Kyle owns more suits than the one he’s wearing, or an argument for a degree-theoretic analysis of gradability and comparison*

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1 Two theories about gradability and comparison

Broadly speaking, there are two types of analyses of gradability and comparison. The first type of analysis characterizes gradable predicates as type-theoretically distinct from non-gradable predicates, such that they relate objects to values in an ordered domain of degrees. The denotation of an arbitrary gradable predicate over individuals on this view looks something like (1) (e.g. Seuren 1973, Cresswell 1976, Bierwisch 1989, and many more).

\[ [[\gamma]] = \lambda d \lambda x. \text{the degree to which } x \text{ is } \gamma \text{ is at least as great as } d \]

The comparative, on this view, saturates the degree argument of the predicate and builds a property that is true of an object just in case the degree to which it is mapped exceeds the degree to which some other object — the standard — is mapped. The syntax and compositional semantics of standards is complex, but there is reason to believe that in many languages (English included), the standard can sometimes be individual-denoting. In such cases, the denotation of more can be stated as in (2) (see e.g. Hankamer 1973, Heim 1985, Kennedy 2007, Bhatt & Takahashi 2011; but see also Lechner 2001 for a different view).

\[ [[\text{more } \gamma]] = \lambda s \lambda t. \max \{ d | [[\gamma]](d)(t) \} \succ \max \{ d | [[\gamma]](d)(s) \} \]

Similar kinds of denotations can be stated for other kinds of degree constructions, with the result that this basic approach to gradable predicate meaning provides a fairly straightforward and extendable account of the semantics of the many complex constructions in which gradable adjectives appear. One analytical wrinkle for the account, however, is that it must stipulate some mechanism for saturating the degree argument in the case of the unmarked, positive form of a gradable predicate. The usual approach is to say that the degree argument is saturated by a variable or some other context-dependent expression whose job is to specify an appropriate threshold that an object must reach in order to count as satisfying the property in the context of utterance. This correctly captures the meaning of the positive form, 

*Thank you, Kyle, for your linguistic and sartorial inspiration.
though some authors have criticized this approach — and the degree-theoretic analysis of gradable adjectives more generally — on the grounds that no language (that we know of) requires overt degree morphology in the positive form (see Klein 1980, Francez & Koontz-Garboden 2015, Grano & Davis to appear). This is indeed somewhat surprising given the denotation in (1).

The second type of analysis starts from the observation that the positive form is unmarked, and assumes that the vague, context-dependent meaning that this form expresses reflects its basic lexical semantics (i.e., is not the result of saturation of a degree argument), and builds a semantics for comparison on top of that meaning. Generalizing quite a bit, in this kind of analysis, an arbitrary gradable predicate is assigned an extension relative to a parameter $\theta$, which serves to determine the threshold that distinguishes the things that satisfy the predicate from the things that don’t, as in (3).

$$\gamma^\theta = \lambda x. \text{the degree to which } x \text{ is } \gamma \text{ is at least as great as } \theta$$

Variants of this analysis differ mainly in what $\theta$ is taken to be: a designated coordinate of the index of evaluation (Lewis 1970, McConnell-Ginet 1973), a parameter that generalizes over alternate precisifications in a supervaluationist framework (Kamp 1975), a comparison class (Wheeler 1972, Klein 1980, van Rooij 2011, Burdett 2016), etc. But the general idea is that this parameter provides a basis for fixing the extension of the predicate in a way that ensures that those objects the predicate is true of have the relevant property to a degree that is greater than the objects that it is false of, which is what the metalanguage characterization of truth conditions in (3) is meant to reflect, and that this parameter can differ from context to context. And with this kind of meaning for the positive form in hand, the semantics of the comparative is straightforward: It denotes a relation between objects such that there is a way of fixing $\theta$ to make the positive form true of the first object and false of the second:

$$\text{[more } \gamma]^\theta = \{\langle t, s \rangle | \exists \theta' [t \in [\gamma]^\theta' \land s \not\in [\gamma]^\theta']$$

Note that I have purposely written the denotation in (3) to make it look as similar as possible to (1), because at the end of the day the truth conditions that the second, threshold parameter analysis derives for the positive form are identical to those derived on the first, degree relation analysis. But the two analyses are distinct in two crucial ways. First, there is a model-theoretic difference: Degrees provide the semantic values of expressions of the object language in the degree relation approach, but not in the threshold parameter analysis. In that sense, the latter is arguably simpler than the former. Second, there is a syntactic difference: Gradable predicates denote properties (type $\langle e, t \rangle$) in the threshold parameter analysis and relations (type $\langle d, et \rangle$) in the degree relation analysis. Here too, the threshold parame-
Kyle owns more suits than the one he’s wearing.

Inference analysis is simpler, and moreover more transparently reflects the cross-linguistic morphosyntactic properties of gradable predicates.

