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BPS Force Balances via Spin-Spin Interactions

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Abstract

We study two systems of BPS solitons in which spin-spin interactions are important in establishing the force balances which allow static, multi-soliton solutions to exist. Solitons in the Israel-Wilson-Perjes (IWP) spacetimes each carry arbitrary, classical angular momenta. Solitons in the Aichelburg-Embacher "superpartner" spacetimes carry quantum mechanical spin, which originates in the zero-modes of the gravitino field of $N = 2$ supergravity in an extreme Reissner-Nordstrom background. In each case we find a cancellation between gravitational spin-spin and magnetic dipole-dipole forces, in addition to the usual one between Newtonian gravitational attraction and Coulombic electrostatic repulsion. In both cases, we analyze the forces between two solitons by treating one of the solitons as a probe or test particle, with the appropriate properties, moving in the background of the other. In the IWP case, the equation of motion for a spinning test particle, originally due to Papapetrou, includes a coupling between the background curvature and the spin of the test particle. In the superpartner case, the relevant equation of motion follows from a κ-symmetric superparticle action.

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1. Introduction

The list of BPS solutions to the classical field equations of $N \geq 2$ supersymmetric field theories seems to grow daily. One of the interesting features of BPS solitons are the, sometimes quite intricate, force cancellations which make static multi-soliton solutions possible. Between BPS magnetic monopoles, for example, magnetostatic repulsion is balanced by an attractive scalar Higgs force [1]. Amongst extreme Reissner-Nordstrom black holes, the electrostatic repulsion is balanced by gravitational attraction in the well known Majumdar-Papapetrou (MP) multi-black hole spacetimes [2][3]. In multi-intersecting brane spacetimes, the cancellation may involve a long list of static gauge and scalar forces as well as gravity (see *e.g.* [4]).

It is interesting to know all the different types of interactions which may contribute to BPS force balances. In this short paper, we will study the cancellation of forces in two systems of BPS solitons and identify two additional interactions, gauge and gravitational spin-spin forces, which contribute in each of these cases. We will focus primarily on the Israel-Wilson-Perjes (IWP) spacetimes [5], which are multi-soliton solutions of 4-dimensional Einstein-Maxwell theory. This theory may be regarded as the bosonic sector of $N = 2$ supergravity and, in this context, it has been shown that the IWP spacetimes preserve half the full spacetime supersymmetry [6].

Like the MP spacetimes, which form a subclass of the IWP solutions, each IWP soliton has charge equal to its mass, giving a balance of electrostatic repulsion and Newtonian gravitational attraction between pairs of solitons. Each IWP soliton, however, may also carry classical angular momentum of arbitrary magnitude and direction. For nonzero angular momentum, an IWP soliton also has a magnetic dipole moment, and at large separation two IWP solitons are subject to the familiar magnetic dipole-dipole force. We will show that in the force balance between spinning IWP solitons this magnetic dipole-dipole force is cancelled by a gravitational spin-spin force. Our strategy is to consider the forces on a BPS probe, or test particle, which has mass equal to its charge and also carries arbitrary classical spin, moving in the background of an IWP spacetime. The equation of motion for such a charged, spinning test particle was derived in [7][8], essentially via taking the probe limit of a finite sized object carrying charge and angular momentum.

We also present a second example in which the solitons carry angular momenta that are quantum mechanical in nature - of order $\hbar$. The multi-soliton solutions in this case are the "superpartner" spacetimes of Aichelburg and Embacher [9] constructed by acting with

broken supersymmetry transformations on the MP solutions[3]. The angular momentum in this case has its origin in the zero modes of the gravitino field in the MP background (see [10] for discussion of this point). The probe we consider to analyse the force balance in this case, following the series of papers [11][12][13][14], is a κ-symmetric superparticle, which has charge equal to its mass, carries intrinsic spin and couples to the background spacetime fields of $N = 2$ supergravity. We will see that the force balance for the superparticle in the superpartner background works out in just the same way it does for the classical BPS test particle in the IWP background, with the spin of the classical probe being replaced by the quantum mechanical spin operator of the superparticle.

