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Kelvin waves (kelvons)—helical waves on quantized vortex lines—are the normal modes of vortices in a superfluid. At zero temperature, the only dissipative channel of vortex dynamics is phonon emission. Starting with the hydrodynamic action, we derive the Hamiltonian of vortex-phonon interaction, thereby reducing the problem of the interaction of Kelvin waves with sound to inelastic elementary excitation scattering. On the basis of this formalism, we calculate the rate of sound radiation by superfluid turbulence at zero temperature and estimate the value of short-wavelength cutoff of the turbulence spectrum.

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Since the early study of phonon scattering by vortex lines [1], much interest has been attracted by the problems of the interaction of Kelvin waves with density-distortion modes [2, 3, 4, 5, 6, 7]. In helium, Kelvin waves are a fundamental ingredient of the evolution of superfluid turbulence [3, 4, 8, 9, 10, 11, 12]. A Kolmogorov-type cascade of Kelvin waves has been argued [3, 10, 12] to be responsible for the decay of superfluid turbulence at zero temperature [11]. As proposed by Vinen [3, 4], the sound radiation by short-wavelength kelvons leads to a dissipative cutoff of the Kolmogorov energy flux. In neutron stars, the excitation of Kelvin waves due to the interaction with the nuclei in the solid crust is suggested to be the main mechanism of pulsar glitches [2]. Not long ago, quantized vortices were created in atomic Bose-Einstein condensate (BEC) [13]. Since then, nonlinear kelvon dynamics have become an attractive topic in the field of ultra-cold gases [5, 6, 7, 14]. Recent experiment in gaseous BEC by Bretin *et. al.* [5] and the subsequent numerical simulation by Mizushima *et. al.* [6] describe the excitation of Kelvin waves by the quadrupole modes. In their theoretical study, Martikainen and Stoof [7] obtained the interaction Hamiltonian of kelvons with the quadrupole modes in a model of a vortex line in a stack of two-dimensional BEC.

We develop a systematic approach to the problem of interaction of phonons with vortices in the hydrodynamic regime, i.e. when any physical length scale is much larger than the vortex core size a_0, which allows one to describe vortices as geometrical lines [8]. We derive the interaction Hamiltonian basing the analysis on the small parameter $\beta = a_0\tilde{k} \ll 1$, where $\tilde{k}$ is the largest wave number among kelvons and phonons. To employ the transparent description in terms of the normal modes, we confine ourselves to the case of weak nonlinearity. For kelvons this implies that the amplitudes b_k of the Kelvin waves of the typical wavelength $\lambda \sim k^{-1}$ are much smaller than λ, which is expressed by the small parameter $\alpha_k = b_k k \ll 1$. For phonons this requires that $\eta \ll n$, where η is the number density fluctuation in a sound wave and n is the average number density. The obtained result allows us to rigorously describe the radiation of sound by kelvons, which we apply to the problem of superfluid turbulence decay at zero temperature. We should mention that, in superfluid turbulence, kelvons are actually superimposed on vortex kinks with curvature radii much larger than kelvon wavelengths [10]. We shall neglect such large-scale curvature and address this issue in the end of the paper.

The condition $a_0 k \sim \beta \ll 1$ implies that the typical vortex line velocities are much smaller than the speed of sound. Along with $\eta \ll n$ it leads to the fact that the vortex-phonon coupling contributes only small corrections to the dynamics of the non-interacting vortex and phonon subsystems. Therefore, a perturbative approach is applicable, provided the interaction energy is written in terms of the canonical variables. Normally, the form of the canonical variables comes from the solution of the interaction-free dynamics. However, when studying vortices separate from phonons, one naturally neglects the compressibility of the fluid [8], since finite compressibility leads only to higher-order "relativistic" corrections. As a result, the dynamics of vortices are described by the Hamiltonian, written in terms of the geometrical configuration of the vortex lines [10]. When the vortices are absent, the phonon modes come from the bilinear Hamiltonian for the density fluctuation $\eta(\mathbf{r}, t)$ and the phase field $\varphi(\mathbf{r}, t)$, which determines the velocity in the density wave (see, e.g., [15]). If finite compressibility of a superfluid is taken into account in order to join the subsystems, the positions of vortices and the fields $\eta(\mathbf{r}, t), \varphi(\mathbf{r}, t)$ are no longer the sets of canonical variables because of the variable-mixing term in the Lagrangian. Introducing the interaction, one has to simultaneously reconsider the canonical variables.

