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We argue that the difficulties associated with the convergence properties of conventional SU(3) chiral perturbation theory can be ameliorated by use of a cutoff, which suppresses the model-dependent short distance effects in such calculations.

1 Introduction

Tony Thomas presented us with a challenge in setting up this workshop—to make the connection between quark and hadronic degrees of freedom. I have accepted this challenge and will go even further by attempting to make contact between the *physics* of Tony Thomas and that of Ulf Meissner! My point of contact will be the study of low energy baryon phyics, studied a number of years ago by Tony and his collaborators in the context of the cloudy bag model and examined in recent years by Ulf and his collaborators using the technique of chiral perturbation theory.

Let's begin with some simple baryon phenomenology which was studied in the 1960's via the quark model, but which has now been updated via SU(3) chiral perturbative studies. In each case the simple quark model picture is reasonably successful but the addition of chiral loops, which are supposed to be essentially model independent, destroys experimental agreement:

i) Baryon masses, wherein the Gell-Mann-Okubo relation obtains at first order in quark mass and is well satisfield experimentally. Chiral loops, however, make significant—O(50-100%)—corrections to individual masses.[1]

ii) Semileptonic hyperon decay, wherein a simple SU(3) representation of the axial couplings in terms of F,D couplings yields an excellent fit to experiment. Chiral loops make O(30-50%) corrections to individual couplings and destroy this agreement.[2]

iii) Nonleptonic hyperon decay, wherein a basic SU(3) fit to s-wave amplitudes provides an excellent representation of the experimental numbers in terms of f,d couplings. Chiral loops make O(30-50%) corrections to individual terms and destroy this agreement. (The situation is somewhat

more confused in the case of the p-waves wherein a significant cancellation between pole terms exists at lowest order and the validity of the chiral expansion is suspect.[3])

iv) Magnetic moments, wherein an SU(3) representation (especially with the strange quark mass accounted for) provides a reasonable fit to measured quantities, but where loop corrections make O(50-90%) corrections, which invalidate the goodness of fit.[4]

Of course, the disagreement brought about by such loop modifications can be eliminated by contributions from yet higher order terms in the chiral expansion. However, this gives one concern about the convergence properties of the series. Thus I would answer the question raised in my title—yes it is an effective field theory but it is not an *effective* field theory!

Below we shall make the case that this problem can be solved by using a cutoff regularization in order to calculate chiral corrections rather than the conventional dimensional regularization. The important point here is that a significant component of the loop correction arises from meson propagation over distances small compared to the ~ 1 fm size of a typical baryon. This piece is not only large but is model dependent and unphysical—the representation of the physics in terms of point particles misrepresents the short distance physics. We shall develop a procedure whereby only the—model independent—long distance component is retained and will show how this resolves the problem with respect to the convergence of the chiral expansion.

In the next section we shall make a somewhat detailed analysis of the problem of baryon mass renormalization, and then in section III we will extend our discussion to include the other aspects of low energy baryon physics referred to above. A short summary appears in a concluding section IV.

2 Baryon Mass

An example of the difficulties of baryon chiral perturbation theory is provided by the analysis of baryon masses. Such masses have an expansion in the masses of the quarks (m_q), or equivalently in terms of the pseudoscalar meson masses (m_M)

$$M_B = M_0 + \Sigma_q\ \bar{b}_q m_q + \Sigma_q\ \bar{c}_q m_q^{3/2} + \Sigma_q\ \bar{d}_q m_q^2 + ... \tag{1}$$

$$= M_0 + \Sigma_M\ b_M m_M^2 + \Sigma_M\ c_M m_M^3 + \Sigma_M\ d_M m_M^4 + ... \tag{2}$$