2. The Israel-Wilson-Perjes Spacetimes

The IWP spacetimes are stationary, solutions to the Einstein-Maxwell equations given by

$$\begin{aligned} ds^2 &= -|f|^{-2}(dt + \omega_i dx^i)^2 + |f|^2 \delta_{ij} dx^i dx^j, \\ F_{ti} &= \partial_i \Phi, \qquad F^{ij} = |f|^{-2} \epsilon^{ijk} \partial_k \xi, \qquad \Phi + i\xi = 1/f. \end{aligned} \tag{1}$$

Here, $f(\vec{x})$ is any complex solution to the flat 3-dimensional Laplacian

$$\nabla^2 f = 0, \tag{2}$$

and the 3-dimensional vector $\vec{\omega}$ is a solution to

$$\vec{\nabla} \times \vec{\omega} = i \left(f \vec{\nabla} f^* - f^* \vec{\nabla} f \right). \tag{3}$$

Taking f to be real and restricting to pointlike singularities, an otherwise arbitrary f may be written as

$$f = 1 + \sum_{i=1}^{N} \frac{M_i}{|\vec{x} - \vec{x}_i|}. \tag{4}$$

These are the MP spacetimes[2]. If f has only one singularity, it is well known that the spacetime is identical to the extreme Reissner-Nordstrom black hole, having mass equal to its charge. For $N > 1$, the real parameters M_i and $\vec{x}_i$ correspond, loosely speaking, to the masses and positions of a collection of extreme black holes, each having charge Q_i equal to its mass M_i. This identification is not precise, in part, because only the total mass of a spacetime is well defined. However, especially in the limit that the objects are widely

[3] See reference [10] for a recent similar treatment of M2-brane superpartners.

separated, the identification seems justified. Hartle and Hawking[3] have shown that the objects in the MP solutions have horizons and so are indeed black holes.

As discussed in [3], there are two independent ways to make the metric function f in (1) complex. The parameters M_i and/or the parameters x_i in (4) may be taken to be complex. Taking $M_i = m_i + in_i$ and keeping the x_i real, yields a collection of solitons with masses m_i and NUT charges n_i. The condition of equal charge and mass for MP solitons is replaced in this case by the relation

$$m_k + in_k = Q_k + iP_k, \tag{5}$$

where Q_k and P_k are the electric and magnetic charges of the kth soliton. The case of a single IWP soliton with M complex is identical to the extremal limit of the charged generalization of Taub-NUT space given by Brill [15].

Keeping the mass parameters M_i real, but allowing the components of the vectors $\vec{x}_i$ to be complex gives a collection of objects, each having electric charge Q_i equal to its mass M_i as in the MP case. Additionally, each object now carries an angular momentum, whose magnitude and direction are encoded in its complex position vector $\vec{x}_i$. For example, as shown in the second reference in [5], taking a single object with complex position vector $\vec{x}_1 = ia\hat{k}$ and real mass parameter M, gives a Kerr-Newman spacetime with electric charge equal to its mass, and angular momentum vector $\vec{J} = aM\hat{k}$ directed along the z-axis. This is, of course, a naked singularity rather than a black hole. It is shown in [3] that this is generically the case when a position vector is taken to be complex in (4)[4].

3. IWP Force Balances

We can think of an IWP solution as a collection of solitons, each of which is characterized by a set of parameters; mass, electric and magnetic charges, NUT charge and angular momentum vector. The positions of these objects stay fixed in time, so that some sort of force balance must hold between them. One can ask what are the different interactions which contribute to this force balance? In order to answer this question, we will look at the forces between two widely separated IWP solitons in the limit that one of the solitons is much lighter than the other and may therefore be treated as a test-object, or probe, moving in the background spacetime fields of the heavier soliton.

[4] Note however that IWP-type solutions in higher dimensions are known to have regular horizons [16].

The force balance which operates in the MP case, when the metric function f is real, is well known. It is simply the cancellation between the Newtonian gravitational attraction and the Coulombic electrostatic repulsion for two solitons, each of which has charge equal to its mass. That this cancellation continues to hold exactly in general relativity, beyond the Newtonian limit, is quite remarkable.

Now, consider taking the parameters M_i to be complex, while keeping the position vectors $\vec{x}_i$ real. In order to analyse the force balance in this case, we would need the equation of motion for a test particle with NUT charge. Some time ago, it was conjectured that NUT charge may play a dual role to mass, in the same way that electric and magnetic charges are dual within Maxwell theory [17] (see also [18]), with mass and NUT charge satisfying a Dirac-like quantization condition. In this context, an equation of motion for a test particle carrying NUT charge (but no mass) was suggested in [19], which is simply the geodesic equation in a dual metric, defined to have Riemann curvature dual to that of the original background metric. This seems to be a good starting point for us. However, we would also need to include ordinary mass for the test particle, as well as, electric and magnetic charges. It seems likely that an ansatz for such an equation of motion could be formulated along the lines of [19]. Whether, or not, this equation produced a force balance for IWP solitons could then be regarded as a test of its validity.