The small parameters allow us to obtain an asymptotic expansion of the canonical variables by means of a systematic iterative procedure. Physically, the procedure restores the retardation in the adjustment of the superfluid velocity field to the evolving vortex configuration. It qualitatively changes the structure of the Hamiltonian with respect to the terms responsible for the radiation of sound and the relativistic corrections to the vortex dynamics.

Long-wave superfluid dynamics at zero temperature are described by the Popov's hydrodynamic action [16]:

$$S = \int dt\, d^3r \left[-(n+\eta)\dot{\Phi} - \frac{(n+\eta)}{2m_0}|\nabla\Phi|^2 - \frac{1}{2\varkappa}\eta^2 \right]. \quad (1)$$

Here the spatial integral is taken over the macroscopic fluid volume, $\Phi(\mathbf{r}, t)$ is the phase field, which determines the velocity according to $\mathbf{v} = (1/m_0)\nabla\Phi$ ($\hbar = 1$), m_0 is the mass of a particle, $\varkappa$ is the compressibility, the dot denotes the derivative with respect to time, and the vortex core radius is given by $a_0 \sim \sqrt{\varkappa/nm_0}$.

The phase Φ is non-single-valued and contains topological defects, the vortex lines. The velocity circulation around each vortex line is quantized:

$$\oint \nabla\Phi(\mathbf{r}) \cdot d\mathbf{r} = 2\pi. \quad (2)$$

The defects can be separated from the regular contribution:

$$\Phi = \Phi_0 + \varphi, \quad (3)$$

where Φ_0 is non-single-valued and satisfies (2), while φ is regular and $\nabla\varphi$ is circulation-free. The standard decomposition into vortices and phonons is done by introducing an additional constraint that to the zeroth approximation eliminates the coupling between Φ_0 and φ in the Hamiltonian, namely

$$\Delta\Phi_0(\mathbf{r}) = 0. \quad (4)$$

(Physically, this parametrization is suggested by the velocity potential of an incompressible fluid.) With Eqs. (3),(4) the Lagrangian becomes

$$L = \int d^3r \left[-n\dot{\Phi}_0 - \eta\dot{\varphi} - \eta\dot{\Phi}_0 \right] - H, \quad (5)$$

where $H = H_{\rm vor} + H_{\rm ph} + H'_{\rm int}$,

$$H_{\rm vor} = \frac{n}{2m_0} \int d^3r \left|\nabla\Phi_0\right|^2, \quad (6)$$

$$H_{\rm ph} = \int d^3r \left[\frac{n}{2m_0}\left|\nabla\varphi\right|^2 + \frac{1}{2\varkappa}\eta^2 \right], \quad (7)$$

$$H'_{\rm int} = \frac{1}{2m_0} \int d^3r \Big[\eta\left|\nabla\Phi_0\right|^2 + 2\eta\nabla\varphi\cdot\nabla\Phi_0 \Big]. \quad (8)$$

The coupling between the vortex variable, Φ_0, and the density waves, $\{\eta, \varphi\}$, is determined by $H'_{\rm int}$ and the time derivative term $\int d^3r\, \eta\dot{\Phi}_0$, both being first-order corrections to the non-interacting parts. Following standard perturbative procedure, we first neglect this coupling to find the non-interacting normal modes. The vortex part of the Lagrangian is then given by $L_{\rm vor} = -n\int d^3r\ \dot{\Phi}_0 - H_{\rm vor}$. For the sake of simplicity, from now on we consider a solitary vortex line; the generalization is straightforward. Let the two-dimensional vector $\boldsymbol{\rho}_0(z_0) = \big(x_0(z_0), y_0(z_0), 0\big)$ describe the position of the vortex line in the plane $z = z_0$ of a Cartesian coordinate system, where the z-direction is chosen along the vortex line. The field Φ_0 is a functional of $\boldsymbol{\rho}_0(z)$, hence

$$\int d^3r\ \dot{\Phi}_0 = \int dz\ \dot{\boldsymbol{\rho}}_0 \cdot \frac{\delta}{\delta\boldsymbol{\rho}_0} \int d^3r\ \Phi_0. \quad (9)$$

