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Utilization And Examination Of A Mass Consistent Wind Flow Model

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UTILIZATION AND EXAMINATION
OF
A MASS CONSISTENT WIND FLOW MODEL

A Masters Project Presented
By
Joanne M. Carroll

Submitted to the Department of Mechanical Engineering in partial fulfillment of the requirements for the degree of

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UTILIZATION AND EXAMINATION
OF
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ABSTRACT

MATHEW, a mass consistent wind flow model is applied to given areas in Princeton and Windsor, Massachusetts for the purpose of determining wind flow fields in those areas and examining the MATHEW program itself. The MATHEW model which was originated by Sherman [1] at Lawrence Livermore Laboratories to give a three-component time-independent nondivergent wind velocity field. The model has been verified by its creators and is accepted as a valid method for calculating wind fields.

A description of the model is given. The analytic foundation of the model is elucidated, the numerical technique is described and the architecture of the MATHEW program is documented. All other pertinent programs required to implement MATHEW are recognized.

Input information required by the MATHEW program is divided into two groups, input (physical) data and input parameters (which define the grid structure used in MATHEW's numerical solution technique algorithm). The input parameters govern the way that the MATHEW model interprets input data. The relationship between input parameters and MATHEW's interpretation of input data is examined. The means by which MATHEW conditions input topographic data is demonstrated. MATHEW's input parameters are confined to lie within a given range. Geometric, data and storage space limitations are defined and investigated.

Five tests (which compare output runs) are performed. The output of a MATHEW run is an adjusted wind velocity field. Input and output isotach maps and topography maps of actual and conditioned contours are test results. The results are analysed visually and mathematically.
Observation of the computational progression of MATHEW's solution algorithm is performed.

Results from the mathematical analyses and algorithm inspection are presented. Conclusions which summarize all observations and visual analyses from Tests I thru V are presented. These conclusions, listed in Table 6, define the relationship between input parameters and MATHEW's interpretation of input data and determine MATHEW's geometric, data and storage space limitations. A recommended schedule procedure (Table 7) for input parameter determination is given. The suggested schedule procedure coupled with the Conclusion Summary Table provides guidelines for the potential MATHEW user, increases the facility of its use and enhances the viability of the model. All conclusions and observations are offered as an effective way to evaluate the merit of the model.
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CHAPTER I

INTRODUCTION

The computer algorithm entitled MATHEW may be used as an analytic tool to determine the optimum location for siting a wind turbine in a large geographic area. Given topographical information, a wind shear profile and instantaneous two-dimensional (horizontal) velocity values for specified positions located at the area in question as physical inputs to the MATHEW program, one may obtain an accurate description of the wind flow field. These adjusted wind flow fields are output at predetermined levels above the ground terrain. The wind prospector may use the output (the adjusted wind flow field) to gain insight as to where the greatest potential for electricity generation exists. Since cost considerations and site availability must be noted as well as high wind speeds, the wind flow field used in conjunction with geographic contour and road maps provide a useful tool for site evaluation.

To obtain a comprehensive understanding of MATHEW, a utilization and examination of the program is performed. Since not only input data (i.e. physical information) but also input parameters must be determined before MATHEW can be run the wind prospector, before using the MATHEW program, must have an understanding of how these input parameters affect the way MATHEW interprets input data.

In this paper, first by describing the model and second by performing test runs on MATHEW, an analysis of the mass consistent wind flow model is performed. Since a complete understanding of MATHEW is a
prerequisite for successfully running the MATHEW program, a model description is first presented. In Chapter II, a description of the model is given. An integral is defined that minimizes the difference between interpolated (original) and adjusted velocities subject to the constraint that flow is non-divergent. From the integral function, the method of least variances is used to derive an elliptic partial differential equation and is the analytic foundation of the model. The equation's formulation and the derivation of subsequent boundary conditions are elucidated. Finite difference approximation is used to solve the boundary value partial differential equation. Coefficients of the numerical differential equation are determined by surrounding terrain conditions. Determination of the coefficients is reviewed. An overrelaxation technique is used to minimize the number of iterations required to solve the differential equation in the algorithm of MATHEW. The technique is outlined. The structure of the MATHEW computer program is demonstrated by a flow chart with documentation for each subroutine function. Solution of not only the numerical differential equation but also of adjusted velocity values are dependent on the surrounding topography. Topographic information input into the MATHEW program is conditioned before it is accepted into the solution algorithm of MATHEW. The means by which topography is conditioned is also examined.

Theoretically, the MATHEW program is limited in the sense that it uses only the equation for conservation of mass as a solution constraint; it does not use other constraints such as the conservation of momentum and the conservation of energy. However, Lawrence Livermore Laboratories believes that quantitatively correct wind fields are produced as long as
realistic wind fields are entered as input to MATHEW. According to Sherman [2], MATHEW assumes that interpolated velocities (derived from instantaneous velocity values) are a fair representation of the actual mean wind field and only need to be minimally adjusted to significantly reduce the resulting divergence. This minimal adjustment assumption justifies the use of calculus of variations. The necessity and importance of using input information that is truly representative of actual physical conditions is then noted.

Input information required by MATHEW may be divided into two groups: (1) input data and (2) input parameters. The input data are physical observations measured at the area in question (i.e. wind velocity values, shear profile, etc.) and the input parameters are to be determined by the MATHEW user. A list of input information is given below.

1. **Input Data**
   1. Instantaneous two dimensional wind velocity vectors at specified locations.
   2. First temperature inversion layer height.
   3. Wind shear profile.
   4. Topographic information.

2. **Input Parameters**
   1. Size of the MATHEW area.
   2. Delta x, delta y and delta z.
   3. M, N and L.
   4. Number of data stations.

Before the solution algorithm of MATHEW can proceed, input data must be conditioned into acceptable form. The input parameters govern how
input data is massaged and thereby determine how the MATHEW program interprets input data. The area in question is first confined to lie in the MATHEW box, (show in Figure 1 on the following page) whose positive x and y directions represent east and north respectively, and whose z direction corresponds to an increase in elevation. The box is divided into grid blocks, which are used as elements of the finite difference approximation routine in MATHEW. Input parameters define the MATHEW box size, the grid block dimensions and the number of positions where velocity values are to be recorded. See section 2.2 for a more detailed discussion of the MATHEW box.

By looking at the flow chart (Figure 21, one may see how the theoretical limitations of a program necessitate establishing a relationship between input parameters and input data. This paper proposes that the way input parameters are defined does indirectly effect how "truly representative" the input information is since it defines how input data is interpreted by the MATHEW program.

Christine Sherman [3] has stated that the dimensions of the MATHEW box are determined by application requirements and computer storage limitations. To comprehend how input parameters may be defined to suit application requirements in an optimal manner, one must recognize that all input parameters are limited (or constrained) to lie within a given range. The areas where MATHEW's range limitations are considered may be categorized as follows: (1) geometric constraints, (2) data constraints and (3) computer storage constraints.

1. **Geometric Constraints**

1. How large a MATHEW area can be used effectively?
\[ M = \text{NUMBER OF GRID BLOCKS IN } x\text{ DIRECTION} \]
\[ N = \text{NUMBER OF GRID BLOCKS IN } y\text{ DIRECTION} \]
\[ L = \text{NUMBER OF GRID BLOCKS IN } z\text{ DIRECTION} \]

\[ (\Delta x)(M) = A \]
\[ (\Delta y)(N) = B \]
\[ (\Delta z)(L) = C \]

FIGURE 1. Mathew Box
FIGURE 2.

The Relationship Between Theoretical Limitations and Appropriate Input Parameters
2. How large may grid blocks be and still give a realistic representation?

3. How does the value of grid level height change the run?

4. Is it better to have a large M and N value or a large L?

2. Data Constraints

1. How few velocity data points may be used to predict a wind field?

2. How does the omission of one data point location alter the adjusted wind velocity field?

3. Storage Space Constraints

1. What is the maximum amount of storage space that the MATHEW program allows?

2. How does this constraint affect determination of parameter values?

By performing a series of tests, these application limitations are confronted. For each site, after input data has been acquired, the MATHEW program is run with various combinations of input parameters to determine an acceptable range for the given constraints and to learn how, by changing input parameters, input information accordingly changes and thereby affects a change in adjusted wind flow fields.

The testing procedure (Chapter III) consists of three parts. The first part, program implementation, is divided into two tasks, (1) data acquisition and (2) program implementation. The data acquired and the United States Geological Survey topography maps for both Princeton and Windsor, Mass. are given in Appendix C. All supplementary programs and all necessary edit changes required for program implementation are given in Appendix B.

Test classification is the second part of testing procedure. Five
individual tests are defined to investigate geometric, data and storage space limitations. The tests are categorized in section 3.2. The MATHEW program is run seventeen times (eight times for Windsor and nine times for Princeton). Input parameters for each run are determined in accordance with test function. MATHEW runs are titled categorically and with respect to location (i.e. WIN1, WIN2 PRINC1, PRINC2, etc.). The Input Parameter Summary Sheet for these runs may be found in section 4.1.

The third part, Examination, is discussed in section 3.3. The results (adjusted wind velocity fields) from each test run are analyzed visually and mathematically. The method of analysis for each test is a function of the test itself. The appropriate analysis procedure is described for each test category.

Application of the MATHEW program to Tests I thru V is given in Chapter IV. Each section of this chapter describes one test performed by defining test function, describing the applicable MATHEW runs, displaying both input information (topography and input isotach maps) and output adjusted wind flow fields, discussing all observations and presenting a condensed listing of conclusions.

In Chapter V, results from mathematical analyses and from observations of the solution algorithm of MATHEW are presented. From these results and a summary of all observations made in Tests I thru V, conclusions on the geometric, data and storage space limitations are made. The conclusions summarized in Table 6 define the relationship between input parameters and MATHEW's interpretation of input data. Finally, a recommended schedule procedure which may be used for all MATHEW users (Table 7) is given as an aid to the potential MATHEW user.
2.1 ANALYTIC FOUNDATION

2.1.1 Formulation. Sasaki [4] introduced a method of objective analysis based on the variational method which optimizes observed values under certain subsidiary conditions. This method allows for the adjusted values to satisfy an imposed strong constraint while changing the observed values by a minimal adjustment. For the MATHEW model, the strong constraint formalism coupled with provisions for no flow through internal (or closed) boundaries is used. Dickerson [5] believes that this formulation has been developed by the need to resolve boundary conditions. It will be discussed in more detail.

As in Sasaki [6], let $U_i^o$ represent observed velocity values at a point $(x,y,z)$ at an instant in time and $U_i$ the correspondingly objectively modified values where subscript $i$ refers to different elements. The sum of the squares of the difference at a point may be expressed as

$$\sum \alpha_i^2 (U_i - U_i^o)^2$$

(1)

where $\alpha_i$ are weighting factors. In order to obtain the modified values we require the integral $J$ of equation (1) over the volume considered, $V$, to be a minimum or the value of its variation to be zero. This may be expressed as
\[ \delta J = \delta \int \sum \alpha_i^2 (U_i - U_i^0)^2 \, dV = 0 \]  

(2)

When the above relationship is subject to the constraint that the three dimensional objectively modified (adjusted) wind field is nondivergent,

\[ J = \int \left( \alpha_1^2 (u - u^0)^2 + \alpha_1^2 (v - v^0)^2 + \alpha_2^2 (w - w^0)^2 \right. \]
\[ + \lambda \left( \frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z} \right) \, dx \, dy \, dz \]

(3)

where:

- \( \delta \) - variational operator
- \( J \) - functional of analysis field
- \( u, v, w \) - adjusted velocity components in the \( x, y \) and \( z \) directions
- \( u^0, v^0, w^0 \) - observed velocity components in the \( x, y \) and \( z \) directions
- \( \alpha_1, \alpha_2 \) - weighting factors
- \( \lambda \) - Lagrangian multiplier

Using the Einstein summation convention where repeated subscripts are summed, the tensor form may be written as

\[ J = \int \left( \alpha_1^2 \delta_{ij} (u_i - u_i^0) (u_j - u_j^0) + \lambda \delta_{ij} \frac{\delta u_i}{\delta x_j} \right) \, dV \]

(4)

where

- \( \delta_{ij} \) - kronecker delta
- \( dV \) - differential volume element

Before proceeding, as in Courant [7], the following relationships are established
1. \( F \) is twice continuously differentiable with respect to \( x_i, u_i \) and \( u_i' \).

