Route Choice Behavior in Risky Networks with Real-Time Information

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ROUTE CHOICE BEHAVIOR IN RISKY NETWORKS WITH REAL-TIME INFORMATION

A Thesis Presented

by

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To Leilani, who has grown with me in so many ways, through so many times, at so many distances

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ABSTRACT

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This research investigates route choice behavior in networks with risky travel times and real-time information. A stated preference survey is conducted in which subjects use a PC-based interactive maps to choose routes link-by-link in various scenarios. The scenarios include two types of maps: the first presenting a choice between one stochastic route and one deterministic route, and the second with real-time information and an available detour. The first type measures the basic risk attitude of the subject. The second type allows for strategic planning, and measures the effect of this opportunity on subjects' choice behavior.

Results from each subject are analyzed to determine whether subjects planned strategically for the en route information or simply selected fixed paths from origin to destination. The full data set is used to estimate route choice models that account for both risk attitude and strategic thinking. Estimation results are used to assess whether models that incorporate strategic behavior more accurately reflect route choice than do simpler path-based models.

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CHAPTER 1

INTRODUCTION

New technologies have opened the door to innovative congestion mitigation strategies such as Advanced Traveler Information Systems (ATIS), which update travelers on current network conditions to help them plan their travel around congestion and other delays [19].

In order to predict the effectiveness of ATIS implementations, it is necessary to accurately model traveler behavior in ATIS-equipped networks. Most models for route choice behavior assume that travelers choose fixed paths from origin to destination, and do not account for changes in network conditions that may occur during their trip [12]. Such models are inadequate for predicting the effects of ATIS implementations, since they cannot model the effects of information travelers receive en route to their destinations.

Recent research has investigated more sophisticated models that do account for en route information [2]. *Adaptive path* models assess route choice as a series of path choices, with the traveler reevaluating at every stage. While such models can account for diversions from an initial path, they assume that the traveler has not planned in advance for information that will become available during the trip.

Routing policy models employ choice sets that consist not of fixed paths, but of sets of decision rules (routing policies) that map network states to routing decisions [12]. Each routing policy in a choice set can be viewed as a distinct strategy for traversing the network.

While a number of studies have addressed the problem of optimal routing policies [14, 23, 18, 25, 20, 21, 30, 10, 11], econometric modeling of routing policy choice is a relatively unexplored topic [12]. A model is proposed in [10] and estimated using synthetic data in [12], but no estimation has yet been performed using empirical data.

This research analyzes choice observations from real human subjects. A stated preference approach is used since it enables experimental control and accuracy that is unavailable with field data. In the interests of context and clarity, an interactive graphical map is the chosen medium for the experiment.

A study using stated-preference data for route choice modeling in stochastic networks can be found in [16]. The study found that the subjects' risk behavior under the cognitive load of a simulated driving task was significantly different from their risk behavior under the relatively low load of a paper-and-pencil stated preference survey. Specifically, subjects under high cognitive load (in the simulator) weighted expected travel time far more heavily than travel time variability in making route decisions.

While the findings from [16] would seem troublesome for a stated-preference approach to route choice modeling, this research focuses mainly on the strategic aspect of route choice behavior, which can occur without cognitive load. For example, a regular commuter has already perceived and internalized the travel times and risks associated with the various links on the traffic network. In this study, travel time ranges are shown to subjects a priori, to indicate general characteristics of a roadway. This simulates a commuter's pre-existing knowledge of the roadway, along with any traffic reports or other information he may have acquired before departing. En route information is presented as specific travel times, which greatly reduces cognitive load and minimizes errors in perception of variability.

Many studies have investigated route choice in networks with real-time information, including adaptive behavior, which describes a subject switching from a previously chosen or experienced route [24, 1, 17, 27], while others use choice sets comprised of fixed paths [8, 22, 2].

The primary objective of this research is to determine whether drivers plan strategically when making routing decisions. For the purposes of this research, "strategic" is defined as accounting for the future availability of information and for any detours that might be taken based on such information. Strategic drivers choose routes according to routing policies, sets of decision rules based on information available at the time of each decision.

The specific questions being addressed are:

1. Do drivers think strategically when planning routes in uncertain networks with real-time, en route information?

2. Can observations of route choice be used to estimate a model which accounts for strategic drivers?

Since strategic choice is only distinguishable in networks with risky travel times, it is necessary to account for the effects of risk on route choice. This research therefore includes a thorough analysis of risk behavior, and assesses the effect of delay probability on risk attitude.

CHAPTER 2

EXPERIMENTAL DESIGN

The experiment was conducted as an interactive survey, using graphical maps with a point-and-click interface. Subjects attended guided survey sessions, during which up to 15 subjects were each stationed at a PC. A study coordinator introduced each session with a presentation outlining the nature and purpose of the study, and providing instructions for completing the survey. After completing several warm-up scenarios, subjects completed six groups of scenarios, with breaks between each. The study coordinator introduced each warm-up secnario and scenario group to provide context and highlight any changes.

Results from each subject were automatically stored by the software. A result file for each user records the details of every scenario and the routes chosen by the user. These data are used in the analysis detailed in Chapter 3.

2.1 Interface

The survey is presented to subjects as a graphical map. This approach is chosen because of the simplicity and clarity as compared to describing the scenarios in written or verbal form. The visual format provides an easily comprehensible representation of each scenario, and helps the user relate to the intended context.

The graphical map interface (shown in Figure 2.1) is an online application designed specifically for this experiment and implemented using Adobe Flash and PHP. The interface consists of a map of the Boston area, with green arrows overlaid to indicate usable links. The user's current position is represented by a large red dot labeled "You". When the user places the cursor over a link that is accessible from the current position, the link will glow bright green. Clicking on a usable link causes the position marker to travel across the link to the destination node of the link. When the final destination is reached, the user is unable to make any other movements, and a button will appear to allow advancement to the next map.

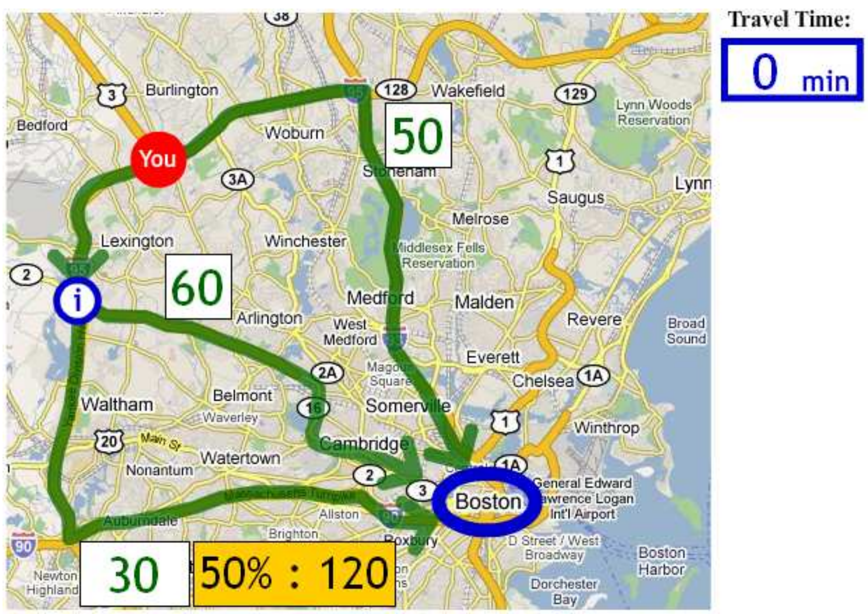

Figure 2.1. Example map interface, with information and detour

Directly adjacent to each link is a white label indicating the usual travel time of the link. Each map has one stochastic link, which has an additional yellow label to indicate the chance of a delay and the full travel time of the link in the event of a delay. If the user travels across this link, the realized travel time will be revealed in the white label, and the yellow label will disappear. This scheme, which is similar to a weather forecast, was deemed a simple and familiar way to present discrete distributions with varying degrees of probability.

