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\textbf{ABSTRACT}

Due to restricted budgets of relief organizations, costs of hiring transportation service providers steer distribution decisions and limit the impact of disaster relief. To improve the success of future humanitarian operations, it is of paramount importance to understand this relationship in detail and to identify mitigation actions, always considering the interdependencies between multiple independent actors in humanitarian logistics. In this paper, we develop a game-theoretic model in order to investigate the influence of transportation costs on distribution decisions in long-term relief operations and to evaluate measures for improving the fulfillment of beneficiary needs. The equilibrium of the model is a Generalized Nash Equilibrium, which has had few applications in the supply chain context to date. We formulate it, utilizing the construct of a Variational Equilibrium, as a Variational Inequality and perform numerical simulations in order to study the effects of three interventions: an increase in carrier competition, a reduction of transportation costs and an extension of framework agreements. The results yield important implications for policy makers and humanitarian organizations (HOs). Increasing the number of preselected carriers strengthens the bargaining power of HOs and improves impact up to a certain limit. The limit is reached when carriers set framework rates equal to transportation unit costs. Reductions of transportation costs have a consistently positive, but decreasing marginal benefit without any upper bound. They provide the highest benefit when the bargaining power of HOs is weak. On the contrary, extending framework agreements enables most improvements when the bargaining power of HOs is strong.

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1. Introduction

Sudden-onset disasters such as the Indian Ocean tsunami, Hurricane Katrina or the Haiti earthquake have had catastrophic consequences. Less covered by the media and academics are slow-onset disasters (Beal, Fernández Barrera, & Mansouri, 2016; Leiras, de Brito Jr, Queiroz Peres, Rejane Bertazzo, & Tsugunobu Yoshiha Yoshizaki, 2014). These can actually cause even more harm to the affected population than sudden-onset ones, even though they allow for longer reaction times (Long & Wood, 1995). One striking example is the 800 million people worldwide suffering from chronic malnutrition due to drought and flooding (World Food Programme, 2017). During disaster relief operations, humanitarian organizations (HOs) alleviate the suffering of victims through the distribution of relief supplies. As the available funds are restricted, HOs have to decide on how to allocate limited supplies to different groups of beneficiaries in order to maximize their impact. Quite often, HOs do not succeed on this critical task and allocate products in a suboptimal manner (Benini, Conley, Dittemore, & Waksman, 2009; Nagurney & Nagurney, 2016; Wardell, 2009; Waters, 2001). Among the reasons for such misallocations are inaccurate need assessments, competition for media attention and to some extent conflicting donor interests. But also costs of transportation often compromise allocation decisions because freight is a major spend category in all relief operations and can heavily reduce the available budget. Given that HOs largely rely on

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external service providers for transportation, transportation costs are, in fact, driven by the freight rates agreed upon with carriers\(^1\). These are extremely high during many relief operations due to poor, and often further disaster compromised, transportation infrastructure, competition between HOs for limited transportation capacity and non-competitive service provider markets (Benini et al., 2009; Cottam, Roe, & Challacombe, 2004; Lall, Wang, & Munthali, 2009; Pedersen, 2001; Rancourt, Bellavance, & Goentzel, 2014; Rizet & Hine, 1993; Samii & Van Wassenhove, 2003; Teravaninthorn & Raballand, 2009). Hence, the negotiation of transportation rates and the selection of service providers are highly critical for the success of relief operations. Doing this for each load individually would, however, cause a tremendous operational effort. Therefore, HOs often set up framework agreements with carriers in advance of distribution, which fix transportation rates for all transportation orders during a specified period of time (up to three years) and are renewed at fixed intervals (Pazirandeh & Herlin, 2014; Rancourt et al., 2014). These are also favored by carriers because they supersede repeated requests for quotation and allow them to build close, long-term business relationships. It is due to the volatile character of disaster relief operations that HOs and carriers have to set up these agreements under very high uncertainty, for example, without knowing the future needs of beneficiaries or the budget available for the operation. In light of this complex environment, HOs and policy makers wonder how they can best intervene in order to mitigate the existing limitations and increase the impact of disaster relief. Predicting the effect of such interventions requires a detailed understanding of how framework agreements are negotiated in disaster relief and how they influence the selection of providers and the allocation of relief items.

Literature in this regard is very limited and does not provide sufficient insights. In general, there is a lack of quantitative models which take into account the huge interdependencies between many independent decision makers (Gutjahr & Nolz, 2016; Leiras et al., 2014; Muggy & L. Heier Stamm, 2014). Studies such as those of Rancourt et al. (2014), Teravaninthorn and Raballand (2009) and Lall et al. (2009) do shed light on the drivers of transportation rates, but are rather macroeconomic and cannot explain individual behavior. The same is true for the research on relief allocation by Benini et al. (2009). Papers by Paul and Wang (2015), Bagchi, Paul, and Maloni (2011) and Trestrail, Paul, and Maloni (2009) provide insights with regard to the selection of transportation service providers, but do not consider the interdependencies between different HOs and their implications for distribution decisions. Furthermore, the literature on framework agreements and option-based contracts for the procurement of relief items (Balcik & Ak, 2014; Iakovou, Vlachos, Keramidas, & Parits, 2014; Liang, Wang, & Gao, 2012; Wang, Feng Li, Liang Liang, Zhimin Huang, & Allan Ashley, 2015) can act as a reference, but results cannot be transferred to transportation services without adaptations because of the absence of storage and obsolescence costs in the service sector. The most promising starting points for developing an answer to the above questions are, therefore, a number of game-theoretic papers dealing with relief allocation (Nagurney, 2018; Nagurney & Nagurney, 2016) and service provider selection (Nagurney, 2016; Nagurney, Daniele, Alvarez Flores, & Caruso, 2018). However, these examine the two connected topics separately and cannot explain the existing interdependencies. Furthermore, they focus on short-term operations and do not take into account important aspects such as framework agreements, budget constraints, sustainability objectives and long-term business interests.

We address this gap in the literature by developing a game-theoretic model of the described setting and analyzing its equilibrium values under various conditions. Game theory is well-suited to study the divergent interests of multiple stakeholders in humanitarian logistics (Gutjahr & Nolz, 2016), and equilibrium models are excellent tools to analyze the outcome of interactions of multiple actors from the perspective of policy makers (Toyasaki, Daniele, & Wakolbinger, 2014). The proposed model consists of two sub-models. In the first sub-model, several HOs and carriers simultaneously negotiate framework agreements for transportation services. In the second sub-model, the same HOs simultaneously make distribution decisions, that is, they all decide at the same time on the volume of relief items to be purchased, on the distribution points to be supplied and on the carriers to be used for transportation. The link between both sub-models is unidirectional. On the one hand, HOs take into account the framework agreements negotiated in the first sub-model for their distribution decisions in the second sub-model. On the other hand, when negotiating framework agreements in the first sub-model, they have no information about key parameters of the second sub-model, for example the available budget or the actual needs of beneficiaries (incomplete information across sub-models). What is more, given the extremely high uncertainty of disaster relief and the potentially long temporal off-set between both sub-models, they are not even able to assign a reasonable probability distribution to the possible parameter values (no beliefs about future states of nature). This lack of information is reinforced by the fact that the decision makers at HOs change from the first to the second sub-model. While the more strategic decisions of Sub-model 1 are taken at headquarters or country offices, the more operational decisions of Sub-model 2 are taken by staff in the field.

For the described reasons (very high uncertainty, long temporal offset, change in decision makers) it is of minor importance and practicality for decision-makers of Sub-model 1 to anticipate the outcomes of Sub-model 2. Moreover, given the complexity of each actor’s behavior and the absence of reasonable ex-ante beliefs, analyzing the two sub-models as a multi-stage game would be technically hardly solvable. Therefore, we investigate each sub-model as a single-stage game. Within both of these single-stage games, all actors have complete information and behave strategically by aligning their decisions to the expected behavior of all other actors. This situation can usually be described by a classical Nash Equilibrium (NE) (Nash, 1951: 1950). Re-formulating a NE as variational inequality and calculating its values based on corresponding algorithms has developed into a common approach for supply chain network problems (cf. Nagurney, 1999 and the references therein). What makes our model mathematically and computationally challenging, however, is the behavior of HOs. These compete for limited transportation capacity and consequently share common constraints. Therefore, the particular part of the model is no longer simply a NE but rather a Generalized Nash Equilibrium (GNE) (see Fischer, Herrich, & Schönefeld, 2014 and von Heusinger, 2009). The applications of GNE models to supply chains are very recent and few in number (see Nagurney, 2018; Nagurney, Yu, & Besik, 2017 and Nagurney & Nagurney, 2016). For calculating the respective GNE of our model, we first appeal to the recently introduced concept of a Variational Equilibrium (cf. Facchinei, Fischer, & Piccialli, 2007 and Kulkarni & Shanbhag, 2012). This concept allows us to also formulate the HOs’ GNE problem as a variational inequality rather than a quasi-variational inequality. Algorithms for variational inequality problems are in a more advanced state than those for quasi-variational inequalities. Consequently, we are able to perform extensive numerical simulations regarding the potential of different interventions by policy makers and HOs with feasible computational effort.

