2009

Value of Traveler Information for Adaptive Routing in Stochastic Time-Dependent Networks

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VALUE OF TRAVELER INFORMATION FOR ADAPTIVE ROUTING IN STOCHASTIC TIME-DEPENDENT NETWORKS

A Thesis Presented

by

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Submitted to the Graduate School of the
University of Massachusetts Amherst in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN CIVIL ENGINEERING

February 2009

Civil & Environmental Engineering

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ACKNOWLEDGMENTS

The author is indebted to the University of Massachusetts (UMass) Amherst, the Department of Civil and Environmental Engineering and the School of Engineering at UMass Amherst, and U.S. Dept. of Transportation (USDOT DTRT06-G-0032) for funding the research.

ABSTRACT

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FEBRUARY 2009

HE HUANG, M.S., Civil & Environmental Engineering

UNIVERSITY OF MASSACHUSETTS AMHERST

Directed by: Professor Song Gao

Real-time information plays an important role in travelers' routing choices in an uncertain network by enabling online adaptation to revealed traffic conditions. The quality of the information affects its effectiveness. Usually there are some limitations in the information provided to the travelers, spatially, temporally or both. In this thesis, three variants of an optimal adaptive routing problem with partial online information problem are introduced: global information with time lag, global pre-trip information and radio information on a subset of links without time lag. A generic description of online information is provided. An algorithm is designed for the optimal routing problem in stochastic time-dependent networks with partial online information and specializations required for each of the three variants are given. A test example is conducted and computationally verifies the non-negative value of information. The work in this thesis is potentially of interest to traveler information systems evaluation and design.

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CHAPTER 1

INTRODUCTION

1.1 Motivation

Congestion is an important worldwide transportation issue. In developed countries where building more infrastructures is usually politically, financially and environmentally constrained, a lot of efforts have been devoted to making full use of current infrastructure system with the help of Intelligent Transportation Systems (ITS).

Advanced Traveler Information System (ATIS), a sub-system of ITS, aims to provide travelers with real-time information about network conditions, in the hope that better informed travelers can make better decisions, and thus collectively the congestion would be relieved. In order to assess an ATIS, a comprehensive model is needed to take into consideration the demand-supply interaction under the influence of ATIS (Gao, 2008). This thesis deals with the demand side of the problem, which describes the optimal routing decisions a traveler can make with the help of ATIS and how much benefit can be obtained from traveler information. Note that no demand-supply interaction is modeled in this thesis, i.e., travel times are not affected by travelers' choices. This is the study of the value of traveler information for a single traveler in an uncertain network.

The value of information provided by ATIS is most evident when traffic conditions are stochastic. In a network where traffic quantities are almost certain, travelers are already quite well informed and ATIS has little to provide. Stochasticity is a basic feature of congested traffic networks. One significant source of the randomness is the disturbances to traffic networks, such as incidents, vehicle breakdown, bad weather, work zones, special events, etc. Traffic networks are treated as stochastic to better reflect reality and enable the modeling of information.

There are various implementations of ATIS, and they differ in the spatial and temporal availability, the quality, the format, and limitation of information provided. For example, a variable message sign (VMS) is usually fixed in location and thus only travelers passing it can obtain the information. It is also limited in the amount of information it can provide. Radio-based systems can provide information to travelers anywhere in the radio coverage. Relatively more detailed information is available compared to VMS, yet still the coverage is usually limited to major highways and arterials. Besides the limitation on the spatial side, there is also limitation on the temporal side. Usually radio broadcast provides traffic condition information every 15 minutes for example, and so for travelers there is a time lag with the information. Internet can also be an access to ATIS, providing travelers with information such as camera images, travel time estimations, work zone and event schedule, and travel advisories. However, once travelers are en route, they can hardly have access to internet, and so internet-based ATIS implementation is usually viewed as a pre-trip planner. More advanced in-vehicle systems are also emerging, possibly with a database of road map, travel times under normal conditions, records of past incidents, etc., and can communicate with information centers to obtain very detailed and updated information.

Travelers' routing decisions in a stochastic network with online information is conceivably different from those in a deterministic network. It is generally believed that adaptive routing will save travel time and enhance travel time reliability. For example, in a network with random incidents, if one does not adapt to an incident scenario, he/she could be stuck in the incident link for a very long time. However, if adequate online information is available about the incident, the traveler can avoid it by switching to an alternative route. The adaptation also ensures that the travel time is not prohibitively high in incident scenarios, and thus provides a more reliable travel time.

It is therefore a very interesting research question how an individual traveler makes adaptive routing decisions based on provided information in a stochastic and time-dependent (STD) network, whose link travel times are random variables with time-dependent distributions. In previous work Gao and Chabini (2006) and Gao (2005), the optimal routing problem with perfect online information is studied and the value of perfect online information evaluated. This study is an extension of Gao and Chabini (2006) and Gao (2005) where a number of partial online information situations are considered. A different algorithm is required, and it is a generic one which can also solve the perfect online information problem in Gao and Chabini (2006) and Gao (2005).

The thesis is organized as follows. First a literature review is given and the contributions of this thesis are summarized. Next the optimal routing policy problem in a stochastic time-dependent network is defined for partial online information situations. Four variants which are particularly pertinent in traffic networks are then studied. A generic algorithm which provides an exact optimal solution to the variants is presented. Computational tests are carried out and the results are given. Finally some conclusions are made and future research directions are given.

1.2 Literature Review

Two of the important characteristics of a network are time-dependency (whether a link cost is dependent on the arrival time at the beginning of the link) and stochasticity (whether link costs are random variables). Routing problems in deterministic networks, both static and dynamic, have been important and well researched topics for a long time (see e.g., Ahuja et al. 1993, Chabini, 1998). Routing problems in stochastic networks are relatively less studied compared to their deterministic counterpart. Two possible types of routing problems exist in a stochastic network: non-adaptive and adaptive. Non-adaptive routing does not consider the fact that information on arrival times on intermediate nodes and/or link travel time realizations will be available during a trip, and thus a fixed path is determined at the origin node and followed regardless of the actual realizations of the stochastic network. On the other hand, adaptive routing considers intermediate decision nodes, and a next link (or sub-path) is chosen based on information collected thus far. The adaptive routing problem is the focus of the review.

Various assumptions are made to define a stochastic network and how the realizations of the stochastic network are revealed to the travelers. Studies in both static and time-dependent (and stochastic) networks are reviewed. In Andreatta and Romeo (1988), the topology of the static network is stochastic; in Polychronopoulos and Tsitsiklis (1996), the whole static network is described by a joint distribution of link travel costs in the dependent case, and by marginal distributions of link travel times in the independent case; in Polychronopoulos (1992), Psaraftis and Tsitsiklis (1993) and Kim et al. (2005), the link costs evolve as Markov processes; in Hall (1988), Chabini (2000), Miller-Hooks and Mahmassani (2000), Pretolani (2000), Miller-Hooks (2001), Yang and

Miller-Hooks (2004), Bander and White (2002), Fan et al. (2005b) and Opasanon and Miller-Hooks (2006), time-dependent networks are described by marginal distributions of link travel times; in Gao and Chabini (2006), time-dependent networks are described by joint distribution of travel times of all links at all times; and in Waller and Ziliaskopoulos (2002), Fan et al. (2005a) and Boyles (2006), conditional probabilities of adjacent link travel costs are utilized. As for the revealing of network conditions, in Andreatta and Romeo (1988), Polychronopoulos and Tsitsiklis (1996), Cheung (1998), Fu (2001), Waller and Ziliaskopoulos (2002) and Provan (2003) it is assumed that one learns the realization of a link travel cost once he/she arrives at the node from which the link emanates; in Chabini (2000), Miller-Hooks and Mahmassani (2000), Miller-Hooks (2001), Yang and Miller-Hooks (2004), Bander and White (2002), Pretolani (2000), Fan et al. (2005b), Opasanon and Miller-Hooks (2006) it is not stated explicitly how travelers learn about the network conditions other than the arrival times at decision nodes, hence the term "time-adaptive"; in Waller and Ziliaskopoulos (2002), Fan et al. (2005a) and Boyles (2006) it is assumed that travelers remember only the travel time on the last link they traverse; in Gao and Chabini (2006) it is assumed that travelers have knowledge about all link travel time realizations up to the current time; and in Psaraftis and Tsitsiklis (1993) and Kim et al. (2005) it is assumed that Markovian travel times and thus travelers learn the current state of the Markovian chain at any time.

The optimal adaptive routing problem studies in stochastic time-dependent (STD) networks are summarized in Table 1 – 1 with a taxonomy developed by Gao and Chabini (2006). A more detailed review follows.

Table 1-1 Taxonomy of the optimal routing policy problem

<table>
<tr><th>Information
Network</th><th>Perfect online</th><th>Partial online</th><th>No online information (time-adaptive)</th></tr>
<tr><td>No link-wise and time-wise dependency</td><td colspan="2"></td><td rowspan="3">Hall (1987), Miller-Hooks and Mahmassani (2000), Chabini (2000), Pretolani (2000), Miller-Hooks (2001), Bander and White (2002), Yang and Miller-Hooks (2004), Fan et al. (2005b), Opasanon and Miller-Hooks (2006)</td></tr>
<tr><td>Complete dependency</td><td>Gao and Chabini (2002, 2006)</td><td>This thesis</td></tr>
<tr><td>Partial dependency</td><td></td><td>Psaraftis and Tsitsiklis (1993), Kim et al. (2005), Boyles (2006)</td></tr>
</table>

Hall (1986) studies for the first time the time-dependent version of the ORP problem. It is shown that in an STD network, routing policies are more effective than paths. Chabini (2000) gives a dynamic programming algorithm based on the concept of decreasing order of time (DOT). The algorithm is optimal in the sense that no algorithms with better theoretical complexity exist. Miller-Hooks and Mahmassani (2000) develop a label-correcting algorithm. Insight into the difference between an optimal routing policy problem and a least expected time path problem is provided. Later Miller-Hooks (2001) compares the said label-correcting algorithm and the dynamic programming algorithm working in decreasing order of time (Chabini, 2000) in both sparse transportation networks and dense telecommunication data networks. Yang and Miller-Hooks (2004) also extend the study of the time-adaptive routing policies to a signalized network.

Pretolani (2000) uses a hyper-path representation of the adaptive routing problem based on arrival times. Bander and White (2002) design a heuristic approach with a promising feature: it will terminate with an optimal solution if one exists, given that the heuristic function underestimates the true cost-to-go. The proposed heuristic has a significant computational advantage compared to dynamic programming, shown through computational tests. Fan et al. (2005b) maximize the probability of arriving on time with continuous probability density functions on link travel times. Later in Fan and Nie (2006), algorithmic issues are explored for the same problem. Opasanon and Miller-Hooks (2006) study the time-adaptive problem with multiple optimization criteria.

Psaraftis and Tsitsiklis (1993) study the problem in acyclic networks, implying that no link would be visited twice, so it is not helpful to keep information of any already traversed links. This assumption along with the infinite horizon assumption makes a polynomial running time algorithm possible. Kim et al. (2005) study a similar problem in a general network with a wider information range. Boyles (2006) studies the problem with minimum expected disutility, which is a general piece-wise polynomial function of arrival time at the destination.

1.3 Contributions

Gao and Chabini (2006) establish a formal framework for the problem and design both exact and approximation algorithms for the problem with perfect online information. This thesis builds on Gao and Chabini (2006) and the contributions to the state of art are threefold:

- A generic representation of online information is provided from which three specializations of partial online information are derived. The generic representation provides a unified view towards routing problems with online information.

- A theoretical proof of the value of information is given which shows that in an adaptive routing context in an STD network, more information is always better (at least not worse) in a flow-independent network.
- A generic algorithm for a number of partial online information variants is designed. This enables the systematic study of the value of traveler information for adaptive routing in an uncertain network where a wide variety of information access situations can be modeled.

CHAPTER 2

PROBLEM DEFINITION

2.1 The Network

Let $G = (N, A, T, \tilde{C})$ denote a stochastic time-dependent network. N is the set of nodes and A is the set of links, with $|N| = n$ and $|A| = m$. It is assumed that there is at most one directional link from node j to k, and thus a link can be denoted as (j, k). T is the set of time periods $\{0, 1, \ldots, K\text{-}1\}$. A support point is defined as a distinct value (vector of values) that a discrete random variable (vector) can take. Therefore a probability mass function (PMF) of a random variable (vector) is a combination of support points and the associated probabilities. Throughout this thesis, a symbol with a ~ over it is a random variable (vector), while the same symbol without the ~ is its support point. The travel time on each link (j, k) at each time period t is a random variable $\tilde{C}_{jk,t}$ with finite number of discrete, positive and integral support points. Beyond time period K-1 travel times are static, i.e., travel times on link (j, k) at any time $t > K$-1 is equal to that at time $K - 1$ for any given support point. The time period from 0 to K-1 is denoted as the dynamic period, while that beyond K-1 static period. It is generally possible to model the peak period as dynamic, while off-peak as static when traffic is more stable. $\{C^1, \ldots, C^R\}$ is the set of support points of the joint probability distribution of all link travel times at all times, where C^r is a vector of time-dependent link travel times with a dimension $K \times m$, $r = 1, 2, \ldots, R$. $C^r_{jk,t}$ is the travel time of link (j, k) at time t in the r-th support point, which has a probability p_r, and $\sum_{r=1}^{R} p_r = 1$.

