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SL Zhu

SJ Puglia

BR Holstein
holstein@physics.umass.edu

MJ Ramsey-Musolf

Chiral Symmetry and the Parity-Violating $NN\pi$ Yukawa Coupling

Shi-Lin Zhu[a], S.J. Puglia[a], B.R. Holstein[c], M. J. Ramsey-Musolf[a,b]

[a] Department of Physics, University of Connecticut, Storrs, CT 06269 USA
[b] Theory Group, Thomas Jefferson National Accelerator Facility, Newport News, VA 23606 USA
[c] Department of Physics and Astronomy, University of Massachusetts, Amherst, MA 01003 USA

Abstract

We construct the complete $SU(2)$ parity-violating (PV) π, N, Δ interaction Lagrangian with one derivative, and calculate the chiral corrections to the PV Yukawa $NN\pi$ coupling constant h_π through $\mathcal{O}(1/\Lambda_\chi^3)$ in the leading order of heavy baryon expansion. We discuss the relationship between the renormalized h_π, the measured value of h_π, and the corresponding quantity calculated microscopically from the Standard Model four-quark PV interaction.

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I. INTRODUCTION

The parity-violating (PV) nucleon-nucleon interaction has been a subject of interest in nuclear and particle physics for some time. To date, PV observables generated by this interaction remain the only experimental windows on the $\Delta S = 0$, nonleptonic weak interaction. Since the 1970's, the PV NN interaction has been studied in a variety of processes, including $\vec{p}-p$ and $\vec{p}-$nucleus scattering, γ-decays of light nuclei, the scattering of epithermal neutrons from heavy nuclei, and atomic PV (for a review, see Refs. [1,2]. The on-going interest in the subject has spawned new PV experiments in few-body systems, including high-energy $\vec{p}-p$ scattering at COSY, $\vec{n}+p \to d+\gamma$ at LANSCE [3], $\gamma+d \to n+p$ at JLab [4], and the rotation of polarized neutrons in helium at NIST.

The theoretical analysis of these PV observables is complicated by the short range of the low-energy weak interaction. The Compton wavelength of the weak gauge bosons (~ 0.002 fm) implies that direct $W^\pm$ and Z exchange between nucleons is highly suppressed by the short-range repulsive core of the strong NN interaction. In the conventional framework, longer range PV effects arise from the exchange of light mesons between nucleons. One requires the exchange of the π, ρ and ω in order to saturate the seven spin-isospin channels associated with the quantum numbers of the underlying four-quark strangeness-conserving PV interaction, $\mathcal{H}_W^{PV}(\Delta S = 0)$ (henceforth, the $\Delta S = 0$ will be understood). These exchanges are parameterized by PV meson-nucleon couplings, h_M, whose values may be extracted from experiment. At present, there appear to be discrepancies between the values extracted from different experiments. In particular, the values of the isovector πNN coupling, h_π, and the isoscalar ρNN coupling, h_ρ^0 – as extracted from $\vec{p}-p$ scattering and the γ-decay of ^{18}F, do not appear to agree with the corresponding values implied by the anapole moment of ^{133}Cs as measured in atomic PV [5].

The origin of this discrepancy is not understood. One possibility is that the use of ρ and ω exchange to describe the short-range part of the PV NN interaction is inadequate. An alternate approach, using effective field theory (EFT), involves an expansion of the short-range PV NN interaction in a series of four-nucleon contact interactions whose coefficients are *a priori* unknown but in principle could be determined from experiment. The use of ρ and ω exchange amounts to adoption of a model – rather than the use of experiment – to determine the coefficients of the higher-derivative operators in this expansion. Whether or not the application of EFT to nuclear PV can yield a more self-consistent set of PV low-energy constants than the meson-exchange approach remains to be seen. A comprehensive analysis of nuclear PV observables using EFT has yet to be performed.

The least ambiguous element – shared by both approaches – involves the long-range π-exchange interaction. At leading order in the derivative expansion, the PV πNN interaction is a purely isovector, Yukawa interaction. The strength of this interaction is characterized by the same constant – h_π – in both the EFT and meson-exchange approaches. At the level of the Standard Model (SM), h_π is particularly sensitive to the neutral current component of $\mathcal{H}_W^{PV}$. In this respect, the result of ^{18}F PV γ-decay measurement is puzzling:

$$h_\pi = (0.73 \pm 2.3) g_\pi \quad , \tag{1}$$

where $g_\pi = 3.8 \times 10^{-8}$ gives the scale of the h_M in the absence of neutral currents [6]. This result is especially significant, since the relevant two-body nuclear parity-mixing matrix element can be obtained by isospin symmetry from the β-decay of ^{18}Ne [2]. The result in Eq. (1) is, thus, relatively insensitive to the nuclear model. Theoretical calculations of h_π starting from $\mathcal{H}_W^{PV}$ have been performed using SU(6)$_w$ symmetry and the quark model [7,8], the Skyrme model [9], and QCD sum rules [10]. As a benchmark for comparison with experiment, we refer the SU(6)$_w$/quark model analysis of Refs. [7,8]—hereafter referred to as DDH,FCDH *. These authors quote a "best value" and "reasonable range" for the h_M:

$$h_\pi(\text{best}) = 7 g_\pi \tag{2}$$

$$h_\pi(\text{range}) \; : \; (0 \rightarrow 30) g_\pi \quad . \tag{3}$$

where here the "best value" is more aptly described as an educated guess, while the "reasonable range" indicates a set of numbers such that theory would be very hard-pressed to explain were the experimental value not found to be within this band. Neverthess, the difference between the "best value" of Eq. (2) and the ^{18}F result would appear to call for an explanation, and in the following note we comment on a possible source of the discrepancy.

In general, the problem of relating the fundamental weak quark-quark interaction to the low-energy constants which parameterize hadronic matrix elements of that interaction is non-trivial. In the framework of EFT, one may define these constants at tree-level in the hadronic effective theory. The quantities extracted from experiment in the conventional analysis, however, are not the tree-level parameters, but rather renormalized couplings. Denoting the latter as h_π^{EFF}, one has

$$h_\pi^{EFF} = Z_N \sqrt{Z_\pi} h_\pi^{BARE} + \Delta h_\pi \quad , \tag{4}$$

where h_π^{BARE} is the coefficient of the leading-order, PV Yukawa interaction in the effective theory, $\sqrt{Z_N}$ and $\sqrt{Z_\pi}$ denotes chiral loop renormalizations of the nucleon and pion wavefunctions, respectively, and Δh_π denotes contributions from chiral loops and higher-dimension operators to the Yukawa interactions (only the finite parts of these couplings are implied; loop divergences are cancelled by the corresponding pole terms in h_π^{BARE} and the $Z_{N,\pi}$). At leading order in $1/\Lambda_\chi$, one has $Z_{N,\pi} = 1$, $\Delta h_\pi = 0$, and $h_\pi^{EFF} = h_\pi^{BARE}$. The renormalized coupling appears as the coefficient in the one-pion-exchange (OPE) PV NN potential

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*Note that although the DDH analysis used the symmetry group SU(6)$_w$ in order to connect weak vector meson and pion couplings the predictions relating pion couplings alone to hyperon decay data rely only SU(3).

$$\hat{H}^{OPE}_{PV} = i\frac{g_{NN\pi}h_\pi^{EFF}}{\sqrt{2}}\left(\frac{\tau_1\times\tau_2}{2}\right)_z(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\left[\frac{\vec{p}_1-\vec{p}_2}{2m_N}, f_\pi(r)\right] \quad , \tag{5}$$

where $g_{NN\pi}$ is the strong πNN coupling and $f_\pi(r) = \exp(-m_\pi r)/4\pi r$. Neglecting the effects of three-body PV forces and 2π-exchange interactions, it is h_π^{EFF} to which the result in Eq. (1) corresponds.

The relationship between h_π^{EFF} and the coupling obtained by computing $\langle N\pi|\mathcal{H}_W^{PV}|N\rangle$ in a microscopic model is not immediately transparent. In what follows, we make several observations about this relationship. We first show that $Z_N\sqrt{Z_\pi}$ and Δh_π are substantial, so that h_π^{EFF} differs significantly from h_π^{BARE}. To that end, we compute all of the chiral corrections to the PV Yukawa interaction through $\mathcal{O}(1/\Lambda_\chi^3)$, where $\Lambda_\chi = 4\pi F_\pi$. We work to leading order in $1/m_N$ in heavy baryon chiral perturbation theory (HBChPT). Of particular significance is the dependence of Δh_π on other low-energy constants parameterizing PV 2π production and the PV $N\to\pi\pi\Delta$ transition. We subsequently reexamine the SU(6)$_w$/quark model calculation of Refs. [7,8] and argue that most – if not all – of the chiral loop effects which renormalize h_π are not included in the microscopic calculation. Thus, the relationship between h_π^{EFF} and microscopic calculations remains ambiguous at best. This ambiguity is unlikely to be resolved until an unquenched lattice QCD calculation of h_π using light quarks becomes tenable. In the meantime, one should not necessarily view a discrepancy between the experimental value of h_π^{EFF} and microscopic model calculations as disturbing.

Our discussion of these observations is organized as follows. In Section 2 we summarize our conventions and notation, including the PV chiral Lagrangians relevant to h_π renormalization. Section 3 gives a discussion of the loop calculations. In Section 4 we comment on the scale of the loop corrections and provide simple estimates of some of the new PV low-energy constants appearing in the analysis. Section 5 gives our discussion of the relationship between h_π^{EFF} and the calculation of Refs. [7,8]. Section 6 summarizes our conclusions. Some technical details are relegated to the Appendices.

II. NOTATIONS AND CONVENTIONS

We follow standard HBChPT conventions [11,12] and introduce

$$\Sigma = \xi^2 \ , \quad \xi = \exp(\frac{i\pi}{F_\pi}) \ , \quad \pi = \frac{1}{2}\pi^a\tau^a \tag{6}$$

with $F_\pi = 92.4$ MeV being the pion decay constant. The chiral vector and axial vector currents are given by

$$\begin{aligned}
\mathcal{D}_\mu &= D_\mu + V_\mu \\
A_\mu &= -\frac{i}{2}(\xi D_\mu\xi^\dagger - \xi^\dagger D_\mu\xi) = -\frac{D_\mu\pi}{F_\pi} + O(\pi^3) \qquad (7)\\
V_\mu &= \frac{1}{2}(\xi D_\mu\xi^\dagger + \xi^\dagger D_\mu\xi) \quad . \qquad (8)
\end{aligned}$$

For the Δ, we use the isospurion formalism [13], treating the Δ field $T^i_\mu(x)$ as a vector spinor in both spin and isospin space with the constraint $\tau^i T^i_\mu(x) = 0$. The components of this field are

$$T^3_\mu = -\sqrt{\frac{2}{3}} \begin{pmatrix} \Delta^+ \\ \Delta^0 \end{pmatrix}_\mu \quad , \; T^+_\mu = \begin{pmatrix} \Delta^{++} \\ \Delta^+/\sqrt{3} \end{pmatrix}_\mu \quad , \; T^-_\mu = -\begin{pmatrix} \Delta^0/\sqrt{3} \\ \Delta^- \end{pmatrix}_\mu \quad . \tag{9}$$

The field T^i_μ also satisfies the constraints for the ordinary Schwinger-Rarita spin-$\frac{3}{2}$ field,

$$\gamma^\mu T^i_\mu = 0 \quad \text{and} \quad p^\mu T^i_\mu = 0 \quad . \tag{10}$$

We eventually convert to the heavy baryon expansion, in which case the latter constraint becomes $v^\mu T^i_\mu = 0$ with v_μ the heavy baryon velocity.

