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Overlapping Branes in M-Theory

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Abstract

We construct new supersymmetric solutions of D=11 supergravity describing n orthogonally "overlapping" membranes and fivebranes for n=2,...,8. Overlapping branes arise after separating intersecting branes in a direction transverse to all of the branes. The solutions, which generalize known intersecting brane solutions, preserve at least 2^{-n} of the supersymmetry. Each pairwise overlap involves a membrane overlapping a membrane in a 0-brane, a fivebrane overlapping a fivebrane in a 3-brane or a membrane overlapping a fivebrane in a string. After reducing n overlapping membranes to obtain n overlapping D-2-branes in D=10, T-duality generates new overlapping D-brane solutions in type IIA and type IIB string theory. Uplifting certain type IIA solutions leads to the D=11 solutions. Some of the new solutions reduce to dilaton black holes in D=4. Additionally, we present a D=10 solution that describes two D-5-branes overlapping in a string. T-duality then generates further D=10 solutions and uplifting one of the type IIA solutions gives a new D=11 solution describing two fivebranes overlapping in a string.

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1. Introduction

It has been conjectured that there exists a consistent quantum theory in D=11 whose low-energy limit is D=11 supergravity [1],[2]. The so called "M-theory" enables one to understand various duality properties of string theory in a unified way. The membrane and fivebrane solutions of D=11 supergravity preserve half of the supersymmetry and have played an important role in elucidating the properties of M-theory.

It is natural to enquire whether there are more general supersymmetric configurations involving membranes and fivebranes. Papadopoulos and Townsend [3] have recently re-interpreted certain solutions of D=11 supergravity [4], which preserve only 2^{-n} of the supersymmetry with n=2,3, as describing n orthogonal membranes intersecting at a point. Dual solutions were also given in [3] describing, for n=2, a pair of fivebranes intersecting on a 3-brane and, for n=3, three fivebranes intersecting pairwise on 3-branes, which in turn intersect on a string.

In this paper, we present generalizations of the solutions of [3],[4] which preserve at least 2^{-n} of the supersymmetry with $n = 2, \ldots, 8$ and include n arbitrary harmonic functions. For $n = 2, 3$ the solutions preserve exactly 2^{-n} of the supersymmetry. These new solutions describe branes which "overlap" orthogonally in a certain number of tangent directions: overlapping branes are obtained from intersecting branes by separating one of the branes in a direction transverse to all of the branes. Branes intersect only if two or more of the n functions are singular at the same point. For $n = 2, 3$ the solutions of [3],[4] are recovered when the n harmonic functions are taken to be equal. Included in our new solutions are membranes and fivebranes overlapping in one tangent direction; in the intersecting case, D=11 membranes and fivebranes are found which intersect on a string[1].

We begin in section (2) by giving overlapping membrane solutions with n=2,3,4. We then show in section (3) that other D=11 solutions, involving fivebranes and collections of membranes and fivebranes all preserving 2^{-n} of the supersymmetry, can be obtained from the membrane solutions of section (2) via dimensional reduction, performing T-duality transformations and then uplifting back to D=11. While carrying out these transformations, we will also find solutions in lower dimensions which describe n overlapping D-branes. Recall that all single, *i.e.*, n=1, D-brane solutions may be obtained starting from any given one, *e.g.*, the D-0-brane, by acting with T-duality [6],[7]. It seems natural to conjecture

[1] While this work was being completed, a paper by Tseytlin [5] appeared which contains many of the results presented in sections (2) - (4) below.

that a similar relation holds between all solutions involving n overlapping D-branes, which preserve the same amount of the supersymmetry. We also discuss solutions with $n \geq 4$, which preserve more than 2^{-n} of the supersymmetry due to the presence of certain special triple overlaps. In section (4), we will discuss the reduction of the n=1,2,3 overlapping brane solutions to dilaton black holes solutions in D=4. We find, for example, that the overlapping membrane (fivebrane) solutions reduce to black holes carrying n types of electric (magnetic) charge. The n=2 solutions belong to a class recently studied in [8],[9]. Further, we find that all the intersecting cases reduce to extreme dilaton black holes with $a = \sqrt{(4/n) - 1}$, as for the cases considered in [3]. We discuss how this comes about at the level of the dimensionally reduced action.

It was pointed out in [3] that the solutions of [3],[4] are not "true" D=11 intersections in the sense that the harmonic function corresponding to a given brane is translationally invariant in the directions tangent to all of the branes. This property is shared by the generalized overlapping solutions of sections (2)-(4). In section (5) we will interpret a solution first constructed by Khuri that preserves 1/4 of the supersymmetry [10] as describing a true overlap of two 5-branes in a string in D=10 type II supergravity. New solutions can be generated from this using $SL(2, Z)$ and T-duality. By uplifting to D=11 we obtain a solution describing two fivebranes overlapping in a string, although it is not a true overlap. Section (6) concludes with some comments.

2. Overlapping Membranes

The bosonic field content of D=11 supergravity consists of a metric and a three-form potential with four-form field strength[2] $\tilde{F}_{MNPQ} = 24\nabla_{[M}\tilde{A}_{NPQ]}$. The action for the bosonic fields is given by

$$S = \int \sqrt{-\tilde{g}} \left\{ R - \frac{1}{12}\tilde{F}^2 - \frac{1}{10,368}\epsilon^{\mu_1 \ldots \mu_{11}} \tilde{F}_{\mu_1 \ldots \mu_4} \tilde{F}_{\mu_5 \ldots \mu_8} \tilde{A}_{\mu_9 \ldots \mu_{11}} \right\}. \tag{1}$$

Supersymmetric solutions to the corresponding equations of motion can be constructed by looking for bosonic backgrounds that admit Killing spinors *i.e.*, backgrounds which admit spinors such that the supersymmetry variation of the gravitino field $\tilde{\psi}_M$ vanishes:

$$\left[D_M + \frac{1}{144}(\Gamma_M{}^{NPQR} - 8\delta_M^N \Gamma^{PQR})\tilde{F}_{NPQR} \right] \epsilon = 0, \tag{2}$$

[2] All D=11 fields have tilde's. All D=10, IIA supergravity fields will have hats and D=10, IIB fields will have bars.

where ϵ is a 32-component Majorana spinor.

