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# A Very Effective String Model?

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## Abstract

Additional evidence is presented for a recently proposed effective string model, conjectured to hold throughout the parameter space of the basic 5 dimensional, triply charged black holes, which includes the effects of brane excitations, as well as momentum modes. We compute the low energy spacetime absorption coefficient  $\sigma$  for the scattering of a triply-charged scalar field in the near extremal case, and conjecture an exact form for  $\sigma$ . It is shown that this form of  $\sigma$  arises simply from the effective string model. This agreement encompasses both statistical factors coming from the Bose distributions of string excitations and a prefactor which depends on the effective string radius. An interesting feature of the effective string model is that the change in mass of the effective string system in an emission process is not equal to the change in the energies of the effective string excitations. If the model is valid, this may hold clues towards understanding back reaction due to Hawking radiation. A number of weak spots and open questions regarding the model are also noted.

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## 1. Introduction

In this paper, we explore further the effective string model introduced in [1] for the quantum microstates of the basic family of 5-dimensional, triply charged black holes [2,3]. In one corner of the black hole parameter space, near extremality and with the momentum charge much smaller than the other two charges, the quantum mechanical properties of the system, entropy and Hawking emission, are well approximated by a model based on momentum excitations of D-branes [2,4,5]. We will refer to this as the momentum dominated limit. It was proposed in [1], however, that a weakly coupled effective string model continues to hold throughout the black hole parameter space, for arbitrary combinations of charges and arbitrarily far from extremality.

The main evidence presented in [1] for this model comes from comparing the thermodynamic properties of the effective string with factors appearing in the frequency (and charge) dependent black hole greybody factors. The low frequency limit of the black hole greybody factors, or equivalently the absorption coefficient,  $\sigma_{abs}$ , was calculated in the momentum dominated limit in [5] and shown to be in striking agreement with the predictions of the D-brane model<sup>1</sup>. These calculations were extended to give a U-duality invariant result for  $\sigma_{abs}$  in [10,6], though still in the near-extreme limit with at most one charge small. It is agreement between these latter results and an appropriate limit of the effective string model in [1] that was cited as evidence for the model. The results of [5] were also extended to the case of two small charges in [11], with a further obvious U-duality invariant extension, noted in [6], giving improved agreement with the effective string model of [1]. We will argue below that this series of increasingly precise results for the low frequency limit of  $\sigma_{abs}$  points towards an ultimate U-duality invariant expression, which would hold throughout the black hole parameter space. This is then an appropriate setting to test the effective string model of [1] and we find exact agreement between the expressions.

A number of further results are presented here. We give a calculation of  $\sigma_{abs}$  for scalar fields carrying arbitrary combinations of the three charges. The presumed extension of this result over the black hole parameter space again agrees with the effective string model of [1]. We observe that the spacetime absorption coefficient  $\sigma_{abs}$  has the structure of an absorption rate which would result from the interaction of two Bose-Einstein distributions of massless, triply charged particles which we label ‘ $R$ ’ and ‘ $L$ ’ which live in a 1+1 dimensional space of radius  $R_{eff}$ . Each distribution is characterised by temperatures  $T_R$  and  $T_L$  and charge

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<sup>1</sup> See [6,7,8,9] for some qualifications to this.

potentials  $\Phi_i$ ,  $i = 1, 2, 3$ . Species  $R$  and  $L$  interact to absorb or emit a third massless boson. The thermodynamic parameters inferred from  $\sigma_{abs} - T_L, T_R$ , and  $\Phi_i$  - agree exactly with the parameters of the effective string thermodynamic model throughout the parameter space. Further, from the thermodynamics of the effective string model, one can extract the effective radii  $R_{L,R}$  for the one dimensional spaces on which the left and right moving excitations move. We show that these three lengths are all the same,  $R_R = R_L = R_{eff}$ .

If the effective string model of [1] is correct, we may be able to learn something interesting about the back reaction due to Hawking radiation. Recall that in the momentum dominated limit, the total mass of the black hole is accounted for by the sum of the masses of the D-branes, plus the energies of the left and right moving excitations  $E_L + E_R$ . In the general case discussed here, one also has  $M = E_L + E_R + m_{bkgr}$ , where  $E_{L,R} = N_{L,R}/R_{eff}$  are the energies of the interacting L and R modes, and  $m_{bkgr}$  is an additional ‘‘background mass’’. In an emission or absorption process, we find that all of these energies change, and that

$$|\Delta M| > |\Delta(E_L + E_R)|, \quad (1)$$

over the range of parameter space which we have checked. In the D-brane picture this would correspond to, e.g., a change in the number of 5-branes and 1-branes, and hence in the effective radius. It would be of interest to understand what this corresponds to in the black hole picture.

