the matrix elements of the bare quark operators: triangle (a), bubble (b) and (c), pole (d) diagrams. Symbol $\times$ corresponds to the source of the external field.

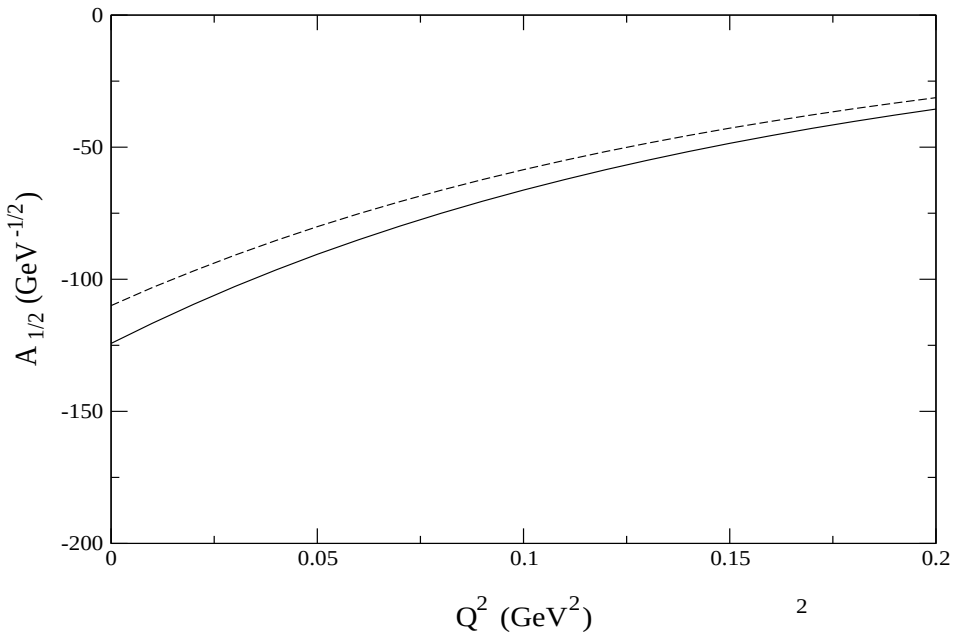

Fig.4 Helicity amplitude $A_{1/2}(Q^2)$. Solid line is the total result, whereas the dashed line corresponds to the valence quark contribution

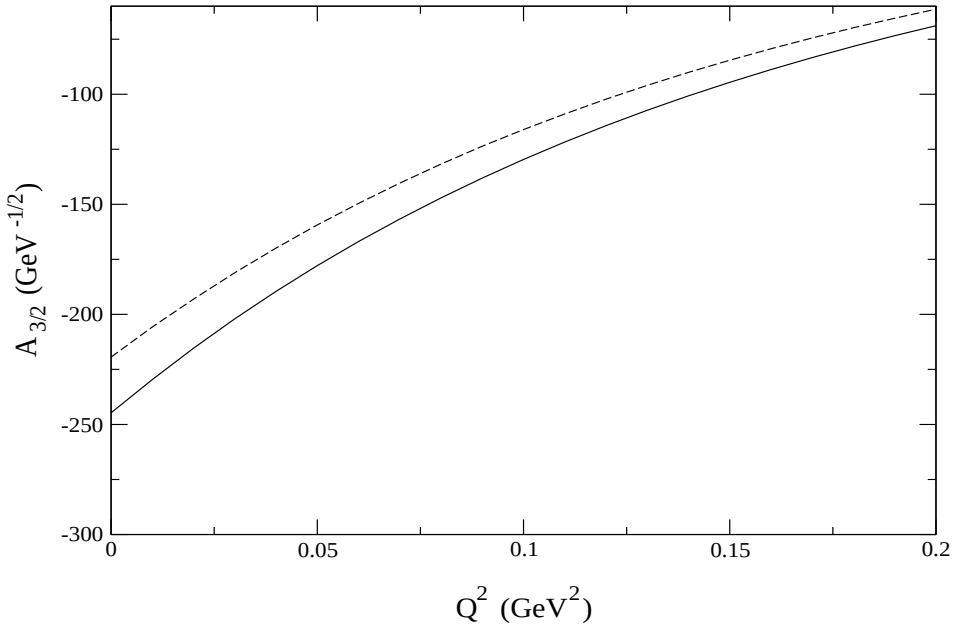

Fig.5 Helicity amplitude $A_{3/2}(Q^2)$. Otherwise as in Fig.4.