We begin by presenting the generalized supersymmetric solution for n=2 orthogonally overlapping membranes[3],

$$\begin{aligned} d\tilde{s}^2 = & -\left(f_1 f_2\right)^{-2/3} dt^2 + f_1^{-2/3} f_2^{1/3}\left(dx_1^2 + dx_2^2\right) + f_1^{1/3} f_2^{-2/3}\left(dx_3^2 + dx_4^2\right) \\ & + \left(f_1 f_2\right)^{1/3}\left(dx_5^2 + \ldots + dx_{10}^2\right), \\ & \tilde{F}_{t12\alpha} = \frac{c_1}{2}\frac{\partial_\alpha f_1}{f_1^2}, \qquad \tilde{F}_{t34\alpha} = \frac{c_2}{2}\frac{\partial_\alpha f_2}{f_2^2}, \qquad \alpha = 5, \ldots, 10. \\ & f_i = f_i(x_5, \ldots, x_{10}), \qquad \nabla^2 f_i = 0, \qquad c_i = \pm 1, \qquad i = 1, 2. \end{aligned} \tag{3}$$

It is straightforward to show that this background preserves 1/4 of the supersymmetry: there are eight Killing spinors of the form $\epsilon = (f_1 f_2)^{-1/6}\eta$, where η is constant and satisfies the algebraic constraints

$$\begin{aligned} \hat{\Gamma}^0\hat{\Gamma}^1\hat{\Gamma}^2\eta &= c_1\eta \\ \hat{\Gamma}^0\hat{\Gamma}^3\hat{\Gamma}^4\eta &= c_2\eta, \end{aligned} \tag{4}$$

where $\hat{\Gamma}^M$ are Gamma matrices in an orthonormal frame. These conditions can be recast, as in [4], in the compact form

$$\hat{\Gamma}_a\eta = \hat{\Gamma}_0 J_a{}^b \hat{\Gamma}_b\eta, \tag{5}$$

where the indices $a, b = 1, \ldots 4$ correspond to the membrane coordinates $x^1, \ldots x^4$ and $J_a{}^b$ is a complex structure on this space (depending on the c_i).

The functions f_i are harmonic in the coordinates $\vec{x} = \{x_5, \ldots, x_{10}\}$ and we first take them to be of the form

$$f_i = 1 + \frac{M_i}{r_i^4}, \qquad r_i = |\vec{x} - \vec{x}_i|\,. \tag{6}$$

The solution then describes a membrane oriented in the $(1,2)$ plane with position $\vec{x}_1$ and another oriented in the $(3,4)$ plane with position $\vec{x}_2$ orthogonally overlapping in a point. In particular note that the directions tangent to the ith set of membranes appear with the power $f_i^{-2/3}$ and those transverse with the power $f_i^{1/3}$, which are the appropriate powers for the n=1 membrane solutions. A membrane with $(1,2)$ orientation intersects one with $(3,4)$ orientation in the degenerate case that $\vec{x}_1 = \vec{x}_2$. Note that the n=2 solutions of Güven [4] are recovered by taking $f_1 = f_2$. These observations clarify the interpretation [3] of the Güven solutions as an intersection of two membranes in the degenerate limit.

[3] We will often refer to both membranes and anti-membranes as membranes.

The most general solution has harmonic functions of the form

$$f_i = 1 + \sum_{I=1}^{k_i} \frac{M_{i,I}}{r_{i,I}^4}, \qquad r_{i,I} = |\vec{x} - \vec{x}_{i,I}| \,. \tag{7}$$

The solution then describes k_1 parallel membranes with $(1,2)$ orientation and positions $\vec{x}_{1,I}$, and k_2 parallel membranes with $(3,4)$ orientation and positions $\vec{x}_{2,I}$. Each membrane of one set orthogonally overlaps all of the membranes in the other set in a point. A membrane with $(1,2)$ orientation intersects one with $(3,4)$ orientation in the degenerate case that $\vec{x}_{1,I} = \vec{x}_{2,J}$, for some combination I, J. Note that in describing the solutions in the rest of the paper we will implicitly take the harmonic functions to be that of a single brane as in (6) for ease of exposition.

Like the intersecting membrane solutions of [3],[4], these new solutions have the property that the metric is invariant not only under translations in common tangent directions[4], *i.e.*, the time direction in this case, but also under translations in all the relative transverse directions. For example, f_1 does not fall off in the x_3, x_4 directions, as one would expect for a D=11 membrane spatially oriented in the $(1,2)$ plane. This suggests that, as in [3], the solutions (3) be considered in a dimensionally reduced context, with all relative transverse directions periodically identified. This implies, *e.g.*, that the membrane with spatial orientation in the $(1,2)$ plane has been wrapped in these directions and directly reduced in the $(3,4)$ directions. The solutions (3) may then be regarded as D=7 black holes. The black holes would each, generically, carry one of two types of electric charge, depending on the 10-dimensional orientation of the corresponding membrane. In the degenerate intersecting case [3], black holes would carry equal amounts of both types of electric charge.

The solutions (3) are generically singular on the surfaces $\vec{x} - \vec{x}_i = 0$, with the scalar curvature diverging. This behavior is different from the n=1 membrane case, where these surfaces are regular event horizons [11]. The singularity in the present case arises because the functions f_i are constant in the relative transverse dimensions, rather than having the fall-off characteristic of a single membrane. Interestingly, in the intersecting case, $f_1 = f_2$, the scalar curvature is finite and this singularity is removed. The singularity reappears if the intersecting solutions are further dimensionally reduced in overall transverse directions.

[4] We will generally follow the terminology of [3] in referring to *common tangent* directions as being tangent directions common to all branes. *Relative transverse* directions are those tangent to at least one but not all branes and *overall transverse* directions are those orthogonal to all branes.

The generalization to n=1,2,3,4 membranes may be written in a unified fashion as

$$
\begin{aligned}
d\tilde{s}^2 =& -\prod_{i=1}^{n} f_i^{-2/3} dt^2 + \sum_{i=1}^{n} \left[f_i^{-2/3} \prod_{j\neq i} f_j^{1/3} \right] \left(dx_{2i-1}^2 + dx_{2i}^2 \right) + \\
&+ \prod_{i=1}^{n} f_i^{1/3} \left(dx_{2n+1}^2 + \ldots + dx_{10}^2 \right) \\
\tilde{F}_{t12\alpha} =& \frac{c_1}{2} \frac{\partial_\alpha f_1}{f_1^2}, \ldots, \tilde{F}_{t,2n-1,2n,\alpha} = \frac{c_n}{2} \frac{\partial_\alpha f_n}{f_n^2}, \qquad \alpha = 2n+1, \ldots, 10. \\
f_i =& f_i(x_{2n+1}, \ldots, x_{10}), \qquad \nabla^2 f_i = 0, \qquad c_i = \pm 1, \qquad i = 1, \ldots, n.
\end{aligned}
\tag{8}
$$

These solutions preserve 2^{-n} of the supersymmetries. The Killing spinors are given by $\epsilon = \prod_{i=1}^{n} f_i^{-1/6} \eta$ where the constant spinor η satisfies the obvious algebraic constraints generalizing (4), which can again be recast in the form (5) with J a complex structure on the relative transverse space of the membranes.