## 2. Black Holes and Greybody Factors

Hawking showed that the energy spectrum emitted by a black hole is given by

$$\frac{dE}{dt}(\omega, q_i) = \frac{\omega \sigma_{abs}(\omega, q_i)}{2\pi(e^{(\omega - q_i \Phi_i)/T_H} - 1)}, \quad (2)$$

where  $\omega$  and  $q_i$  are the emitted energy and charge,  $T_H$  and  $\Phi_i$  are the Hawking temperature and chemical potential and  $\sigma_{abs}$  is the spacetime absorption coefficient for the mode, as required by detailed balance. Since  $\sigma_{abs}$  modifies the black body form of the emitted spectrum, it is known as a ‘‘greybody’’ factor.

The 5-dimensional metric and gauge fields are

$$ds^2 = -f^{-2/3} h dt^2 + f^{1/3} (h^{-1} dr^2 + r^2 d\Omega^2), \quad A_{ti} = \frac{\mu \sinh \delta_i \cosh \delta_i}{f_i r^2}, \quad (3)$$

$$f = \prod_{i=1}^3 f_i, \quad f_i = 1 + \frac{\mu \sinh^2 \delta_i}{r^2}, \quad h = 1 - \frac{\mu}{r^2}.$$

In most of what follows, we will follow [1] and set the string coupling and the sizes of the internal dimensions<sup>2</sup> to one,  $g = R = V = 1$ . The mass  $M$ , charges  $Q_i$  ( $i=1,2,3$ ) and entropy  $S$  of the black holes can then be expressed in terms of three boost parameters  $\delta_i$  and a nonextremality parameter  $\mu$  as<sup>3</sup>

$$M = \frac{1}{2}\mu \sum_i \cosh 2\delta_i, \quad Q_i = \frac{1}{2}\mu \sinh 2\delta_i, \quad S = 2\pi\mu^{\frac{3}{2}} \prod_i \cosh \delta_i. \quad (4)$$

The inverse Hawking temperature and chemical potentials are given by

$$\beta_H = 2\pi\mu^{\frac{1}{2}} \prod_i \cosh \delta_i, \quad \Phi_i = A_{ti}(\mu) = \tanh \delta_i \quad (5)$$

First, consider scattering by an uncharged scalar field satisfying  $\nabla^2\Phi = 0$  and restrict to the S-wave sector  $\Phi = e^{-i\omega t}\phi(r)$ . Then in terms of the radial coordinate  $v = \frac{r_0^2}{r^2}$ ,

$$(1-v)\frac{d}{dv}((1-v)\phi'(v)) + \left[ C_0 + \frac{C_1}{v} + \frac{C_2}{v^2} + \frac{C_3}{v^3} \right] \phi = 0, \quad (6)$$

where the coefficients  $C_k$  are given by

$$\begin{aligned} C_3 &= \frac{\omega^2\mu}{4}, & C_2 &= \frac{\omega^2\mu}{4} \left( \sum_i \sinh^2 \delta_i \right), \\ C_1 &= \frac{\omega^2\mu}{4} \left( \sum_{i<j} \sinh^2 \delta_i \sinh^2 \delta_j \right), & C_0 &= \frac{\omega^2\mu}{4} \prod_i \sinh^2 \delta_i. \end{aligned} \quad (7)$$

A scalar carrying momentum charge also satisfies an equation of this form [5,12,10,6]. We show below that this form of the equation, with different coefficients  $C_k$ , holds in the case of a scalar field carrying general values of the three charges as well.

As discussed in the introduction, a sequence of continually improving results for the absorption coefficient  $\sigma_{abs}$  have appeared in the literature [4,5,10,6,11]. The first of these references [4] gives the leading term in the power law expansion of  $\sigma_{abs}$  in frequency  $\omega$ , which is proportional to the area of the black hole horizon (see Appendix and [13]). The

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<sup>2</sup> The 10-dimensional and 5-dimensional gravitational couplings are given by  $\kappa_{10}^2 = 8\pi G_{10} = 64\pi^7 g^2$  and  $\kappa_5^2 = 8\pi G_5 = \frac{2\pi^2 g^2}{RV}$ .

<sup>3</sup> This notation for the charges simplifies the formulas. We will sometimes, however, refer to the charges  $Q_i$ ,  $i = 1, 2, 3$  as the 1-brane, 5-brane and momentum charges, respectively. We will also make use of the notation  $\mu = r_0^2$ ,  $r_i = r_0 \sinh \delta_i$ .

later calculations, starting with [5], all give a form for the absorption coefficient (still in a low frequency approximation)<sup>4</sup>.

$$\sigma_{abs} = \pi^2 \omega^2 \mu a_L a_R \frac{e^{2\pi(a_L+a_R)} - 1}{(e^{2\pi a_L} - 1)(e^{2\pi a_R} - 1)}, \quad (8)$$

with the constants  $a_{L,R}$  determined in terms of the coefficients  $C_k$  in the wave equation (6).