\(^1\) When using the term “carrier” in this paper, we refer to logistics service providers offering transportation services.
In summary, we make the following contributions. First, we add depth to the existing studies of transportation markets in disaster relief by shedding light on the negotiation of framework agreements between multiple self-interested actors and the interrelation of these framework agreements with distribution decisions of competing organizations. This deepened understanding of individual decision-making is extremely helpful for predicting the effect of interventions by policy makers and HOs. A numerical comparison of the effect of multiple such interventions is another contribution of this paper. Furthermore, the presented model is the first in this context to incorporate budget constraints, sustainability objectives and long-term business interests, which are essential characteristics of long-term relief operations. Accordingly, we help to develop the under-researched field of slow-onset and long-term disasters. Finally, we address the lack of quantitative models for humanitarian logistics which take into account the interdependencies between many independent actors, and widen the rare applications of GNE to supply chain problems. The paper is structured as follows: In the next section, we present the game-theoretic model and the equilibrium conditions. In Section 3, we illustrate the model with a brief numerical example. Then, in Section 4, we perform simulations to assess the effects of different interventions by policy makers and HOs on the equilibrium values. Finally, we close the paper with a summary and an outlook in Section 5.

2. Game-theoretic model

The model that we construct in this section describes the distribution of relief supplies as part of long-term relief operations; for example, as in response to slow-onset disasters such as chronic famine in Africa. However, we do not intend to model a specific relief operation, but rather provide an abstraction of the decision processes within such operations in order to derive generic conclusions. As illustrated in Fig. 1, the model includes $H$ humanitarian organizations, with a typical humanitarian organization denoted by $h$, $L$ preselected carriers, with a typical carrier denoted by $l$, and $D$ distribution points, with a typical distribution point denoted by $d$. Furthermore, besides the $L$ specific carriers, we consider a general spot market for transportation services which is denoted by $L+1$.

HOs purchase relief items and contract carriers for the transportation of these products to different distribution points. From there, multiple individuals affected by a disaster are supplied. In order to contract carriers, HOs, on the one hand, set up framework agreements with preselected carriers in advance of distribution decisions. On the other hand, HOs contract carriers ad-hoc on the spot market if they cannot or do not want to make use of the framework agreements. While framework rates are the result of negotiations, spot market rates are externally given market prices and beyond the influence of single HOs.

We model this situation using two temporally offset sub-models. In the preceding Sub-model 1, $H$ HOs negotiate and sign framework agreements for transportation services with $L$ preselected carriers. They do this under vast uncertainty, not knowing the values or probability distributions of future spot market rates, beneficiary needs, carrier capacities or financial budgets. These parameters are only revealed in the subsequent Sub-model 2, in which the same $H$ HOs decide on the volume of relief items to be purchased, on the distribution points to be supplied and on the carriers to be used for transportation. For the latter, they consider the prevailing spot market rates and the framework agreements from Sub-model 1. While framework rates are fixed for all orders during the duration of the agreements, framework volumes are only projections and non-binding for any of the parties (Pazirandeh & Herlin, 2014; Rancourt et al., 2014). Nevertheless, HOs use the agreed volumes as upper bounds for carrier assignments in Sub-model 2 because, in the interest of sustainability, they want to limit the dependency of local companies on business with the humanitarian sector (for details see Section 2.1.1). Given the long duration of framework agreements, Sub-model 2 takes place repeatedly after the finalization of Sub-model 1, which is only rerun after the expiration of the agreements. We will now present both sub-models and their respective equilibrium conditions separately.

The variables, parameters, functions and weights are given in Tables 1–3 respectively. Recall that all actors behave strategically and, in the equilibrium of the sub-models, align their decisions to the equilibrium decisions of all other actors. We use the superscript * whenever referring to such equilibrium values.

---

**Table 1**: Variables.

<table>
<thead>
<tr>
<th>Sub-model 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{hd}^{l}$</td>
</tr>
<tr>
<td>$x_{l}^{d}$</td>
</tr>
<tr>
<td>$x^{l,d}$</td>
</tr>
<tr>
<td>$p_{hd}^{l}$</td>
</tr>
<tr>
<td>$p_{l}^{d}$</td>
</tr>
<tr>
<td>$p_{l+1}^{d}$</td>
</tr>
<tr>
<td>$\lambda_{h}^{d}$</td>
</tr>
<tr>
<td>$\lambda_{l}^{d}$</td>
</tr>
<tr>
<td>$\lambda_{l+1}^{d}$</td>
</tr>
</tbody>
</table>

**Sub-model 2**

| $y_{hd}^{l}$                                     |
| $y_{l}^{d}$                                      |
| $y^{l,d}$                                        |
| $\lambda_{h}^{d}$                               |
| $\lambda_{l}^{d}$                               |
| $\lambda_{l+1}^{d}$                             |

**Table 2**: Parameters.

<table>
<thead>
<tr>
<th>Sub-model 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_{hd}$</td>
</tr>
<tr>
<td>$C_{l}^{d}$</td>
</tr>
<tr>
<td>$p_{hd}^{l}$</td>
</tr>
<tr>
<td>$t_{l}^{d}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sub-model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{h}$</td>
</tr>
<tr>
<td>$K_{l}^{d}$</td>
</tr>
<tr>
<td>$C_{l}^{d}$</td>
</tr>
<tr>
<td>$n_{d}$</td>
</tr>
<tr>
<td>$p_{l+1}^{d}$</td>
</tr>
</tbody>
</table>

---
Table 3
Functions and weights.

<table>
<thead>
<tr>
<th>Sub-model 1</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_h(X_h, P^*_h))</td>
<td>Transportation costs of (h)</td>
</tr>
<tr>
<td>(R_h(X))</td>
<td>Dependency risk of (h)</td>
</tr>
<tr>
<td>(\omega_h^R)</td>
<td>Relative importance of risk compared to costs for (h)</td>
</tr>
<tr>
<td>(E_l(P, X^*))</td>
<td>Expected profit of (l)</td>
</tr>
<tr>
<td>(S_l(P))</td>
<td>Satisfaction with (l)</td>
</tr>
<tr>
<td>(\omega_l^S)</td>
<td>Relative importance of satisfaction compared to profit for (l)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sub-model 2</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I_h(Y))</td>
<td>Impact of (h)</td>
</tr>
<tr>
<td>(A_l(Y))</td>
<td>Activity signal of (h)</td>
</tr>
<tr>
<td>(\omega_h^A)</td>
<td>Relative importance of signaling compared to impact for (h)</td>
</tr>
</tbody>
</table>

2.1. Sub-model 1: Negotiation of framework agreements

In this sub-model, \(H\) HOs negotiate framework agreements with \(L\) preselected carriers. Similar to competitive bidding, carriers compete in terms of the rates offered for transportation to different distribution points. They set rates \(P_{bd}\) in order to maximize the weighted sum of expected profit and customer satisfaction, considering the transportation volumes projected by HOs. These, in turn, are interested in ensuring a certain projected transportation capacity by means of framework agreements and target to minimize the weighted sum of the related costs and risk. Anticipating the rates of carriers, they decide on the breakdown \(X_h\) of projected volumes across carriers. Both fixed rates \(P^*_bd\) and projected volumes \(X^*bd\) are then recorded as part of the framework agreements.

2.1.1. Behavior of HOs

Each HO \(h\), \(h = 1, \ldots, H\) knows that its financial budget in Sub-model 2 will be restricted, but it does not know at which level. Therefore, when setting up framework agreements, it seeks to ensure a predefined service level at minimal costs (Balci & Ak, 2014). In other words, it targets to enter into framework agreements which collectively secure the transportation capacity \(M_{bd}\) for each distribution point \(d, d = 1, \ldots, D\) while minimizing the costs of transportation \(C_h\). Being a cost minimizer, HO \(h\) will in general favor carriers which offer low transportation rates. However, the more relative volume it assigns to single carriers, the more it becomes dependent on these carriers and the higher is the impact if some of these carriers are not able or willing to provide the promised transportation capacity in Sub-model 2. This could happen when carriers decide to pursue more profitable business opportunities (Rancourt et al., 2014) or plead “force majeur” in the case of disaster-related disruptions (Egan, 2010), because their interests might not be aligned with the humanitarian objectives (Carland, Goentzel, & Montibeller, 2018). Therefore, HO \(h\) also strives to minimize its dependency risk \(R_h\) by distributing the projected volumes in an appropriate manner across multiple providers (Balci & Ak, 2014). This is reasonable behavior for organizations operating in environments with high service and supply risk (Meena, Sarmah, & Sarkar, 2011). In addition, HOs also want to avoid an immoderate dependency of carriers on business with humanitarian organizations, which would hinder a sustainable development of the local transportation market. Therefore, they make sure that no carrier \(l, l = 1, \ldots, L\) is assigned volumes exceeding an upper threshold \(G_l\). For example, the Red Cross does not procure more than 30% of a supplier’s total production volume (Rosenkranz, 2017) and the WFP assigns transportation volumes to carriers proportionally to their total transportation capacity (Rancourt et al., 2014). Then, each \(h, h = 1, \ldots, H\) faces the following optimization problem:

\[
\begin{align*}
\text{minimize} & \quad C_h(X_h, P^*_h) + \omega_h^R \cdot R_h(X) \\
\text{subject to} & \quad \sum_{l=1}^L X_{hd} \geq M_{bd}, \quad d = 1, \ldots, D \\
& \quad \sum_{d=1}^D X_{hd} \leq G_l, \quad l = 1, \ldots, L \\
& \quad X_{hd} \geq 0, \quad l = 1, \ldots, L; \quad d = 1, \ldots, D.
\end{align*}
\]

(1a)

Objective function (1a) minimizes the weighted sum of transportation costs and dependency risk. Constraint (1b) ensures that HO \(h\) assigns sufficient volume to cover the desired transportation volume for each distribution point. Constraint (1c) secures that the maximum volume is not exceeded for any carrier. Constraint (1d) guarantees the non-negativity of \(X_{hd}\).

