A Brief Introduction to Chiral Perturbation Theory

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A brief introduction to the subject of chiral perturbation theory (χpt) is presented, including a discussion of effective field theory and applications of χpt in the arena of purely mesonic interactions as well as in the πN sector.

1 Introduction

For the past three decades or so the holy grail sought by particle/nuclear knights has been to verify the correctness of the "ultimate" theory of strong interactions—quantum chromodynamics (QCD). The theory is, of course, deceptively simple on the surface. Indeed the form of the Lagrangian[1)]

$$\mathcal{L}_{\rm QCD} = \bar{q}(i\not{D} - m)q - \frac{1}{2}{\rm tr}\, G_{\mu\nu}G^{\mu\nu}\,. \tag{3}$$

is elegant, and the theory is renormalizable. So why are we still not satisfied? While at the very largest energies, asymptotic freedom allows the use of perturbative techniques, for those who are interested in making contact with low energy experimental findings there exist at least three fundamental difficulties:

i) QCD is written in terms of the "wrong" degrees of freedom—quarks and gluons—while low energy experiments are performed with hadronic bound states;

ii) the theory is non-linear due to gluon self interactions;

iii) the theory is one of strong coupling—$g^2/4\pi \sim 1$—so that perturbative methods are not practical.

Nevertheless, there has been a great deal of recent progress in making contact between theory and experiment using the technique of "effective field theory", which exploits the chiral symmetry of the QCD interaction. In order to understand how

[1)] Here the covariant derivative is

$$iD_\mu = i\partial_\mu - gA_\mu^a\frac{\lambda^a}{2}\,, \tag{1}$$

where λ^a (with $a = 1, \ldots, 8$) are the SU(3) Gell-Mann matrices, operating in color space, and the color-field tensor is defined by

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - g[A_\mu, A_\nu]\,, \tag{2}$$

this is accomplished, we first review symmetry breaking as well as the concept of effective interactions. Then we show how these ideas can be married via chiral perturbation theory and indicate a few contemporary physics applications.

2 Symmetry and Symmetry Breaking

The importance of symmetry in physics is associated with Noether's theorem, which states that associated with any symmetry in physics is a corresponding conservation law. Thus, for example,

i) translation invariance implies conservation of momentum;

ii) time translation invariance implies conservation of energy;

iii) rotational invariance implies conservation of angular momentum

These, however, are perhaps the only exact symmetries in nature. All others are broken in some way and there are in general only three types of symmetry breaking which can occur:

Explicit: The most familiar is *explicit* symmetry breaking, wherein the breaking occurs in the Lagrangian itself. As an example, consider the harmonic oscillator Lagrangian

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\omega^2 x^2 \tag{4}$$

which is clearly symmetric under spatial inversion—$x \to -x$. Correspondingly, the ground state (lowest energy) configuration—$x = 0$, which is found via $\frac{\partial L}{\partial x} = 0$—also shares this symmetry. On the other hand, if we add a linear potential (constant force) into the system, the Lagrangian becomes

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\omega^2 x^2 + \lambda x \tag{5}$$

The ground state is now given by $x = \lambda/m\omega^2$, which no longer is invariant under spatial inversion, but this is to be expected because of the *explicit* symmetry breaking term—λx—in the Lagrangian.

Spontaneous: Less familiar but still relatively common is *spontaneous* symmetry breaking, wherein the Lagrangian of a system possesses a symmetry but the ground state does not. A simple classical physics example of this is the case of a thin hoop of radius R immersed in a gravitational field and rotating about a vertical axis at fixed angular velocity ω. A bead which can move without friction along the hoop is then described by the Lagrangian[1]

$$L = \frac{1}{2}m(R^2\dot{\theta}^2 + \omega^2 R^2 \sin^2\theta) + mgR\cos\theta \tag{6}$$

where θ is the angle subtended by the bead from the bottom of the hoop. Clearly the Lagrangian is invariant under reflection—$L(\theta) \to L(-\theta)$—but the ground state configuration is given by

$$\frac{\partial L}{\partial \theta} = m\omega^2 R^2 \sin\theta(\cos\theta - \frac{g}{m\omega^2 R}) = 0, \tag{7}$$

which has the stable equilibirium solution $\theta = +\cos^{-1}\frac{g}{\omega^2 R}$ *or* $\theta = -\cos^{-1}\frac{g}{\omega^2 R}$ if $\frac{g}{\omega^2 R} < 1$. Obviously in this situation the ground state breaks the symmetry under $\theta \to -\theta$, even though the Lagrangian does not—this is an example of *spontaneous* symmetry breaking.