Maldacena and Strominger [5] worked directly in the momentum dominated, near extreme limit

$$r_0, r_3 \ll r_1, r_2, \quad r_0^3 \ll r_1 r_2 r_3. \quad (9)$$

and by matching solutions of the hypergeometric equation near the horizon to solutions of Bessel's equation near infinity found

$$a_{L,R} = \sqrt{C_0 + C_1} \mp \sqrt{C_0}, \quad (10)$$

with  $C_0, C_1$  approximated in the limit (9). In [10,6] it was noted that the restriction to the limit  $r_3 \ll r_1, r_2$  was not necessary, and that, so long as *at most* one of the charges was small, (10) holds with the *exact values* of  $C_0, C_1$ . Since each coefficient  $C_k$  is symmetric in the three charges, this gives a U-duality invariant extension of the results of [5]. Klebanov and Mathur [11] found an improved mapping of the near horizon regime to the hypergeometric equation giving

$$a_{L,R} = \sqrt{C_0 + C_1 + C_2} \mp \sqrt{C_0}, \quad (11)$$

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<sup>4</sup> We note that the expressions of this form which have appeared in the literature, are not strictly consistent in keeping powers of the small parameter  $\epsilon$  ( $\epsilon = \omega \max\{r_i\}$  for neutral emission). One problem, noted in [11], is that to get the form of the ratio of Bose factors in (8), gamma functions have to be expanded as *e.g.*  $\Gamma(1 - C_2) \approx \Gamma(1)$ , so that terms of order  $\epsilon^2$  are dropped. However, the exponentials in (8) contain all powers of  $\epsilon$ , and hence have been selectively kept. Also, in the matching procedures used in [11], the second Bessel function has been simply dropped. However, this can and should be incorporated, and leads to additional order  $\epsilon^2$  corrections in the prefactor of (8). Again, dropping these terms while keeping all the terms in the exponential function is inconsistent. The justification here for studying the form (8) for  $\sigma_{abs}$  is that the Bose factors have central physical significance. This constitutes an educated guess about which higher order terms to keep.

with the constants  $C_k$  evaluated in the limit of two small charges

$$r_0, r_2, r_3 \ll r_1, \quad r_0^3 \ll r_1 r_2 r_3. \quad (12)$$

An obvious U-duality invariant extension of their result holds as well, with the exact coefficients  $C_0, C_1, C_2$  [6].

These evolving results for the coefficients  $a_{L,R}$  in the papers [5,10,6,11] point towards a possible ultimate form

$$a_{L,R} = \sqrt{C_0 + C_1 + C_2 + C_3} \mp \sqrt{C_0}. \quad (13)$$

In terms of the boost parameters this combination simplifies greatly to give

$$a_{L,R} = \frac{\omega \mu^{\frac{1}{2}}}{2} \left( \prod_i \cosh \delta_i \mp \prod_i \sinh \delta_i \right). \quad (14)$$

This form, we will see, fits in well with the effective string model of [1]. One virtue of the expression (13) is that it has the correct limit as  $\omega$  goes to zero. The leading order term in a power law expansion of (8) is

$$\sigma_{abs} \simeq \frac{1}{2} \pi \mu (a_L + a_R) \omega^2, \quad (15)$$

which is proportional to the exact black hole area for  $a_{L,R}$  as in (13). On the other hand, the low frequency limit of (11), for example, misses  $A_H$  by a term of order  $\frac{r_0^6}{r_1^2 r_2^2 r_3^2}$ . It seems likely that the correct result is of the form (8) with coefficients (13) times a function  $f(\omega)$ , with  $f(0) = 1$ . For the purposes of discussion, we will assume that the low energy absorption coefficient has the form (8) with coefficients (13) in some meaningful approximation.

### 3. General Charges

We have also calculated the absorption coefficient for a scalar field which carries an arbitrary combination of the three charges. Such a generally charged scalar does not arise via dimensional reduction of the 10-dimensional supergravity lagrangian, since it does not correspond to a perturbative degree of freedom. Only scalars carrying KK charge will arise in this way. Rather it is an effective scalar field describing the propagation of 5-D particles which come from combinations of strings and 5-branes wrapped around the internal dimensions and boosted along the common string. The leading coupling of the

scalar to the gauge fields is through the standard gauge covariant derivative. The coupling to the 5-D moduli scalars, which act as a mass term, can be inferred via U-duality from the KK charged case where the equation of motion is known from dimensional reduction.

We find that the 5-D, U-duality invariant wave equation for a scalar carrying charges  $q_i$  is given by

$$D_\mu D^\mu \phi - f^{2/3} \left( \sum_i \frac{q_i^2}{f_i^2} \right) \phi = 0, \quad D_\mu \phi = \left( \nabla_\mu - i \sum_j q_j A_\mu^j \right) \phi. \quad (16)$$

The mass term displays the weighted sum of squares of the charges appropriate for a BPS scalar field. Taking the scalar  $\phi$  to be a function of  $r, t$  only leads to an equation of the form (6) with a rather complicated set of coefficients  $C_k$ . However, combinations which give the constants  $a_{L,R}$  according to (13) simplify considerably to give

$$a_{L,R} = \frac{1}{2} \mu^{\frac{1}{2}} \left\{ \left( \omega - \sum_i q_i \tanh \delta_i \right) \prod_i \cosh \delta_i \mp \left( \omega - \sum_i q_i \coth \delta_i \right) \prod_i \sinh \delta_i \right\} \quad (17)$$

and hence, using (5), we have

$$2\pi(a_L + a_R) = \beta_H \left( \omega - \sum_i q_i \Phi_i \right). \quad (18)$$

We will see below that this gives an absorption rate in agreement with the effective string model.

