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# Type-1.5 superconductivity in two-band systems

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In the usual Ginzburg-Landau theory the critical value of the ratio of two fundamental length scales in the thery $\kappa_c = 1/\sqrt{2}$ separates regimes of type-I and type-II superconductivity. The latter regime possess thermodynamically stable vortex excitations which interact with each other repulsively and tend to form vortex lattices. It was shown in [5] that this dichotomy in broken in $U(1) \times U(1)$ Ginzburg-Landau models which possess three fundamental length scales which results in the exisrtence of a distinct phase with vortex excitations which interact attractively at large length scales and repulsively at shorter distances. Here we briefly review these results in particular discussing the role of interband Josephson coupling and the case where only one band is superconducting while superconductivity in another band is induced by interband proximity effect. The report is partially based on E. Babaev, J. Carlström, J. M. Speight arXiv:0910.1607. [a]

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The textbook classifications of superconductors divide them in two classes: type-I and type-II, according to their behavior in an external field. Type-I superconductors expel low magnetic fields, while elevated fields produce macroscopic normal domains in the interior of superconductor. Type-II superconductors possess much richer magnetic response by supporting stable vortex excitations. Lattices of these vortices form as the energetically preferred state when the applied magnetic field exceeds a certain threshold called the lower critical magnetic field. This picture of type-II superconductivity relies on the fact that interaction between co-directed vortices is purely repulsive [2]. In [5] is was demonstrated that in two-component superconductors, there are vortex solutions in a very wide parameter range which are on one hand thermodynamically stable, and on the other hand, possess interaction potential which is non-monotonic: repulsive at short distances but attractive at larger distances. The longer range attractive interaction part originates from the fact that in these solutions, the size of the core of one of the components is the largest length scale of the problem: i.e. the core of one of the components extends beyond the current carrying region. In general, the precise conditions for the appearance of non-monotonic interaction are quite complicated. However, in the simplest case the following description is quite accurate: When two vortices are situated at a distance smaller than the extended core size, but larger than the effective magnetic field penetration length, then the vortices attract each other. At shorter distances the interaction mediated by currents and magnetic field wins and the vortices start to repel each other. This is schematically shown on Fig. 1. It should be stressed that in the one-component Ginzburg-Landau theory co-directed vortices have attractive interaction they are thermodynamically unstable because the first critical magnetic field in that case is typically larger than the thermodynamical critical magnetic field. However it was shown that in two-component superconductors there is a large range of parameters where the vortices with long-range attractive, and short-range repulsive interaction are thermodynamically stable (i.e. can be produced by magnetic fields with strengths smaller than the thermodynamical critical magnetic field [5]).

Indeed such a vortex interaction, along with thermodynamic stability, should cause the system response to external field to be entirely different from vortex states of traditional type-II Ginzburg-Landau model. Namely, the attraction between vortices should, at low fields, produce the "semi-Meissner state" [5]). The implications of it include (i) formation of voids of vortex-less states, where there are *two well developed superconducting components* and (ii) *vortex clusters where one of the components would typically dominate* because the second component would be suppressed (in fact significantly suppressed for a range of parameters) due to overlapping of outer cores of the vortices. The "phase separation", of this nature, which, from the point of view of the second component resembles a mixed state of type-I superconductors makes this system principally different from the inhomogeneous vortex states of single-component superconductors where inhomogeneity can be induced by small corrections beyond the Ginzburg-Landau theory in regimes where $\kappa$ is close to $1/\sqrt{2}$ see remark [3].

The two-band superconductor $MgB_2$ [8] was regarded in many early theoretical and experimental works as a standard type-II superconductor which should possess regular Abrikosov vortex lattices [11]. However, an objection to this scenario was raised in the recent experimental works by Moshchalkov et al. [6, 7] where a formation of highly inhomogeneous states was observed with vortex clusters and vortex-less Meissner domains strikingly similar to the picture of the semi-Meissner state [5] which results from vortices having a longer range attractive part in the interaction potential in the two-component model. In the ref. [6] which was based on Bitter dec-

[a] A talk given at the "Vortex VI" conference on 17 September 2009, Rhodes, Greece.