An example network is shown in Figure 2-1 and Table 2-1 with 3 nodes, 3 links and 2 time periods. There are 3 support points, each with a probability of 1/3, for the joint distribution of 6 travel time random variables (links (*a, b*), (*b, c*) and (*a, c*) over time periods 0 and 1). A support point can be conveniently viewed as a day. Travel times beyond time 1 are the same as those at time 1 for each of the 3 support points.

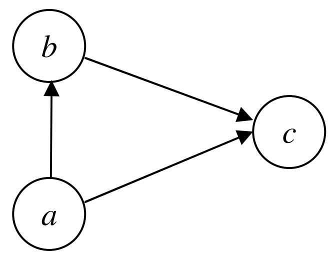


Figure 2-1 Algorithm DOT-PART: A Small Network

Table 2-1 Support points for the Small Network

Time	Link	C^1	C^2	C^3
0	(*a, b*)	1	1	1
	(*b, c*)	2	2	1
	(*a, c*)	3	3	2
1	(*a, b*)	1	1	2
	(*b, c*)	1	2	1
	(*a, c*)	3	2	2

$p_1 = p_2 = p_3 = 1/3$

2.2 Online Information

Let H be a trajectory of (node, time) pairs a traveler could experience in the network to the current node j and time t: $H = \{(j_0, t_0), \ldots, (j, t)\}$, where j_0 is the origin, t_0 is the departure time, j is the current node and t is the current time. Denote the information coverage on links and time periods as $Q \subseteq A \times T$. Information is represented as the travel time realizations on time-dependent links in Q. It is assumed there is no error in revealing the true travel times, i.e., a 1 minute travel time will be revealed as 1 minute, not any other value. An information scheme is defined as a mapping from trajectory H to coverage Q, that is, information depends on traversed locations and times. Here are examples of online information schemes with trajectory $H = \{(j_0, t_0), \ldots, (j, t)\}$:

- Perfect online information (Gao and Chabini, 2006): $Q^{\mathrm{POI}}(H) = A \times \{0,1,\ldots,t\}$ (all links up to the current time)
- Global information with time lag Δ: $Q^{\mathrm{LAG}}(H) = A \times \{0,1,\ldots,t - \Delta\}$ (all links up to Δ time ago)
- Global pre-trip information with departure time t_0: $Q^{\mathrm{PRE}}(H) = A \times \{0,1,\ldots,t_0\}$ (all links up to the departure time t_0)
- Radio information on $B \subseteq A$ with no time lag: $Q^{\mathrm{RADIO}}(H) = B \times \{0,1,\ldots,t\}$ (a subset of links up to the current time)
- No online information (see e.g., Gao and Chabini, 2006): $Q^{\mathrm{NOI}}(H) = \varnothing$ (no information on any link at any time)

The example in Figure 2-1 and Table 2-1 is used to illustrate the different information schemes. At time 0 and any node, a traveler with POI knows the travel time realizations of $\{\tilde{C}_{ab,0}, \tilde{C}_{bc,0}, \tilde{C}_{ac,0}\}$ which could be either {1,2,3} or {1,1,2}; a traveler with global information with a lag of 1 minute does not know any travel time realization yet; a traveler with global pre-trip information with departure time 0 has the same knowledge as with POI; a traveler with radio information on link (*a, b*) with no time lag knows the travel time realization of $\tilde{C}_{ab,0}$ which is always 1; and a traveler with NOI simply does not know any travel time realization.

As the time moves from 0 to 1, more information could be obtained while that from time 0 is kept. A traveler with POI knows the travel time realizations of { $\tilde{C}_{ab,0}$, $\tilde{C}_{bc,0}$, $\tilde{C}_{ac,0}$, $\tilde{C}_{ab,1}$, $\tilde{C}_{bc,1}$, $\tilde{C}_{ac,1}$ } which could be each of the 3 support points; a traveler with global information with a lag of 1 minute knows what happened at time 0: the travel time realizations of { $\tilde{C}_{ab,0}$, $\tilde{C}_{bc,0}$, $\tilde{C}_{ac,0}$ } which could be either {1,2,3} or {1,1,2}; a traveler with global pre-trip information with departure time 0 does not gain any more information *en route* and thus his/her information remains unchanged ; a traveler with radio information on link (*a, b*) with no time lag knows the travel time realization of { $\tilde{C}_{ab,0}$, $\tilde{C}_{ab,1}$ } which could be {1,1} or {1,2}; and a traveler with NOI still does not know any travel time realization.

As the time moves from 1 to 2, only the traveler with global information with a lag of 1 minute will gain more useful information, as he/she now knows what happened in time 1. A traveler with POI, pre-trip or radio information does not gain any more useful information because his/her information is always up-to-date and the information he/she had at time 1 is enough for any time periods beyond 1 due to the static period assumption. A traveler with NOI does not gain any more information by definition.

2.3 Event Collection

The concept of event collection is generalized from that defined in Gao and Chabini (2006) to the case of a general information scheme. Let $\tilde{C}_Q$ be the vector of random travel times of all time-dependent links in *Q*. For a given support point C_Q, there exists one or more support points C of the network, such that the travel time on any time-dependent link in Q is the same in both C_Q and C. In other words, for any possible revealed link travel times in Q, a set of support points of the network that are compatible with the information can be identified. Such a set is defined as an event collection, *EV*. As more information is collected, information coverage Q grows and the size of *EV* decreases or remains unchanged. When *EV* becomes a singleton, a deterministic network (not necessarily static) is revealed to the traveler. If a traveler has perfect online information with $Q^{\mathrm{POI}} = A \times \{0, 1, \ldots, t\}$, the network becomes deterministic no later than the start of the static period, i.e., $K - 1$. When travelers have less than perfect online information, it is possible that the network remains stochastic beyond the dynamic period.

In the example of Figure 2-1 and Table 2-1, it is assumed that a traveler has POI. At time 0 he/she received the information that travel times on links (a, b), (b, c) and (a, c) are 1, 2 and 3 respectively. By utilizing his/her *a priori* knowledge of the joint distribution of link travel times, he/she can infer that support points C^1 or C^2 are possible as both provide compatible travel times with what is revealed, while support point C^3 is not. Therefore his/her event collection is $\{C^1, C^2\}$. As the time moves from 0 to 1, the traveler obtains more information. If the newly revealed travel times on links (a, b), (b, c) and (a, c) are 1, 1 and 3 respectively, the traveler knows for sure that support point C^1 will be realized and his/her event collection is $\{C^1\}$. Similarly, If the newly revealed travel times on links (a, b), (b, c) and (a, c) are 1, 2 and 2 respectively, the traveler knows for sure that support point C^2 will be realized and his/her event collection is $\{C^2\}$.

Similarly a traveler with global information with a lag of 1 minute has no idea which support point will be realized at time 0 and his/her event collection is $\{C^1, C^2, C^3\}$. At time 1, he/she knows link travel times realized at time 0, and is faced with the same situation as a traveler with POI did at time 0. If the revealed travel times on links (*a, b*), (*b, c*) and (*a, c*) at time 0 are 1, 2 and 3 respectively, his/her event collection is $\{C^1, C^2\}$. At time 2, he/she will have an event collection $\{C^1\}$ or $\{C^2\}$. The same logic can be applied to other information schemes. Note that for NOI, the event collection remains as $\{C^1, C^2, C^3\}$ for any time period.

All the possible event collections with information coverage Q, denoted as $\boldsymbol{EV}(Q)$, can be generated by performing a partition of $\{C^1, \ldots, C^R\}$ based on $\tilde{C}_Q$. $\boldsymbol{EV}(Q) = \{EV_1, EV_2, \ldots\}$, where $C^r_{jk,t}$ is invariant over $r \in EV_i$, $\forall((j, k), t) \in Q$, $\forall i$, and $\exists$ $((j, k), t) \in Q$ such that $C^r_{jk,t} \neq C^{r'}_{jk,t}$, for $r \in EV_i$, $r' \in EV_j$, $j \neq i$, $\forall i$, $\forall j$. In other words, support points in an EV are undistinguishable in terms of revealed travel times on links in Q, but are distinctive from those in another EV. All the possible event collections for a given information scheme can be generated in preprocessing. Here are some important facts about event collections:

- There is no overlapping among elements of $\boldsymbol{EV}(Q)$, so there are at most R event collections at any certain time and location ($|\boldsymbol{EV}(t)| \leq R$);
- Any element EV of $\boldsymbol{EV}(Q)$ is a subset of one and only one element EV' of a later $\boldsymbol{EV}(Q')$: $EV' \cap EV = \varnothing$ or EV';
- $|\boldsymbol{EV}(Q)| \geq |\boldsymbol{EV}(Q')|$;

The generation of event collection can be carried out in increasing order of time, as the information coverage can only grow and later partitions can be done based on earlier ones. An example from Figure 2-1 and Table 2-1 is shown here for a traveler with up-to-date radio information on link (*a, b*). Since the information coverage depends only on the current time *t*, not the trajectory, *Q* (*H*) can be simplified as *Q* (*t*) and ***EV*** (*Q*) as ***EV*** (*t*). At time 0, information coverage *Q* (0) = {(*a, b*)} × {0}. The travel time on link (*a, b*) at time 0 is 0 for all 3 support points, so the partition yields only one event collection and ***EV*** (0) = {{C^1, C^2, C^3}}. At time 1, information coverage *Q* (1) = {(*a,b*)} × {0, 1} where the incremental information is on {(*a, b*)} × {1}. The partition can then be carried out on ***EV***(0) based on travel time realizations of link (*a, b*) at time 1, which can be either 1 or 2. Therefore ***EV***(1) = {{C^1, C^2}, {C^3}}. During the static period, no more useful information will be available, so ***EV*** (*t*) = {{C^1, C^2}, {C^3}} for all $t > 1$.

Another example is shown for a traveler with global information with a lag of 1 minute. At time 0, *Q* (0) =∅, and thus ***EV*** (0) = {{C^1, C^2, C^3}}. At time 1, *Q* (1) = {(*a,b*), (*b,c*), (*a,c*)} × {0}. First check link-time pair ((*a,b*), 0) with only 1 possible value, and {{C^1, C^2, C^3}} remains unchanged. Next check ((*b,c*),0) with 2 possible values and {{C^1, C^2, C^3}} is partitioned as {{C^1, C^2}, {C^3}}. Lastly check ((*a,c*),0) and {{C^1, C^2}, {C^3}} remains unchanged because $C^1{}_{ac,0}$ and $C^2{}_{ac,0}$ are the same, while {C^3} is already a singleton. Therefore ***EV*** (1) = {{C^1, C^2}, {C^3}}. Similarly ***EV*** ($t \geq 2$) = {{C^1}, {C^2}, {C^3}}.

2.4 The Decisions and the Optimal Routing Policy Problem

It is assumed that travelers can make decisions only at nodes. The decision is what node *k* to take next at each node, based on the current state *x* = {*j, t, EV*}, where *j* is the current node, *t* is the current time, and *EV* is the current event collection. A routing policy μ is defined as a mapping from state to decision on the next node, for all possible states and all possible next nodes out a given state, $\mu : x = \{j, t, EV\} \mapsto k$.

The next state $y = \left\{k, \tilde{t}', \tilde{EV}'\right\}$ of the traveler is uncertain. The travel time on link (*j, k*) at time *t* given *EV* could be uncertain, resulting in an uncertain arrival time $\tilde{t}'$ at node *k*. The next event collection $\tilde{EV}'$ is uncertain because: 1) $\tilde{t}'$ is uncertain and thus the next information coverage $\tilde{Q}'$ is uncertain, e.g., at 8:00 with a possible travel time of 1 or 2 minute(s) on the next link, $\tilde{Q}'$ could cover either 8:01 or both 8:01 and 8:02; 2) Even with a given *Q'* and a given *t'*, travel times of links in *Q'* between *t* and t' are uncertain.

For a traveler with up-to-date radio information on link (*a,b*) in Figure 2-1 and Table 2-1, let $\mu\{a, 0, \{C^1, C^2, C^3\}\} = c$. The travel time on link (*a, c*) could be either 3 or 2 given the event collection {C^1, C^2, C^3}, with a probability of 2/3 or 1/3. If the travel time is 3, the event collection at node *c* will be an element of ***EV***(3); if the travel time is 2, the event collection at node *c* will be an element of ***EV***(2). In this specific example, ***EV***(3) = ***EV***(2), but generally they are not equal.

The traveler makes another decision at state *y*, and continues the process until the destination node is reached. The travel time of a routing policy from any initial state to a destination is a random variable; a routing policy can be manifested as different paths in different support points.

Definition 1: (Optimal routing policy problem). The optimal routing policy (ORP) problem in a stochastic time-dependent network is to find the routing policy that optimizes an objective function to a given destination *d*, for all possible states, i.e., all possible combinations of origins, departure times and event collections.

The objective function can be expected travel time, travel time variance and expected travel time schedule delay, or a combination of some of the criteria.

2.5 The Value of Information

Let $e(\mu,x)$ be the objective function of following routing policy μ from an initial state x, and $e^*(x) = \min_{\mu} e(\mu,x)$. The value of information is to be investigated theoretically in obtaining $e^*(x)$ and a theorem is to be proved that "more information is always better (or at least not worse) in flow-independent networks".

Two information schemes 1 and 2 in the same network are to be studied. It is assumed that for any trajectory H, information scheme 2 has a larger coverage Q_2 than that of information scheme 1, Q_1: $Q_1 \subseteq Q_2$.

Definition 1 (S_1 contains S_2)**.** A partition of set S is a set of subsets which are mutually exclusive and collectively exhaustive of S. Let S_1 and S_2 be two partitions of S. S_1 contains S_2 if for any $y \in S_2$, there exists $z \in S_1$, such that $y \subseteq z$ and $y \cap z' = \varnothing$, $\forall z' \neq z$. In other words, any element of S_2 is a subset of one and only one element of S_1. See Figure 2 for a schematic representation.