For maps that include real-time information, a blue "i" icon is shown at the node where the user will receive the information. When the user arrives at that node, the actual travel time of the stochastic link is revealed, and the yellow label disappears. Due to the location of the information node, this simulates travelers being informed of conditions on the stochastic link before they actually traverse it.

At all times, an accumulated travel time counter is displayed at the top-right of the screen. This counter indicates the travel time spent so far on the current map. It updates after every link traversal, and resets to zero after each map is completed. Since each map is considered as an independent trip, no measure of accumulated travel time over multiple maps is displayed, nor is any long term "score" kept. Subjects were

2.2 Map Types

Two different map types are used in the survey: simple risk maps and strategy maps. Each risk scenario is presented once in both map types (though not in the same order), so that differences in choice behavior can be observed by direct comparison.

2.2.1 Simple Risk

The simple risk maps are aimed at determining a subject's risk attitude without the influence of real-time information or detours. The subject decides between two routes, one with a deterministic travel time, and the other with a stochatic travel time. The form of this network type is represented in Figure 2.2.

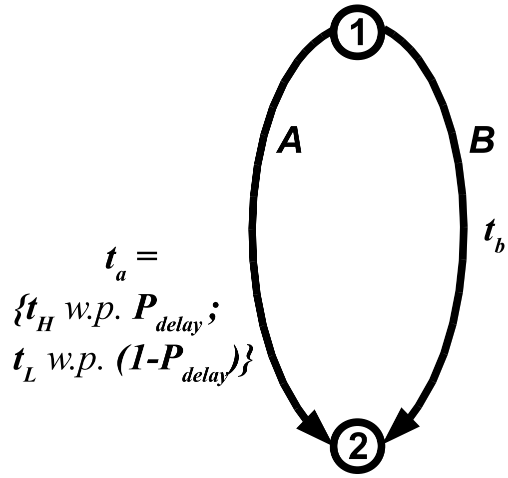

Figure 2.2. Abstract network for simple risk map type

The exact travel time of the deterministic route (route B) is shown to the user before the trip begins. The stochatic travel time on A has two possible outcomes, t_L and t_H, with the probability of the high travel time, $P_{delay} = P(t_a = t_H)$ indicated to the user before the trip begins. The realized travel time of route A is revealed to the subject only if and when he uses the stochastic link.

Risk scenarios are generated by a factorial design, with t_H, t_b, and P_{delay} as the design factors. t_L is fixed at 30 minutes throughout all scenarios. t_H can take the values 40, 50 and 60; t_b can take values from 35 to $t_H - 5$, such that the deterministic route is not dominated by the stochastic route; P_{delay} can take the values 0.2, 0.5 and 0.8. In the strategy map scenarios, the high time on the stochastic link, t_M, is fixed at 120 minutes. Figure 2.1 shows travel times for several example scenarios. $E[t_a]$ represents the expected travel time of the stochastic alternative, and $E[t_b - t_a]$ represents expected time savings of the stochastic alternative over the deterministic alternative.

t_b	t_L	t_H	P_{delay}	$E[t_a]$	$E[t_b - t_a]$
35	30	40	0.2	32	3
40	30	60	0.5	45	-5
45	30	50	0.2	34	9
50	30	60	0.5	45	5
55	30	60	0.8	54	1

Table 2.1. A sample of scenario travel times

2.2.2 Strategy Maps

The strategy maps measure the extent to which a subject recognizes and utilizes strategically advantageous real-time information. In these scenarios, subjects choose between a deterministic route and a route which branches into a stochastic link and a deterministic detour. Subjects who choose the latter will learn the realized travel time of the stochastic link before they must decide whether to use it. This allows the subject to choose the faster of the two links.

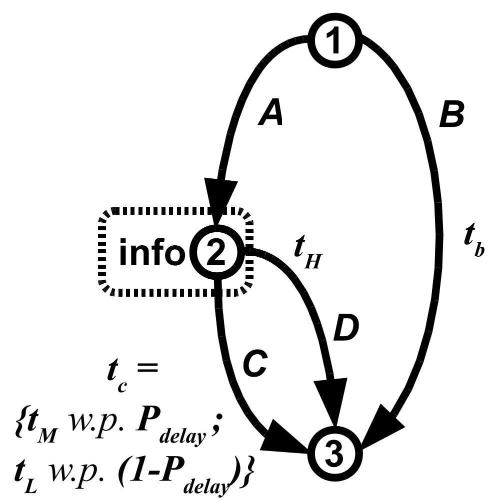

Figure 2.3. Abstract network for routing strategy tests

Subjects who do not plan in advance for this informed choice will view the stochastic branch as two individual paths, with expected travel times $E[t_c]$ and t_d. Subjects who recognize the strategic advantage of the provided information, however, will assess the stochastic branch based on the expected minimum travel time of the two options available (links C and D). That is, they plan in advance for the informed choice they will make later in the trip.

Each set of travel times t_L and t_H used in a simple risk map is also used a corresponding strategy map. t_L, however, appears as the low travel time for link C, while t_H is the deterministic travel time for link D. Since the travel time of link C is shown while the opportunity to detour to link D is still available, subjects who make use of this information will avoid any delay on link C and instead use link D, with travel time t_H. Strategic-thinking subjects will therefore assess the stochastic branch in the strategy map as equivalent to the stochastic path in the corresponding simple risk map. Such subjects are expected to exhibit very similar risk attitudes between the two map types.

The high travel time t_M for link C is chosen such that $E[t_c]$ always exceeds t_b. The value of t_M is further inflated to ensure that, if a subject directly compares link C and and link B (i.e., does NOT account in advance for the information or detour), the perceived risk on link C is unreasonably high. Since the travel time of link D (t_H) is deterministic and always higher than t_b, it is extremely unlikely that non-strategic users will choose the stochastic branch, and such users are expected to consistently choose the deterministic branch.