The relativistic parity-conserving (PC) Lagrangian for π, N, Δ interactions needed here is

$$\begin{aligned}
\mathcal{L}^{PC} = &\frac{F_\pi^2}{4} Tr D^\mu \Sigma D_\mu \Sigma^\dagger + \bar{N}(i\mathcal{D}_\mu\gamma^\mu - m_N)N + g_A \bar{N} A_\mu \gamma^\mu \gamma_5 N \\
&-\bar{T}^\mu_i[(i\mathcal{D}^{ij}_\alpha \gamma^\alpha - m_\Delta \delta^{ij})g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma^\lambda(i\mathcal{D}^{ij}_\alpha\gamma^\alpha - m_\Delta\delta^{ij})\gamma_\lambda\gamma^\nu \\
&+\frac{g_1}{2} g_{\mu\nu} A^{ij}_\alpha \gamma^\alpha\gamma_5 + \frac{g_2}{2}(\gamma_\mu A^{ij}_\nu + A^{ij}_\mu\gamma_\nu)\gamma_5 + \frac{g_3}{2}\gamma_\mu A^{ij}_\alpha\gamma^\alpha\gamma_5\gamma_\nu]T^\nu_j \\
&+g_{\pi N\Delta}[\bar{T}^\mu_i(g_{\mu\nu} + z_0\gamma_\mu\gamma_\nu)\omega^\nu_i N + h.c.] \quad ,
\end{aligned} \tag{11}$$

where $\omega^i_\mu = \text{tr}[\tau^i A_\mu]/2$ while D_μ and $\mathcal{D}_\mu$ are the gauge and chiral covariant derivatives, respectively. Explicit expressions for the fields and the transformation properties can be found in [14]. Here, z_0 is an off-shell parameter, which is not relevant in the present work [13].

In order to obtain proper chiral counting for the nucleon, we employ the conventional heavy baryon expansion of $\mathcal{L}^{PC}$, and in order to cosistently include the Δ we follow the small scale expansion proposed in [13]. In this approach energy-momenta and the delta and nucleon mass difference δ are both treated as small expansion parameters in chiral power counting. The leading order vertices in this framework can be obtained via $P_+\Gamma P_+$ where Γ is the original vertex in the relativistic Lagrangian and

$$P_\pm = \frac{1 \pm \not{v}}{2} \quad . \tag{12}$$

are projection operators for the large, small components of the Dirac wavefunction respectively. We collect some of the relevant terms below:

$$\begin{aligned}
\mathcal{L}^{PC}_v = &\bar{N}[iv \cdot D + 2g_A S \cdot A]N - i\bar{T}^\mu_i[iv \cdot D^{ij} - \delta^{ij}\delta + g_1 S \cdot A^{ij}]T^j_\mu \\
&+g_{\pi N\Delta}[\bar{T}^\mu_i \omega^i_\mu N + \bar{N}\omega^{i\dagger}_\mu T^\mu_i]
\end{aligned} \tag{13}$$

where S_μ is the Pauli-Lubanski spin operator and $\delta \equiv m_\Delta - m_N$.

The PV analog of Eq. (11) can be constructed using the chiral fields $X^a_{L,R}$ defined as [15]:

$$X^a_L = \xi^\dagger \tau^a \xi \ , \quad X^a_R = \xi \tau^a \xi^\dagger \ , \quad X^a_\pm = X^a_L \pm X^a_R \quad . \tag{14}$$

We find it convenient to follow the convention in Ref. [15] and separate the PV Lagrangian into its various isospin components.

The hadronic weak interaction has the form

$$\mathcal{H}_W = \frac{G_\mu}{\sqrt{2}} J_\lambda J^{\lambda\,\dagger} \ + \ \text{h.c.} \quad , \tag{15}$$

where J_λ denotes either a charged or neutral weak current built out of quarks. In the Standard Model, the strangeness conserving charged currents are pure isovector, whereas the neutral currents contain both isovector and isoscalar components. Consequently, $\mathcal{H}_W$ contains $\Delta I = 0, 1, 2$ pieces and these channels must all be accounted for in any realistic hadronic effective theory.

We quote here the relativistic Lagrangians, but employ the heavy baryon projections, as described above, in computing loops. It is straightforward to obtain the corresponding heavy baryon Lagrangians from those listed below, so we do not list the PV heavy baryon terms below. For the πN sector we have

$$\mathcal{L}^{\pi N}_{\Delta I=0} = h^0_V \bar{N} A_\mu \gamma^\mu N \tag{16}$$

$$\begin{aligned}\mathcal{L}^{\pi N}_{\Delta I=1} = {} & \frac{h^1_V}{2} \bar{N} \gamma^\mu N Tr(A_\mu X^3_+) - \frac{h^1_A}{2} \bar{N} \gamma^\mu \gamma_5 N Tr(A_\mu X^3_-) \\ & - \frac{h_\pi}{2\sqrt{2}} F_\pi \bar{N} X^3_- N\end{aligned} \tag{17}$$

$$\begin{aligned}\mathcal{L}^{\pi N}_{\Delta I=2} = {} & h^2_V \mathcal{I}^{ab} \bar{N} [X^a_R A_\mu X^b_R + X^a_L A_\mu X^b_L] \gamma^\mu N \\ & - \frac{h^2_A}{2} \mathcal{I}^{ab} \bar{N} [X^a_R A_\mu X^b_R - X^a_L A_\mu X^b_L] \gamma^\mu \gamma_5 N \ ,\end{aligned} \tag{18}$$

where $\mathcal{I}^{ab}$ is a matrix coupling the $X^{a,b}$ to $I = 2, I_3 = 0$. The above Lagrangian was first given by Kaplan and Savage (KS) [15]. However, the coefficients used in our work are slightly different from those of Ref. [15] since our definition of A_μ differs by an overall phase.

The term proportional to h_π contains no derivatives. At leading-order in $1/F_\pi$, it yields the PV $NN\pi$ Yukawa coupling traditionally used in meson-exchange models for the PV NN interaction [7,2]. Unlike the PV Yukawa interaction, the vector and axial vector terms in Eqs. (16-18) contain derivative interactions. The terms containing h^1_A and h^2_A start off with $NN\pi\pi$ interactions, while all the other terms start off as $NN\pi$. Such derivative interactions have not been included in conventional analyses of nuclear and hadronic PV experiments. Consequently, the experimental constraints on the low-energy constants h^i_V, h^i_A are unknown.

It is useful to list the first few terms obtained by expanding the Lagrangians in Eqs. (16-18) in $1/F_\pi$. For the present purposes, the following terms are needed:

$$\mathcal{L}^{\pi NN}_{\rm Yukawa} = -ih_\pi(\bar{p}n\pi^+ - \bar{n}p\pi^-)[1 - \frac{1}{3F_\pi^2}(\pi^+\pi^- + \frac{1}{2}\pi^0\pi^0)] \tag{19}$$

$$\mathcal{L}^{\pi NN}_V = -\frac{h_V^0 + 4/3h_V^2}{\sqrt{2}F_\pi}[\bar{p}\gamma^\mu nD_\mu\pi^+ + \bar{n}\gamma^\mu pD_\mu\pi^-] \tag{20}$$

$$\begin{aligned}\mathcal{L}^{\pi NN}_A &= i\frac{h_A^1 + h_A^2}{F_\pi^2}\bar{p}\gamma^\mu\gamma_5 p(\pi^+D_\mu\pi^- - \pi^-D_\mu\pi^+) \\ &\quad + i\frac{h_A^1 - h_A^2}{F_\pi^2}\bar{n}\gamma^\mu\gamma_5 n(\pi^+D_\mu\pi^- - \pi^-D_\mu\pi^+) \\ &\quad + i\frac{\sqrt{2}h_A^2}{F_\pi^2}\bar{p}\gamma^\mu\gamma_5 n\pi^+D_\mu\pi^0 - i\frac{\sqrt{2}h_A^2}{F_\pi^2}\bar{p}\gamma^\mu\gamma_5 n\pi^+D_\mu\pi^0 \; .\end{aligned} \tag{21}$$

For the PV πNN Yukawa coupling we have also kept terms with three pions.

The corresponding PV Lagrangians involving a $N\to\Delta$ transition are somewhat more complicated. We relegate the complete expressions to Appendix A, and give here only the leading terms required for our calculation. As noted in Ref. [14], the one-pion $\pi N\Delta$ PV Lagrangian vanishes at leading order in the heavy baryon expansion. The two-pion terms are

$$\begin{aligned}\mathcal{L}^{\pi N\Delta}_A &= -\frac{ih_A^{p\Delta^{++}\pi^-\pi^0}}{F_\pi^2}\bar{p}\Delta^{++}_\mu D^\mu\pi^-\pi^0 - \frac{ih_A^{p\Delta^{++}\pi^0\pi^-}}{F_\pi^2}\bar{p}\Delta^{++}_\mu D^\mu\pi^0\pi^- \\ &\quad -\frac{ih_A^{p\Delta^{+}\pi^0\pi^0}}{F_\pi^2}\bar{p}\Delta^{+}_\mu D^\mu\pi^0\pi^0 - \frac{ih_A^{p\Delta^{+}\pi^+\pi^-}}{F_\pi^2}\bar{p}\Delta^{+}_\mu D^\mu\pi^+\pi^- \\ &\quad -\frac{ih_A^{p\Delta^{+}\pi^-\pi^+}}{F_\pi^2}\bar{p}\Delta^{+}_\mu D^\mu\pi^-\pi^+ - \frac{ih_A^{p\Delta^{0}\pi^+\pi^0}}{F_\pi^2}\bar{p}\Delta^{0}_\mu D^\mu\pi^+\pi^0 \\ &\quad -\frac{ih_A^{p\Delta^{0}\pi^0\pi^+}}{F_\pi^2}\bar{p}\Delta^{0}_\mu D^\mu\pi^0\pi^+ - \frac{ih_A^{p\Delta^{-}\pi^+\pi^+}}{F_\pi^2}\bar{p}\Delta^{-}_\mu D^\mu\pi^+\pi^+ \\ &\quad -\frac{ih_A^{n\Delta^{++}\pi^-\pi^-}}{F_\pi^2}\bar{n}\Delta^{++}_\mu D^\mu\pi^-\pi^- - \frac{ih_A^{n\Delta^{+}\pi^-\pi^0}}{F_\pi^2}\bar{n}\Delta^{+}_\mu D^\mu\pi^-\pi^0 \\ &\quad -\frac{ih_A^{n\Delta^{+}\pi^0\pi^-}}{F_\pi^2}\bar{n}\Delta^{+}_\mu D^\mu\pi^0\pi^- - \frac{ih_A^{n\Delta^{0}\pi^0\pi^0}}{F_\pi^2}\bar{n}\Delta^{0}_\mu D^\mu\pi^0\pi^0 \\ &\quad -\frac{ih_A^{n\Delta^{0}\pi^+\pi^-}}{F_\pi^2}\bar{n}\Delta^{0}_\mu D^\mu\pi^+\pi^- - \frac{ih_A^{n\Delta^{0}\pi^-\pi^+}}{F_\pi^2}\bar{n}\Delta^{0}_\mu D^\mu\pi^-\pi^+ \\ &\quad -\frac{ih_A^{n\Delta^{-}\pi^+\pi^0}}{F_\pi^2}\bar{n}\Delta^{-}_\mu D^\mu\pi^+\pi^0 - \frac{ih_A^{n\Delta^{-}\pi^0\pi^+}}{F_\pi^2}\bar{n}\Delta^{-}_\mu D^\mu\pi^0\pi^+ \quad + \text{h.c.}\;,\end{aligned} \tag{22}$$

where the couplings $h_A^{p\Delta^{++}\pi^-\pi^0}$ are defined in terms of the various SU(2) PV low-energy constants in Appendix A.