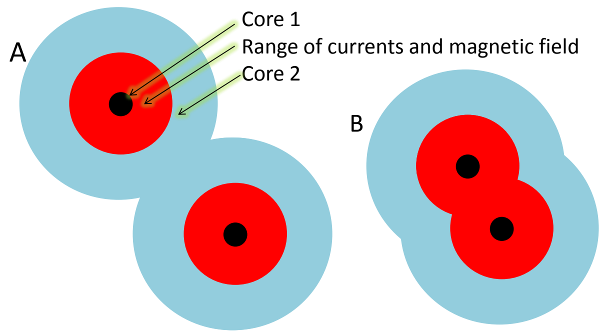



Figure 1. A schematic illustration of the origin of the non-monotonic interaction potential between vortices in the superfluid mixture without intercomponent Josephson coupling discussed in [5]. A: attractive interaction mediated by outer cores overlap B: domination of the repulsive interaction mediated by currents and magnetic field.

oration methods and [7] based on scanning SQUID microscopy a statistically preferred intervortex separation was reported. Moshchalkov et al. proposed that this phase separation is an intrinsic property of $MgB_2$ and is associated with the mentioned above three fundamental length scales in a two-component superconductor which in that case represents a new kind of superconducting states outside the usual type-I/type-II dichotomy. The term type-1.5 superconductivity was coined for this scenario in [6]. Let us stress that if there appear several fundamental length scales at the level of Ginzburg-Landau theory such a state is indeed entirely different from the states of single-component supercondtors. In the latter case, although a variety of different non-universal microcopic corrections may indeed produce a weak intervortex attraction [3], it does not alter the classification of single-component superconductors at the level of fundamental length scales in the Ginzburg-Landau theory.

The theory in [5] with added intercomponent Josephson coupling (briefly considered below) directly applies to the case where there is fully developed superconductivity in both bands. However in general in a two-band superconductor, at elevated temperatures there can be a regime where only one band is superconducting while superconductivity in another band is induced by inter-band proximity effect (also called inter-band Josephson effect). In particular this was argued to be the case in $MgB_2$ above a certain temperature [9].

So it is an important generic question whether type-1.5 superconductivity is possible in the case where one of the bands does not have a coherence length in the Ginzburg-Landau sense, and has a non-zero density of superconducting condensate only because of the inter-band proximity effect.

To study the essential properties of vortex physics in two-component systems we use the following free energy density functional

$$\begin{aligned}\mathcal{F} = &\frac{1}{2}\Big(|\psi_1|^2-1\Big)^2+\alpha|\psi_2|^2+\frac{1}{2}\beta|\psi_2|^4 \\ &+\frac{1}{2}|(\nabla+ie\mathbf{A})\psi_1|^2+\frac{1}{2}|(\nabla+ie\mathbf{A})\psi_2|^2 \\ &-\eta|\psi_1||\psi_2|\cos(\theta_2-\theta_1)+\frac{1}{2}(\nabla\times\mathbf{A})^2.\end{aligned} \tag{1}$$

The regime with $\eta = 0, \alpha < 0, \beta > 0$ corresponds to the situation of two independent superconducting components coupled only by vector potential, studied in [5]. In the case of two bands with well-developed superconductivity, the inter-band Josephson coupling $\eta \neq 0$ works against the type-1.5 regimes.

We studied numerically the effect of the Josephson coupling on the vortex-vortex interaction energy in a system with two superconducting bands (i.e. $\eta \neq 0, \alpha < 0, \beta > 0$). The results of numerical calcualions of the intervortex interaction energy in the model (1) are shown on Fig. 2. In the first curve $\eta = 0$ and the condensates interact only through the shared vector potential, the parameters $\alpha, \beta, e$ were choosen to yield a disparity of coherence lengths and penetration depth to produce a type-1.5 regime. Adding a moderate Josephson coupling $\eta = 0.05$ increases ground state densities of the condensates, and decreases penetration length (which depends on superfluid densities in both bands and thus on $\eta$) which results in fact in a *deeper* minimum of the interaction potential. However this coupling decreases the disparity of the recovery rates of the condensates, resulting in a decreased range of the attractive interaction. Even though a sufficiently strong Josephson coupling in the GL model can eliminate type-1.5 behavior, this example shows that the type-1.5 behavior services even in case of a rather substantial interband Josephson coupling. Similarly type-1.5 regime exists also in the presence of mixed gradient terms [10].