We assume the objective function (1a) to be twice continuously differentiable and strictly convex. Furthermore, we define the feasible set \(\mathcal{X}_h^1\) for each HO \(h\) as

\[
\mathcal{X}_h^1 = \{X_h | (1b) \text{ and } (1d) \text{ hold}\}
\]

and we let \(\mathcal{S}^1 = \bigcap_{h=1}^H \mathcal{X}_h^1\). In addition, we define the feasible set \(\mathcal{S}^1\) consisting of the shared constraints as

\[
\mathcal{S}^1 = \{X | (1c) \text{ holds}\}
\]

Observe that not only does the disutility of each HO (1a) depend on the strategies of the other HOs, that is, the projected transportation volumes, but so does the feasible set because of the common constraints (1c). Hence, the above game-theoretic model, in the context of Sub-model 1, for adaptations to other disaster relief models see Nagurney, Alvarez Flores, and Soylu (2016) and Nagurney et al. (2018).

Definition 1 (Generalized Nash Equilibrium). A vector of all transportation volumes projected by HOs, \(X^* \in \mathcal{S}^1 \cap \mathcal{S}^1\), constitutes a Generalized Nash Equilibrium if for each HO \(h, h = 1, \ldots, H\):

\[
U_h(X_h, \bar{X}_h) \geq U_h(X_h, \bar{X}_h), \quad \forall X_h \in \mathcal{X}_h^1 \cap \mathcal{S}^1,
\]

(4)

where \(\bar{X}_h = (X^*_1, \ldots, X_{h-1}^*, X_{h+1}^*, \ldots, X_L^*)\) and \(U_h(X) = -[C_h(X_h, P^*_h) + \omega_h^R \cdot R_h(X)]\); \(h = 1, \ldots, H\).

Hence, a Generalized Nash Equilibrium is established if no HO can unilaterally improve upon its utility by changing its projection of transportation volumes in the network, given the transportation volume projections of the other HOs, and subject to the volume requirement constraints (1b), the shared/coupling constraints (1c) and the non-negativity constraints (1d). We remark that \(\mathcal{S}^1 = \mathcal{X}^1 = \bigcup_{h=1}^H \mathcal{X}_h^1\) and \(\mathcal{S}^1 = \mathcal{S}^1\) are convex sets.

If there are no coupling, that is, shared, constraints in the above model, then \(X_h^*\) defined in Definition 1 need only lie in the set \(\mathcal{X}^1\), and, under the assumption of convexity of the disutility functions and that they are continuously differentiable, we know that (cf. Gabay & Moulin, 1980 and Nagurney, 1999) the solution to what would then be a Nash equilibrium problem (see Nash, 1951; Nash, 1950) would coincide with the solution of the following variational inequality (VI) problem: determine \(X^* \in \mathcal{X}^1\), such that

\[
- \sum_{h=1}^H \sum_{d=1}^D \left(\nabla X_h U_h(X^*) \cdot X_h - X_h^*\right) \geq 0, \quad \forall X \in \mathcal{X}^1.
\]

(5)
where \((\cdot, \cdot)\) denotes the inner product in the corresponding Euclidean space and \(\nabla U_h(U_h(X))\) denotes the gradient of \(U_h(X)\) with respect to \(X_h\).

As emphasized in Nagurney et al. (2017), a refinement of the Generalized Nash Equilibrium is what is known as a variational equilibrium and it is a specific type of GNE (see Faccchinei et al., 2007 and Kulkarni & Shanbhag, 2012). Specifically, in a GNE defined by a variational equilibrium, the Lagrange multipliers associated with the shared/coupling constraints are all the same. This implies that all humanitarian organizations share a common perception of the sustainable upper demand limit of a service provider and behave similarly in order to respect it. Given that sustainability is a common objective of all HOSs, we consider this a reasonable assumption. More precisely, we have the following definition:

**Definition 2 (Variational Equilibrium).** A strategy vector \(X^*\) is said to be a variational equilibrium of the above Generalized Nash Equilibrium game if \(X^* \in \mathbb{R}^1 \cap S^1\) is a solution of the variational inequality

\[
\begin{align*}
H & \sum_{h=1}^{H} \{ \nabla U_h(U_h(X^*), X_h - X_h^*) \geq 0, \quad \forall X_h \in \mathbb{R}^1 \cap S^1. \tag{6}
\end{align*}
\]

By utilizing a variational equilibrium, we can take advantage of the well-developed theory of variational inequalities, including algorithms which are in a more advanced state of development and application than algorithms for quasi-variational inequality problems (cf. Nagurney, 1999 and the references therein).

We now expand the terms in variational inequality (6).

**Theorem 1** (VI Formulation of the GNE in Sub-model 1). Specifically, we have that (6) is equivalent to the variational inequality: determine \(X^* \in \mathbb{R}^1 \cap S^1\), such that

\[
\begin{align*}
& \sum_{h=1}^{H} \sum_{l=1}^{L} \sum_{d=1}^{D} \left[ \frac{\partial C_h(X_h^*, P_h^*)}{\partial X_h} + \alpha_h \cdot \frac{\partial R_h(X^*)}{\partial X_h} \right] \times \left[ X_{hd} - X_h^* \right] \geq 0, \\
& \forall X_h \in \mathbb{R}^1 \cap S^1. \tag{7}
\end{align*}
\]

Proof of the above follows through the use of the definition and the expansion of the gradient terms.

**Remark 1** (Existence and Uniqueness of Solution). A solution to (7) is guaranteed to exist from the classical theory of variational inequalities (cf. Kinderlehrer & Stampacchia, 1980 and Nagurney, 1999) since the function that enters the variational inequality is continuous and the feasible set is compact. Furthermore, since the function entering (7) is strictly monotone, the solution to the variational inequality (7) is unique.

**Remark 2** (Alternative Variational Inequality to (7)). We now utilize the Lagrange multipliers associated with the constraints as defined in Table 1. Then, an equivalent variational inequality to that of (7), which we will use to construct the variational inequality for the complete supply chain network (see, e.g., Nagurney, 2018), is the following one:

Find \((X^*, \lambda^M, \lambda^c, \lambda^c) \in \mathbb{R}^{H+H+D+D+L}\):

\[
\begin{align*}
& \sum_{h=1}^{H} \sum_{l=1}^{L} \sum_{d=1}^{D} \left[ \frac{\partial C_h(X_h^*, P_h^*)}{\partial X_h} + \alpha_h \cdot \frac{\partial R_h(X^*)}{\partial X_h} \right] + \lambda^M \cdot \left[ X_{hd} - X_h^* \right] - \lambda^c \cdot \left[ \sum_{h=1}^{H} \sum_{d=1}^{D} X_{hd} \right] \geq 0, \\
& \forall (X, \lambda^M, \lambda^c) \in \mathbb{R}^{H+H+D+D+L}. \tag{8}
\end{align*}
\]

2.1.2. Behavior of carriers

A carrier \(l; l = 1, \ldots, L\) aims to maximize the weighted sum of its expected profit \(E_l\) and customer satisfaction \(S_l\). Profit results from charging transportation rates \(p_{hid}\) greater than the unit transportation costs \(c_{lid}\) and will be realized once HOSs make use of the framework agreements in Sub-model 2. Therefore, carrier \(l\) estimates its profit \(E_l\) based on the projected transportation volumes \(X^*\) and the transportation rates \(P\). Besides optimizing the profit for the current framework agreement, carriers are also interested in extending the business beyond the current horizon. HOSs’ willingness to further use the carrier in the future depends on their customer satisfaction, which is determined by the HOSs’ perception of price fairness (Bolton & Lemon, 1999). High transportation rates might be acceptable in the short-term, but can lead to dissatisfaction and a search for alternative carriers in the long-term (Oum, Waters, & Yong, 1992). As a consequence, service providers use prices to secure the loyalty of customers (Cram, 1996). This could, for example, be observed during the Afghanistan crisis when local carriers decided to lower transportation rates to avoid the deployment of a UN transport fleet (Samii & Van Wassenhove, 2003). Therefore, we consider the customer satisfaction \(S_l\) with carrier \(l\) as part of carrier \(l\)’s objective function and assume that \(S_l\) depends on the agreed transportation rates\(^4\). Then, each carrier \(l; l = 1, \ldots, L\) faces the following optimization problem:

\[
\begin{align*}
& \text{maximize} \quad E_l(P, X^*) + \alpha^l \cdot S_l(P) \tag{9a} \\
& \text{subject to} \quad c_{lid} \leq p_{hid} \leq p_{hid}^*, \quad h = 1, \ldots, H, \quad d = 1, \ldots, D. \tag{9b}
\end{align*}
\]

Objective function (9a) maximizes the weighted sum of expected profit and customer satisfaction. Constraint (9b) guarantees that the rate \(p_{hid}\) carrier \(l\) charges \(h\) for transport to \(d\) will not be less than its transportation unit costs \(c_{lid}\) and that no humanitarian organization \(h\) will pay a rate beyond its reservation price \(p_{hid}^*\). The reservation price \(p_{hid}^*\) is defined as the maximum price organization \(h\) is willing to pay for transportation with carrier \(l\) to distribution point \(d\) (Kalish & Nelson, 1991).

We assume the objective function to be twice continuously differentiable and strictly concave. Furthermore, we define the feasible set \(\mathbb{K}^2\) for each carrier \(l\) as:

\[
\mathbb{K}^2 = \{ \mathbb{P}_{l} \mid (9b) \text{ holds} \}. \tag{10}
\]

and we let \(\mathbb{K} = \prod_{l=1}^{L} \mathbb{K}^2\). We remark that \(\mathbb{K}^2; l = 1, \ldots, L\) and \(\mathbb{K}\) are convex sets.