Anomalous: Finally, we consider *anomalous* or quantum mechanical symmetry breaking wherein the Lagrangian at the classical level is symmetric, but the symmetry is broken upon quantization. Obviously there are no classical physics examples of this phenomenon and, to my knowledge, every manifestation except one is in the arena of quantum field theory. The one example from ordinary quantum mechanics involves the breaking of scale invariance by a two-dimensional delta function potential[2]. To set the stage, first consider a *free* particle of mass m, which satisfies the time-independent Schrodinger equation

$$-\frac{1}{2m}\nabla^2\psi = \frac{k^2}{2m}\psi \tag{8}$$

A general partial wave solution can be written as

$$\psi_\ell(\vec{r}) = \frac{1}{r}\sum_\ell a_\ell \chi_\ell(r) P_\ell(\cos\theta) \tag{9}$$

where $\chi(r)$ satisfies the differential equation

$$\left(-\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2} + k^2\right)\chi_\ell(r) = 0 \tag{10}$$

Obviously there exists a scale invariance here—the Schrodinger equation is invariant under the scale transformation $r \to \lambda r, k \to k/\lambda$, a consequence of which is that the solution must be a function only of k *times* r and not of k or r alone. For example, a free particle solution can be written in the form

$$\psi(\vec{r}) = e^{ikz} = e^{ikr\cos\theta} \stackrel{r\to\infty}{\longrightarrow} \frac{1}{2ikr}\sum_\ell (2\ell+1)P_\ell(\cos\theta)(e^{ikr} - e^{-i(kr-\ell\pi)}) \tag{11}$$

Note that there exists a phase shift $\ell\pi$ of the outgoing spherical wave with respect to its incoming counterpart. This phase shift is, however, $k-independent$ as required by scale invariance. On the other hand if we include a potential $V(r)$ then the solution has the asymptotic form

$$\psi^{(+)}(\vec{r}) \stackrel{r\to\infty}{\longrightarrow} e^{ikz} + \frac{e^{ikr}}{r}f_k(\theta) \tag{12}$$

where the scattering amplitude $f_k(\theta)$ is given by

$$f_k(\theta) = \sum_\ell (2\ell+1)\frac{e^{2i\delta_\ell(k)} - 1}{2ik}P_\ell(\cos\theta) \tag{13}$$

In this case there exists an additional phase shift $\delta_\ell(k)$ in each partial wave, which breaks the scale invariance, but this is to be expected because of the presence of the (symmetry violating) potential.

In the case of two dimensions, one can write the scattering wave function in the asymptotic form

$$\psi^{(+)}(\vec{r}) \stackrel{r\to\infty}{\longrightarrow} e^{ikz} + \frac{1}{\sqrt{r}} e^{i(kr+\frac{\pi}{4})} f_k(\theta) \tag{14}$$

with scattering amplitude

$$f_k(\theta) = -i \sum_{m=-\infty}^{\infty} \frac{e^{2i\delta_m(k)} - 1}{\sqrt{2\pi k}} e^{im\theta} \tag{15}$$

where now we expand in terms of exponentials $e^{im\theta}$ instead of Legendre polynomials. What is special about two dimensions is that it is possible to introduce a *scale invariant* potential $V(\vec{r}) = g\delta^2(\vec{r})$. The associated differential cross section is found to be

$$\frac{d\sigma}{d\Omega} \stackrel{k\to\infty}{\longrightarrow} \frac{\pi}{2k\ln(\frac{k^2}{\mu^2})} \tag{16}$$

which corresponds to pure $m = 0$ scattering with an energy dependent phase shift

$$\cot\delta_0(k) = \frac{1}{\pi}\ln\frac{k^2}{\mu^2} - \frac{2}{g} \tag{17}$$

The scale invariance present at the classical level (no scattering cross section since we have a delta function potential) is then violated upon quantization.

Interestingly, QCD makes use of all three forms of symmetry breaking!

3 Effective Field Theory

The power of effective field theory is associated with the feature that there exist many situations in physics involving two scales, one heavy and one light. Then if one is working at energies small compared to the heavy scale, one can fully describe the interactions in terms of an "effective" picture, which is written only in terms of the light degrees of freedom, but which fully includes the influence of the heavy mass scale through virtual effects. A number of good review articles exist concerning the subject[3].

Before proceeding to QCD, however, it is useful to study this idea in the simpler context of ordinary quantum mechanics, in order to get familiar with the concept. Specifically, we examine the question of why the sky is blue, whose answer can be found in an analysis of the scattering of photons from the sun by atoms in the atmosphere—Compton scattering[4]. First we examine the problem using traditional quantum mechanics and consider elastic (Rayleigh) scattering from (for simplicity) single-electron (hydrogen) atoms. The appropriate Hamiltonian is then

$$H = \frac{(\vec{p} - e\vec{A})^2}{2m} + e\phi \tag{18}$$

and the leading—$\mathcal{O}(e^2)$—amplitude for Compton scattering is given by the Kramers-Heisenberg form

$$\begin{aligned}\text{Amp} &= -\frac{e^2/m}{\sqrt{2\omega_i 2\omega_f}}\left[\hat{\epsilon}_i\cdot\hat{\epsilon}_f^* + \frac{1}{m}\sum_n\left(\frac{\hat{\epsilon}_f^*\cdot <0|\vec{p}e^{-i\vec{q}_f\cdot\vec{r}}|n>\hat{\epsilon}_i\cdot <n|\vec{p}e^{i\vec{q}_i\cdot\vec{r}}|0>}{\omega_i+E_0-E_n}\right.\right.\\ &+ \left.\left.\frac{\hat{\epsilon}_i\cdot <0|\vec{p}e^{i\vec{q}_i\cdot\vec{r}}|n>\hat{\epsilon}_f^*\cdot <n|\vec{p}e^{-i\vec{q}_f\cdot\vec{r}}|0>}{E_0-\omega_f-E_n}\right)\right] \qquad (19)\end{aligned}$$

where $|0>$ represents the hydrogen ground state having binding energy E_0.

