Supercurrent stability in a quasi-one-dimensional weakly interacting Bose gas

Supercurrent Stability in a Quasi-1D Weakly Interacting Bose Gas

Yu. Kagan, N.V. Prokof'ev, and B.V. Svistunov
Russian Research Center "Kurchatov Institute," 123182 Moscow, Russia

We discuss a possibility of observing superfluid phenomena in a quasi-1D weakly interacting Bose gas at finite temperatures. The weakness of interaction in combination with generic properties of 1D liquids can result in a situation when relaxational time of supercurrent is essentially larger than the time of experimental observation, and the behavior of the system is indistinguishable from that of a genuine superfluid.

PACS numbers: 03.75.Fi, 05.30.Jp, 67.65.+z

The discovery of Bose-Einstein condensation in ultra-cold dilute gases [1] has opened up a unique possibility for the study of fine aspects of superfluidity, including the specifics of lower dimensionality. Primarily, the new opportunities come from the availability of isolated systems, free of contact with walls or any sort of substrate. A special interest is associated with the prospects of creating one-dimensional systems. Actually, one deals here with a quasi-1D situation, when the configuration of a trap yields toroidal spatial particle distribution with small transverse localization size $l_\perp$. At low temperature, and small interaction energy per particle, ϵ_0, as compared to the transverse-motion level spacing, the system kinematics and collective dynamics are purely one-dimensional, though the value of the effective interaction is determined by 3D pair collisions (see below).

Strictly speaking, the 1D superfluidity cannot exist at $T \neq 0$. - At finite temperature the relaxational time for supercurrent is finite, i.e., independent of the system size in the limit $L \to \infty$. Nevertheless, as will be shown below, analyzing the situation with a dilute gas one finds that the relaxation rate of the supercurrent is *exponentially* reduced with decreasing interaction, and that in a *quasi*-1D *weakly* interacting system this can lead to a new quality when at a finite temperature the lifetime of a supercurrent state considerably exceeds the lifetime of the isolated gas sample.

We start with rendering basic notions and results for the low-energy properties of 1D (super)fluids. First, we remind that in 1D there exists a condition on superfluid density, n_s, and compressibility, κ, determining whether the superfluid groundstate is stable against either single impurity, or disorder, or an external commensurate potential. This condition requires that the dimensionless parameter (m is the particle mass, $\hbar$=1)

$$K = \frac{1}{\pi}\sqrt{\frac{m}{\kappa n_s}} \equiv \frac{cm}{\pi n_s} \tag{1}$$

be less than corresponding critical value K_c (here $c = \sqrt{n_s/m\kappa}$ is the sound velocity). For the most important for our purposes cases of single impurity and random potential the critical values of K are $K_c = 1$ [2] and $K_c = 2/3$, respectively [3]. As we will see below, the case of quasi-1D weakly interacting Bose gas corresponds to $K \ll 1$, so that the stability of the superfluid groundstate is guaranteed.

The low-energy spectrum of any one-component 1D system in the superfluid phase is exhausted by bosonic sound-like excitations and a collective state supporting non-zero supercurrent, $J_I = 2\pi n_s I/mL$, labeled by an integer index I [4] (we consider a system of ring geometry)

$$H_0 = \sum_{q\neq 0} \omega_q b_q^\dagger b_q + \sum_I E_I d_I^\dagger d_I \ , \tag{2}$$

where $b_q^\dagger$ creates a boson with energy $\omega_q = cq$ and momentum $q = 2\pi k/L$, where k is integer. We have used a standard short-hand notation to specify the current state - obviously $d_I^\dagger d_I$ is either 1 or 0 and $\sum_I d_I^\dagger d_I = 1$. The energy of the current state is given by

$$E_I = \frac{2\pi^2 n_s}{mL} I^2 \ . \tag{3}$$

Note, that although the energy of the current state scales like $\sim 1/L$, the momentum associated with it is system-size independent, $P_I = 2\pi n_s I$.

Within the long-wavelength effective Hamiltonian [describing the superfluid state in terms of the phase field], $H = \int dx[(n_s/2)(d\Phi/dx)^2 + (\kappa/2)\dot{\Phi}^2]$, where the original Bose field is written as $\Psi = \sqrt{n}e^{i\Phi}$, bosonic excitations and current states (sometimes called "zero mode" states [4]) correspond to the following decomposition

$$\Phi = \phi(b) + 2\pi I x/L \ , \qquad \oint (d\phi/dx)dx = 0 \ ,$$

where x is a coordinate along the circle. Thus I is nothing but a phase field circulation

$$I = \frac{1}{2\pi} \oint (d\Phi/dx)dx \ . \tag{4}$$

The very notion of the superfluid phase means that I is a well-defined quantum number even when rotational invariance is violated. In higher dimensions at $T < T_c$ relaxation times τ_I are extremely long with exponential dependence on the system size, and for all relevant experimental time-scales, $\tau_{\rm exp}$, the condition $\tau_{\rm exp} \ll \tau_I$ is satisfied. Special arrangements are to be made to shorten τ_I (e.g., weak-link or tunnel-junction systems). To find relaxation times in 1D we may use the transition Hamiltonian [5] which gives explicitly transition amplitudes between states $|I, \{N_q\}\rangle$ and $|I', \{N'_q\}\rangle$ for arbitrary rotational invariance breaking terms. For the case of a local perturbation $V(x)$ with spatial dimension smaller than the correlation radius $1/mc$, which without loss of generality may be replaced by

$$V(x) \to V_0 \, \delta(x - x_0) \ ,$$

the transition Hamiltonian acquires a form

$$H_{\rm int} = gV_0 n \sum_{II'} d_I^\dagger d_{I'} e^{i2\pi n(I-I')x_0} \Lambda_{II'}(b) + h.c. \ . \tag{5}$$