<table>
<tr><td>S</td><td colspan="8">a b c d e f g h</td></tr>
<tr><td>S_1</td><td colspan="5">a b c d e</td><td colspan="3">f g h</td></tr>
<tr><td>S_2</td><td>a</td><td colspan="2">b c</td><td colspan="2">d e</td><td>f</td><td colspan="2">g h</td></tr>
</table>

Figure 2-2 A schematic view of S_1 containing S_2

Lemma 1. $\boldsymbol{EV}(Q_1)$ contains $\boldsymbol{EV}(Q_2)$, for any trajectory H.

Proof: By definition, $\boldsymbol{EV}(Q_1)$ and $\boldsymbol{EV}(Q_2)$ are two partitions of the set of network support points $\{C^1, \ldots, C^R\}$. Assume by contradiction that $\boldsymbol{EV}(Q_1)$ does not contain $\boldsymbol{EV}(Q_2)$, then there exists $EV_2 \in \boldsymbol{EV}(Q_2)$, such that for any $EV_1 \in \boldsymbol{EV}(Q_1)$, $EV_2 \cap EV_1 \neq EV_2$. As $\boldsymbol{EV}(Q_1)$ and $\boldsymbol{EV}(Q_2)$ are partitions of the same set, there must exist EV_1, such that $EV_2 \cap EV_1 \neq \varnothing$. By definition travel times on all time-dependent links in Q_2 is invariant across support points in EV_2. As $Q_1 \subseteq Q_2$, travel times on all time-dependent links in Q_1 is also invariant across support points in EV_2, specifically from $EV_2 \cap EV_1$ to $EV_2 \backslash (EV_1 \cap EV_2)$. Since $EV_1 \cap EV_2$ and $EV_2 \backslash (EV_1 \cap EV_2)$ are subsets of two distinctive elements of $\boldsymbol{EV}(Q_1)$, by definition travel times on all time-dependent links in Q_1 vary from $EV_1 \cap EV_2$ to $EV_2 \backslash (EV_1 \cap EV_2)$. There is a contradiction and this completes the proof.

Theorem 1. The optimal objective function value under information scheme 2 is no worse than that under information scheme 1, for the same origin j_0, departure time t_0, and event collections EV_2 and EV_1, where EV_2 is a subset of EV_1. Mathematically

$$e_2^*(j_0, t_0, EV_2) \leq e_1^*(j_0, t_0, EV_1),\ \ \forall j_0 \in N, \forall t_0 \in T,\ \ \forall EV_1, EV_2 \mid EV_2 \subseteq EV_1.$$

Proof: It is to be shown that any feasible routing policy μ_1 under information scheme 1 is equivalent to at least one feasible routing policy μ_2 under information scheme 2. It is proved by construction. At the initial state, set $\mu_2(j_0, t_0, EV_2) = \mu_1(j_0, t_0, EV_1)$. Upon arrival at the next node j_1 at time t_1, the information coverage Q_1 is a subset of Q_2 from the trajectory $\{(j_0, t_0), (j_1, t_1)\}$. By Lemma 1, $\boldsymbol{EV}(Q_1)$ contains $\boldsymbol{EV}(Q_2)$, therefore set $\mu_2(j_1, t_1, EV_2^{'}) = \mu_1(j_1, t_1, EV_1^{'}), \forall EV_2^{'} \in EV(Q_2), EV_2^{'} \subseteq EV_1^{'}$. The process continues and a routing policy μ_2 is obtained defined over information scheme 2 which produces exactly the same trajectory as μ_1, and thus the same objective function value. Therefore there exists a feasible routing policy under information scheme 2 with the same objective function value as the optimal routing policy under information scheme 1 and the optimal objective function under scheme 2 is at least as good as that under scheme 1. This completes the proof.

CHAPTER 3

ALGORITHM DESIGN

3.1 Partial Online Information Problem Variants

In order to study the value of information in the context of optimal adaptive routing in a flow-independent network, algorithms are designed to solve the optimal routing policy (ORP) problem with partial online information. The variants considered are pertinent in a traffic network:

- Global information with time lag Δ (LAG). For example, at 7:00 travelers only have information about traffic conditions up to 6:45.
- Global pre-trip information with departure time t_0 (PRE). For example, travelers get pre-trip information from internet before they start the journey. Once departed, they can no longer get online for more information. Therefore $\boldsymbol{EV}(t) = \boldsymbol{EV}(t_0), \forall t \geq t_0$.
- Information on a subset of links with no time lag (RADIO). For example, only traffic conditions on several major highways and arterials will be reported in a radio broadcast.
- No online information (NOI). This can be viewed as a special case of partial online information.

A generic algorithm is presented based on generic optimality conditions for the four partial online information problem variants and perfect online information (POI) variant. It can be shown that the generic algorithm is equivalent to Algorithm DOT-SPI in Gao and Chabini (2006) which is designed to solve the POI variant only.

Some characteristics of the five variants are:

- In all variants, information coverage Q is determined by the current time, instead of the whole trajectory, therefore $\boldsymbol{EV}(t)$ is used instead of $\boldsymbol{EV}(Q)$. Note that time lag Δ in LAG, departure time t_0 in PRE and radio coverage B in RADIO are treated as exogenous system parameters.
- With the exception of LAG, in all other variants travelers receive no more useful information during the static period, i.e., Q does not grow beyond time K–1, either because no information is provided by definition (PRE and NOI), or additional information will not enlarge Q (RADIO and POI);
- In the case of LAG with a time lag Δ, a traveler continues receiving information beyond the static period until $K - 1 + \Delta$, at which time $Q = A \times T$.

Let T^* denote the time beyond which a traveler receives no more information, and thus $T^* = K - 1 + \Delta$ for LAG, and $T^* = K - 1$ for all other four variants (PRE, RADIO, POI and NOI). Consider the routing decision making beyond T^*. The event collection will remain the same during all future time periods as that at time T^*, $EV \in \boldsymbol{EV}(T^*)$, since no more information will be received. The travel times are also static by definition. It is like traveling in a static and stochastic network defined by EV with no information. An optimal routing problem in such a network can be solved by a classical static shortest path algorithm in a converted deterministic network by taking link travel time means.

3.2 The Optimality Conditions

Since link travel times are random variables, there exist multiple optimization criteria. The expected travel time is used in the remaining of the thesis, as generally it is the primary criterion in routing choices. Other criteria regarding travel reliability, such as travel time variance and expected travel time schedule delay, and a combination of some of the criteria, will be explored in future researches.

Let e_μ *(j, t, EV)* be the expected travel time to the destination node *d* if the departure from node *j* happens at time *t* with the event collection *EV* by following routing policy μ. S_μ *(j, t, r)* is the travel time to the destination node *d* if support point *r* is realized with a departure from node *j* at time *t* by following routing policy μ. The relationship between e_μ *(j, t, EV)* and S_μ *(j, t, r)* is as follows:

$$e_\mu(j,t,EV) = \sum_{r\in EV} S_\mu(j,t,r)\Pr(r \mid EV) \tag{1}$$

The routing policy is defined on event collection, not support point. However, for each support point, a routing policy is manifested as a path with a certain travel time. For example, for a traveler with up-to-date radio information on link (*a,b*) in Figure 2-1 and Table 2-1, the routing decision at node *a* at time 0 can only be made based on the event collection $\{C^1, C^2, C^3\}$. Let $\mu\{a,0,\{C^1,C^2,C^3\}\} = c$. The travel time by following routing policy μ starting from node *a* at time 0 is a random variable with possible different outcomes in different support points: $S_\mu(a,0,C^1) = 3$, $S_\mu(a,0,C^2) = 3$, and $S_\mu(a,0,C^3) = 2$.

The relationship between S_μ at node *j* and the succeeding node *k* by following μ is critical to solving the ORP problem. S_μ (*j, t, r*) is defined for a trip departing at time *t*. For the variants POI, LAG, RADIO and NOI, the information coverage is not a function of departure time, and thus the event collections at time *t* is the same no matter whether *t* is the departure time or not. In this case,

$$S_\mu(j,t,r) = C^r_{jk,t} + S_\mu(k, t + C^r_{jk,t}, r) \text{ , where } k = \mu(j,t,EV),\ r \in EV \qquad (2)$$

For the PRE variant, however, the information coverage does depend on departure time, and thus in general (2) does not hold. A different variable $S_\mu(j,t,r;t_0)$ can then be defined as the travel time from node *j* and time *t* to the destination node if support point *r* is realized by following routing policy μ, with a departure time t_0. Similarly $e_\mu(j,t,EV;t_0)$ and $\mu(j,t,EV;t_0)$ can be defined. In this case,

$$S_\mu(j,t,r;t_0) = C^r_{jk,t} + S_\mu(k, t + C^r_{jk,t}, r;t_0) \text{, where } k = \mu(j,t,EV;t_0),\ r \in EV$$

$$e_\mu(j,t,EV;t_0) = \sum_{r \in EV} S_\mu(j,t,r;t_0)\Pr(r \mid EV)$$

Proposition 1: For the POI, LAG, RADIO and NOI variants, the minimum expected travel time $e_{\mu^*}(j,t,EV)$, $\forall j \in N \backslash \{d\}$, $\forall t$, $\forall EV \in \boldsymbol{EV}(t)$ and optimal routing policy μ^* are solutions to the following system of equations:

$$e_{\mu^*}(j,t,EV) = \min_{k \in A(j)} \left\{ \sum_{r \in EV} \left(C^r_{jk,t} + S_{\mu^*}(k, t + C^r_{jk,t}, r) \right) \Pr(r \mid EV) \right\} \qquad (3)$$

$$\mu^*(j,t,EV) = \arg\min_{k \in A(j)} \left\{ \sum_{r \in EV} \left(C^r_{jk,t} + S_{\mu^*}(k, t + C^r_{jk,t}, r) \right) \Pr(r \mid EV) \right\} \qquad (4)$$

where $S_{\mu^*}(j,t,r) = C^r_{jk^*,t} + S_{\mu^*}(k^*, t + C^r_{jk^*,t}, r),\ k^* = \mu^*(j,t,EV),\ r \in EV$. *A* (*j*) is the set of downstream nodes out of node *j*. The boundary conditions are:

1) At the destination: $e_{\mu^*}(d,t,EV) = 0,\ \mu^*(d,t,EV) = d$, $\forall t$, $\forall EV \in \boldsymbol{EV}(t)$.

2) Beyond T^*: $\mu^*(j, t \geq T^*, EV) = \mu^*(j, T^*, EV)$, $\forall j$, $\forall EV \in \boldsymbol{EV}(T^*)$, where $T^*=K-1+\Delta$ for LAG, and $T^*= K-1$ for all other 3 variants (RADIO, POI and NOI).

Proof: (Necessity). The necessity can be proved by showing that if $e_{\mu^*}(j, t, EV)$ is the minimum expected travel time and μ^* the optimal routing policy, they must satisfy the system of equations (3) ~ (4).

Trivially, if the boundary conditions at the destination node are not satisfied, μ^* is not optimal.

The optimal routing policy beyond T^* is not a function of time t, because both the travel times and event collections do not change over time. Thus $\mu^*(j, t \geq T^*, EV) = \mu^*(j, T^*, EV)$, $\forall j$, $\forall EV \in \boldsymbol{EV}$ (T^*). Further making use of (1) in (3) and (5) and the following are obtained:

$$e_{\mu^*}(j, T^*, EV) = \min_{k \in A(j)} \left\{ \sum_{r \in EV} C^r_{jk,T^*} \Pr(r \mid EV) + e_{\mu^*}(k, T^*, EV) \right\} \quad (5)$$

$$\mu^*(j, T^*, EV) = \arg\min_{k \in A(j)} \left\{ \sum_{r \in EV} C^r_{jk,T^*} \Pr(r \mid EV) + e_{\mu^*}(k, T^*, EV) \right\} \quad (6)$$

These are the optimality conditions of a static shortest path problem in a converted deterministic network where link travel times are replaced by their means $\sum_{r \in EV} C^r_{jk,T^*} \Pr(r \mid EV)$ at T^* given an event collection $EV \in \boldsymbol{EV}(T^*)$. In a static stochastic network, the expected path travel time is equal to the sum of the expected link travel times along the path, and therefore the minimum expected time path is the same as the shortest path in a converted deterministic network where link times are replaced by their means. If μ^* is optimal, it must manifest as the shortest path in each of the converted deterministic network defined by *EV*, and thus (5) and (6) must be satisfied.

Assume by contradiction that (3) and (4) are not satisfied for some state with a departure time earlier than T^*. Let (j, t, *EV*) be such a state. Therefore there must exist an outgoing node $k' \in A(j)$, such that

$$\sum_{r\in EV}\left(C^r_{jk',t} + S_{\mu^*}(k', t + C^r_{jk',t}, r)\right)\Pr(r \mid EV) < \sum_{r\in EV}\left(C^r_{jk^*,t} + S_{\mu^*}(k^*, t + C^r_{jk^*,t}, r)\right)\Pr(r \mid EV)$$

Therefore a different routing policy μ' can be constructed such that $\mu'(j, t, EV) = k'$, and $\mu' = \mu^*$ for all other states. Then the following is obtained:

$$\begin{aligned}
&e_{\mu'}(j,t,EV) = \sum_{r\in EV} S_{\mu'}(j,t,r)\Pr(r \mid EV) = \sum_{r\in EV}\left(C^r_{jk',t} + S_{\mu'}(k', t + C^r_{jk',t}, r)\right)\Pr(r \mid EV) \\
&= \sum_{r\in EV}\left(C^r_{jk',t} + S_{\mu^*}(k', t + C^r_{jk^*,t}, r)\right)\Pr(r \mid EV) \\
&< \sum_{r\in EV}\left(C^r_{jk^*,t} + S_{\mu^*}(k^*, t + C^r_{jk^*,t}, r)\right)\Pr(r \mid EV) = e_{\mu^*}(j,t,EV)
\end{aligned}$$

which is contradicted with fact that μ^* is optimal.