Here the leading component is the familiar ω-independent Thomson amplitude and would appear naively to lead to an energy-independent cross-section. However, this is not the case. Indeed, by expanding in powers of ω and using a few quantum mechanical tricks, then provided that the energy of the photon is much smaller than a typical excitation energy—as is the case for optical photons—the cross section can be written as

$$\frac{d\sigma}{d\Omega} = \lambda^2\omega^4|\hat{\epsilon}_f^*\cdot\hat{\epsilon}_i|^2\left(1+\mathcal{O}\left(\frac{\omega^2}{(\Delta E)^2}\right)\right) \qquad (20)$$

where

$$\lambda = \alpha_{em}\sum\frac{2|z_{n0}|^2}{E_n-E_0} \qquad (21)$$

is the atomic electric polarizability, $\alpha_{em} = e^2/4\pi$ is the fine structure constant, and $\Delta E \sim m\alpha_{em}^2$ is a typical hydrogen excitation energy. We note that $\alpha_{em}\lambda \sim a_0^2 \times \frac{\alpha_{em}}{\Delta E} \sim a_0^3$ is of order the atomic volume, as will be exploited below, and that the cross section itself has the characteristic ω^4 dependence which leads to the blueness of the sky—blue light scatters much more strongly than red[5].

Now while the above derivation is certainly correct, it requires somewhat detailed and lengthy quantum mechanical manipulations which obscure the relatively simple physics involved. One can avoid these problems by the use of effective field theory methods. The key point is that of scale. Since the incident photons have wavelengths $\lambda \sim$ 5000A much larger than the $\sim$ 1A atomic size, then at leading order the photon is insensitive to the presence of the atom, since the latter is electrically neutral. If χ represents the wavefunction of the atom, then the effective leading order Hamiltonian is simply

$$H_{eff}^{(0)} = \chi^*\left(\frac{\vec{p}^2}{2m}+e\phi\right)\chi \qquad (22)$$

and there is *no* interaction with the field. In higher orders, there *can* exist such atom-field interactions and this is where the effective Hamiltonian comes in to play. In order to construct the effective interaction, we demand certain general principles—this Hamiltonian must satisfy fundamental symmetry requirements. In particular H_{eff} must be gauge invariant, must be a scalar under rotations, and must be even under both parity and time reversal transformations. Also, since we are dealing with

Compton scattering, H_{eff} should be quadratic in the vector potential. Actually, from the requirement of gauge invariance it is clear that the effective interaction can utilize only the electric and magnetic fields

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial}{\partial t}\vec{A}, \qquad \vec{B} = \vec{\nabla} \times \vec{A} \tag{23}$$

since these are invariant under a gauge transformation

$$\phi \to \phi + \frac{\partial}{\partial t}\Lambda, \qquad \vec{A} \to \vec{A} - \vec{\nabla}\Lambda \tag{24}$$

while the vector and/or scalar potentials are not. The lowest order interaction then can involve only the rotational invariants $\vec{E}^2, \vec{B}^2$ and $\vec{E}\cdot\vec{B}$. However, under spatial inversion—$\vec{r} \to -\vec{r}$—electric and magnetic fields behave oppositely—$\vec{E} \to -\vec{E}$ while $\vec{B} \to \vec{B}$—so that parity invariance rules out any dependence on $\vec{E}\cdot\vec{B}$. Likewise under time reversal—$t \to -t$—we have $\vec{E} \to \vec{E}$ but $\vec{B} \to -\vec{B}$ so such a term is also ruled out by time reversal invariance. The simplest such effective Hamiltonian must then have the form

$$H_{eff}^{(1)} = \chi^*\chi[-\frac{1}{2}c_E\vec{E}^2 - \frac{1}{2}c_B\vec{B}^2] \tag{25}$$

(Terms involving time or spatial derivatives are much smaller.) We know from electrodynamics that $\frac{1}{2}(\vec{E}^2 + \vec{B}^2)$ represents the field energy per unit volume, so by dimensional arguments, in order to represent an energy in Eq. 25, c_E, c_B must have dimensions of volume. Also, since the photon has such a long wavelength, there is no penetration of the atom, so only classical scattering is allowed. The relevant scale must then be atomic size so that we can write

$$c_E = k_E a_0^3, \qquad c_B = k_B a_0^3 \tag{26}$$

where we anticipate $k_E, k_B \sim \mathcal{O}(1)$. Finally, since for photons with polarization $\hat{\epsilon}$ and four-momentum q_μ we identify $\vec{A}(x) = \hat{\epsilon}\exp(-iq\cdot x)$, then from Eq. 23, $|\vec{E}| \sim \omega$, $|\vec{B}| \sim |\vec{k}| = \omega$ and

$$\frac{d\sigma}{d\Omega} \propto |<f|H_{eff}|i>|^2 \sim \omega^4 a_0^6 \tag{27}$$

as found in the previous section via detailed calculation. Clearly the effective interaction method provides and efficient and insightful way in which to perform the calculation.