Polaronic-type exponential operator $\Lambda(b)$ in this expression equals to

$$\Lambda_{II'}(b) = \exp\left\{ \frac{i(I-I')}{\sqrt{K}} \sum_{q\neq 0} \left(\frac{2\pi}{L|q|}\right)^{1/2} {\rm sgn}(q)(b_q - b_q^\dagger) e^{iqx_0} \right\} \ . \tag{6}$$

The exact value of the numeric coefficient g in Eq. (5) is outside the scope of the long-wavelength treatment used to derive H_{int} (it is of order unity for weakly-interacting Fermi gas).

Formally, the problem has reduced to the problem of particle dynamics on a 1D lattice with Ohmic coupling to the oscillator bath environment (see, e.g., review [6]) after polaronic on-site transformation. The dimensionless Ohmic coupling parameter

$$\alpha = (I - I')^2/K \tag{7}$$

is large in our case, $\alpha > 1$, and "particle"/current dynamics is incoherent at any finite temperature. We may then immediately utilize the well-known expression for the transition probability between states I and I' which derives from the Golden-rule expression for $H_{\rm int}$ (see, e.g., Ref. [7])

$$W = |gV_0 n|^2 e^{-Z} \frac{2\sqrt{\pi}\Omega}{\xi_{II'}^2 + \Omega^2} \frac{|\Gamma[1 + \alpha + i\xi_{II'}/(2\pi T)]|^2}{\Gamma[1+\alpha]\Gamma[1/2+\alpha]} e^{\xi_{II'}/2T} \ , \tag{8}$$

where

$$Z = 2\alpha \ln\left(\frac{\gamma \epsilon_0}{2\pi T}\right) \ , \tag{9}$$

$$\Omega = 2\pi\alpha T \ , \tag{10}$$

$$\xi_{II'} = E_I - E_{I'} \ , \tag{11}$$

γ is a numeric coefficient of order unity, and ϵ_0 plays the role of the high-energy cutoff (at higher energies the dispersion curve is no longer sound-like and physics is determined by single-particle processes). Equation (8) was first derived in Refs. [8,9].

To find the decay rate τ_I^{-1} one has to sum Eq. (8) over I', but since we are interested in the parameter range $\alpha \gg 1$ and $T \ll \epsilon_0$ the dominant contribution comes from $I' = I \pm 1$. It follows then from Eq. (3) that $\xi \sim 4\pi^2 n_s I/mL \ll T$

even for $E_I \sim T$, and we may neglect current energy transfer to the bosonic environment in Eq. (8). Also, for $\alpha \gg 1$ we may write approximately $\Gamma[1+\alpha]/\Gamma[1/2+\alpha] \approx \sqrt{\alpha}$. The final result may be written then as

$$\tau_I^{-1} \sim \frac{|gV_0 n|^2}{T\sqrt{\alpha}} \left(\frac{2\pi T}{\gamma \epsilon_0}\right)^{2\alpha} . \tag{12}$$

We see that for large α the decay rate is severely suppressed at low temperatures $T < T_0 \sim \epsilon_0/2\pi$. Physically, this effect reflects small overlap between different vacuum states corresponding to quantum numbers I and I'. In practice, for $\alpha \gg 1$ there is a "kinetic crossover" between the superfluid and normal behavior, i.e., between frozen and fast current relaxation, at $T \sim T_0$.

To estimate realistic parameters for ultra-cold atomic gases we have to start from the original quasi-1D geometry. Let the transverse motion is confined by the symmetric parabolic potential with frequency $\omega_\perp$. Assuming that the 3D scattering length a is small as compared with the oscillator length $l_\perp = 1/\sqrt{m\omega_\perp}$, we may derive the potential energy per particle from the 3D relation

$$\epsilon_0 = n\frac{4\pi a}{m} \int d\mathbf{r}_\perp \, \varphi_0^4(r_\perp) \equiv nU_{\text{eff}} , \tag{13}$$

where φ_0 is the wavefunction of the transverse motion (here n is the 1D particle density). For the parabolic potential we have

$$\epsilon_0 = n2a\omega_\perp . \tag{14}$$

Purely 1D kinematics requires two conditions to be satisfied

$$T \ll \omega_\perp , \qquad\qquad \epsilon_0 \ll \omega_\perp . \tag{15}$$

The second condition requires that a specific gas parameter, na, be small: $na \ll 1$. When deriving Eq. (12) we assumed that $T \ll \epsilon_0$ to guarantee that transition rates between the supercurrent states are negligible. Obviously, this is a more severe restriction on the temperature range than the first inequality in Eq. (15).