2. The second derivative of \( u_i'' \) is also assumed continuous.

3. Let \( u_i = u_i(x_i) = f(x_i) \) be the desired extremal function yielding the minimum. Suppose that in a sufficiently small neighborhood \( g \) of the function \( f(x_i) \), the integral \( J[u_i] \) is smallest when \( u = f(x_i) \).

4. Let \( \eta(x_i) \) be defined over the volume, \( V \), possess a continuous second derivative and vanish at the end points but is otherwise arbitrary.

5. \( \overline{u}_i = u_i + \varepsilon \eta(x_i) = u_i + \delta u_i \). \( (6) \)

6. \( \delta y = \varepsilon \eta(x_i) \) is known as the variation function.

7. \( \varepsilon \) is a parameter with a small absolute value.

8. Let \( J(\overline{u}_i) = J(u_i + \varepsilon \eta_i) = I(0) \). \( (7) \)

9. \( J(\overline{u}_i) \) has a minimum at \( \varepsilon = 0 \).

10. The variation of \( J \) equals zero when \( J \) possesses a minimum at \( \varepsilon = 0, J'(\overline{u}_i) = 0 \) or \( I'(0) = 0 = \delta J \). \( (8) \)

Some useful terminology is now introduced. We called

\[ \varepsilon \eta_i = \delta (u_i) \]

In this way

\[ \varepsilon \eta_x = \delta (u) \]
\[ \varepsilon \eta_y = \delta (v) \]
\[ \varepsilon \eta_z = \delta (w) \]

Now to minimize the integral \( J \), we know that

\[
J(u_i) = \int_V F(x_i, u_i, u_i', \lambda) \, dx_i
\]
\[ I(e) = \int \left( \alpha^2 \delta_{ij} (\mathbf{\bar{u}}_i - \mathbf{\bar{u}}_j)^2 + (\mathbf{\bar{u}}_i - \mathbf{\bar{u}}_j) + \lambda \delta_{ij} \frac{\partial \mathbf{u}_i}{\partial x_j} \right) dV \]  

(9)

Let

\[ h(x_i, \mathbf{\bar{u}}_i, \mathbf{\bar{u}}'_i) = \alpha^2 \delta_{ij} (\mathbf{\bar{u}}_i - \mathbf{\bar{u}}_j)^2 \]  

(10)

Then

\[ I'(e) = \int \delta_{ij} \left( \frac{\partial h}{\partial \mathbf{u}_i} \frac{\partial \mathbf{u}_i}{\partial e} + \frac{\partial h}{\partial \mathbf{u}_j} \frac{\partial \mathbf{u}_j}{\partial e} \right) dV \]  

(11)

\[ I'(e) = \int \delta_{ij} \left( \frac{\partial h}{\partial \mathbf{u}_i} \eta_i + \frac{\partial h}{\partial \mathbf{u}_j} \eta'_i \right) dV \]  

(12)

Substituting

\[ I'(0) = \int \delta_{ij} \left( \frac{\partial h}{\partial \mathbf{u}_i} \eta_i + \frac{\partial h}{\partial \mathbf{u}_j} \eta'_i \right) dV \]  

(13)

Using the chain rule, one may note

\[ \delta_{ij} \delta_{ik} \frac{d}{d\mathbf{x}_k} \left( \frac{\partial h}{\partial \mathbf{u}_i} \eta_i \right) = \delta_{ij} \left( \delta_{ik} \frac{d}{d\mathbf{x}_k} \frac{\partial h}{\partial \mathbf{u}_i} \eta_i + \frac{\partial h}{\partial \mathbf{u}_i} \frac{\partial \mathbf{u}_i}{\partial x_k} \eta'_i \right) \]  

(14)

Substituting,

\[ \delta J = \int \delta_{ij} \left( \frac{\partial h}{\partial \mathbf{u}_i} \eta_i - \delta_{ik} \frac{d}{d\mathbf{x}_k} \left( \frac{\partial h}{\partial \mathbf{u}_i} \eta_i \right) + \delta_{ik} \frac{d}{d\mathbf{x}_k} \left( \frac{\partial h}{\partial \mathbf{u}_i} \eta'_i \right) \right) dV \]  

(15)

Since

\[ = \int \delta_{ij} \left( \frac{\partial h}{\partial \mathbf{u}_i} \eta_i - \delta_{ik} \frac{d}{d\mathbf{x}_k} \left( \frac{\partial h}{\partial \mathbf{u}_i} \eta_i \right) \right) dV + \delta_{ij} \frac{\partial h}{\partial \mathbf{u}_i} \eta_i \bigg|_V \]  

(16)

and

\[ = \int \delta_{ij} \left( \frac{\partial h}{\partial \mathbf{u}_i} - \delta_{ik} \frac{d}{d\mathbf{x}_k} \left( \frac{\partial h}{\partial \mathbf{u}_i} \right) \right) \eta_i dV + \delta_{ij} \frac{\partial h}{\partial \mathbf{u}_i} \eta_i \bigg|_V \]  

(17)
Since
\[ \epsilon \frac{\partial h}{\partial u_i} \eta_j = \epsilon \frac{\partial h}{\partial v} \eta_j \hat{r} + \epsilon \frac{\partial h}{\partial w} \eta_j \hat{r} \]
and
\[ \epsilon \eta_i = \delta(i) \]
then
\[ \delta_{ij} \epsilon \frac{\partial h}{\partial u_i} \eta_j \left|_{v} \right. = \delta(u) \lambda n_x \left|_{x} \right. + \delta(v) \lambda n_y \left|_{y} \right. + \delta(w) \lambda n_z \left|_{z} \right. \] (18)
and we have
\[ 0 = \delta J = \epsilon \int \delta_{ij} \left( \frac{\partial h}{\partial u_i} - \delta_{jk} \frac{\partial}{\partial x_k} \left( \frac{\partial h}{\partial u_i} \right) \right) \eta_j \partial V \]
\[ + \delta(u) \lambda n_x \left|_{x} \right. + \delta(v) \lambda n_y \left|_{y} \right. + \delta(w) \lambda n_z \left|_{z} \right. \] (19)

Since \( \eta_j \) is arbitrary, for the integral to vanish one must set the terms inside the brackets equal to zero. One obtains the Euler–Lagrange equation
\[ 0 = \delta_{ij} \left( \frac{\partial h}{\partial u_i} - \delta_{jk} \frac{\partial}{\partial x_k} \left( \frac{\partial h}{\partial u_i} \right) \right) \] (20)
where
\[ \frac{\partial h}{\partial u_i} = 2 \alpha^2 (u_i - u^*_i) \]
\[ \frac{\partial}{\partial x_i} \left( \frac{\partial h}{\partial u_i} \right) = \frac{\partial}{\partial x_i} (\lambda) = \frac{\partial \lambda}{\partial x_i} \]

In this way
\[ u = u^* + \frac{1}{2 \alpha^2} \frac{\partial \lambda}{\partial x} \] (21)
\[ v = v^* + \frac{1}{2 \alpha^2} \frac{\partial \lambda}{\partial y} \] (22)
2.1.2 Boundary Conditions. Above, it was shown by using Courant [8] that for \( J \) to be stationary, the first variation must vanish and that the Euler-Lagrange equation must be satisfied for all interior points. One must also consider that the boundary conditions

\[
\delta(u) \lambda n_x = 0 \\
\delta(v) \lambda n_y = 0 \\
\delta(w) \lambda n_z = 0
\]

exist. As noted by Sasaki [9], the MATHEW program uses the strong constraint formalism (i.e. \( \varepsilon = 0 \)) coupled with provisions for no flow though internal boundaries. This allows for a channeling effect and for a diversion flow around complex topographic features.

On the boundary, either the normal variation of velocity equals zero \( (n_x \delta(u) = 0) \) or \( \lambda \) equals zero. Sherman [10] states that specifying both conditions overdetermines the problem. In this way, there are essentially two cases to consider. When \( \delta(u) = 0 \), the variation of the function at the boundary is set equal to zero. The function has a prescribed value at the boundary. The strong constraint formalism uses the set of prescribed closed boundary conditions \( \delta(u) = 0 \). This implies that the variation of \( u-u^* \) equals zero on a boundary (i.e. \( u=u^* \)). From equations (21)-(23) one may see that then \( \frac{\partial \lambda}{\partial n} \) must equal zero also.

If the adjusted wind velocity has the same value as the observed velocity, the boundary at that point is called "no flow through". In this way, for closed boundaries, the boundary condition \( \frac{\partial \lambda}{\partial x} = 0 \) is prescribed by assigning \( \delta(u) = 0 \).
Lateral boundary conditions are satisfied by $\lambda = 0$ around the outside of the MATHEW box; on the interior boundaries where topography is blocking the flow, "no flow through" conditions exist.

On the outside of the MATHEW box, there do exist open boundaries where no apriori conditions may be assumed. Since $\delta(u)$ is arbitrary at the boundary in these cases, one obtains as a necessary boundary condition the natural boundary condition $\lambda = 0$. When $\lambda = 0$ on a boundary, $\frac{\partial \lambda}{\partial n}$ in general does not equal zero. This means that the observed velocities are adjusted for the open "flow through" case. Sherman [11] then determines that the normal velocity variation does not equal zero here. The flow through boundary allows in-flow and out-flow flux adjustments to take place.

Substituting the Euler-Lagrange equation into the non-diversion equation, one obtains

\[
\frac{\partial^2 \lambda}{\partial x^2} + \frac{\partial^2 \lambda}{\partial y^2} + \frac{\partial^2 \lambda}{\partial z^2} (\frac{\partial \lambda}{\partial z})^2 = -2 \alpha_1^2 (\frac{\partial v^0}{\partial x} + \frac{\partial v^0}{\partial y} + \frac{\partial w^0}{\partial z})
\]  
(25)

Letting

\[
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}
\]

then

\[
\nabla^2 \lambda = -2 \alpha_1^2 (\frac{\partial v^0}{\partial x} + \frac{\partial v^0}{\partial y} + \frac{\partial w^0}{\partial z})
\]  
(26)

The terms on the right-hand side of the above equation may be evaluated directly from observed data using standard techniques. Sasaki [12] concludes that if this sum is equal to zero, the adjusted quantities
are equal to the observed quantities. When the sum of these terms is not zero, the solution of the above equation gives the deviation of the Lagrangian multiplier, $\lambda$. The adjusted velocity values may be then obtained from the Euler-Lagrange equations.
2.2 NUMERICAL TECHNIQUE

Finite difference approximations are used to solve the elliptic partial differential equation given in equation 25. The MATHEW box encloses the area in question. The Universal Transverse Mercator coordinate system is used to define the box. The positive x and y directions correspond to east and north respectively and the positive z direction defines an increase in elevation. A numerical transformation of the equations derived from objective analysis is performed to solve for the Lagrangian multiplier, \( \lambda \), which is used to determine the adjusted velocity field. Coefficients of the differential equation as well as boundary conditions, are specific to grid block location since they are functions of surrounding terrain conditions. The iteration routine used to solve the numerical differential equation applies an overrelaxation technique to speed up convergence. The weights, \( \alpha_1 \) and \( \alpha_2 \) are determined with respect to grid block geometry; they are defined.