#### 4. Modeling the Absorption Coefficient

Before turning to the model of [1], we want to see what general features of the presumed black hole microstates are suggested by the low energy form (8) of the absorption coefficient  $\sigma_{abs}$ . In particular, we find that the form of  $\sigma_{abs}$  is consistent with a picture of left and right moving Bose gases confined to a compact one-dimensional space, which can interact to emit or absorb excitations, which we will call loops, propagating in the bulk of spacetime. In addition, we can infer the effective radius  $R_{eff}$  of the one-dimensional space. Consider the expression (8) to have the form  $\sigma_{abs} = P \bar{\sigma}_{abs}$ , where

$$P = \pi^2 \omega^2 \mu a_L a_R, \quad \bar{\sigma}_{abs} = \frac{(e^{2\pi(a_L+a_R)} - 1)}{(e^{2\pi a_L} - 1)(e^{2\pi a_R} - 1)}. \quad (19)$$

Then the combination of exponential factors  $\bar{\sigma}_{abs}$  has a statistical interpretation in terms of a pair of Bose distributions and the prefactor  $P$  contains information about the energy

dependence of the interaction vertex and the dimension and size of the space on which the excitations move.

Consider uncharged emission. Let  $\rho_{L,R}(\omega_{L,R}) = 1/(e^{\beta_{L,R}\omega_{L,R}} - 1)$  be the distributions for two Bose gases at inverse temperatures  $\beta_{L,R}$ . We suppose that a left-mover and a right-mover, with  $\omega_L = \omega_R = \omega/2$ , can annihilate to form a loop of energy  $\omega$  and that the reverse process in which a loop is converted to a left and right moving pair is also possible. The net absorption coefficient is the difference between the microphysical probabilities for the system to absorb or emit a loop. These two processes have different Bose enhancement factors which combine to give the form of  $\bar{\sigma}_{abs}$ . Fix the initial state to have  $l$  incident loops, then to leading order in the coupling between the two gases, the relevant statistical factors are

$$l [\rho_L(\omega/2) + 1] [\rho_R(\omega/2) + 1] - (l + 1)\rho_L(\omega/2)\rho_R(\omega/2) \quad (20)$$

In the case where the number of loops is large  $l \gg 1$ , as is necessary for the classical limit,  $l \simeq l + 1$ . The number of loops  $l$  then factors out and will ultimately be divided out when normalizing by the incident flux, yielding the factor

$$\frac{e^{\beta_L\omega/2 + \beta_R\omega/2} - 1}{(e^{\beta_L\omega/2} - 1)(e^{\beta_R\omega/2} - 1)}, \quad (21)$$

which matches<sup>5</sup> the form of  $\bar{\sigma}_{abs}$ , if we identify

$$2\pi a_{L,R} = \beta_{L,R}\omega/2. \quad (22)$$

In order to understand the physical information in the prefactor  $P$ , we need to work with a more detailed model. Assume that the two interacting species, which we've labeled 'L' and 'R', live on a d-dimensional subspace of the compact dimensions with volume  $V_{in}$ . Assume also that the degrees of freedom have an associated d+1-momentum, which is conserved in the interactions, *i.e.*  $p_L^a + p_R^a = p_{loop}^a$  in an interaction, where  $a$  is a direction tangent to the space on which the excitations move. Following the conventions of [4], write the interaction vertex in the form

$$\kappa_{10}^2 \sqrt{2} (2\pi)^2 (-iA), \quad (23)$$

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<sup>5</sup> This was noted in [11] without including the loops. Putting the loops in a coherent state with high occupation number gives the same result.

where the amplitude  $A$  is left general<sup>6</sup>. If left and right movers with  $\omega_L = \omega_R = \omega/2$  and  $\vec{p}_L = -\vec{p}_R$  combine to give a neutral loop of energy  $\omega$ , then Fermi's Golden Rule gives the rate for emission of a loop from the system to be

$$\Gamma_{em}d\omega = \frac{1}{2\pi}\kappa_5^2 \frac{V_{in}}{(2\pi)^{2d}} \frac{|A|^2}{\omega_L\omega_R\omega} \rho_L(\omega_L)\rho_R(\omega_R)(\rho_l(\omega) + 1)d^4k. \quad (24)$$