The n=3 solutions describe three membranes with orientations in the $(1,2)$, $(3,4)$ and $(5,6)$ directions respectively. If we consider these relative transverse dimensions to be compact, we get [3] n=3 D=5 black holes, each carrying, generically, one of three different types of electric charge. The n=4 solutions give D=3 objects, each carrying one of four types of electric charge. In this case the solutions have bad asymptotics, the functions f_i diverging logarithmically. However, they share many formal properties with the $n < 4$ solutions, including (a lesser amount of) unbroken supersymmetry, and Sen [12] has made use of similar objects in his study of strong-weak coupling duality in three-dimensional string theory. We note that the components of the stress tensor in an orthonormal frame fall off faster than $1/r$ for these objects. One may also consider n=5, however, in this case, the total transverse space has shrunk to zero dimensions and the functions f_i are constant, giving D=11 Minkowski space.

3. New Overlapping Branes From Old Via T-Duality

3.1. Dimensional Reduction and Type II T-Duality Rules

Rather than present the new D=11 solutions involving fivebranes in a similar direct fashion, in this section we will show how they may be obtained from the membrane solutions (8), making use of dimensional reduction and T-duality. In particular, starting from the n=2 membrane solution, we will obtain a fivebrane overlapping a fivebrane on a 3-brane and also a fivebrane and a membrane overlapping on a string. Starting from the n=3

membrane solution, we will find solutions with three fivebranes, solutions with a membrane and two fivebranes and solutions with a fivebrane and two membranes. Similar overlapping solutions may be constructed from the n=4 membrane solution, but we will not write them down explicitly. However, we will explain why for $n \geq 4$ there are also overlapping solutions that preserve more than 2^{-n} of the supersymmetry. In addition to these D=11 solutions we will also see how one can construct various overlapping brane solutions of type IIA and type IIB string theory in D=10.

There are two distinct ways to dimensionally reduce the membrane solutions (8) to get solutions of D=10, IIA supergravity. We will choose to reduce along one of the overall transverse directions, taking the functions f_i to be independent of, *e.g.*, x_{10}. This gives n membranes overlapping in a point in D=10, which are D-2-branes of the IIA theory. If instead one reduces along one of the relative transverse directions one obtains a fundamental string and n-1 D-2-branes overlapping in a point. For present purposes, we need only a limited form of the relations between the fields of D=11 supergravity and those of D=10, IIA supergravity, given by[5]

$$\tilde{g}_{10,10} = e^{4\hat{\phi}/3}, \qquad \tilde{g}_{\mu\nu} = e^{-2\hat{\phi}/3}\hat{g}_{\mu\nu}, \qquad \tilde{F}_{\mu\nu\rho\sigma} = \hat{F}_{\mu\nu\rho\sigma}, \tag{9}$$

where $\mu, \nu = t, 1, \ldots, 9$ and we have assumed that $\tilde{g}_{\mu,10} = F_{10,\mu,\nu,\rho} = 0$. The n=2 membrane solution (3) then reduces along x_{10} to give

$$\begin{aligned} d\hat{s}^2 =& -\left(f_1 f_2\right)^{-1/2} dt^2 + f_1^{-1/2} f_2^{1/2}\left(dx_1^2 + dx_2^2\right) + f_1^{1/2} f_2^{-1/2}\left(dx_3^2 + dx_4^2\right) \\ &+ \left(f_1 f_2\right)^{1/2}\left(dx_5^2 + \ldots + dx_9^2\right), \qquad f_i = f_i(x_5, \ldots, x_9), \\ e^{-2\hat{\phi}} =& \left(f_1 f_2\right)^{-1/2}, \qquad \hat{F}_{t12\alpha} = \frac{c_1}{2}\frac{\partial_\alpha f_1}{f_1^2}, \qquad \hat{F}_{t34\alpha} = \frac{c_2}{2}\frac{\partial_\alpha f_2}{f_2^2}. \end{aligned} \tag{10}$$

We can now use type II T-duality to find new overlapping brane solutions. Recall that type II T-duality is a map from solutions of IIA supergravity into solutions of IIB supergravity, and vice-versa. For diagonal metrics, the action of type II T-duality, with respect to a symmetry direction α, on the dilaton and metric is simply given by [13],

$$\bar{g}_{\alpha\alpha} = 1/\hat{g}_{\alpha\alpha}, \qquad e^{-2\bar{\varphi}} = \hat{g}_{\alpha\alpha} e^{-2\hat{\phi}}, \tag{11}$$

[5] A complete derivation of the reduction from D=11 to D=10, IIA and the Type II T-duality rules is given in [13].

where $\hat{\phi}$ is the IIA dilaton and $\bar{\varphi}$ is the IIB dilaton. The action of T-duality on the gauge fields is more involved [13] and we display only the final results below.

There are two different ways to act with T-duality on (10). Acting on one of the overall transverse directions maps it into a common tangent direction, increasing the dimension of both branes by one. In this way, we can get two D-3-branes of type IIB overlapping on a string, two D-4-branes of type IIA overlapping on a membrane and so on. Alternatively, we can act with T-duality on one of the relative transverse dimensions, increasing the dimension of one brane and decreasing the dimension of the other. In this way, we can construct a D-string overlapping a D-3-brane of type IIB in a point and a D-0-brane overlapping a D-4-brane of type IIA in a point. By combining both sorts of transformations we can construct a variety of overlapping brane solutions which we record here:

$$\begin{array}{lllll} IIA: & 0\cap 4(0); & 2\cap 2(0); & 2\cap 4(1); & 2\cap 6(2); & 4\cap 4(2); \\ & 4\cap 6(3); & 4\cap 8(4); & 6\cap 6(4); & 6\cap 8(5) & \\ IIB: & 1\cap 3(0); & 1\cap 5(1); & 3\cap 3(1); & 3\cap 5(2); & 3\cap 7(3); \\ & 5\cap 5(3); & 5\cap 7(4); & 5\cap 9(5); & 7\cap 7(5); & \end{array} \tag{12}$$

where $3\cap 5(2)$ denotes a D-3-brane overlapping a D-5-brane in a membrane and so on.