(Sufficiency) The sufficiency can be proved by showing that if $e_{\mu^*}(j,t,EV)$ and μ^* satisfy the system of equations (3) ~ (4), they must be the minimum expected travel time and the optimal routing policy respectively.

Assume by contradiction that μ^* is not optimal, therefore there must exist a routing policy μ such that $e_\mu(j, t, EV) < e_{\mu^*}(j, t, EV)$ for some (j, t, *EV*). From the proof of necessity, (5) ~ (6) are also the sufficient condition for μ^* to be optimal at or beyond T^*. Therefore $t < T^*$. Assume t is the latest time when the inequality occurs, and thus $e_\mu(j, t', EV) = e_{\mu^*}(j, t', EV)$, $\forall t' > t$. Assume $\mu(j, t, EV) = k$:

$$
\begin{aligned}
e_{\mu}(j,t,EV) &= \sum_{r\in EV} S_{\mu}(j,t,r)\Pr(r \mid EV) = \sum_{r\in EV}\left(C^{r}_{jk,t} + S_{\mu}(k,t+C^{r}_{jk,t},r)\right)\Pr(r \mid EV) \\
&= \sum_{r\in EV} C^{r}_{jk,t}\Pr(r \mid EV) + \sum_{r\in EV} S_{\mu}(k,t+C^{r}_{jk,t},r)\Pr(r \mid EV) \\
&= \sum_{r\in EV} C^{r}_{jk,t}\Pr(r \mid EV) + e_{\mu}(k,t+C^{r}_{jk,t},EV) \\
&= \sum_{r\in EV} C^{r}_{jk,t}\Pr(r \mid EV) + e_{\mu^*}(k,t+C^{r}_{jk,t},EV) \\
&= \sum_{r\in EV}\left(C^{r}_{jk,t} + S_{\mu^*}(k,t+C^{r}_{jk,t},r)\right)\Pr(r \mid EV) \\
&\ge \sum_{r\in EV}\left(C^{r}_{jk^*,t} + S_{\mu^*}(k^*,t+C^{r}_{jk^*,t},r)\right)\Pr(r \mid EV) \\
&= e_{\mu^*}(j,t,EV)
\end{aligned}
$$

Therefore the assumption is not valid and μ^* is optimal.

Proposition 2: For the PRE variant with departure time t_0, the minimum expected travel time $e_{\mu^*}(j,t,EV;t_0)$, $\forall j \in N \backslash \{d\}$, $\forall EV \in \boldsymbol{EV}(t)$ and optimal routing policy μ^* are solutions to the following system of equations:

$$
e_{\mu^*}(j,t,EV;t_0) = \min_{k\in A(j)}\left\{\sum_{r\in EV}\left(C^{r}_{jk,t} + S_{\mu^*}(k,t+C^{r}_{jk,t},r;t_0)\right)\Pr(r \mid EV)\right\} \tag{7}
$$

$$
\mu^*(j,t,EV;t_0) = \arg\min_{k\in A(j)}\left\{\sum_{r\in EV}\left(C^{r}_{jk,t} + S_{\mu^*}(k,t+C^{r}_{jk,t},r;t_0)\right)\Pr(r \mid EV)\right\} \tag{8}
$$

where $S_{\mu^*}(j,t,r;t_0) = C^{r}_{jk^*,t} + S_{\mu^*}(k^*,t+C^{r}_{jk^*,t},r;t_0)$, $k^* = \mu^*(j,t,EV;t_0)$, $r \in EV$. $A(j)$ is the set of downstream nodes out of node j. The boundary conditions are:

1) At the destination: $e_{\mu^*}(d,t,EV;t_0) = 0$, $\mu^*(d,t,EV;t_0) = d$, $\forall t$, $\forall EV \in \boldsymbol{EV}(t)$.

2) Beyond T^*: $\mu^*(j,t \ge T^*,EV;t_0) = \mu^*(j,T^*,EV;t_0)$, $\forall j$, $\forall EV \in \boldsymbol{EV}(T^*)$, where $T^* = K-1$.

The proof of Proposition 2 is similar to that of Proposition 1 with notation change only.

Proposition 3: Optimality conditions (3) ~ (4) are equivalent to the optimality conditions for the perfect online information (POI) variant in Gao and Chabini (2006).

The proof of Proposition 3 is done by making use of the fact that with POI, the travel time on an outgoing link is a deterministic value given an event collection.

3.3 Algorithm DOT-PART

The evaluation of $e_{\mu^*}(j,t,EV)$ only depends on $S_{\mu^*}(j,t',r)$ from a later time $t' > t$, due to the positive and integral link travel time assumption. Therefore the labels can be optimally set in a decreasing order of time, making use of the acyclic property of the network along the time dimension. At time T^* and beyond, any deterministic static shortest path algorithm can be used to compute $e_{\mu^*}(j,t,EV)$, $\forall j \in N$, $\forall t \geq T^*$, $\forall EV \in EV(T^*)$. The procedure to generate event collections carry out partitions of the network support points in an increasing order of time. At time t, a partition is made on $\boldsymbol{EV}(t\text{-}1)$ based on each (link, time) pair in the incremental information coverage, $Q(t) \backslash Q(t\text{-}1)$. Note that Q is written as a function of t, because in all the 5 variants, Q only depends on t, not the trajectory.

The algorithm solves the ORP problem from all initial states for POI, LAG, RADIO and NOI, but only from departure time 0 for PRE. In order to solve PRE variant with all departure times, an outer loop over all departure times t_0 has to be added to the main loop, and the main loop over time t will be from T^*-1 down to t_0. This is because the event collection at time t when t is the departure time is different from when t is not.

For any other variant, the event collection at t is the same regardless of the departure time. Adding an outer loop is not the most efficient implementation to solve the PRE variant. However since the focus of this thesis is the study of the value of traveler information, a correct implementation is enough. More efficient implementations will be explored in future research.

Algorithm DOT-PART

(Generic for the 5 variants: POI, LAG, PRE with departure time 0, RADIO and NOI)

Initialization

Step 1:

If information scheme = LAG with a lag of Δ then

$T^* = \mathrm{K} - 1 + \Delta$

else

$T^* = K - 1$

Construct $\boldsymbol{EV}(t)$, $t = 0, \ldots, T^*$ by calling Generate_Event_Collection (see the statement below)

Step 2:

Compute $e_{\mu^*}(j, T^*, EV)$ and $\mu^*(j, T^*, EV)$, $\forall j \in N, \forall EV \in \boldsymbol{EV}(T^*)$ with a static deterministic shortest path algorithm in a converted static deterministic network where link travel times are replaced by their means at time T^*.

Compute $S_{\mu^*}(j, T^*, r)$ by executing μ^* in the original static stochastic network, $\forall j \in N, \forall r \in EV$; $S_{\mu^*}(j, t > T^*, r) = S_{\mu^*}(j, T^*, r)$.

Step 3:

$e_{\mu^*}(j, t, EV) \leftarrow +\infty,\ \forall j \in N \setminus \{d\}, \forall t < T^*, \forall EV \in \boldsymbol{EV}(t)$

$e_{\mu^*}(d, t, EV) \leftarrow 0,\ \forall t < T^*, \forall EV \in \boldsymbol{EV}(t)$

Main Loop

For $t = T^*$-1 down to 0

For each $EV \in \boldsymbol{EV}(t)$

For each link $(j, k) \in A$

$$temp = \sum_{r \in EV} \left(C^r_{jk,t} + S_{\mu^*}(k, t + C^r_{jk,t}, r) \right) \Pr(r \mid EV)$$

If $temp\_e < e_{\mu^*}(j, t, EV)$ then

$e_{\mu^*}(j, t, EV) = temp\_e$

$\mu^*(j, t, EV) = k$

For each $r \in EV$ and each $j \in N$

$k^* = \mu^*(j, t, EV)$

$$S_{\mu^*}(j, t, r) = C^r_{jk^*,t} + S_{\mu^*}(k^*, t + C^r_{jk^*,t}, r)$$

Generate_Event_Collection

$D = \{C^1, \ldots, C^R\}$

For $t = 0$ to T^*

If information scheme = POI

$Q(t) = A \times \{0, 1, \ldots, t\}$

If information scheme = LAG with a lag Δ

$Q(t) = A \times \{0,1,\ldots,t - \Delta\}$

If information scheme = PRE with departure time 0

$Q(t) = A \times \{0\}$

If information scheme = RADIO with link set B

$Q(t) = B \times \{0,1,\ldots,t\}$

$Q(-1) = \varnothing$ //a proxy for the convenience of representation

For $t = 0$ to T^*

For each (link, time) pair $((j,k),t') \in Q(t) \setminus Q(t\text{-}1)$

For each disjoint subset $S \in D$

D' ← A partition of S based on $\tilde{C}_{jk,t}$

D ← Union of all D'

EV*(t)* ← D;

Following a similar analysis as in Gao and Chabini (2006), it can be derived that Algorithm DOT-PART has a complexity of $O(mKR\ln R + R\times\text{SSP})$ and $\Omega(mKR + \text{SSP})$, where SSP is the complexity of the static deterministic shortest path algorithm. The algorithm is strongly polynomial in R, the number of support points of the link travel time joint distribution. For real applications, time-dependent travel time observations on all links from each day can be viewed as one support point.

A running time test is conducted with randomly generated networks on a Dell Optiplex with 2.40GHz Intel Core 2 CPU and 2.00GB of RAM. Details of the random network generator can be found in Gao (2005). The number of nodes (n), the number of time periods (K), and the number of support points (R) are chosen as input variables; the number of links (m) is three times as great as the number of nodes. Random numbers from multivariate normal distributions are generated for link travel times. The relationship between running time of the algorithm and the input variables for the LAG variant is shown in Table 3-1. It can be seen that the relationship between running time and each of the 3 input variables is close to linear. Similar tests are conducted for other variants and the relationships are similar.

Table 3-1 Relationship between CPU time (sec) and input variables in LAG variant

Running time of Generate_Event_Collection									
m	30			60			90		
K / R	600	1200	1800	600	1200	1800	600	1200	1800
50	0.239211	0.461097	0.665552	0.47334	0.925444	1.352627	0.708472	1.392897	2.008585
100	0.48257	0.916187	1.337647	0.952481	1.8222	2.700408	1.434959	2.753483	3.999361
300	1.416003	2.701326	3.951077	2.810238	5.359506	7.861029	4.206747	7.990318	11.76882
Running time of DOT-PART for LAG variant (excluding Generate_Event_Collection)									
m	30			60			90		
K / R	600	1200	1800	600	1200	1800	600	1200	1800
50	0.652758	1.167605	1.693616	1.456281	2.555125	3.682067	2.282774	4.047678	5.795551
100	1.43934	2.469138	3.511649	3.185009	5.387602	7.667592	4.998235	8.52984	12.06706
300	5.652469	8.856279	12.11531	12.45083	19.62694	27.11089	19.78276	31.1017	43.87052

For applications in real life size networks, the computational time is not the constraint, but the memory is. One possible solution is to change the representation of link travel times from discrete time based to continuous time based. A piece-wise linear representation of link travel times has been implemented and computational tests have been conducted in a Swedish city network with about 7500 directional links, 39600 time periods and 30 support points. The results in real-life networks will be reported in succeeding researches.

CHAPTER 4

COMPUTATIONAL TESTS

4.1 Objectives

The objectives of the computational tests are to: 1) compare computationally the optimal expected travel times of each of the three partial online information variants and no online information (NOI) with perfect online information (POI); 2) compare computationally the optimal expected travel times of partial online information variants with the same type of information but different system parameters; 3) show the value of information and verify the theoretical result derived in Section 2.5 that more information is always better (or at least not worse) in a flow-independent network.

4.2 The Test Network

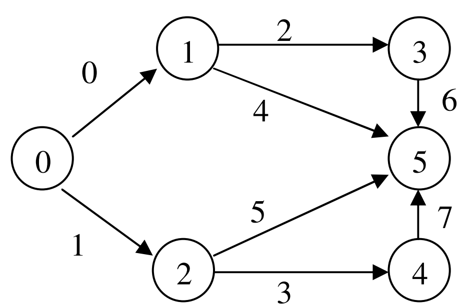


Figure 4-1 The test network

The test network is shown in Figure 4-1 with 6 nodes and 8 directed links. There are diversion possibilities at nodes 0, 1 and 2. The study period is from 6:30am to 8:00am. The time resolution is 1 minute for departures and arrivals at intermediate nodes, and there are 90 time periods in total. The travel time is in seconds.

The link travel time distribution is generated through an exogenous simulation with the mesoscopic supply simulator of DynaMIT from Ben-Akiva, et al. (2001). The demand between OD (0, 5) is low between 6:30am to 7:00am and higher later on. There are random incidents in the network defined as follows: 1) There is at most one incident for any given day; 2) The incident has a positive probability of occurrence on link 0, 2, 4 and 6, but zero on link 1, 3, 5 and 7; and 3) If an incident occurs on a link, the start time can be every 10 minutes with equal probability. The 4 possible locations and 9 possible start times result in 4×9+1 (no incident) =37 support points. Details of the network can be found in Gao (2005).