To obtain $\int d^3r\ \dot{\Phi}_0$, it is sufficient to calculate the integral $\int d^3r\ \delta\Phi_0$, where $\delta\Phi_0$ is the variation of the phase field due to the distortion of the vortex line by $\delta\boldsymbol{\rho}_0(z)$. Using the identity $\Delta(\boldsymbol{\rho}^2) = 4$, with $\boldsymbol{\rho} = (x, y, 0)$, and Eq. (4), obtain

$$\int d^3r\ \delta\Phi_0 = \frac{1}{4} \int d^3r\ \nabla\cdot\left[\ \delta\Phi_0\nabla(\boldsymbol{\rho}^2)\ \right]. \quad (10)$$

The variation $\delta\Phi_0(\mathbf{r})$ can be viewed as being produced by two vortex lines with opposite circulation quanta and separated by $\delta\boldsymbol{\rho}_0(z)$. In view of Eq. (2) the field $\delta\Phi_0(\mathbf{r})$ experiences a jump of 2π across the surface $\mathcal{S}$ that extends between these vortex lines along the vector field $\delta\boldsymbol{\rho}_0(z)$, therefore the integration volume must have a cut along $\mathcal{S}$. Applying the Gauss theorem, we rewrite Eq. (10) as the surface integral $(1/2)\oint_{\mathcal{S}} \delta\Phi_0\ (\boldsymbol{\rho}\cdot d\mathbf{S})$ yielding

$$\int d^3r\ \dot{\Phi}_0 = \pi \int dz\ [\hat{\mathbf{z}} \times \boldsymbol{\rho}_0(z)] \cdot \dot{\boldsymbol{\rho}}_0(z). \quad (11)$$

Introducing the complex variable $w(z) = \sqrt{nm_0\kappa/2}\ \big[x(z) + iy(z)\big]$, where $\kappa = 2\pi/m_0$ is the velocity circulation quantum, obtain

$$L_{\rm vor} = \frac{1}{2} \int dz\ \big[iw^*\dot{w} - i\dot{w}^*w\big] - H_{\rm vor}[w, w^*], \quad (12)$$

which implies that $w(z)$ and $w^*(z)$ are the canonical variables with respect to $L_{\rm vor}$. The energy (6), rewritten in terms of $w(z)$ and $w^*(z)$ gives the vortex Hamiltonian [10]. In this paper we keep only the bilinear term of the expanded with respect to $\alpha_k \ll 1$ Hamiltonian, the remaining terms determine the interaction of kelvons [12]. The result is [17]

$$H_{\rm vor} \approx \sum_k \varepsilon_k a_k^\dagger a_k\ ,\ \ \varepsilon_k = (\kappa/4\pi)\ln(1/ka_0)k^2, \quad (13)$$

where a_k and $a_k^\dagger$ are the kelvon creation and annihilation operators, obtained, assuming the periodicity along z, by the Fourier transforms $a_k = L^{-1/2}\int w(z)\exp(-ikz)\,dz$, where L is the system size in the z-direction, and $w(z)$ is understood as a quantum field.

The sound waves are described by the Lagrangian $L_{\rm ph} = \int d^3r\,(-\eta\dot{\varphi}) - H_{\rm ph}[\eta, \varphi]$, with $H_{\rm ph}$ given by (7). We assume that the system is contained in a cylinder of radius R with the symmetry axis along the z-direction and that the system is periodic along z with the period L. In the cylindrical geometry, the phonon fields $\eta(r, \theta, z)$, $\varphi(r, \theta, z)$ are parametrized by phonon creation and annihilation operators c_s, $c_s^\dagger$ as

$$\eta = \sum_s \sqrt{\omega_s\,\varkappa\,/\,2}\;\left[\,\chi_s\,c_s + \chi_s^*\,c_s^\dagger\,\right],$$
$$\varphi = -i\sum_s \sqrt{1\,/\,2\,\omega_s\,\varkappa}\;\left[\,\chi_s\,c_s - \chi_s^*\,c_s^\dagger\,\right], \qquad (14)$$
$$\chi_s = \chi_s(r,\theta,z) = \mathcal{R}_{mq_r}(r)\;Y_m(\theta)\;\mathcal{Z}_{q_z}(z),$$

where $\mathcal{R}_{mq_r}(r) = (\pi q_r/R)^{1/2} J_m(q_r r)$, $Y_m(\theta) = (2\pi)^{-1/2}\exp(im\theta)$, $\mathcal{Z}_{q_z}(z) = L^{-1/2}\exp(iq_z z)$, s stands for $\{q_r, m, q_z\}$, and $J_m(x)$ are the Bessel functions of the first kind. The phonon Hamiltonian then reads

$$H_{\rm ph} = \sum_s \omega_s c_s^\dagger c_s, \quad \omega_s = c\,q, \qquad (15)$$

with $q = \sqrt{q_r^2 + q_z^2}$ and $c = \sqrt{n/\varkappa m_0}$.