The net absorption coefficient is again the difference between the microphysical absorption minus emission rates, normalized by the number of incident loops. For  $l \gg 1$ , one then finds

$$\begin{aligned} \sigma_{abs} &= \Gamma_{abs} - \Gamma_{em} \\ &= \frac{\kappa_5^2}{(2\pi)^{2d-1}} \frac{V_{in}|A|^2}{2} \frac{e^{\beta_L\omega/2+\beta_R\omega/2} - 1}{(e^{\beta_L\omega/2} - 1)(e^{\beta_R\omega/2} - 1)} \end{aligned} \quad (25)$$

Now compare the expression for the prefactor  $P$  in (25) to that for the spacetime absorption coefficient in (19). Plugging the explicit expressions (14) for  $a_{L,R}$  into the prefactor  $P$  in (19) gives

$$\begin{aligned} P &= \frac{\pi^2}{4}\omega^4\mu^2 \left( \prod_i \cosh^2 \delta_i - \prod_i \sinh^2 \delta_i \right) \\ &\equiv \frac{\kappa_5^2}{2\pi} (2\pi R_{eff}) \frac{\omega^4}{8} \end{aligned} \quad (26)$$

The last equality above, which has been written in a way to facilitate comparison with (25), serves as a definition of  $R_{eff}$ . Recall that the 5-dimensional gravitational coupling  $\kappa_5^2$  has the dimension of length cubed, so that the units work out to give a one dimensional volume (*i.e.*  $d = 1$ )  $V_{in}$  proportional to  $2\pi R_{eff}$ . Similarly, we see that the amplitude  $A$  has energy dependence proportional to  $\omega^2$ . There is an overall undetermined constant in the determinations of  $A$  and  $V_{in}$ , which may be fixed by looking in the momentum dominated limit. If we take

$$V_{in} = 2\pi R_{eff}, \quad |A|^2 = \frac{\omega^4}{4}. \quad (27)$$

then the effective radius is given by

$$R_{eff} = \mu^2 \left( \prod_i \cosh^2 \delta_i - \prod_i \sinh^2 \delta_i \right), \quad (28)$$

and in the momentum dominated limit  $R_{eff}$  reduces to

$$R_{eff} \simeq RN_1N_5, \quad (29)$$

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<sup>6</sup> In the momentum dominated limit, the amplitude is given by  $|A|^2 = (p_L \cdot p_R)^2 = 4\omega_L^2\omega_R^2$ .

where, in the notation of [3],  $N_{1,5}$  are the number of one-branes and five-branes. This is the ‘fat black hole’ result of [14,15]. We will see below that the same full value of the effective radius  $R_{eff}$  in (28) emerges in the effective string model of [1] as the ratio of the entropy to the temperature, in the usual thermodynamic relation for a one dimensional gas.

## 5. Thermodynamics of the Effective String Model

The effective string proposal made in [1] is initially based on the observation that the entropy  $S$  of the black hole system given in (4) can be exactly broken up into the sum of two terms<sup>7</sup>, which may be interpreted as the contributions of right and left handed massless excitations moving in 1-dimension,

$$S = 2\pi \left( \sqrt{N_R} + \sqrt{N_L} \right), \quad N_{L,R} = \frac{\mu^3}{4} \left( \prod_i \cosh \delta_i \pm \prod_i \sinh \delta_i \right)^2. \quad (30)$$

$N_{L,R}$  are interpreted as the excitation levels of the right and left handed sectors of a  $c = 6$  conformal field theory<sup>8</sup>. For reference, in the limit where momentum excitations dominate,  $N_{L,R} = N_1 N_5 N_{L,R}^{mom}$ . It could be that this split is an arbitrary one and has no physical relevance. However, evidence that there is something interesting going on comes from a comparison made in [1] between the thermodynamic properties of the effective left and right movers inferred from  $S_{L,R}$  and the form of the spacetime absorption coefficient. We will see that this agreement, made at an approximate level in [1] based on the results for the spacetime absorption coefficient presented in [5,10,6], becomes exact for the form of the spacetime absorption coefficient assumed in section (2).

The thermodynamic properties of effective left and right movers derived in [1] are as follows. The left and right contributions to the entropy are given by

$$S_{L,R} = \pi \mu^{\frac{3}{2}} \left( \prod_i \cosh \delta_i \pm \prod_i \sinh \delta_i \right) = 2\pi \sqrt{N_{L,R}}. \quad (31)$$

Inverse temperatures  $\beta_{L,R}$  are computed for the left and right movers as

$$\beta_{L,R} = \left( \frac{\partial S_{L,R}}{\partial M} \right)_{Q_i} = 2\pi \mu^{\frac{1}{2}} \left( \prod_i \cosh \delta_i \mp \prod_i \sinh \delta_i \right), \quad (32)$$

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<sup>7</sup> This split is particularly compelling in the rotating case, which we do not consider here.