Given these considerations, a case can certainly be made on both theoretical and simplicity grounds that the threshold parameter analysis should be preferred over the degree relation analysis. Proponents of the degree relation analysis have, at various times, attempted to present empirical arguments in favor of degrees, but it is in fact exceedingly difficult to find points where the analyses make different empirical predictions, given that the core semantics provided by the threshold analysis provides a basis for imposing orderings that mirror the structure of degree scales. The goal of this squib is to provide a new empirical argument in favor of the degree relation analysis, though I will acknowledge at the outset that it may very well be an “argument of limited cleverness”: I will discuss a particular class of comparative constructions that have a straightforward analysis on the degree relation approach, but no obvious, non-ad hoc analysis on the threshold approach that I can see.

2 Subset comparatives

Grant (2013) and Aparicio (2013) observe that comparatives like (5a), to which Grant gives the name subset comparatives, differ from from regular amount comparatives like (5b) in their presuppositions.

(5) a. Kyle owns more suits than the one he’s wearing.
   b. Kyle owns more suits than me.

Both (5a) and (5b) entail that the cardinality of the set of suits that Kyle owns is greater than the cardinality of a second set, which is introduced by the standard phrase, and spelled out in (6a) and (6b) respectively.

(6) a. \{x \mid x \text{ is the suit that Kyle is wearing} \}
   b. \{x \mid x \text{ is a suit that I own} \}

But these two comparatives differ in their presuppositions: (5a) presupposes that the set introduced by the standard is a subset of the corresponding target set of objects picked out by the rest of the clause — that the suit that Kyle is wearing is a subset of the suits Kyle owns — which gives rise to the inference that Kyle owns the suit he is wearing, an inference that is preserved under negation and question formation:

(7) a. Kyle doesn’t own more suits than the one he’s wearing.
   b. Does Kyle own more suits than the one he’s wearing?

Thus (5a) presupposes that the set in (6a) is a subset of the set of suits Kyle owns, and asserts that it is a proper subset; hence the name “subset comparative.”
Christopher Kennedy

(5b) carries no such presupposition, neither in terms of objects, which would imply that Kyle and I share ownership of suits (if only!), nor in terms of quantities, which would imply that the set of numbers that represent pluralities of suits that I own is a subset of the set of numbers that represent pluralities of suits that Kyle owns. If this were the case, then (5b) would presuppose that Kyle and I have at least the same number of suits, and would assert that he has more. But this is not the case: Both (7a) and (7b) are compatible with me owning more suits than Kyle (contrary to actual fact, of course).

(8) a. Kyle doesn’t own more suits than me.
b. Does Kyle own more suits than me?

Grant (2013) writes the presupposition of subset comparatives into the denotation of a special comparative morpheme, but Aparicio (2013) shows that the presupposition follows automatically from the interaction of the “phrasal” semantics for the comparative morpheme illustrated above in (2) and the particular syntactic properties of subset comparatives, which involve configurations in which the target of comparison is the comparative-marked argument. To see how this works, let me first illustrate the analysis of (5b). Assume that the gradable predicate in this example is a cardinality predicate MANY that relates plural individuals to their cardinalities, which composes with the plural noun suits to derive the degree relation in (9).

(9) \[[\text{MANY suits}]] = \lambda d \lambda x. \#(x) \geq d \land \text{suits}(x)

This is not the meaning that we need to derive the correct truth conditions for (5b), however, given the semantics for comparatives in (2); instead we need a relation between degrees \(d\) and individuals \(y\) such that \(d\) is the number of suits that \(y\) owns. Such a relation can be derived by scoping the comparative morpheme to a position above existential closure of the internal argument of owns (which composes with the verb via something like Chung and Ladusaw’s (2004) Restrict operation), as shown in (10) (where \(i\) marks the scope of the comparative, as in Heim & Kratzer 1998).

(10) \[[ i [ \exists\text{-clo} [\text{owns} [t_i \text{MANY suits}]urator]]]] = \lambda d \lambda y. \exists x [\text{owns}(x)(y) \land \#(x) \geq d \land \text{suits}(x)]

This expression corresponds to \(\gamma\) in (2); composing it with the comparative morpheme, then the standard (than me), and finally the target (Kyle) derive the truth conditions in (11), which are just what we want: the maximal \(d\) such that \(d\) is a number of suits that Kyle owns exceeds the maximal \(d\) such that \(d\) is a number of suits that I own.
Kyle owns more suits than the one he’s wearing.

\[
(11) \quad \max \{ d \mid \exists x[\text{owns}(x)(\text{kbj}) \land \#(x) \geq d \land \text{suits}(x)] \} > \max \{ d \mid \exists x[\text{owns}(\text{ck})(x) \land \\
\#(x) \geq d \land \text{suits}(x)] \}
\]

