

## Comment on 'Analysis of $O(p^2)$ Corrections to $h\bar{d}\bar{d}|Q_{7,8}|K_i$ '

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# Comment on ‘Analysis of $\mathcal{O}(p^2)$ Corrections to $\langle \pi\pi | \mathcal{Q}_{7,8} | K \rangle$ ’

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## Abstract

We extend in several respects our earlier work on  $\mathcal{O}(p^2)$  corrections to matrix elements of the electroweak penguin operator  $\mathcal{O}_{\text{ewp}}$ . First, to facilitate comparison with certain lattice studies we calculate  $\mathcal{O}(p^2)$  corrections to  $\langle \pi | \mathcal{O}_{\text{ewp}} | K \rangle$  in the SU(3) limit of equal light quark masses. Next, we demonstrate how an apparent disagreement in the literature regarding whether higher order chiral contributions increase or decrease  $\langle (\pi\pi)_{I=2} | \mathcal{O}_{\text{ewp}} | K \rangle$  is simply a consequence of how the leading order chiral amplitude is defined. Finally, we address an aspect of the  $\epsilon'/\epsilon$  problem by estimating  $\mathcal{O}(p^2)$  corrections to recent determinations of  $\langle (\pi\pi)_{I=2} | \mathcal{Q}_{7,8} | K \rangle$  which were carried out in the chiral limit.

## I. INTRODUCTION

There is currently great interest in matrix elements of the four-quark operators  $\mathcal{Q}_{7,8}$ , both in the phenomenology of  $\epsilon'/\epsilon$  and in lattice studies. Not surprisingly, chiral perturbation theory (ChPT) provides an important theoretical context for progress in this area. At chiral order  $\mathcal{O}(p^0)$ , both  $\mathcal{Q}_7$  and  $\mathcal{Q}_8$  are represented uniquely by the electroweak penguin operator  $\mathcal{O}_{\text{ewp}}$ . In a recent paper [1], we performed a ChPT analysis of one-loop corrections to  $K \rightarrow \pi$  and  $K \rightarrow 2\pi$  matrix elements of  $\mathcal{O}_{\text{ewp}}$ .<sup>1</sup> The purpose of this paper is to expand upon several aspects of Ref. [1].

First, the calculation in Ref. [1] employed physical values for the meson masses  $m_\pi$ ,  $m_K$  and  $m_\eta$ . This turns out to be rather general compared to what some current lattice calculations need [2]. Work on kaon-to-pion matrix elements done with domain wall quarks has been performed in the SU(3) symmetric limit [3,4]. In fact, reference to the SU(3) limit has been a common strategy in certain lattice simulations for quite some time [5]. Below, we shall report the (nontrivial) restriction of our  $K$ -to- $\pi$  matrix elements to the equal quark mass case of  $m_u = m_d = m_s$ . In this work we present results valid in the case of unquenched QCD. Analogous results in the quenched and partially quenched case can be found in Ref. [6].

Another feature of Ref. [1] was the determination of the fractional shift  $\Delta_2$  for the  $\mathcal{O}(p^2)$  corrections to the chiral limit determination of  $\langle(\pi\pi)_{I=2}|\mathcal{Q}_{7,8}|K^0\rangle$ . In particular, we discussed why the sign for  $\Delta_2$  is opposite to that expected from unitarization approaches (*e.g.* see Ref. [7]) based on the Omnès equation. Results in Ref. [8] would appear to contradict this finding. It turns out, however, that Ref. [1] and Ref. [8] normalize the so-called ‘chiral limit result’ (*i.e.* the leading order term in a chiral perturbation theory expansion) in different ways. In order to eliminate any undue confusion in future literature that this issue might cause, we carefully identify the source of the difference.

Finally, using the chiral limit normalization for  $\langle(\pi\pi)_{I=2}|\mathcal{Q}_{7,8}|K^0\rangle$  appearing in Refs. [9–11], we discuss the  $\mathcal{O}(p^2)$  corrections to such determinations. Knowing the size of such corrections is important in order to compare the predictions of Refs. [9–11] with recent lattice QCD determinations [12].

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<sup>1</sup>In Ref. [1] the following substitutions should be made to correct typographical errors:

1. Replace  $F_0$  by  $F$  in Eq. (13).
2. Omit  $F^2$  from Eq. (20).
3. In  $I_{K+\pi^+}(q^2)$  of Eq. (29), replace  $\dots + \bar{A}(m_K^2)\dots$  by  $\dots - \bar{A}(m_K^2)\dots$ .
4. Divide the final line of Eq. (29) by two.

These errors occurred entirely in the printing process; none of the results of the paper are changed.