4 Application to QCD: Chiral Perturbation Theory

Now let's apply these ideas to the case of QCD. The relevant invariance in this case is "chiral symmetry." The idea of "chirality" is defined by the operators

$$\Gamma_{L,R} = \frac{1}{2}(1 \pm \gamma_5) = \frac{1}{2}\begin{pmatrix} 1 & \mp 1 \\ \mp 1 & 1 \end{pmatrix} \tag{28}$$

which project left- and right-handed components of the Dirac wavefunction via

$$\psi_L = \Gamma_L \psi \qquad \psi_R = \Gamma_R \psi \quad \text{with} \quad \psi = \psi_L + \psi_R \tag{29}$$

In terms of these chirality states the quark component of the QCD Lagrangian can be written as

$$\bar{q}(i \not{D} - m)q = \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R - \bar{q}_L m q_R - \bar{q}_R m q_L \tag{30}$$

The reason that these chirality states are called left- and right-handed can be seen by examining helicity eigenstates of the free Dirac equation. In the high energy (or massless) limit we note that

$$u(p) = \sqrt{\frac{E+m}{2E}} \begin{pmatrix} \chi \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi \end{pmatrix} \stackrel{E \gg m}{\longrightarrow} \sqrt{\frac{1}{2}} \begin{pmatrix} \chi \\ \vec{\sigma}\cdot\hat{p}\chi \end{pmatrix} \tag{31}$$

Left- and right-handed helicity eigenstates then can be identified as

$$u_L(p) \sim \sqrt{\frac{1}{2}} \begin{pmatrix} \chi \\ -\chi \end{pmatrix}, \qquad u_R(p) \sim \sqrt{\frac{1}{2}} \begin{pmatrix} \chi \\ \chi \end{pmatrix} \tag{32}$$

But

$$\begin{array}{ll} \Gamma_L u_L = u_L & \Gamma_R u_L = 0 \\ \Gamma_R u_R = u_R & \Gamma_L u_R = 0 \end{array} \tag{33}$$

so that in this limit chirality is identical with helicity—

$$\Gamma_{L,R} \sim \text{helicity!}$$

With this background, we now return to QCD and observe that in the limit as $m \to 0$

$$\mathcal{L}_{\rm QCD} \stackrel{m=0}{\longrightarrow} \bar{q}_L i \not{D} q_L + \bar{q}_R i \not{D} q_R \tag{34}$$

would be invariant under *independent* global left- and right-handed rotations

$$q_L \to \exp(i \sum_j \lambda_j \alpha_j) q_L, \qquad q_R \to \exp(i \sum_j \lambda_j \beta_j) q_R \tag{35}$$

(Of course, in this limit the heavy quark component is also invariant, but since $m_{c,b,t} >> \Lambda_{\rm QCD}$ it would be silly to consider this as even an approximate symmetry in the real world.) This invariance is called $SU(3)_L \bigotimes SU(3)_R$ or chiral $SU(3) \times SU(3)$. Continuing to neglect the light quark masses, we see that in a chiral symmetric world one would expect to have sixteen—eight left-handed and eight right-handed—conserved Noether currents

$$\bar{q}_L \gamma_\mu \frac{1}{2} \lambda_i q_L \,, \qquad \bar{q}_R \gamma_\mu \frac{1}{2} \lambda_i q_R \tag{36}$$

Equivalently, by taking the sum and difference, we would have eight conserved vector and eight conserved axial vector currents

$$V^i_\mu = \bar{q}\gamma_\mu \frac{1}{2}\lambda_i q, \qquad A^i_\mu = \bar{q}\gamma_\mu\gamma_5 \frac{1}{2}\lambda_i q \tag{37}$$

In the vector case, this is just a simple generalization of isospin (SU(2)) invariance to the case of SU(3). There exist *eight* (3^2-1) time-independent charges

$$F_i = \int d^3x V^i_0(\vec{x},t) \tag{38}$$

and there exist various supermultiplets of particles having identical spin-parity and (approximately) the same mass in the configurations—singlet, octet, decuplet, *etc.* demanded by SU(3) invariance.