<sup>8</sup> This comes from  $S = 2\pi \sqrt{(N_B + N_F/2)N_{L,R}/6} = 2\pi \sqrt{cN_{L,R}/6}$ , where  $N_B$  and  $N_F$  are the number of 1-dimensional bosons and fermions and  $c = N_B + N_F/2$ .

and chemical potentials are given by

$$(\beta\Phi_i)_{L,R} = - \left( \frac{\partial S_{L,R}}{\partial Q_i} \right)_{Q_{j \neq i}, M} = 2\pi\mu^{\frac{1}{2}} \left( \tanh \delta_i \prod_{j=1}^3 \cosh \delta_j \mp \coth \delta_i \prod_{j=1}^3 \sinh \delta_j \right). \quad (33)$$

The comparison with the form of the absorption coefficient given in section (2) is now straightforward. In the neutral case we have using (14)

$$2\pi a_{L,R} = \beta_{L,R} \omega / 2, \quad (34)$$

and hence the exponential terms in the absorption cross section for the effective string calculated as in section (4) agree with the exponential terms in the spacetime absorption coefficient. The generally charged case works similarly. We have from (17),

$$2\pi a_{L,R} = \frac{1}{2} [\beta (\omega - q_i \Phi_i)]_{L,R}, \quad (35)$$

and again the the exponential terms in the absorption coefficients agree.

In order for the prefactors to agree as well, the radius of the effective string must match with the effective radius deduced from the spacetime absorption coefficient in section (4). Here, we determine  $R_{eff}$  by the observation that the entropy and energy are both extensive quantities, and so proportional to the length of the string. For a one dimensional ideal Bose gas,

$$\begin{aligned} E_{L,R} &= \frac{c}{6} \pi^2 R_{L,R} T_{L,R}^2 \\ S_{L,R} &= \frac{c}{6} \pi^2 R_{L,R} T_{L,R} \end{aligned} \quad (36)$$

Given the expressions for  $S_{L,R}$  and  $\beta_{L,R}$  in equations (31) and (32) one finds the same effective length for both the  $R$  and  $L$  gases,  $R_L = R_R$  as would be needed for the consistency of the model, and the value matches that determined from the prefactor  $P$  in (28),

$$R_{eff} = \mu^2 \left( \prod_i \cosh^2 \delta_i - \prod_i \sinh^2 \delta_i \right). \quad (37)$$

We also note that, in the momentum dominated limit

$$\delta_3 \ll \delta_{1,2} \quad \delta_1, \delta_2 \gg 1 \quad (38)$$

we have  $\cosh \delta_i \approx \sinh \delta_i$  for  $i = 1, 2$ , leading to  $R_{eff} = N_1 N_5 R$  as previously in (29).

Another way to express the Bose statistical factors for the effective string excitations is in terms of the changes  $\Delta N_{L,R}$  in an absorption or emission process. It follows from the thermodynamics of the system, that we must have

$$2\pi a_{L,R} = \beta_{L,R} \frac{\Delta N_{L,R}}{R_{eff}}, \quad (39)$$

under arbitrary variations of  $\mu$  and  $\delta_i$ . This can be verified by direct calculation. The sum of these two equations gives a relation which will be of later use. Using (18) we have

$$\beta_H(\omega - \sum q_i \Phi_i) = \frac{1}{R_{eff}}(\beta_R \Delta N_R + \beta_L \Delta N_L) \quad (40)$$

From the first law, the left hand side is the change in the entropy of the black hole when it emits or absorbs energy  $\omega$  and charges  $q_i$ . The right hand side is the change in the entropy of the brane system,  $\Delta S_R + \Delta S_L$ .

## 6. Some Weak Spots or Clues?

In the last section, we found impressive agreement between the thermodynamic properties of the the effective string model and features of the spacetime absorption coefficients. In this final section, we would like to point out some other features of the proposed correspondence which do not show such obvious agreement and hence stand as open questions. We begin by discussing the relationship between the combined energies of the right and left moving excitations compared to the black hole ADM mass.

The relations (32), (36) and (37) are standard thermodynamic relations for a pair of 1-dimensional ideal gases. For some of the thermodynamic quantities there is a direct relation to analogous properties of the corresponding black hole. For example, by construction, the entropy of the black hole is  $S_{bh} = S_R + S_L$ , and also the inverse Hawking temperature is  $\beta_H = (\beta_L + \beta_R)/2$ . We can ask what are the meanings at the spacetime level of the total energies  $E_{L,R}$  carried by the left and right movers? Combining the formulas above, one finds the additional standard relation

$$E_{L,R} = \frac{N_{L,R}}{R_{eff}}. \quad (41)$$

However, it is easily checked that the sum  $E = E_R + E_L$  does not equal the ADM mass of the black hole given in (4). In the momentum dominated limit, the difference between the ADM mass and the sum of the energies carried by the left and right moving excitations is

attributed to the mass of a background soliton on which they propagate. In this limit, the excitation energies reduce to  $E_{L,R} \simeq N_{L,R}^{mom}/R$ . The mass of the background is then

$$\begin{aligned} m_{bkgr} &= M - (E_R + E_L) \\ &\simeq \frac{R}{g}N_1 + \frac{RV}{g}N_5, \end{aligned} \tag{42}$$

where we have again restored  $R, V, g$ . The soliton mass  $m_{bkgr}$  is then the sum of the masses of the 1-branes and 5-branes. Similarly, in the model considered in [11], the energy of the excitations correspond to the momentum and 1-brane energies, and the the difference between these and the ADM mass is just the mass of the 5-branes. In general, however,  $m_{bkgr}$  defined according to the first line of (42) does not appear to have a simple interpretation. All three types of constituents, momentum, 1-branes and 5-branes are intertwined in the both the excitations and the background on which they propagate.