CHAPTER 3

ANALYSIS OF RESULTS

In total, the dataset includes over 3400 observations from 74 individual subjects. Subjects were recruited from the University of Massachusetts student and staff community, as well as the surrounding area. The mean age of subjects is 24.2 years and mean driving experience is 6.9 years. 40 (or 54%) of the subjects are male, and 34 (or 46%) female. In an exit questionnaire taken after all maps were completed, subjects answered a free-response question regarding strategic behavior. Each response was read carefully and categorized as strategic, non-strategic, or indeterminate. 46 responses (62%) were considered clearly strategic, while 18 responses (24%) were considered clearly non-strategic. 10 responses (14%) were indeterminate.

3.1 Risk Attitude Analysis

Strategic behavior is only identifiable in networks with risky alternatives. Therefore, in order to accurately measure strategic thinking, it is necessary to control for the effects of risk attitude. The simple risk maps are designed to measure subjects' risk attitudes without the complication of strategic alternatives. The findings from these maps are used to develop a risk attitude model form that is used in the strategic thinking analysis to account for the effects of risk.

Before estimating a risk model, a less formal analysis is used to develop an intuitive understanding of the results. Reasonable measures of benefit and risk are defined and applied to the results to analyze individual risk behavior. This qualitative analysis seeks to address two important questions:

1. What proportions of subjects exhibited risk prone, risk averse, and risk neutral behavior?

2. Did subjects' risk attitudes vary with P_d, the probability of delay?

The measure of benefit is here defined as $Ben = E[t_b - t_a]$, which represents the expected savings in travel time by choosing the stochastic alternative in a given scenario. A positive value of Ben indicates that the stochastic alternative has a lower expected travel time than that of the deterministic alternative.

The measure of risk used here is the standard deviation of outcomes for the stochastic alternative t_a, defined as $Risk = \sigma_{t_a}$. The ratio of benefit to risk $BR = Ben/Risk$ is therefore used to measure risk attitude. We are primarily concerned with the sign of BR, which determines risk attitude. A subject who accepts a stochastic alternative with $BR < 0$ exhibits risk-prone behavior. A subject who refuses any $BR \leq 0$ exhibits risk-averse behavior. A subject who refuses all $BR < 0$ and accepts all $BR > 0$ exhibits risk-neutral behavior.

The scenarios presented to subjects represented a range of BR values from -1.38 to 1.38. Ideally, if BR is indeed the sole criterion in subject's decision-making, all subjects would exhibit a clear threshold BR value. They would always choose a stochastic alternative with a BR value above the threshold, and always refuse a stochastic alternative with a BR value below the threshold. Most subjects did not exhibit a perfect threshold, but did exhibit a relatively small "ambivalent range". The bounds of this range, BR_{high} and BR_{low}, are determined by the highest BR for which the subject did not choose the stochastic alternative, and the lowest BR for which the subject did choose the stochastic alternative. A subject with a positive BR_{low} is risk averse, since he never chose the stochastic alternative unless there was a positive expected benefit. A subject with a negative BR_{high} is risk prone, since he would accept the risk of the stochastic option even if the expected benefit is negative. For a subject whose ambivalent range includes both positive and negative BR values, no definite assessment can be made regarding risk attitude. Changes in each subject's range across different values of P_d, however, are still meaningful in evaluating the effect of P_d on risk attitude.

Figures 3.1, 3.2, and 3.3 illustrate these ranges for each of the 74 individual subjects. It is clear that the probability of delay on the stochastic alternative, P_d, has a strong effect on risk attitude. The vast majority (88%) of subjects exhibit perfectly risk-averse behavior when $P_d = 0.2$, with only 12% of subjects ever choosing the stochastic alternative when $BR \leq 0$. Subjects are mainly risk-neutral or risk-averse when $P_d = 0.5$, with 86% of subjects choosing the stochastic alternative when $BR = 0$, but only 12% accepting $BR < 0$. For $P_d = 0.8$, the majority of subjects (80%) exhibit perfectly risk-prone behavior, with only 7% of subjects never choosing the stochastic alternative when $BR \leq 0$.