The PV $\pi\Delta\Delta$ Lagrangians, also listed in Appendix A, contain terms analogous to the Yukawa, V, and A terms in Eqs. (16-18). Since we compute corrections up to one-loop order only, and since the initial and final states are nucleons, the PV $\pi\pi\Delta\Delta$ terms (A-type) are not relevant here. The leading, single-π Yukawa and V-type interactions are

$$\begin{aligned}\mathcal{L}^{\pi\Delta\Delta}_{\text{Yukawa}} = & -i\frac{h_\Delta}{\sqrt{3}}(\bar{\Delta}^{++}\Delta^{+}\pi^{+} - \bar{\Delta}^{+}\Delta^{++}\pi^{-}) \\ & -i\frac{h_\Delta}{\sqrt{3}}(\bar{\Delta}^{0}\Delta^{-}\pi^{+} - \bar{\Delta}^{-}\Delta^{0}\pi^{-}) \\ & -i\frac{2h_\Delta}{3}(\bar{\Delta}^{+}\Delta^{0}\pi^{+} - \bar{\Delta}^{0}\Delta^{+}\pi^{-})\end{aligned} \tag{23}$$

$$\begin{aligned}\mathcal{L}^{\pi\Delta\Delta}_{V} = & -\frac{h_V^{\Delta^{++}\Delta^{+}}}{F_\pi}(\bar{\Delta}^{++}\gamma_\mu\Delta^{+}D^\mu\pi^{+} + \bar{\Delta}^{+}\gamma_\mu\Delta^{++}D^\mu\pi^{-}) \\ & -\frac{h_V^{\Delta^{+}\Delta^{0}}}{F_\pi}(\bar{\Delta}^{+}\gamma_\mu\Delta^{0}D^\mu\pi^{+} + \bar{\Delta}^{0}\gamma_\mu\Delta^{+}D^\mu\pi^{-}) \\ & -\frac{h_V^{\Delta^{0}\Delta^{-}}}{F_\pi}(\bar{\Delta}^{0}\gamma_\mu\Delta^{-}D^\mu\pi^{+} + \bar{\Delta}^{-}\gamma_\mu\Delta^{0}D^\mu\pi^{-})\end{aligned} \tag{24}$$

where the coefficients are given in Appendix A.

One may ask whether there exist additional PV effective interactions that could contribute at the order to which we work. In the pionic sector there exists one CP-conserving, PV Lagrangian:

$$\mathcal{L}_\pi^{PV} = \epsilon_{ijk}\omega_\mu^i\omega_\nu^j(D^\mu\omega_k^\nu - D^\nu\omega_k^\mu) \quad . \tag{25}$$

At leading order in $1/F_\pi$, $\mathcal{L}_\pi$ contains five pions. Its lowest order contribution appears at two-loop order at best, so we do not consider it here.

Similarly, one may consider possible contributions from two-derivative operators. There exists one CP-conserving, PV operator:

$$\frac{1}{\Lambda_\chi}\bar{N}\sigma^{\mu\nu}[D_\mu A_\nu - D_\nu A_\mu]N \quad . \tag{26}$$

There exist three independent PC, two-derivative operators [16]. For example, one may choose the following three:

$$\frac{1}{\Lambda_\chi}\bar{N}i\gamma_5 D_\mu A^\mu N \ , \tag{27}$$

$$\frac{1}{\Lambda_\chi}\bar{N}A^\mu A_\mu N \ , \tag{28}$$

$$\frac{1}{\Lambda_\chi}\bar{N}\sigma^{\mu\nu}[A_\mu, A_\nu]N \ . \tag{29}$$

As we discuss in Appendix B, none of the two-derivative operators in Eqs. (26) - (29) contribute to the renormalization of h_π at the order to which we work in the present analysis.

III. THE LOOP CORRECTIONS

The leading order loop corrections to the Yukawa interaction of Eq. (19) are generated by the diagrams of Figs. 1-2. As we discuss in Appendix B, the contributions from many of the diagrams which nominally renormalize h_π vanish at the order at which we truncate. In particular, none of the vector (V-type) πNN and $\pi\Delta\Delta$ terms contribute to this order. In what follows, we discuss only the non-vanishing Yukawa and A-type contributions. Details regarding the vanishing of the other contributions appear in Appendix B. Following the conventional practice, we regulate the loop integrals using dimensional regularization. The pole terms proportional to $1/D-4$ are cancelled by appropriate counterterms. We identify only the terms nonanalytic in quark masses with the loops. All other analytic terms are indistinguishable from finite parts of the corresponding counterterms.

The nonvanishing contribution from Fig. 1(a) arises from the insertion of the 3π part of the Yukawa interaction of Eq. (17). The nonanalytic term is

$$iM_{(a)} = \frac{5}{6}\frac{m_\pi^2}{\Lambda_\chi^2}\ln(\frac{\mu}{m_\pi})^2 h_\pi \tau^+ \ , \tag{30}$$

where $\Lambda_\chi = 4\pi F_\pi$ and μ is the subtraction scale introduced in dimensional regularization. For simplicity, we show here only the contributions for $n \to p\pi^-$. The terms for $p \to n\pi^+$ are equal in magnitude and opposite in sign since it is the hermitian conjugate of the $n \to p\pi^-$ piece. This property holds to all orders of chiral expansion.

The nonvanishing contribution from Fig. 1 (b) arises from strong vertex correction to the leading order πNN Yukawa interaction:

$$iM_{(b)} = \frac{3}{4}g_A^2\frac{m_\pi^2}{\Lambda_\chi^2}\ln(\frac{\mu}{m_\pi})^2 h_\pi \tau^+ \ . \tag{31}$$

The terms in Figs. 1(c1)-(c2) are generated by the PV axial $\pi\pi NN$ couplings proportional to the h_A^i. We have

$$iM_{(c1)+(c2)} = 2\sqrt{2}\pi g_A \frac{m_\pi^3}{F_\pi \Lambda_\chi^2} h_A^1 \tau^+ \ . \tag{32}$$

The contribution from h_A^2 to these two diagrams cancels out, leaving only the dependence on h_A^1. We note that although this term is propotional to $m_q^{3/2}$ and, thus, nominally suppressed, the coefficient of h_A^1 is fortuitously large ($\sim 1/4$). The two pion vertex in FIG. 1 (d1)-(d2) comes from the chiral connection V_μ:

$$iM_{(d1)+(d2)} = -\frac{1}{2}\frac{m_\pi^2}{\Lambda_\chi^2}\ln(\frac{\mu}{m_\pi})^2 h_\pi \tau^+ \ . \tag{33}$$

The leading contribution involving Δ intermediate states arises from Fig. 2(a). The corresponding amplitude receives contributions from three different isospin combinations for the Δ intermediate states. Their sum reads

$$iM_{2(a)} = -\frac{20}{9}\frac{g_{\pi N\Delta}^2 h_\Delta}{\Lambda_\chi^2}[(2\delta^2 - m_\pi^2)\ln(\frac{\mu}{m_\pi})^2 - 4\delta\sqrt{\delta^2 - m_\pi^2}\ln\frac{\delta + \sqrt{\delta^2 - m_\pi^2}}{m_\pi}]\tau^+ \ . \quad (34)$$

The corrections generated by the PV $\pi\pi N\Delta$ vertices are

$$iM_{2(b1)+2(b2)} = \frac{2}{3}\frac{g_{\pi N\Delta}}{F_\pi \Lambda_\chi^2}[(\delta^2 - \frac{3}{2}m_\pi^2)\delta\ln(\frac{\mu}{m_\pi})^2 - 2(\delta^2 - m_\pi^2)^{3/2}\ln\frac{\delta + \sqrt{\delta^2 - m_\pi^2}}{m_\pi}]h_A^\Delta \tau^+ \ , \quad (35)$$

where h_A^Δ is defined as

$$h_A^\Delta = \frac{1}{\sqrt{3}}(h_A^{n\Delta^0\pi^+\pi^-} + h_A^{p\Delta^+\pi^-\pi^+}) + \sqrt{\frac{2}{3}}(h_A^{n\Delta^+\pi^0\pi^-} - h_A^{p\Delta^0\pi^0\pi^+}) - h_A^{n\Delta^{++}\pi^-\pi^-} - h_A^{p\Delta^-\pi^+\pi^+} \ . \quad (36)$$

Summing all the non-vanishing loop contributions yields the following expression for Δh_π:

$$\begin{aligned} \Delta h_\pi = & \frac{1}{3}\frac{m_\pi^2}{\Lambda_\chi^2}\ln(\frac{\mu}{m_\pi})^2 h_\pi + \frac{3}{4}g_A^2\frac{m_\pi^2}{\Lambda_\chi^2}\ln(\frac{\mu}{m_\pi})^2 h_\pi + 2\sqrt{2}\pi g_A\frac{m_\pi^3}{F_\pi\Lambda_\chi^2}h_A^1 \\ & -\frac{20}{9}\frac{g_{\pi N\Delta}^2 h_\Delta}{\Lambda_\chi^2}[(2\delta^2 - m_\pi^2)\ln(\frac{\mu}{m_\pi})^2 - 4\delta\sqrt{\delta^2 - m_\pi^2}\ln\frac{\delta + \sqrt{\delta^2 - m_\pi^2}}{m_\pi}] \\ & +\frac{2}{3}\frac{g_{\pi N\Delta}}{F_\pi\Lambda_\chi^2}[(\delta^2 - \frac{3}{2}m_\pi^2)\delta\ln(\frac{\mu}{m_\pi})^2 - 2(\delta^2 - m_\pi^2)^{3/2}\ln\frac{\delta + \sqrt{\delta^2 - m_\pi^2}}{m_\pi}]h_A^\Delta \end{aligned} \quad (37)$$

The final nonvanishing corrections arise from N and π wavefunction renormalization. These corrections, which have been computed previously [17], generate deviations from unity of Z_N and $\sqrt{Z_\pi}$ appearing in the expression for h_π^{EFF} in Eq. (4). In the case of Z_N, the nonvanishing contributions arise from Figs. 1(e1)-(e2) and 2(c1)-(c2):

$$\begin{aligned} Z_N - 1 = & \frac{9}{4}g_A^2\frac{m_\pi^2}{\Lambda_\chi^2}\ln(\frac{\mu}{m_\pi})^2 - 4g_{\pi N\Delta}^2[\frac{2\delta^2 - m_\pi^2}{\Lambda_\chi^2}\ln(\frac{\mu}{m_\pi})^2 \\ & - 4\frac{\delta\sqrt{\delta^2 - m_\pi^2}}{\Lambda_\chi^2}\ln\frac{\delta + \sqrt{\delta^2 - m_\pi^2}}{m_\pi}] \ . \end{aligned} \quad (38)$$

The pion's wavefunction renormalization arises from Fig. 2 (k) [18]:

$$\sqrt{Z_\pi} - 1 = -\frac{1}{3}\left(\frac{m_\pi}{\Lambda_\chi}\right)^2 \ln\left(\frac{\mu}{m_\pi}\right)^2 \ . \quad (39)$$

Numercially, the loop contributions to $\sqrt{Z_\pi}$ are small compared to those entering Z_N.