## II. ANALYSIS

In this section, we shall be concerned with both  $K$ -to- $\pi$  and  $K$ -to- $2\pi$  matrix elements of  $\mathcal{O}_{\text{ewp}}$ . We recall the ChPT definition  $\mathcal{O}_{\text{ewp}} \equiv g \text{Tr} [\lambda_6 U Q U^\dagger]$  where  $Q = \text{diag} (2/3, -1/3, -1/3)$  is the quark charge matrix and  $U \equiv \exp(i\lambda_k \Phi_k/F)$  is the matrix of light pseudoscalar fields. We denote the pseudoscalar meson decay constant in lowest order by  $F$ .

### A. The SU(3) Limit of $\langle \pi | \mathcal{O}_{\text{ewp}} | K \rangle$

Let us denote any amplitude evaluated in the SU(3) limit with a superbar ( $\overline{\mathcal{M}}_i$ ) and likewise for the meson masses,

$$\overline{m}^2 = m_\pi^2 = m_K^2 = m_\eta^2 \quad (\text{SU(3) limit}) . \quad (1)$$

The  $\mathcal{O}(p^0)$  amplitudes are unaffected by passage to the SU(3) world,

$$\overline{\mathcal{M}}_{K^+ \rightarrow \pi^+}^{(0)} = \mathcal{M}_{K^+ \rightarrow \pi^+}^{(0)} = \frac{2g}{F^2} , \quad \overline{\mathcal{M}}_{K^0 \rightarrow \pi^0}^{(0)} = \mathcal{M}_{K^0 \rightarrow \pi^0}^{(0)} = 0 \quad (2)$$

and

$$-i\overline{\mathcal{M}}_{K^0 \rightarrow \pi^+ \pi^-}^{(0)} = -i\mathcal{M}_{K^0 \rightarrow \pi^+ \pi^-}^{(0)} = -\frac{\sqrt{2}g}{F^3} , \quad -i\overline{\mathcal{M}}_{K^0 \rightarrow \pi^0 \pi^0}^{(0)} = -i\mathcal{M}_{K^0 \rightarrow \pi^0 \pi^0}^{(0)} = 0 . \quad (3)$$

It is for the  $\mathcal{O}(p^2)$  amplitudes that the SU(3) limit is nontrivial. Calculation reveals the full next-to leading order amplitudes to be

$$\begin{aligned} \overline{\mathcal{M}}_{K^+ \rightarrow \pi^+}^{(0+2)} &= \frac{2g}{F^2} \left[ 1 - \frac{3\overline{m}^2}{(4\pi F)^2} \log \frac{\overline{m}^2}{\mu_\chi^2} + \frac{\overline{m}^2}{F^2} \left[ K_{++}^r(\mu_\chi) - 24L_4^r(\mu_\chi) - 8L_5^r(\mu_\chi) \right] \right] \\ &= \frac{2g}{F_\pi F_K} \left[ 1 - \frac{6\overline{m}^2}{(4\pi F)^2} \log \frac{\overline{m}^2}{\mu_\chi^2} + \frac{\overline{m}^2}{F^2} K_{++}^r(\mu_\chi) \right] , \\ \overline{\mathcal{M}}_{K^0 \rightarrow \pi^0}^{(0+2)} &= \frac{\sqrt{2}g}{F^2} \left[ -\frac{\overline{m}^2}{(4\pi F)^2} \left( 1 + \log \frac{\overline{m}^2}{\mu_\chi^2} \right) + \frac{\overline{m}^2}{F^2} K_{00}^r(\mu_\chi) \right] . \end{aligned} \quad (4)$$

In the above,  $\mu_\chi$  is an arbitrary energy scale,  $L_{4,5}^r(\mu_\chi)$  are finite, scale-dependent low energy constants (LECs) of the  $\mathcal{O}(p^4)$  strong chiral lagrangian [14] and  $K_{++,00}^r(\mu_\chi)$  are finite, scale-dependent combinations of the counterterms ( $\{c_i\}$ ) defined in Ref. [1],

$$\begin{aligned} K_{++} &= \frac{2F^2}{g} \left[ \frac{1}{3}(c_1 - c_3) + \frac{7}{3}c_4 + 2c_5 + 3c_6 \right] , \\ K_{00} &= \frac{F^2}{g} \left[ \frac{1}{3}c_1 + c_2 + \frac{2}{3}c_3 - \frac{2}{3}c_4 \right] . \end{aligned} \quad (5)$$