Here M_0 is a common mass and b_M and d_M contain adustable parameters representing terms in the effective Lagrangian. However, the non-analytic $m_q^{3/2}$

terms come from loop diagrams, and the coefficients are not adjustable—they are given in terms of the baryon-meson coupling constants. The leading SU(3) breaking terms involving b_M go back to Gell-Mann and Okubo[5], the non-analytic corrections from one-loop diagrams, represented above by c_M, were first calculated by Langacker and Pagels[6], and the complete m_M^4 corrections (including diagrams up to two loops) were calculated by Borasoy and Meissner[1]. The convergence difficulties in the expansion are demonstrated by the resulting fit for the nucleon mass where, in the same sequence, the different contributions are given, in GeV, by[1]

$$M_N = 0.711 + 0.202 - 0.272 + 0.298 + \ldots \tag{3}$$

or, more dramatically for the Ξ,

$$M_\Xi = 0.767 + 0.844 - 0.890 + 0.600 + \ldots \tag{4}$$

The non-analytic terms appear unavoidably large and the expansion has certainly not converged at this order. The final fit also violates the Gell-Mann-Okubo relation by an amount that is five times larger than the experimentally observed value.

To one-loop order, the explicit form of the contributions to the baryon masses is given by[7]

$$\begin{aligned} M_N &= \hat{M}_0 - 4m_K^2 b_D + 4(m_K^2 - m_\pi^2) b_F + L_N \\ M_\Lambda &= \hat{M}_0 - \frac{4}{3}(4m_K^2 - m_\pi^2) b_D + L_\Lambda \\ M_\Sigma &= \hat{M}_0 - 4m_\pi^2 b_D + L_\Sigma \\ M_\Xi &= \hat{M}_0 - 4m_K^2 b_D - 4(m_K^2 - m_\pi^2) b_F + L_\Xi \end{aligned} \tag{5}$$

where

$$\hat{M}_0 = M_0 - 2(2m_K^2 + m_\pi^2) b_0 \tag{6}$$

with M_0, b_D, b_F and b_0 as free parameters. (Note that M_0 and b_0 do not have separate effects, but only enter in the combination $\hat{M}_0$.) The ingredients L_B contain the nonanalytic contributions from loop diagrams, and have the form

$$L_i = -\frac{1}{24\pi F_\pi^2} \sum_j \kappa_i^j m_j^3 \tag{7}$$

where κ_i^j are given in terms of the D and F baryon axial-vector couplings in ref. 1. These non-analytic terms are quite large, having values

$$L_N = -0.31\ GeV,$$

$$
\begin{aligned}
L_\Lambda &= -0.66\ GeV, \\
L_\Sigma &= -0.67\ GeV, \\
L_\Xi &= -1.02\ GeV.
\end{aligned} \tag{8}
$$

using $D = 0.806$, $F = 0.46$ and $F_\pi = 93$MeV. In particular, the Ξ mass shift is clearly unphysically large. It is not possible to obtain a reasonably convergent fit to the masses with such large non-analytic terms to this order.

2.1 Cutoff Regularization

The mass analysis is especially simple in the heavy baryon formalism[8,9,10] using dimensional regularization—all of the mass shifts are proportional to a single integral

$$
\int \frac{d^4k}{(2\pi)^4} \frac{k_i k_j}{(k_0 - i\epsilon)(k^2 - m_P^2 + i\epsilon)} = -i\delta_{ij} \frac{I(m_P^2)}{24\pi} \quad \text{with} I(m_P^2) = m_P^3 \tag{9}
$$

where m_P is the mass of the Goldstone boson that is involved in the loop. Here we note a peculiarity of dimsional regulariztion—although the integral appears cubically divergent by power counting arguments, the dimensionally regularized result is finite. There us also the disturbing feature that while the long-distance pion exchange component is most model-independent and believable, Eq. 9 emphasizes the contributions from the short distance K and η exchange by a factor of nearly 30! It is this feature which is responsible for the large loop corrections and the consequent problems with SU(3) chiral convergence. This problem can be solved by use of a regularization scheme which eliminates the large and model-dependent short distance effects. What one needs is to multiply the relevant loop integrals by a function which is unity for long distance propagation (small loop momenta) but which vanishes for short distance (large loop momenta). The precise form of this function is unimportant, only that it is present.[a] For the purposes of this paper, we shall employ a dipole cutoff which, when included in the integration in Eq. 9, yields

$$
\begin{aligned}
\bar{I}(m_P^2) &= -8\pi i \int \frac{d^4k}{(2\pi)^4} \frac{\mathbf{k}\cdot\mathbf{k}}{(k_0 - i\epsilon)(k^2 - m_P^2 + i\epsilon)} \left(\frac{\Lambda^2}{\Lambda^2 - k^2}\right)^2 \\
&= \left[\frac{\Lambda^4}{(\Lambda^2 - m_P^2)^2}(m_P^3 - \Lambda^3) + \frac{3}{2}\frac{\Lambda^5}{\Lambda^2 - m_P^2}\right]
\end{aligned} \tag{10}
$$

