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Nucleon Polarizabilities

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Nucleon Axial Matrix Elements

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Abstract

Current issues associated with nucleon axial matrix elements are studied, including the Goldberger-Treiman discrepancy, the induced pseudoscalar, and SU(3) chiral perturbation theory.

1 Introduction

I have been given the task of speaking about the nucleon axial matrix elements. In comparison with the many exciting things being discussed at this meeting this may seem rather prosaic. However, I will try to convince you otherwise by discussing issues associated with the Goldberger-Treiman discrepancy, recent and future measurements of the induced pseudoscalar, and the renormalizations of the axial couplings within SU(3) chiral perturbation theory.

2 The Goldberger-Treimen Discrepancy OR Time Dependence of Fundamental Constants

Many years ago Dirac noticed that the ratio of electrical to gravitational forces between a pair of electrons was equal to $\alpha/(Gm_e^2) \sim 10^{40}$. In asking himself how such an enormous dimensionless number could arise he noticed that 10^{40} is also the age of the universe measured in fundamental units of time—*i.e.* the time it takes light to traverse an elementary particle—10^{17} sec./ 10^{-23} sec.! He then asked if it were possible that the electrical to gravitational ratio might change as the universe evolved. It turns out on further analysis that this is extremely unlikely—the consequences of even relatively small changes to either the fine structure or gravitational constants turn out to be significant (the anthropic principle),[1] but I am prepared today to point out an arena where the changes in a fundamental coupling have been major, and they have occured within a generation—-the nucleon axial coupling. Below I give a list of values which I have gathered from various sources, and it is clear that there has been a seven percent increase in g_A within a decade!

$$\begin{array}{ll} 1959: g_A/g_V = 1.17 \pm 0.02[2] & 1965: g_A/g_V = 1.18 \pm 0.02[3] \\ 1967: g_A/g_V = 1.24 \pm 0.01[4] & 1969: g_A/g_V = 1.26 \pm 0.02[5] \end{array} \tag{1}$$

In fact this number continues to increase—the latest published experiments from Grenoble give $g_A/g_V = 1.266 \pm 0.004$,[6] which is the value I shall employ in this note.

Now my facetious discussion in the previous paragraph is at one level amusing, but at another has some important ramifications when considered in terms of the Goldberger-Treiman relation[15]

$$M_N g_A(0) = F_\pi g_{\pi NN}(0) \tag{2}$$

which is required by chiral invariance and hence by QCD. Now I have indicated in Eq. 2 that the axial coupling g_A and the pion-nucleon coupling constant $g_{\pi NN}$ are *both* to be evaluated at zero momentum transfer. However while the former is the ("time-dependent") number quoted above, the latter is *not* a physical quantity. What *is* directly measurable is the pion-nucleon coupling evaluated at the pion mass-squared—$g_{\pi NN}(m_\pi^2)$, so it is useful to examine the Goldberger-Treiman *discrepancy*

$$\Delta_\pi = 1 - \frac{M g_A(0)}{F_\pi g_{\pi NN}(m_\pi^2)} = 1 - \frac{g_{\pi NN}(0)}{g_{\pi NN}(m_\pi^2)} \tag{3}$$

In the venerable text Bjorken and Drell this number is described as being less than 0.1, but I want to argue that it must be *much* less. Indeed, while $g_{\pi NN}(0)$ is not an observable, one can show that in reasonable models such as the linear sigma model or via correlated $\pi - \rho$ exchange one should expect $\Delta_\pi \simeq 0.02$. However, there is another interesting approach via the so-called Dashen-Weinstein theorem,[7] which uses the fact that while there does not exist a prediction for Δ_π in chiral SU(2), since it is given in terms of an a priori unknown counterterm, in SU(3) this quantity is given in terms of a sum of quark masses times an SU(3) octet operator. Thus a relation exists—the Dashen–Weinstein theorem—*between* corresponding kaon and pion quantities[8, 9]

$$\Delta_\pi = \frac{\sqrt{3}}{2}\frac{F_K}{F_\pi}\frac{m_u + m_d}{m_u + m_s}\left(\frac{g_{\Lambda KN}}{g_{\pi NN}}\Delta_K^\Lambda - \frac{1}{\sqrt{6}}\frac{g_{\Sigma KN}}{g_{\pi NN}}\Delta_K^\Sigma\right) \tag{4}$$

where

$$\Delta_K = 1 - \frac{(M_N + M_{\Sigma,\Lambda})g_A(0)}{\sqrt{2}F_K g(m_K^2)} = \left\{\begin{array}{ll} 0.32 & \Lambda \\ -0.05 & \Sigma \end{array}\right. \tag{5}$$

where I have used the values[11, 12]

$$\begin{array}{ccc} & g(m_K^2) & g_A(0) \\ \Lambda & -13.5 & -0.72 \\ \Sigma & 4.3 & 0.34 \end{array} \tag{6}$$