Note the one loop renormalization of h_π from the PV Yukawa πNN and $\pi\Delta\Delta$ vertices is already at the order $1/\Lambda_\chi^2$. An additional loop will introduce a factor of $1/\Lambda_\chi^2$. Loops containing the axial vector $NN\pi\pi$ and $N\Delta\pi\pi$ vertices and one strong $NN\pi$ or $N\Delta\pi$ vertex are of $\mathcal{O}(1/\Lambda_\chi^2 F_\pi)$. To obtain contributions of $\mathcal{O}(1/\Lambda_\chi^3)$, one would require the insertion of operators carrying explicit factors of $1/\Lambda_\chi$ into one loop graphs. We find no such contributions.

IV. THE SCALE OF LOOP CORRECTIONS

We may estimate the numerical importance of the loop corrections to h_π^{BARE} by taking $\delta = 0.3$ GeV, $g_A = 1.267$ [19] and $g_{\pi N\Delta} = 1.05$ [13] and by choosing $\mu = \Lambda_\chi = 1.16$ GeV†. With these inputs, the value of $Z_N\sqrt{Z_\pi}$ is completely determined. The vertex corrections, which appear as Δh_π in Eq. (4), depend on the PV couplings h_π, h_A^1, h_Δ, and h_A^Δ. We obtain

$$h_\pi^{EFF} = 0.5h_\pi + 0.25h_A^1 - 0.24h_\Delta + 0.079h_A^\Delta \ . \tag{40}$$

Note that the effect of the wavefunction renormalization corrections is to reduce the dependence on h_π^{BARE} by roughly 50%. In addition, the dependence of h_π^{EFF} on h_A^1 and h_Δ is non-negligible. Their coefficients are only a factor of two smaller than that of h_π^{BARE}. Although these contributions arise at $\mathcal{O}(p^2,\ p^3)$, they are fortuitously enhanced numerically. Thus, in a complete anaysis of the OPE PV interaction one should not ignore these constants.

At present, one has no direct experimental constraints on the parameters h_A^1, h_Δ, and h_A^Δ, as a comprehensive analysis of hadronic PV data including the full chiral structure of the PV hadronic interaction has yet to be performed. Consequently, one must rely on theoretical input for guidance regarding the scale of the unknown constants. Estimates of h_A^1 are given by the authors of Ref. [15]. These authors observe that the usual pole dominance approximation for P-wave non-leptonic hyperon decays typically underpredicts the experimental amplitudes by a factor of two. The difference may be resolved by the inclusion of local, parity-conserving operators having structures analogous to the A-type terms in Eq. (17). The requisite size of the $\Delta S = 1$ contact terms may imply a scale for the analogous $\Delta I = 1$ PV terms. If so, one might conclude that h_A^1 should be on the order of $10g_\pi$. On the other hand, a simple factorization estimate leads to $h_A^1 \sim 0.2g_\pi$. While the sign of h_A^1 is fixed in the factorization approximation, the sign of the larger value is undetermined. Thus, it is reasonable to conclude that h_A^1 may be large enough to significantly impact h_π^{EFF}, though considerably more analysis is needed to yield a firm conclusion.

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†Since the dependence on μ is logarithmic, one may choose other values, such as $\mu = m_\rho$, without affecting the numerical results significantly

The $\pi\Delta\Delta$ Yukawa coupling h_Δ has been estimated in Ref. [8] using methods similar to those of Ref. [7]. The authors quote a "best value" of $h_\Delta = -20 g_\pi$, with a "reasonable range" of $(-51 \to 0) \times g_\pi$.[‡] Naïvely, subsitution of the best value into Eq. (40) would increase the value of h_π^{EFF}, whereas the ^{18}F result would seem to require a reduction. As we argue below, however, the relationship between the couplings computed in Refs. [7,8] and the parameters appearing in Eq. (40) is somewhat ambiguous. Direct substitution of the theoretical value into h_π^{EFF} may not be entirely appropriate.

To date, no theoretical estimate of the A-type $\pi\pi N\Delta$ coupling has been performed. A simple estimate of the scale is readily obtained using the factorization approximation. To that end, we work with tree-level form of $\mathcal{H}_W^{PV}$. Neglecting short-distance QCD corrections and terms containing strange quarks, one has

$$\mathcal{H}_W^{PV}(\Delta S = 0) = \frac{G_F}{\sqrt{2}}\{\cos^2\theta_c \bar{u}\gamma_\lambda(1-\gamma_5)d\bar{d}\gamma^\lambda(1-\gamma_5)u \tag{41}$$

$$-2(1-2\sin^2\theta_W)V_\lambda^{(3)}A^{(3)\ \lambda} + \frac{4}{3}\sin^2\theta_W V_\lambda^{(0)}A^{(3)\ \lambda}\} \quad , \tag{42}$$

where $V_\lambda^{(3)}$ and $A_\lambda^{(3)}$ denote the third components of the octet of vector and axial vector currents, respectively, and

$$V_\lambda^{(0)} = \frac{1}{2}(\bar{u}\gamma_\lambda u + \bar{d}\gamma_\lambda d) \quad . \tag{43}$$

Consider now the first term in the expression for h_A^Δ given in Eq. (36). In the factorization approximation, $\mathcal{H}_W^{PV}$ contributes only to the antisymmetric combination

$$\frac{1}{2}(h_A^{n\Delta^0\pi^+\pi^-} - h_A^{n\Delta^0\pi^-\pi^+}) \quad . \tag{44}$$

The neutral current contribution to this combination, which arises only from the term containing $V_\lambda^{(3)}$, is

$$\sqrt{2}G_F F_\pi^2(1-2\sin^2\theta_W)C_5^A(n\Delta^0) \approx 2g_\pi C_5^A(n\Delta^0) \quad , \tag{45}$$

where $C_5^A(n\Delta^0) \sim \mathcal{O}(1)$ is the axial vector $n \to \Delta^0$ form factor at the photon point. After Fierz re-ordering, the charged current component of $\mathcal{H}_W^{PV}$ contributes roughly

$$-(4g_\pi/3)C_5^A(n\Delta^0), \tag{46}$$

yielding a total factorzation contribution of about $(2g_\pi/3)C_5^A(n\Delta^0)$. Thus, one would expect the scale of the axial vector $\pi\pi N\Delta$ couplings to be on the order of a few $\times\ g_\pi$.

In the particular case of the combination appearing in h_A^Δ, however, the sum of factorization contributions cancels identically. As one sees from the expressions for the $h_A^{N\Delta\pi\pi}$ given in Appendix A, isospin requires

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‡This coupling is denoted $f_{\Delta\Delta\pi}$ in Ref. [8].

$$h_A^{n\Delta^0\pi^+\pi^-} + h_A^{p\Delta^+\pi^-\pi^+} = 0 \quad . \tag{47}$$

The factorization contributions independently satisfy this sum rule. The second combination of constants appearing in Eq. (36),

$$h_A^{n\Delta^+\pi^0\pi^-} - h_A^{p\Delta^0\pi^0\pi^+} \quad , \tag{48}$$

also vanishes in the factorization approximation, even though the individual couplings do not. The third pair of couplings received no factorization contributions. Thus, one has $h_A^\Delta = 0$ in this approximation. In principle, non-factorization contributions yield a non-zero value for h_A^Δ. Although we have not evaluated these contributions, we do not expect the scale to be significantly larger than the factorization value for the individual $h_A^{N\Delta\pi\pi}$ couplings. Consequently, we estimate a reasonable range for h_A^Δ of $(0 \to \text{few}) \times g_\pi$.

These theoretical estimates suggest considerable ambiguity in the prediction for h_π^{EFF}. In principle, some of this ambiguity might be removed by performing the comprehensive analysis of hadronic PV suggested above, in which the various constants would be determined entirely by experiment. The viability of such a program remains to be seen.

V. COMPARING WITH MICROSCOPIC CALCULATIONS

The results in Eqs. (37-39) embody the full SU(2) chiral structure at $\mathcal{O}(p^3)$ of $\langle N\pi|\mathcal{H}_W^{PV}|N\rangle$ at leading order in the pion momentum. Any microscopic calculation of this matrix element which respects the symmetries of QCD should display the dependence on light quark masses appearing in h_π^{EFF}. In principle, an unquenched lattice QCD calculation with light quarks would manifest this chiral structure. In practice, however, unquenched calculations remain difficult, and even quenched calculations require the use of heavy quarks. For a lattice determination of $\langle N\pi|\mathcal{H}_W^{PV}|N\rangle$, the expressions in Eqs. (37-39) could be used to extrapolate to the light quark limit, much as the chiral structure of baryon mass and magnetic moment can be used for similar extrapolations [20].

In the absence of a first principles QCD calculation, one must rely on symmetries and/or models to obtain the PV $NN\pi$ coupling. A variety of such approaches have been undertaken, including the SU(6)$_w$/quark model calculation of Refs. [7,8], the Skyrme model [9], and QCD sum rules [10]. To date, the DDH/FCDH analysis remains the most comprehensive and has become the benchmark for comparison between experiment and theory. Consequently, we focus on this work as a "case study" in the problem of matching microscopic calculations onto hadronic effective theory.

The DDH/FCDH approach relies heavily on symmetry methods in order to relate the PV $\Delta S = 0$ matrix elements to experimental $\Delta S = 1$ nonleptonic hyperon decay amplitudes. All the charged current (CC) contributions to the $\Delta S = 0, 1$ $B \to B'M$ amplitudes, where M is a pseudoscalar meson, can be related using SU(3) arguments. Likewise, the neutral current (NC) component of the effective weak Hamiltonian belonging to the same multiplets as the CC components (*i.e.* those arising from a product of purely left-handed currents) can also be related via SU(3). The remaining NC contributions to the $\Delta S = 0$ PV amplitudes are computed using factorization and the MIT bag model. The DDH approach also employs SU(6)$_w$ symmetry arguments in order to calculate parity-violating vector meson couplings. Although one requires only SU(3) to determine the pseudoscalar couplings, we refer below to the general SU(6)$_w$ formalism used in Refs. [7,8].

The general SU(6)$_w$ analysis employed by DDH/FCDH introduces five reduced matrix elements: $a_{t,v}$, $b_{t,v}$, and c_v. These constants correspond to SU(6)$_w$ components of the weak Hamiltonian:

$$[(\bar{B}B)_{35} \otimes M_{35}]_{35} \sim c_v \tag{49}$$

$$[(\bar{B}B)_{405} \otimes M_{35}]_{280,\overline{280}} \sim b_t, b_v \tag{50}$$

$$[(\bar{B}B)_{405} \otimes M_{35}]_{280,\overline{280}} \sim a_t, a_v \tag{51}$$

One may represent these different components of $\mathcal{H}_W^{PV}$ diagramatically as in Fig. 3. The components shown in Fig. 3a,b correspond to $b_{t,v}$ and c_v, respectively. In practice, these contributions are determined entirely from empirical hyperon decay data. The term in Fig. 3a corresponds to $a_{t,v}$ and is computed in Refs. [7,8] using factorization.