[a] There exist a few subtleties concerning the maintainance of various symmetries such as chiral invariance within a cutoff regularization. However, one can deal with these problems by the addition of appropriate noncovariant terms to the perturbation expansion.[11]

Table 1: Given are numerical values of the integral $I(m_P^2)$ (Eq. 9) in GeV^3 for various values of the cutoff Λ given in MeV.

	dim.	$\Lambda = 300$	$\Lambda = 400$	$\Lambda = 500$	$\Lambda = 600$
$\bar{I}_\pi$	0.0025	0.0061	0.0157	0.0320	0.0567
$\bar{I}_K$	0.1213	0.0024	0.0077	0.0183	0.0363
$\bar{I}_\eta$	0.1815	0.0020	0.0069	0.0163	0.0329

Table 2: Given (in GeV) are the nonanalytic contributions to baryon masses in dimensional regularization and for various values of the cutoff parameter Λ in MeV.

	dim.	$\Lambda = 300$	$\Lambda = 400$	$\Lambda = 500$	$\Lambda = 600$
N	-0.31	-0.04	-0.11	-0.22	-0.40
Σ	-0.67	-0.03	-0.08	-0.18	-0.34
Λ	-0.66	-0.03	-0.08	-0.18	-0.34
Ξ	-1.02	-0.02	-0.06	-0.15	-0.29

In Table 1 is shown the size of this integral for various cutoff values of order inverse baryon size. Note that unlike the case of dimensional regularization, the (long distance) pion contribution is now emphasized, as we desire. For baryon masses, the previous formulae are unchanged except for the substitution $m_P^3 \to \bar{I}(m_P^2)$, and in Table 2 we list the resulting loop corrections Obviously, there are no longer any large SU(3) violating loop pieces and the chiral expansion is now under control.

That we have not altered the chiral invariance via this procedure can by examining the form of the one loop contribution in the limit $\Lambda >> m_P$

$$\delta M_i = -\frac{1}{24\pi F_\pi^2}\sum_j(\frac{1}{2}\Lambda^3 - \frac{1}{2}\Lambda m_j^2 + m_j^3 + \ldots) \tag{11}$$

Obviously the term in m_j^3 is identical to that arising in conventional dimensional regularization, but more interesting are the contributions proportional to Λ^3 and to Λm_P^2. The piece cubic in Λ has the form

$$\delta M_i^{\Lambda^3} = -\frac{\Lambda^2}{48\pi F_\pi^2}\sum_j \kappa_i^j \tag{12}$$

and is independent of baryon type—it may be absorbed into a renormalization of M_0—

$$M_0^r = M_0 - (5d_A^2 + 9f_A^2)\frac{\Lambda^3}{48\pi F_\pi^2} \tag{13}$$

On the other hand the terms linear in Λ

$$\delta M_i^\Lambda = \frac{\Lambda}{48\pi F_\pi^2} \sum_j \kappa_i^j m_j^2 \tag{14}$$

must be able to be absorbed into renormalizations of the coefficients involving m_q, and indeed this is found to be the case—one verifies that[12]

$$\begin{aligned} d_m^r &= = d_m - \frac{3f_A^2 - d_A^2}{128\pi F_\pi^2}\Lambda \\ f_m^r &= f_m - \frac{5d_A f_A}{192\pi F_\pi^2}\Lambda \\ b_0^r &= b_0 - \frac{13d_A^2 + 9f_A^2}{576\pi F_\pi^2}\Lambda \end{aligned} \tag{15}$$

That this renormalization can occur involves a highly constrained set of conditions and the fact that they are satisfied is a significant verification of the chiral invariance of the cutoff procedure. Of course, once one has defined renormalized coefficients, since they are merely phenomenological parameters which must be determined empirically, the procedure is identical to the results of the usual dimensionally regularized technique when the masses are smaller than the cutoff.