Using the results of Ref. [1] one can verify that the combinations  $K_{++,00}^r(\mu_\chi)$  compensate for the explicit scale dependence of the chiral logarithms and the implicit scale dependence

of the finite LECs  $L_{4,5}^r$ . The presence of  $L_{4,5}$  can be understood as affecting the  $K^+ \rightarrow \pi^+$  transition via wavefunction renormalization. We note that the two forms for  $\overline{\mathcal{M}}_{K^+ \rightarrow \pi^+}^{(0+2)}$  displayed in Eq. (4) correspond to the choice of either keeping  $L_{4,5}$  explicit or absorbing them in the renormalization of  $F_\pi, F_K$  (as done in Ref. [1]). Recall that in the SU(3) limit the relation between  $F_\pi, F_K$  and  $F$  is given by [14]

$$F_{\pi,K} = F \left[ 1 - \frac{3}{2} \frac{\overline{m}^2}{(4\pi F)^2} \log \frac{\overline{m}^2}{\mu_\chi^2} + \frac{\overline{m}^2}{F^2} \left( 12 L_4^r(\mu_\chi) + 4 L_5^r(\mu_\chi) \right) \right]. \quad (6)$$

## B. Alternative Definitions of ‘The Leading Chiral Term’

For the remainder of this paper, we leave the SU(3) limit and hereafter employ physical values for all particle masses. Consider K-to-2 $\pi$  matrix elements of the operators  $\mathcal{Q}_{7,8}$  written as

$$\mathcal{M}_I \equiv \langle (\pi\pi)_I | \mathcal{Q}_{7,8} | K \rangle = \mathcal{M}_I^{(0)} \cdot (1 + \Delta_I) \quad (I = 0, 2), \quad (7)$$

where  $\mathcal{M}_I^{(0)}$  is evaluated in the chiral world and  $\Delta_I$  gives the fractional  $\mathcal{O}(p^2)$  correction. In particular we found in Ref. [1] for the isospin I=2 case that chiral corrections *increase* the chiral limit value by about 27% ( $\Delta_2^{\text{CG}} = +0.27 \pm 0.27$ ), in seeming contrast with the recent claim [8] that chiral loops *reduce* the chiral limit value by about 50%.

We wish to explain the origin of this discrepancy. It is not due to mistakes in either Ref. [1] or Ref. [8] but rather to the fact that the chiral limit result is normalized differently in these two papers. Our first observation is that in Ref. [1] we work with a dimensionful coupling  $g$  (of dimension six in mass), while in Ref. [8] a dimensionless coupling is used (we denote it here by  $\overline{g}$ ),

$$\mathcal{O}_{\text{ewp}} = \begin{cases} g \langle \lambda_6 U Q U^\dagger \rangle & (\text{Ref. [1]}) \\ F^6 \overline{g} \langle \lambda_6 U Q U^\dagger \rangle & (\text{Ref. [8]}) \end{cases}, \quad (8)$$

implying the leading order matrix elements

$$i\mathcal{M}_2^{(0)} = \begin{cases} 2g/(3F^3) & (\text{Ref. [1]}) \\ 2F^3 \overline{g}/3 & (\text{Ref. [8]}) \end{cases}. \quad (9)$$

Moreover, the chiral loop corrections are defined in the two references as

$$i\mathcal{M}_2^{(0+2)} = \begin{cases} \frac{2}{3} \frac{g}{F_\pi^2 F_K} (1 + \Delta_2^{\text{CG}}) & (\text{Ref. [1]}) \\ \frac{2}{3} F_\pi^3 \overline{g} (1 + \Delta_2^{\text{PPS}}) & (\text{Ref. [8]}) \end{cases}. \quad (10)$$

That is, both analyses shift some one-loop terms (the ratios  $F/F_\pi$  and  $F/F_K$ ) into the definition of the leading order matrix element. Clearly this makes no difference at all if one sums the leading and next-to-leading terms. However, this will affect what the two references call the ‘next-to-leading term’ ( $\Delta_2^{\text{CG}}$  vs  $\Delta_2^{\text{PPS}}$ ) and explains the difference in their

stated ‘chiral corrections’.<sup>2</sup> The large difference in the stated results is accounted for by the large powers of  $F/F_\pi$  and  $F/F_K$  needed to relate  $\Delta_2^{\text{CG}}$  to  $\Delta_2^{\text{PPS}}$ . Moreover, neither of the two definitions coincides with the one given in Eq. (7), where  $\Delta_I$  includes *all* the corrections of  $\mathcal{O}(p^2)$ .