Now we address the coupling between phonons and vortices. In terms of the obtained variables, the Lagrangian (5) takes on the form

$$L = \sum_k i\,\dot{a}_k a_k^\dagger + \sum_s i\,\dot{c}_s c_s^\dagger - T - H, \qquad (16)$$

where $T = \int d^3r\;\eta\dot{\Phi}_0 \equiv T\{a_k, \dot{a}_k, a_k^\dagger, \dot{a}_k^\dagger,\; c_s, \dot{c}_s, c_s^\dagger, \dot{c}_s^\dagger\}$, and $H = H_{\rm vor} + H_{\rm ph} + H'_{\rm int}$. The coupling term T plays a special role in the Lagrangian (16). This term is linear in time derivatives of the variables and thus can not contribute to the energy in accordance with the Lagrangian formalism. Moreover, because of the time derivatives in T, the equations of motion in terms of $\{a_k, a_k^\dagger\}, \{c_s, c_s^\dagger\}$ take on a non-Hamiltonian form. This implies that the chosen variables become non-canonical in the presence of the interaction, and therefore the total energy, H, *in these variables* can not be identified with the Hamiltonian.

There must exist such a variable transformation $\{a_k, a_k^\dagger\}, \{c_s, c_s^\dagger\} \to \{\tilde{a}_k, \tilde{a}_k^\dagger\}, \{\tilde{c}_s, \tilde{c}_s^\dagger\}$ that restores the canonical form of the Lagrangian, $L = \sum_k i\,\dot{\tilde{a}}_k \tilde{a}_k^\dagger + \sum_s i\,\dot{\tilde{c}}_s \tilde{c}_s^\dagger - H\{\tilde{a}_k, \tilde{a}_k^\dagger, \tilde{c}_s, \tilde{c}_s^\dagger\}$. The canonical variables are obtained by the following iterative procedure. The term T is expanded with respect to $\alpha_k \ll 1$, $\beta \ll 1$ and $\eta \ll n$ yielding $T = T^{(1)} + T^{(2)} + \cdots$. Then the variables are adjusted by $a_k \to a_k + a_k^{(1)}$, $c_s \to c_s + c_s^{(1)}$, where $a_k^{(1)}(\{a_k, a_k^\dagger, c_s, c_s^\dagger\})$ and $c_s^{(1)}(\{a_k, a_k^\dagger, c_s, c_s^\dagger\})$ are chosen to eliminate the term $T^{(1)}$ in (16). As a result, $T \to 0 + T'^{(2)} + \cdots$, where the prime means that the structure of the remaining terms has changed. At the next step, $T'^{(2)}$ is eliminated by $a_k \to a_k + a_k^{(2)}$, $c_s \to c_s + c_s^{(2)}$ and so on. By construction, the canonical variables are given by $\tilde{a}_k = a_k + a_k^{(1)} + a_k^{(2)} + \cdots$, $\tilde{c}_s = c_s + c_s^{(1)} + c_s^{(2)} + \cdots$, and likewise for the conjugates. In practice, only the first few terms are enough, as the rest ones give just higher-order corrections.

The explicit expression for T is obtained following the steps of the derivation of Eq. (11). The only difference here is that the role of the auxiliary function $\boldsymbol{\rho}^2$ is played by Q, defined by $\Delta Q(\mathbf{r}) \equiv \eta(\mathbf{r})$:

$$T = 2\pi \int dz\;[\hat{\mathbf{z}} \times \nabla Q(\boldsymbol{\rho}_0(z), z)] \cdot \dot{\boldsymbol{\rho}}_0(z). \qquad (17)$$