3 Additional Applications

Having seen how to perform the long-distance regularization in the case of the mass renormailization, it is now straightforward to carry through a similar analyis in the other cases of low energy baryon phenomenology mentioned above.[13] In the case of the axial couplings and the S-wave hyperon decay the relevant heavy baryon integral is of the form

$$\int \frac{d^4k}{(2\pi)^4} \frac{k_i k_j}{(k_0 - i\epsilon)^2 (k^2 - m^2 + i\epsilon)} = -i\delta_{ij} \frac{J(m^2)}{16\pi^2} \tag{16}$$

In dimensional regularization the integral has the value

$$J_{dim-reg}(m^2) = m^2 \ln \frac{m^2}{\mu^2} \tag{17}$$

while the cutoff version is given by

$$\int \frac{d^4k}{(2\pi)^4} \frac{k_i k_j}{(k_0 - i\epsilon)^2 (k^2 - m^2 + i\epsilon)} (\frac{\Lambda^2}{\Lambda^2 - k^2})^2 = -i\delta_{ij} \frac{J_\Lambda(m^2)}{16\pi^2}. \tag{18}$$

Here

$$J_\Lambda(m^2) = \frac{\Lambda^4 m^2}{(\Lambda^2 - m^2)^2} \ln \frac{m^2}{\Lambda^2} + \frac{\Lambda^4}{\Lambda^2 - m^2} \tag{19}$$

and has the form $m^2 \ln m^2$ in dimensional regularization. However, using long distance regularization the short distance effects are under control and the $\sim$ (50%) corrections from loop diagrams are reduced substantially, allowing agreement with experimental findings to be reproduced without large next order counterterms. For both axial couplings and hyperon decay it can be verified that chiral invariance is maintained in the limit that $m_p^2 << \Lambda^2$ via the renormalizations

$$\begin{aligned} d_A^r &= d_A - \frac{3}{2} d_A (3d_A^2 + 5f_A^2 + 1) \frac{\Lambda^2}{16\pi^2 F_\pi^2} \\ f_A^r &= f_A - \frac{1}{6} f_A (25d_A^2 + 63f_A^2 + 9) \frac{\Lambda^2}{16\pi^2 F_\pi^2} \end{aligned} \tag{20}$$

for f, d axial couplings and

$$\begin{aligned} d_w^r &= d_w - \frac{1}{2}[d_w(1 + 13d_A^2 + 9f_A^2) + 18 f_w d_A f_A] \frac{\Lambda^2}{16\pi^2 F_\pi^2} \\ f_w^r &= f_w - \frac{1}{2}[f_w(1 + 5d_A^2 + 9f_A^2) + 10 d_w d_A f_A] \frac{\Lambda^2}{16\pi^2 F_\pi^2} \end{aligned} \tag{21}$$

in the case of hyperon decays. Likewise one can verify that the fit to semileptonic and to nonleptonic hyperon decay amplitudes is quite satisfactory and that agreement with the Lee-Sugawara relation is restored for the latter.

Finally, in the case of the magnetic moments, the relevant heavy quark integral is

$$\int \frac{d^4k}{(2\pi)^4} \frac{k_i k_j}{(k_0 - i\epsilon)(k^2 - m^2 + i\epsilon)^2} = -i\delta_{ij} \frac{K(m^2)}{16\pi} \tag{22}$$

The dimensionally regularized form is given by $K_{dim-reg}(m^2) = m$. Once again, the integral shows no sign of its true linear divergence, and grows at large values of m, indicating short distance dominance at large m. The use of the dipole cutoff yields

$$K(m^2) = -\frac{1}{3}\Lambda^4 \frac{1}{(\Lambda + m)^3} \quad , \tag{23}$$

In this case use of the long-distance regularized version of the loop integral also restores agreement between the experimental magnetic moments and their one-loop corrected values and chiral invariance is maintained in the case $m_P^2 << \Lambda^2$ via the renormalizations of the lowest order parameters f_μ, d_μ via

$$\begin{aligned} d_\mu^r &= d_\mu + \frac{M_0\Lambda}{4\pi F_\pi^2} d_A f_A \\ f_\mu^r &= f_\mu + \frac{M_0\Lambda}{24\pi F_\pi^2}(\frac{5}{3}d_A^2 + 3f_A^2) \end{aligned} \tag{24}$$

Thus in each case we are able include chiral loop corrections without destroying either chiral invariance *or* experimental agreement.