Figure 1 in Chapter I gives an illustration of the MATHEW box. The MATHEW box sets as its lowest point (i.e. \( z = 0 \)) equal to minimum terrain level. Lengths A and B define the east-west and north-south lengths of the area in question. The height of the rectangular box corresponds (approximately) to the height of the first temperature inversion layer. It is noted here that although the value C might be considered as input data, since it (the first temperature inversion layer height) does reflect physical conditions, for all cases studied in this report the inversion layer height is assumed equal to between 550 and 600 m above terrain height. M, N and L are defined as the number of grid points in
the x, y and z directions respectively.

\( \Delta x \), \( \Delta y \) and \( \Delta z \) are the respective distances between grid points in the x, y and z directions. One grid block has the volume \( \Delta x \times \Delta y \times \Delta z \) and all grid blocks are of equal dimensions. Exterior grid blocks are those located on \( x = 1 \), \( x = M \), \( y = 1 \), \( y = N \), \( z = L \) and \( z \) = topography height level. All interior grid blocks are those enclosed by exterior grid block boundaries. Observed velocity values in the x and y direction (u and v velocity components) are assigned to each grid block location. These values are obtained by interpolating observed instantaneous velocity values measured at specified locations and by using a shear coefficient (also measured) to determine the relationship between velocity and height. Since measurements of the vertical velocity components are rarely available, Sherman [13] believes that initial vertical velocity should be set equal to zero.

The topography must be massaged or conditioned into acceptable form so that it is in accordance with finite difference technique. Actual elevations of an area are averaged within double width grid cells to provide an elevation of the double cell. Sherman [14] says that the double cell width restriction results from the difference scheme used in the model. The topography height at each x-y grid position is assigned to a given level. The manner by which topography is conditioned is described in more detail in section 2.4.

At every interior grid point equation (1) is numerically transformed into

\[
\frac{\lambda_{ijk} - 2\lambda_{ijk} + \lambda_{i-1,jk}}{\Delta x^2} + \frac{\lambda_{ijk} - 2\lambda_{ijk} + \lambda_{i,jk-1}}{\Delta y^2} + \left( \frac{\alpha_1}{\alpha_2} \right)^2 \frac{\lambda_{ijk} - 2\lambda_{ijk} + \lambda_{ijk-1}}{\Delta z^2} = 2c_i^2 \epsilon_g
\]  

(27)
where the divergence is

$$\varepsilon = \frac{u^0_{i+1,k} - u^0_{i,j,k}}{2\Delta x} + \frac{v^0_{i+1,j,k} - v^0_{i,j+1,k}}{2\Delta y} + \frac{w^0_{i,j,k+1} - w^0_{i,j,k}}{2\Delta z}$$  \hspace{1cm} (28)

The above equation may be written as

$$-(A_m + B_m \frac{\Delta x}{\Delta y}) \lambda_{i,j,k} + C_m \frac{\Delta x^2}{\Delta y} \lambda_{i,j,k} + D_m \lambda_{i+1,j,k} + E_m \lambda_{i,j+1,k}$$

$$+ F_m \frac{\Delta x}{\Delta y} \lambda_{i,j+1,k} + G_m \frac{\Delta x^2}{\Delta y} \lambda_{i,j+1,k} + H_m \frac{\Delta x^2}{\Delta y} \lambda_{i,j,k+1}$$

$$+ I_m \frac{\Delta x^2}{\Delta y} \lambda_{i+1,j,k} = 2\alpha^2\Delta x \varepsilon_{i,j,k}$$  \hspace{1cm} (29)

where subscript $m$ refers to the $ijk$ position.

The constants $A_m$ through $H_m$ are defined in Christine Sherman's thesis [15] for all topographic descriptors. The topographically dependent constants are defined in terms of boundary conditions at each grid cell position. In this way their values depend on the topography surrounding the grid cell. Determination of these coefficients is discussed in Appendix A.

At the boundaries, the normal first derivative is approximated by a first forward difference

$$\left. \frac{\partial \lambda}{\partial n} \right|_{x=1} = \frac{3\lambda_{x=1} - 4\lambda_x + \lambda_{x=0}}{2\Delta n}$$  \hspace{1cm} (30)

or by a backward first difference

$$\left. \frac{\partial \lambda}{\partial n} \right|_{x=0} = \frac{-3\lambda_{x=0} + 4\lambda_x - \lambda_{x=1}}{2\Delta n}$$  \hspace{1cm} (31)
The adjusted wind velocities are calculated from

$$u_{ijk} = \frac{1}{4} \left( u^0_{i+1,jk} + 2u^0_{ijk} + u^0_{i-1,jk} \right) + \frac{1}{2\alpha_1^2} \left( \frac{\lambda_{i+1,jk} - \lambda_{i-1,jk}}{2\Delta x} \right)$$

(32)

$$v_{ijk} = \frac{1}{4} \left( v^0_{ijk+1} + 2v^0_{ijk} + v^0_{ijk-1} \right) + \frac{1}{2\alpha_2^2} \left( \frac{\lambda_{ijk+1} - \lambda_{ijk-1}}{2\Delta y} \right)$$

(33)

$$w_{ijk} = \frac{1}{4} \left( w^0_{ijk+1} + 2w^0_{ijk} + w^0_{ijk-1} \right) + \frac{1}{2\alpha_3^2} \left( \frac{\lambda_{ijk+1} - \lambda_{ijk-1}}{2\Delta z} \right)$$

(34)

The values of $\lambda$ for all interior grid points is calculated by using equation 27. The entire system of difference equation (equation 29) are solved simultaneously for $\lambda_{ijk}$ at each interior grid point. As already noted, the coefficients of equation 29 are specific to grid point location. The coefficient matrix is diagonally dominant and nonsymmetric; the equation set is solve iteratively.

In recent years much study has been given to ways for speeding up the convergence of the iterative method. The Gauss Seidel numerical method (used above) is iterative and has been studied intensively in this regard. The overrelaxation technique has been found to be profitable in speeding up the convergence. An overrelaxation factor of 1.78 has been determined experimentally by Sheman [16].

For equation 29, if we let

$$GR = -(A_m + B_m \left( \frac{\Delta x}{\Delta y} \right)^2 + C_m \left( \frac{\alpha_1}{\alpha_2} \right) \left( \frac{\Delta x}{\Delta y} \right)^2$$

and

$$Z = D_m \lambda_{ijk} + E_m \lambda_{i+1,jk} + F_m \left( \frac{\Delta x}{\Delta y} \right)^2 \lambda_{i+1,jk} + G_m \left( \frac{\Delta x}{\Delta y} \right)^2 \lambda_{ijk-1}$$

$$+ H_m \left( \frac{\alpha_1}{\alpha_2} \right)^2 \left( \frac{\Delta x}{\Delta z} \right)^2 \lambda_{ijk+1} + I_m \left( \frac{\alpha_1}{\alpha_2} \right)^2 \left( \frac{\Delta x}{\Delta z} \right)^2 \lambda_{ijk-1}$$

and

$$\text{orig}_{ijk} = 2 \alpha_1^2 \Delta x^2 \varepsilon_{ijk}$$
then equation 29 may be written as

\[ GR \lambda_{ijk} + Z = \text{orig}_{ijk} \]

Solving for \( \lambda_{ijk} \)

\[ \lambda_{ijk} = \frac{\text{orig}_{ijk}}{GR} + \frac{Z}{GR} \]  

(35)

Using the overrelaxation technique designed by Forsythe and Wasaw (17) in the iterative routine, one obtains

\[ \lambda_{ijk}^{ne} = \omega \left( \frac{\text{orig}_{ijk}}{GR} + \frac{Z}{GR} \right) + (1 - \omega) \lambda_{ijk}^{ne-1} \]  

(36)

Reducing the above

\[ \lambda_{ijk}^{n+1} = \lambda_{ijk}^{n} + \omega \left( \frac{\text{orig}_{ijk}}{GR} + \frac{Z}{GR} - \lambda_{ijk}^{n} \right) \]  

(37)

If we let

\[ \text{RES} = \frac{1}{GR} (Z + \text{orig}_{ijk} - GR \lambda_{ijk}) \]

Then

\[ \lambda_{ijk}^{n+1} = \lambda_{ijk}^{n} + \omega(\text{RES}) \]  

(38)

is used. RES is termed the Residual. In section 2.3 a more detailed description of the iteration procedure used by the MATHEW program is given.

Having solved for \( \lambda_{ijk} \) at all interior grid points, the values of
\( \lambda_{ijk} \) on the topographic boundaries are solved for by using equations 30 and 31. On open boundaries, the procedure used for calculating the Lagrangian multiplier is documented in Appendix A. Having solved for \( \lambda_{ijk} \) at all grid points, the adjusted velocity components are calculated using equations 32, 33 and 34.

The weight ratios \( \frac{\alpha_1}{\alpha_2} \) are determined as inputs to the MATHEW program. In the project report of Marilyn Pelosi [18], the weight ratio is discussed. She says that the particular weight ratio used for a specific application of MATHEW should be related to grid block geometry. MATHEW adjusts the initial wind field to yield a mass consistent wind field. By keeping the volume flow in the horizontal and vertical direction the same order of magnitude, the algorithm works towards a goal of mass consistency in an efficient manner. Since the flow is assumed incompressible, the mass flow in the horizontal direction is proportional to (area) x (velocity) = \( q_1 \times v_y \). (Figure 3) The mass flow in the vertical direction = \( q_2 \times v_y \). Therefore a weighting scheme is used to keep \( \frac{\alpha_1}{\alpha_2} \) = \( \frac{q_1}{q_2} \) equal to one. The ratio \( \frac{\alpha_1}{\alpha_2} \) should be of the same order of magnitude as \( \frac{q_2}{q_1} \). Since the MATHEW box is rectangular, the weight ratio, \( \frac{\alpha_1}{\alpha_2} \), is calculated from the average of \( \frac{A-B}{2C} \) and \( \frac{A-B}{2C} \); the weight ratios corresponding to the x and y directions respectively. A, B, and C are lengths shown on Figure 3. Using the appropriately weighted scheme allows rapid convergence of the iteration described above. If \( \frac{\alpha_1}{\alpha_2} \) is too large, the vertical velocity dominates and if \( \frac{\alpha_1}{\alpha_2} \) is too small the horizontal velocity dominates.
$V_h = \text{Horizontal Velocity}$

$V_v = \text{Vertical Velocity}$

$A_1 = \text{Vertical Area}$

$A_2 = \text{Horizontal Area}$

**FIGURE 3.**

Weighting Factor Calculation for Mathew Box
2.3 COMPUTER FORMAT

The architecture of the MATHEW program is described in this section. A flow chart of the MATHEW program (Figure 4) and an explanation of input tapes are given. Input information required by MATHEW is listed in Chapter I. Also a brief documentation each subroutine (and its function) is given. For a more detailed account of the internal structure of MATHEW, see Appendix A, entitled the Documentation of MATHEW Subroutines.

Both Tape 20 and Tape 47 are outputs from subsidiary programs required to implement MATHEW (see Appendix B). Tape 47 is the input topography tape and Tape 20 the input velocity tape. Tape 47 gives averaged topography heights (dimensioned in meters) for double width grid squares dimensioned 2 x delta x by 2 x delta y. The input velocity tape gives interpolated two dimensional (x and y velocity components) velocity values at each grid block location at a reference height. In all MATHEW runs, a reference height of 50 m is used.

Input parameters are defined before the algorithm of MATHEW can proceed. The values defined are:

M, N, L - the number of grid points in the x, y and z directions respectively.