In accordance with $q_r\rho_0 \sim \beta k\rho_0 \ll 1$, we expand the radial functions, retaining only the greatest term for each particular angular momentum m. Switching to the cylindrical coordinates, $\boldsymbol{\rho}_0 = (\rho_0\cos\gamma,\;\rho_0\sin\gamma,\;0)$, and noticing that

$$\left[\dot{\gamma}\,\rho_0\,\frac{\partial}{\partial r} - \frac{\dot{\rho}_0}{\rho_0}\,\frac{\partial}{\partial\theta}\right]\mathcal{R}_{mq_r}(r) Y_m(\theta)\Big|_{r=\rho_0,\theta=\gamma} \propto \left\{\begin{array}{ll} d(w^m)/dt, & m \ge 0 \\ d(w^{*|m|})/dt, & m < 0 \end{array}\right., \qquad (18)$$

obtain

$$T \approx \sum_{s,k_1\ldots k_m}\Big[-iA_{s,k_1\ldots k_m}\,c_s\,\frac{d}{dt}(a_{k_1}\ldots a_{k_m}) - iB_{s,k_1\ldots k_m}\,c_s\,\frac{d}{dt}(a_{k_1}^\dagger\ldots a_{k_m}^\dagger)\Big] + \text{H.c.}, \qquad (19)$$

where the sum is over all s with $m \neq 0$ and

$$A_{s,k_1\ldots k_m} = -\Theta(m)\,A_s\,\delta_{k_1+\ldots+k_m,-q_z},$$
$$B_{s,k_1\ldots k_m} = (-1)^{|m|}\Theta(-m-1)\,A_s\,\delta_{k_1+\ldots+k_m,q_z},$$
$$A_s = \frac{\sqrt{a_0 q}}{2^{|m|/2+1}\;|m|!}\,\frac{n^{\frac{1-|m|}{2}}(m_0\kappa)^{\frac{2-|m|}{2}}\;q_r^{|m|+\frac{1}{2}}\;q^{-2}}{L^{(|m|-1)/2}\;R^{1/2}}, \qquad (20)$$

where $\Theta(m) = \left\{\begin{array}{ll} 1, & m \ge 0 \\ 0, & m < 0 \end{array}\right.$. Thus,

$$a_k = \tilde{a}_k \qquad (21)$$

(we omitted the terms that do not contain phonon operators and thus result only in relativistic corrections to the kelvon spectrum and kelvon-kelvon interactions),

$$c_s = \tilde{c}_s + (1-\delta_{m,0})\sum_{k_1\ldots k_m}\Big[A_{s,k_1\ldots k_m}\tilde{a}_{k_1}^\dagger\ldots\tilde{a}_{k_m}^\dagger + B_{s,k_1\ldots k_m}\tilde{a}_{k_1}\ldots\tilde{a}_{k_m}\Big], \quad s = \{q_r, m, q_z\}. \qquad (22)$$

The Hamiltonian H is then given by the energy (6)-(8) in terms of the variables $\{\tilde{a}_k, \tilde{a}_k^\dagger\}, \{\tilde{c}_s, \tilde{c}_s^\dagger\}$. Up to neglected relativistic corrections, the variable transformation does not change the spectrum of the elementary modes: the zero-order Hamiltonians are given by (13), (15) in terms of $\{\tilde{a}_k, \tilde{a}_k^\dagger\}, \{\tilde{c}_s, \tilde{c}_s^\dagger\}$. The transform (22) applied to (15) generates the interaction term

$$H_{\rm int}^{\rm (rad)} = \sum_{s,\{k_i\}}(1-\delta_{m,0})\Big[\,\omega_s A_{s,k_1\ldots k_m}\;\tilde{a}_{k_1}^\dagger\ldots\tilde{a}_{k_m}^\dagger\tilde{c}_s^\dagger + \omega_s B_{s,k_1\ldots k_m}\;\tilde{a}_{k_1}\ldots\tilde{a}_{k_m}\tilde{c}_s^\dagger\,\Big] + \text{H.c.} \qquad (23)$$

Remarkably, the energy term $\propto \int d^3r\, \eta \left|\nabla\Phi_0\right|^2$ in (8), which results in the same operator structure as Eq. (23), is irrelevant, being smaller in $\beta \ll 1$. It can be checked straightforwardly, that the term $\propto \int d^3r\ \eta\nabla\varphi \cdot \nabla\Phi_0$ in Eq. (8) gives Fetter's amplitudes of the elastic and inelastic scattering of phonons [1] (see, however, the remark at the end of the paper). In addition, this term leads to a macroscopically small splitting of the phonon spectrum due to the superimposed fluid circulation.