If chiral symmetry were realized in the conventional fashion one would expect there also to exist corresponding nearly degenerate same spin but *opposite* parity states generated by the action of the time-independent axial charges $F^5_i = \int d^3x A^i_0(\vec{x},t)$ on these states. Indeed since

$$\begin{aligned} H|P\rangle &= E_P|P\rangle \\ H(Q_5|P\rangle) &= Q_5(H|P\rangle) = E_P(Q_5|P\rangle) \end{aligned} \tag{39}$$

we see that $Q_5|P\rangle$ must also be an eigenstate of the Hamiltonian with the same eigenvalue as $|P>$, which would seem to require the existence of parity doublets. However, experimentally this does not appear to be the case. Indeed although the $J^p = \frac{1}{2}^+$ nucleon has a mass of about 1 GeV, the nearest $\frac{1}{2}^-$ resonance lies nearly 600 MeV higher in energy. Likewise in the case of the 0^- pion, which has a mass of about 140 MeV, the nearest corresponding 0^+ state (if it exists at all) is nearly 700 MeV or so higher in energy.

The resolution of this apparent paradox is that the axial symmetry is spontaneously broken, in which case Goldstone's theorem requires the existence of eight massless pseudoscalar bosons, which couple derivatively to the rest of the universe. That way the state $Q_5|P>$ is equivalent to $|Pa>$, where a signifies one of these massless bosons, and in this way the problem of parity doublets is avoided. Of course, in the real world such massless 0^- states do not exist. This is because in the real world exact chiral invariance is broken by the small quark mass terms which we have neglected up to this point. Thus what we have in reality are eight very light (but not massless) pseudo-Goldstone bosons which make up the pseudoscalar octet. Since such states are lighter than their other hadronic counterparts, we have a situation wherein effective field theory can be applied—provided one is working at energy-momenta small compared to the ~ 1 GeV scale which is typical of hadrons, one can describe the interactions of the pseudoscalar mesons using an effective Lagrangian. Actually this has been known since the 1960's, where a good deal of work was done with a *lowest order* effective chiral Lagrangian[6]

$$\mathcal{L}_2 = \frac{F_\pi^2}{4}\mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{m_\pi^2}{4}F_\pi^2 \mathrm{Tr}(U + U^\dagger)\,. \tag{40}$$

where the subscript 2 indicates that we are working at two-derivative order or one power of chiral symmetry breaking—*i.e.* m_π^2. Here $U \equiv \exp(\sum \lambda_i\phi_i/F_\pi)$, where $F_\pi = 92.4$ is the pion decay constant. This Lagrangian is *unique*—if we expand to lowest order in $\vec{\phi}$

$$\begin{aligned}
\mathrm{Tr}\partial_\mu U\partial^\mu U^\dagger &= \mathrm{Tr}\frac{i}{F_\pi}\vec{\tau}\cdot\partial_\mu\vec{\phi}\times\frac{-i}{F_\pi}\vec{\tau}\cdot\partial^\mu\vec{\phi} = \frac{2}{F_\pi^2}\partial_\mu\vec{\phi}\cdot\partial^\mu\vec{\phi} \\
\mathrm{Tr}(U+U^\dagger) &= \mathrm{Tr}(2-\frac{1}{F_\pi^2}\vec{\tau}\cdot\vec{\phi}\vec{\tau}\cdot\vec{\phi}) = const. - \frac{2}{F_\pi^2}\vec{\phi}\cdot\vec{\phi}
\end{aligned} \tag{41}$$

we reproduce the free pion Lagrangian, as required.

At the SU(3) level, including a generalized chiral symmetry breaking term, there is even predictive power—one has

$$\frac{F_\pi^2}{4}\mathrm{Tr}\partial_\mu U\partial^\mu U^\dagger = \frac{1}{2}\sum_{j=1}^{8}\partial_\mu\phi_j\partial^\mu\phi_j + \cdots \tag{42}$$

$$\begin{aligned}
\frac{F_\pi^2}{4}\mathrm{Tr}2B_0m(U &+ U^\dagger) = \mathrm{const.} - \frac{1}{2}(m_u+m_d)B_0\sum_{j=1}^{3}\phi_j^2 \\
&- \frac{1}{4}(m_u+m_d+2m_s)B_0\sum_{j=4}^{7}\phi_j^2 - \frac{1}{6}(m_u+m_d+4m_s)B_0\phi_8^2 + \cdots
\end{aligned} \tag{43}$$

where B_0 is a constant and m is the quark mass matrix. We can then identify the meson masses as

$$\begin{aligned}
m_\pi^2 &= 2\hat{m}B_0 \\
m_K^2 &= (\hat{m}+m_s)B_0 \\
m_\eta^2 &= \frac{2}{3}(\hat{m}+2m_s)B_0\,,
\end{aligned} \tag{44}$$

where $\hat{m} = \frac{1}{2}(m_u+m_d)$ is the mean light quark mass. This system of three equations is *overdetermined*, and we find by simple algebra

$$3m_\eta^2 + m_\pi^2 - 4m_K^2 = 0\ . \tag{45}$$

which is the Gell-Mann-Okubo mass relation and is well-satisfied experimentally[7]. Expanding to fourth order in the fields we also reproduce the well-known and experimentally successful Weinberg $\pi\pi$ scattering lengths.