In regular comparatives like (5b), the target and standard expressions saturate distinct argument positions from the comparative expression. What makes subset comparatives special, Aparicio points out, is that the standard and target expressions correspond to the same argument position as the comparative expression: in (5a), the direct object of \textit{owns}. Aparicio shows that the correct truth conditions and presuppositions for this example can be derived by scoping the comparative morpheme above the subject and holding off on existential closure of the internal argument position (the target) until after the standard has been saturated. This derives the following structure and interpretation for the argument of \textit{more}:

\[
(12) \quad [\![i \text{ Kyle owns } t_i \text{ MANY suits}]) = \lambda d \lambda x. \text{owns}(x)(\text{kbj}) \land \#(x) \geq d \land \text{suits}(x)
\]

This expression corresponds to \(\gamma\) in (2); composition with the comparative morpheme, followed by composition with the standard expression (\textit{than the one he’s wearing} = \text{skw}) and finally existential closure of the target argument (the individual argument introduced by \textit{MANY suits}) derives (13) as the denotation of the subset comparative.

\[
(13) \quad \exists x[\max \{ d \mid \text{owns}(x)(\text{kbj}) \land \#(x) \geq d \land \text{suits}(x) \} > \max \{ d \mid \text{owns}(\text{skw})(\text{kbj}) \land \\
\#(\text{skw}) \geq d \land \text{suits}(\text{skw}) \}]
\]

In prose: There is an \(x\) such that the maximal \(d\) such that \(d\) is a number of \(x\)s that Kyle owns and \(x\) is a suit is greater than the maximal \(d\) such that Kyle owns the suit he is wearing and the number of the suit he is wearing is \(d\). This will be true as long as Kyle owns more than one suit (since one is the number of the suit he is wearing), and crucially it will be \textit{undefined} if Kyle does not own the suit he is wearing, since in that case the standard set would be empty, and there would be nothing to maximize over. In this way, the presuppositions of subset comparatives are derived.

3 An argument for degrees

In my illustration of “regular” (non-subset) phrasal comparatives above, I used a degree relation semantics for gradable predicates and comparatives. However, I could have just as well used a threshold parameter analysis. First, assume a threshold-sensitive variant of \textit{MANY} which returns the following denotation for \textit{MANY suits}:

\[
(14) \quad [\![[\text{MANY suits}}])^\theta = \lambda x. \#(x) \geq \theta \land \text{suits}(x)
\]
Then assume that the grammar provides some mechanism for scoping a threshold-manipulating comparative morpheme (see Larson 1988). Scoping the comparative above existential closure of the internal argument, exactly as in the analysis described above, derives threshold-sensitive expression in (15).

\[
[[\exists\text{-clo}[\text{owns}[\text{MANY suits}]]]]^\theta = \lambda y. \exists x[\text{owns}(x)(y) \land \#(x) \geq \theta \land \text{suits}(x)]
\]

Taking this expression as \(\gamma\) in (4) and composing first with the comparative morpheme and then with the standard and target derives the truth conditions in (16), which also accurately capture the meaning of (5b): there’s a threshold \(\theta'\) such that there’s a group of suits that Kyle owns whose count is at least as great as \(\theta'\) and there’s no group of suits that I own whose count is at least as great as \(\theta'\).

\[
\exists \theta'[\exists x[\text{owns}(x)(kbj) \land \#(x) \geq \theta' \land \text{suits}(x)] \land \neg \exists x[\text{owns}(x)(ck) \land \#(x) \geq \theta' \land \text{suits}(x)]
\]

When we try to generalize this approach to subset comparatives, however, we fail to derive the same results as the degree theoretic analysis. Assuming as above that the comparative morpheme can take scope above the subject, and holding off on existential closure over the internal argument of \text{owns}, we derive (17) as our threshold-sensitive \(\gamma\)-term:

\[
[[\text{Kyle owns [MANY suits]]]}]^\theta = \lambda x. \text{owns}(x)(kbj) \land \#(x) \geq \theta \land \text{suits}(x)
\]

Composing this with \textit{more}, then the standard (\textit{the one he’s wearing} = \text{skw}), and then existentially closing the target argument, just as in Aparicio’s analysis, derives (18).

\[
\exists \theta'[\exists x[\text{owns}(x)(kbj) \land \#(x) \geq \theta' \land \text{suits}(skw)] \land \neg \exists x[\text{owns}(skw)(kbj) \land \#(skw) \geq \theta' \land \text{suits}(skw)]
\]

These truth conditions are satisfied whenever the number of suits Kyle owns is greater than the number one, which is the number of the suit he is wearing. However, they are also satisfied if he doesn’t own the suit he is wearing, so the meaning does not derive the presupposition of (5a). Such a presupposition can of course be stipulated, but it does not follow in any way from the architecture of the threshold parameter analysis, in the way that it does follow automatically from the degree relation analysis, given otherwise fully parallel assumptions about the compositional analysis of phrasal comparatives. To the extent that we want our analyses to derive facts like these with a minimum of \textit{ad hoc} stipulations, then, subset comparatives provide an argument for a degree-theoretic approach to the semantics of gradability and comparison.
Kyle owns more suits than the one he’s wearing

References