The PV $NN\pi$ Yukawa coupling can be expressed in terms of these SU(6)$_w$ reduced matrix elements plus an additional factorization/quark model term. Temporarily neglecting short-distance QCD corrections to $\mathcal{H}_W^{PV}$, one has

$$\begin{aligned}\langle p\pi^-|\mathcal{H}_W^{PV}|n\rangle &= \frac{1}{3\sqrt{2}}\tan\theta_c c_v \\ &\quad - \frac{2}{9\sqrt{2}}\csc 2\theta_c \sin^2\theta_W(2c_v - b_t) + \frac{1}{3}\sin^2\theta_W y \quad ,\end{aligned} \tag{52}$$

where θ_c and θ_W are the Cabibbo and Weinberg angles, respectively, and y denotes a Fierz/factorization contribution. The first term on the RHS of Eq. (52) gives the CC contribution, while the remaining terms arise from weak NC's. Including short-distance QCD renormalization of $\mathcal{H}_W^{PV}$ leads to a modification of Eq. (52):

$$\begin{aligned}\langle p\pi^-|\mathcal{H}_W^{PV}|n\rangle &= \Big\{[1-2\sin^2\theta_W]\gamma(K) + \sin^2\theta_c\Big\}\frac{\rho}{\sin^2\theta_c}g_\pi \\ &\quad + \sin^2\theta_c(B_1 + B_2) \quad ,\end{aligned} \tag{53}$$

where

$$g_\pi = \frac{1}{3\sqrt{2}}\tan\theta_c c_v \tag{54}$$

$$B_1 = \frac{4}{9\sqrt{2}}\eta E(K)\left(\frac{1}{\sin\theta_c\cos\theta_c}\right)(b_v/6 - b_t/12 - c_v/2) \tag{55}$$

$$B_2 = \frac{1}{3}F(K)y \quad , \tag{56}$$

and $\gamma(K)$, $E(K)$ and $F(K)$ are summed leading log (renormalization group) factors dependent on

$$K = 1 - \frac{\alpha_s(\mu)}{\pi}[11 - \frac{2}{3}N_f] \ln \frac{M_W^2}{\mu^2} \quad . \tag{57}$$

The overall scale factor ρ appearing in Eq. (53) was introduced in Ref. [7] in order to account for various theoretical uncertainties entering the analysis.

The appearance of c_v, b_t, and b_v in g_π and B_1 relies on *tree-level* $\mathrm{SU}(6)_w$ symmetry—long-distance chiral corrections of the types shown in Fig. 4 have not been explicitly included. Inclusion of such corrections would necessitate a reanalysis of the $\Delta S = 1$ amplitudes in much the same way that one treats the octet of baryon axial vector currents [11] or magnetic moments [21]. For example, letting $A(\Lambda^0_-)$ denote the amplitude for $\Lambda \to p\pi^-$ one has at tree-level

$$A(\Lambda^0_-) = \frac{1}{\sqrt{3}}(b_v/6 - b_t/12 - c_v/2) \quad . \tag{58}$$

Including the leading chiral corrections would yield the modification

$$A(\Lambda^0_-) = \frac{1}{\sqrt{3}}\sqrt{Z_\Lambda Z_p Z_\pi}(b_v/6 - b_t/12 - c_v/2) + \Delta A(\Lambda^0_-) \quad , \tag{59}$$

where $\Delta A(\Lambda^0_-)$ denotes vertex corrections and possible contributions from higher-dimension operators. Similar corrections would appear in the $\mathrm{SU}(6)_w$ symmetry terms in Eqs. (52, 53). Given the absence of these corrections from the DDH/FCDH analysis, the symmetry components $\langle p\pi^-|\mathcal{H}_W^{PV}|n\rangle$ do not formally embody the subleading chiral structure of h_π^{EFF}. The *numerical* impact of applying chiral corrections to the DDH/FCDH tree-level $\mathrm{SU}(6)_w$ analysis is much less clear, since some of the chiral modifications can be absorbed into renormalized values of the chiral couplings, which are determined empirically. Nevertheless, the potentially sizeable effect of the SU(2) chiral corrections on h_π^{EFF} should give one pause.

A related issue is the degree to which ambiguities introduced by kaon and η loops in SU(3) HBChPT could plague an analysis of the $\Delta S = 1$ amplitudes. Here recent work by Donoghue and Holstein argues that finite nucleon size call for long-distance regularization of such heavy meson loops, which substantially reduces their effects [22]. Results are then similar to what arises from use of a cloudy bag approach to such matrix elements [23]. A comprehensive study of such issues – and their impact on the DDH/FCDH calculation of h_π – goes beyond the scope of the present work. Nevertheless, the sizeable impact of the chiral corrections in h_π^{EFF} and the use of tree-level symmetry arguments in Refs. [7,8] points to a possibly significant mismatch between h_π^{EFF} and h_π^{DDH}.

The remaining terms in the DDH/FCDH analysis – involving the parameters η and y – are determined by explicit MIT bag model calculations. One may ask whether the latter effectively includes any part of the subleading chiral structure of h_π^{EFF}. In order to address this question, we make three observations:

1. Sea quarks and gluons generate c_v. The parameter c_v vanishes identically in any quark model in which baryons consist solely of three constituent quarks. The $\Delta S = 1$ hyperon decay data, however, clearly implies that $c_v \neq 0$. In order to obtain a nonzero value in a quark model, one requires the presence of sea quarks and gluons. It is shown in [24] for example, that $c_v \neq 0$ when gluons are added to the MIT bag model. Similarly, one would expect contributions from the $q\bar{q}$ pairs in the sea. Since relativistic quark models already contain $q\bar{q}$ pairs in the form of "Z-graphs" [25], it is likely that disconnected $q\bar{q}$ insertions (see Fig. 5b) give the dominant sea quark contribution to c_v. In a chirally corrected analysis of nonleptonic decays, the long-distance part of the disconnected $q\bar{q}$ insertions appear explicitly in the guise of pseudoscalar loops, while the short-distance contributions are subsumed into the value of c_v and possible higher dimension operators. "Quenched" quark models without explicit pionic degress of freedom generally do not include the long-distance physics of disconnected insertions.

2. The m_q-dependence is different. In conventional HBChPT analyses of hadronic observables, one only retains the loop contributions non-analytic in the light quark mass. The constituent quark model (without explicit pions) has a difficult time producing these non-analytic contributions. The simplest, illustrative example is the nucleon isovector charge radius, $\langle r^2 \rangle_{T=1}$, which is singular in the chiral limit [26]. This chiral singularity, of the form $\ln m_\pi^2 \sim \ln m_q$, is produced by π loops. Relativistic quark models, such as the MIT bag model, yield a finite value for $\langle r^2 \rangle_{T=1}$ as $m_q \to 0$. One cannot produce the chiral singularity in a quark model without including disconnected $q\bar{q}$ insertions dressed as mesons.

The corresponding argument in the case of h_π^{EFF} is less direct, but still straightforward. In the limit of a degenerate N and Δ, the non-analytic terms in h_π^{EFF} have quark mass-dependences of the form $m_q \ln m_q$ or $m_q^{3/2}$. As we show in Appendix C, bag model matrix elements of the four quark operators appearing in $\mathcal{H}_W^{PV}$ have a Taylor series expansion about $m_q = 0$. Thus, the parameters η and y cannot contain the non-analytic structures generated by the diagrams in Figs. 1-2.

3. Graphs are missing. This observation is simply a diagrammatic summary of the previous two observations. For simplicity, consider a subset of the quark-level diagrams associated with the appearance of h_A^i in h_π^{EFF}. Typical contributions to the axial $NN\pi\pi$ PV vertex are shown in Fig. 5a. The corresponding loop contributions to h_π^{EFF} appear in Fig. 5b,c. Those in Fig. 5b involve disconnected $q\bar{q}$ insertions, which do not occur in the constituent quark model. The contribution of Fig. 5c involves Z-graphs, which are produced in a relativistic quark model[§]. In principle, the $3q + q\bar{q}$ intermediate state could contain an $N\pi$ pair. As argued previously, however, the Z-graphs implicit in the MIT bag model calculation of h_π do not produce the nonanalytic structure of the corresponding π loop. Apparently, only an unquenched quark model, which generates the disconnected insertions of Fig. 5b, could produce the requisite nonanalytic terms.

[§]e.g., as a correction to the $b_{t,v}$ terms of Fig. 3b

From this "case study" of the DDH/FCDH calculation of h_π, we conclude that the $\mathrm{SU}(6)_w$/quark model approach used in Refs. [7,8] does not incorporate the chiral structure of h_π^{EFF}. Were the numerical impact of the chiral corrections negligible, this observation would not be bothersome. The actual impact of the chiral corrections, however, is significant. Thus, it is perhaps not surprising that the ^{18}F result and the DDH/FCDH "best value" do not agree.

VI. CONCLUSIONS

With the confirmation of the electroweak sector of the Standard Model at the 1% level or better in a variety of leptonic and semi-leptonic processes, one has little reason to doubt its validity in the purely hadronic domain. Similarly, the predictions of QCD in the perturbative regime have been confirmed with a high degree of confidence. Thus, one may justifiably consider $\mathcal{H}_W^{PV}$, the effective Hamiltonian including its perturbative strong interaction correction, to be well understood. Moreover, the precision available with present and future hadronic PV experiments is unlikely to match the levels achieved in leptonic and semileptonic processes. Consequently, one has little hope of detecting small deviations in $\mathcal{H}_W^{PV}$ from its SM structure due to "new physics". On the other hand, much about QCD in the non-perturbative regime remains mysterious: the mechanism of confinement, the dynamics of chiral symmetry breaking, the role or sea quarks in the low-energy structure of the nucleon, and so forth. Each of these issues bears on one's understanding of *matrix elements* of $\mathcal{H}_W^{PV}$. In this sense, the low-energy, PV hadronic weak interaction constitutes a probe of the dynamics of low-energy QCD, in a manner analogous to the probe provided by the electromagnetic interaction.

From a phenomenological standpoint, the matrix element one may hope to extract from hadronic PV observables with the least ambiguity is $\langle N\pi|\mathcal{H}_W^{PV}|N\rangle$. In this study, we have argued that any theoretical interpretation of this matrix element must take into account the consequences of chiral symmetry. Indeed the chiral corrections to the tree-level, PV πNN Yukawa coupling are not small. At $\mathcal{O}(1/\Lambda_\chi^3)$, the effective coupling measured in experiments, h_π^{EFF}, depends not only on the bare coupling, h_π^{BARE}, but also on new (and experimentally undetermined) PV low-energy constants, h_A^1, h_A^Δ, and h_Δ, as well. Furthermore, the coefficients of h_π^{BARE}, h_A^1, and h_Δ are comparable in magnitude. At present, one has only simple theoretical estimates of the magnitudes of the h_A^1 and h_A^Δ in addition to the FCDH calculation of h_Δ. These estimates suggest that the new PV couplings appearing in h_π^{EFF} could be as large as h_π^{BARE}. Since no experimental constraints have been obtained for the new couplings, there exists considerable latitude in the theoretical expectation for h_π^{EFF}.

For two decades now, the benchmark theoretical calculation of $\langle N\pi|\mathcal{H}_W^{PV}|N\rangle$ has been the $\mathrm{SU}(6)_w$/quark model approach of Ref. [7], updated in Ref. [8]. We have argued, however, that the DDH/FCDH calculation does not manifest the general strictures of chiral invariance obtained in the present analysis. At the quark level, this chiral structure reflects the importance of the "disconnected" $q\bar{q}$ components of the sea. While relativistic quark models contain $q\bar{q}$ sea quark effects in the guise of Z-graphs or lower-component wavefunctions, the most common "quenched" versions do not include explicit disconnected pairs**. Given the size of the chiral corrections associated in part with the disconnected insertions, it may then be not so surprising to find a possible discrepancy between the experimental value for h_π^{EFF} and the DDH/FCDH "best value".