This leads to a further interpretational question. In the momentum dominated limit (38), if  $\mu$  is essentially fixed, then when a loop is emitted from the brane, it is only the excitation energies  $E_{L,R}$  which change. The mass of the branes  $m_{bkgr}$  remains unchanged, as one would expect. In the general case, however, both the excitation energies and  $m_{bkgr}$  change with emission. We have

$$\Delta(E_L + E_R) = \frac{\Delta(N_L + N_R)}{R_{eff}} + (N_L + N_R)\Delta\left(\frac{1}{R_{eff}}\right). \tag{43}$$

Consider neutral emission, in which a left and right mover, each with energy  $\omega/2$  combine to form a loop of energy  $\omega$ , and  $\Delta N_L = \Delta N_R = \Delta N$ . It then follows from the thermodynamic relations above, or directly from (40), that for neutral emission

$$\delta M = \omega = \frac{2\Delta N}{R_{eff}}. \tag{44}$$

and therefore from (44) and (43)

$$\Delta M = \Delta(E_L + E_R) - (N_L + N_R)\Delta\left(\frac{1}{R_{eff}}\right) \tag{45}$$

So if we let  $M = E_L + E_R + m_{bkgr}$ , when the ADM mass changes, in addition to a change in  $E_L + E_R$ , there is a change in the background mass of  $\Delta m_{bkgr} = (E_L + E_R)\frac{\Delta R_{eff}}{R_{eff}}$ .

It is of interest to know what the sign of this extra change is. While it is straightforward to write down the variation of  $R_{eff}$ , it seems difficult in general to determine it's sign

because  $\Delta\delta_i$  and  $\Delta\mu$  can have general signs. However for neutral emission the condition that the charges  $Q_i$  are fixed relates the variations. In this case, one can check the behavior of the variation in  $R_{eff}$  in various limits. Checking for equal  $\delta_i$ , one large  $\delta_i$ , and two large  $\delta_i$ , we find

$$\Delta\frac{1}{R_{eff}} = -2\frac{\Delta\mu}{\mu}\frac{1}{R_{eff}}\alpha, \quad 0 \leq \alpha \leq 1 \quad (46)$$

where  $\alpha$  is a function of the  $\delta_i$  and is found to be a positive number, between zero and one, in the above mentioned limits. Since for neutral emission,  $\Delta\mu$  has the same sign as  $\Delta M$ , this means that

$$|\Delta M| > |\Delta(E_L + E_R)| \quad (47)$$

Of the cases checked the largest value for  $\alpha$ , and hence the largest additional contribution to the change in the ADM mass, was for equal  $\delta_i$  or one large  $\delta_i$ , which gives  $\alpha$  close to one.

In the effective string picture, we interpret equation (47) as saying that in an emission process, there is some adjustment of the background soliton structure, which leads to an additional contribution to the total emitted energy (and likewise for absorption). An interesting question is what does this mean in the black hole spacetime? That is, if the generalized string model is correct, can we learn something about the backreaction in Hawking emission? Is there also a division of field energies in the spacetime, corresponding to the division between  $E_L + E_R$  and  $m_{bkg}$ ? Does the additional emitted energy come from inside or outside of the horizon?

Another area which needs to be explored more fully is how charge is carried by the effective string excitations. For agreement with the emission of generally charged scalars, the effective string excitations must have statistical distributions of the form

$$\rho_{L,R}(\omega, q_i) = \frac{1}{e^{\beta_{L,R}\omega - (\beta\Phi_i)_{L,R}q_i} - 1}. \quad (48)$$

In particular, it is clear that the excitations must carry all three varieties of charge. This raises a number of questions. Is momentum of the left and right moving excitations along the string still to be identified with Kaluza-Klein charge, as in the momentum dominated limit? If so, then the U-duality invariance of the model is compromised. If not, to what does this momentum now correspond? Without an understanding of how charge is carried by the excitations, the effective string model of [1] is incomplete. Rather, it seems to be a set of interesting thermodynamic relations still in search of a microscopic model.

Finally, it remains to be seen whether the effective string model of [1] can be made to agree with results for the emission of fixed scalars [16,17,18,7,9], intermediate scalars [19], higher angular momentum [20,8] and higher spin modes [21].