Δx, Δy - delta x and delta y in meters. It should be noted that delta x and delta y must always be divisible by 63.4 m since this distance separates topographic height information.

\[ \frac{\sigma_l}{\sigma_z} = \frac{\sigma_y}{\sigma_x} \] - predetermined weights. (See section 2.2.)

REF - reference height in meters for which interpolated velocity values are given.
Start

Define Input Parameters

Input Interpolated Velocity Field (TAPE 20)

Input Topography (TAPE 47)

Go to Subroutine Define minimum surface height and calculate surface height in terms of z. Expand Mathew square to full dimensions.

Go to Subroutine Topog

Go to Subroutine Expand

Set surface height equal to largest of four surrounding grid cells.

Go to Subroutine Mattopo

Determine grid description based on topography of four surrounding grid cells.

Go to Subroutine Grides

Go to Subroutine Setup

Set up coefficients of D.E.

Go to Subroutine Set

Use powerlaw to expand initial input matrices to all grid points.

Go to Subroutine Pwrl

Calculate RHS of D.E.

Go to Subroutine Setorig

Main Program

Calculate $A$ at all interior grid points.

Go to Subroutine Setorig

Go to Subroutine Adjuvw

Calculate $A$ values at closed boundaries; $A$ at open boundaries; $A$ at topography.

Output adjusted three-component velocity values at all levels (TAPE 86) and two-component interpolated velocity values at all levels (TAPE 21).

FIGURE 4. Flow Chart of Mathew Program
\[ \Delta z = \text{delta } z \text{ in meters} \]
\[ \alpha = \text{shear coefficient which describes vertical velocity variations.} \]

The subroutines shown in Figure 4 are documented below.

1. **Subroutine Topog.**

   1. All heights (in meters) are read from Tape 47 and translated into levels. Each level is equally distanced by delta z. The minimum surface level height is determined.

   2. Level heights are adjusted so that lowest surface height corresponds to zero level.

   3. The MATHEW array is expanded so that the number of x and y coordinates are doubled. Each grid cell level assigns its level to its neighboring right, upper and upper right grid cell so that every four grid cell blocks has the same level height.

2. **Subroutine Mattopo.**

   1. The surface level at each grid point is set equal to the largest surface level value of the surrounding grid cells. In this way one more coordinate is added to the x and y directions.

   2. One is added to each level. Surface level height at each grid point is given as KMATTOP(i,j). This array describes the conditioned contour (section 5.1) and is the output onto Tape 50 (section 2.4).

3. **Subroutine Grides.**

   1. The array IGRIDS(i,j,k) is initially equal to -1 for all points in the MATHEW box.

   2. Grid descriptor values are assigned to all points in the MATHEW box. Grid descriptor values are assigned according to grid points relationship to surface level height of neighboring grid
cells. First, grid descriptor values are assigned to interior points, second to x and y open boundaries, third to the top level and finally to KMATTOP.

4. Subroutine Setup.

For each interior grid position, the coefficients of the differential equation (equation 29) are determined on the basis of grid descriptor value. The differential equation will be used to calculate for all interior grid points. Combining equations 27, 28 and 29, one obtains for the x direction

\[ \lambda_{i+1,j,k} - 2\lambda_{i,j,k} + \lambda_{i-1,j,k} = D_m \lambda_{i+1,j,k} - A_m \lambda_{i,j,k} + E_m \lambda_{i-1,j,k} \tag{39} \]

where \( D_m, A_m \) and \( E_m \) are constants whose values are based on the surrounding terrain condition. Grid descriptor values which have been assigned to all interior grid positions determine the values of the coefficients \( D_m, A_m \) and \( E_m \) and the values of all coefficients used in the above equation. The Lagrangian multiplier, \( \lambda \), at position \( i+1 \) might be written as \( \lambda_{i,j,k} \) or might be written in terms of values at neighboring points. For example, if in the \( x \) direction a closed boundary exists between position \( i \) and position \( i+1 \) (both on the same level), at \( i+1 \) will be written

\[ \lambda_{i+1} = \frac{4\lambda_i - \lambda_{i-1}}{3} \tag{40} \]

This equation is obtained by using the first backward difference equation (equation 31) with the knowledge that \( \frac{\partial \lambda}{\partial x} = 0 \) at position (equation 31) \( i+1 \). For the above case (case C3 in Appendix A) \( D_m = 0, A_m = 2/3 \) and \( E_m = 2/3 \).

5. Subroutine Purlaw.

Takes initial velocity values and expands to all levels using powerlaw coefficients. The relationship used is

\[ u = u_0 \left( \frac{Z}{Z_0} \right)^\alpha \tag{41} \]
where \( u \) is the velocity at height \( z \). The velocity \( u \) is calculated for height \( z \) using the shear coefficient \( \alpha \).


1. Sets up the right-hand side up the differential equation using interpolated velocity values.

7. Main program.

Uses Gauss iteration technique and overrelaxation method to solve for \( \lambda \) at all interior grid cells. The Lagrangian multiplier is calculated by using equation 27 (section 2.2) A flow chart (Figure 5) for the iteration procedure is given on the following pages. The variables used are:

1. \( \omega \)-overrelaxtion factor. After 600 iterations, the overrelaxation factor changes from 1.78 to 1.0

2. RCerr-maximum allowable difference between \( \lambda \) calculated during the \( n^{th} \) and \( n^{th}-1 \) iterations = .002

3. Maximum number of iterations = 8000

4. NOGTERR-counter used to find the number of grid points at which accuracy check fails.

8. Subroutine Setuvw.

1. Sets up first part of right-hand side of the equation used to calculate adjusted velocity values. From equation 32,

\[
\frac{u_{\omega jk}}{2} + 2u_{ijk} + u_{\omega jk}
\]

is calculated for the x direction and likewise for the y direction at all interior grid points. Note that for interior grid points located adjacent to closed boundaries the velocity values at the closed boundaries is assigned equal to zero.
$w = 1.78$

DO $N = 1, 8000$

NOGTERR = 0

DO I = 2, 24
  J = 2, 24
  $K = \text{KMAT} + 1, 10$

DO LOOP

lamold = $\lambda_{jk}$

Calculate new $\lambda_{jk}$

lamold = 0?

yes

Err = $\frac{\lambda_{jk} - \text{lamold}}{\text{lamold}}$

ERR $\geq$ RCERR?

no

no

N = 600?

yes $w = 1.0$

no

$N = 50$ - Constant?

yes

Write N, & NOGTERR

NOGTERR = 0?

no

continue to next subroutine.

Write iter, RCERR

yes

iter not converged;

end

FIGURE 5.
Flow Chart of the Mathew Iteration Procedure
9. **Subroutine Ajstuvw.**

Calculates adjusted velocity values at all grid points.

1. Defines variables used in adjustment equation. (Equations 21, 22 and 23).

2. Calculates \( \lambda \) values for closed boundaries adjacent to interior points. Method of calculating \( \lambda \) values is based on grid descriptor values at interior grid points.

3. Calculates adjusted velocity values at all interior grid points.

4. Calculate \( \frac{\partial \lambda}{\partial n} \) on edge points. Since edge points considered are only the open ones, \( \lambda = 0 \) at these points. Values are calculated on \( x = 1, x = M, y = 1, y = N \) and \( z = L \).

5. Calculates adjusted velocity values on edge points.

6. Calculate \( \lambda \) values at position of interest on topography. The diagram below illustrates examples where \( i \) is the position of interest. Note that on the topography, the boundary is always considered to be closed in the \( z \) direction but may be open in the \( x \) and \( y \) direction.

7. Calculate \( \lambda \) values at grid points neighboring the position of interest on topography.

   1. If a neighboring grid point, \( a \), has the same topographic level, then the neighboring point is considered to be closed in the \( z \) direction and open in the \( x \) or \( y \) direction. In this way is calculated from the condition that \( \frac{\partial \lambda}{\partial z} \) equals zero there.

   2. If neighboring grid point, \( b \), is assigned a higher level, then the neighboring grid point is closed in the \( x \) or \( y \) direction and the value is calculated in terms of \( \frac{\partial \lambda}{\partial x} = 0 \).

   3. If neighboring grid point,
c, is assigned a lower topographic level the position is then an interior grid point and boundary conditions need not be applied.

8. Calculate adjusted velocity values at topography.
2.4 TOPOGRAPHY CONDITIONING

The means by which topography is conditioned is examined. All topographical information is available from the U.S. Geological Survey Digital Terrain Tape. After having determined the area in question, terrain height (ft) levels are given for each assigned x and y position in the MATHEW box. (See Figure 1.) The digital terrain tape uses a spacing of 63.4 m between each x and y grid position. In this way, the delta x and delta y values given by the MATHEW grid structure must be multiples of 63.4 m. Input parameters, M, N, L, delta x and delta y are all determined with respect to the size of the area in question, the height of the first inversion layer (assumed approximately equal to 550 m) and the maximum number of blocks allowed in the MATHEW box. It is noted that some variability exists here. For example, choosing a large delta x and delta y to account for a smaller delta z value is one of the many options available to the user. These options are explored in the form of tests.

Figure 6 shows the actual topography of an area in Princeton, Massachusetts. Although the topography has been drawn free-hand, it shows to scale approximately the contours of the chosen area. In the z direction, the distance from sea level is given in ft and in the x and y directions associated digital terrain tape descriptors for Massachusetts are listed. One unit equals 63.4 m. The area shown in the figures displayed is the most south-west corner of the area in question for all Princeton runs. The figures presented here are used as an aid to demonstrate how topography is conditioned in the MATHEW model. Input
parameters described here correspond to those given for actual runs as may be seen in Table 3, the Input Parameter Summary Sheet.

Figure 7 displays the digital terrain tape interpretation of the given topography. The digital terrain tape gives integer values for all heights so that the minimum difference in height levels given from the tape equals one ft. Figure 7 shows a resolution of ten feet since height levels taken from the terrain tape have been rounded off. Although the contour drawing does not precisely display terrain tape values, the representation is accurate. Topographic information is taken from the digital terrain tape and stored in Tape 45. Contour plots of Tape 45 are labeled "terrain contour" (section 4.1). (It should be noted here that in section 2.3 a detailed flow chart listing all programs and tapes is given.)

The topography is then averaged into double width grid cells. In this way, for assigned input parameter (as is used in the PRINCl thru PRINC9 runs) delta x = 190.2 m and delta y = 317. m (Figure 8). One topographic level height from Tape 48 is averaged from 60 grid positions from the digital terrain tape. Contour plots of Tape 48 are titled "Input Contour" since this information is input into the MATHREW program (section 4.1). If the delta x and delta y input parameters are assigned (as in the PRINC1 run) equal to 443.8 and 697.4 m respectively, (Figure 9), then assigned level height on Tape 48 is averaged from 308 grid positions from the digital terrain tape. Grid level height values are then entered into the MATHEW program where more conditioning is performed.

Next, heights associated with each grid cell are leveled or rounded
FIGURE 8.
Input Topography Using Delta x = 190.2 m and Delta y = 317 m.
off to correspond to a predetermined level height. Level heights increase from 1 to L and are separated by a distance of delta z. In this way if delta z is assigned equal to 50 m and a terrain height at a given grid position equals 328 ft, the terrain height level assigned to that grid position is 2. All height levels are stored on Tape 51. Then, MATHEW looks at height levels assigned to grid cells on Tape 51. For each grid cell the height levels at all surrounding grid cells are observed and the largest neighboring level height value is assigned to that cell. Tape 50 contains the new grid level heights. Contour plots of Tape 50 are titled "Conditioned Contour". During this process, the contour of the entire area may be effectively shifted. For a detailed documentation of the conditioning procedure, see Appendix A. The conditioned contour is now acceptable to the algorithm of MATHEW. Figure 10 shows the contour plot of Tape 50 which is derived from information shown in Figure 8. Delta z equals 50 m as is used in the PRINC2 run. Figure 11 also shows the contour plot of Tape 50 which is derived from the Tape 48 diagram shown in Figure 8. In this case, however, delta z equals 37.5 m (as is used in PRINC3 thru PRINC9 runs). Figure 12 shows Tape 50 derived from Tape 48 of Figure 9. In this case, (as in PRINC1) delta z = 25 m.