4 Conclusions

We have argued that a significant component of the poor convergence seen in previous calculations in SU(3) baryon chiral perturbation theory is due to the inclusion of large and spurious short-distance contributions when loop processes are regularized dimensionally. The use of a momentum space cutoff keeps only the long distance portion of the loops and leads to a greatly improved behavior. Indeed although we have formulated our discussion in terms of merely a different sort of regularization procedure, it is interesting to note that our results are quite consistent with the sort of SU(3) breaking effects found in chiral confinement models such as the cloudy bag, when the effects of kaon and/or eta loops is isolated.[14]

One might ask why baryon chiral perturbation theory has this problem while mesonic chiral theories do not. (Most applications in mesons work perfectly well using dimensional regularization.) At first sight one might argue that the separation scale in baryons corresponds to lower energies because the physical size of baryons is larger than mesons. While this is a true statement, it does not really answer the question, since the baryon problem surfaces entirely within the point particle theory. For some reason, given the same meson masses, the loop corrections are larger in the baryonic point particle theory compared to a mesonic point particle theory. This feature can perhaps be blamed on the baryon propagator in the loop integral which, being linear in the momentum, suppresses high momentum contributions less than a corresponding quadratic mesonic propagator. However, the existence of the problem is beyond doubt, given the troubles discussed in the introduction. Fortunately, we do not as a consequence have to abandon all such chiral calculations—a revised regularization scheme seems capable of resolving the dilemma.

The simplicity that underlies baryon physics is more evident when chiral loops are calculated with a long-distance regularization. In this context, we hope that baryon chiral perturbation theory will become more phenomenologically useful.

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References

1. B. Borasoy and U. G. Meissner, Phys. Lett. **B365**, 285 (1996); Ann. Phys. (NY) **254**, 192 (1997).
2. J. Bijnens, H. Sonoda, and M.B. Wise, Nucl. Phys. **B261**, 185 (1985); J.F. Donoghue and B.R. Holstein, Phys. Lett. **160B**, 173 (1985); E. Jenkins and A. Manohar, Phys. Lett. **259B**, 353 (1991).
3. E. Jenkins and A. Manohar, ref. 4; B. Borasoy and B.R. Holstein, UMass preprint (1998).
4. U-G. Meissner and S. Steininger, Nucl. Phys. **B499**, 349 (1997).
5. M. Gell-Mann in M. Gell-Mann and Y. Ne'eman, **the Eightfold Way**, Benjamin, New York (1962) and Phys. Rev. **125**, 1067 (1962); S. Okubo, Prog. Theo. Phys. **27**, 949 (1962).
6. P. Langacker and H. Pagels, Phys. Rev. **D8**, 4595 (1975).
7. V. Bernard, N. Kaiser, and U.-G. Meissner, Int. J. Mod. Phys. **E4**, 193 (1995).
8. H. Georgi, Phys. Lett. **B240**, 447 (1990).
9. N. Isgur and M. Wise, Phys. Lett. **B232**, 113 (1989).
10. E. Jenkins and A.V. Manohar, Phys. Lett. **B255**, 558 (1991).
11. I. Gerstein, R. Jackiw, B.W. Lee, and S. Weinberg, Phys. Rev. **D3**, 2486 (1971).
12. J.F. Donoghue and B.R. Holstein, hep-ph/9803312.
13. J.F. Donoghue, B.R. Holstein, and B.Borasoy, to appear.
14. See, *e.g.* T. Yamaguchi et al., Nucl. Phys. **A500**, 429 (1989); K. Kubodera et al., Nucl. Phys. **A439**, 695 (1985); S. Theberge et al., Phys. Rev. **D22**, 2838 (1980); A.W. Thomas, J. Phys **G7**, L283 (1981); R.E. Stuckey and M.C. Birse, J. Phys. **G23**, 29 (1997).