Applying chiral corrections to the SU(3) analysis of $\Delta S = 1$ hyperon decays may help to close the gap between h_π^{EFF} and h_π^{DDH}. Presumably, similar corrections should be applied in other approaches not containing explicit pionic degress of freedom. In the longer run, one may be able to use the chiral structure of h_π^{EFF} to extrapolate an unquenched lattice calculation with heavy quarks into the physical regime.

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**Some effects of disconnected $q\bar{q}$ pairs may, however, hide in the effective parameters of the quark model, such as the string tension [27].

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APPENDIX A: PV LAGRANGIANS

Here we present the full expressions for some of the PV Lagrangians not included in the main body of the paper. The analogues of Eqs. (16-18) are

$$\begin{aligned}\mathcal{L}^{\pi\Delta N}_{\Delta I=0} &= f_1\epsilon^{abc}\bar{N}i\gamma_5[X^a_L A_\mu X^b_L + X^a_R A_\mu X^b_R]T^\mu_c \\ &+g_1\bar{N}[A_\mu, X^a_-]_+T^\mu_a + g_2\bar{N}[A_\mu, X^a_-]_-T^\mu_a + \text{h.c.}\end{aligned} \tag{A1}$$

$$\begin{aligned}\mathcal{L}^{\pi\Delta N}_{\Delta I=1} &= f_2\epsilon^{ab3}\bar{N}i\gamma_5[A_\mu, X^a_+]_+T^\mu_b + f_3\epsilon^{ab3}\bar{N}i\gamma_5[A_\mu, X^a_+]_-T^\mu_b \\ &+\frac{g_3}{2}\bar{N}[(X^a_L A_\mu X^3_L - X^3_L A_\mu X^a_L) - (X^a_R A_\mu X^3_R - X^3_R A_\mu X^a_R)]T^\mu_a \\ &+\frac{g_4}{2}\{\bar{N}[3X^3_L A^\mu(X^1_L T^1_\mu + X^2_L T^2_\mu) + 3(X^1_L A^\mu X^3_L T^1_\mu + X^2_L A^\mu X^3_L T^2_\mu) \\ &-2(X^1_L A^\mu X^1_L + X^2_L A^\mu X^2_L - 2X^3_L A^\mu X^3_L)T^3_\mu] - (L \leftrightarrow R)\} + \text{h.c.}\end{aligned} \tag{A2}$$

$$\begin{aligned}\mathcal{L}^{\pi\Delta N}_{\Delta I=2} &= f_4\epsilon^{abd}\mathcal{I}^{cd}\bar{N}i\gamma_5[X^a_L A_\mu X^b_L + X^a_R A_\mu X^b_R]T^\mu_c \\ &+f_5\epsilon^{ab3}\bar{N}i\gamma_5[X^a_L A_\mu X^3_L + X^3_L A_\mu X^a_L + (L \leftrightarrow R)]T^\mu_b \\ &+g_5\mathcal{I}^{ab}\bar{N}[A_\mu, X^a_-]_+T^\mu_b + g_6\mathcal{I}^{ab}\bar{N}[A_\mu, X^a_-]_-T^\mu_b + \text{h.c.} \quad ,\end{aligned} \tag{A3}$$

where the terms containing f_i and g_i start off with one- and two-pion vertices, respectively. In the heavy baryon expansion, the terms containing the f_i start to contribute at $\mathcal{O}(1/m_N)$. The leading order term vanishes since $P_+ \cdot i\gamma_5 \cdot P_+ = 0$. Since we work only to lowest order in the $1/m_N$ expansion, we obtain no contribution from the terms containing the f_i.

For the pv $\pi\Delta\Delta$ effective Lagrangians we have

$$\mathcal{L}^{\pi\Delta}_{\Delta I=0} = j_0\bar{T}^i A_\mu\gamma^\mu T_i \ , \tag{A4}$$

$$\begin{aligned}\mathcal{L}^{\pi\Delta}_{\Delta I=1} &= \frac{j_1}{2}\bar{T}^i\gamma^\mu T_i Tr(A_\mu X^3_+) - \frac{k_1}{2}\bar{T}^i\gamma^\mu\gamma_5 T_i Tr(A_\mu X^3_-) \\ -\frac{h^1_{\pi\Delta}}{2\sqrt{2}}f_\pi\bar{T}^i X^3_- T_i &- \frac{h^2_{\pi\Delta}}{2\sqrt{2}}f_\pi\{3\bar{T}^3(X^1_- T^1 + X^2_- T^2) + 3(\bar{T}^1 X^1_- + \bar{T}^2 X^2_-)T^3 \\ -2(\bar{T}^1 X^3_- T^1 + \bar{T}^2 X^3_- T^2 &- 2\bar{T}^3 X^3_- T^3)\} + j_2\{3[(\bar{T}^3\gamma^\mu T^1 + \bar{T}^1\gamma^\mu T^3)Tr(A_\mu X^1_+) \\ +(\bar{T}^3\gamma^\mu T^2 + \bar{T}^2\gamma^\mu T^3)Tr(A_\mu X^2_+)] &- 2(\bar{T}^1\gamma^\mu T^1 + \bar{T}^2\gamma^\mu T^2 - 2\bar{T}^3\gamma^\mu T^3)Tr(A_\mu X^3_+)\} \\ +k_2\{3[(\bar{T}^3\gamma^\mu\gamma_5 T^1 + \bar{T}^1\gamma^\mu\gamma_5 T^3)Tr(A_\mu X^1_-) &+ (\bar{T}^3\gamma^\mu\gamma_5 T^2 + \bar{T}^2\gamma^\mu\gamma_5 T^3)Tr(A_\mu X^2_-)] \\ &-2(\bar{T}^1\gamma^\mu\gamma_5 T^1 + \bar{T}^2\gamma^\mu\gamma_5 T^2 - 2\bar{T}^3\gamma^\mu\gamma_5 T^3)Tr(A_\mu X^3_-)\} \\ &+j_3\{\bar{T}^a\gamma^\mu[A_\mu, X^a_+]_+T^3 + \bar{T}^3\gamma^\mu[A_\mu, X^a_+]_+T^a\} \\ &+j_4\{\bar{T}^a\gamma^\mu[A_\mu, X^a_+]_-T^3 - \bar{T}^3\gamma^\mu[A_\mu, X^a_+]_-T^a\} \\ &+k_3\{\bar{T}^a\gamma^\mu\gamma_5[A_\mu, X^a_-]_+T^3 + \bar{T}^3\gamma^\mu\gamma_5[A_\mu, X^a_+]_+T^a\} \\ &+k_4\{\bar{T}^a\gamma^\mu\gamma_5[A_\mu, X^a_-]_-T^3 - \bar{T}^3\gamma^\mu\gamma_5[A_\mu, X^a_+]_-T^a\} \ ,\end{aligned} \tag{A5}$$

$$\mathcal{L}^{\pi\Delta}_{\Delta I=2} = j_5 \mathcal{I}^{ab}\bar{T}^a\gamma^\mu A_\mu T^b + j_6\mathcal{I}^{ab}\bar{T}^i[X^a_R A_\mu X^b_R + X^a_L A_\mu X^b_L]\gamma^\mu T_i$$
$$+k_5\mathcal{I}^{ab}\bar{T}^i[X^a_R A_\mu X^b_R - X^a_L A_\mu X^b_L]\gamma^\mu\gamma_5 T_i$$
$$+k_6\epsilon^{ab3}[\bar{T}^3 i\gamma_5 X^b_+ T^a + \bar{T}^a i\gamma_5 X^b_+ T^3] \,, \tag{A6}$$

where we have suppressed the Lorentz indices of the Δ field, i.e., $\bar{T}^\nu \cdots T_\nu$. The vertices with k_i, h_Δ contain two pions. All other vertices contain one pion when expanded to the leading order. At first sight the leading order term with k_6 in (A6) has no pions. However such a term cancels its hermitian conjugate exactly. The constants $h^i_{\pi\Delta}$ are the PV $\pi\Delta\Delta$ Yukawa coupling constants.

In Section 2, the leading terms in the above Lagrangians were expressed in terms of effective $\pi\pi N\Delta$ and $\pi\Delta\Delta$ coupling constants. These constants may be expressed in terms of the f_i, g_i, k_i, j_i and $h^i_{\pi\Delta}$ as follows:

$$h_A^{p\Delta^{++}\pi^-\pi^0} = -2g_1 + 2g_2 - g_3 - 3g_4 - \frac{2}{3}g_5 + \frac{2}{3}g_6$$
$$h_A^{p\Delta^{++}\pi^0\pi^-} = 2g_1 + g_3 + 6g_4 + \frac{2}{3}g_5$$
$$h_A^{p\Delta^{+}\pi^0\pi^0} = -\frac{\sqrt{6}}{9}(6g_2 + 9g_4 + 2g_6)$$
$$h_A^{p\Delta^{+}\pi^+\pi^-} = -\frac{\sqrt{6}}{9}(-6g_1 - 9g_4 + 4g_5 + 6g_6)$$
$$h_A^{p\Delta^{+}\pi^-\pi^+} = -\frac{\sqrt{6}}{9}(6g_1 - 6g_2 - 4g_5 + 4g_6)$$
$$h_A^{p\Delta^{0}\pi^+\pi^0} = -\frac{\sqrt{3}}{9}(6g_1 + 6g_2 - 3g_3 + 9g_4 + 2g_5 + 2g_6)$$
$$h_A^{p\Delta^{0}\pi^0\pi^+} = -\frac{\sqrt{3}}{9}(-6g_1 + 12g_2 + 3g_3 + 18g_4 - 2g_5 - 8g_6)$$
$$h_A^{p\Delta^{-}\pi^+\pi^+} = \frac{\sqrt{2}}{3}(6g_2 - 9g_4 + 2g_6)$$
$$h_A^{n\Delta^{++}\pi^-\pi^-} = \frac{\sqrt{2}}{3}(6g_2 - 9g_4 + 2g_6)$$
$$h_A^{n\Delta^{+}\pi^-\pi^0} = -\frac{\sqrt{3}}{9}(6g_1 + 6g_2 + 3g_3 - 9g_4 + 2g_5 + 2g_6)$$
$$h_A^{n\Delta^{+}\pi^0\pi^-} = -\frac{\sqrt{3}}{9}(-6g_1 + 12g_2 - 3g_3 - 18g_4 - 2g_5 - 8g_6)$$
$$h_A^{n\Delta^{0}\pi^0\pi^0} = -\frac{\sqrt{6}}{9}(-6g_2 + 9g_4 - 2g_6)$$
$$h_A^{n\Delta^{0}\pi^+\pi^-} = -\frac{\sqrt{6}}{9}(-6g_1 + 6g_2 + 4g_5 - 4g_6)$$
$$h_A^{n\Delta^{0}\pi^-\pi^+} = -\frac{\sqrt{6}}{9}(6g_1 - 9g_4 - 4g_5 - 6g_6)$$
$$h_A^{n\Delta^{-}\pi^+\pi^0} = 2g_1 - 2g_2 + g_3 + 3g_4 + \frac{2}{3}g_5 - \frac{2}{3}g_6$$

$$h_A^{n\Delta^-\pi^0\pi^+} = -2g_1 - g_3 - 6g_4 - \frac{2}{3}g_5 \tag{A7}$$

$$h_\Delta = h^1_{\pi\Delta} + h^2_{\pi\Delta}$$

$$\begin{aligned} h_V^{\Delta^{++}\Delta^+} &= \frac{1}{\sqrt{6}}(j_0 + \frac{4}{3}j_6) - 2\sqrt{6}j_2 - \frac{2\sqrt{6}}{3}(j_3 + j_4) + \frac{j_5}{3\sqrt{6}} \\ h_V^{\Delta^+\Delta^0} &= \frac{\sqrt{2}}{3}(j_0 + \frac{4}{3}j_6) - \frac{2\sqrt{2}}{9}j_5 \\ h_V^{\Delta^0\Delta^-} &= \frac{1}{\sqrt{6}}(j_0 + \frac{4}{3}j_6) + 2\sqrt{6}j_2 + \frac{2\sqrt{6}}{3}(j_3 + j_4) + \frac{j_5}{3\sqrt{6}} \end{aligned} \tag{A8}$$

It is interesting to note there is only one independent PV Yukawa coupling constant h_Δ for $\pi\Delta\Delta$ interactions.