4.3 Test Results

Algorithm DOT-PART is run for the three partial online information schemes, no online information (NOI) and perfect online information (POI) to derive the minimum expected travel times from each of the variants from node 0 to node 5 for all departure times and all event collections. The results are aggregated by departure time, by taking an expectation over all event collections at a given time.

Figures 3.a and 3.b show the result for the LAG (global information with time lag Δ) variant: LAG5 indicates there is a 5 minutes information time lag, and LAG10 and LAG15 respectively a 10 minutes and 15 minutes lag. It can be seen that the following relationship holds:

$$\text{POI} \leq \text{LAG5} \leq \text{LAG10} \leq \text{LAG15} \leq \text{NOI}.$$

Figure 3.c shows the results for the PRE (global pre-trip information) variant. It can be seen that the following relationship holds:

$$\text{POI} \leq \text{PRE} \leq \text{NOI}.$$

Figures 3.d and 3.e show the results for the RADIO (information on a subset of links with no time lag) variant: RADIO4 indicates only traffic condition information on link 4 is provided and RADIO45 on both link 4 and 5. Note that link 4 has a positive incident probability while link 5 does not. In fact, the lower half of the network is incident free and serves as potential diversions in case of incidents in the upper half the network. Reporting traffic conditions on links with incident is definitely helpful, but if alternative routes are also congested then there is probably no benefit to divert. If combined with reports on alternative routes, the value of information can be enhanced. It can be seen that the following relationship holds:

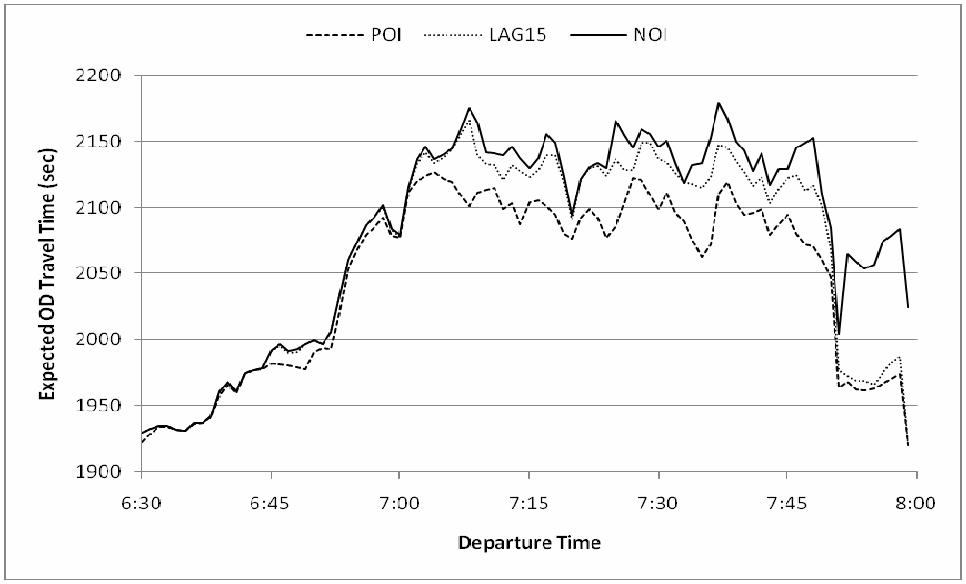
$$\text{POI} \le \text{RADIO45} \le \text{RADIO4} \le \text{NOI}.$$

It also holds for other radio coverage situations on shown here. For example, POI ≤ RADIO23 ≤ RADIO2 ≤ NOI, where RADIO2 indicates only traffic condition information on link 2 is provided and RADIO23 on both link 2 and 3.

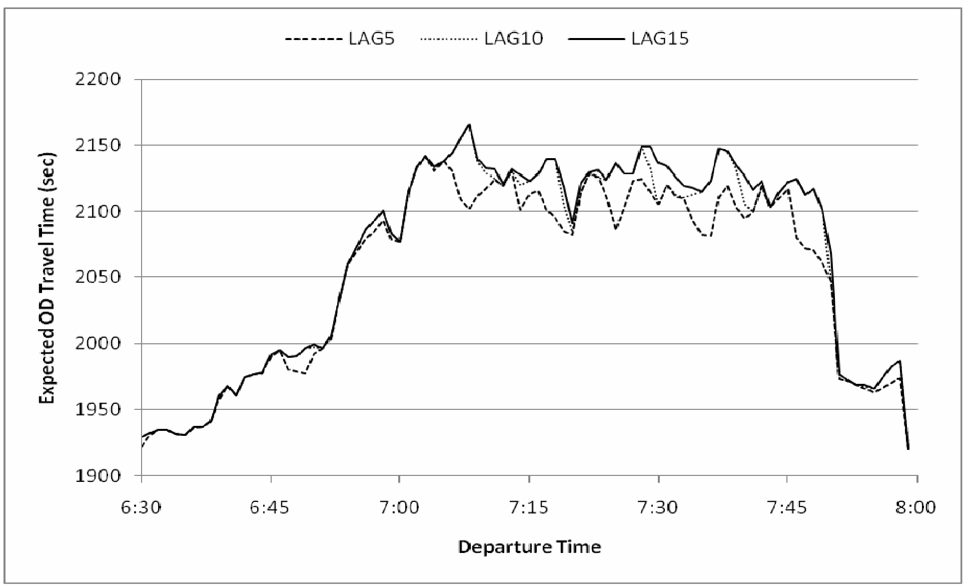
In summary, the following conclusions can be drawn:

- The results from the situations with partial online information are better than that of no online information, but worse than that of POI: POI ≤ LAG, PRE, RADIO≤ NOI;
- The less information time lag, the better the results: LAG5 ≤ LAG10 ≤ LAG15;
- The more information provided, either temporal or spatial, the better the results: PRE ≤ NOI; RADIO45 ≤ RADIO4.

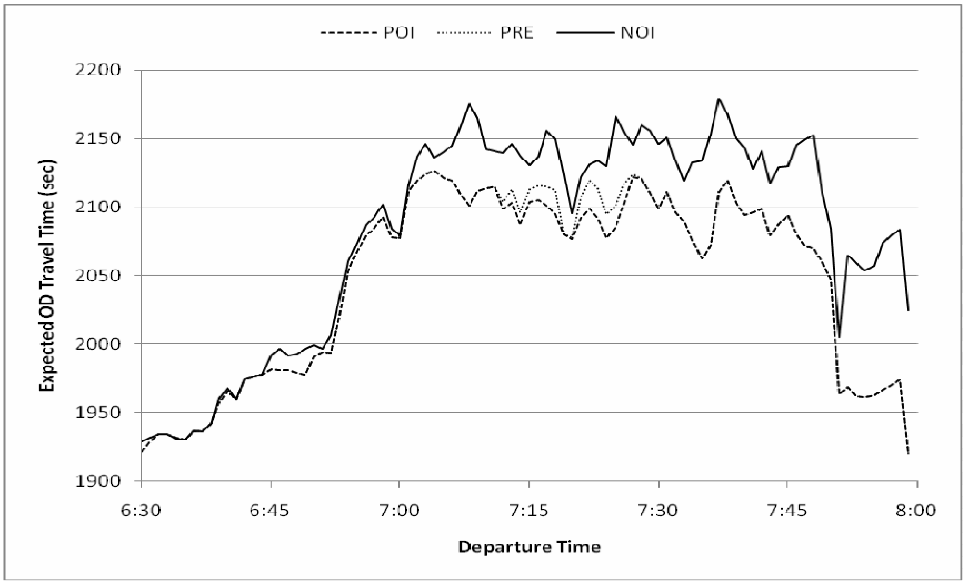
These results show the value of information for optimal adaptive routing on both temporal and spatial dimensions in terms of reducing expected travel time.



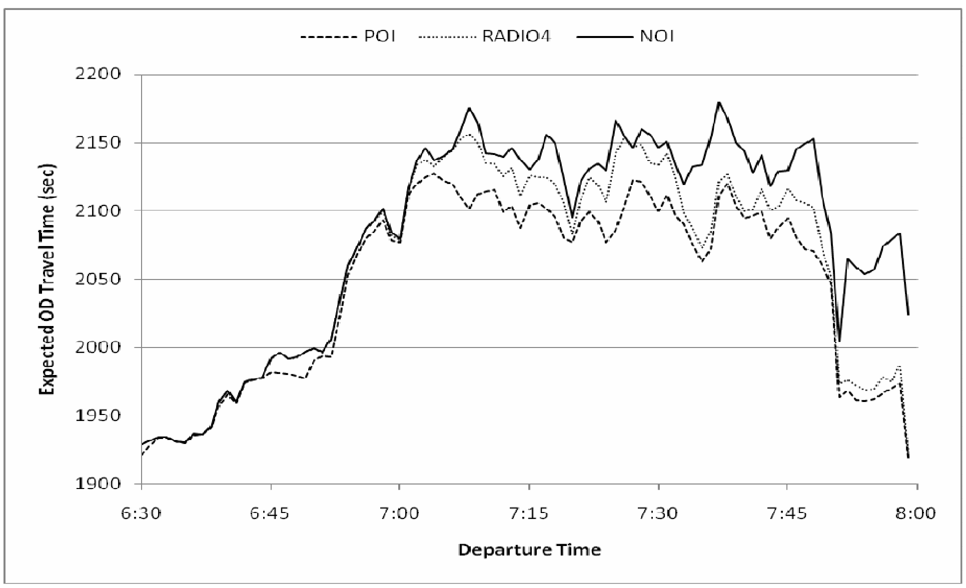
a) Results for the LAG variant vs. POI and NOI



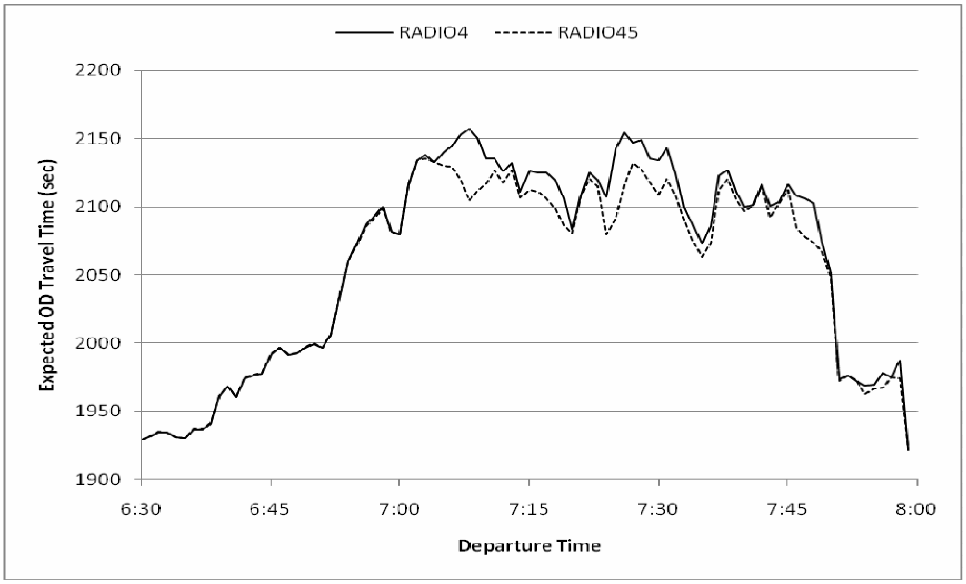
b) Results for the LAG variant with different *Δ*



c) Results for the PRE variant vs. POI and NOI



d) Results for the RADIO variant vs. POI and NOI



e) Results for the RADIO variant with different *B*

Figure 4-2 The test results

CHAPTER 5

CONCLUSION AND FUTERE DIRECTION

5.1 Conclusion

The optimal routing policy (ORP) problems in stochastic time-dependent (STD) networks with partial online information are studied. A unified view is described towards online information based on which generic optimality conditions for the optimal routing problem with partial online information are derived. Three variants that are particularly pertinent to the modeling and routing applications in traffic networks are then studied in detail: global information with time lag, global pre-trip information, and radio information on a subset of links without time lag. The three variants take into account the partial online information with limitations on both temporal and spatial sides, which are realistic depictions of traffic systems equipped with ATIS. An exact algorithm (Algorithm DOT-PART) is designed for the partial online information problem and implemented for the three variants and no online information (NOI) variant. Computational results show that more information is generally better (or at least not worse) in the specific test setting. A theoretical proof of the non-negative value of traveler information for adaptive routing in a flow-independent stochastic network can be found in Gao and Huang (2008).

5.2 Future Direction

Future researches on ORP problems in STD networks with partial online information can be in the following directions:

- As an implementation of ATIS, VMS discussed in the introduction is an interesting variant of partial online information problem and probably one of the mostly used. The VMS case is more involved than those discussed in this thesis, as the information is trajectory-based.
- In this work, the three variants contain information limitation on either temporal or spatial dimension. In the future research, a combination of limitations on both dimensions can be considered, e.g., a more realistic radio broadcast variant with time lag.
- In this work expected travel time is used as the criterion in routing choices. Other criteria, such as travel time variance and expected travel time schedule delay, and a combination of some of the criteria, will be explored.