Then using $(m_u+m_d)/(m_u+m_s)=m_\pi^2/m_K^2$ we find $\Delta_\pi^{theo}\simeq 0.028$. Besides changes in the size of the axial coupling, however, the size of F_π decreased by 1% in 1990 when it was realized that previous evaluations had not included the running of the weak coupling constant,[10] and there has been continuous debate about the size of $g_{\pi NN}(m_\pi^2)$, with current analyses favoring either the Karsruhe value 13.4[13] or the VPI number 13.05.[14] Thus we find

$$\begin{aligned} g_{\pi NN}=13.4 &\longrightarrow \Delta_\pi = 4.1\%, \quad \text{or} \quad m_s/\hat{m}\approx 48 \\ g_{\pi NN}=13.05 &\longrightarrow \Delta_\pi = 1.5\%, \quad \text{or} \quad m_s/\hat{m}\approx 17 \end{aligned} \tag{7}$$

so that if the low value of $g_{\pi NN}$ is confirmed, the Goldberger-Treiman discrepancy would strongly favor the conventional χpt picture ($m_s/\hat{m}=25$) over its generalized version, which predicts $m_s/\hat{m}<25$.[9]

3 The Induced Pseudoscalar

The axial matrix element of the nucleon consists in general of two pieces, the usual axial coupling and the induced pseudoscalar[1]

$$<p(p')|A_\mu|n(p)>=\bar{u}(p')(g_A(q^2)\gamma_\mu\gamma_5+g_P(q^2)\frac{q_\mu}{2M}\gamma_5)u(p) \tag{8}$$

and chiral considerations require that this new piece is dominated by its pion pole contribution

$$g_P(q^2)=\frac{4MF_\pi}{m_\pi^2-q^2}g_{\pi NN}(q^2)\simeq\frac{4MF_\pi}{m_\pi^2-q^2}g_{\pi NN}(m_\pi^2)-\frac{2M^2}{3}g_A(0)r_A^2 \tag{9}$$

where r_A is the axial radius. This result is generally used in the combination

$$r_P=\frac{m_\mu}{2Mg_A(0)}g_P(q^2=-0.9m_\mu^2)=6.7 \tag{10}$$

relevant for muon capture. This is the standard approach and is used because the contraction of the four-vector q_μ with the lepton tensor results in a factor of the lepton mass accompanied by the nucleon matrix element

[1] In general one could also allow an axial tensor coupling, but this "second class current" is disallowed by G-invariance.

of γ_5, which brings in an additional suppression $|\vec{q}|/2M$, meaning that despite the extraordinary precision of modern nuclear beta decay experiments, any effects from g_P arise only at $\mathcal{O}[r_P m_e^2/(2Mm_\mu)] \sim 10^{-5}$!. On the other hand in muon capture this factor becomes $r_P m_\mu/2M$, which means that the pseudosclar contributes at the same order as weak magnetism and becomes in principle measurable. The one problem here is that typically one has only a single number—the capture rate—to work with so that in order to extract the desired value of r_P one must make three reasonable, but still model-dependent, assumptions—i) the validity of CVC in order to extract $f_V(q^2), f_M(q^2)$ from electron; scattering data; ii) the validity of the impulse approximation to evaluate $g_A(q^2)$; and iii) the assumption of G-invariance to rule out the presence of second class currents. Using these assumptions one finds the experimental values

$$r_P = \begin{cases} 6.5 \pm 2.4 & H[17] \\ 6.9 \pm 0.2 & {}^3He[18] \\ 9.0 \pm 1.7 & {}^{12}C[19] \end{cases} \tag{11}$$

which are in agreement with the chiral expectations. It should be noted that the extraordinary precision associated with the ^3He number is allowed because of a spectacular new PSI experiment which measured the capture rate to 3%

$$\Gamma_\mu(^3He) = 1496 \pm 4 sec.^{-1} \tag{12}$$

In order to eliminate some of this model dependence, there are additional approaches which have been and which are being pursued

i) Radiative muon capture on hydrogen: This is the approach which has received the most recent attention, because the result[20]

$$r_P = 9.8 \pm 0.7 \pm 0.3 \tag{13}$$

is at variance with the chiral prediction at the 3σ level. Now this TRIUMF measurement is extraordinarily difficult both because of the tiny 10^{-8} branching fraction compared to ordinary capture and because of the presence of many possible experimental backgrounds. However, it has the advantage that at the maximum photon energy $k_{max} = 100 MeV$ the momentum transfer is $q_{max} = m_\mu^2$ compared to the value $q_\mu^2 = -0.9 m_\mu^2$ which obtains in the ordinary muon capture case.