APPENDIX B: VANISHING LOOP CONTRIBUTIONS

As noted in Section 3, a large number of graphs which nominally contribute to h_π^{EFF} actually vanish up to $\mathcal{O}(1/\Lambda_\chi^3)$. Here, we summarize the the reasons why.

Consider first the corrections due to the PV vector πNN vertices. For FIG. 1 (b) we have

$$iM_{(b)} = i\frac{g_A^2}{\sqrt{2}F_\pi^3}\tau^+(h_v^0 + \frac{4}{3}h_V^2)(v\cdot q)\int\frac{d^Dk}{(2\pi)^D}\frac{(S\cdot k)^2}{v\cdot kv\cdot(k+q)(k^2-m_\pi^2)} \sim \mathcal{O}(1/m_N\Lambda_\chi^3)\ , \tag{B1}$$

where we have used $v\cdot q \sim \mathcal{O}(1/m_N)$. Since we are working to leading order in the $1/m_N$ expansion, this amplitude does not contribute. The PV vector interactions also appear in Figs. 1(j1,j2). The corresponding amplitude is

$$iM_{(j1)+(j2)} = -i\frac{g_A^2}{\sqrt{2}F_\pi^3}\tau^+(h_v^0 + 2h_V^1 - \frac{8}{3}h_V^2)\int\frac{d^Dk}{(2\pi)^D}\frac{[(S\cdot k),(S\cdot q)]_+}{v\cdot kv\cdot(k+q)(k^2-m_\pi^2)} = 0 \tag{B2}$$

This integral vanishes because it is proportional to $[(S\cdot v),(S\cdot q)]_+$, which vanishes because $S\cdot v = 0$. All other possible insertions of PV vector πNN vertices vanish for similar reasons as either (B1) or (B2). In what follows, we refer only to insertions involving the PV πNN Yukawa and $\pi\pi NN$ axial couplings.

The propagator corrections in FIG. 1 (g1)-(h2) vanish after integration since their amplitude of (g1)-g2) goes as

$$\sim h_\pi\int\frac{d^Dk}{(2\pi)^D}\frac{v\cdot k}{k^2-m_\pi^2} = 0 \quad . \tag{B3}$$

while the amplitude of (h1)-(h2) goes as

$$\sim h_A^i \int \frac{d^D k}{(2\pi)^D} \frac{S\cdot k}{k^2 - m_\pi^2} = 0 \quad . \tag{B4}$$

The amplitude of FIG. 1 (i1)-(i4) contains a vanishing integral

$$\sim h_\pi \int \frac{d^D k}{(2\pi)^D} \frac{S\cdot k}{v\cdot k(k^2 - m_\pi^2)} = 0 \quad . \tag{B5}$$

Figs. 1 (j1)-(j2) do not contribute for the PV Yukawa coupling h_π due to charge conservation. The remaining non-zero diagrams are Figs. 1 (a)-(f2) where the insertions in loops are of the Yukawa or axial interations. Figs. (f1), (f2) arises from the insertion of the counter terms of mass and wave function renormalization. Fig. 1 (e1)-(e2) and Fig. 2 (c1)-(c2) contribute to the wave function renormalization in Eq. (38).

Due to the heavy baryon projection $P_+ \cdot i\gamma_5 \cdot P_+ = 0$ the one pion PV $\pi N\Delta$ vertex does not contribute in the leading order of heavy baryon expansion. Hence, the chiral loop corrections from FIG. 2 (d1)-(g4) are of higher order. Fig. 2 (h1)-(j2) vanishes after integration for reasons similar to (B2). The remaining, non-vanishing diagrams are discussed explicitly in Section 3.

As pointed out in Section 2, both PC and PV two-derivative operators which conserve CP do not contribute to h_π renormalization. For example, there exists one CP-conserving, PV such operator:

$$\frac{1}{\Lambda_\chi}\bar{N}\sigma^{\mu\nu}[D_\mu A_\nu - D_\nu A_\mu]N \quad . \tag{B6}$$

After expansion, the leading term starts with three pions. It contributes via Figure 1 (a), at the order of $1/\Lambda_\chi F_\pi^3$. Moreover the loop integration yields a factor $g_{\mu\nu}$ and leads to zero after contraction with $\sigma^{\mu\nu}$.

Another possibility comes from insertions of PC two-derivative nucleon pion operators. There are three PC operators which conserve CP:

$$\frac{1}{\Lambda_\chi}\bar{N} i\gamma_5 D_\mu A^\mu N \ , \tag{B7}$$

$$\frac{1}{\Lambda_\chi}\bar{N} A^\mu A_\mu N \ , \tag{B8}$$

$$\frac{1}{\Lambda_\chi}\bar{N}\sigma^{\mu\nu}[A_\mu, A_\nu] N \ . \tag{B9}$$

Note the first two operators are symmetric in the Lorentz indices. Only the last one arises from the antisymmetric operators listed in Eq. (29). The first one starts off with one pion. The relevant Feynman diagrams are Figure 1 (c1)-(c2), where the PV vertex is associated with h_A^i. Note these diagrams do not contribute at leading order of HBChPT due to the presence of the $i\gamma_5$. The remaining two operators start off with two pions. The relevant diagrams are Figure 1 (d1)-(d2). After integration the contribution of the third operator reads

$$\sim h_\pi \epsilon^{\mu\nu\alpha\beta} v_\alpha S_\beta v^\mu q^\nu m_\pi^2 \ln m_\pi / \Lambda_\chi F_\pi^2 \ . \tag{B10}$$

So its contribution is zero. In contrast the second operator yields

$$h_\pi (v \cdot q) m_\pi^2 \ln m_\pi / (\Lambda_\chi F_\pi^2) \ . \tag{B11}$$

Note $v \cdot q \sim 1/m_N$. So its contribution is of order $1/(\Lambda_\chi^3 m_N)$. In short, none of the two-derivative operators contribute to the renormalization of h_π at the order to which we work.

APPENDIX C: BAG MODEL INTEGRALS

Here, we show that the four-quark bag model integrals relevant to the calculation of the DDH/FCDH parameters η and y have a Taylor expansion in light quark mass around $m_q = 0$. We write a bag model quark wavefunction as [28,18]

$$\psi(x) = \begin{pmatrix} iu(r)\chi \\ \ell(r)\vec{\sigma} \cdot \vec{r}\chi \end{pmatrix} \exp(-iEt) \quad , \tag{C1}$$

where χ denotes a two-component Pauli spinor and where wave function normalization yields

$$\int d^3r (u(r)^2 + \ell(r)^2) = 1 \quad , \tag{C2}$$

where the the radial integration runs from 0 to the bag radius, R. The four quark matrix elements of interest here can depend three different integrals:

$$\int d^3r u(r)^4 \ , \quad \int d^3r \ell(r)^4 \ , \quad \int d^3r \ u(r)^2 \ell(r)^2 \quad . \tag{C3}$$

The quark radial wave functions are

$$u(r) = N j_0(\frac{p_n r}{R}) \tag{C4}$$

$$\ell(r) = -N \left(\frac{\omega_n - m_q R}{\omega_n + m_q R} \right)^{1/2} j_1(\frac{p_n r}{R}) \quad , \tag{C5}$$

where

$$\tan p_n = -\frac{p_n}{\omega_n + m_q R - 1} \qquad (n = 1, 2, \cdots) \tag{C6}$$

$$p_n = \sqrt{\omega_n^2 - m_q^2 R^2} \tag{C7}$$

$$N = \sqrt{\frac{p_n^4}{R^3(2\omega_n^2 - 2\omega_n + m_q R)\sin^2 p_n}} \tag{C8}$$

$$R^4 = \frac{N\omega_n - Z_0}{4\pi B} \tag{C9}$$

B is the bag constant and Z_0 is a phenomenological parameter involved with the center of mass motion of the bag.

For light quarks and lowest eigenmode

$$\omega_0 \approx (2.043 + 0.493 m_q R) \tag{C10}$$

$$N \approx \quad 2.27/\sqrt{4\pi R^3} \quad . \tag{C11}$$

It is straightforward to show that the bag model integrals in Eq. (C3) have a Taylor expansion about $m_q = 0$. The argument proceeds by noting that the quantities N, R, p_n, ω_n and the argument of the spherical Bessel functions all have Taylor series in m_q about $m_q = 0$. The existence of this expansion can be seen be an explicit, iterative construction. First, expand ω_n and R:

$$\omega_n = \sum_{n=0}^{\infty} \omega_{n,k} m_q^k \tag{C12}$$

$$R = \sum_{n=0}^{\infty} R_k m_q^k \quad . \tag{C13}$$

Now let $m_q = 0$ in Eqs. (C6,C7). Doing so eliminates all dependence on R and determines $\omega_{n,0}$. Next, set $m_q = 0$ in Eqs. (C8, C9) with $\omega_n \to \omega_{n,0}$. Doing so determines R_0. Now expand Eqs. (C6,C7) to first order in m_q. This step yields $\omega_{n,1}$ in terms of $\omega_{n,0}$ and R_0. Expanding Eqs. (C8, C9) to first order in m_q then determines R_1 in terms of $\omega_{n,0}$, $\omega_{n,1}$, and R_0 and so forth. Note that at any step of the recursion, the argument of any transcendental function is $\omega_{n,0}$. Hence, at any order, a solution for the $\omega_{n,k}$ and R_k exists.

The expansion of the bag model integrals continues by computing their derivatives with respect to m_q and using the expansions of N, R, etc. in terms of m_q as constructed above. Taking n derivatives of one of the integrals in Eq. (C3) yields new intregrals involving powers of r/R times products of the Bessel functions and their derivatives. Using the standard Bessel function recursion relations, the derivatives of the j_k can always be expressed in terms of other spherical Bessel functions. Since the j_k and their derivatives are finite at the origin, and since the radial bag integration is bounded above by R, the nth derivative of any of the integrals in Eq. (C3) is finite. Thus, each of the integrals in Eq. (C3) can be expanded in a Taylor series about $m_q = 0$.