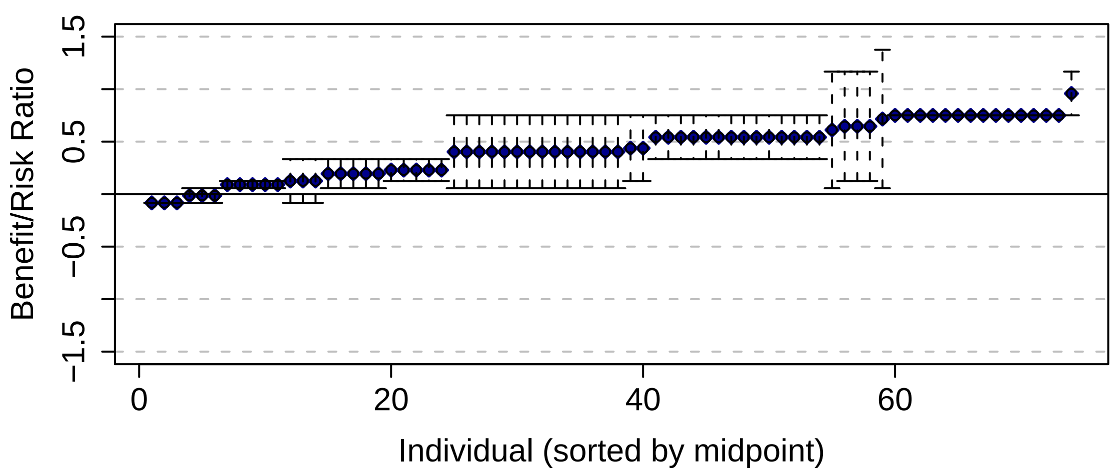

Figure 3.1. Ambivalent range for each user, for scenarios with $P_d = 0.2$

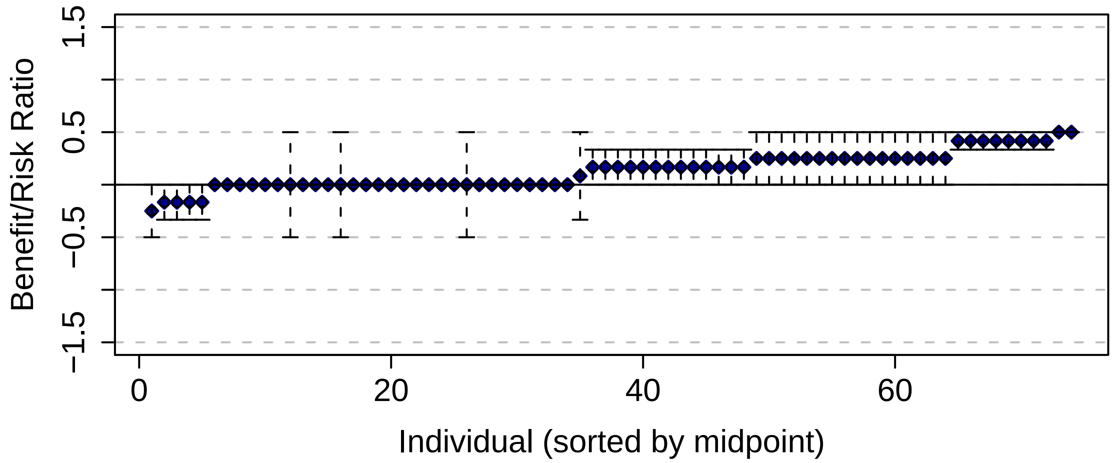

Figure 3.2. Ambivalent range for each user, for scenarios with $P_d = 0.5$

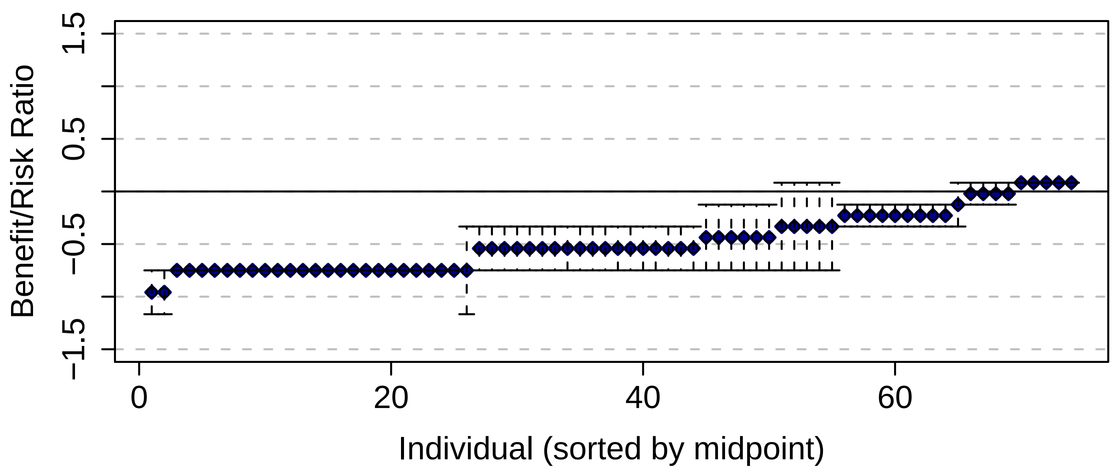

Figure 3.3. Ambivalent range for each user, for scenarios with $P_d = 0.8$

These results strongly support the findings of Kahneman and Tversky [15], namely the certainty effect and the overweighting of low probabilities. The lower travel time t_L on the stochastic alternative can be viewed as a reference point, and delays are assessed relative to this reference point. The subject risks a relatively large delay when choosing the stochastic alternative, but is guaranteed a relatively small delay when choosing the deterministic alternative. The certainty effect is observed when a subject underweights an outcome that is merely probable (e.g. an 80% chance of being delayed on the stochastic alternative) relative to an alternative with a certain outcome (e.g. a 100% certain delay on the deterministic alternative). This effect tends to increase risk proneness at high probabilities. For low probabilities, subjects overweight the possible delay on the stochastic alternative, and thus exhibit risk-averse behavior. Avineri and Prashker [3] confirm the two phenomena in a stated-preference route choice context, via a series of experiments similar to those in [15].

The appearance of these effects is likely due in part to the description-based approach used in the experiment. Since the subjects' only knowledge of traffic conditions is derived from the travel times and probabilities shown on the map, their perception is dependent on their interpretation of the information provided, rather than on previous experience. Earlier research, such as that of Ben-Elia, et al. [6], and Barron and Erev [4] [9], has shown that the effects can be diminished or even reversed when subjects have repeated experience with the particular situation(s) presented.

3.2 Strategic Behavior Analysis

Since each travel time scenario is adapted for both simple and strategy map types, it is possible to directly compare a subject's results for each map type in a given scenario to classify behavior as strategic, non-strategic, and/or detour-biased.

The strategy maps are designed such that only two *reasonable* routing policies are available, hereafter called the "deterministic policy" and the "stochastic policy". The deterministic policy involves simply taking Link B as shown in Figure 2.3. The stochastic policy involves taking Link A to Node 2, learning the actual travel time of link C, and choosing the faster of links C and D. The arrangement of the travel times ensures that the distribution of outcomes for the stochastic policy is identical to that of the stochastic path in the corresponding simple map. The fixed-path stochastic alternative (link A to link C), however, is clearly worse than that in simple map. The inflated high outcome t_M ensures that the expected travel time of the stochastic link is much higher – and thus the expected benefit much lower. A non-strategic subject would only choose this alternative if he/she were extremely risk-prone. The other fixed-path alternative of the stochastic branch, the deterministic detour, always has a higher travel time than the deterministic branch, and the deterministic branch dominates.

If a subject's choices differ between the two map types for a given scenario, it is likely that he/she perceived the risk differently between the two map types. For example, a subject who chooses the stochastic path in the simple map and the deterministic branch in the strategy map may be perceiving a decreased benefit and/or higher risk in the strategy map. This is consistent with a non-strategic assessment of the network, and the number of such choice mismatches provides some indicator of whether a subject is thinking strategically.

Table 3.1 enumerates the possible inferences from choice discrepancies. The first two columns indicate the subject's recorded choice in the two map types of a particular scenario. Subjects who accept the the same risk in the strategy map that they accepted in the simple risk map are deemed "strategic", meaning they recognize the potential advantage of future information. Users who are unwilling to accept the risk in the strategy map are deemed "non-strategic", as they are assumed not to be recognizing the future detour availability, but rather assessing the network in terms of fixed paths.

The third row addresses subjects who accept greater risk in a strategy map than they do in a simple risk map. Such behavior would suggest the effect of a "detour bias", or preference for routes with information and opportunities for delay-mitigating detours. Such subjects are also likely strategic thinkers, since it is very unlikely

Simple Risk	Strategy	Inference
Stoch.	Stoch.	Strategic
Stoch.	Determ.	Non-Strat.
Determ.	Stoch.	Detour Bias
Determ.	Determ.	N/A

Table 3.1. Inference table for qualitative analysis of strategic behavior

that the detour bias would outweigh the perceived extreme risk in a non-strategic evaluation of the stochastic branch.

Lastly, no inference can be made from cases in which the subject chooses the deterministic alternative in both map types. Such a subject has not accepted the simple risk presented, and is not expected to accept the perceived risk associated with either a strategic or non-strategic assessment of the strategy map.

Since it is unreasonable to expect perfectly consistent behavior from any subject, qualitative analysis of strategic behavior is carried out by counting the number of each mismatch type for each subject. The histograms in Figure 3.4 show proportions of strategy map choices which matched the choice in the corresponding strategy map. Subjects with a higher percentage exhibited more conistent choice behavior between map types than those with a lower percentage.