Let us pause to make a few comments on this table. Firstly, to implement the T-duality transformations properly for $p \geq 7$ one must use the recently formulated massive type II supergravity [6]. Secondly, we note that acting with $SL(2,Z)$ duality will give new solutions describing overlapping branes carrying R-R and NS-NS charges. Thirdly, the solution $1\cap 5(1)$ was constructed in [14] (see also [15]) in the context of determining black hole entropy using D-brane techniques. Finally, note that for the cases where the branes overlap in a 5-brane the overall transverse directions have shrunk to a point and the solution is just Minkowski space. However, in the case $5\cap 9(5)$, since at the level of classical solutions a D-9-brane is just Minkowski space, this overlap is just a D-5-brane which obviously does correspond to a non-trivial solution. This suggests that non-trivial overlapping solutions may also exist in the other cases in this class, possibly of the type discussed in section 5.

We can compare the list (12) of overlapping D-brane field theory solutions with supersymmetric superpositions of D-branes allowed at the string theory level. Recall that open strings have Neumann (N) boundary conditions in directions tangent to a D-brane and Dirichlet (D) boundary conditions in the transverse directions. In a configuration of

overlapping D-branes, certain open string coordinates may have Dirichlet boundary conditions at one end of the string and Neumann boundary conditions at the other end. It is argued in [16], that in order for a superposition of D-branes to have some amount of unbroken supersymmetry, the number of these ND directions must be a multiple of four [6]. In our solutions, the number of ND directions is equal to the size of the relative transverse space, which has dimension four for all the solutions in (12) and one can check that this is a complete list involving two branes. Solutions with eight ND directions will be presented in section (5).

The main purpose of this paper is to construct supersymmetric solutions of M-theory describing overlapping fivebranes and membranes. This can be achieved by uplifting the type IIA overlapping solutions involving D-4-branes and D-2-branes and the results are presented in the next subsections. Uplifting the other type IIA branes to D=11 is left for future work.

3.2. Fivebrane and Fivebrane Overlap

The following series of steps yields overlapping fivebranes in D=11. Taking both functions f_i in (10) to be independent of x_5 and x_6, act successively with T-duality in these two directions. This gives a IIA solution with two D-4-branes, oriented in the (1256) and (3456) hyperplanes, overlapping on a membrane in the (56) plane, with metric and dilaton given by

$$\begin{aligned} d\hat{s}^2 = &(f_1 f_2)^{-1/2}\left(-dt^2 + dx_5^2 + dx_6^2\right) + f_1^{-1/2} f_2^{1/2}\left(dx_1^2 + dx_2^2\right) + f_1^{1/2} f_2^{-1/2}\left(dx_3^2 + dx_4^2\right) \\ &+ (f_1 f_2)^{1/2}\left(dx_7^2 + dx_8^2 + dx_9^2\right), \qquad e^{-2\hat{\phi}} = (f_1 f_2)^{1/2}. \end{aligned} \tag{13}$$

Lifting back to D=11, the dilaton turns into a common tangent direction and we get

$$\begin{aligned} d\tilde{s}^2 = &(f_1 f_2)^{-1/3}\left(-dt^2 + dx_5^2 + dx_6^2 + dx_{10}^2\right) + f_1^{-1/3} f_2^{2/3}\left(dx_1^2 + dx_2^2\right) \\ &+ f_1^{2/3} f_2^{-1/3}\left(dx_3^2 + dx_4^2\right) + (f_1 f_2)^{2/3}\left(dx_7^2 + dx_8^2 + dx_9^2\right). \\ \tilde{F}_{34\alpha\beta} = &\frac{c_1}{2}\epsilon_{\alpha\beta\gamma}\partial_\gamma f_1, \qquad \tilde{F}_{12\alpha\beta} = \frac{c_2}{2}\epsilon_{\alpha\beta\gamma}\partial_\gamma f_2, \qquad f_i = f_i(x_7, x_8, x_9), \end{aligned} \tag{14}$$

[6] This condition can also be arrived at by determining which D-brane configurations satisfy a zero force condition [17],[18]. The special case of "embedded" branes, where the dimension of the intersection is the same as one of the branes, has been studied by Douglas [19].

where $\epsilon_{\alpha\beta\gamma}$ is the D=3 flat space alternating symbol. The solution preserves 1/4 of the supersymmetry and the Killing spinors are given by $\epsilon = (f_1 f_2)^{-1/12}\eta$ with the constant spinor η satisfying the constraints

$$\begin{aligned} \hat{\Gamma}^3\hat{\Gamma}^4\gamma^*\eta &= c_1\eta \\ \hat{\Gamma}^1\hat{\Gamma}^2\gamma^*\eta &= c_2\eta, \end{aligned} \tag{15}$$

with $\gamma^* = \hat{\Gamma}^7\hat{\Gamma}^8\hat{\Gamma}^9$. This can also be recast, as in [3] in the form

$$\hat{\Gamma}_a\eta = J_a{}^b\hat{\Gamma}_b\gamma^*\eta, \tag{16}$$

where J is again the appropriate complex structure on the relative transverse space of the fivebranes.

This solution describes two fivebranes, oriented in the $(1,2,5,6,10)$ and $(3,4,5,6,10)$ hyperplanes, overlapping on a 3-brane in the $(5,6,10)$ hyperplane. Setting $f_1 = f_2$, it reduces to the intersecting fivebrane solution of [3]. Reducing (14) along all four relative transverse dimensions gives, as in [3], D=7 3-branes, which are magnetic duals to the D=7 black holes, which came from reducing the n=2 membrane solution (3). As with (3), the solution (14) is singular at the positions of the branes due to the lack of fall-off of the functions f_i in the relative transverse directions. Unlike in the case of n=2 membranes, this singularity is not removed in the intersecting case.