**Definition 3** (Nash Equilibrium). A price pattern \(P^* \in \mathbb{K}\) is a Nash Equilibrium if for each carrier \(l; l = 1, \ldots, L\):

\[
U^l(P^*, P^*_{-l}) \geq U^l(P_l, P^*_l), \quad \forall P_l \in \mathbb{K}^2, \tag{11}
\]

where \(U^l(P) = E_l(P, X^*) + \alpha^l \cdot S_l(P)\) and \(P^*_l \equiv (P_1^*, \ldots, P_{l-1}^*, P_l^*, P_{l+1}^*, \ldots, P_L^*)\).

Then, following Gabay and Moulin (1980) and Nagurney (1999), the below result is immediate under our assumptions:

**Theorem 2** (VI Formulation of NE in Sub-model 1). A price vector \(P^*\) is a Nash Equilibrium if and only if \(P^* \in \mathbb{K}\) is a solution of the variational inequality:

\[
\begin{align*}
& \sum_{l=1}^{L} \sum_{h=1}^{H} \sum_{d=1}^{D} \left[ \frac{\partial E_l(P, X^*)}{\partial p_{hid}} + \alpha^l \cdot \frac{\partial S_l(P^*)}{\partial p_{hid}} \right] \times [p_{hid} - p_{hid}^*] \geq 0, \\
& \forall P_l \in \mathbb{K}. \tag{12}
\end{align*}
\]

\(^4\) According to the service management literature, customer satisfaction is the result of perceived service quality relative to the price (Hallowell, 1996). We leave service quality out of our analysis (by assuming it to be homogeneous across carriers) and focus on the effects of prices.
Remark 3 [Existence and Uniqueness of Solution]. A solution $P^*$ of prices to the variational inequality problem (12) is guaranteed to exist since the function entering (12) is continuous under the imposed assumptions and the feasible set is compact. Furthermore, since the function entering (12) is strictly monotone, the solution to the variational inequality (12) is unique.

2.1.3. Supply chain network equilibrium

After characterizing the equilibrium conditions for HOs and carriers in Sub-model 1 separately, we now provide the equilibrium conditions for the complete, multi-tiered supply chain (SC) network. In order to formalize the agreement between the tiers, the transportation volume projections and the price patterns have to coincide and to satisfy the sum of the two variational inequalities (8) and (12). Such a consideration leads us to the following definition.

Definition 4 [SC Network Equilibrium]. The equilibrium state of the supply chain network consisting of HOs and carriers is one where the volume projections and the transportation rates coincide and they and the Lagrange multipliers satisfy the sum of inequalities (8) and (12).

The following theorem is, hence, immediate.

Theorem 3 [VI Formulation of SC Network Equilibrium in Sub-model 1]. A pattern of volume projections, transportation rates and Lagrange multipliers is a supply chain network equilibrium according to the above definition if and only if it satisfies the following variational inequality:

Find $(X^*, \lambda^M, \lambda^G, p^*) \in \mathbb{R}^{H_L + H_D + H_L \times \mathbb{R}^2}$,

\[
\sum_{h=1}^{H} \sum_{l=1}^{L} \sum_{d=1}^{D} \left[ \frac{\partial C_h(x_h, p_h)}{\partial x_{hld}} + \alpha_h \frac{\partial R_h(x^*)}{\partial x_{hld}} - \lambda^M_{hd} + \lambda^G_{hd} \right] 
\times [x_{hld} - x^*_{hld}] + \sum_{l=1}^{L} \sum_{d=1}^{D} \left[ -M_{hd} + \sum_{i=1}^{I} x_{i,d} \right] \times [\lambda^M_{hd} - \lambda^M_{hd}'] + \sum_{h=1}^{H} \sum_{l=1}^{L} \sum_{d=1}^{D} \left[ \frac{\partial E_l(p^*, X^*)}{\partial p_{hld}} + \alpha_l \frac{\partial S_l(p^*)}{\partial p_{hld}} \right] \times [p_{hld} - p^*_{hld}] \geq 0.
\]

\[
\forall (X^*, \lambda^M, \lambda^G, p^*) \in \mathbb{R}^{H_L + H_D + H_L \times \mathbb{R}^2}. \quad (13)
\]

We now put variational inequality (13), which is the unified variational inequality, into standard form (see Nagurney, 1999), that is: determine $z^* \in K \subset \mathbb{R}^N$, such that:

\[
\langle f(z^*), z - z^* \rangle \geq 0, \quad \forall z \in K, \quad (14)
\]

where $f$ is a continuous function from $K \to \mathbb{R}^N$, and $K$ a closed, convex set, with both the vectors $f(z)$ and $z$ being column vectors.

We define $z_1$ as the vector: $z_1 \equiv (X^*, \lambda^M, \lambda^G, p^*)$; the feasible set $K^1$ as $K^1 \equiv [H_L + H_D + H_L \times \mathbb{R}^2]$; and $f_1(z_1) \equiv (F_1(z_1), F_2(z_1), F_3(z_1), F_4(z_1))$, where component $hld$ of $F_1(z_1) = \frac{\partial C_h(x_h, p_h)}{\partial x_{hld}} + \alpha_h \frac{\partial R_h(x^*)}{\partial x_{hld}} - \lambda^M_{hd} + \lambda^G_{hd} \quad (h=1, \ldots, H, l=1, \ldots, L, d=1, \ldots, D)$; and component $hld$ of $F_2(z_1) = -M_{hd} + \sum_{i=1}^{I} x_{i,d}$

2.2. Sub-model 2: Distribution decisions

In this sub-model, humanitarian organizations decide on the volume of relief items to be purchased, on the distribution points to be supplied and on the carriers to be used for transportation\(^5\). They can either contract carriers ad hoc on the spot market or make use of the previously negotiated framework agreements. Different from Sub-model 1, they now have visibility on spot market rates, beneficiary needs, disposable financial budgets and available carrier capacities. Due to opportunistic decisions of carriers, the available capacity might, in fact, be different from what was assumed in Sub-model 1. In the interest of focus, however, we do not explicitly model these decisions of carriers in Sub-model 2. Instead, we assume the decision, how much of their available capacity they dedicate to business with humanitarian organizations, as externally given. In Sub-model 2, carriers are then considered “order-takers” who accept all transportation orders up to their offered capacity limit.

A humanitarian organization $h; h = 1, \ldots, H$ seeks to maximize the weighted sum of its impact $I_h$ and activity signal $A_h$. HOs have primarily altruistic motivations (Wardell, 2009) and target to produce impact by reducing the suffering and deprivation of people (Holguín-Veras, Pérez, Jaller, Van Wassenhove, & Aros-Vera, 2013). Using output indicators as proxies (Hoffmann, Roberts, Shoham, & Harvey, 2004), they estimate the achieved impact based on the needs of beneficiaries $n_h$ at distribution point $d$, their own volume of goods provided and the volume of goods provided by other HOs. However, HOs have also other organizational objectives (Benni et al., 2009). For example, they are continuously competing for donations (Altay & Pal, 2014). In this context, they use distribution volumes to signal their performance to donors as these base their donation decisions on the observed relief volumes (Wardell, 2009).

Consequently, by distributing products to beneficiaries, HO $h$ generates an activity signal $A_h$, which positively influences future donation amounts and therefore provides utility for $h$.

Obviously, HOs cannot decide on distribution volumes without limitations. Although costs are of lower concern in disaster relief (Graña, Goentzel, & Fine, 2014), especially when beneficiaries’ deprivation costs are very high (Holguín-Veras et al., 2013), HOs typically have to deal with limited and earmarked budgets $B_h$, which can strongly influence distribution decisions (Berkur, Besiou, & Wakolbinger, 2016; Gutjahr & Nolz, 2016). Accordingly, HOs take into account purchase prices $c^h_k$, negotiated framework rates $p^h_{hld}$ ($l = 1, \ldots, L$) and externally given spot market rates $p^*_{hld}$ ($l = 1, \ldots, L$) for the simplicity of notation) when deciding on purchasing and distribution volumes\(^6\). Similarly, transportation capacities of carriers are often limited in disaster relief (Benni et al., 2009). Hence, HOs also take into account the available capacity $K_d$ of carrier $l$ for transportation to distribution point $d$. Then, each HO $h; h = 1, \ldots, H$ faces the following optimization problem in Sub-model 2:

maximize $I_h(Y) + \alpha_h A_h(Y)$

subject to $\sum_{l=1}^{L} \sum_{d=1}^{D} (c^h_k + p^h_{hld}) \cdot y_{hld} \leq B_h$

$\sum_{l=1}^{L} \sum_{d=1}^{D} y_{hld} \leq K_d, \quad l = 1, \ldots, L + 1, \quad d = 1, \ldots, D$

\[
(15a)
\]

\[
(15b)
\]

\[
(15c)
\]

\(^5\) We assume that all purchased volumes are also transported to distribution points. Therefore, we only use the variable $y_{hld}$ as defined in Table 1 and highlight that volumes purchased by $h$ can be calculated as $\sum_{l=1}^{L} \sum_{d=1}^{D} y_{hld} = y_{hld}$.

\(^6\) Please note that also purchase prices might be regulated by framework agreements with product suppliers. However, due to our specific research interest, we do not model the negotiation of purchase prices, but consider them as externally given in our model.
\[0 \leq y_{hld} \leq x_{hld}^* \quad l = 1, \ldots, L \quad d = 1, \ldots, D \]  
(15d)

\[0 \leq y_{h(l+1)d} \quad d = 1, \ldots, D \]  
(15e)

The objective function (15a) maximizes the weighted sum of impact \(I_h\) and activity signal \(A_h\). Constraint (15b) ensures that HO \(h\) does not spend more than the available budget\(^7\). Constraint (15c) secures that no carrier transports more than its market transportation capacity \(K_{id}\). Moreover, constraint (15d) guarantees the non-negativity of \(y_{hld}\) and makes sure that HO \(h\) does not assign more volumes to a carrier than originally agreed in Sub-model 1. Finally, constraint (15e) ensures the non-negativity of \(y_{h(l+1)d}\).