We leave this direction to future work and concentrate instead on IWP solitons carrying angular momentum. However, we do note here that the pairing in equation (5) of mass with electric charge and NUT charge with magnetic charge for IWP solitons fits in well with the conjectured duality of mass and NUT charge described above. It also seems likely that a proper understanding of the motion of test particles with NUT charge would contribute to a similar understanding of the motion of Kaluza-Klein monopoles [20][21] and D6-branes (as described in [22]), which involve a Euclidean Taub-NUT space in their construction.

We now turn to IWP solitons which carry angular momentum (but zero NUT charge). In this case, we require a spinning probe in the background of a single IWP soliton to carry out a force balance analysis. Fortunately, the equation of motion for a spinning test particle has been investigated at great length in the literature. It was first derived in 1951 by Papapetrou [7] by starting with finite sized objects carrying angular momentum and

taking a test particle limit. This work was extended by Dixon [8] to include electromagnetic interactions giving the equation of motion

$$\begin{aligned} u^a \nabla_a p^b &= q F^b{}_c u^c - \tfrac{1}{2}\left(R^b{}_{cde} u^c + \frac{g}{2}\frac{q}{m} \nabla^b F_{de} \right) S^{de} \\ &\equiv F^b_{maxwell} + F^b_{spin} + F^b_{dipole} \end{aligned} \tag{6}$$

Here p^a and u^a are the four-momentum and 4-velocity of a test-particle[5] which has mass m, charge q, gyromagnetic ratio g and angular momentum tensor S^{ab}. The angular momentum tensor is related to an angular momentum vector by $S_{ab} = \epsilon_{abcd} u^c S^d$. The term $F_{maxwell}$ in (6) gives the usual electromagnetic force. F_{spin} and F_{dipole}, we will see, give gravitational and electromagnetic spin-spin forces.

Wald [23] has computed F_{spin} in (6) for a test particle in the background of a Kerr black hole (*i.e.* charge zero) with angular momentum $\vec{J}$. In the limit of large separation, there is a spin-spin force on the test particle given by

$$\vec{F}_{spin} = -\vec{\nabla} \left\{ \frac{-\vec{S}\cdot\vec{J} + 3(\vec{S}\cdot\hat{r})(\vec{J}\cdot\hat{r})}{r^3} \right\}. \tag{7}$$

Wald [23] notes that up to an overall sign change, this gravitational spin-spin force has the same form as the familiar magnetic dipole-dipole force from basic electrodynamics, if the spin vectors are replaced by magnetic moment vectors.

The spacetime fields for a single IWP soliton with vanishing NUT charge are simply those of a Kerr-Newman spacetime with $Q = M$. It is easily checked that Wald's result (7) for F_{spin} is unchanged by the addition of charge for the background spacetime. For a charged test particle in the Kerr-Newman spacetime F_{dipole} in equation (6) will also be nonzero. To calculate this we plug into (6) the far field limit of the spatial components of the gauge potential in Kerr-Newman

$$A_i \simeq \frac{Q J_{ik} x^k}{M r^3}. \tag{8}$$

We then find that the magnetic spin-spin force, as expected from Wald's observation, combines simply with the gravitational spin-spin force to give

$$\vec{F}_{spin} + \vec{F}_{dipole} = -\left(1 - \frac{gqQ}{2mM}\right) \vec{\nabla} \left\{ \frac{r^2 \delta^k_m - 3x^k x^m}{r^5} \right\} J_{kn} S^{mn}. \tag{9}$$

[5] Far from the background soliton one has the usual relation $p^a = m u^a$. However, in general, p^a and u^a are not collinear (see [23] for a discussion of this point).

One sees immediately that the sum of these forces vanishes, if both the test particle and the black hole have charges equal to their masses, $Q = M$ and $q = m$, and the test particle has gyromagnetic ratio $g = 2$. These are precisely the conditions for both the background and the probe to be IWP solitons. Note that the gyromagnetic ratio $g = 2$ for the background is already built into the gauge potential (8) for the Kerr-Newman background.

For the static probe, $F_{maxwell}$ in (6) contributes a Coulombic force, which is balanced by the Newtonian gravitational force contained in the the Christoffel symbol terms on the left hand side as in the MP case. Putting this together with the cancellation of the gauge and gravitational spin-spin forces in (9), we then have a complete understanding of the force balance between spinning IWP solitons.

4. Force Balance for BPS Superparticles

We now turn to our second example. In reference [9] Aichelberg and Embacher constructed a class of multi-soliton solutions, which they called "superpartner" spacetimes, in the following way. Start with the MP spacetimes. Since these break half the supersymmetries of $N = 2$ supergravity, one can generate new solutions by acting with the broken supersymmetry generators. The resulting superpartner spacetimes have nontrivial gravitino fields and carry nonzero, quantum mechanical, spin angular momenta, filling out a BPS multiplet of spin states[6].