Consider now the the case of nonzero Josephson coupling $\eta \neq 0$ but with one of the bands being *above* its critical temperature [1]. In that case the effective potential for $\psi_2$ has *only positive* coefficients $\alpha, \beta > 0$. Thus the second band has a nonzero density of Cooper pairs only because of inter-band tunneling represented by the term $-\eta|\psi_1||\psi_2|\cos(\theta_2-\theta_1)$. This term also locks phases $\theta_1 = \theta_2$. So in the following, we consider only solutions with the winding in the total "locked" phase. These vortices have finite energy and carry one flux quantum. If there is a phase winding only in one phase, one gets a Josephson vortex with linearly diverging energy [12]

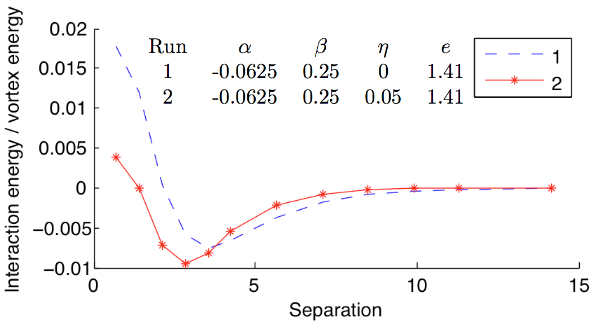



Figure 2. Intervortex interaction energy in a system with two active bands. In the first case, the Josephson coupling is zero, and the ground state densities of the condensates are 1 and 0.25. In the second case, nonzero but moderate Josephson coupling $\eta = 0.05$ decreases the range of the attractive part of the interaction potential but at the same time it increases the ground state densities to approximately 1 and 0.4, yielding a slightly deeper minimum.

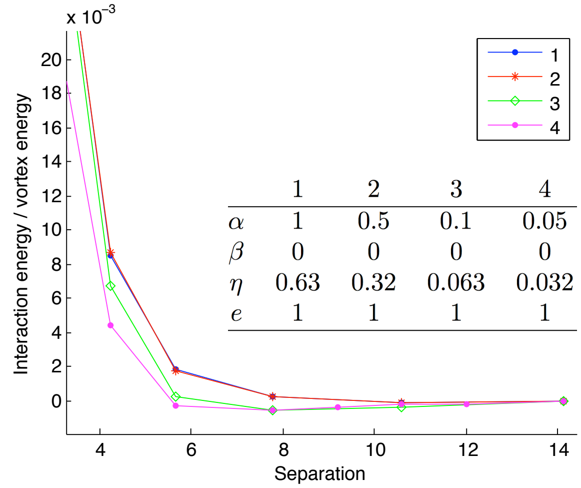



Figure 3. Intervortex interaction energy for a density ratio of 0.1.

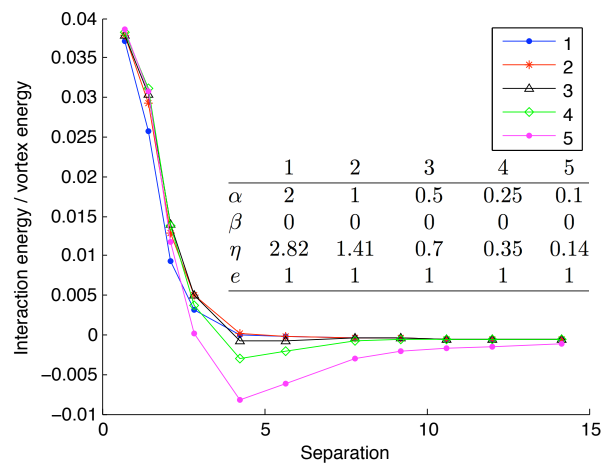



Figure 4. Intervortex interaction energies for density ratio of 0.5.