We assume the objective function to be twice continuously differentiable and strictly concave. Furthermore, we define the feasible set \(\mathbb{R}_+^3\) for each HO \(h\) as:

\[\mathbb{X}_h^3 \equiv \{Y_h\} \quad (15d) \text{ and (15e) hold} \]  
(16)

and we let \(\mathbb{X}^3 \equiv \prod_{h=1}^H \mathbb{X}_h^3\). In addition, we define the feasible set \(S^2\) consisting of the shared constraints as

\[S^2 \equiv \{Y\} \quad (15c) \text{ holds}. \]  
(17)

We remark that both \(\mathbb{X}^3\) and \(S^2\) are convex sets. Applying the same logic as for Sub-model 1, we can derive the variational inequality formulation for the problem.

**Theorem 4** (VI Formulation of Sub-model 2). A strategy vector \(Y^*\) is said to be a variational equilibrium of the above Generalized Nash Game if \(Y^* \in \mathbb{X}^3 \cap S^2\) is a solution of the variational inequality:

\[\begin{align*}
\frac{1}{H} \sum_{h=1}^H \sum_{l=1}^L \sum_{d=1}^D & \left[ \frac{\partial H_h(Y^*)}{\partial y_{hld}} - \alpha_h^A \cdot \frac{\partial A_h(Y^*)}{\partial y_{hld}} + (c_h^p + p_{hld}) \cdot \lambda_{h} \right] \cdot [y_{hld} - y^*_{hld}] \geq 0, \\
\forall Y & \in \mathbb{X}^3 \cap S^2.
\end{align*} \]  
(18)

**Remark 4** (Existence and Uniqueness of Solution). A solution \(Y^*\) to the variational inequality problem (18) is guaranteed to exist since the function entering (18) is continuous and the feasible set is compact. Furthermore, since the function entering (18) is strictly monotone, the solution to the variational inequality (18) is unique.

**Remark 5** (Alternative Variational Inequality to (18)). Recall the Lagrange multipliers associated with the constraints as defined in Table 1. Then, an equivalent variational formulation of problem (15a) under constraints (15b)–(15e) is the following one:

\[\begin{align*}
\frac{1}{H} \sum_{h=1}^H \sum_{l=1}^L \sum_{d=1}^D & \left[ \frac{\partial H_h(Y^*)}{\partial y_{hld}} - \alpha_h^A \cdot \frac{\partial A_h(Y^*)}{\partial y_{hld}} + (c_h^p + p_{hld}) \cdot \lambda_{h} \right] \cdot [y_{hld} - y^*_{hld}] \\
&\times [y_{hld} - y^*_{hld}] + \sum_{h=1}^H \left( B_h - \sum_{l=1}^L \sum_{d=1}^D (c_h^p + p_{hld}) \cdot y_{hld} \right) \cdot [\lambda_{h}^B - \lambda_{h}^B^*] \geq 0, \\
\forall (Y, \lambda^B, \lambda^K) & \in \mathbb{X}^3 \cap S^2.
\end{align*} \]  
(19)

As we did for the variational inequality formulation for Sub-model 1, we now provide the standard form of variational inequality (19). In particular, if we let \(z_2\) now be such: \(z_2 = (Y, \lambda^B, \lambda^K)\), and we have the feasible set \(\mathbb{X}^2\) be defined as: \(\mathbb{X}^2 \equiv \mathbb{R}_+^H \cap (H+1)D+H+(L+1)D\)

with the function \(F_2(z_2)\) that enters the standard VI format (14) now defined as: \(F_2(z_2) = (F_2^1(z_2), F_2^2(z_2), F_2^3(z_2))\) with:

- component \(F_2^1(z_2) = -\frac{\partial H_h(Y^*)}{\partial y_{hld}} - \alpha_h^A \cdot \frac{\partial A_h(Y^*)}{\partial y_{hld}} + (c_h^p + p_{hld}) \cdot \lambda_{h}^B + \lambda_{h}^K \) \(h = 1, \ldots, H; l = 1, \ldots, L + 1, d = 1, \ldots, D; \) component \(h\) of \(F_2^2(z_2) = B_h - \sum_{l=1}^L \sum_{d=1}^D (c_h^p + p_{hld}) \cdot y_{hld} \) \(h = 1, \ldots, H; \) and component \(l\) of \(F_2^3(z_2) = K_{id} - \sum_{l=1}^L y_{hld} \) \(l = 1, \ldots, L + 1, d = 1, \ldots, D; \) then (19) can be put into standard form (14).

### 3. An illustrative example

In order to illustrate the mathematical model we now provide a brief example consisting of two HOs \(H = 2\), two carriers \((L = 2)\) and two distribution points \((D = 2)\). In Sub-model 1, each HO wants to sign framework agreements covering a volume of \(M_{hld} = 1500\) tons per distribution point and wants to limit the total volume signed per carrier to \(G_j = 3000\) tons. The unit cost of transportation \(c_{ij}^p\) is 0.300 €UR per ton for each carrier and distribution point, and the reservation price \(p_{hld}^*\) is 0.900 €UR per ton for each HO, carrier and distribution point. In Sub-model 2, each HO has a budget \(B_h = 5000\) €UR, and each carrier offers a transportation capacity \(K_{id} = 2500\) tons for each distribution point. Furthermore, each distribution point has needs \(N_j = 5000\) tons and transportation spot market rates \(p_{hld}^{SP}\) = 0.600 €UR per ton. These five parameters of Sub-model 2 are, however, unknown to HOs and carriers in Sub-model 1. For the functional forms used in this example and the values of the remaining parameters, please refer to Tables 5 and 6 in Appendix A.

We calculated the equilibrium solution of this illustrative example (and of all its variants in Section 4) by solving successively variational inequalities (13) and (19) with the iterative projection method of Solodov and Tseng (1996). For details on the solution algorithm, please refer to Appendix B. Given the symmetry of parameter values, the equilibrium solution is symmetric as well (see Table 4). In the equilibrium of Sub-model 1, the negotiated framework agreements include for all combinations of HOs, carriers and distribution points identical volumes \(x_{hld}^* = 0.750\) tons and identical rates \(p_{hld}^* = 0.506 €UR per ton. Then, in the equilibrium of Sub-model 2, each HO fully uses the volumes agreed upon in the framework agreements \(y_{hld}^* = 0.750\) tons and contracts further volumes of \(y_{hld}^* = 0.456\) tons per distribution point on the spot market. Thus, each HO ships to each distribution point a total volume of 15956 tons. However, given the needs \(N_j = 5000\) at each distribution point, the total volume of 3912 tons shipped to each distribution point only fulfills 78.2% of actual needs. In this case, carrier capacity is not the reason for the shortcoming \((\lambda^B_{id} = 0.000; i = 1, \ldots, L + 1, d = 1, \ldots, D)\). Instead, both organizations are restricted by their budgets \((\lambda^K_{id} = 0.309; h = 1, \ldots, H)\). This shows how tight budgets can limit the impact of disaster relief.

### 4. Effect of interventions

In this section, we quantitatively analyze three interventions which have been identified as opportunities for improving the impact of disaster relief: an increase in competition between carriers, a reduction of transportation costs and an extension of framework agreements. As limited competition is a root cause for high transportation rates, Lal et al. (2009) propose to develop and strengthen service provider markets. This can, for example, be achieved by increasing the number of preselected carriers \(L\). Furthermore, Tevrainthorn and Rabball (2009) find that reducing vehicle operating costs for fuel, tires, maintenance, labor and capital, that is, reducing unit transportation costs \(c_{ij}^p\) should yield considerably lower transportation rates. Finally, according to
practitioners are convinced of the positive effects of framework agreements on disaster relief and hope to see them more widely used. This can be simulated by increasing the volumes $M_{ld}$ which are covered by framework agreements.

To assess the effects of an intervention, we modify the particular parameter of the model and measure the change to the average percentage of fulfilled needs under equilibrium conditions. We first demonstrate the implications of each intervention for one specific scenario, the illustrative example from the previous section. Then, we analyze the robustness of the results by investigating the effects of each intervention for further 21 scenarios (see Fig. 8 in Appendix C). In the first robustness analysis, we vary the number of HOs in order to assess the influence of competition between HOs ($H = 1, \ldots, 4$). The second robustness analysis examines different levels of symmetry between HOs. In the symmetric scenario, all HOs have the same demand ($M_{ld}$) and budget ($B_{ld}$). In the asymmetric scenarios, one HO has 1.5 times (slight asymmetry) to 3.0 times (strong asymmetry) more demand and budget than the other HOs. The third robustness analysis deals with different numbers of preselected carriers and accordingly with different levels of carrier competition ($L = 1, \ldots, 5$). Transportation markets are often very heterogeneous and consist of both very big, professional service providers and small, owner-operated carriers (Teravaninthorn & Raballand, 2009). Therefore, the fourth robustness analysis investigates different levels of symmetry between carriers. While in the symmetric scenario all carriers have the same capacity ($G_{l}$ and $K_{ld}$) and cost structure ($c_{ld}^{t}$), in the asymmetric scenario one carrier has 1.5 (slight asymmetry) to 3.0 times (strong asymmetry) more capacity and 1.5 to 3.0 times lower transportation unit costs than the other carriers. Different types of carriers use different types of trucks and this is easily visible to HOs. Therefore, and different from studies such as Mahadevan, Hazra, and Jain (2017) or Nam, Chaudhury, and Rao (1995), we assume that HOs have complete information about these asymmetries. Finally, the fifth and sixth robustness analyses examine different levels of spot market rates $P_{l+1}$ and different types of relief items, simulated by different levels of purchase prices $c_{ld}^{F}$. Unless specified differently, we keep the total volume of framework agreements ($\sum_{h=1}^{H} \sum_{l=1}^{L} M_{ld}$), the total available budget ($\sum_{h=1}^{H} B_{ld}$), the total volume limit ($\sum_{l=1}^{L} G_{l}$) and the total transportation capacity ($\sum_{l=1}^{L} \sum_{d=1}^{D} K_{ld}$) constant for all robustness scenarios. This enables the comparability of results and allows to focus our analysis on the effects of competition, instead of mixing them with the consequences of increased budgets or extended capacities.