### C. Estimate of Pure Next-to-Leading Order Corrections

Some recent papers [9–11] are devoted to evaluating the  $K \rightarrow \pi\pi$  electroweak penguin matrix elements in the chiral limit. The procedure used there is to relate the dimensionful constant  $g$  to vacuum expectation values of appropriate dimension six operators. The  $K \rightarrow \pi\pi$  matrix elements are then obtained by normalizing with the appropriate numerical factors and  $1/F^3$ , corresponding to the first line in Eq. (9). The chiral corrections to these determinations (see also Ref. [13]) are therefore given by  $\Delta_2$  of Eq. (7) and upon adopting the convenient reference scale as the  $\rho$ -meson mass ( $\mu_\chi = m_\rho$ ) we find

$$\Delta_2 = -0.118 - 0.727 \frac{L_4^r(m_\rho)}{10^{-3}} - 0.134 \frac{L_5^r(m_\rho)}{10^{-3}} + \Delta_2^{(\text{ct})}(m_\rho) \quad , \quad (11)$$

where  $\Delta_2^{(\text{ct})}(m_\rho)$  is the contribution from the finite  $\mathcal{O}(p^2)$  electroweak counterterms,

$$\Delta_2^{(\text{ct})}(m_\rho) = -\frac{1}{g} \left[ m_K^2 (c_2^r + c_3^r - 2c_4^r - 2c_5^r - 4c_6^r) - m_\pi^2 (c_1^r + c_2^r + 4c_4^r + 4c_5^r + 2c_6^r) \right]_{\mu_\chi=m_\rho} \quad . \quad (12)$$

In Eq. (11), the first numerical factor comes from chiral loops evaluated at scale  $m_\rho$  and the LECs  $L_{4,5}^r$  (which enter via wavefunction renormalization) are normalized to  $10^{-3}$ . Analogous to the procedure adopted in Ref. [1], we can estimate the size of  $\Delta_2^{(\text{ct})}(m_\rho)$  by varying the scale of the pure chiral loop term between 0.6 GeV and 1 GeV. This procedure yields  $|\Delta_2^{(\text{ct})}(m_\rho)| \leq 0.20$ . We then use the value  $L_5^r(m_\rho) = (1.4 \pm 0.5) \cdot 10^{-3}$  to arrive at the conservative estimate

$$\Delta_2 = - \left( 0.30 \pm 0.21 + 0.727 \frac{L_4^r(m_\rho)}{10^{-3}} \right) \quad . \quad (13)$$

Since  $L_4^r$  is poorly known, it is not possible to estimate  $\Delta_2$  more precisely. We note that  $\Delta_2^{\text{CG,PPS}}$  can be related to  $\Delta_2$  by using the appropriate expressions for  $F_\pi/F$  and  $F_K/F$ , and we have explicitly checked the agreement of Ref. [1] and Ref. [8] on this point.

Finally, in the large  $N_c$  limit one has  $L_4 = 0$  and an explicit expression of  $\Delta_2^{(\text{ct})}$  in terms of  $L_5^r$  [8]. This term is seen to cancel almost exactly the  $L_5$  contribution from wavefunction renormalization [8], and one obtains<sup>3</sup>  $\Delta_2^{N_c \rightarrow \infty} = -0.08$ .

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<sup>2</sup>For a toy version of this point, if ChPT analyses of researchers A,B are written as  $\mathcal{M}_A = 10-2$  and  $\mathcal{M}_B = 5+3$ , then the percentage corrections will be very different,  $\Delta_A = -20\%$  and  $\Delta_B = +60\%$ .

<sup>3</sup>The large  $N_c$  estimate reported here only refers to the operator  $\mathcal{Q}_8$ , which is of considerable phenomenological interest.

### III. SUMMARY OF RESULTS

We enumerate our conclusions as follows:

1. We have displayed in Eq. (4) the K-to- $\pi$  matrix elements of the electroweak penguin operator  $\mathcal{O}_{\text{ewp}}$  in the limit of SU(3) flavor symmetry. This will allow comparison with lattice studies which work in this kinematic regime.
2. We have pointed out how different definitions of ‘leading chiral order’ amplitudes (*viz* Eq. (10)) can lead to numerically distinct ‘chiral corrections’, which differ not only in magnitude but even in sign. Although such distinctions are a matter of chiral book-keeping and have no intrinsic meaning, one must be careful to avoid misinterpretation.
3. We have provided in Eq. (13) a numerical estimate of  $\mathcal{O}(p^2)$  corrections to  $\langle(\pi\pi)_{I=2}|\mathcal{Q}_{7,8}|K\rangle$ . Such corrections would modify recent chiral determinations of this matrix element. Our results indicate that even in the extreme scenario of  $\Delta_2 \simeq -0.50$  allowed by our uncertainties, the results of Refs. [9,11] for  $\langle(\pi\pi)_{I=2}|\mathcal{Q}_8|K^0\rangle$  remain significantly larger than cited lattice values [12].

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