Figure Captions

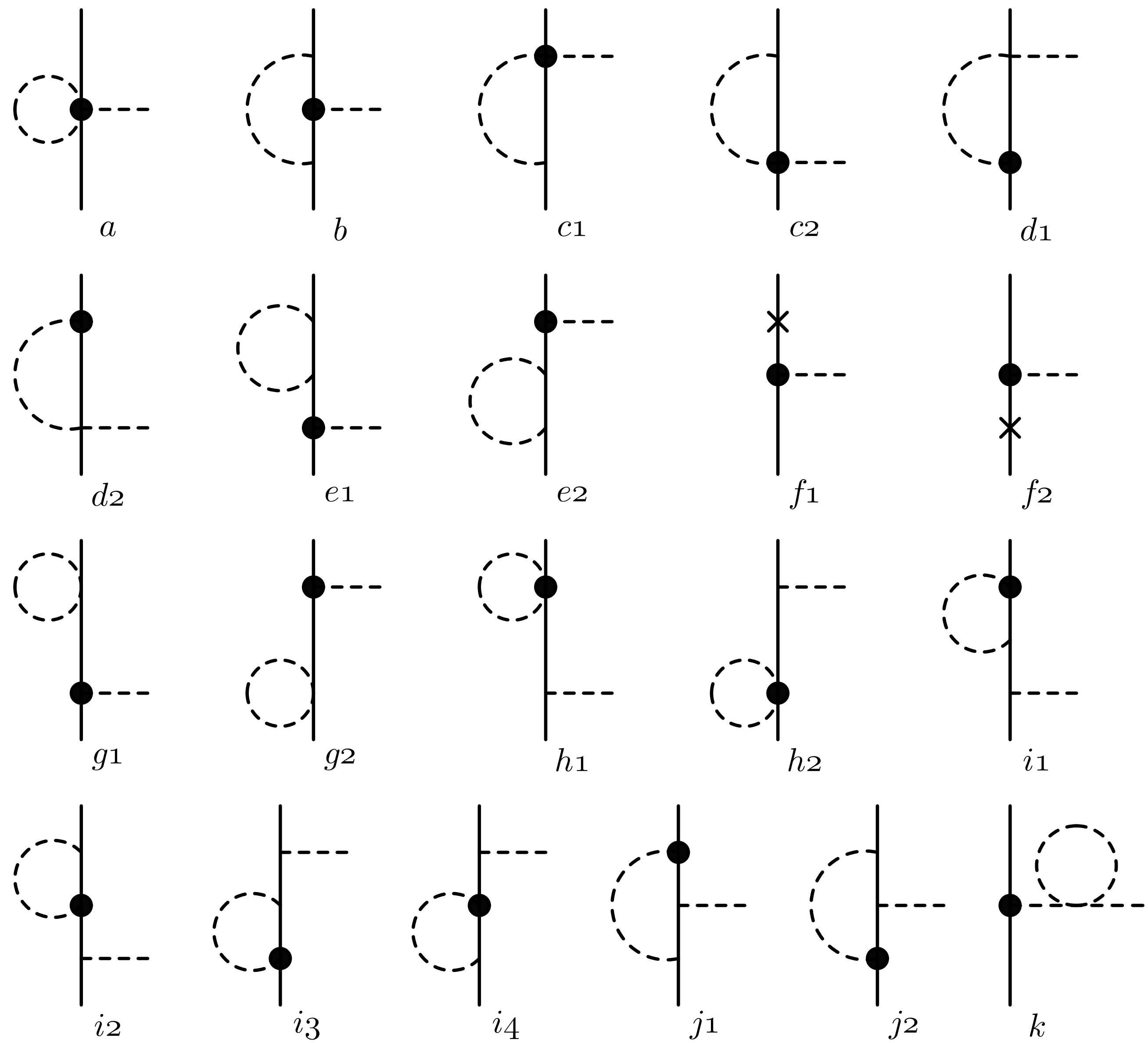
Figure 1. Meson-nucleon intermediate state contributions to the PV πNN vertex h_π. The shaded circle denotes the PV vertex. The solid and dashed lines correspond to the nucleon and pion respectively.

Figure 2. The chiral corrections from Δ intermediate state, which is denoted by the double line.

Figure 3.Diagrammatic representation of the SU(6)$_w$ components of $\langle B'M|\mathcal{H}_W^{PV}(\Delta S = 0, 1)|B\rangle$. Figs. 3a-c correspond, respectively, to $b_{t,v}$, c_v, and $a_{t,v}$. The wavy line denotes the action of $\mathcal{H}_W^{PV}$.

Figure 4. Chiral corrections to the $B \to B'M$ nonleptonic weak decay.

Figure 5. Quark line diagrams for the renormalization of h_π due to the axial PV $\pi\pi NN$ interaction. As in Fig. 3, the wavey line denotes the action of $\mathcal{H}_W^{PV}$. Fig. 5a shows a typical contribution to h_A^i. Figs 5b,c denote the corresponding loop corrections to h_π. Fig. 5(b) contains the disconnected $q\bar{q}$ insertions, while Fig. 5(c) gives a Z-graph contribution.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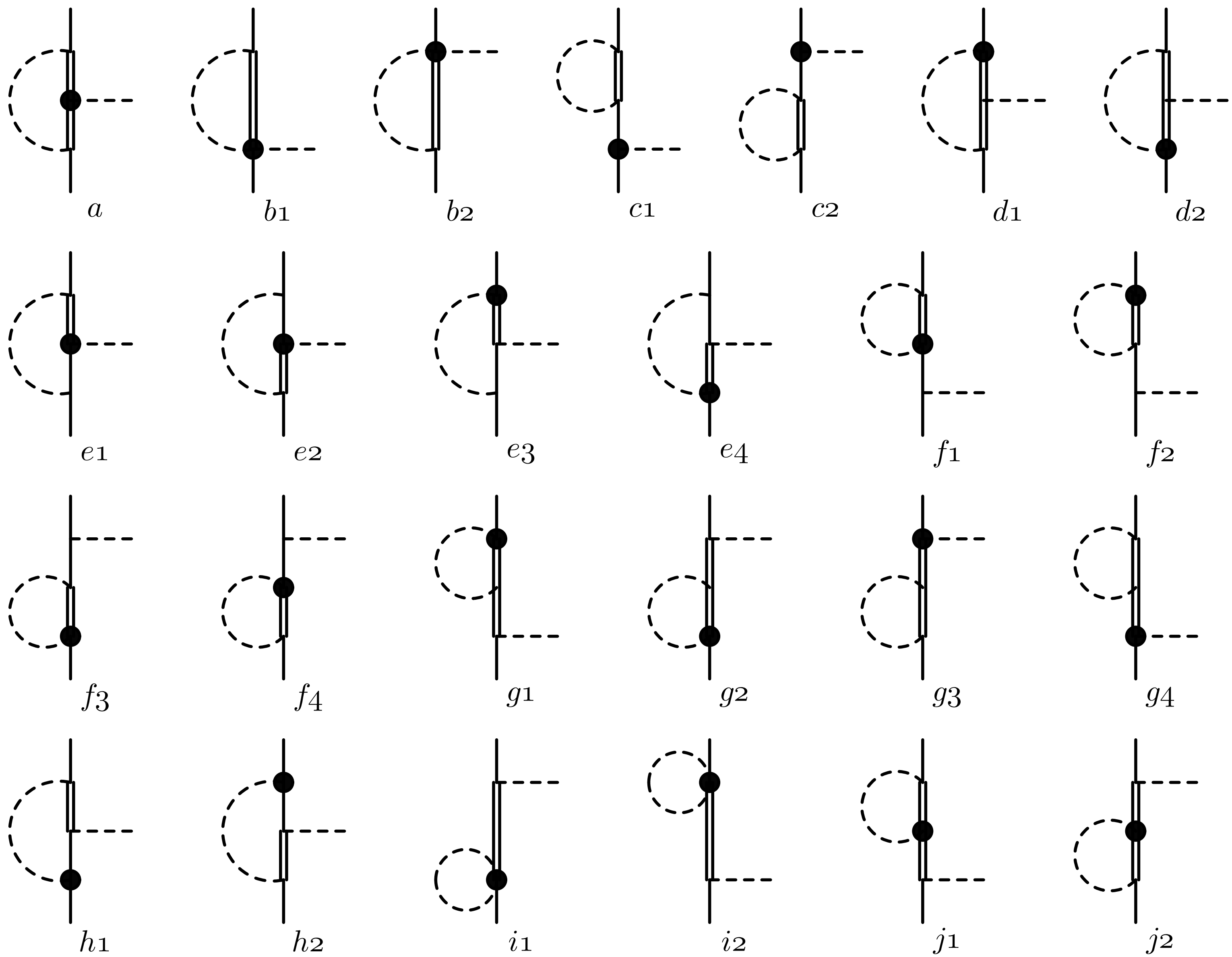

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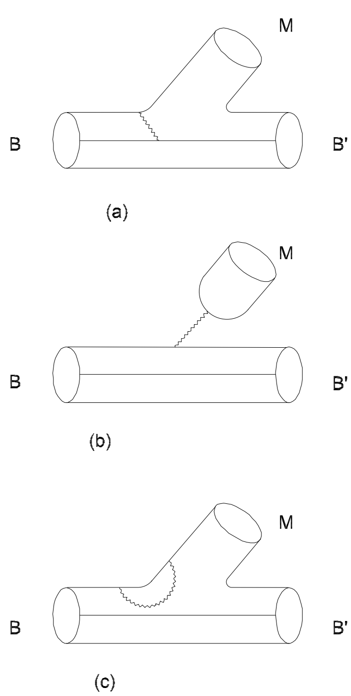
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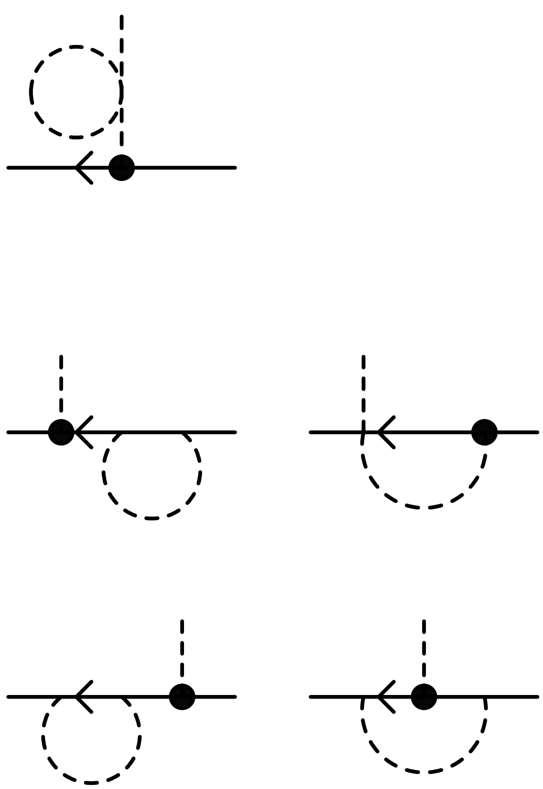

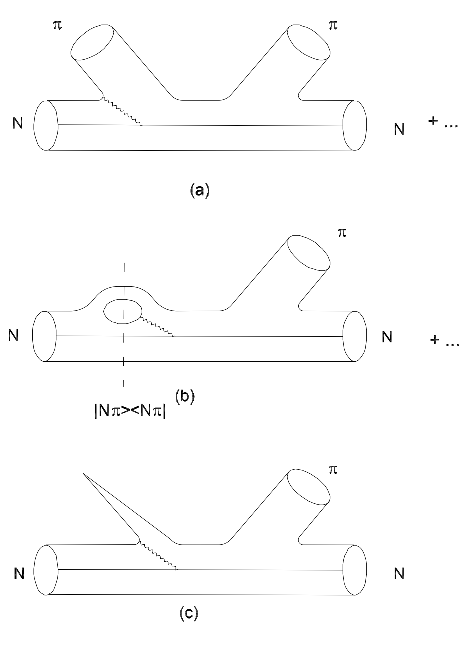

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