(which cannot be produced by external field under usual circumstances).

Since the phases in this regime are locked to equal values which minimizes the Josephson term $\theta_1 = \theta_2 = \phi$, our effective model becomes

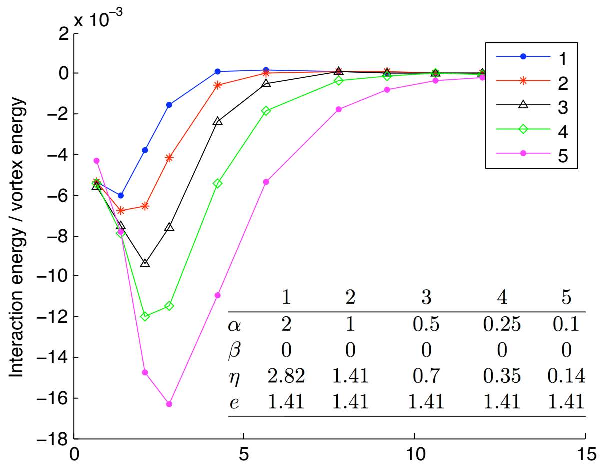
$$\begin{aligned} \mathcal{F} = &\frac{1}{2}\Big(|\psi_1|^2 - 1\Big)^2 + \alpha|\psi_2|^2 + \frac{1}{2}\beta|\psi_2|^4 \\ &+\frac{1}{2}|(\nabla + ie\mathbf{A})\psi_1|^2 + \frac{1}{2}|(\nabla + ie\mathbf{A})\psi_2|^2 \\ &-\eta|\psi_1||\psi_2| + \frac{1}{2}(\nabla\times\mathbf{A})^2. \end{aligned} \tag{2}$$

We present accurate numerical solutions for one-quanta vortices (i.e. with the phase winding $\Delta\phi = 2\pi$) and vortex-vortex interaction in the model (2) (for other details including analytical theory see [1]). The numerical solutions were obtained using a local relaxation method. A two-vortex configuration is initially generated fixing only the positions of the vortex cores and phase windings. Then this multiple vortex configuration is relaxed with respect to all the other degrees of freedom in the system, thus producing highly accurate solutions of the Ginzburg-Landau equations of motion with given phase windings and vortex separation. The procedure is repeated for a different vortex separation yielding a highly accurate vortex interaction potential.

First lets consider the regime where the fourth order term in $|\psi_2|$ can be neglected. In this case we conducted simulations with the density ratios $|\psi_2|^2/|\psi_1|^2$ being 0.1 and 0.5 [1]. The numerical results are presented in Figs. 3-5. The computed interaction energy is given in units of $2E_v$ where $E_v$ is the energy of a single vortex.

In the first case with the density ratio 0.1, we find that in general, the recovery lengths of the condensates can be quite different, even though one of the bands has proximity-induced superconductivity. We find that as a consequence of the disparity in the recovery lengths, the system crosses over from the Type-II to the Type-1.5 regime when $\alpha$ and $\eta$ are sufficiently small (Fig. 3). The low density of condensate in the band with proximity-induced superconductivity means that the attractive part of the interaction is weak. In the curves 3 and 4, we find a slight long range attraction, yielding a minimum energy at a separation of approximately 8. The curves 3 and 4 correspond to the smallest values of $\alpha$ and $\eta$, yielding quite large cores in the band with proximity induced superconductivity.

In the second case (Fig. 4), the density ratio is 0.5. The




| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\alpha$ | 2 | 1 | 0.5 | 0.25 | 0.1 |
| $\beta$ | 0 | 0 | 0 | 0 | 0 |
| $\eta$ | 2.82 | 1.41 | 0.7 | 0.35 | 0.14 |
| $e$ | 1.41 | 1.41 | 1.41 | 1.41 | 1.41 |

Figure 5. Intervortex interaction energies at a density ratio of 0.5 and an increased charge of $e = 1.41$.

vortex-vortex binding energy is now much larger, and the minimum energy occurs at a smaller separation. Long range attraction occur in curves 3-5 with a maximum $\alpha$ of 0.5, in contrast to $\alpha \approx 0.1$ in the previous case.