### 4.1. Increase in carrier competition

A limited number of service providers achieving disproportionately high profit margins is one key reason for high transportation costs (Lali et al., 2009). According to Lukassen and Wallenburg (2010), stronger competition between service providers decreases short-term profit maximization in favor of more long-term oriented objectives. Therefore, we analyze the effect of increasing the number of service providers, with which framework agreements are set up, by calculating the equilibrium values for $L = 1, \ldots, 5$.

#### 4.1.1. Illustrative example

As Fig. 2 shows, increasing the number of carriers also increases the average need fulfillment. If more carriers participate in the negotiations, the bargaining power of HOs is strengthened and lower transportation rates are agreed upon as part of the framework agreements. Consequently, the limited budget can be used for purchasing more relief items instead of paying service providers. If both HOs only negotiate with one carrier ($L = 1$), this carrier uses his monopoly-like position and sets framework rates equal to the reservation price of both HOs ($p_{ld}^{r} = 0.900$). As these in the end turn out to be higher than the spot market rates $p_{h(l+1)ld}^{r}$ (which is not foreseeable for neither the HOs nor the carrier), both HOs decide to only make use of the spot market. If both HOs negotiate with two carriers ($L = 2$), framework rates fall slightly below the spot market level ($p_{hld}^{r} = 0.506$) and, thanks to the use of framework agreements, more relief can be provided. Conducting negotiations with a third carrier then further strengthens the bargaining power of HOs. Consequently, framework rates are set close to the marginal transportation costs ($p_{l+1ld}^{r} = 0.337$ and $c_{l+1ld}^{r} = 0.300$) and a considerable improvement in need fulfillment is achieved. Given our assumption that carriers will never charge rates below their costs, increasing the number of carriers beyond three does not strongly improve the average need fulfillment anymore, because carriers have almost no room left for price reductions.

#### 4.1.2. Robustness analysis

Fig. 3 shows that the results of all scenarios are in principle comparable to the illustrative example. A variation of $H$ highlights the negative effect of competition among HOs and the influence of demand concentration on bargaining power. If only one HO is in demand of transportation, it can achieve framework rates equal to transportation unit costs already by involving a second provider into the negotiations. However, if three or four HOs require
transportation services, this price level can only be achieved by conducting negotiations with five providers. Investigating different levels of HO and carrier symmetry, we can draw two main conclusions. On the one hand, slight asymmetries in demands, budgets, capacities or cost structures do not imply any relevant changes compared to the symmetric example. On the other hand, strong asymmetries only imply a relevant difference when exactly two carriers are involved into the framework negotiations. In case of strongly asymmetric HOs, the bigger HO has a quasi-monopolistic demand share and can use its bargaining power to obtain prices close to marginal costs. In the case of strongly asymmetric carriers, the bigger carrier has major costs advantages which are passed on as price reductions to the HOs. These economies of scale even outweigh the shift of bargaining power caused by the asymmetries. We also assessed the implications of different levels of purchase prices and spot market rates. With regard to different levels of purchase prices, the results from the illustrative example are qualitatively robust. While the level of need fulfillment is consistently different, because cheaper products strain the restricted budget less, the qualitative course of the graphs remains the same. This is also true for different levels of spot markets rates whenever \( L \) exceeds two. The implications of involving a second carrier, however, differ in these scenarios. It allows to switch from spot market based transportation to framework based transportation in the case of medium or high spot market rates. In the case of low spot market rates, however, framework rates still exceed spot market rates and framework agreements are not used for transportation. In summary, we can state that increasing the number of preselected carriers strengthens the bargaining power of HOs and improves impact up to a certain limit. The limit is reached when carriers, driven by competition, set framework rates equal to transportation unit costs. In the case of two HOs, this is achieved when four carriers are involved into negotiations. Increasing competition beyond this level does not bring additional benefits. This result is robust for different levels of HO symmetry, carrier symmetry, purchase prices and spot market rates, but not for different numbers of HOs.

4.2. Reduction of transportation costs

High transportation unit costs, caused for example by bad roads, unskilled drivers or old trucks, are another important constraint for disaster relief (Teravaninthorn & Raballand, 2009). To simulate the effect of reducing transportation unit costs, for example, by investments in driver training or fleet modernization, we reduce the parameter \( c_{rij}^u \) symmetrically for all preselected carriers and distribution points by up to 75%. We assume that HOs have full transparency over this change and react to it with decreasing reservation prices. This level of transparency can be achieved by appropriate contract designs and remuneration forms (Lim, 2000).

4.2.1. Illustrative example

Fig. 4 shows that the reduction of transportation unit costs leads to an increase in need fulfillment. As costs decrease, carriers...
pass on parts of the savings to HOs in order to secure their customer satisfaction. According to the principle of dual entitlement (Bolton, Warlop, & Alba, 2003), HOs would perceive prices as unfair if carriers would not pass on at least parts of the savings. Interestingly, for cost reductions up to approximately 41%, the observed cuts in rates exceed the actual cost reductions. For example, a cost reduction of 8.5% (−0.026) leads to a cut in rates of 0.082. As the general cost level falls, carriers need to compensate the increasing price sensitivity of HOs by lowering rates disproportionately. These disproportionate adjustments occur at a decreasing rate, because also the profit margin of carriers shrinks steadily and price cuts strain carrier profits more and more. At a cost reduction of approximately 41%, HOs are price sensitive to such an extent that carriers set transportation rates equal to transportation unit costs. From this point on, all further price reductions lead to a linear improvement in need fulfillment, because carriers have no further room for disproportionate cuts in rates. Different to the previous intervention, however, no upper limit for the improvement in need fulfillment exists when reducing transportation costs.

The insights from this illustrative example support the claim of Lall et al. (2009). The authors suggest that policy makers should prioritize the improvement of low quality feeder roads over the enhancement of established road networks on international corridors. Given our finding with respect to the dependency of price sensitivity on the general cost level, we also expect basic corrections of low quality roads to yield higher improvements than further modernizations of acceptable infrastructures.

4.2.2. Robustness analysis

As it is illustrated in Fig. 5, in the case of \( H = 1 \) carriers set rates equal to transportation unit costs for all cost levels. Therefore, cost reductions always imply linear improvements in need fulfillment. For \( H = 3 \) and \( H = 4 \) the insights from the illustrative example apply with two adaptations. First, small cost reductions in sub-model 1 do not yield any benefit, because framework rates still turn out to be higher than spot market rates in sub-model 2. Second, the transition to a linear development of improvements occurs at higher levels of cost reduction, because HOs have less bargaining power and, in consequence, are less price sensitive. The same relationship holds in the opposite direction for \( L = 3, \ldots, 5 \). Due to the shift of bargaining power towards HOs, carriers are forced to price at marginal costs already for lower levels of cost reduction compared to the illustrative example. If two HOs negotiate framework agreements with only one carrier (\( L = 1 \)), the carrier sets rates equal to reservation prices of HOs for all scenarios with cost reductions below 55%. In this range, cost reductions have first no effect, because spot market rates still turn out to undercut framework rates in sub-model 2, and then a linear effect, because reservation prices fall proportionally to the general price level. Surprisingly, when cost reductions exceed 55%, the carrier stops to capitalize on his quasi-monopolistic position and sets rates below reservation prices instead. Due to the extremely high price sensitivity of HOs at this cost level, the carrier behaves comparably to the quasi-dupoly of carriers in the illustrative example and tolerates disproportionately high price cuts. As for the previous intervention, slight asymmetries in demands, budgets, capacities or cost structures do not imply any relevant changes compared to the symmetric example. In the case of strong asymmetries and cost reductions of up to approximately 30%, the improvement in need fulfillment is lower than in the illustrative example for reasons of bargaining power and economies of scale (see previous intervention). Comparing different levels of purchase prices and spot market rates, we identified two noteworthy differences to the illustrative example. First, if spot market rates turn out to be low in sub-model 2, small reductions of transportation unit costs in sub-model 1 do not yield any improvement for beneficiaries, because HOs will continue to only use the spot market for transportation. Second, in the case of low purchase prices and spot market rates, decreasing the transportation unit costs beyond a certain threshold yields an average need fulfillment greater than 100%. Making investments in transportation cost reductions under such circumstances does consequently not bring further benefits for beneficiaries, but instead might lead to waste by HOs who want to signal their performance to donors. In summary, we can state that transportation cost reductions have a positive, but decreasing marginal benefit with respect to need fulfillment for all types of relief items and without any upper limit. Under specific circumstances, which are dependent on the number of HOs, number of carriers and level of spot market rates, they, however, need to exceed a certain threshold before coming to full effect. This effect is highest, when the bargaining power of HOs is low.

4.3. Extension of framework agreements

Practitioners emphasize the positive influence of framework agreements on disaster relief (Rosenkranz, 2017). For example, they help to reduce administrative workload and to safeguard against price fluctuations. Therefore, we investigate the effect of extending the volume covered in framework agreements by increasing the parameter \( M_{hd} \) by up to 66%, symmetrically for all HOs and distribution points.