(a) Simple Map Choice = Stochastic (b) Simple Map Choice = Deterministic

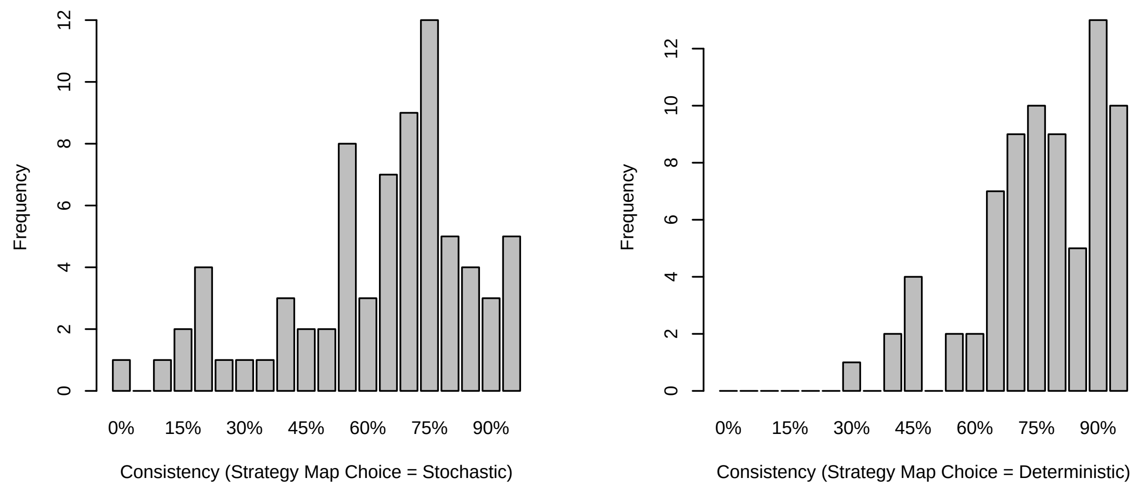

Figure 3.4. Frequency of choice consistency proportions.

The percentages in Figure 3.4(a) measure how closely subjects' choices correspond to a strategic outlook. Each scenario for which the subject chose the stochastic alternative in both map types is considered a strategic choice. The percentages in Figure 3.4(b) measure any detour bias subjects may have, favoring the stochastic branch in the strategy map. Users with low percentages frequently chose the stochastic branch in the strategy map even when they had chosen the deterministic alternative in the corresponding simple map.

Since very few subjects are entirely consistent or entirely inconsistent, a threshold of 75% consistency is used to informally assess subjects as strategic or detour-biased. A subject who is consistent in choices of the stochastic branch (Figure 3.4(a)) *at least* 75% of the time is assessed as strategic. A subject who is consistent in choices of the deterministic branch (Figure 3.4(b)) *at most* 75% of the time is assessed as detour-biased. Note that a subject may be assessed as both strategic and detour-biased. Applying this threshold, approximately 39% of subjects are found to be strategic thinkers, and approximately 32% are found to be detour-biased.

A more general way to measure strategic behavior is to classify choice observations rather than subjects. Since the models presented in Section 4.1 distinguish strategic behavior on a per-observation basis rather than a per-user basis, it is useful to calculate per-observation percentages for comparison. By this measure, 31% of all observations are distinguishably strategic (though not necessarily detour-biased), while 15% are distinguishably non-strategic. 11% are distinguishably detour biased, and 43% are indistinguishable.

CHAPTER 4

MODELS

Several model specifications under the discrete choice modeling framework [5] are estimated using the data collected in the experiment. Two main types of simple risk model are estimated, both of mixed Logit [28] form. The first type uses expected travel time ETT_x and risk as explanatory attributes. For these models, risk is measured by the standard deviation of outcomes, σ_x. The second type is a non-expected-utility model based on Cumulative Prospect Theory (CPT). This model calculates utilties based on the potential outcomes for each alternative.

The strategy models are latent-class logit models, with classes for strategic and non-strategic choice. Each class has a distinct utility function. While both functions are of the same form for a given latent-class model, the explanatory attributes are changed to reflect differeng perceptions of the alternatives.

All estimation is performed using BIOGEME [7], with latent-class estimation making use of the DONLP2 [26] algorithm.

The risk models treat the data as panel data, since multiple observations are collected for each subject. Since risk attitudes are expected to vary across subjects, parameters measuring risk attitude (e.g. travel time standard deviation) are estimated as random parameters distributed across subjects. Independent normal (or log-normal, where appropriate) distributions are assumed.

The mixed logit model is defined as follows:

Consider a single subject's sequence of chosen alternatives, one for each scenario, $x = \{x_1, \ldots, x_S\}$, where S is the number of scenarios. If the utility random errors are assumed to be i.i.d. extreme value type I over scenarios, subjects and alternatives, the probability conditional on B that a subject n makes this sequence of choices is the product of logit functions:

$$L_n(X|B) = \prod_{s=1}^{S} \frac{exp(V_{x_s ns}|B)}{\sum_j exp(V_{jns}|B)} \tag{4.1}$$

$L_n(X\|B)$	:	Conditional probability of individual n selecting the set of alternatives X
B	:	Vector of model parameters to be estimated
$f(B)$	:	probability density function of random parameters B
$V_{x_s ns}$	:	Systematic utility of alternative x_s, for individual n in scenario s

The unconditional probability of individual n selecting the set of alternatives X is the integral of this product over the entire distribution of B:

$$P_n(X) = \int L_n(x|B) f(B) dB \tag{4.2}$$

4.1 Simple Risk Models

In order to verify and quantify the findings from the qualitative analysis, a model is developed to measure the effects of travel time and risk, in the absence of any strategic alternatives. The model is estimated using data from the simple risk maps only, so no distinction is necessary between strategic and non-strategic behavior.

4.1.1 Benefit/Risk Models

The simplest form of the model, named *SR1*, uses the utility functions specified below. V_{dtm} (Equation 4.3) and V_{stc} (Equation 4.4) are the systematic utilities of the deterministic and stochastic alternatives, repsectively.

$$V_{dtm} = \beta_{ett} ETT_{dtm} \tag{4.3}$$

$$V_{stc} = \beta_{ett} ETT_{stc} + \beta_{std} \sigma_{stc} + ASC_{stc} \tag{4.4}$$

ETT_x : Expected travel time of alternative x

σ_{stc} : standard deviation of possible outcomes for the stochastic alternative

ASC_{stc} : Alternative-specific constant for the stochastic alternative

Estimation for this model predicts negative mean values for β_{ett} and β_{std}, which are sensible since both increased travel time and increased risk are generally expected to diminish the utility of the alternative. However, since the results of the quantitative analysis suggest that the probability of delay (P_d) affects risk attitude, an expanded model *SR2* is estimated, which includes separate parameters for each level of probability (0.2, 0.5, and 0.8). Equations 4.5 and 4.6 define the systematic utility functions for model *SR2*.

$$V_{dtm} = \beta_{ett} ETT_{dtm} \tag{4.5}$$

$$V_{stc} = \beta_{ett} ETT_{stc} + \theta_{p02} \sigma_{stc} * (P02) + \theta_{p05} \sigma_{stc} * (P05) + \theta_{p08} \sigma_{stc} * (P08) + ASC_{stc} \tag{4.6}$$

$P02$: 1 if $P_d = 0.2$; 0 otherwise

$P05$: 1 if $P_d = 0.5$; 0 otherwise

$P08$: 1 if $P_d = 0.8$; 0 otherwise

Results from models *SR1* and *SR2* are compared in Table 4.1. While estimates for both models are reasonable, a likelihood-ratio test shows that *SR2* is significantly better than *SR1* at a 1% significance level. The effect of P_d is therefore included in all subsequent models presented. The values of the estimates also agree very well with the findings from the quantitative analysis, specifically that higher-probability risks are underweighted and lower-probability risks are overweighted. The results also suggest a general risk proneness when $P_d = 0.8$. Note that all t-statistics presented with estimation results are robust.