3.3. Membrane and Fivebrane Overlap

We get n=2 solutions with a membrane and a fivebrane in D=11 by a slightly different path. Starting again with (10), T-dualize on one dimension parallel to a membrane, *e.g.* x_3, to get a IIB solution describing a string overlapping a 3-brane in a point,

$$\begin{aligned} d\bar{s}^2 = &(f_1f_2)^{-1/2}(-dt^2) + f_1^{-1/2}f_2^{1/2}\left(dx_1^2+dx_2^2+dx_3^2\right) + f_1^{1/2}f_2^{-1/2}\left(dx_4^2\right) \\ &+ (f_1f_2)^{1/2}\left(dx_5^2+dx_6^2+dx_7^2+dx_8^2+dx_9^2\right), \qquad e^{-2\bar{\varphi}} = f_2^{-1}. \end{aligned} \tag{17}$$

Next, take both functions f_i to be independent of the coordinate x_5 and T-dualize in that direction to give a IIA solution describing a D-4-brane overlapping a membrane in a string,

$$\begin{aligned} d\hat{s}^2 = &(f_1f_2)^{-1/2}(-dt^2+dx_5^2) + f_1^{-1/2}f_2^{1/2}\left(dx_1^2+dx_2^2+dx_3^2\right) + f_1^{1/2}f_2^{-1/2}\left(dx_4^2\right) \\ &+ (f_1f_2)^{1/2}\left(dx_6^2+dx_7^2+dx_8^2+dx_9^2\right), \qquad e^{-2\hat{\phi}} = f_1^{1/2}f_2^{-1/2}. \end{aligned} \tag{18}$$

Finally, lift back to D=11 to get a fivebrane overlapping a membrane in a string

$$\begin{aligned} d\tilde{s}^2 =& f_1^{-1/3} f_2^{-2/3}(-dt^2 + dx_5^2) + f_1^{-1/3} f_2^{1/3} \left(dx_1^2 + dx_2^2 + dx_3^2 + dx_{10}^2\right) \\ &+ f_1^{2/3} f_2^{-2/3} \left(dx_4^2\right) + f_1^{2/3} f_2^{1/3} \left(dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2\right), \\ \tilde{F}_{4\alpha\beta\gamma} =& \frac{c_1}{2} \epsilon_{\alpha\beta\gamma\delta} \partial_\delta f_1, \qquad \tilde{F}_{t45\alpha} = \frac{c_2}{2} \frac{\partial_\alpha f_2}{f_2^2}, \qquad f_i = f_i(x_6, \ldots, x_9), \end{aligned} \tag{19}$$

where $\epsilon_{\alpha\beta\gamma\delta}$ is the D=4 flat space alternating symbol. The eight Killing spinors have the form $\epsilon = f_1^{-1/12} f_2^{-1/6} \eta$ with the constant spinor η satisfying

$$\begin{aligned} \hat{\Gamma}^0 \hat{\Gamma}^4 \hat{\Gamma}^5 \eta &= c_1 \eta \\ \hat{\Gamma}^4 \hat{\Gamma}^6 \hat{\Gamma}^7 \hat{\Gamma}^8 \hat{\Gamma}^9 \eta &= c_2 \eta. \end{aligned} \tag{20}$$

This solution describes a membrane in the $(4, 5)$ direction and a fivebrane in the $(1, 2, 3, 5, 10)$ direction, overlapping in a string in the (5) direction. Reducing (19) along all relative transverse dimensions gives electric and magnetic strings in D=6. Electric-magnetic duality then maps within this class of solutions, as expected based on the D=11 duality between membranes and fivebranes .

3.4. Fivebrane, Fivebrane and Fivebrane Overlap

We now turn to n=3 solutions involving fivebranes. Begin by reducing the n=3 membrane solution (8) to D=10, IIA by taking the functions f_i independent of x_{10}, giving

$$\begin{aligned} d\hat{s}^2 =& -\prod_{i=1}^{3} f_i^{-1/2} dt^2 + \sum_{i=1}^{3} \left[f_i^{-1/2} \prod_{j \neq i} f_j^{1/2} \right] \left(dx_{2i-1}^2 + dx_{2i}^2\right) \\ &+ \prod_{i=1}^{3} f_i^{1/2} \left(dx_7^2 + dx_8^2 + dx_9^2\right), \qquad e^{-2\hat{\phi}} = \prod_{i=1}^{3} f_i^{-1/2}, \end{aligned} \tag{21}$$

which describes three D-2-branes in D=10. Acting with T-duality in different ways generates a set of solutions of type II supergravity, which describe three overlapping branes, which all preserve 1/8 of the supersymmetry. We will not present this list here but we note that the intersections of any pair of D-branes is necessarily one from (12). To get intersections of membranes and fivebranes in M-theory we again want to uplift the solutions of type IIA supergravity involving D-2-branes and D-4-branes. The uplifted $D = 11$ solutions involving $n = 3$ branes also preserve 1/8 of the supersymmetry.

If we act with T-duality on (21) successively in all six relative transverse dimensions to get a IIA solution and then lift back to D=11, we get

$$\begin{aligned} d\hat{s}^2 =& \prod_{i=1}^{3} f_i^{-1/3}(-dt^2 + dx_{10}^2) + \sum_{i=1}^{3}\left[f_i^{2/3}\prod_{j\neq i} f_j^{-1/3}\right]\left(dx_{2i-1}^2 + dx_{2i}^2\right) \\ &+ \prod_{i=1}^{3} f_i^{2/3}\left(dx_7^2 + dx_8^2 + dx_9^2\right), \\ \tilde{F}_{12\alpha\beta} =& \frac{c_1}{2}\epsilon_{\alpha\beta\gamma}\partial_\gamma f_1, \qquad \tilde{F}_{34\alpha\beta} = \frac{c_2}{2}\epsilon_{\alpha\beta\gamma}\partial_\gamma f_2, \qquad \tilde{F}_{56\alpha\beta} = \frac{c_3}{2}\epsilon_{\alpha\beta\gamma}\partial_\gamma f_3. \end{aligned} \tag{22}$$

This now describes three fivebranes oriented in the $(3,4,5,6,10)$, $(1,2,5,6,10)$ and $(1,2,3,4,10)$ hyperplanes respectively. Each pair of fivebranes overlaps on a 3-brane. The 3-branes finally all overlap on a string. Reducing these solutions along all relative transverse dimensions gives [3] strings in D=5 carrying three types of magnetic charge, which are dual to the D=5 electric black holes coming from (8) with n=3.

There are two additional solutions corresponding to three overlapping fivebranes. The first may be obtained from (21) by first performing T-duality in one overall transverse direction and on one direction each from the membranes, *e.g.*, the x_7, x_2, x_4 and x_6 directions. Uplifting this solution to D=11 gives fivebranes in the $(1,4,6,7,10)$, $(2,3,6,7,10)$ and $(2,4,5,7,10)$ directions. Pairwise the fivebranes overlap in 3-branes and the latter overlap in a 2-brane. For this solution there are two overall transverse directions and the harmonic functions logarithmically diverge. The second solution can be obtained from (21) by T-dualizing on two overall transverse directions, *e.g.*, x_7, x_8. Uplifting to D=11 gives fivebranes in the $(1,2,7,8,10)$, $(3,4,7,8,10)$ and $(5,6,7,8,10)$ directions. In this case the overlap of the pairwise overlaps is a 3-brane. The harmonic functions now grow linearly as there is only one overall transverse direction.