APPENDIX

AN ILLUSTRATIVE EXAMPLE FOR ALGORITHM DOT-PART

An example is to be used to illustrate how Algorithm DOT-PART works. The small network in Figure A-1 is the same as the example network in Figure 2-1. It has three nodes, three links and the number of time periods is 2. The values of the travel time realizations are in Table A-1, the same as those in Table 2-1. Each of the three support points has a probability of 1/3. The network is designed to be very small to make the understanding of the algorithm easier. Note that travelers starting from node *b* or node *c* have no choice but to take node *c* as the next node. It is suggested that the reader pay attention to how routing decision at node *a* is affected by time and online information.

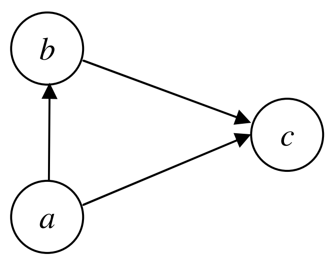


Figure A-1 Algorithm DOT-PART: A Small Network

Table A-1 Support points for the Small Network

Time	Link	C^1	C^2	C^3
0	(*a*, *b*)	1	1	1
	(*b*, *c*)	2	2	1
	(*a*, *c*)	3	3	2
1	(*a*, *b*)	1	1	2
	(*b*, *c*)	1	2	1
	(*a*, *c*)	3	2	2

1. POI variant

$T^* = K - 1 = 1$. Beyond $T^* = 1$, travelers receive no more information.

Step 1: Construct $\boldsymbol{EV(t)}$, $t = 0, 1$ by calling Generate_Event_Collection

$D = \{\{C^1, C^2, C^3\}\}$

$t = 0$ $\quad Q^{\mathrm{POI}}(0) = A \times \{0\}$ //all links at time 0

$((j, k), t') = ((a, b), 0)$ //link (a, b) at time 0

$S = \{C^1, C^2, C^3\}$

$D' = \{\{C^1, C^2, C^3\}\}$

$D = \{\{C^1, C^2, C^3\}\}$

$((j, k), t') = ((b, c), 0)$ //link (b, c) at time 0

$\mathrm{S} = \{C^1, C^2, C^3\}$

$D' = \{\{C^1, C^2\}, \{C^3\}\}$

$D = \{\{C^1, C^2\}, \{C^3\}\}$

$((j, k), t') = ((a, c), 0)$ //link (a, c) at time 0

$S = \{C^1, C^2\}$

$D' = \{\{C^1, C^2\}, \{C^3\}\}$

$S = \{C^3\}$

$D' = \{\{C^1, C^2\}, \{C^3\}\}$

$D = \{\{C^1, C^2\}, \{C^3\}\}$

$\boldsymbol{EV}(0) = \{\{C^1, C^2\}, \{C^3\}\}$

$t = 1$ $\quad Q^{\mathrm{POI}}(1) = A \times \{0,1\}$ //all links at time 0 and 1

$Q^{\mathrm{POI}}(1) \setminus Q^{\mathrm{POI}}(0) = A \times \{1\}$ //all links at time 1

$((j, k), t') = ((a, b), 1)$ //link (a, b) at time 1

$S = \{C^1, C^2\}$

$D' = \{\{C^1, C^2\}, \{C^3\}\}$

$S = \{C^3\}$

$D' = \{\{C^1, C^2\}, \{C^3\}\}$

$D = \{\{C^1, C^2\}, \{C^3\}\}$

$((j, k), t') = ((b, c), 1)$ //link (b, c) at time 1

$S = \{C^1, C^2\}$

$D' = \{\{C^1\}, \{C^2\}, \{C^3\}\}$

$S = \{C^3\}$

$D' = \{\{C^1\}, \{C^2\}, \{C^3\}\}$

$D = \{\{C^1\}, \{C^2\}, \{C^3\}\}$

$((j, k), t') = ((a, c), 1)$ //link (a, c) at time 1

$S = \{C^1\}$

$D' = \{\{C^1\}, \{C^2\}, \{C^3\}\}$

$S = \{C^2\}$

$D' = \{\{C^1\}, \{C^2\}, \{C^3\}\}$

$S = \{C^3\}$

$D' = \{\{C^1\}, \{C^2\}, \{C^3\}\}$

$D = \{\{C^1\}, \{C^2\}, \{C^3\}\}$

$\boldsymbol{EV}(1) = \{\{C^1\}, \{C^2\}, \{C^3\}\}$

A summary of the results of constructing event collections is as follows.

$\boldsymbol{EV}(0) = \{\{C^1, C^2\}, \{C^3\}\}$

$\boldsymbol{EV}(1) = \{\{C^1\}, \{C^2\}, \{C^3\}\}$

Step 2: Compute $e_{\mu^*}(j, 1, EV),\ \forall j \in N, \forall EV \in \boldsymbol{EV}(1)$

This step involves solving deterministic static shortest path problems with each single support point C^r, $r = 1, 2, 3$. Any classical shortest path algorithm can be used. In our small network, this can be done by observation. The results are listed in Table A-2. In each result cell, the minimum expected travel time is given and the corresponding next node is in the parenthesis. Since event collections at static time period are all singleton, $S_{\mu^*}(a, 1, C^i) = e_{\mu^*}(a, 1, \{C^i\})$, $i = 1, 2, 3$.

Step 3: $e_{\mu^*}(j, t, EV) \leftarrow +\infty, \ \forall j \in \{a, b\}, \forall t < 1, \forall EV \in \boldsymbol{EV}(t)$

$e_{\mu^*}(c, t, EV) \leftarrow 0, \ \forall t < 1, \forall EV \in \boldsymbol{EV}(t)$

Step 4: (main loop)

$t = 0$

$EV = \{C^1, C^2\}$

$(j, k) = (a, b)$

$$temp\_e = [1 + S_{\mu^*}(b, 0{+}1, C^1)] \Pr(C^1 \mid EV) + [1 + S_{\mu^*}(b, 0{+}1, C^2)]\, Pr(C^2 \mid EV)$$

$$= (1 + 1) \times 0.5 + (1 + 2) \times 0.5 = 2.5 < +\infty$$

$e_{\mu^*}(a, 0, \{C^1, C^2\}) = 2.5$, $\mu^*(a, 0, \{C^1, C^2\}) =$ node b

$S_{\mu^*}(a, 0, C^1) = 1 + S_{\mu^*}(b, 0{+}1, C^1) = 1 + 1 = 2$

$S_{\mu^*}(a, 0, C^2) = 1 + S_{\mu^*}(b, 0{+}1, C^2) = 1 + 2 = 3$

$(j, k) = (b, c)$

$$temp\_e = [2 + S_{\mu^*}(c, 0{+}2, C^1)] \Pr(C^1 \mid EV) + [2 + S_{\mu^*}(c, 0{+}2, C^2)] \Pr(C^2 \mid EV)$$

$= (2 + 0) \times 0.5 + (2 + 0) \times 0.5 = 2 < +\infty$

$e_{\mu^*}(b, 0, \{C^1, C^2\}) = 2,\ \ \mu^*(b, 0, \{C^1, C^2\}) = \text{node } c$

$S_{\mu^*}(b, 0, C^1) = 2 + S_{\mu^*}(c, 0{+}2, C^1) = 2 + 0 = 2$

$S_{\mu^*}(b, 0, C^2) = 2 + S_{\mu^*}(c, 0{+}2, C^2) = 2 + 0 = 2$

$(j, k) = (a, c)$

$temp\_e = [3 + S_{\mu^*}(c, 0{+}3, C^1)]\ \Pr(C^1 \mid EV)$

$+ [3 + S_{\mu^*}(c, 0{+}3, C^2)]\ \Pr(C^2 \mid EV)$

$= (3 + 0) \times 0.5 + (3 + 0) \times 0.5 = 3 < e_{\mu^*}(a, 0, \{C^1, C^2\})$

$EV = \{C^3\}$

$(j, k) = (a, b)$

$temp\_e = [1 + S_{\mu^*}(b, 0{+}1, C^3)]\ \Pr(C^3 \mid EV)$

$= (1 + 1) \times 1 = 2 < +\infty$

$e_{\mu^*}(a, 0, \{C^3\}) = 2,\ \ \mu^*(a, 0, \{C^3\}) = \text{node } b$

$S_{\mu^*}(a, 0, C^3) = 1 + S_{\mu^*}(b, 0{+}1, C^3) = 1 + 1 = 2$

$(j, k) = (b, c)$

$temp\_e = [1 + S_{\mu^*}(c, 0{+}1, C^3)]\ \Pr(C^3 \mid EV)$

$= (1 + 0) \times 1 = 1 < +\infty$

$e_{\mu^*}(b, 0, \{C^3\}) = 1,\ \ \mu^*(b, 0, \{C^3\}) = \text{node } c$

$S_{\mu^*}(b, 0, C^3) = 1 + S_{\mu^*}(c, 0{+}1, C^3) = 1 + 0 = 1$

$(j, k) = (a, c)$

$temp\_e = [2 + S_{\mu^*}(c, 0{+}2, C^3)]\ \Pr(C^3 \mid EV)$

$= (2 + 0) \times 1 = 2 = e_{\mu^*}(a, 0, \{C^3\})$

The summary of results at time 1 and 0 is in Table A-2.

Table A-2 Results in the Static Period and at Time 0 in POI variant

Time $t \geq 1$, $\boldsymbol{EV}(1) = \{\{C^1\}, \{C^2\}, \{C^3\}\}$			
Node \ EV	$\{C^1\}$	$\{C^2\}$	$\{C^3\}$
b	1(node *c*)	2(node *c*)	1(node *c*)
a	2(node *b*)	2(node *c*)	2(node *c*)

Time $t = 0$, $\boldsymbol{EV}(0) = \{\{C^1, C^2\}, \{C^3\}\}$		
Node \ EV	$\{C^1, C^2\}$	$\{C^3\}$
b	2(node *c*)	1(node *c*)
a	2.5(node *b*)	2(node *c*)

2. NOI variant

$T^* = K - 1 = 1$. Beyond $T^* = 1$, travelers receive no more information.

Step 1: Construct $\boldsymbol{EV}(t)$, $t = 0, 1$ by calling Generate_Event_Collection

$D = \{\{C^1, C^2, C^3\}\}$

$t = 0$ $\quad Q^{\text{LAG}}(0) = \varnothing$ //no information available

$\boldsymbol{EV}(0) = \{\{C^1, C^2, C^3\}\}$

$t = 1$ $\quad Q^{\text{LAG}}(1) = \varnothing$ //no information available

$\boldsymbol{EV}(1) = \{\{C^1, C^2, C^3\}\}$

A summary of the results of constructing event collections is as follows.

$\boldsymbol{EV}(0) = \{\{C^1, C^2, C^3\}\}$

$\boldsymbol{EV}(1) = \{\{C^1, C^2, C^3\}\}$

Step 2: Compute $e_{\mu^*}(j, 1, EV)$, $\forall j \in N, \forall EV \in \boldsymbol{EV}(1)$

This step involves solving deterministic static shortest path problems with each single support point C^r, $r = 1, 2, 3$. Any classical shortest path algorithm can be used. In our small network, this can be done by observation. The results are listed in Table A-3. In each result cell, the minimum expected travel time is given and the corresponding next node is in the parenthesis. $S_{\mu^*}(b, 1, C^1) = 1$, $S_{\mu^*}(b, 1, C^2) = 2$, $S_{\mu^*}(b, 1, C^3) = 1$, $S_{\mu^*}(a, 1, C^1) = 3$, $S_{\mu^*}(a, 1, C^2) = 2$, $S_{\mu^*}(a, 1, C^3) = 2$.

Step 3: $e_{\mu^*}(j, t, EV) \leftarrow +\infty, \ \forall j \in \{a, b\}, \forall t < 1, \forall EV \in \boldsymbol{EV}(t)$

$e_{\mu^*}(c, t, EV) \leftarrow 0, \ \forall t < 1, \forall EV \in \boldsymbol{EV}(t)$

Step 4: (main loop)

$t = 0$

$EV = \{C^1, C^2, C^3\}$

$(j, k) = (a, b)$

$$temp\_e = [1 + S_{\mu^*}(b, 0{+}1, C^1)]\, \Pr(C^1 \mid EV)$$

$$+ [1 + S_{\mu^*}(b, 0{+}1, C^2)]\, Pr(C^2 \mid EV)$$

$$+ [1 + S_{\mu^*}(b, 0{+}1, C^3)]\, Pr(C^3 \mid EV)$$

$$= (1 + 1) \times 1/3 + (1 + 2) \times 1/3 + (1 + 1) \times 1/3 = 7/3 < +\infty$$

$e_{\mu^*}(a, 0, \{C^1, C^2, C^3\}) = 7/3$, $\mu^*(a, 0, \{C^1, C^2, C^3\})$ = node b

$S_{\mu^*}(a, 0, C^1) = 1 + S_{\mu^*}(b, 0{+}1, C^1) = 1 + 1 = 2$

$S_{\mu^*}(a, 0, C^2) = 1 + S_{\mu^*}(b, 0{+}1, C^2) = 1 + 2 = 3$

$S_{\mu^*}(a, 0, C^3) = 1 + S_{\mu^*}(b, 0{+}1, C^3) = 1 + 1 = 2$

$(j, k) = (b, c)$

$$temp\_e = [2 + S_{\mu^*}(c, 0{+}2, C^1)]\ \Pr(C^1 \mid EV)$$

$$+ [2 + S_{\mu^*}(c, 0{+}2, C^2)]\ \Pr(C^2 \mid EV)$$

$$+ [1 + S_{\mu^*}(c, 0{+}1, C^3)]\ \Pr(C^3 \mid EV)$$

$$= (2 + 0) \times 1/3 + (2 + 0) \times 1/3 + (1 + 0) \times 1/3 = 5/3 < +\infty$$

$$e_{\mu^*}(b, 0, \{C^1, C^2, C^3\}) = 5/3,\ \ \mu^*(b, 0, \{C^1, C^2, C^3\}) = \text{node } c$$

$$S_{\mu^*}(b, 0, C^1) = 2 + S_{\mu^*}(c, 0{+}2, C^1) = 2 + 0 = 2$$

$$S_{\mu^*}(b, 0, C^2) = 2 + S_{\mu^*}(c, 0{+}2, C^2) = 2 + 0 = 2$$

$$S_{\mu^*}(b, 0, C^3) = 1 + S_{\mu^*}(c, 0{+}1, C^3) = 1 + 0 = 1$$

$(j, k) = (a, c)$

$$temp\_e = [3 + S_{\mu^*}(c, 0{+}3, C^1)]\ \Pr(C^1 \mid EV)$$

$$+ [3 + S_{\mu^*}(c, 0{+}3, C^2)]\ \Pr(C^2 \mid EV)$$

$$+ [2 + S_{\mu^*}(c, 0{+}2, C^2)]\ \Pr(C^2 \mid EV)$$

$$= (3 + 0) \times 1/3 + (3 + 0) \times 1/3 + (2 + 0) \times 1/3$$

$$= 8/3 > e_{\mu^*}(a, 0, \{C^1, C^2, C^3\})$$

The summary of results at time 1 and 0 is in Table A-3.