However, when one attempts to go beyond tree level, in order to unitarize the results, divergences arise and that is where the field stopped at the end of the 1960's. The solution, as proposed a decade later by Weinberg[8] and carried out by Gasser and Leutwyler[9], is to absorb these divergences in phenomenological constants, just as done in QED. What is different in this case is that the theory is nonrenormalizabile in that the forms of the divergences are *different* from the terms that one started with. That means that the form of the counterterms that are used to absorb these divergences must also be different, and Gasser and Leutwyler wrote down the most general counterterm Lagrangian that one can have at one loop, involving *four-derivative* interactions

$$\begin{aligned}\mathcal{L}_4 &= \sum_{i=1}^{10} L_i\mathcal{O}_i = L_1\left[\mathrm{tr}(D_\mu U D^\mu U^\dagger)\right]^2 + L_2\mathrm{tr}(D_\mu U D_\nu U^\dagger)\cdot\mathrm{tr}(D^\mu U D^\nu U^\dagger)\\ &+ L_3\mathrm{tr}(D_\mu U D^\mu U^\dagger D_\nu U D^\nu U^\dagger) + L_4\mathrm{tr}(D_\mu U D^\mu U^\dagger)\mathrm{tr}(\chi U^\dagger + U\chi^\dagger)\\ &+ L_5\mathrm{tr}\left(D_\mu U D^\mu U^\dagger\left(\chi U^\dagger + U\chi^\dagger\right)\right) + L_6\left[\mathrm{tr}\left(\chi U^\dagger + U\chi^\dagger\right)\right]^2\\ &+ L_7\left[\mathrm{tr}\left(\chi^\dagger U - U\chi^\dagger\right)\right]^2 + L_8\mathrm{tr}\left(\chi U^\dagger \chi U^\dagger + U\chi^\dagger U\chi^\dagger\right)\\ &+ iL_9\mathrm{tr}\left(F^L_{\mu\nu}D^\mu U D^\nu U^\dagger + F^R_{\mu\nu}D^\mu U^\dagger D^\nu U\right) + L_{10}\mathrm{tr}\left(F^L_{\mu\nu}UF^{R\mu\nu}U^\dagger\right)\end{aligned} \tag{46}$$

where the covariant derivative is defined via

$$D_\mu U = \partial_\mu U + \{A_\mu, U\} + [V_\mu, U] \tag{47}$$

the constants $L_i, i = 1, 2, \ldots 10$ are arbitrary (not determined from chiral symmetry alone) and $F^L_{\mu\nu}, F^R_{\mu\nu}$ are external field strength tensors defined via

$$F^{L,R}_{\mu\nu} = \partial_\mu F^{L,R}_\nu - \partial_\nu F^{L,R}_\mu - i[F^{L,R}_\mu, F^{L,R}_\nu], \qquad F^{L,R}_\mu = V_\mu \pm A_\mu. \tag{48}$$

Now just as in the case of QED the bare parameters L_i which appear in this Lagrangian are not physical quantities. Instead the experimentally relevant (renormalized) values of these parameters are obtained by appending to these bare values the divergent one-loop contributions—

$$L^r_i = L_i - \frac{\gamma_i}{32\pi^2}\left[\frac{-2}{\epsilon} - \ln(4\pi) + \gamma - 1\right] \tag{49}$$

By comparing predictions with experiment, Gasser and Leutwyler were able to determine empirical numbers for each of these ten parameters. Typical values are shown in Table 1, together with the way in which they were determined.

The important question to ask at this point is why stop at order four derivatives? Clearly if two-loop amplitudes from $\mathcal{L}_2$ or one-loop corrections from $\mathcal{L}_4$ are calculated, divergences will arise which are of six-derivative character. Why not include these? The answer is that the chiral procedure represents an expansion in energy-momentum. Corrections to the lowest order (tree level) predictions from one loop corrections from $\mathcal{L}_2$ or tree level contributions from $\mathcal{L}_4$ are $\mathcal{O}(E^2/\Lambda_\chi^2)$ where $\Lambda_\chi \sim 4\pi F_\pi \sim 1$ GeV is the chiral scale[10]. Thus chiral perturbation theory is a

Coefficient	Value	Origin
L_1^r	0.65 ± 0.28	$\pi\pi$ scattering
L_2^r	1.89 ± 0.26	and
L_3^r	-3.06 ± 0.92	$K_{\ell 4}$ decay
L_5^r	2.3 ± 0.2	F_K/F_π
L_9^r	7.1 ± 0.3	π charge radius
L_{10}^r	-5.6 ± 0.3	$\pi \to e\nu\gamma$

Table 1. Gasser-Leutwyler counterterms and the means by which they are determined.

low energy procedure. It is only to the extent that the energy is small compared to the chiral scale that it makes sense to truncate the expansion at the one-loop (four-derivative) level. Realistically this means that we deal with processes involving $E < 500$ MeV, and, for such reactions the procedure is found to work very well.