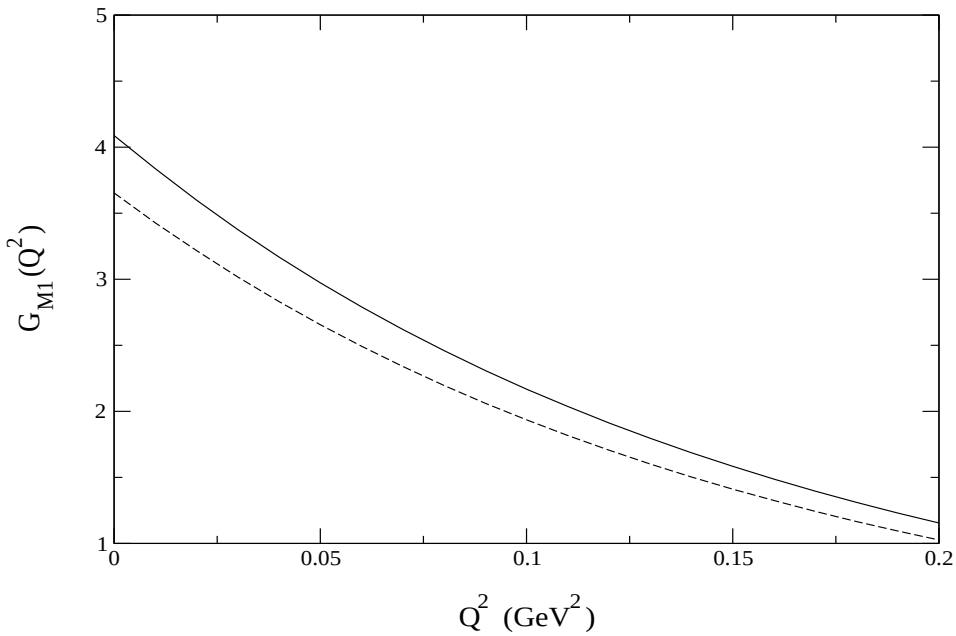

Fig.6 Form factor $G_{M1}(Q^2)$. Otherwise as in Fig.4.

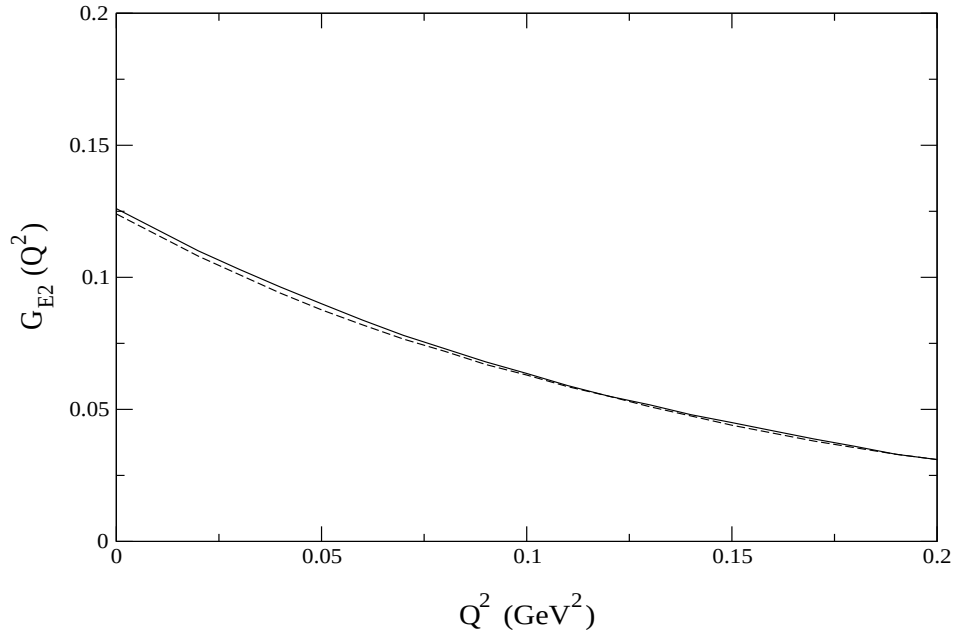

Fig.7 Form factor $G_{E2}(Q^2)$. Otherwise as in Fig.4.

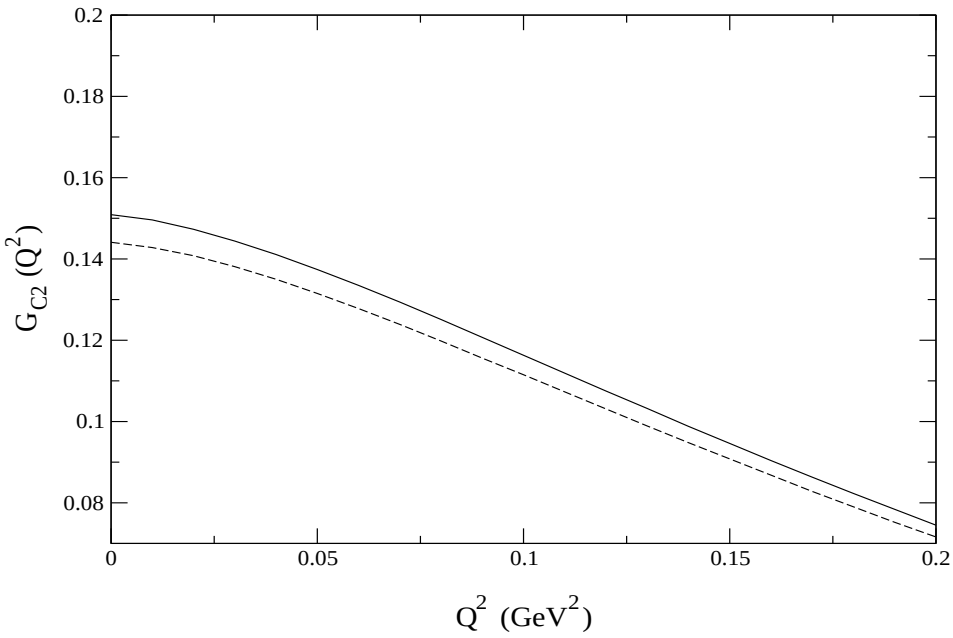

Fig.8 Form factor $G_{C2}(Q^2)$. Otherwise as in Fig.4.