4.3.1. Illustrative example

According to Fig. 6, increasing the framework volumes also increases the average need fulfillment. By fixing transportation rates in advance, HOs do not have to rely on uncertain spot market rates. In this illustrative example, HOs and carriers agree on framework rates \( p_{hd} = 0.506 \) in sub-model 1 which in sub-model 2 turn out to be lower than the spot market rates \( p_{hd} = 0.600 \). Therefore, HOs can save budget by making use of the framework agreements, which in turn can be spent on purchasing more relief items. In the initial situation, framework volumes \( M_{hd} \) are limited to 1.500. Given their budget \( b_h = 5.000 \), HOs can afford to make complete use of the framework agreements \( (X_{hd} = 1.500) \) and additionally procure transportation services on the spot market \( (y_{hd} = 0.456) \). With framework volumes increasing, HOs procure less and less services on the spot market and improve the level of need fulfillment proportionally (linear development). For \( M_{hd} = 1.988 \), which constitutes a 33% increase, HOs spend their entire budget for transportation on framework based services \( (y_{hd} = 0.000) \). Further increases of \( M_{hd} \) do not yield additional improvements, because HOs lack budget to make use of the extended volumes.
4.3.2. Robustness analysis

As Fig. 7 shows, increasing $M_{hd}$ does not provide any benefits for $H = 3$, $H = 4$ and $L = 1$. In these cases, the bargaining power of HOs in Sub-model 1 is lower than in the illustrative example. This leads to framework rates which exceed spot market rates in Sub-model 2. Under these circumstances, HOs do not make use of the framework agreements and only procure services on the spot market. Therefore, the volume of framework agreements does not have any effect on the need fulfillment. For $H = 1$ and $L = 3, \ldots, 5$ the bargaining power of HOs is stronger than in the illustrative example. Accordingly, framework rates are smaller, the improvements from increasing $M_{hd}$ are comparatively higher and budget limitations only occur at higher levels of $M_{hd}$. The same is true for all asymmetric scenarios (demands, budgets, capacities or cost structures), where the effect is considerably higher for strong asymmetries than for slight asymmetries. In the case of asymmetric HOs, the bigger HO has quasi-monopolistic bargaining power, and in the case of asymmetric carriers, the bigger carrier can leverage major economies of scale. In both cases, framework rates are disproportionately lower than in the illustrative example and, therefore, increases of $M_{hd}$ lead to higher improvements. The three scenarios with different levels of purchase prices qualitatively confirm the insights from the illustrative scenario. Due to different expenses for the procurement of products, different levels of need fulfillment can be observed in all three scenarios, though. Finally, if spot market rates turn out to be low in Sub-model 2, framework agreements are not used and changes to $M_{hd}$ do not have any effect. If spot market rates are found to be high, the improvements from increasing $M_{hd}$ are higher than in the illustrative example. However, the threshold imposed by the limited budget remains exactly the same. In summary, we can state that increasing $M_{hd}$ yields linear improvements in need fulfillment whenever framework rates are below spot market rates and HOs have enough budget left to make use of these increases. The improvements are highest when the bargaining power of HOs is high, or when spot market rates are high. In these cases, fixing rates ahead of time
allows organizations to secure their impact despite potential price increases on the spot market.

5. Summary and outlook

In this paper, we developed a game-theoretic model to investigate the influence of transportation rates and framework agreements on distribution decisions in long-term relief operations and to evaluate measures for increasing the impact of humanitarian organizations. In order to do so, we have analyzed the equilibrium states of our model under different conditions, leveraging the concepts of (Generalized) Nash Equilibrium, Variational Equilibrium and Variational Inequalities.

We investigated three interventions to improve the fulfillment of beneficiary needs: an increase in carrier competition, a reduction of transportation costs and an extension of framework agreements. According to our results, all initiatives provide promising improvements. Increasing the number of preselected carriers, with which framework agreements are set up, strengthens the bargaining power of HOs and improves impact up to a certain limit. The limit is reached when carriers set framework rates equal to transportation unit costs. In the case of two HOs, this is achieved when four carriers are involved into negotiations. Increasing competition beyond this level does not bring additional benefits. This result is robust for different levels of HO symmetry, carrier symmetry, purchase prices and spot market rates, but not for different numbers of HOs. No such upper limit exists for reductions of transportation costs. These have a positive, but decreasing marginal benefit for all types of relief items. Under specific circumstances, which are dependent on the number of HOs, number of carriers and level of spot market rates, they, however, need to exceed a certain threshold before coming to full effect. This effect is highest, when the bargaining power of HOs is weak. The opposite relationship is true when extending the volumes of framework agreements. This measure provides the highest benefits when the bargaining power of HOs is strong, or when spot market rates are high. In these cases, fixing lower rates ahead of time then allows organizations to achieve the same impact as they would in the case of lower levels of spot market rates. In general, such extensions of framework volumes yield linear improvements in need fulfillment whenever framework rates are below spot market rates and HOs have enough budget left to make use of these volume increases. For all interventions we found that slight asymmetries of demands, budgets, capacities or cost structures do not cause results which are considerably different from the symmetric scenario. Strong asymmetries, however, can entail relevant differences and need to be considered by decision makers when assessing the improvement potential of interventions. Finally, all interventions can also lead to inefficiencies when humanitarian organizations do not use lower transportation rates to save money, but instead over-fulfill the needs of beneficiaries to signal their performance to donors.
The presented research has certain limitations and can be extended in several directions. Generally, we have focused on commercial providers of logistics services. In practice, humanitarian service providers, such as the UNHRD, have been gaining more and more importance (Vega & Roussat, 2016). An extension of our model to humanitarian service providers, who do not pursue profitability objectives, and a comparison with the commercial model, would add valuable insights to the related discussion. Similarly, our paper investigates situations in which HOs act in an uncoordinated way and reduce their bargaining power through competitive behavior. This is a commonly reported issue, which coordination bodies such as the Logistics Cluster would like to solve (Cottam et al., 2004). Our model could also be adjusted to simulate how such coordinators can best help to increase the impact of disaster relief and to understand which organizations should best cooperate to leverage maximum synergies (Crujissjen, Borm, Fleuren, & Hamers, 2010). Future research could, moreover, extend our analysis with respect to the uncertainty under which framework agreements are negotiated. On the one hand, simulations could examine the optimality of the decisions in Sub-model 1 for different levels of budgets, needs and carrier capacities in Sub-model 2. On the other hand, the model could be adjusted in such a way that decision-making in Sub-model 1 explicitly considers the uncertainty of Sub-model 2, for example the risk that a carrier will not deliver the ordered amount. Both approaches would help to gain a better understanding of the interdependencies between both sub-models. Likewise, our model in principle allows one to analyze the effect of conflicting HO priorities, asymmetric beneficiary needs and HO competition for media and donor attention. For reasons of complexity, we have left these aspects out of our analysis. However, it could be interesting for future research to simulate the implications of these issues for the optimal setup of framework agreements. This could also involve an analysis of the results on the level of single actors, as interventions might not have a positive effect for all HOs in the case of budget or demand asymmetries. Finally, our model assumed that carriers base their pricing decisions on profit considerations and the expected customer satisfaction with respect to reservation prices. In fact, the literature does not yet fully agree on the determinants which should be considered for the design of pricing models (Lukassen & Wallenburg, 2010). Studying the impact of different determinants and pricing models on transportation rates would therefore be highly interesting as well.

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Appendix A. Functional forms and parameter values

Tables 5 and 6 summarize the functional forms and parameter values used for the numerical simulations in this paper. While transportation costs $C_i(x_i, P_i)$, expected profit $E_i(P, X^i)$ and activity signal $A_i(Y)$ are linear functions, the dependency risk $R_{nl}(x)$ is a quadratic function. We provide a brief reasoning for the functional forms of satisfaction $S_i(P)$ and impact $I_h(x)$ below as these are less intuitive than the other functional forms and require some further background.

Satisfaction We estimate the satisfaction $S_{nld}(p_{nld})$ of HO $h$ related to the transport by carrier $l$ to distribution point $d$ based on the ratio of transportation rate $p_{nld}$ and HOs’ particular reservation price $p_{nld}^{s,t,h}$ (see the concept of consumer surplus as utility, e.g. Marshall, 1927). We assume the satisfaction with a price of zero to be 100% and the satisfaction with a price equal to the reservation price to be 0%. Furthermore, we assume the satisfaction to be monotonically decreasing at an increasing rate with respect to the transportation rate $p_{nld}$. We then calculate the satisfaction $S_i$ with carrier $l$ as the sum over the satisfaction of all HOs $h$ with the transportation of this carrier to all distribution points $d$, weighted by the particular transportation requirements $M_{nld}$. This implies that HOs’ satisfaction on high volume lanes is more important than on small volume lanes. It is a challenging task to estimate the reservation price $p_{nld}^{s,t,h}$ as it is influenced by a multitude of factors (see e.g. Kohli & Mahajan, 1991). For simplification, we focus on two main drivers: bargaining power and fairness perception. On the one hand, the reservation price decreases with an increase in bargaining power (McKibben, 2015) and bargaining power is positively correlated to the size of an organization and its relative demand share (Pazirandeh & Herlin, 2014). Accordingly, we estimate the relative bargaining power $b_{i}$ of HO $h$ as $b_{i} = \frac{\sum_{d=1}^{D} M_{nld} \sum_{l=1}^{L} 1_{l=1} Y_{nld}}{\sum_{l=1}^{L} M_{nld}}$. On the other hand, the reservation price is shaped by perceptions of price fairness. According to the principle of dual entitlement, HO $h$ considers a reasonable surcharge on top of the transportation costs $I_i$ as fair (Bolton et al., 2003). Therefore, we call $s_{h}$ the surcharge accepted by HO $h$ and assume $p_{nld}^{s,t,h} = (1 + s_{h}) \cdot c_{l}^{s,t,h}$ as the price perceived as fair by $h$ for transportation by $l$ to $d$, that is, we also assume that HO $h$ has full transparency over the unit transportation costs of carrier $l$. This could be achieved by appropriate contract designs and remuneration forms (Lim, 2000). We then calculate the reservation price $p_{nld}^{s,t,h}$ as $p_{nld}^{s,t,h} = (2 - b_{i}) \cdot p_{nld}^{s,t,h}$. Consequently, an organization with a relative demand share of 100% will have a reservation price equal to the fair price. An organization with a relative demand share of 0% will have a reservation price twice as high as the fair price.