In fact Gasser and Leutwyler, besides giving the form of the $\mathcal{O}(p^4)$ chiral Lagrangian, have also performed the one loop integration and have written the result in a simple algebraic form. Users merely need to look up the result in their paper and, despite having ten phenomenological constants, the theory is quite predictive. An example is shown in Table 2, where predictions are given involving quantities which arise using just two of the constants—L_9, L_{10}. The table also reveals at least one intruguing problem—a solid chiral prediction, that for the charged pion polarizability, is possibly violated although this is not clear since there are three experimental results, only one of which is in disagreement. Clearing up this discrepancy should be a focus of future experimental work. Because of space limitations we shall have to be content to stop here, but interested readers can find applications to other systems in a number of review articles[16].

Reaction	Quantity	Theory	Experiment
$\pi^+ \to e^+\nu_e\gamma$	$h_V(m_\pi^{-1})$	0.027	0.029 ± 0.017[11]
$\pi^+ \to e^+\nu_e e^+e^-$	r_V/h_V	2.6	2.3 ± 0.6[11]
$\gamma\pi^+ \to \gamma\pi^+$	$(\alpha_E + \beta_M)\,(10^{-4}\,\mathrm{fm}^3)$	0	1.4 ± 3.1[12]
	$\alpha_E\,(10^{-4}\,\mathrm{fm}^3)$	2.8	6.8 ± 1.4[13]
			12 ± 20[14]
			2.1 ± 1.1[15]

Table 2. Chiral Predictions and data in radiative pion processes.

5 χpt and Nucleons

For applications invlolving nucleons it is important to note that the same ideas can be applied within the sector of meson-nucleon interactions, although with a bit more difficulty. Again much work has been done in this regard[17], but there remain important challenges[18]. Writing the lowest order chiral Lagrangian at the SU(2) level is straightforward—

$$\mathcal{L}_{\pi N} = \bar{N}(i\not{D} - m_N + \frac{g_A}{2}\not{u}\gamma_5)N \tag{50}$$

where g_A is the usual nucleon axial coupling in the chiral limit, the covariant derivative $D_\mu = \partial_\mu + \Gamma_\mu$ is given by

$$\Gamma_\mu = \frac{1}{2}[u^\dagger, \partial_\mu u] - \frac{i}{2}u^\dagger(V_\mu + A_\mu)u - \frac{i}{2}u(V_\mu - A_\mu)u^\dagger, \tag{51}$$

and u_μ represents the axial structure

$$u_\mu = iu^\dagger\nabla_\mu U u^\dagger \tag{52}$$

Expanding to lowest order, we find

$$\begin{aligned}\mathcal{L}_{\pi N} &= \bar{N}(i\not{\partial} - m_N)N + g_A\bar{N}\gamma^\mu\gamma_5\frac{1}{2}\vec{\tau}N\cdot(\frac{i}{F_\pi}\partial_\mu\vec{\pi} + 2\vec{A}_\mu)\\ &- \frac{1}{4F_\pi^2}\bar{N}\gamma^\mu\vec{\tau}N\cdot\vec{\pi}\times\partial_\mu\vec{\pi} + \ldots\end{aligned} \tag{53}$$

which yields the Goldberger-Treiman relation, connecting strong and weak couplings of the nucleon system[19]

$$F_\pi g_{\pi NN} = m_N g_A \tag{54}$$

Using the present best values for these quantities, we find

$$92.4\text{MeV}\times 13.05 = 1206\text{MeV} \quad \text{vs.} \quad 1189\text{MeV} = 939\text{MeV}\times 1.266 \tag{55}$$

and the agreement to better than two percent strongly confirms the validity of chiral symmetry in the nucleon sector. Actually the Goldberger–Treiman relation is only strictly true at the unphysical point $g_{\pi NN}(q^2 = 0)$ and one *expects* about a 1% discrepancy to exist. An interesting "wrinkle" in this regard is the use of the so-called Dashen-Weinstein relation, which accounts for lowest order SU(3) symmetry breaking effects, to predict this discrepancy in terms of corresponding numbers in the strangeness changing sector[20].

However, any realistic approach must also involve loop calculations as well as the use of a Foldy-Wouthuysen transformation in order to assure proper power counting. This approach goes under the name of heavy baryon chiral perturbation theory (HBχpt) and interested readers can find a compendium of such results in the review article[18]. For our purposes we shall have to be content to examine just one application—measurement of the *generalized* proton *polarizability* via virtual Compton scattering. First recall from section 3 the concept of polarizability as the constant of proportionality between an applied electric or magnetizing field and the resultant induced electric or magnetic dipole moment—

$$\vec{p} = 4\pi\alpha_E\vec{E}, \qquad \vec{\mu} = 4\pi\beta_M\vec{H} \tag{56}$$