	SR1	**SR2**
$\boldsymbol{\beta_{ett}}$ (t-stat)	-0.242 (-17.89)	-0.439 (-15.07)
$\boldsymbol{\beta_{std}}$ mean (t-stat) stdev (t-stat)	 -0.0549 (-5.80) 0.0014 (1.47*)	
$\boldsymbol{\theta_{p02}}$ mean (t-stat) stdev (t-stat)		 -0.0637 (-2.46) 0.0861 (3.71)
$\boldsymbol{\theta_{p05}}$ mean (t-stat) stdev (t-stat)		 -0.0286 (-1.41*) 0.0006 (0.19*)
$\boldsymbol{\theta_{p08}}$ mean (t-stat) stdev (t-stat)		 0.235 (8.26) 0.0794 (3.58)
$\boldsymbol{ASC_{stc}}$ mean (t-stat) stdev (t-stat)	 -0.0452 (-0.48) 0.447 (3.93)	 -0.621 (-2.66) 0.563 (4.24)
Total Param.	5	9
Observations	1767	1767
Individuals	74	74
No. of Draws	1000	1000
Adj. ρ^2	0.252	0.345
Final LL	-922.498	-793.533
LR Test Stat.		**257.93**

Table 4.1. Comparison of estimates and statistics for models SR1 and SR2.

Also notable are the estimates for the standard devations of θ_{p02} , θ_{p05}, and θ_{p08}. The standard deviations of θ_{p02} and θ_{p08} are significantly different from 0, suggesting that there is variation in risk attitude between subjects. The standard devation of θ_{p05} is not significant, suggesting that subjects are more uniform in their risk attitudes when $P_d = 0.5$.

4.1.2 Cumulative Prospect Theory (CPT) Model

While model *SR2* is capable of accounting for the 3 levels of delay probability presented in this study, it is of limited use in explaining or predicting general choice behavior. In reality, a single link or route can have any number of possible outcomes, with any level of probability. Model *SR2* is incapable of predicting behavior in these cases, since it accounts for 2 outcomes at most per alternative, and only at the probability levels specified.

The model results and the findings from the direct analysis suggest that subjects' risk attitudes change according to the delay probability. In order to more generally explain this behavior, a parameterized functional form must be employed.

The Cumulative Prospect Theory (CPT) developed by Tversky and Kahneman [29], and applied to strategic route choice by Gao et al. [13], proposes a functional form that accounts for weighted perceptions of probabilities. The CPT choice model rates each alternative based on its set of potential outcomes and the probability of each. Under CPT, subjects evaluate outcomes in reference to a "status quo", viewing some outcomes as gains and others as losses. Subjects' sensitivity to these gains and losses diminishes as the difference from the status quo increases. In other words, the difference between a 10-minute delay and an 11-minute delay would weigh more heavily in a subject's mind than the difference between a 100-minute delay and 101-minute delay.

Each outcome is transformed by a value function $v(\Delta t)$, where Δt represents the difference between the outcome and the reference point. The value function models the subject's diminishing sensitivity to more extreme outcomes, and is defined as follows:

$$v(\Delta t) = \begin{cases} \Delta t^{\alpha}, & \text{if } \Delta t > 0 \\ -\lambda(-\Delta t)^{\beta}, & \text{if } \Delta t \leq 0 \end{cases} \tag{4.7}$$

α : Value parameter for gain outcomes

β : Value parameter for loss outcomes

The probabilities for each prospect are also transformed by a weighting function $w(P_j)$ – where P_j is the objective probability of outcome j – to account for the subject's perception.

$$w^{+}(P_j) = \frac{P_j^{\rho}}{\left(P_j^{\rho} + (1-P_j)^{\rho}\right)^{1/\rho}}, \quad w^{-}(P_j) = \frac{P_j^{\delta}}{\left(P_j^{\delta} + (1-P_j)^{\delta}\right)^{1/\delta}} \tag{4.8}$$

ρ : Probability weighting parameter for gain outcomes

δ : Probability weighting parameter for loss outcomes

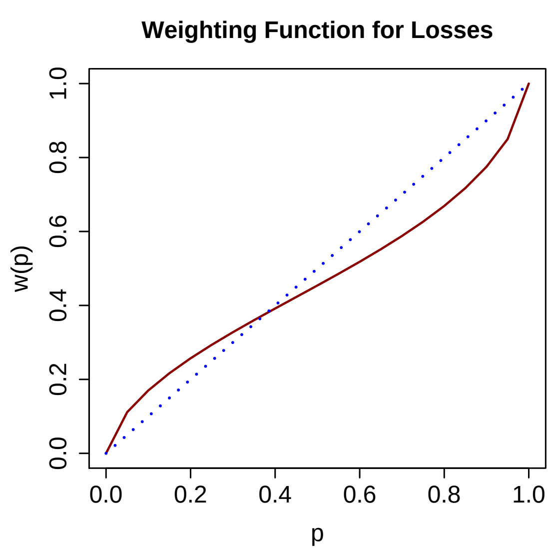

Figure 4.1. Weighting Function $w^{-}(p)$ with $\delta = 0.69$

The loss-domain weighting function $w^{-}(P_j)$ defined in figure 4.8 is illustrated in figure 4.1. The dotted line represents a perfectly objective perception of probability

($w^-(P_j) = P_j$). The solid curve represents a more realistic perception, in which low probabilities are overweighted and high probabilities are underweighted.

The CPT model is rank-dependent, which means that the weighted probability of each outcome also accounts for all outcomes worse than the reference point. The utility of a prospect f^- with $m+1$ loss outcomes is defined as

$$Y(f^-) = \sum_{j=-m}^{0} \pi_j^- v(x_j), \tag{4.9}$$

where π_j^- is the decision weight for loss outcome j. The outcomes are arranged in increasing order and the negative subscripts indicate negative (loss) outcomes. π_j^- is calculated from the weighting functions of cumulative probabilities:

$$\pi_j^- = w^-(p_{-m} + \cdots + p_j) - w^-(p_{-m} + \cdots + p_{j-1}), \quad -m+1 \leq j \leq 0, \tag{4.10}$$

and $\pi_{-m}^- = w^-(p_{-m})$.