Table A-3 Results in the Static Period and at Time 0 in NOI variant

Time $t \geq 1$, $\boldsymbol{EV}(0) = \{\{C^1, C^2, C^3\}\}$	
Node \ EV	$\{C^1, C^2, C^3\}$
b	4/3(node *c*)
a	7/3(node *c*)
Time $t = 0$, $\boldsymbol{EV}(0) = \{\{C^1, C^2, C^3\}\}$	
Node \ EV	$\{C^1, C^2, C^3\}$
b	5/3(node *c*)
a	7/3(node *b*)

3. LAG variant

Set: $\Delta = 1$. $T^* = K - 1 + \Delta = 2$. Beyond $T^* = 2$, travelers receive no more information.

Step 1: Construct $\boldsymbol{EV}(t)$, $t = 0, 1, 2$ by calling Generate_Event_Collection

$D = \{\{C^1, C^2, C^3\}\}$

$t = 0$ $\quad Q^{\mathrm{LAG}}(0) = \varnothing$ //no information available yet

$\boldsymbol{EV}(0) = \{\{C^1, C^2, C^3\}\}$

$t = 1$ $\quad Q^{\mathrm{LAG}}(1) = A \times \{0\}$ //all links at time 0

$Q^{\mathrm{LAG}}(1) \setminus Q^{\mathrm{LAG}}(0) = A \times \{0\}$ //all links at time 0

$((j, k), t') = ((a, b), 0)$ //link (a, b) at time 0

$S = \{C^1, C^2, C^3\}$

$D' = \{\{C^1, C^2, C^3\}\}$

$D = \{\{C^1, C^2, C^3\}\}$

$((j, k), t') = ((b, c), 0)$ //link (b, c) at time 0

$S = \{C^1, C^2, C^3\}$

$D' = \{\{C^1, C^2\}, \{C^3\}\}$

$D = \{\{C^1, C^2\}, \{C^3\}\}$

$((j, k), t') = ((a, c), 0)$ //link (a, c) at time 0

$S = \{C^1, C^2\}$

$D' = \{\{C^1, C^2\}\}$

$S = \{C^3\}$

$D' = \{\{C^3\}\}$

$D = \{\{C^1, C^2\}, \{C^3\}\}$

$\boldsymbol{EV}(1) = \{\{C^1, C^2\}, \{C^3\}\}$

$t = 2$ $\quad Q^{\text{LAG}}(2) = A \times \{0,1\}$ //all links at time 0 and 1

$Q^{\text{LAG}}(2) \setminus Q^{\text{LAG}}(1) = A \times \{1\}$ //all links at time 1

$((j, k), t') = ((a, b), 1)$ //link (a, b) at time 1

$S = \{C^1, C^2\}$

$D' = \{\{C^1, C^2\}\}$

$S = \{C^3\}$

$D' = \{\{C^3\}\}$

$D = \{\{C^1, C^2\}, \{C^3\}\}$

$((j, k), t') = ((b, c), 1)$ //link (b, c) at time 1

$S = \{C^1, C^2\}$

$D' = \{\{C^1\}, \{C^2\}\}$

$S = \{C^3\}$

$D' = \{\{C^3\}\}$

$D = \{\{C^1\}, \{C^2\}, \{C^3\}\}$

$((j, k), t') = ((a, c), 1)$ //link (a, c) at time 1

$S = \{C^1\}$

$D' = \{\{C^1\}\}$

$S = \{C^2\}$

$D' = \{\{C^2\}\}$

$S = \{C^3\}$

$D' = \{\{C^3\}\}$

$D = \{\{C^1\}, \{C^2\}, \{C^3\}\}$

$\boldsymbol{EV}(2) = \{\{C^1\}, \{C^2\}, \{C^3\}\}$

A summary of the results of constructing event collections is as follows.

$\boldsymbol{EV}(0) = \{\{C^1, C^2, C^3\}\}$

$\boldsymbol{EV}(1) = \{\{C^1, C^2\}, \{C^3\}\}$

$\boldsymbol{EV}(2) = \{\{C^1\}, \{C^2\}, \{C^3\}\}$

Step 2: Compute $e_{\mu^*}(j, 2, EV), \ \forall j \in N, \forall EV \in \boldsymbol{EV}(2)$

This step involves solving deterministic static shortest path problems with each single support point C^r, $r = 1, 2, 3$. Any classical shortest path algorithm can be used. In our small network, this can be done by observation. The results are listed in Table A-4 for nodes b and a only. In each result cell, the minimum expected travel time is given and the corresponding next node is in the parentheses. Since event collections at static time period are all singleton, $S_{\mu^*}(a, 2, C^i) = e_{\mu^*}(a, 2, \{C^i\})$, $i = 1, 2, 3$.

Step 3: $e_{\mu^*}(j, t, EV) \leftarrow +\infty, \ \forall j \in \{a, b\}, \forall t < 2, \forall EV \in \boldsymbol{EV}(t)$

$e_{\mu^*}(c, t, EV) \leftarrow 0, \ \forall t < 2, \forall EV \in \boldsymbol{EV}(t)$

Step 4: (main loop)

$t = 1$

$EV = \{C^1, C^2\}$

$(j, k) = (a, b)$

$$temp\_e = [1 + S_{\mu^*}(b, 1{+}1, C^1)] \Pr(C^1 \mid EV) + [1 + S_{\mu^*}(b, 1{+}1, C^2)] \, Pr(C^2 \mid EV)$$

$= (1 + 1) \times 0.5 + (1 + 2) \times 0.5 = 2.5 < +\infty$

$e_{\mu*}(a, 1, \{C^1, C^2\}) = 2.5,\ \ \mu^*(a, 1, \{C^1, C^2\}) = \text{node } b$

$S_{\mu*}(a, 1, C^1) = 1 + S_{\mu*}(b, 1{+}1, C^1) = 1 + 1 = 2$

$S_{\mu*}(a, 1, C^2) = 1 + S_{\mu*}(b, 1{+}1, C^2) = 1 + 2 = 3$

$(j, k) = (b, c)$

$temp\_e = [1 + S_{\mu*}(c, 1{+}1, C^1)]\ \Pr(C^1 \mid EV)$

$+ [2 + S_{\mu*}(c, 1{+}2, C^2)]\ \Pr(C^2 \mid EV)$

$= (1 + 0) \times 0.5 + (2 + 0) \times 0.5 = 1.5 < +\infty$

$e_{\mu*}(b, 1, \{C^1, C^2\}) = 1.5,\ \ \mu^*(b, 1, \{C^1, C^2\}) = \text{node } c$

$S_{\mu*}(b, 1, C^1) = 1 + S_{\mu*}(c, 1{+}1, C^1) = 1 + 0 = 1$

$S_{\mu*}(b, 1, C^2) = 2 + S_{\mu*}(c, 1{+}2, C^2) = 2 + 0 = 2$

$(j, k) = (a, c)$

$temp\_e = [3 + S_{\mu*}(c, 1{+}3, C^1)]\ \Pr(C^1 \mid EV)$

$+ [2 + S_{\mu*}(c, 1{+}2, C^2)]\ \Pr(C^2 \mid EV)$

$= (3 + 0) \times 0.5 + (2 + 0) \times 0.5 = 2.5 = e_{\mu*}(a, 1, \{C^1, C^2\})$

$EV = \{C^3\}$

$(j, k) = (a, b)$

$temp\_e = [2 + S_{\mu*}(b, 1{+}2, C^3)]\ \Pr(C^3 \mid EV)$

$= (2 + 1) \times 1 = 3 < +\infty$

$e_{\mu*}(a, 1, \{C^3\}) = 3,\ \ \mu^*(a, 1, \{C^3\}) = \text{node } b$

$S_{\mu*}(a, 1, C^3) = 2 + S_{\mu*}(b, 1{+}2, C^3) = 2 + 1 = 3$

$(j, k) = (b, c)$

$$temp\_e = [1 + S_{\mu^*}(c, 1{+}1, C^3)]\ \Pr(C^3 \mid EV)$$

$$= (1 + 0) \times 1 = 1 < +\infty$$

$$e_{\mu^*}(b, 1, \{C^3\}) = 1,\ \ \mu^*(b, 1, \{C^3\}) = \text{node } c$$

$$S_{\mu^*}(b, 1, C^3) = 1 + S_{\mu^*}(c, 1{+}1, C^3) = 1 + 0 = 1$$

$(j, k) = (a, c)$

$$temp\_e = [2 + S_{\mu^*}(c, 1{+}2, C^3)]\ \Pr(C^3 \mid EV)$$

$$= (2 + 0) \times 1 = 2 < e_{\mu^*}(a, 1, \{C^3\})$$

$$e_{\mu^*}(a, 1, \{C^3\}) = 2,\ \ \mu^*(a, 1, \{C^3\}) = \text{node } c$$

$$S_{\mu^*}(a, 1, C^3) = 2 + S_{\mu^*}(c, 1{+}2, C^3) = 2 + 0 = 0$$

Similar calculations can be carried out at time 0 and a summary of results is in Table A-4.

Table A-4 Results in the Static Period and at Time 1 and 0 in LAG variant

<table>
<tr><td colspan="4">Time $t \geq 2$, $\boldsymbol{EV}(2) = \{\{C^1\}, \{C^2\}, \{C^3\}\}$</td></tr>
<tr><td>EV / Node</td><td>$\{C^1\}$</td><td>$\{C^2\}$</td><td>$\{C^3\}$</td></tr>
<tr><td>b</td><td>1(node c)</td><td>2(node c)</td><td>1(node c)</td></tr>
<tr><td>a</td><td>2(node b)</td><td>2(node c)</td><td>2(node c)</td></tr>
<tr><td colspan="4">Time $t = 1$, $\boldsymbol{EV}(1) = \{\{C^1, C^2\}, \{C^3\}\}$</td></tr>
<tr><td>EV / Node</td><td colspan="2">$\{C^1, C^2\}$</td><td>$\{C^3\}$</td></tr>
<tr><td>b</td><td colspan="2">1.5(node c)</td><td>1(node c)</td></tr>
<tr><td>a</td><td colspan="2">2.5(node b)</td><td>2(node c)</td></tr>
<tr><td colspan="4">Time $t = 0$, $\boldsymbol{EV}(0) = \{\{C^1, C^2, C^3\}\}$</td></tr>
<tr><td>EV / Node</td><td colspan="3">$\{C^1, C^2, C^3\}$</td></tr>
<tr><td>b</td><td colspan="3">5/3(node c)</td></tr>
<tr><td>a</td><td colspan="3">7/3(node b)</td></tr>
</table>

4. PRE variant

Set: $t_0 = 0$. $T^* = K - 1 = 1$. Beyond $T^* = 1$, travelers receive no more information.

Step 1: Construct $\boldsymbol{EV}(t)$, $t = 0, 1$ by calling Generate_Event_Collection

$D = \{\{C^1, C^2, C^3\}\}$

$t = 0$ $Q^{\text{PRE}}(0) = A \times \{0\}$ //all links at time 0

$((j, k), t') = ((a, b), 0)$ //link (a, b) at time 0

$S = \{C^1, C^2, C^3\}$

$D' = \{\{C^1, C^2, C^3\}\}$

$D = \{\{C^1, C^2, C^3\}\}$

$((j, k), t') = ((b, c), 0)$ //link (b, c) at time 0

$S = \{C^1, C^2, C^3\}$

$D' = \{\{C^1, C^2\}, \{C^3\}\}$

$D = \{\{C^1, C^2\}, \{C^3\}\}$

$((j, k), t') = ((a, c), 0)$ //link (a, c) at time 0

$S = \{C^1, C^2\}$

$D' = \{\{C^1, C^2\}, \{C^3\}\}$

$S = \{C^3\}$

$D' = \{\{C^1, C^2\}, \{C^3\}\}$

$D = \{\{C^1, C^2\}, \{C^3\}\}$

$\boldsymbol{EV}(0) = \{\{C^1, C^2\}, \{C^3\}\}$

$t = 1$ $Q^{\text{PRE}}(1) = Q^{\text{PRE}}(0)$ //no more information

$Q^{\text{POI}}(1) \setminus Q^{\text{POI}}(0) = \varnothing$

$\boldsymbol{EV}(1) = \{\{C^1, C^2\}, \{C^3\}\}$

A summary of the results of constructing event collections is as follows.