The MATHEW model uses topographical information from Tape 50 to establish boundary conditions and derive constants used in the elliptical partial differential equation (equation 29). When comparing Figure 10 thru Figure 12 drawings to Figure 6 it can be seen that all versions of conditioned topography seem to neglect small gradients in height. By choosing large delta x and delta y values, averaging is performed over a
FIGURE 10.

Conditioned Topography Using Delta x = 190.2 m, Delta y = 317. m.
and Delta z = 50. m.
Conditioned Topography Using Delta x = 190.2 m, Delta y = 317 m, and Delta z = 37.5 m.
large area. By choosing a large delta z, gradients in topography will be large and occur abruptly. Testing results given in Chapter IV give further insight to topographical conditioning.
CHAPTER III

TESTING PROCEDURE

3.1 PROGRAM UTILIZATION

In order to use the MATHEW program, two tasks must be clearly understood. These are data acquisition (i.e. the acquisition of topographic information from digital terrain tape and the acquisition of input velocity values for associated sites) and program implementation. A flow chart outlining program utilization tasks may be found on the following page. (Figure 13) The flow chart seen here, with minor edit changes, duplicates that in Marilyn Pelosi's [19] project report. Because the subject of her work is another version of the MATHEW program developed by Mario Estoque of the University of Miami, the reader may refer to her thesis (pages 79-99) for a detailed description of the data acquisition procedure. The data to be acquired is described below.

1. Topographical information for the site is taken from a digital terrain tape obtained from the National Cartiographic Information Center in Reston, Virginia.

2. Time interval: Wind speeds and directions are observed and recorded concurrently at all locations for consecutive 15 second intervals using one minute periods in Princeton and five minute periods in Windsor. The vectors are averaged over the assigned time period and the most representative time period is chosen as input data for MATHEW.

3. Data stations: There are to be five data station for each site, to be evenly dispersed over the MATHEW area. Data stations are to be located in clear areas so that wind turbulence due to trees is minimized.
Determine Exact Boundaries, Select Data, Determine Site Locations and Grid Structure

Collect Wind Speed and Direction Data

Make Necessary Corrections and Decompose Velocities into Components

Collected Data

Create Collected Data Tape

Interpolated Velocity Field

Reduce Topography into Two Δx by Two Δy Grid Squares.

Run Mathew

Adjusted Velocity Field

Input Velocity Field

Plot Isotach Curves and Mathematically Analyze Results

FIGURE 13. Program Utilization of Mathew
4. **Shear profile**: Wind speed and direction are recorded at a specific location for four to five different heights, evenly spaced at approximately one hundred ft apart. Recordings are taken at fifteen second intervals for the allotted time periods given above. This is done three times. Average velocity values may be calculated for each averaged height.

A Data Station Summary Sheet for Princeton and Windsor may be found in Appendix C. Because the Estoque version of MATHEW differs from the Lawrence Livermore Laboratories version used here, program implementation differs and is described in Appendix B.

In Appendix B may be found a listing of all programs used, a flow chart defining their interrelationship and a schedule of editing procedures required for implementing a specific MATHEW run. It is noted that a major portion of editing procedure listings are the result of the independent work of Marilyn Pelosi.
3.2 TEST CLASSIFICATION

For each of the two sites chosen, (Windsor and Princeton, Mass.), the MATHEW program is run a variety of times. Specified input information for each run is varied in accordance with tests listed below. Input parameters (delta x, delta y, etc.) are visually displayed in Figure 1. These tests which are designed to investigate geometric data and storage space constraints are categorized below.

**Test I:** reduce the MATHEW area in two stages. Delta x, delta y and delta z are the same for each but M and N change.

**Test II:** The MATHEW area is the same for all runs. Delta x and delta y are the same but delta z changes. L is varied. The input velocity field is the same.

**Test III:** The MATHEW area is the same for all runs. Delta x and delta y change but delta z remains the same. M and N are varied. The input velocity field is the same.

**Test IV:** The MATHEW area is the same for all runs. Delta x, delta y and delta z change correspondingly as M, N and L are varied. The input velocity field is the same.

**Test V:** The MATHEW area is the same for all runs. Delta x, delta y and delta z are the same. M, N and L are the same. The number of data stations specifying velocity vectors is changed. In this way, the input or interpolated velocity
field differs for each run. First one data station is deleted, then two, etc. until the program is run with only one data station velocity value.

The Test Classification Summary Chart (Table 1) which follows demonstrates how each test category compares changes in input parameters and thereby affects input information.
<table>
<thead>
<tr>
<th>Test #</th>
<th>Test Compares Changes in</th>
<th>Input Information Affected by Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Area ° Data Stations</td>
<td>Delta x, Delta y, Delta z Area interpretation of Topography Input Velocity Field</td>
</tr>
<tr>
<td>Test 1</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Test 2</td>
<td></td>
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<td>X</td>
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<tr>
<td>Test 5</td>
<td></td>
<td>X</td>
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</table>
3.3 EXAMINATION

MATHEW test results are analysed visually and mathematically. In this section visual and mathematical analyses are defined. The Examination Procedure Summary Table, Table 2, is also given to demonstrate how results from each test category are examined.

Visual analysis entails comparing contours and isotach curves. For each run made, the following plots may be produced:

1. Contour plot of the MATHEW area (taken from digital terrain tape information).
2. Contour plot of topographic information input to the MATHEW program.
3. Contour plot of Tape 51 used in the algorithm of MATHEW (section 2.4).
4. Contour plot of KMATTOP array (section 2.2). This is a plot of the final interpretation to topography used by MATHEW to define boundary conditions in solving for adjusted wind fields.
5. Isotach plot of interpolated wind field at 50 and 100 m above ground.
6. Isotach plot of adjusted wind field at 50 and 100 m above ground level.

Visual comparisons may be made between:

1. Adjusted velocity fields and the interpolated velocity fields for each MATHEW run.
2. Adjusted velocity fields produced from several MATHEW runs.
3. Adjusted velocity fields and corresponding terrain contours.

Velocity fields are mathematically analysed in three ways:

1. Calculate adjusted shear coefficient, $\alpha$. 
2. Calculate velocity differences between interpolated and adjusted velocity values.

3. Calculate velocity differences between adjusted velocity values produced from two different runs.

The adjusted shear coefficient is simply the curve fit powerlaw wind shear exponent (equation 41) for adjusted velocity values at all levels above ground for one specified grid point. The adjusted shear coefficient is calculated for every grid position in the MATHEW area. The minimum value, maximum value and averaged value of may then be given for each run.

For each level height above ground, the difference in the magnitude of input and output velocity vectors is found for every grid position. The maximum difference and the minimum difference are determined. Also, the URMS of the root-mean-square velocity difference is also given. It is defined as

\[ URMS = \sqrt{\frac{\sum_{i=1}^{n} (u_i - u_i^o)^2}{n}} \]  

(42)

where

- \( u \) - magnitude of adjusted velocity vector
- \( u^o \) - magnitude of the interpolated velocity vector
- \( n \) - number of grid positions at the specified above ground level

A graph (Figure 112) which shows how the maximum difference (UMAX),
<table>
<thead>
<tr>
<th>Test #</th>
<th>Visual Analysis</th>
<th>Mathematical Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Visual Comparison Between</td>
<td>Calculate URMS</td>
</tr>
<tr>
<td></td>
<td>Adjusted Velocity and Corresponding Terrain Contours</td>
<td>Adjusted Velocity and Interpolated Velocity</td>
</tr>
<tr>
<td>Test 1</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Test 2</td>
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<td>Test 4</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Test 5</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>
the minimum difference (UMIN) and the URMS values vary with height above ground level may be obtained. See section 5.1.1. For each MATHEW run, URMS values for the first five levels above ground are obtained. From these values, the minimum URMS value, the maximum URMS value and the averaged URMS value is given. (Table 5)

In the same way adjusted velocity values obtained from runs used in the same test category may be compared and similarly URMS values calculated. An isotach curve which plots velocity difference values for each grid position at specified above ground level positions may be acquired (See Figure 111 in section 5.6.) The URMS values for levels corresponding to 50 m above ground are presented for each set of runs to which they apply.

Table 2 tells which of the aforementioned analysis techniques apply to each test category.

Observation of solution technique variables (i.e. the Lagrangian multiplier value and values of the divergence of interpolated velocity fields during their computational progression) are intermittently examined to monitor trends in the iteration algorithm. Qualitative observations of the trends noted during iteration procedure and during calculation of adjusted velocity values given in section 5.1.2.
CHAPTER IV

APPLICATION

4.1 OVERVIEW

The MATHEW model is applied to sites in Windsor and Princeton, Massachusetts. In this chapter each of the five tests described in section 3.2 are applied and examined individually. A total of seventeen MATHEW runs are performed, nine of which are applied to Princeton and eight to Windsor. With each run is associated specified input parameters and input information. Because of the abundance of information required for and produced by so many MATHEW runs, a schematization of all MATHEW runs precludes individual test description.

Contour plots of the areas of interest for Princeton and Windsor may be seen in Figures 63 and 14 respectively. Input data for the sites has been acquired in accordance with the guidelines given in Marilyn Pelosi's thesis. Wind speed magnitudes and directions are obtained for five locations or data stations at each site. Appendix C records the wind velocity vectors chosen as input to MATHEW, the height at which wind speed was measured, the location of data stations in terms of digital terrain tape coordinates, the grid coordinates of MATHEW and associated wind shear coefficients for both Princeton and Windsor. United States Geological Survey Topography maps are also given. Data stations are numbered one to five and are located on all plots.

MATHEW test runs are titled chronologically and in terms of their
site. In this way the first run made for Princeton is called PRINC1, the second PRINC2, etc., and those for Windsor are titled WIN1, WIN2, etc. An input parameter summary sheet for all runs may be found on the following page. (Table 3) The size of the area in question, \( \text{delta} \; x \), \( \text{delta} \; y \), \( \text{delta} \; z \), \( M \), \( N \), and \( L \) are given. See Figure 1. The total number of grid cells is simply \( M \times N \times L \) and is considered when examining storage space limitations. The ratio \( \frac{\alpha_1}{\alpha_2} \) is the ratio of weights in equation 25. As may be seen from the table, for runs PRINC3 to PRINC9 all geometric input parameters are equivalent but wind speed information is deleted.

Since tests compare runs corresponding to designated input information (see Test Classification Summary Chart), some runs performed are used in more than one test category. A summary of tests and corresponding runs is given in Table 4.