Integrated over the photon spectrum this leads to an enhancement of about a factor of three for pseudoscalar effects in RMC over those in OMC.[21] Clearly this is an experiment that should be repeated.

ii) Threshold pion photoproduction on hydrogen: This might seem a strange place to study nucleon *axial* matrix elements, but this is possible because of the PCAC relation[22]

$$< \pi^+ n_{p'} | V_\mu^{em} | p_p > \stackrel{q\to 0}{\longrightarrow} \frac{-i}{\sqrt{2}F_\pi} < n_{p'} | A_\mu^- | p_p > \qquad (14)$$

The variation in q^2 which is allowed by the use of electroproduction rather than photoproduction permits a check of the q^2-variation of both axial matrix elements. A recent Saclay experiment produced in this way a measurement of the axial radius r_A which was in good agreement with parallel neutrino scattering measurements, when a small chiral symmetry offset is included, but more importantly for our case for the first time a measurement was made of the *shape* of $g_P(q^2)$ which was in good agreement with the pion pole dominance assumption.[23]

iii) Correlations in polarized muon capture on ^3He: The final method which is being pursued at present goes back to an old idea to measure the correlation of the final neutrino direction with initial state polarization in the case that the muon and target are polarized.[24] Of course, the muon is almost completely longitudinally polarized at the time of its capture, but unless the target too is polarized this polarization for the most part lost as the muon cascades down through the various atomic levels before finally reaching the ground state–1S–level from which it is captured. In the general case, when one has muon polarization $P'\hat{n}$ and (spin 1/2) target polarization $P\hat{n}$ the decay distribution is found to be of the form

$$\frac{d^2\Gamma_\mu}{d\Omega_{\hat{k}}} = A - 2PP'B - \frac{1}{2}(P+P')C\hat{n}\cdot\hat{k} + 2PP'D\left[(\hat{n}\cdot\hat{k})^2 - \frac{1}{3}\right] \quad (15)$$

where the structure functions A, B, C, D are functions of the weak formfactors g_v, g_M, g_A, g_P whose specific forms can be found in the literature. The important feature for our case is that D has a strong dependence on g_P, and thus measurement of the angular correlations allows one to pick out the induced pseudoscalar. This measurement was attempted unsuccessfully many years ago at LAMPF, but only recently has the ability to polarize ^3He at high levels given hope that this experiment can actually be carried out. Preliminary results at TRIUMF are encouraging, but the precision is not yet at a level where anything definitive can be said.[25]

4 Axial Couplings and SU(3)

The existence of semileptonic hyperon decays such as $\Lambda, n \rightarrow pe^-\bar{\nu}_e, \Sigma^-, \Xi^- \rightarrow \Lambda e^-\bar{\nu}_e, \Xi^- \rightarrow \Sigma^0 e^-\bar{\nu}_e$, etc. allows the probing of axial matrix elements in SU(3). Indeed it has long been known that to leading order in chiral symmetry one can describe such decays in terms of simple f,d parameters, *e.g.*

$$\begin{array}{ll} g_V^{pn} = f_V & g_A^{np} = f_A + d_A \\ g_V^{p\Lambda} = -\sqrt{\frac{3}{2}} f_V & g_A^{p\Lambda} = -\sqrt{\frac{1}{6}}(3f_A + d_A) \\ g_V^{n\Sigma^-} = -f_V & g_A^{n\Sigma^-} = -f_A + d_A \end{array} \tag{16}$$

This type of fit yields remarkably good results—$\chi^2_{d.o.f} \approx 8.5$ for ten degrees of freedom, when small $\leq 5\%$ quark model symmetry breaking effects are added.[26] One can try to do even better by including chiral loops using heavy baryon chiral perturbation theory. At one loop one finds results[27]

$$g_A^{ij} = (f_A, d_A)^{ij} + \sum_m \beta_m^{ij} m_m^2 \ln \frac{m_m^2}{\mu^2} \tag{17}$$