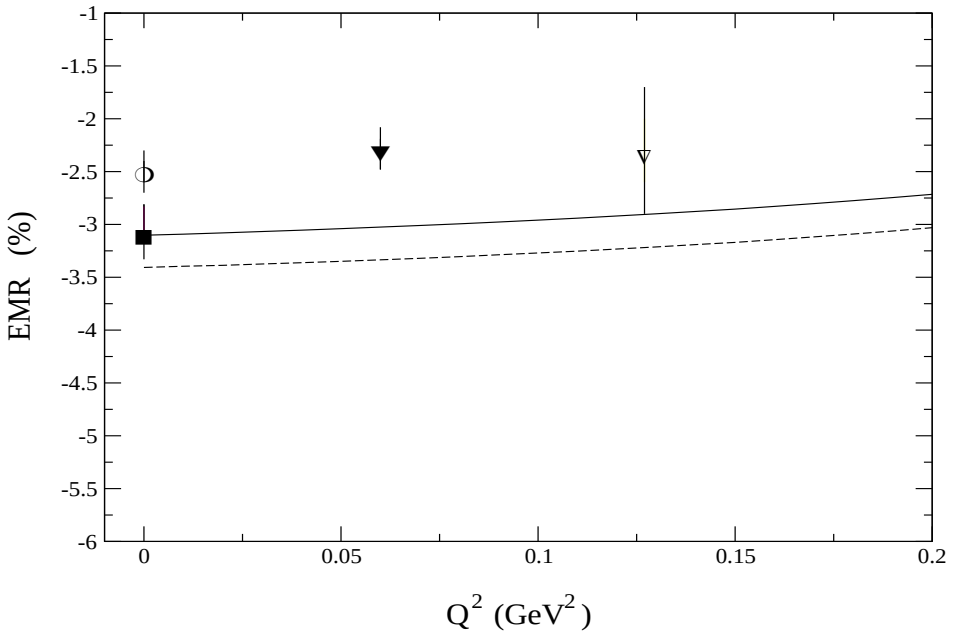

Fig.9 Ratio EMR$(Q^2) = -G_{E2}(Q^2)/G_{M1}(Q^2)$. Data are taken from Refs. [38] (filled triangle), [39] (filled box), [41] (opened circle) and [42] (opened triangle). Otherwise as in Fig.4.

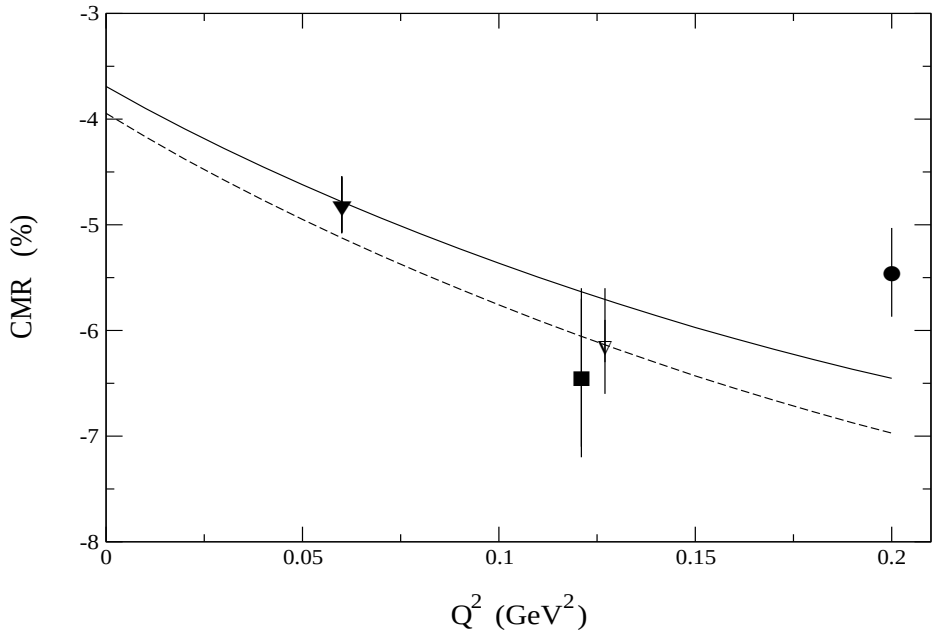

Fig.10 Ratio CMR$(Q^2) = G_{C2}(Q^2)/G_{M1}(Q^2)$. Data are taken from Refs. [38] (filled triangle), [42] (opened triangle), [43] (filled box) and [44] (filled circle). Otherwise as in Fig.4.