A listing of the potential plots produced from a MATHEW run is given in section 3.3. The manner by which plots are titled is described. All plot titles are initially identified by the associated run name. Contour plots of the area in question whose topographic information is derived directly from the digital terrain tape are given the title "Terrain Contour". Contour plots which represent topographic information input into MATHEW are entitled "Input Contour" (i.e. Tape 48 in section 2.4) The final version of the topography have plots titled "Conditioned Contour". Tape 51 (section 2.4) which stores "leveled off" height values is an intermediate recording of information between the input information (Tape 43) and the final version of conditioned topography (Tape 50) used to establish boundary conditions in the algorithm. Since Tape 51 and
TABLE 3. Input Parameter Summary Sheet

<table>
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<tr>
<th>Mathew Run</th>
<th>Area (km²)</th>
<th>Delta x (m.)</th>
<th>Delta y</th>
<th>Delta z</th>
<th>M</th>
<th>N</th>
<th>L</th>
<th>( \frac{c_2}{\sigma^2} )</th>
<th>Data Station Omitted</th>
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<td>PRINC1</td>
<td>37.45</td>
<td>443.8</td>
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<td>317.</td>
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<td>25</td>
<td>25</td>
<td>11</td>
<td>5.0</td>
<td>-</td>
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<td>317.</td>
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<td>15</td>
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<td>15</td>
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<td>$#5$</td>
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<td>6.4</td>
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<td>50.</td>
<td>35</td>
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<td>1.9</td>
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<td>126.8</td>
<td>63.4</td>
<td>50.</td>
<td>17</td>
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<td>126.8</td>
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<td>190.2</td>
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<td></td>
<td>PRINC3 &amp; PRINC6</td>
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<td>PRINC4 &amp; PRINC8</td>
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<tr>
<td></td>
<td>PRINC7 &amp; PRINC9</td>
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</tbody>
</table>
Tape 50 plots are very similar, all contour plots of Tape 51 need not be presented. However, in Appendix D, one may find plots of Tape 51 for WIN1 and PRINC1. As may be observed from comparing plots, the process of choosing the largest surrounding grid cell essentially shifts the large topographic gradient. See Appendix A for more detail.

Isotach plots describing initial two dimensional velocity fields are entitled "Interpolated Velocity at 50 (or 100) m" and plots describing the three dimensional output fields are similarly entitled "Adjusted Velocity at 50 (or 100) m". Two-dimensional plots (which neglect the w velocity component) of adjusted velocity fields for WIN1 and PRINC1 may be found in Appendix D. Since they closely resemble the three-dimensional outputs, they are presented for WIN1 and PRINC1 runs only. All isotach plots are presented for two levels above ground (usually the first and second) since adjusted wind flow patterns for each level appear to be quite different. On all plots, the x and y axes are assigned digital terrain tape coordinates (a one unit change represents 63.4 m). For each test, input information pertaining to the corresponding MATHEW run is reviewed, all plots are presented and observations on trends and insights obtained from visual analysis are given.
4.2 TEST I

In Test I, the MATHEW area of interest is reduced in two stages. This test may be divided into two parts. In the first part, runs for MATHEW areas of 6.83 km$^2$ (Figure 14) and 12.9 km$^2$ (Figure 15) are compared and in the second part runs for the MATHEW area of 6.83 km$^2$ and for an area four times its size (29.1 km$^2$) (Figure 16) are compared. For both parts, runs compared are divided into blocks with the same grid size (i.e. delta $x$, delta $y$ and delta $z$ are the same) and the same input data (wind velocity vectors and shear coefficients). The test is designed to compare the effect of changing the data station density (the number of data stations per unit area). Ideally, one would expect that adjusted velocity values calculated for the smaller areas will duplicate adjusted velocity fields for the larger areas in the locale of the smaller MATHEW area.

One may see from Figure 14 which shows the terrain contour of Windsor, Mass., that the area presented covers the minimum amount of space required to contain all five data stations. The topographic information represented by this plot is used for runs WIN4 (part 1) and WIN6 (part 2). In Figures 15 and 16, the data station density is one half and one quarter as large, respectively, as for Figure 14. In both the larger terrain contours, the enclosed smaller MATHEW area is shown.

In the first part of this test, the two test runs compared are WIN4 and WIN7. For both runs, delta $x = delta y = 190.2$ m and delta $z = 50$ m. Figures 17 and 18 represent topographic information input for WIN4 and WIN7 runs respectively. By comparing these plots to terrain contours,
FIGURE 14.
WIN3-Terrain Contour
FIGURE 15.
WIN7-Terrain Contour
FIGURE 16.
WIN8-Terrain Contour
FIGURE 17.
WIN4-Input Contour
FIGURE 18.
WIN7-Input Contour
one may see that the topography has been smoothed but is none-the-less a recognizable replica of the area. Data stations 1 and 3 which are located at hill tops may still be identified as located at the highest positions in the area. Figure 17 and the smaller area outlined in Figure 18 obviously describe the same area and differ in appearance from each other only slightly. In the area which surrounds site 3 (Windsor Hill) details of the contour pattern are seen to differ.

Figures 19 and 20 give contour plots of the conditioned topography for WIN4 and WIN7 runs respectively. These plots mark the transition from topographic heights to levels. By comparing Figure 19 to the smaller area in Figure 20, one can see slight differences in the topographic gradient lines pattern although overall they conform well with each other. To the south of site 3 in Figure 19, one is given a hint of the topographic drop from 2132' to 1968.5'. The negative gradient is seen to exist east of the smaller area outlined in Figure 20.

The input wind fields for WIN4 and WIN7 runs at level 1 above ground are given by Figures 21 and 22 respectively and at level 2 above ground by Figures 25 and 26 respectively. Since the magnitudes of wind vectors are calculated from a least-squares interpolation method using wind data given for stations 1 through 5, the corresponding plots match precisely.

Figures 23 and 24 are isotach plots of adjusted velocity fields at level 1 above ground for WIN4 and WIN7 runs respectively. In the area surrounding site 5 it may be seen that the isotach lines form five concentric circular patterns. When the location of these circular patterns is compared to conditioned topography plots it is seen that steep velocity gradients occur where steep topography gradients exist.
FIGURE 19.
WIN4-Conditioned Contour
FIGURE 21.
WIN3-Interpolated Velocity Field at 50 m. Above Ground Level
FIGURE 22.
WIN7-Interpolated Velocity Field at 50. m. Above Ground Level
FIGURE 23.
WIN4-Adjusted Velocity Field at 50 ft. Above Ground Level
FIGURE 24.

WIN7-Adjusted Velocity Field at 50. m. Above Ground Level
FIGURE 25.
WIN3-Interpolated Velocity Field
at 100. m. Above Ground Level
FIGURE 26.
WIN7-Interpolated Velocity Field at 100. m. Above Ground Level
When comparing the locations of these five gradients on Figure 23 and 24, it is seen that although their general relationship to each other is the same, in Figure 24 the gradients are weaker. This is due to the fact that for the WIN7 run, the pattern of the topographic gradient lines which separate level heights equal to 1804' and equal to 1968.5' is also more widely separated (Figure 20). The larger decline in surface height seen to the west of the smaller area is considered in the WIN7 run and accounts for larger spacing of topographic gradient line patterns.

At level 2, the adjusted wind fields for WIN4 and WIN7 runs are given by Figures 27 and 28 respectively. Steep velocity gradients do not exist here. Wind flow patterns surrounding site 3 are seen to appear to be quite different. It appears that in Figure 28 the area for which wind speeds of 14 mph or greater exist is much larger than in Figure 27. Since the WIN7 run covers a larger area than the WIN4 run, the hills encompassed by sites 1 and 3 are seen to be more as hills (Figure 20) and not as plateaus (Figure 19). Because of this, the windspeeds are adjusted to have a larger value at site 3.

In part two of Test I, the two runs compared are WIN6 and WIN8. The geometry of the grid block size is the same for both runs (i.e. \( \delta x = \delta y = 190.2 \text{ m} \) and \( \delta z = 50 \text{ m} \)). Input topography contours for the two runs are given in in Figures 29 and 30. The smaller MATHEW area shown in Figure 29 is outlined in Figure 30. Topographic information describing the conditioned contour is given by Figures 31 and 32.

Interpolated wind fields for WIN6 and WIN8 runs at 50 m above ground are shown in Figures 33 and 34 respectively and at 100 m above ground in Figures 37 and 38 respectively. The adjusted wind fields output from
FIGURE 27.
WING-Adjusted Velocity Field at 100 m Above Ground Level
FIGURE 28.
WIN7-Adjusted Velocity Field at 100 m. Above Ground Level
FIGURE 29.
WIN6-Input Contour
FIGURE 30.
WIN8-Input Contour
FIGURE 31.

WIN6-Conditioned Contour
FIGURE 32.
WIN8-Conditioned Contour
FIGURE 33.
WIN6-Interpolated Velocity Field at 50. m. Above Ground Level
FIGURE 34.

WIN8 - Interpolated Velocity Field at 50 m. Above Ground Level
FIGURE 25.

WIN6-Adjusted Velocity Field at 50. m. Above Ground Level
FIGURE 35.
WIN8-Adjusted Velocity Field at 50. m. Above Ground Level
FIGURE 37.

WIN6-Interpolated Velocity Field at 100 m Above Ground Level.
FIGURE 38.

WIN8-Interpolated Velocity Field at 100 m. Above Ground Level
WIN5 runs are shown at 50 m above ground in Figure 35 and at 100 m above ground in Figure 39. For the WIN8 run, adjusted velocity fields are given at 50 and 100 m above ground in Figures 36 and 40 respectively.

When one compares topographic gradient line patterns in Figures 31 and 32 one may see that the largest discrepancies occur in the area between site 1 and site 5 and in the area to the north of site 3. In Figure 31 for the WIN5 run, a steep step from 1804' to 2132' occurs whereas in Figure 19 between positions 1 and 5 there exists an intermediate sweeping plane of 1968.5' that is channeled between site 5 (at 1804') and site 1 (at 2132'). Referring back to the terrain contour shown in Figure 16, it seems that the WIN 6 run conditions contours more accurately since the increase in topographic height from site 5 to site 1 is in reality quite gradual.

Velocity adjustment shows its consideration of conditioned contours in Figures 35 and 36. One can see that for the WIN8 run between sites 5 and 1 two local velocity gradients exist which are not shown for the WIN5 run. The first is located directly north of site 1 and the second to the northwest. At the first, a minimum wind speed of 8 mph is shown and the second has a minimum wind speed of 3.5 mph. These negative velocity gradients are seen to exist in the 1968.5' plane region described in Figure 32. Since the WIN6 algorithm does not experience the flat plane region sweeping between sites 1 and 5, the isoloch lines experience a more gradual transition from 10.0 mph to 15.5 mph at site 1. When observing this same area on Figure 39 and 40, one may see, however, that the adjusted isoloch lines follow very much the same pattern. It is noted that at the second level above ground isoloch lines follow a more
FIGURE 39.
WIN6-Adjusted Velocity Field
at 100. m. Above Ground Level
FIGURE 40.

WIN8-Adjusted Velocity Field at 100 m. Above Ground Level
smooth pattern and the two local velocity gradients disappear.

In Figure 36, a large local velocity gradient exists north of site 3 corresponding to the topographic gradient that is shown in Figure 32. Since Figure 31 does not see this topography gradient from 1968.5' to 2132' north of site 3, the local velocity gradient is not seen in Figure 22. At 100 m above ground in Figure 40, the local velocity gradient is more slight but still exists to the north of site 3 whereas in Figure 39, the velocity gradient is again assumed nonexistent.

For both parts of Test I, the following observations may be made:

1. Adjusted wind velocities calculated for the smaller area do not duplicate adjusted wind velocity values calculated for the larger area in the locale of the smaller area. However, greater discrepancies are seen on level 1 than on level 2.

2. Decreasing data station density does not appear to inhibit wind adjustment calculation.

3. By using a large MATHEW area, it seems that the conditioned topography in the locale of the smaller MATHEW area accurately represents actual topography. For runs using the large area, topographic information provided for locations north, south, east and west of the smaller MATHEW area is used when terrain contour information is averaged. Because of this, input contour information for the larger MATHEW area better describes the actual contour; it follows that the conditioned contour also better describes actual conditions.