In the third case (Fig. 5), the charge has been increased by a factor $\sqrt{2}$. The resulting shorter penetration length decreases the magnetic repulsion between vortices. Observe that now the energy of an axially symmetric vortex solution with two flux quanta is smaller than the energy of two infinitely separated one-quanta vortices, nonetheless the axially-symmetric two-quanta vortex is not stable since the minimum energy corresponds to a nonzero vortex separation.

Figure 6 shows the effect of the addition of a fourth order term with $\beta = 0.1$ in the free energy of the band with Josephson-induced superconductivity. This image is further reinforced in Fig. 7 showing pronounced non-monotonic interaction and thus type-1.5 superconductivity in this system. Figure 8 shows a system with larger density ratio than the previous systems. The increased condensate density, especially in the band with Josephson-induced condensate provides a dominating attractive interaction potential that pushes the system into the Type-I regime.

The figures 9-10 show cross-sections of vortices in two cases exhibiting Type-1.5 superconductivity. The first case is the fifth curve of Fig. 4. The right image correspond to the energy minimum. Here, the cores overlap is significant in the induced band, but almost nonexistent in the active band. There is a moderate overlap of magnetic fields. Decreasing the separation produces slightly more cores overlap in the band with induced superconductivity, but the condensation energy gained is more than compensated by the increasing magnetic and current-current interaction-driven repulsion, resulting in increased total energy. In the second case, correspond-

| Run | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $\alpha$ | 2 | 1 | 0.5 | 0.25 | 0.1 |
| $\beta$ | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| $\eta$ | 2.82 | 1.41 | 0.7 | 0.35 | 0.14 |
| $e$ | 1.41 | 1.41 | 1.41 | 1.41 | 1.41 |
| $\|\psi_2\|^2/\|\psi_1\|^2$ | 0.46 | 0.44 | 0.41 | 0.36 | 0.29 |
| $\|\psi_2\|^2+\|\psi_1\|^2$ | 2.84 | 2.11 | 1.73 | 1.5 | 1.34 |

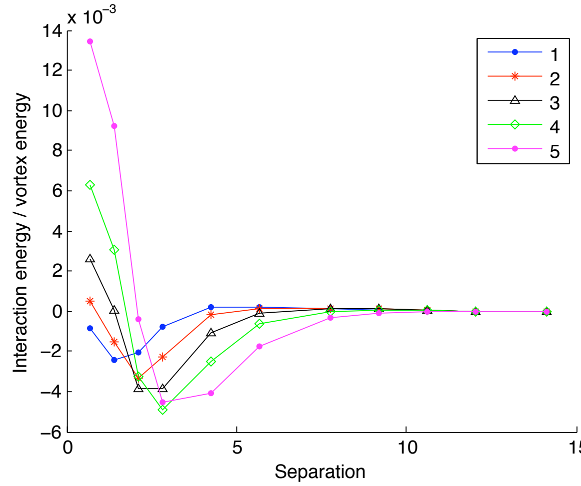



Figure 6. Intervortex interaction energy. Model parameters are given in the inset. Observe that the density ratios are different for different curves.

| Run | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 0.1 | 0.5 | 2 | 0.1 | 0.5 | 2 | 0.1 | 0.5 | 2 |
| $\beta$ | 0.1 | 0.1 | 0.1 | 0.5 | 0.5 | 0.5 | 1 | 1 | 1 |
| $\eta$ | 0.215 | 0.8 | 3 | 0.55 | 1.2 | 3.6 | 1.09 | 1.89 | 4.8 |
| $e$ | 1.41 | 1.41 | 1.41 | 1.41 | 1.41 | 1.41 | 1.41 | 1.41 | 1.41 |
| $\|\psi_2\|^2/\|\psi_1\|^2$ | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| $\|\psi_2\|^2+\|\psi_1\|^2$ | 1.61 | 1.93 | 3.12 | 1.78 | 2.12 | 3.38 | 2.06 | 2.5 | 4.1 |