We now turn to the parameter K for a translationally invariant system. (Weak disorder does not change the value of K dramatically.) For the weakly interacting gas compressibility is given simply by inverse U_{eff}, and, in the low-energy limit, $n_s = n$. Thus,

$$K = \frac{1}{\pi}\sqrt{\frac{mU_{\text{eff}}}{n}} = \frac{\sqrt{2}}{\pi}\left(\frac{a}{l_\perp}\right)^{1/2} \frac{1}{(nl_\perp)^{1/2}} . \tag{16}$$

Experimentally it causes no difficulty to choose $nl_\perp > 1$, and to make index K very small or parameter $\alpha = 1/K \gg 1$.

Let us consider an example of sodium gas assuming 1D density $n = 10^6$ cm^{-1}. Then, $na \approx 0.25$ and $\epsilon_0 \approx 0.5 \times \omega_\perp$. For the magnetic trap frequency $\omega_\perp = 3 \times 10^4$ s^{-1}, we find $nl_\perp \approx 30$, and extremely small $K < 0.01$ (!). The crossover temperature for the frozen current dynamics is roughly $T_0 = \epsilon_0/2\pi \approx 2.5 \times 10^{-8}$ K, and within the experimental range.

We note that the effect of supercurrent stability considered above is essentially collective in nature and non-perturbative. Our transition Hamiltonian is valid only if initial and final states are given in terms of sound-like bosonic excitations - its validity may be questioned if the energy change in the transition, ξ, exceeds ϵ_0. This imposes the restriction that interaction is not macroscopically small, $U_{\text{eff}} \gg 2\pi^2/mL$. Also, even formally, for $\xi > \epsilon_0$ we find very fast relaxation rates from Eq. (8) (for estimates one may roughly replace in Eq. (12) $2\pi T \to \xi$). We would like thus to verify that with all the above parameters we still may neglect energy (and momentum) transfer between the current and bosonic modes. Since $\xi = E_I - E_{I-1} = 4\pi^2 n(I-1/2)/mL$, for $L = 1$ cm we find $\xi \approx 8 \times 10^{-9}(I-1/2)$ K, that is in the temperature range between 4×10^{-9} K and T_0 we still have $\xi < T$ for thermal values of I and system-size dependence is not crucial. On another hand, even if the energy has been dissipated into a single bosonic mode (in fact roughly $\alpha \gg 1$ bosons are emitted/absorbed in the transition), the momentum of the boson would be only $q = 4\pi(I-1/2)/KL$ and much smaller than the momentum change of the current state $2\pi n$.

Thus, a quasi-1D interacting Bose gas with realistic parameters can support a supercurrent at $T \neq 0$. The stability of such a state is guaranteed by extremely large relaxational time as compared to any reasonable experimental time. In connection with the question of preparation of supercurrent state, it is worthwhile to note that for an equilibrium 1D system at finite temperature there is a finite probability to be found in a supercurrent state. This naturally follows from the fact that the energy of a supercurrent state scales like $1/L$, so that at finite T the statistical ensemble involves a large set of different numbers I. After a deep cooling, the system will be found in a state with some particular I, randomly varying from one experiment to another [10].

This work was supported by the Russian Foundation for Basic Research (Grant No. 98-02-16262) and by the Grant INTAS-97-0972 [of the European Community].

---

[1] M.H. Anderson, J.R. Ensher, M.R. Matthews, C.E. Wieman, and E.A. Cornell, Science, **269**, 198 (1995).
[2] Kane C.L. and Fisher M.P.A., Phys. Rev. Lett. **68**, 1220 (1992).
[3] Giamarchi T. and H.J.Schulz, Phys. Rev. B **37**, 325 (1988).
[4] F. D. M. Haldane, Phys. Rev. Lett. **47**, 1840 (1981); J. Phys. C **14**, 2585 (1981).
[5] V.A. Kashurnikov, A.I. Podlivaev, N.V. Prokof'ev, and B.V. Svistunov, Phys. Rev. B, **53**, 13091 (1996).
[6] A.J. Leggett, S. Chakravarty, A.T. Dorsey, M.P.A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987).
[7] Yu. Kagan and N.V. Prokof'ev, Zh. Eksp. Teor. Fiz., **96**, 2209 (1989) [Sov. Phys. JETP - **69**, 1350]; Yu. Kagan and N.V. Prokof'ev, in *Quantum Tunneling in Condensed Matter*, eds. Yu. Kagan and A.J. Leggett, North-Holland, Elsevier, 37-143 (1992)
[8] H. Grabert and U. Weiss, Phys. Rev. Lett. **54**, 1605 (1985).
[9] M.P.A. Fisher and A.T. Dorsey, Phys. Rev. Lett. **54**, 1609 (1985).
[10] N.V. Prokof'ev and B.V. Svistunov, submitted to Phys. Rev. B (cond-mat/9907203).