4. Since velocity adjustment is dependent on the conditioned contour, and because runs using the large MATHEW area provide conditioned contours that more closely approximate actual topography, one may conclude that adjusted velocity fields produced by WIN7 and WIN8 runs (which use the large MATHEW area) more strongly resemble actual wind fields.
4.3 TEST II

In Test II, the grid block height dimension, delta z is compared for two separate values while all other input parameters are held constant. This test is divided into two parts. In part one, by comparing WIN3 and WIN4 runs, one is comparing a delta z = 25 m with a delta z = 50 m. In part two, the WIN5 run which uses an input delta z = 12.5 m is compared to the WIN6 run which uses a delta z = 50 m. The test is designed to determine the effect of changing the grid block height levels on the adjusted velocity field.

In the first part, both WIN3 and WIN4 runs use an area of interest shown in Figure 1. Input parameters delta x = delta y = 139.4 m, M = 17, and N = 25 for both runs. Since the first temperature inversion layer is constant, values of L = 22 and L = 11 are given as input to WIN3 and WIN4 runs respectively. The terrain contour and input contours are the same (Figures 17 and 41) for both runs since the areas, M and N values are the same. Conditioned contour plots of WIN3 and WIN4 runs may be found in Figures 42 and 19 respectively. Comparison of these two plots shows that both plots are alike in the respect that an increase in topography is shown to exist from the west to east. But in Figure 19 this transition is seen to occur abruptly and occur only in the southern half of the area described. The WIN3 run conditioned contour shows that the elevation increases more smoothly in the easterly direction, that the highest topographic point is located in the southeastern portion of the area and that topography increases in the north portion also exist. The WIN3 interpretation of terrain contour more closely approximates actual
FIGURE 41.
WIN3-Input Contour
FIGURE 42.
WIN3-Conditioned Contour
conditions than the WIN4 interpretation.

Since the interpolation scheme used to determine input velocity field requires only input velocity values, delta x, delta y, M and N input fields are the same for both WIN3 and WIN4 runs. For both runs, the interpolated velocity field for 50 and 100 m above ground level may be seen in Figures 21 and 35 respectively.

Figure 43 shows the adjusted velocity field given by the WIN3 run at 25 m above ground level, or the first level above ground. One may observe the large density of local wind velocity gradients in this diagram. Since local isotach gradients occur where topographic gradients exist, the abundance of local gradients is not surprising. It may also be noted that the magnitude of local gradients is smaller than gradients found in Figure 17 (adjusted velocity field at 50 m above ground for WIN4) since the magnitude of the topography gradients is smaller. The velocity gradients shown in Figure 23 are also larger since this wind field is twice the height above ground as the field shown in Figure 43.

Figure 44 shows the adjusted wind field at 50 m above ground for the WIN3 run. Dramatic differences in isotach line patterns are seen when comparing this figure to Figure 23. Although both output wind fields are at the same level height, the output velocity field associated with the WIN3 run give a smooth and continuous pattern, while that output by the WIN4 run shows a disjointed abrupt pattern. The magnitudes of velocity values at site locations are approximately the same for both Figures 44 and 23: at site 1 output wind speed equals 15 mph for both WIN3 and WIN4 runs. At site 3 a wind speed of 10 mph for WIN3 compares with a wind speed of 10.5 mph for WIN4.
FIGURE 43.
WIN3-Adjusted Velocity Field at 25 m. Above Ground Level
FIGURE 44.
WIN3-Adjusted Velocity Field at 50. m. Above Ground Level
FIGURE 45.
Isotach Difference Between Adjusted Velocity Values Output from WIN3 and WIN4 Runs at 50. m. Above Ground Level
FIGURE 46.

WIN3-Adjusted Velocity Field at 100 m. Above Ground Level
area between site 5 and 1 in Figure 46 while in Figure 27 distance between isotach curves changes abruptly. Areas of rapid and/or slow wind change are seen in Figure 27 whereas wind change is more consistent in Figure 33.

In part 2, both WIN5 and WIN6 runs use an area of interest shown in Figure 14. Input parameters delta x and delta y both equal 190.2 m, M = 11 and N = 17 for both runs. For WIN5 delta z = 12.5 m and L = 44 and for WIN6 delta z = 50 m and L = 11.

The input topography contour for both runs is the same and may be seen in Figure 29. The conditioned topography for WIN5 is shown in Figure 47 and for WIN6 in Figure 41. Figures 33 and 37 give isotach plots of the input velocity field for 50 and 100 m above ground respectively and may be applied to both WIN5 and WIN6 runs. Figure 48 and Figure 35 show adjusted velocity fields at 50 m above ground level for WIN5 and WIN6 runs respectively. Adjusted wind velocity fields for WIN5 and WIN6 runs at 100 m above ground may be seen in Figures 49 and 39 respectively. The difference between adjusted velocity values given by WIN5 and WIN6 runs at 50 m above ground is shown for each grid position on the isotach plot given in Figure 50.

By comparing Figures 48 and Figure 35, one may see that the adjusted velocity fields at 50 m above ground given by WIN5 and WIN6 runs appear quite different, especially in the area surrounding site 5. This observation is verified by Figure 50 which shows that maximum velocity magnitude difference of 2.4 mph, 2.5 mph and 1.7 mph occur in the areas where local velocity gradients exist (Figure 35). The conditioned topography given by the WIN5 (Figure 47) run shows a smoother elevation
FIGURE 47.

WIN5-Conditioned Contour
FIGURE 48.
WIN5-Adjusted Velocity Field
at 50 m. Above Ground Level
FIGURE 49.

WIN5-Adjusted Velocity Field
at 100. m. Above Ground Level
FIGURE 50.

Isotach Difference Between Adjusted Velocity Values output from WIN5 and WIN6 Runs at 50. m. Above Ground Level
increase from west to east than that given by the WIN6 run (Figure 31). In the area described by $x = 1210$ to $x = 1216$ and $y = 869$ to $y = 893$, the difference in contour patterns is quite dramatic. Figure 31 shows a rapid elevation increase from 1804' to 2132' in the central portion of this area while Figure 47 shows a gradual elevation increase from 1804' to 1927' to 1928' to 2009' in the locale of this same area. The adjusted isotach curve for the WIN5 run describes a gradual transition from 8 mph to 14 mph in the area extending from site 5 to site 1 while the WIN6 run shows the transition as being rapid for the same area.

Again for 100 m above ground, Figures 49 and 39 look extremely similar with exception to the area west of site 3. Figure 50 shows this area as being a locale of major differences between adjusted wind values at 50 m above ground, and it seems that these differences have been continued to the next 50' increment. A point to note here is that Figure 49 represents wind velocity fields at the eighth level above ground and looks very similar to the input velocity field shown in Figure 37. It seems that because the adjusted velocity field is very distant from the ground (in terms of levels), the effects due to topography are considered less strongly and the output wind field begins to resemble the input field.

Conclusions from both parts of Test II are as follows:

1. Using a smaller delta z value allows the MATHEW program to interpret actual terrain contours in a more detailed way. Plots of the conditioned contours with a smaller delta z value more closely approximate actual terrain conditions.

2. Since velocity adjustment is dependent on topographic information described by the conditioned contour, the adjusted velocity fields derived from a more accurate interpretation of
actual terrain will themselves more accurately describe actual conditions.

3. Adjusted velocity fields described for the first level above ground contain areas where high local velocity gradients exist.

4. Adjusted velocity fields described for several levels above ground (i.e. above eight) begin to closely resemble interpolated velocity fields for that level.
4.4 TEST III

For all runs used in Test III, the MATHEW area of interest is the same. A terrain contour plot of this area is seen in Figure 14. Grid block height is set equal to 50 m for all runs. Test III compares the effect made by changing delta x and delta y grid block parameters. Four runs are examined and compared with each other. To eliminate confusion, it has been determined that the most viable method for visually examining these runs is to compare corresponding plots from each run individually. Grid block area for the WIN1 run has a delta x = 63.0 m and delta y = 126.8 m, M = 35 and N = 25. Alternatively, for WIN2, delta x = 126.8 m and delta y = 63.4 m, M = 17 and N = 49. The WIN4 run uses delta x = 126.8 = delta y = 126.8 m, M = 17 and N = 25 while the WIN6 run uses delta x = delta y = 190.2 m, M = 11 and N = 17.

Input contour plots for WIN1 and WIN2 runs are shown in Figures 51 and 52 respectively. For WIN4, the contour plot of input topographic information is given by Figure 17 and for WIN6 by Figure 29. Plots given by Figures 17, 51 and 52 all appear very similar. The averaging of terrain tape values has little effect on changing input information for all areas of the plot excepting that located south of site 3. For the WIN2 and WIN4 runs, the height at 2025' covers a much smaller area than for the WIN1 run where this plateau claims the southwest portion of the MATHEW area. The input topography given by the WIN6 run appears somewhat different though. Since the grid block area size is more than doubled for this run, topographic averages give a more general representation of the MATHEW area contours. The fact that both sites 1 and 3 reside on
FIGURE 51.
WIN1-Input Contour
FIGURE 52.

WIN2-Input Contour
hilltops is disguised in Figure 29. For all plots, the most prominent trend are identified; gradual increase in elevation from the southwest to the east and northeast is seen in all figures.

Plots describing conditioned contour information for WIN1 and WIN2 runs are given by Figures 53 and 54 respectively. Although they both show the same overall trend, details of the plots differ immensely. In Figure 17 contour lines describing the elevation increase from 1968.5' to 2132' are examined. The pattern of these contour lines describes a step rise equal to 63.4' and a step run equal to 126.8'. For the WIN1 run, a step pattern is also described (Figure 53). The rise equals 126.8' and the run distance equals 63.4'. The legnths of the rise and run correspond to grid square dimensions. It is noted that for both plots an equivalent amount of right angles defining the step patterns are used. Since both delta x and delta y are small, M and N are relatively large so that there exists a greater likelihood that more corners will appear.

Inversely, the larger the grid dimensions (Figure 31), the fewer the number of corners are described. For the WIN5 run, conditioned contour plot shows little detail as to where elevation gradients occur. It is to be noted here that WIN4 and WIN2 conditioned contours (Figures 19 and 54) look very similar except, as to be expected, the WIN4 plot (Figure 19) has fewer right angle turns in the contour line pattern. Again, to the south of site 3 both WIN2 and WIN4 conditioned contour (Figures 54 and 19) plots show a smaller area where elevation decreases occur while the WIN1 plot (Figure 53) shows the lower elevation extending further southward. It appears that the WIN1 conditioned contour plot (Figure 53) most closely approximates actual conditions (Figure 14). Regarding the
FIGURE 53.
WIN1-Conditioned Contour
FIGURE 54.
WIN2-Conditioned Contour
other three plots, it may be stated that the WIN4 plot (Figure 19) gives
a more detailed version of the WIN6 plot (Figure 31) and the WIN2 plot
(Figure 54) gives a more detailed version of the WIN4 plot.

At 50 m above ground level the input velocity fields for WIN1, WIN2
and WIN4 runs are given by Figures 55, 56 and 21 respectively. Since the
least-squares method interpolation scheme uses the delta x and delta y
parameters, the three figures all look quite different. The same input
velocities at each data station are used for all three runs. Since WIN1
uses a delta y equal to twice its delta x value, the input field appears
stretched in the y direction. The input field associated with the WIN2
run appears stretched in the x direction since it uses a delta x value
equal to twice its delta y value. Figure 21, though, since it uses a
delta x value which equals its delta y value, is a compromise between
Figure 55 and Figure 56. Figures 33 which shows the input field used for
the WIN6 run almost duplicates that given by Figure 8. For the WIN6
plot, delta x equals delta y. The WIN4 and WIN6 input fields give a more
acceptable interpretation of actual wind fields.