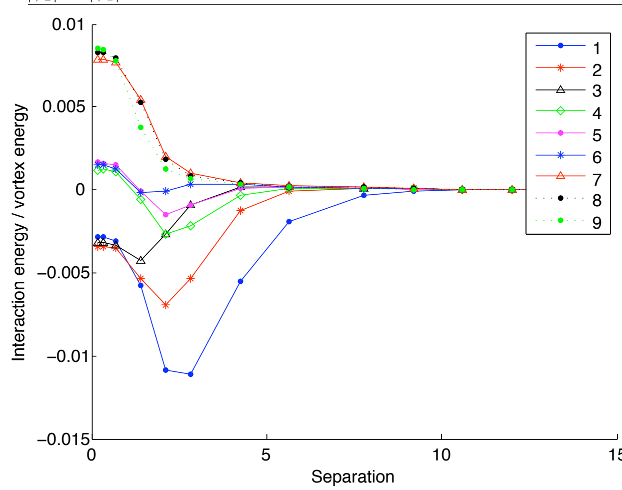
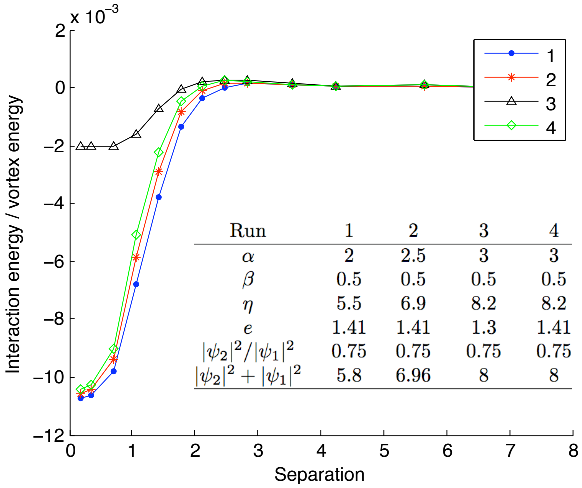



Figure 7. Intervortex interaction energy in different regimes described in the inset.




| Run | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $\alpha$ | 2 | 2.5 | 3 | 3 |
| $\beta$ | 0.5 | 0.5 | 0.5 | 0.5 |
| $\eta$ | 5.5 | 6.9 | 8.2 | 8.2 |
| $e$ | 1.41 | 1.41 | 1.3 | 1.41 |
| $\lvert\psi_2\rvert^2/\lvert\psi_1\rvert^2$ | 0.75 | 0.75 | 0.75 | 0.75 |
| $\lvert\psi_2\rvert^2+\lvert\psi_1\rvert^2$ | 5.8 | 6.96 | 8 | 8 |

Figure 8. Intervortex interaction energy in different regimes described in the inset.

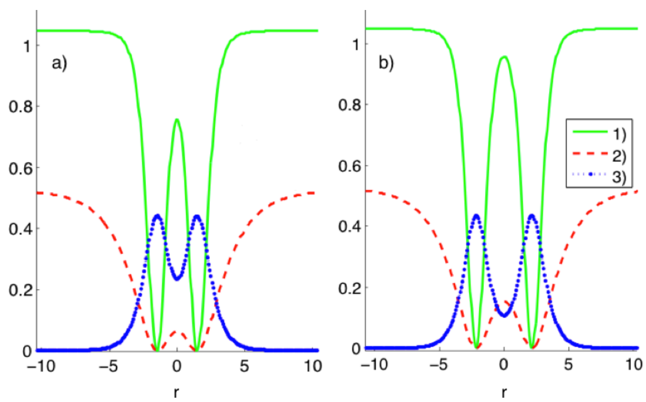



Figure 9. Cross section of interacting vortices for the case $\alpha = 0.1$, $\beta = 0$, $\eta = 0.14$ and $e = 1$ (the fifth curve in fig 4). Curves "1" and "2" show the behaviour of $|\psi_{1,2}|$ and the curve "3" represents the magnetic field. The right image displays the system at vortex separation of $\approx 4.2$ (energy minimum), the left image displays the system at a separation of $\approx 2.8$

ing to the fourth curve in Fig. 6, the charge is larger, resulting in a more sharply peaked magnetic field. The minimum energy does in this case occur at a smaller separation (2.8 instead of 4.2). The overlap of the cores in the main band, as well as in the magnetic field/current carrying regions is larger in this case. Observe that the increase in $\alpha$ and $\eta$ clearly results in a faster recovery of the condensate density in the band with proximity effect induced superconductivity.