However, this inclusion of supposedly *model-independent* corrections brings in modifications to the axial couplings at the level of 30-50% which results in a vastly increased χ^2. Of course, one can restore experimental agreement by the addition of appropriately chosen higher order counterterms, but then one worries about the convergence of the chiral expansion and is certainly justified in asking what is going on. Our answer is that this simple chiral picture omits an important piece of physics, which is finite hadronic size.[28] The simple chiral expansion assumes (at lowest order) propagation of mesons between *point* baryons, while in the real world any such propagation takes place between objects about a fermi or so in size. That means that only the *long*-distance component of the meson loop is really model-independent and to be trusted. One can eliminate such short distance components by use of a cutoff regularization with scale $\sim$300 MeV $\leq \Lambda \leq\sim$ 600 MeV of order inverse baryon size rather than the usual dimensional regularization which mixes both long and short distance effects.[29] The specific form of the cutoff function is unimportant, so for calculational purposes it is useful to use a simple dipole. The result is that the heavy baryon integral responsible for loop corrections to axial couplings

$$I_{ij}(m^2) = \int \frac{d^4k}{(2\pi)^4} \frac{k_i k_j}{(k_0 - i\epsilon)^2(k^2 - m^2 + i\epsilon)} = \frac{-i\delta_{ij}}{16\pi^2} m^2 \ln \frac{m^2}{\mu^2} \tag{18}$$

is replaced by

$$\tilde{I}_{ij} = \int \frac{d^4k}{(2\pi)^4} \frac{k_i k_j}{(k_0 - i\epsilon)^2(k^2 - m^2 + i\epsilon)} \left(\frac{\Lambda^2}{\Lambda^2 - k^2}\right)^2 = \frac{-i\delta_{ij}}{16\pi^2} J(m^2) \tag{19}$$

where

$$J(m^2) = \frac{\Lambda^4}{(\Lambda^2 - m^2)^2} m^2 \ln \frac{m^2}{\Lambda^2} + \frac{\Lambda^4}{\Lambda^2 - m^2} \tag{20}$$

We see then that unlike Eq. 18 which emphasizes heavy meson (short-distance) propagation over that of light mesons, in the large mass limit

$$J(m^2) \stackrel{m^2 >> \Lambda^2}{\longrightarrow} \frac{\Lambda^4}{m^2} \ln \frac{m^2}{\Lambda^2} \to 0 \tag{21}$$

On the other hand in the large cutoff limit we have

$$J(m^2) \stackrel{\Lambda^2 >> m^2}{\longrightarrow} \Lambda^2 + m^2 \ln \frac{m^2}{\Lambda^2} \tag{22}$$

which reproduces the usual dimensional regularization result plus a quadratic term in Λ. That this latter piece does not destroy the chiral invariance can be seen from the feature that it can be absorbed in a renormalization of the basic couplings[30]

$$\begin{aligned} d_A^r &= d_A^{(0)} - \frac{3}{2} d_A (3d_A^2 + 5f_A^2 + 1) \frac{\Lambda^2}{16\pi^2 F_\pi^2} \\ f_A^r &= f_A^{(0)} - \frac{1}{6} f_A (25 d_A^2 + 63 f_A^2 + 9) \frac{\Lambda^2}{16\pi^2 F_\pi^2} \end{aligned} \tag{23}$$

	dim.	Λ=300	Λ=400	Λ=500	Λ=600
$g_A(\bar{p}n)$	1.72	0.37	0.53	0.69	0.84
$g_A(\bar{p}\Lambda)$	-1.78	-0.34	-0.51	-0.67	-0.84
$g_A(\bar{\Lambda}\Sigma^-)$	1.17	0.23	0.34	0.45	0.56
$g_A(\bar{n}\Sigma^-)$	0.36	0.07	0.10	0.14	0.17
$g_A(\bar{\Lambda}\Xi^-)$	0.83	0.15	0.23	0.31	0.39
$g_A(\bar{\Sigma}^0\Xi^-)$	2.46	0.45	0.68	0.91	1.15

Table 1: Given are the nonanalytic contribtions to g_A for various transitions in dimensional regularization and for various values of the cutoff parameter Λ in MeV.

However, in this procedure with reasonable values of the cutoff, the SU(3) chiral expansion is now under control, as can be seen in Table 1.

This brings such results into agreement with typical chiral bag calculations, such as the cloudy bag,[31] and there is no longer any need to append large counterterm contributions in higher orders.

5 Conclusion

We have above considered an old subject—that of nucleon axial matrix elements—from the point of view of modern experiments. I hope that I have convinced you that despite the age of the field, the new results in the areas of Goldberger-Treiman discrepancies, induced pseudoscalar measurements, and SU(3) chiral perturbative studies promise continued interest even as we approach the millenium.

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