Since subjects in this experiment have no prior experience with the network, and thus no "usual" route, the shortest possible travel time, t_L, is chosen as the reference point for the CPT model. As presented to subjects, this is the usual travel time of the stochastic alternative, with t_H presented as a potential delay. With t_L as the reference, all other travel times are considered delays, or losses. The utility functions for the alternatives are:

$$\begin{aligned} Y_{determ} &= -\lambda(t_D - t_L)^\beta w^-(1) \\ &= -\lambda(t_D - t_L)^\beta \end{aligned} \tag{4.11}$$

$$\begin{aligned} Y_{stoch} &= -\lambda(t_H - t_L)^\beta w^-(P_d) - \lambda(t_L - t_L)^\beta (w^-(1) - w^-(P_d)) + ASC_{stoch} \\ &= -\lambda(t_H - t_L)^\beta \frac{P_d^\delta}{\left(P_d^\delta + (1-P_d)^\delta\right)^{1/\delta}} + ASC_{stoch} \end{aligned} \tag{4.12}$$

The CPT model is estimated assuming that all parameters are random variables across subjects. The parameters λ and ASC_{stoch} are distributed normally, while β and δ are distributed log-normally.

	Estimate
$\boldsymbol{\lambda}$	
mean (t-stat w.r.t. 0)	0.386 (5.61)
stdev (t-stat w.r.t. 0)	0.0447 (0.84*)
$\boldsymbol{\beta}$	
mean (t-stat w.r.t. 1)	1.000 (0.00*)
stdev (t-stat w.r.t. 0)	0.1213 (4.41)
$\boldsymbol{\delta}$	
mean (t-stat w.r.t. 1)	0.628 (-10.15)
stdev (t-stat w.r.t. 0)	0.212 (5.44)
$\boldsymbol{ASC_{stoch}}$	
mean (t-stat w.r.t. 0)	-0.682 (-3.40)
stdev (t-stat w.r.t. 0)	0.390 (1.08*)
Observations	1767
Individuals	74
Total Param.	8
Adj. ρ^2	0.307
Final LL	-840.586

Table 4.2. Estimation results for the CPT simple risk model

The estimates for the mean and standard devation of δ support the finding that subjects have varying perceptions of probability, but generally follow the pattern of overweighting low probabilities and underweighting high probabilities. While this finding was already well-supported by model *SR2*, the CPT model offers a functional form for describing this behavior.

Curiously, the estimate for the mean of β suggests that, on average, subjects' sensitivity to loss remains fairly constant for all magnitude of loss. The estimate for the standard deviation of β, however, suggests that a considerable portion of subjects did exhibit some level of increasing or decreasing sensitivity.

The overall fit of the model, while not strictly comparable with that of model *SR2*, suggests that subjects' thinking largely matched the expectations of CPT.

4.2 Strategic Choice Model

In analyzing choice behavior in the strategy maps, it must be recognized that subjects may be perceiving different choice sets. Specifically, subjects viewing the network strategically will choose from a set of routing policies, while subjects viewing the network non-strategically will choose from a set of fixed paths.

Model *SR2* can be applied to the data from the strategy maps, if it is assumed momentarily that all users uniformly think either strategically or non-strategically. The difference between the two assumptions is in the utility function for the stochastic branch. Referring to the network shown in Figure 2.3, a strategic subject would see the expected travel time of the stochastic branch as $ETT_{stc,strat} = P_d{*}t_H{+}(1{-}P_d){*}t_L$, since the worst-case travel time is the detour on link D. A non-strategic subject, on the other hand, would view the stochastic branch as two separate paths. One path (link A - link D)is deterministic with travel time t_D. Since t_D is always greater than t_B, the route using link B dominates and the link A - link D path is not likely to be chosen. The other path on the stochastic branch (link A - link C) is stochastic, with expected travel time $ETT_{stc,path} = P_d * t_M + (1 - P_d) * t_L$, and standard deviation $\sigma_{stc,path} = (t_H - t_L)\sqrt{P_d(1 - P_d)}$. Since the non-strategic subject does not recognize the availability of the detour, the perceived worst-case travel time is t_M, which is always much greater than t_D.

The systematic utility of the stochastic branch is therefore

$$V_{stc} = \beta_{ett}ETT_{stc,strat} + \theta_{p02}\sigma_{stc,strat} * (P02) + \theta_{p05}\sigma_{stc,strat} * (P05) + \theta_{p08}\sigma_{stc,strat} * (P08) \tag{4.13}$$

for strategic subjects, and

$$V_{stc} = \beta_{ett} ETT_{stc,path} + \theta_{p02}\sigma_{stc,path} * (P02) + \theta_{p05}\sigma_{stc,path} * (P05) + \theta_{p08}\sigma_{stc,path} * (P08) \quad (4.14)$$

for non-strategic subjects.

Model *SR2* is estimated once using the utility function in Equation 4.13, implicitly assuming all subjects are strategic, and again using the utility function in Equation 4.14, implicitly assuming all users are non-strategic. Estimation results are presented in Table 4.3. Note that the term ASC_{stoch} has been removed for the purpose of this comparison. Due to the particular travel times used in this experiment, ASC_{stoch} is not estimable for non-strategic users when using only observations from the strategic maps.

	Strat.	**Non-Strat.**
$\boldsymbol{\beta_{ett}}$ (t-stat)	-0.467 (-18.19)	-0.237 (-17.95)
$\boldsymbol{\theta_{p02}}$		
mean (t-stat)	-0.324 (-12.26)	0.0432 (9.31)
stdev (t-stat)	0.132 (6.50)	0.0228 (5.23)
$\boldsymbol{\theta_{p05}}$		
mean (t-stat)	-0.0261 (-2.01)	0.169 (16.93)
stdev (t-stat)	0.0712 (4.33)	0.0108 (2.50)
$\boldsymbol{\theta_{p08}}$		
mean (t-stat)	0.190 (8.04)	0.365 (16.93)
stdev (t-stat)	0.139 (6.47)	0.0273 (5.62)
Total Param.	7	7
Observations	1699	1699
Individuals	74	74
Adj. ρ^2	0.329	0.225
Final LL	-783.670	-905.991

Table 4.3. Comparison of estimates and statistics for strategic and non-strategic models

While the fit of the strategy-only model is better, it is not possible to conclude that the non-strategic model is entirely inaccurate. Rather, it is expected that the most accurate model would recognize both strategic and non-strategic behavior. This hypothesis can be tested by a J-test, a specification test which measures the significance of a designated subset of variables. The test is performed by first estimating the non-strategic model, obtaining $\hat{V}_{stc,path}$, then estimating a mixed model and evaluating the mixing parameter γ for signficance. The systematic utility of the stochastic branch is therefore expanded to that shown in Equation 4.15.

$$V_{stc} = (1-\gamma)(V_{stc,strat}) + \gamma(\hat{V}_{stc,path}) \tag{4.15}$$

The estimate obtained for γ is 0.0778, with a robust standard error of 0.0212. The estimate is therefore significantly different from both 0 and 1 at the 1% level. This suggests that both $V_{stc,strat}$ and $V_{stc,path}$ are significant components of the true model. A latent-class model is therefore desirable, as it can be used to estimate both utility functions, as well as the probability that each function applies to any given observation. The formulation used for estimation is a mixed logit model with discrete mixing distribution. This is equivalent to a latent-class model since the class membership function is a non-conditional probability. The model is specified in Equation 4.16. κ is a binary variable with distribution $\{1 \text{ w.p. } W, 0 \text{ w.p. } (1-W)\}$

$$V_{stc} = (1-\kappa)(V_{stc,strat}) + \kappa(V_{stc,path}) \tag{4.16}$$

The choice probability is then

$$\begin{aligned} P_n(x_s|B) \quad = \quad & W\left(\frac{exp(V_{stc,strat}|B)}{exp(V_{stc,strat}|B) + exp(V_{det}|B)}\right) \\ & +(1-W)\left(\frac{exp(V_{stc,path}|B)}{exp(V_{stc,path}|B) + exp(V_{det}|B)}\right) \end{aligned} \tag{4.17}$$

Estimation results are presented in Table 4.4. For the latent-class models, the utility function parameters are estimated as fixed values rather than random variables. The estimates very closely match the estimated means in the simple risk model, however, so model accuracy is still sufficient. The latent-class model is estimated using the entire dataset, including simple risk and strategy maps.