**Note Added:** After this work was complete, the papers [22,23] appeared which have some overlap with our results. In particular, using a different radial coordinate in the near horizon region, these authors were able to show [22] that the absorption coefficient has the form (13) which we have conjectured. They also extract the effective string radius from the prefactor to the absorption coefficient.

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## Appendix A. $\sigma_{abs} \rightarrow A_H$ at low frequencies

In this appendix we outline the calculation showing that the low frequency limit of the absorption coefficient for a charged scalar field in the spacetime (3) is the horizon area, and derive how low the frequency must be. An analogous result has, of course, appeared previously for scalar fields in four dimensions. In [13] part of the argument was given for a black hole in any dimension. However, conflicting approximations are made at different points in the calculation in [13], and one must additionally show that there is a parameter range over which the result actually holds. The main result of this appendix are the conditions (A.11) and (A.12).

We consider the wave equation for a scalar field carrying a single charge in the background (3) written in terms of the tortoise coordinate

$$\chi''(r_*) + [\omega_\infty^2 - V_{coul} - V_{grav}] \chi = 0, \quad (\text{A.1})$$

where  $\lambda = r^{3/2} f^{1/4}$ ,  $\chi = \lambda \phi$ ,  $dr_* = \sqrt{f}(1 - \frac{r_0^2}{r^2})^{-1} dr$ , and

$$V_{coul} = r_n^2 \frac{\omega_\infty^2 - \mu^2}{r^2 + r_n^2}, \quad V_{grav} = \frac{\lambda''(r_*)}{\lambda}, \quad (\text{A.2})$$

Here  $\omega_\infty^2 = \omega^2 - k_5^2$  and  $\mu = \omega - k_5(1 + \frac{r_0^2}{r_3^2})^{\frac{1}{2}}$ .

Let  $\kappa$  be the surface gravity,  $r_1$  be the largest charge in the metric, and  $k_5$  be the Kaluza-Klein charge. Then near the horizon  $r_* \rightarrow -\infty$ , the scattering potentials go to zero exponentially fast, and

$$\phi \rightarrow Ae^{-i\omega_H r_*}, \quad (\text{A.3})$$

where  $\omega_H = \omega - k_5(1 + \frac{r_o^2}{r_n^2})^{-1/2}$  is the frequency of the wave near the horizon. This assumes that

$$\kappa^3 r_1 e^{2\kappa r_*} \ll \omega_H^2, \quad (\text{A.4})$$

$$\text{and } e^{2\kappa r_*} \ll 1. \quad (\text{A.5})$$

In the asymptotically flat region  $r_* \gg r_1$  the solutions are Bessel functions,

$$\phi_\infty(r) = \sqrt{\frac{\omega_\infty \pi}{2}} \frac{e^{-i\pi/4}}{r} \left( H_\nu^{(2)}(\omega_\infty r) + i S H_\nu^{(1)}(\omega_\infty r) \right), \quad \nu = 1 - \omega_\infty^2 (r_1^2 + r_5^2 + r_0^2) + \mu^2 r_n^2. \quad (\text{A.6})$$

Since there is no overlap between these two regions, we use a third region to “bridge the gap”. The wave equation can be solved exactly when  $\omega_\infty = \mu = 0$ . One can then do a double power series expansion for  $\phi$  in these two parameters. The expansion is valid in a “middle region”, away from  $r_* \rightarrow \pm\infty$ . One finds

$$\phi_o = B[\ln(r - r_o) + \ln(r + r_o) - 2\ln r] + C. \quad (\text{A.7})$$

To match the near horizon solution (A.3) to the middle solution one expands the plane wave, which requires

$$\omega_H |r_*| \ll 1 \quad (\text{A.8})$$

in the matching region. Note that the condition (A.4) requires a *large* negative value of  $r_*$ , while (A.5) requires a *small* value of  $|r_*|$ . The large argument expansion of the near horizon solution can be matched onto the middle solution, which then can be matched onto the small argument expansion of the Bessel function can be matched onto the middle solution; see [6] for details. The latter expansion requires

$$\omega_\infty r \ll 1. \quad (\text{A.9})$$

Matching gives the the coefficients  $A$  and  $S$ . The absorption coefficient can be computed either as  $1 - |S|^2$ , or as the ratio of the flux of the field  $\phi$  crossing the horizon to the incident flux from infinity. Either way gives

$$\sigma_{abs} = \frac{\omega_H \omega_\infty^2}{4\pi} A_H. \quad (\text{A.10})$$

Finally, let us combine the conditions for validity of the result (A.10). Choose values for  $\kappa, r_1$ . Then (A.8) and (A.5) require that

$$\omega_H \ll 2\kappa. \tag{A.11}$$

Now, if  $-\infty < \ln(\kappa r_1) < 1$ , one can check that (A.4) is already satisfied. So it is sufficient to satisfy (A.11). If  $\ln(\kappa r_1) > 1$ , in order to satisfy (A.4), one needs

$$\omega_H \ll \frac{2\kappa}{\ln(\kappa r_1)}, \tag{A.12}$$

which is more stringent than (A.11).

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