3.5. Membrane, Fivebrane and Fivebrane Overlap

Again starting from (21), act successively with T-duality on both spatial tangent directions of one D-2-brane and one each of the tangent directions of the two other branes, *e.g.*, the x_1, x_2, x_3 and x_5 directions. Lifting back to D=11 then gives

$$\begin{aligned} d\tilde{s}^2 =& - f_1^{-2/3}(f_2 f_3)^{-1/3}dt^2 + f_1^{1/3}(f_2 f_3)^{-1/3}\left(dx_1^2 + dx_2^2 + dx_{10}^2\right) \\ &+ f_1^{-2/3} f_2^{2/3} f_3^{-1/3} dx_3^2 + f_1^{1/3} f_2^{-1/3} f_3^{2/3} dx_4^2 \\ &+ f_1^{-2/3} f_2^{-1/3} f_3^{+2/3} dx_5^2 + f_1^{1/3} f_2^{2/3} f_3^{-1/3} dx_6^2 \\ & f_1^{1/3} f_2^{2/3} f_3^{2/3}\left(dx_7^2 + dx_8^2 + dx_9^2\right), \\ \tilde{F}_{t35\alpha} =& \frac{c_1}{2}\frac{\partial_\alpha f_1}{f_1^2}, \qquad \tilde{F}_{36\alpha\beta} = \frac{c_2}{2}\epsilon_{\alpha\beta\gamma}\partial_\gamma f_2, \qquad \tilde{F}_{45\alpha\beta} = \frac{c_3}{2}\epsilon_{\alpha\beta\gamma}\partial_\gamma f_3, \end{aligned} \tag{23}$$

which describes a membrane in the $(3,5)$ plane and two fivebranes in the $(1,2,4,5,10)$ and $(1,2,3,6,10)$ hyperplanes. The fivebranes overlap on a 3-brane. The membrane overlaps each of the fivebranes on a string and the strings overlap each other and the 3-brane at a point. Reducing all the relative transverse directions gives D=4 black holes carrying one type of electric and two types of magnetic charge.

There exists another solution where the pairwise overlaps overlap in a string. It has only two overall transverse directions so the harmonic functions diverge logarithmically. To obtain the solution start with (21) and perform T-duality on one overall transverse direction and on one of the directions along one of the membranes, *e.g.*, x_2 and x_7. After uplifting to D=11 one finds a membrane in the $(1,7)$ direction and fivebranes in the $(2,3,4,7,10)$ and $(2,5,6,7,10)$ directions.

3.6. Membrane, Membrane and Fivebrane Overlap

Again, start with (21). Act successively with T-duality on one direction each from two of the membranes, *e.g.*, x_1 and x_3, and lift back to D=11 to get

$$\begin{aligned} d\tilde{s}^2 =& -(f_1 f_2)^{-2/3} f_3^{-1/3} dt^2 + f_1^{1/3} f_2^{-2/3} f_3^{-1/3} dx_1^2 + f_1^{-2/3} f_2^{1/3} f_3^{-1/3} dx_3^2 \\ &+ f_1^{-2/3} f_2^{1/3} f_3^{2/3} dx_2^2 + f_1^{1/3} f_2^{-2/3} f_3^{2/3} dx_4^2 \\ &+ f_1^{1/3} f_2^{1/3} f_3^{-1/3} (dx_5^2 + dx_6^2 + dx_{10}^2) \\ & f_1^{1/3} f_2^{1/3} f_3^{2/3} \left(dx_7^2 + dx_8^2 + dx_9^2\right), \\ \tilde{F}_{t23\alpha} =& \frac{c_1}{2} \frac{\partial_\alpha f_1}{f_1^2}, \qquad \tilde{F}_{t14\alpha} = \frac{c_2}{2} \frac{\partial_\alpha f_2}{f_2^2}, \qquad \tilde{F}_{24\alpha\beta} = \frac{c_3}{2} \epsilon_{\alpha\beta\gamma} \partial_\gamma f_3. \end{aligned} \tag{24}$$

This describes two membranes in the $(2,3)$ and $(1,4)$ planes and a fivebrane in the $(1,3,5,6,10)$ hyperplane.

3.7. Four Overlapping Branes or More

We have not carried out a systematic search for all overlapping brane solutions with four or more sets of branes. We expect that any configuration of branes which satisfies the conditions for pairwise overlaps should yield a solution of the sort described above. We would like to call attention, however, to an interesting phenomenon associated with triple overlaps. For certain special triple overlaps, the three gamma matrix projections multiply together to give a projection appropriate for another set of branes. This raises

the possibility of adding an additional set of branes with this new orientation, without further breaking supersymmetry[7].

The first special triple intersection is that of two membranes and a fivebrane. Taking the branes to be oriented in the (12), (3, 4) and (1, 3, 5, 6, 7) directions, the product of the three projections gives the projection for a fivebrane in the (2, 4, 5, 6, 7) hyperplane. The resulting configuration gives a solution with $2\cap2\cap5\cap5$ preserving 1/8 of the supersymmetry, which has been noted by Klebanov and Tseytlin [20][8]. This solution has three overall transverse directions and can be reduced to give a black hole solution in four dimensions with finite horizon area, making it interesting for studies of black hole entropy [20],[21]. Note that the polarization of the fourth set of branes (*i.e.*, whether they are branes or anti-branes) is determined by the polarizations of the first three sets.

The second special triple overlap can be found from the above case by deleting one of the membranes. This gives two fivebranes each overlapping a membrane in distinct strings. The deleted membrane can clearly be added back in without further breaking supersymmetry. The third special case involves three fivebranes. As noted above, three fivebranes can overlap either in a common 3-brane, a 2-brane or a string. In the 2-brane case, the product of the three projections gives another fivebrane projection, allowing for a solution with four fivebranes and two overall transverse directions, which preserves 1/8 of the supersymmetry. The four fivebranes overlap in a common 2-brane. All three special triple overlap solutions are related by T-duality after reduction to IIA D-branes.