$\boldsymbol{EV}(0) = \{\{C^1, C^2\}, \{C^3\}\}$

$\boldsymbol{EV}(1) = \{\{C^1, C^2\}, \{C^3\}\}$

Step 2: Compute $e_{\mu^*}(j, 1, EV),\ \forall j \in N, \forall EV \in \boldsymbol{EV}(1)$

This step involves solving deterministic static shortest path problems with each single support point C^r, $r = 1, 2, 3$. Any classical shortest path algorithm can be used. In our small network, this can be done by observation. The results are listed in Table A-5 for nodes b and a only. In each result cell, the minimum expected travel time is given and the corresponding next node is in the parentheses. $S_{\mu^*}(b, 1, C^1) = 1$, $S_{\mu^*}(b, 1, C^2) = 2$, $S_{\mu^*}(b, 1, C^3) = 1$, $S_{\mu^*}(a, 1, C^1) = 1{+}1 = 2$, $S_{\mu^*}(a, 1, C^2) = 1{+}2 = 3$, $S_{\mu^*}(a, 1, C^3) = 2$.

Step 3: $e_{\mu^*}(j, t, EV) \leftarrow +\infty,\ \forall j \in \{a,b\}, \forall t < 1, \forall EV \in \boldsymbol{EV}(t)$

$e_{\mu^*}(c, t, EV) \leftarrow 0,\ \forall t < 1, \forall EV \in \boldsymbol{EV}(t)$

Step 4: (main loop)

$t = 0$

$EV = \{C^1, C^2\}$

$(j, k) = (a, b)$

$$temp\_e = [1 + S_{\mu^*}(b, 0{+}1, C^1)]\ \Pr(C^1 \mid EV)$$

$$+ [1 + S_{\mu^*}(b, 0{+}1, C^2)]\ Pr(C^2 \mid EV)$$

$$= (1 + 1) \times 0.5 + (1 + 2) \times 0.5 = 2.5 < +\infty$$

$e_{\mu^*}(a, 0, \{C^1, C^2\}) = 2.5$, $\mu^*(a, 0, \{C^1, C^2\}) =$ node b

$S_{\mu^*}(a, 0, C^1) = 1 + S_{\mu^*}(b, 0{+}1, C^1) = 1 + 1 = 2$

$S_{\mu^*}(a, 0, C^2) = 1 + S_{\mu^*}(b, 0{+}1, C^2) = 1 + 2 = 3$

$(j, k) = (b, c)$

$temp\_e = [2 + S_{\mu^*}(c, 0{+}2, C^1)]\ \Pr(C^1 \mid EV)$

$+ [2 + S_{\mu^*}(c, 0{+}2, C^2)]\ \Pr(C^2 \mid EV)$

$= (2 + 0) \times 0.5 + (2 + 0) \times 0.5 = 2 < +\infty$

$e_{\mu^*}(b, 0, \{C^1, C^2\}) = 2,\ \ \mu^*(b, 0, \{C^1, C^2\}) = \text{node } c$

$S_{\mu^*}(b, 0, C^1) = 2 + S_{\mu^*}(c, 0{+}2, C^1) = 2 + 0 = 2$

$S_{\mu^*}(b, 0, C^2) = 2 + S_{\mu^*}(c, 0{+}2, C^2) = 2 + 0 = 2$

$(j, k) = (a, c)$

$temp\_e = [3 + S_{\mu^*}(c, 0{+}3, C^1)]\ \Pr(C^1 \mid EV)$

$+ [3 + S_{\mu^*}(c, 0{+}3, C^2)]\ \Pr(C^2 \mid EV)$

$= (3 + 0) \times 0.5 + (3 + 0) \times 0.5 = 3 > e_{\mu^*}(a, 0, \{C^1, C^2\})$

$EV = \{C^3\}$

$(j, k) = (a, b)$

$temp\_e = [1 + S_{\mu^*}(b, 0{+}1, C^3)]\ \Pr(C^3 \mid EV)$

$= (1 + 1) \times 1 = 2 < +\infty$

$e_{\mu^*}(a, 0, \{C^3\}) = 2,\ \ \mu^*(a, 0, \{C^3\}) = \text{node } b$

$S_{\mu^*}(a, 0, C^3) = 1 + S_{\mu^*}(b, 0{+}1, C^3) = 1 + 1 = 2$

$(j, k) = (b, c)$

$temp\_e = [1 + S_{\mu^*}(c, 0{+}1, C^3)]\ \Pr(C^3 \mid EV)$

$= (1 + 0) \times 1 = 1 < +\infty$

$e_{\mu^*}(b, 0, \{C^3\}) = 1,\ \mu^*(b, 0, \{C^3\}) = \text{node } c$

$S_{\mu^*}(b, 0, C^3) = 1 + S_{\mu^*}(c, 0{+}1, C^3) = 1 + 0 = 1$

$(j, k) = (a, c)$

$temp\_e = [2 + S_{\mu^*}(c, 0{+}2, C^3)]\ \Pr(C^3 \mid EV)$

$= (2 + 0) \times 1 = 2 = e_{\mu^*}(a, 0, \{C^3\})$

Similar calculations can be carried out for departure time $t_0 = 1$ and a summary of results is in Table A-5.

Table A-5 Results in the Static Period and at Time 0 in PRE variant

<table>
<tr><td colspan="4">Time $t \geq 1$, $\boldsymbol{EV}(1) = \{\{C^1\}, \{C^2\}, \{C^3\}\}$</td></tr>
<tr><td>EV / Node</td><td>$\{C^1\}$</td><td>$\{C^2\}$</td><td>$\{C^3\}$</td></tr>
<tr><td>b</td><td>1(node c)</td><td>2(node c)</td><td>1(node c)</td></tr>
<tr><td>a</td><td>2(node b)</td><td>2(node c)</td><td>2(node c)</td></tr>
<tr><td colspan="4">Time $t = 0$, $\boldsymbol{EV}(0) = \{\{C^1, C^2\}, \{C^3\}\}$</td></tr>
<tr><td>EV / Node</td><td colspan="2">$\{C^1, C^2\}$</td><td>$\{C^3\}$</td></tr>
<tr><td>b</td><td colspan="2">2(node c)</td><td>1(node c)</td></tr>
<tr><td>a</td><td colspan="2">2.5(node b)</td><td>2(node b)</td></tr>
</table>

5. RADIO variant

Set: $B = \{(a,b)\}$. $T^* = K - 1 = 1$. Beyond $T^* = 1$, travelers receive no more information.

Step 1: Construct $\boldsymbol{EV(t)}$, $t = 0, 1$ by calling Generate_Event_Collection

$D = \{\{C^1, C^2, C^3\}\}$

$t = 0$ $\quad Q^{\text{RADIO}}(0) = B \times \{0\}$ //link set B at time 0

$((j, k), t') = ((a, b), 0)$ //link (a, b) at time 0

$S = \{C^1, C^2, C^3\}$

$D' = \{\{C^1, C^2, C^3\}\}$

$D = \{\{C^1, C^2, C^3\}\}$

$\boldsymbol{EV}(0) = \{\{C^1, C^2, C^3\}\}$

$t = 1$ $Q^{\text{RADIO}}(1) = B \times \{0,1\}$ //link set B at time 0 and 1

$Q^{\text{POI}}(1) \setminus Q^{\text{POI}}(0) = B \times \{1\}$ //link set B at time 1

$((j, k), t') = ((a, b), 1)$ //link (a, b) at time 1

$S = \{C^1, C^2, C^3\}$

$D' = \{\{C^1, C^2\}, \{C^3\}\}$

$D = \{\{C^1, C^2\}, \{C^3\}\}$

$\boldsymbol{EV}(1) = \{\{C^1, C^2\}, \{C^3\}\}$

A summary of the results of constructing event collections is as follows.

$\boldsymbol{EV}(0) = \{\{C^1, C^2, C^3\}\}$

$\boldsymbol{EV}(1) = \{\{C^1, C^2\}, \{C^3\}\}$

Step 2: Compute $e_{\mu^*}(j, 1, EV)$, $\forall j \in N, \forall EV \in \boldsymbol{EV}(1)$

This step involves solving deterministic static shortest path problems with each single support point C^r, $r = 1, 2, 3$. Any classical shortest path algorithm can be used. In our small network, this can be done by observation. The results are listed in Table A-6 for nodes b and a only. In each result cell, the minimum expected travel time is given and the corresponding next node is in the parentheses. $S_{\mu^*}(b, 1, C^1) = 1$, $S_{\mu^*}(b, 1, C^2) = 2$, $S_{\mu^*}(b, 1, C^3) = 1$, $S_{\mu^*}(a, 1, C^1) = 1+1 = 2$, $S_{\mu^*}(a, 1, C^2) = 1+2 = 3$, $S_{\mu^*}(a, 1, C^3) = 2$.

Step 3: $e_{\mu^*}(j, t, EV) \leftarrow +\infty$, $\forall j \in \{a, b\}, \forall t < 1, \forall EV \in \boldsymbol{EV}(t)$

$e_{\mu^*}(c, t, EV) \leftarrow 0, \ \forall t < 1, \forall EV \in \boldsymbol{EV}(t)$

Step 4: (main loop)

$t = 0$

$EV = \{C^1, C^2, C^3\}$

$(j, k) = (a, b)$

$temp\_e = [1 + S_{\mu^*}(b, 0{+}1, C^1)] \Pr(C^1 \mid EV)$

$+ [1 + S_{\mu^*}(b, 0{+}1, C^2)] Pr(C^2 \mid EV)$

$+ [1 + S_{\mu^*}(b, 0{+}1, C^3)] Pr(C^3 \mid EV)$

$= (1 + 1) \times 1/3 + (1 + 2) \times 1/3 + (1 + 1) \times 1/3 = 7/3 < +\infty$

$e_{\mu^*}(a, 0, \{C^1, C^2, C^3\}) = 7/3, \ \mu^*(a, 0, \{C^1, C^2, C^3\}) = \text{node } b$

$S_{\mu^*}(a, 0, C^1) = 1 + S_{\mu^*}(b, 0{+}1, C^1) = 1 + 1 = 2$

$S_{\mu^*}(a, 0, C^2) = 1 + S_{\mu^*}(b, 0{+}1, C^2) = 1 + 2 = 3$

$S_{\mu^*}(a, 0, C^3) = 1 + S_{\mu^*}(b, 0{+}1, C^3) = 1 + 1 = 2$

$(j, k) = (b, c)$

$temp\_e = [2 + S_{\mu^*}(c, 0{+}2, C^1)] \Pr(C^1 \mid EV)$

$+ [2 + S_{\mu^*}(c, 0{+}2, C^2)] \Pr(C^2 \mid EV)$

$+ [1 + S_{\mu^*}(c, 0{+}1, C^3)] \Pr(C^3 \mid EV)$

$= (2 + 0) \times 1/3 + (2 + 0) \times 1/3 + (1 + 0) \times 1/3 = 5/3 < +\infty$

$e_{\mu^*}(b, 0, \{C^1, C^2, C^3\}) = 5/3, \ \mu^*(b, 0, \{C^1, C^2, C^3\}) = \text{node } c$

$S_{\mu^*}(b, 0, C^1) = 2 + S_{\mu^*}(c, 0{+}2, C^1) = 2 + 0 = 2$

$S_{\mu^*}(b, 0, C^2) = 2 + S_{\mu^*}(c, 0{+}2, C^2) = 2 + 0 = 2$

$S_{\mu^*}(b, 0, C^3) = 1 + S_{\mu^*}(c, 0{+}1, C^3) = 1 + 0 = 1$

$(j, k) = (a, c)$

$$temp\_e = [3 + S_{\mu^*}(c, 0{+}3, C^1)]\ \Pr(C^1 \mid EV)$$

$$+ [3 + S_{\mu^*}(c, 0{+}3, C^2)]\ \Pr(C^2 \mid EV)$$

$$+ [2 + S_{\mu^*}(c, 0{+}2, C^2)]\ \Pr(C^2 \mid EV)$$

$$= (3 + 0) \times 1/3 + (3 + 0) \times 1/3 + (2 + 0) \times 1/3$$

$$= 8/3 > e_{\mu^*}(a, 0, \{C^1, C^2, C^3\})$$

The summary of results at time 1 and 0 is in Table A-6.

Table A-6 Results in the Static Period and at Time 0 in RADIO variant

Time $t \geq 1$, $\boldsymbol{EV}(1) = \{\{C^1, C^2\}, \{C^3\}\}$		
Node \ EV	$\{C^1, C^2\}$	$\{C^3\}$
b	1.5(node *c*)	1(node *c*)
a	2.5(node *b*)	2(node *c*)
Time $t = 0$, $\boldsymbol{EV}(0) = \{\{C^1, C^2, C^3\}\}$		
Node \ EV	$\{C^1, C^2, C^3\}$	
b	5/3(node *c*)	
a	7/3(node *b*)	

The summary of the expected travel time from node *a* to node *c* for each departure time in all variants (POI, NOI, LAG, PRE, and RADIO) is in Table A-7.

Table A-7. The expected travel time from node a to node c in all variants

Departure time *t*	POI	NOI	LAG ($\Delta = 1$)	PRE	RADIO ($B = \{(a,b)\}$)
≥ 2	2	7/3	2	2	7/3
= 1	2	7/3	7/3	2	7/3
= 0	7/3	7/3	7/3	7/3	7/3

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