	Estimate
$\boldsymbol{\beta_{ett}}$ (t-stat)	-0.390 (-24.70)
$\boldsymbol{ASC_{stc}}$ (t-stat)	-1.02 (-8.79)
$\boldsymbol{\theta_{p02}}$ (t-stat)	-0.0338 (-3.73)
$\boldsymbol{\theta_{p05}}$ (t-stat)	0.0458 (3.98)
$\boldsymbol{\theta_{p08}}$ (t-stat)	0.275 (15.02)
$\boldsymbol{W}$ (t-stat)	0.873 (40.80)
$\boldsymbol{1-W}$ (t-stat)	0.127 (5.93)
Total Param.	7
Observations	3466
Adj. ρ^2	0.299
Final LL	-1676.313

Table 4.4. Estimation results for latent-class model

The estimate for W, the probability of a strategic choice, suggests that 87.3% of observations were likely to have resulted from strategic assessment of the alternatives. Conversely, 12.7% are estimated to have resulted from non-strategic behavior. This is not far from the assessment from the quantitative analysis, that 15% of observed choices were distinguishably non-strategic. The t-statistics for W and $1 - W$ show that the number of both strategic and non-strategic choices are statistically significant at the 1% level. Furthermore, the latent-class model closely reproduces the estimates of risk attitude obtained from estimation on the simple risk maps. This suggests that the latent-class model is capable of measuring both risk attitude and strategic behavior in the simple networks used in this study.

A more generally applicable strategic choice model incorporates the CPT model rather than the benefit/risk model used above. The latent-class structure is identical to that described in equation 4.16, but the systematic utility functions are of the CPT form defined in equations 4.11 and 4.12. Estimation results are shown in table 4.5

	Estimate
$\boldsymbol{\lambda}$ (t-stat w.r.t. 0)	0.451 (9.54)
$\boldsymbol{ASC_{stoch}}$ (t-stat w.r.t. 0)	-1.21 (-9.25)
$\boldsymbol{\beta}$ (t-stat w.r.t. 1)	0.939 (2.27)
$\boldsymbol{\delta}$ (t-stat w.r.t. 1)	0.543 (35.70)
$\boldsymbol{W}$ (t-stat w.r.t. 0)	0.893 (39.73)
$\mathbf{1} - \boldsymbol{W}$ (t-stat w.r.t 0)	0.107 (4.77)
Total Param.	6
Observations	3466
Adj. ρ^2	0.289
Final LL	-1702.765

Table 4.5. Estimation results for the CPT latent-class model

While the fit of the latent-class CPT model is not as good as that of the latent-class benefit/risk model, the estimate of strategic behavior probability, 89.3%, is in line with the estimate of 87.3% from the benefit/risk model. As in the simple risk CPT model, the estimate for the value parameter β is close to 1 and the estimate for the probability weighting parameter δ indicates a substantial distortion of percieved probability. These results show that a reasonable latent-class CPT model is estimable, but further research into the form of the model, including the choice of reference point, may be necessary to improve accuracy in future applications.

CHAPTER 5

CONCLUSIONS

The qualitative and quantitative analyses performed in this work have made significant progress in answering the reasearch questions:

Do drivers think strategically when planning routes in uncertain networks with real-time, en route information?

The per-scenario comparison of choice behavior between the two map types indicated that many subjects (63%) frequently rejected risks in the strategy maps that they accepted in the simple risk maps. This is likely due to an exaggerated perception of the risk as presented in the strategy map, which is expected from a fixed-path assessment of the network.

The significance tests and comparisons of the several models estimated show statistically significant portions of both strategic and non-strategic behavior. While users may not be easily classified by behavior type, the modeling results show that both types of behavior are likely present, and should be considered in future modeling work.

Can observations of route choice be used to estimate a model which accounts for strategic behavior?

The latent-class models succeed at accounting for both risk attitude and strategic behavior simultaneously. The close agreement with the qualitative measurement of risk attitude provides strong support for this finding.

The latent-class CPT model is the most useful, since it is capable of discerning strategic behavior and predicting a generally applicable model for risk attitude.

Another important aspect of risk in route choice is addressed in this research:

Do drivers' risk attitudes change according to the probability of delay?

The qualitative analysis shows clearly that subjects are more risk-prone as the value of P_d increases. These results emphasize the importance of accounting for P_d in the quantitative models.

Modeling results agree well with the qualitative findings. The simple risk model which accounts for P_d performed significantly better than the model which neglected this variable. In addition, the estimates for each probability level support the finding that subjects are generally risk prone for $P_d = 0.8$

The fit of the CPT model further supports this finding by establishing a functional form with a significant probability weighting factor. This model also provides a more general way to measure and predict this effect in applications beyond this experiment.

5.1 Future Work

The purpose of this research is to use a controlled experiment to investigate strategic thinking and risk attitudes in route choice. The careful design and limited context of the experiment allow for the identification of statistically significant factors, which can guide future research involving more complex networks and real-world choices.

One key challenge in working with real-world data from complex networks is that strategic choices are not directly observable. In a given trip, a driver's routing policy will be manifested as a single path from origin to destination. Usually, there are multiple policies that would result in the same observed path. A *latent-choice* model has been developed by Gao et al. in [12], and experimental validation of such model is a necessary step toward reliably analyzing real-world data.

Another critical avenue of investigation is more general and accurate modeling of risk attitude. While the estimation results in this research establish some validity for CPT-based models, data from more diverse networks with wider ranges of delay probabilities and travel times is needed to generate more realistic estimates.

Of major importance in every aspect of this topic is the practical applicability of the findings. This research is designed to identify important factors in route choice. While stated preference results are not ideal for direct practical application, these results will guide future research efforts, including driving simulator experiments and analysis of real-world data.

The Regional Traveler Information Center (RTIC) at the University of Massachusetts is one important example of a platform well-suited to real-world experimentation and data collection. The RTIC currently collects real-time information regarding traffic conditions on area roads. With expansion of the center currently under way, there is great opportunity for development of research-minded projects involving new methods of communicating information to drivers en route to their destinations.

Future findings from this research area will have direct impact on new and existing ATIS implementations, by offering more advanced and accurate predictions of the effectiveness of specific treatments or entire systems. This will eventually accelerate and improve the design process and allow for low-cost development of advanced ATIS designs.

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