In the further series of papers [11]-[14] Aichelburg and Embacher went on to study the motion of a κ-symmetric superparticle probe in the background of a superpartner soliton. The superparticle represents the test particle limit of a second superpartner soliton. This is precisely the setup we need to study the force balance between superpartner solitons We now show that it is a simple application of the results of [11]-[14] that, in the limit of large separation between a superparticle and some number of superpartner solitons, a cancellation between gauge and gravitational spin-spin forces takes place which allows static configurations to exist.

A lengthy calculation in [13] yields the equation of motion for the superparticle, expanded out to quadratic order in its fermionic superspace coordinates θ^k. Aichelberg and Embacher identify within this result a gravitational spin-spin interaction term, in which the classical angular momentum of the probe in Papepetrou's equation of motion [7] is

[6] As discussed in [10] the angular momentum is carried by the quantized states of the gravitino zero-modes

replaced by the quantum mechanical spin operator of the superparticle. Here we additionally note that the superparticle equation of motion in [13] also contains a magnetic dipole-dipole force term, which cancels with the gravitational spin-spin force as part of the force balance between superpartners. From equation (6.7) of [11] we have[7]

$$F_\mu^{spin} + F_\mu^{dipole} = \frac{1}{4}\bar{\pi}^j u^\nu \left[\delta^{jk} R_{\mu\nu}{}^{mn}\gamma_{mn} - \frac{iq}{m}\epsilon^{jk}\gamma^{mn}(\partial_\mu F_{mn}\gamma_\nu - \partial_\nu F_{mn}\gamma_\mu)\right]\theta^k \qquad (10)$$

Here θ^j , $j = 1, 2$ are the two (Majorana) fermionic coordinates of the superparticle, π^j are their conjugate momenta, γ_μ are four dimensional gamma matrices, and γ_{mn} is the antisymmetric product of Dirac matrices. Latin letters are frame indices and greek letters are spacetime coordinate indices. We refer the reader to [11] for detailed definitions and conventions[8].

The spin operator of the superparticle S^{op}_{mn}, to order θ^2, is given by [9]

$$S^{op}_{mn} = -\bar{\pi}^j \gamma_{mn}\theta^j. \qquad (11)$$

With this identification, the first term in equation (10) has exactly the same form as the classical gravitational spin-spin force F_{spin} in equation (6) above. To study how the force balance occurs in the superpartner spacetimes, consider a static probe, as before, with four velocity $u^m = (1, 0, 0, 0)$ and look at the gauge field strength terms in (10). The spinor coordinates θ^k in [11], for this four velocity, satisfy the gauge condition $(1+\gamma_0)(\theta^1+i\theta^2) = 0$ and using the fact that γ_0 is pure imaginary in [11], it then follows that $\bar{\pi}^j\epsilon_{jk}\gamma_0\theta^k = i\bar{\pi}^j\delta_{jk}\theta^k$. Substituting this and the static property of the background $\partial_0 F_{mn} = 0$ into (10), we finally find that the sum of the gravitational and magnetic spin-spin forces for the superparticle in equation (10) reduces *identically* to the expression (9) for the classical spinning particle, with the spin tensor of the classical particle replaced by the spin operator S^{op}_{mn} of the superparticle. The gyromagnetic ratio for the superparticle automatically comes out to be $g = 2$.

[7] Additonal force terms, besides those we display here also appear in the equation of motion of the superparticle in [11]. That all of these terms give zero net interaction between superpartners is noted at the level of the superparticle lagrangian in [14] without a specific discussion of the nature of the cancelling forces.

[8] We have substituted in the relation between the superparticle supercharge Q^k and the fermionic coordinates θ^k, $Q^k = \pi^k + O(\theta^2)$ into equation (6.7) of [9] to obtain (10).

The asymptotic forms of the metric and gauge potential of the superpartner spacetimes of [9] are the same as that of Kerr-Newman with $Q = M$. Therefore, the evaluation of the Riemann tensor and electromagnetic field strength in (9) is as before. The gravitational spin-spin and magnetic dipole-dipole forces then balance for the superparticle in these spacetimes in the same way they do for the classical spinning particle in the IWP spacetimes. We note that the superparticle spin in (11) is operator valued. To get actual numerical values for the individual forces in equation (10), one must evaluate the force in a particular spin state. This and related issues are discussed in [14][10].

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