Impact. Deprivation costs of disaster victims are monotonic, nonlinear and convex with respect to the deprivation time (Holguín-Veras et al., 2013). A key effect of deprivation time is the accumulation of deprivation volume. Therefore, we assume deprivation costs of individuals to also be convex with respect to the deprivation volume. Accordingly, the impact of relief supplies, which reduce the deprivation volume of individuals, is monotonic, nonlinear and concave with respect to the volume. A simplified estimation for the impact $l$ of delivering a volume of $Y_{l}$ to a distribution point $d$ with the total need $n_{d}$ is then given by $l(Y_{l}) = Y_{l} - \frac{n_{d}}{Y_{l}} - Y_{l}^{2}$. According to the humanitarian principle of equity, no distribution point should be systematically disadvantaged (Gutjahr & Nolz, 2016). However, the time without relief supplies
can make an important difference between distribution points (Holguín-Veras et al., 2013) and organizations need to balance egalitarian and utilitarian objectives (Tolfghi, Torabi, & Mansouri, 2016). To take into account the time-dependencies between periods we differentiate distribution points based on the impact of needs and weight the impact at each distribution point $d$ with an urgency factor $u_d$. Consequently, we can calculate the impact of a relief operation, in which $H$ HOs provide the shipment vector $Y$ via $L$ preselected carriers and the spot market $L + 1$ to $D$ distribution points, as follows:

$$I(Y) = \sum_{d=1}^{D} u_d \cdot \left( \sum_{h=1}^{H} \sum_{l=1}^{L+1} y_{hlid} \right) - \frac{1}{2 \cdot n_d} \left( \sum_{h=1}^{H} \sum_{l=1}^{L+1} \sum_{i=1}^{L} \left( y_{hid} \right)^2 \right).$$

The impact of a single HO $h$ is then the difference between the impact of the relief operation with HO $h$ and the impact of the relief operation without HO $h$. Let $Y_h$ be the vector of shipments by all organizations except $h$. The impact $I_h$ of one organization $h$ in the relief operation is consequently $I_h(Y) = I(Y) - I(Y_h)$ and can be calculated as follows:

$$I_h(Y) = \sum_{d=1}^{D} u_d \cdot \left( \sum_{l=1}^{L+1} y_{hlid} \right) - \frac{1}{2 \cdot n_d} \cdot \left( \sum_{l=1}^{L+1} \left( \sum_{i=1}^{L} y_{hid} \right)^2 \right).$$

### Table 6
Parameter values for numerical simulations (alphabetically).

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Indices</th>
<th>Example</th>
<th>Scenarios</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_h$</td>
<td>Budget of HO $h$</td>
<td>$(h = 1, \ldots , H)$</td>
<td>5.000</td>
<td>2.500, …, 7.500</td>
</tr>
<tr>
<td>$c_{ld}$</td>
<td>Unit cost of $l$ for transport to $d$</td>
<td>$(l = 1, \ldots , L)$ \ $D$ \ $(d = 1, \ldots , D)$</td>
<td>0.300</td>
<td>0.075, …, 0.300</td>
</tr>
<tr>
<td>$c_{ld}^h$</td>
<td>Purchase price for $h$</td>
<td>$(h = 1, \ldots , H)$</td>
<td>0.750</td>
<td>0.600, 0.750, 0.900</td>
</tr>
<tr>
<td>$D$</td>
<td>Number of distribution points</td>
<td></td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$G_l$</td>
<td>Volume limit for carrier $l$</td>
<td>$(l = 1, \ldots , L)$</td>
<td>3.000</td>
<td>4.500</td>
</tr>
<tr>
<td>$H$</td>
<td>Number of HOs</td>
<td></td>
<td>2</td>
<td>1, …, 4</td>
</tr>
<tr>
<td>$i_{ld}$</td>
<td>Relative importance of $d$ for $h$</td>
<td>$(h = 1, \ldots , H) \ $ $(d = 1, \ldots , D)$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$K_d$</td>
<td>Capacity of carrier $l$ for transport to $d$</td>
<td>$(l = 1, \ldots , L)$ \ $(d = 1, \ldots , D)$</td>
<td>2.500</td>
<td>3.750</td>
</tr>
<tr>
<td>$K_{d+1}$</td>
<td>Capacity of spot market for transport to $d$</td>
<td>$(d = 1, \ldots , D)$</td>
<td>$\infty$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$M_{ld}$</td>
<td>Target volume of $h$ for $d$</td>
<td>$(h = 1, \ldots , H) \ $ $(d = 1, \ldots , D)$</td>
<td>1.500</td>
<td>1.500, 2.498</td>
</tr>
<tr>
<td>$n_d$</td>
<td>Needs at $d$</td>
<td>$(d = 1, \ldots , D)$</td>
<td>5.000</td>
<td>5.000</td>
</tr>
<tr>
<td>$p_{d+1+l}$</td>
<td>Spot market rates for transport by $h$ to $d$</td>
<td>$(h = 1, \ldots , H) \ $ $(d = 1, \ldots , D)$</td>
<td>0.600</td>
<td>0.450, 0.600, 0.750</td>
</tr>
<tr>
<td>$r_{ld}$</td>
<td>Relative risk $h$ associates with $l$</td>
<td>$(h = 1, \ldots , H) \ $ $(l = 1, \ldots , L)$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$s_h$</td>
<td>Surcharge accepted by $h$</td>
<td>$(h = 1, \ldots , H)$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$u_h$</td>
<td>Relative urgency of $d$ for $h$</td>
<td>$(d = 1, \ldots , D)$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$\alpha_h$</td>
<td>Weight of signaling for $h$</td>
<td>$(h = 1, \ldots , H)$</td>
<td>0.200</td>
<td>0.200</td>
</tr>
<tr>
<td>$\omega_h$</td>
<td>Weight of risk for $h$</td>
<td>$(h = 1, \ldots , H)$</td>
<td>0.200</td>
<td>0.200</td>
</tr>
<tr>
<td>$\gamma_h$</td>
<td>Weight of satisfaction for $l$</td>
<td>$(l = 1, \ldots , L)$</td>
<td>0.400</td>
<td>0.400</td>
</tr>
</tbody>
</table>

### Appendix B. Solution algorithm

The Solodov and Tseng (1996) method is an iterative projection-contraction method, where the second projection is a more general operator. The solution vector $z^*_{m}$ of sub-model $m \in \{1, 2\}$ in iteration $\tau$ of the algorithm is the result of the second projection and calculated as:

$$z^*_{m} = z^{\tau-1}_{m} - \gamma M^{-1} (T_{\alpha_{m}} (z^{\tau-1}_{m}) - T_{\alpha_{m}} (Pr_{X} (z^{\tau-1}_{m}))),$$

with $\gamma \in \mathbb{R}^+$. The scaling matrix $M$ must be a symmetric positive matrix and is used to accelerate the convergence. Furthermore, $T_{\alpha_{m}} = I - \alpha_{m} F_{m}$, where $I$ is the identity function, $\alpha_{m}$ is chosen dynamically such that $T_{\alpha_{m}}$ is strongly monotone, and $F_{m}$ is the function entering the variational inequalities (12) and (19) when they are formulated in standard form. Finally, $z^*_{m}$ is the first projection in each iteration and is calculated as:

$$z^*_{m} = Pr_{X} (\varepsilon z^{\tau-1}_{m} - \alpha_{m} F_{m} (z^{\tau-1}_{m})).$$

The Solodov and Tseng (1996) algorithm has less restrictive conditions for convergence than many variational inequality algorithms, and requires only monotonicity of the function $F_{m}$, with the rate of convergence also established in Solodov (2003) for this and related algorithms. We implemented the algorithm in MATLAB R2016a setting $\alpha_{m} = 0.3 \cdot 10^{-7}$, $\gamma = 1.0$ and $M = I$. We solved, in total, 4626 different instances on an Intel(R) Core(TM) i5 CPU with 2.60 gigahertz and 8.00 gigabytes RAM. Let $d^*_{m} = z^*_{m} - z^{\tau-1}_{m}$ be the vector of differences between the solutions of two consecutive iterations for sub-model $m$. We stopped the algorithm when the Euclidean norm of $d^*_{m}$ fell below $\epsilon = 1 \cdot 10^{-5}$. Initializing all variables as zero, on average 390 iterations were required to solve Sub-model 1 and 433 iterations were required to solve Sub-model 2. On average, an iteration took 0.005 seconds.
Appendix C. Scenarios investigated with numerical solutions

<table>
<thead>
<tr>
<th>Interventions</th>
<th>Increase of carrier competition ($L = 1 \ldots 5$)</th>
<th>Reduction of transportation cost ($\ell^2 = 0.075 \ldots 0.300$)</th>
<th>Increase of framework volumes ($M_{\text{up}} = 1.600 \ldots 2.698$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Different numbers of HOs ($H = 1 \ldots 4$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Different numbers of carriers ($L = 1 \ldots 5$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Different levels of HO symmetry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Different levels of carrier symmetry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Different levels of spot market ratios ($\ell = 0.46 \ldots 0.60$)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 8. Scenarios investigated with numerical simulations.

References