In conclusion, type-1.5 superconductivity is a magnetic response possible in multicomponent systems because of the existence of several fundamental length scales associated with the masses of the fields which is distinct from the type-I/type-II dichotomy found in usual single-component Ginzburg-Landau model. Here we discuss

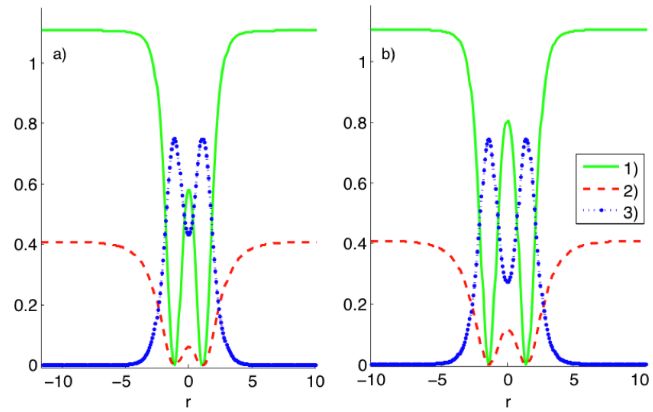



Figure 10. Cross section of interacting vortices for the case $\alpha = 0.25$, $\beta = 0.1$, $\eta = 0.35$ and $e = 1.41$ (the fourth curve in fig 6). Curves "1" and "2" show the behaviour of $|\psi_{1,2}|$ and the curve "3" represents the magnetic field. The right image displays the system at a separation of $\approx 2.8$ (energy minimum), the left image displays the system at a separation of $\approx 2.1$

that this kind of superconductivity may be present for a rather large range of parameters in two-band systems becasue it persists in the presence of intercomponent Josephson coupling and even can take place in the case where only one of the bands has true superconductivity while superconductivity in the other band is induced by interband proximity effect [1].

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[3] The single component Ginzburg-Landau theory is parameterized by a ratio $\kappa$ of the fundamental length scales in the theory: the penetration length $\lambda$ and the coherence length $\xi$. The critical value of the Ginzburg-Landau parameter $\kappa_c = 1/\sqrt{2}$ separates the regimes of repulsively interacting thermodynamically stable vortices ($\kappa_c > 1/\sqrt{2}$) and attractively interacting thermodynamically unstable vortex solutions in type-I regime ($\kappa_c < 1/\sqrt{2}$). Precisely at $\kappa_c = 1/\sqrt{2}$ in the Ginzburg-Landau theory vortices do not interact. Note that, strictly speaking, the fundamental-length-scales-based type-I/type-II classification of superconductors which yields noninteracting vortices at the critical value of $\kappa$ is a property of a description at the level of Ginzburg-Landau theory in the sense that more complexity can arise as a consequence of inhomogeneity, boundary effect and indeed at some more microscopic level one can identify a variety of small effects

which would introduce small corrections to the universal GL form of vortex interaction. This is quite relevant for known single-component superconductors where is $\kappa$ very close to $1/\sqrt{2}$, because in that regime vortex interaction is very weak and the effects beyond the Ginzburg-Landau theory are more pronounced and can in principle produce a weak intervortex attraction [4]. Here however we are interested only in a fundamental classification of magnetic responce multicomponent superconductors, that is, a classification based on the fundamental length scales associated with fields in a Ginzburg-Landau theory and we do not consider small corrections arising from specific microscopic physics or higher order corrections. The effects discussed in the present work and [1, 5] are therefore entirely different from the attractive interaction due to non-GL effects in superconductors with $\kappa_c \approx 1/\sqrt{2}$ [4]. For early experiment on superconductors with $\kappa$ very close to $1/\sqrt{2}$ see [13].


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