Starting with an $n = 4$ solution, which preserves 1/16 of the supersymmetry, the special triple overlaps contained in it can be used to construct new solutions with $n > 4$ also preserving 1/16 of the supersymmetry. To illustrate this procedure consider starting with a solution with four fivebranes, in which there are three separate triple overlaps of dimension two. This solution can then be extended three times to give a configuration with a total of seven fivebranes and two overall transverse dimensions. We expect, although it would

[7] We assume, for the purposes of this discussion, that such a configuration will be a solution, if it satisfies the pairwise overlap conditions. We have checked this explicitly in an example of n=5 overlapping fivebranes preserving 1/16 of the supersymmetry, though not in general.

[8] Upon dimensional reduction, this solution can be related via T-duality to *e.g.*, a symmetric configuration of four overlapping 3-branes [20],[21].

be tedious to check, that this gives a new solution preserving 1/16 of the supersymmetry with fivebranes oriented as follows,

$$\begin{array}{llll} (1,2,3,4,5), & (3,4,5,6,7), & (1,2,3,6,7), & (1,3,4,6,8), \\ (2,3,4,7,8), & (1,3,5,7,8), & (2,3,5,6,8), & \end{array} \tag{25}$$

all overlapping in a common string. The polarizations of the last three sets of branes would be determined by those of the first four. As for the other cases with two overall transverse dimensions, this would reduce to give black hole type solutions in $D = 3$. The $2 \cap 2 \cap 5 \cap 5$ given in [20] is the only case of $n \geq 4$ overlapping branes with three overall transverse directions. Note that a membrane in the $(3, 9)$ plane can be added to (25) to give a solution with $n = 8$ preserving only 1/32 of the supersymmetry.

4. Reduction to D=4

The new D=11 solutions with fall off in at least three overall transverse directions may be dimensionally reduced to give black hole solutions in D=4. Membranes and fivebranes in D=11 give electrically and magnetically charged black holes in D=4, respectively. Branes aligned along different eleven-dimensional hyperplanes carry charge under different four-dimensional gauge fields. Write the D=11 metric as

$$\begin{aligned} d\tilde{s}^2 =& e^{-\Phi} g_{\mu\nu} dx^\mu dx^\nu + \sum_i e^{2\varphi_i} \left(dx^i\right)^2, \\ \Phi =& \sum_i \varphi_i, \qquad \mu, \nu = 0, \ldots, 3, \qquad i = 4, \ldots, 10, \end{aligned} \tag{26}$$

with all the fields independent of the x_i. Then $g_{\mu\nu}$ is the D=4 Einstein metric, which for the overlapping brane solutions with n=1,2,3 is given by

$$ds^2 = -\prod_{i=1}^{n} f_i^{-1/2} dt^2 + \prod_{i=1}^{n} f_i^{1/2} \left(dx_1^2 + dx_2^2 + dx_3^2\right). \tag{27}$$

In the intersecting cases in which the functions f_i are equal, the metric (27) is that of a single dilaton black hole [22] with dilaton coupling $a = \sqrt{(4/n) - 1}$, as noted in [3]. We briefly illustrate how this comes about below, by considering the dimensionally reduced action in two examples.

Reduce the gauge field by taking only components of the form $\tilde{F}_{ij\mu\nu}$ to be nonzero. This gives a collection of 21 two-form field strengths in D=4, which we label $F_{(ij)}$, with $i < j$. We further restrict to field configurations satisfying

$$F_{(ij)} \wedge F_{(kl)} = 0, \tag{28}$$

for all i, j, k, l in order for the cubic term in the D=11 action to vanish. The reduced action is then given by

$$S = \int \sqrt{-g} \left\{ R - \tfrac{1}{2} (\nabla\Phi)^2 - \sum_k (\nabla\varphi_k)^2 - \sum_{j<k} F^2_{(jk)} e^{\Phi - 2\varphi_j - 2\varphi_k} \right\}. \tag{29}$$

4.1. n=1

For n=1, assume that only one of the 21 D=4 field strengths, $F_{(45)}$, is nonvanishing and satisfies (28). The equations of motion can then be used to relate all seven scalar fields φ_i to a single scalar field ψ according to

$$\varphi_4 = \varphi_5 = \psi, \qquad \varphi_6 = \ldots = \varphi_{10} = \Phi = -\frac{\psi}{2}. \tag{30}$$

Substituting into (29) and rescaling $\psi = \frac{4}{3\sqrt{3}}\phi$ then gives the standard form of the dilaton gravity action

$$S = \int d^4x \sqrt{-g} \left\{ R - 2(\nabla\phi)^2 - e^{-2a\phi} F^2 \right\}, \tag{31}$$

with $a = \sqrt{3}$.

4.2. n=2

Consider now the n=2 membrane (fivebrane) solutions in which only two of the four dimensional field strengths, $F_{(45)}, F_{(67)}$, are nonzero. Since these are pure electric (magnetic), the condition (28) is satisfied. The equations of motion allow a truncation to two scalar fields ψ and η given by

$$\varphi_4 = \varphi_5 = \psi, \qquad \varphi_6 = \varphi_7 = \eta, \qquad \varphi_8 = \varphi_9 = \varphi_{10} = \Phi = -\psi - \eta. \tag{32}$$

Further taking the combinations $\lambda = \psi + \eta$ and $\Omega = \psi - \eta$, the reduced action (29) becomes

$$S = \int \sqrt{-g} \left\{ R - \frac{9}{2}(\nabla\psi)^2 - (\nabla\Omega)^2 - e^{-(3\lambda+2\Omega)} F^2_{(45)} - e^{-(3\lambda-2\Omega)} F^2_{(67)} \right\}. \tag{33}$$

In the intersecting case with $f_1 = f_2$, $F_{45} = F_{67}$ and we see that the source for Ω vanishes. Setting $\Omega = 0$ and rescaling $\lambda = \frac{2}{3}\phi$ then gives the standard dilaton gravity action with $a = 1$. Similar considerations show how the $a = \sqrt{(4/n) - 1}$ dilaton gravity action arises in the remaining n=2,3 intersecting cases in which the functions f_i are taken to be equal.

5. More Overlapping Branes

The overlapping brane solutions we have considered so far are all translationally invariant in the relative transverse directions. As we pointed out earlier, this suggests that these solutions be considered after dimensional reduction in the relative transverse directions. It is natural to ask if "true" overlapping brane solutions exist where this restriction is relaxed. In ten dimensions the answer turns out to be yes. Specifically, one can have two 5-branes that overlap in a string. In fact this type of solution was first constructed by Khuri in [10] (see also [23]) but the interpretation as overlapping branes was not discussed. After first considering new solutions generated from this solution by T-duality we will see that there exists a new solution of M-theory describing two fivebranes overlapping in a string.