The corresponding interaction energy is

$$E = -\frac{1}{2}4\pi\alpha_E E^2 - \frac{1}{2}4\pi\beta_M H^2 \tag{57}$$

which, upon quantization, leads to a proton Compton scattering cross section

$$\begin{aligned}\frac{d\sigma}{d\Omega} &= \left(\frac{\alpha_{em}}{m}\right)^2\left(\frac{\omega'}{\omega}\right)^2[\frac{1}{2}(1+\cos^2\theta) \\ &- \frac{m\omega\omega'}{\alpha_{em}}[\frac{1}{2}(\alpha_E+\beta_M)(1+\cos\theta)^2 + \frac{1}{2}(\alpha_E-\beta_M)(1-\cos\theta)^2 + \ldots].\end{aligned} \tag{58}$$

It is clear from Eq.(58) that, from careful measurement of the differential scattering cross section, extraction of these structure dependent polarizability terms is possible provided that

i) the energy is large enough that these terms are significant compared to the leading Thomson piece and

ii) that the energy is not so large that higher order corrections become important

and this has been accomplished recently at SAL and MAMI, yielding[21]

$$\alpha_E^{exp} = (12.1 \pm 0.8 \pm 0.5) \times 10^{-4}\mathrm{fm}^3, \qquad \beta_M^{exp} = (2.1 \mp 0.8 \mp 0.5) \times 10^{-4}\mathrm{fm}^3 \tag{59}$$

A chiral one loop calculation has also been performed by Bernard, Kaiser, and Meissner and yields a result in good agreement with these measurements[22]

$$\alpha_E^{theo} = 10\beta_M^{theo} = \frac{5e^2g_A^2}{384\pi^2F_\pi^2m_\pi} = 12.2 \times 10^{-4}\mathrm{fm}^3 \tag{60}$$

The idea of *generalized* polarizability can be understood from the analogous venue of electron scattering wherein measurement of the charge form factor as a function of $\vec{q}^2$ leads, when Fourier transformed, to a picture of the *local* charge density within the system. In the same way the virtual Compton scattering process—$\gamma^* + p \to \gamma + p$ can provide a measurement of the $\vec{q}^2$-dependent electric and magnetic polarizabilities, whose Fourier transform provides a picture of the *local polarization density* within the proton. On the theoretical side our group has performed a one loop HBχpt calculation and has produced a closed from expression for the predicted polarizabilities[23]

$$\bar{\alpha}_E^{(3)}(\bar{q}) = \frac{e^2 g_A^2 m_\pi}{64\pi^2 F_\pi^2} \frac{4 + 2\frac{\bar{q}^2}{m_\pi^2} - \left(8 - 2\frac{\bar{q}^2}{m_\pi^2} - \frac{\bar{q}^4}{m_\pi^4}\right)\frac{m_\pi}{\bar{q}} \arctan\frac{\bar{q}}{2m_\pi}}{\bar{q}^2\left(4 + \frac{\bar{q}^2}{m_\pi^2}\right)},$$

$$\bar{\beta}_M^{(3)}(\bar{q}) = \frac{e^2 g_A^2 m_\pi}{128\pi^2 F_\pi^2} \frac{-\left(4 + 2\frac{\bar{q}^2}{m_\pi^2}\right) + \left(8 + 6\frac{\bar{q}^2}{m_\pi^2} + \frac{\bar{q}^4}{m_\pi^4}\right)\frac{m_\pi}{\bar{q}} \arctan\frac{\bar{q}}{2m_\pi}}{\bar{q}^2\left(4 + \frac{\bar{q}^2}{m_\pi^2}\right)}. \quad (61)$$

In the electric case the structure is about what would be expected—a gradual falloff of $\alpha_E(\bar{q})$ from the real photon point with scale $r_p \sim 1/m_\pi$. However, the magnetic generalized polarizability is predicted to *rise* before this general falloff occurs—chiral symmetry requires the presence of both a paramagnetic and a diamagnetic component to the proton. Both predictions have received some support in a soon to be announced (and tour de force) MAMI measurement at $\bar{q} = 600$ MeV[24]. However, since parallel kinematics were employed in the experiment the desired generalized polarizabilities had to be identified on top of an enormous Bethe-Heitler background. A Bates measurement, to be performed by the OOPS collaboration next spring, will take place at $\bar{q} = 240$ MeV and will use the cababilities of the OOPS detector system to provide a 90 degree out of plane measurement, which will be *much* less sensitive to the Bethe-Heitler blowtorch. We anxiously await the results.

6 Summary

Above we have discussed some of the consequences of the feature that the $SU(3)_L \times SU(3)_R$ chiral symmetry of QCD is broken spontaneously in the axial sector, implying the existence of eight pseudo-Goldstone bosons which, because they are considerably lighter than the remaining hadronic spectrum, can be treated via an effective field theory—chiral perturbation theory. The predictions arising from such calculations are rigorous consequences of the underlying chiral symmetry of QCD and are subject to experimental tests using TAPS or other detectors at low energy machines. A taste of the predictive power is given above, but readers seeking a more substantial meal can find extensive summaries in various other sources[16].

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