A kelvon is known to carry a quantum of (negative) angular momentum projection [2]. The interaction (23) explicitly conserves the angular momentum: a real process of the emission of a phonon with the angular momentum $(-m)$ requires an annihilation of m kelvons. Since $\varepsilon_k \sim (a_0k)\omega_k$, the total momentum transferred to phonons in a radiation event should be small in order to satisfy the energy conservation. Thus, the radiation by one kelvon is kinematically suppressed. The leading radiation process is the emission of the $m = -2$ (quadrupole) phonon mode, the events involving more than two kelvons being suppressed by $\alpha_k \ll 1$. First-order processes of the two-phonon emission come from the term $\propto \int d^3r\ \eta\nabla\varphi \cdot \nabla\Phi_0$ of (8). The amplitude of these processes is suppressed by the relativistic parameter $\beta \ll 1$.

The Hamiltonian (23) can be used to obtain the rate of sound radiation by superfluid turbulence at zero temperature. At large wave numbers, where the radiation is appreciable, Kelvin-wave turbulence is characterized by $\alpha_k \ll 1$ and features the spectrum $n_k = \langle a_k^\dagger a_k \rangle \propto k^{-17/5}$ [12]. The occupation number decay rate is $\dot{n}_k = -\sum_{s,k_1} W_{s,k,k_1}$, where W_{s,k,k_1} is the probability of the event $|0_s,\, n_k,\, n_{k_1}\rangle \to |1_s,\, n_k{-}1,\, n_{k_1}{-}1\,\rangle$ per unit time. Applying the Fermi Golden Rule to W_{s,k,k_1} with the interaction (23) and replacing the sums by integrals, obtain

$$\dot{n}_k = -\frac{(m_0\kappa/2\pi)^5}{15\pi n m_0} \ln^5(1/a_0k)\,(a_0k)^5\, k^5\, n_k^2. \qquad (24)$$

Following Ref. [12], this formula allows us to obtain the momentum scale $k_{\rm ph}$, at which the kelvon cascade is cut off by the radiation of sound:

$$k_{\rm ph} \sim \frac{[a_0/R_0]^{6/31}}{\left[\ln(R_0/a_0)\right]^{24/31}} a_0^{-1}, \qquad (25)$$

where R_0 is the typical distance between the vortex lines in the tangle.

Finally, we comment on Vinen's estimate of the power radiated per unit length of the vortex line, Eq. (2.24) of Ref. [4]. The power at the momentum k can be determined by $\Pi_k = -\sum_{k'\sim k} \varepsilon_{k'}\dot{n}_{k'}/L$, with $\dot{n}_{k'}$ given by (24). In Ref. [4], the retarded potential method is employed giving the estimate $\Pi_k' \propto b_k^2 \propto n_k$, while $\Pi_k \propto n_k^2$. In the quasi-particle language, Π_k' implies that the radiation is governed by the conversion of *one* kelvon into a phonon. Vinen argues that this process becomes allowed in superfluid turbulence, where kelvons are actually superimposed on vortex kinks of typical size $\sim R_0 \gg k^{-1}$, or, equivalently, the kelvon coupling to a kink lifts the ban on single-kelvon radiative processes by effectively removing the momentum conservation constraint. We note that the probabilities of such elementary events are likely to be suppressed *exponentially*, as it is generically the case, say, for soliton-phonon interactions (see, e.g., [18] and references therein.) The processes involving kelvon-kink coupling should contain an exponentially small factor $\sim \exp(-R_0k)$, which arises from the convolution of the smooth kink profile with the oscillating kelvon mode.

In the problem of phonon scattering from a vortex, the answer depends on whether the vortex is pinned or free [19]. In fact, the elastic scattering matrix element of Ref. [1] corresponds to the pinned case only. In the Hamiltonian formalism developed here, the difference is due to an *additional* matrix element generated by the $|m| = 1$ term of the interaction Hamiltonian (23) in the second order of perturbation theory. We are indebted to Edouard Sonin for raising this question. Furthermore, a second-order amplitude generated by a combination of the $|m| = 2$ term with the $|m| = 1$ one is of the same order as the direct first-order amplitude of inelastic phonon scattering. Thus, the result of Ref. [1] for inelastic scattering needs to be corrected accordingly.

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