By setting the gauge fields to zero in the heterotic solution in [10] one gets type IIA and type IIB solutions describing two NS-NS 5-branes overlapping in a string:

$$\begin{aligned} ds^2 =& - dt^2 + dx_1^2 + f_2(dx_2^2 + dx_3^2 + dx_4^2 + dx_5^2) + f_1(dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2) \\ H_{mnp} =& - \frac{c_1}{2}\epsilon_{mnpq}\partial_q f_1 \qquad H_{\mu\nu\lambda} = -\frac{c_2}{2}\epsilon_{\mu\nu\lambda\rho}\partial_\rho f_2, \qquad e^{2\phi} = f_1 f_2 \\ f_1 =& f_1(x_6, x_7, x_8, x_9), \qquad f_2 = f_2(x_2, x_3, x_4, x_5), \qquad \nabla^2 f_i = 0, \end{aligned} \tag{34}$$

where $x^\mu, \mu = 2, \ldots 5$ are the spatial coordinates of one 5-brane, $x^m, m = 6, \ldots 9$ are the spatial coordinates on the second brane and the epsilon tensors are those of the corresponding flat space. Note that the harmonic function of each brane has non-trivial falloff in the spatial directions of the other brane. As shown in [10] the solution preserves 1/4 of the supersymmetry.

Using $SL(2, Z)$ duality of type IIB theory we can generate a solution describing two D-5-branes overlapping in a string:

$$\begin{aligned} d\bar{s}^2 =& (f_1 f_2)^{-1/2}(-dt^2 + dx_1^2) + f_1^{-1/2} f_2^{1/2}(dx_2^2 + dx_3^2 + dx_4^2 + dx_5^2) \\ &+ f_1^{1/2} f_2^{-1/2}(dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2), \qquad e^{2\bar{\phi}} = f_1^{-1} f_2^{-1} \\ H'_{mnp} =& - \frac{c_1}{2}\epsilon_{mnpq}\partial_q f_1 \qquad H'_{\mu\nu\lambda} = -\frac{c_2}{2}\epsilon_{\mu\nu\lambda\rho}\partial_\rho f_2 \\ f_1 =& f_1(x_6, x_7, x_8, x_9), \qquad f_2 = f_2(x_2, x_3, x_4, x_5), \qquad \nabla^2 f_i = 0, \end{aligned} \tag{35}$$

where the H' is the R-R three form field strength. Acting now with T-duality we can generate a set of overlapping brane solutions of type II supergravity given by

$$\begin{aligned} IIA: &\quad 0 \cap 8(0); \quad 2 \cap 6(0); \quad 2 \cap 8(1); \quad 4 \cap 4(0); \quad 4 \cap 6(1); \\ IIB: &\quad 1 \cap 7(0); \quad 1 \cap 9(1); \quad 3 \cap 5(0); \quad 3 \cap 7(1); \quad 5 \cap 5(1); \end{aligned} \tag{36}$$

It is worth pointing out that this list corresponds at a string theory level to superpositions of branes with eight ND directions and thus, according to [16], should be supersymmetric. It is not hard to convince oneself that (36) provides a complete list of cases involving two branes with eight ND directions.

To obtain the new D=11 solution describing two fivebranes overlapping in a string, we can uplift the $4 \cap 4(0)$ solution of type IIA. Alternatively we can uplift the NS-NS overlapping 5-branes (35) directly. In either case we are led to the solution

$$\begin{aligned} d\tilde{s}^2 =& (f_1 f_2)^{-1/3}(-dt^2 + dx_1^2) + f_1^{-1/3} f_2^{2/3}(dx_2^2 + dx_3^2 + dx_4^2 + dx_5^2) \\ &+ f_1^{2/3} f_2^{-1/3}(dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2) + f_1^{2/3} f_2^{2/3} dx_{10}^2 \\ \tilde{F}_{mnp10} =& -\frac{c_1}{2}\epsilon_{mnpq}\partial_q f_1, \qquad \tilde{F}_{\mu\nu\lambda 10} = -\frac{c_2}{2}\epsilon_{\mu\nu\lambda\rho}\partial_\rho f_2. \end{aligned} \tag{37}$$

The solution preserves 1/4 of the D=11 supersymmetry. The Killing spinors are of the form $\epsilon = (f_1 f_2)^{-1/12}\eta$ with the constant spinor η satisfying

$$\begin{aligned} \hat{\Gamma}^6\hat{\Gamma}^7\hat{\Gamma}^8\hat{\Gamma}^9\hat{\Gamma}^{10} =& c_1\epsilon \\ \hat{\Gamma}^2\hat{\Gamma}^3\hat{\Gamma}^4\hat{\Gamma}^5\hat{\Gamma}^{10} =& c_2\epsilon. \end{aligned} \tag{38}$$

This solution describes two fivebranes in the $(1,2,3,4,5)$ and $(1,6,7,8,9)$ directions overlapping in a string in the (1) direction. Note that although the functions do depend on the relative transverse directions, they are translationally invariant in the overall transverse direction x_{10}. So unlike the D=10 solution (35) this overlapping solution is not a true overlap.

6. Conclusion

In this paper we have constructed a class of supersymmetric solutions of D=11 supergravity describing n=2,...,8 overlapping membranes and fivebranes. The pairwise overlaps of the branes in this class can either be: two fivebranes in a threebrane, a fivebrane and a membrane in a string or two membranes in a point. The solutions preserve at least 2^{-n} of the supersymmetry and generalize those presented in [3],[4]. For the cases $n = 2, 3$ the solutions preserve exactly 2^{-n} of the supersymmetry. We explicitly showed that the n=2 solutions and certain of the n=3 solutions reduce to collections of extremal dilaton black holes in D=4.

In section (5) we constructed an additional D=11 solution describing two fivebranes intersecting in a string that preserves 1/4 of the supersymmetry. This solution includes

non-trivial dependence on the relative transverse coordinates. It would be interesting to know if this feature can be further generalised in this case and to other configurations of membranes and fivebranes.

By reducing the D=11 solutions to D=10 and using T-duality we also showed how a large class of solutions of type IIA and IIB supergravity could be constructed. It would be interesting to further study these and additional solutions involving NS branes, which may be obtained by acting with $SL(2,Z)$. Finally, one may also try to generalize the overlapping solutions to include travelling waves along the branes as in [24],[25],[26].

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