For each run, at 100 m above ground level, interpolated wind fields
duplicate trends seen for plots at 50 m above ground level. The
magnitudes of corresponding isolach curves is calculated using a shear
coefficient equal to .4. Plots for WIN1 and WIN2 are given in Figures 57
and 58 respectively. For WIN4 and WIN6 runs, plots may be seen in
Figures 25 and 39 respectively.

Adjusted velocity fields output from WIN5 and WIN6 runs at 50 m above
ground level are shown in Figures 23 and 35 respectively. The number of
local velocity gradients calculated from the WIN4 run is greater than
FIGURE 55.
WIN1-Interpolated Velocity Field at 50 m. Above Ground Level
FIGURE 56.
WIN2-Interpolated Velocity Field at 50 m. Above Ground Level
FIGURE 57.
WINME-Interpolated Velocity Field
at 100. m. Above Ground Level
FIGURE 58.
WIN2-Interpolated Velocity Field at 100 m. Above Ground Level
that calculated from the WIN6 run. Since velocity gradients occur on the first level above ground at positions where contour lines are at right angles to each other, it follows that more local gradients exist in the output from the WIN4 run than from the WIN6 run. It is noted that the area covered by local velocity gradients is always proportional to grid area. At the locations where major discrepancies exist between conditioned contour plots (Figures 19 and 31), especially south of site 3 and between sites 1 and 5, major discrepancies also exist between the output velocity fields (Figures 23 and 35). South of site 3, the WIN4 run predicts a local velocity gradient (Figure 23) at the position where a decrease in elevation exists (Figure 19), while the WIN6 run predicts a smooth transition (Figure 35) since it sees a flat surface (Figure 31). For WIN4, a choppy, disjointed wind speed transition is shown (Figure 23) from sites 5 to 1 while WIN6 displays a smooth but steep wind speed increase (Figure 35). Since input velocities for both runs are the same and since conditioned contour plots show the same trends, adjusted velocity fields give the same general information.

The adjusted velocity field provided by the WIN2 run is shown in Figure 60. This plot most closely resembles Figure 23. For both plots, the locations of local velocity gradients is approximately the same. Since input fields for WIN2 (Figure 56) and WIN4 (Figure 21) runs differ, the overall trends shown by both plots differ. In Figure 23, north of site 1 the 10 mph isotach line extends farther north than the corresponding 10 mph isotach line shown north of site 1 in Figure 60.

Figure 59 shows the output wind field from the WIN1 run. When comparing this plot to that given by the WIN2 run (Figure 60) one sees
FIGURE 59.
WINI-Adjusted Velocity Field at 50 m Above Ground Level
FIGURE 60.
WIN2-Adjusted Velocity Field at 50 m. Above Ground Level
that the density of local velocity gradients is equivalent for both runs. Grid areas used for both runs are also equal. The two plots, however, look very different. The area located to the south of site 3 is noted to demonstrate this difference. Figure 59 shows a large negative velocity gradient which corresponds to the elevation decrease from 2132' to 1968.5' (Figure 53) for the WIN1 run. For the WIN2 run only a small local velocity gradient exists (Figure 60) since the area is assumed mostly flat (Figure 54). In the northern half of the area in question, for Figure 59, isotach lines defining velocity gradients exist, whereas wind velocity magnitude is shown to be constant for a major portion of the said area in Figure 60. By referring to the input velocity plots, one may trace the existence of isotach lines shown for WIN1 (Figure 59) back to its origin (Figure 55). Similarly, observance of Figure 56 show that the northern portion of the MATHEW area is originally assumed flat for WIN2.

The output velocity fields given by WIN4 (Figure 27) and WIN6 (Figure 39) runs appear very similar at 100 m above ground level. Since local velocity gradients disappear at the second level above ground, discrepancies due to contour details are less prominent here. Figures 61 and 62 show adjusted velocity fields output from WIN1 and WIN2 runs respectively. Again, the area south of site 3 shows that major differences in not only wind speed gradient trends but also wind speed magnitudes exist. These differences are born from major contrasts shown in Figures 59 and 60. In the central portion of the MATHEW area between \( x = 1216 \) and \( x = 1220 \) and \( y = 880 \) and \( y = 890 \), the two plots obviously disagree. For WIN2 the adjusted wind speed value remains relatively
FIGURE 61.
WIN1-Adjusted Velocity Field at 100 m. Above Ground Level
FIGURE 62

MINI-ADJACENT VELOCITY FIELD

at 100' m. above ground level
constant at 12 mph (Figure 62) but for WIN1 wind speed values range from 14 to 18 mph. One may note that this area is the maximum distance away from all data stations and thereby is strongly affected by differences in the interpolation algorithm (Figures 57 and 58).

Conclusions made from Test III runs are as follows:

1. Changing delta x and delta y grid block parameters changes the way MATHEM interprets topography. The smaller the grid block value, the more detail as to exactly where terrain gradients exist is given. By using a larger delta x and delta y value, the general topographic trends are recorded but details are omitted.

2. Changing delta x and delta y values changes the means by which input data is interpolated. Using a delta x value which is larger than delta y tends to stretch the field in the x direction and visa versa. The use of equivalent delta x and delta y values gives a more evenly dispersed input velocity scheme.

3. The smaller the delta x and delta y value, the more local velocity gradients will occur on the first level above ground.

4. Since velocity adjustment depends on not only the conditioned contour but also the input velocity field, for the most realistic output it is suggested that the delta x and delta y not only be small (so that terrain gradients locations are detailed) but also be equal (since input fields should show an even dispersion of velocity values).
4.5 **TEST IV**

Test IV compares two runs, PRINC1 and PRINC2 for the MATHEW area of interest outlined in Figure 63. Figure 64 shows a larger, less detailed terrain contour plot of the MATHEW area used. In Test IV, by comparing two runs with different grid block volumes (not only delta x and delta y but also delta z is changed), the means by which the MATHEW program interprets input data via input parameters is examined. Essentially, Test IV combines Test II and Test III to show the complete effect of grid block geometry changes. PRINC1 which uses a delta x = 443.8 m, delta y = 697.4 m and delta z = 25 m with M, N and L values equal to 11, 11 and 22 respectively. The PRINC2 run uses delta x = 190.2 m, delta y = 317 m and delta z = 50 m with M, N and L = 25, 25 and 11 respectively.

Input topographic information for PRINC1 is shown in Figure 65 and for PRINC2 in Figure 66. The terrain contour plot shows that the MATHEW area has a gently sloping valley stretching from the northwest (at 1000') with decreasing elevation towards the southeast (at 800'). In the southwest an average ground level height of 1100' exists. The topography decreases to a value of 900' in the valley and then rises to 1200' in the northeast direction. The highest terrain point is located at a hilltop in the northeast sector of the MATHEW area. Figure 65 shows that a great deal of averaging has occurred. In fact for every MATHEW grid area, 308 digital terrain tape points have been used. The size of the highest topped hill has been made disproportionally large while the 800' plateau in the southeast has been virtually neglected. Overall, however, the most prominent trends are still retained. Since PRINC2 uses a small grid
FIGURE 64.

Prineton-Terrain Contour
FIGURE 65.
PRINCI-Input Contour
FIGURE 66.

PRINC2-Input Contour
block area (only 60 digital terrain tape values are used in averaging topographic height), as shown in Figure 66 more topography details may be retained. The hilltop mentioned earlier is now at its proper size but its location is somewhat misplaced. The 800' plateau in the southeast is recognized and all trends are recorded with adequate detail.

Topographic information detailing the conditioned contours are represented in Figure 67 for PRINC1 and in Figure 68 for PRINC2. The plots are extremely different. The major factor contributing to their unlike appearances is the grid level height, delta z. The fact that input topography information for both plots are not the same also contributes. In Figure 68, most of the topography is seen as flat. The plane which travels from the northwest to the southeast is elevated at 984', and the major portion of the south by southeast is at 820'. In Figure 67, however, the gradients are more smoothly defined. Plateaus are portrayed as large (approximately .8 km²) segments. As compared to Figure 68, it is seen that the 820' plateau covers one quarter the area. The 984' level plane described in Figure 68 is now divided into five segments, three of which are at 984', one at 902' and one at 1066'. A more gentle gradient is seen in the southeastwardly decreasing valley. Figure 67 gives a more accurate interpretation of actual topography.

Isotach curves which define the interpolated velocity field at 25 m above ground for PRINC1 are shown in Figure 69. The adjusted velocity field for the corresponding level may be found on Figure 70. It is noted that the local velocity gradients appear to be so large since MATHEW grid area is large. These local gradients occur at the edges of topographic gradients. Since there exist so many contour lines and because the grid
FIGURE 67.

PRINCI-Conditioned Contour
FIGURE 68.
PRINC2-Conditioned Contour
FIGURE 69.
PRINC1-Interpolated Velocity Field at 25. m. Above Ground Level
FIGURE 70.

PRINCl-Adjusted Velocity Field at 25. m. Above Ground Level
area is so large, the major portion of the MATHEW area experiences the local velocity gradients. The area at which the highest wind speed is said to occur on Figure 70 is that surrounding site 5. On Figure 69, this site also sees the highest wind speed. Since Figure 67 shows site 5 located on a plane the windspeed was not adjusted a great deal. Sites 1, 2 and 3 occur where topographic gradients exist, according to Figure 67 and velocity magnitudes for these stations are seen to decrease from Figure 69 to Figure 70.

Figures 71 and 72 show input wind fields for PRINC1 and PRINC2 runs respectively. Although input velocity data and data station locations are the same for both runs, the input fields do not duplicate each other. This is because data station locations must be positioned in terms of MATHEW grid positions. Since grid position is a function of grid area, the station locations for each run differ.

Adjusted wind fields for PRINC1 and PRINC2 runs are shown in Figures 73 and 74 respectively at 50 m above ground level. The two plots are seen to appear extremely different although similar trends may be observed. Since PRINC2 recognizes steep topography gradients, local velocity gradients are concentrated. PRINC1, (Figure 73), however, displays adjusted wind values on the second level above ground. The isotach curves show a smooth and flowing wind pattern that only minimally resembles its corresponding input plot (Figure 71). For PRINC2 at the locations where topography is assumed flat (Figure 72) velocity adjustment is small and the adjusted isotach curves duplicate the corresponding input curves. For both plots certain similarities exist. In the central south portion of the MATHEW area for both plots is shown
FIGURE 71.
PRINC1-Interpolated Velocity Field at 50. m. Above Ground Level
FIGURE 72.
PRINC2-Interpolated Velocity Field at 50. m. Above Ground Level
FIGURE 73.
PRINCI-Adjusted Velocity Field at 50. m. Above Ground Level
FIGURE 74.

PRINC2-Adjusted Velocity Field at 50. m. Above Ground Level
an 11 mph velocity isotach curve located where elevation decreases from 984' to 820' (Figures 67 and 68). Also, in both Figures 73 and 74, the three areas which located the maximum winds speeds are found to be positioned the same. The highest velocity values may be found in the areas of stations 5, 2 and 3. The maximum wind speeds occur in the area of site 5 (Figures 73 and 74) which is in the northwest section of the valley (Figures 67 and 68). Referring to Figure 61, one may note that for PRINC2, the area surrounding station 3 contains the maximum density of local velocity gradients. Comparison of adjusted wind values for the PRINC1 run (Figure 73) for the same area shows that here also, the velocity has been greatly adjusted. Wind speed values for PRINC1 range from 11 to 12 mph (Figure 73) while wind speed values as low as 9 mph are seen for PRINC2 in the area surrounding site 3. The large values of negative velocity gradients seen for PRINC2 are attributed again to the large topographic gradients experienced. The adjusted wind field given for PRINC run shows a smoother transition of adjusted velocity gradients and gives a more acceptable interpretation of the interpolated wind field.

Figures 75 and 76 contour input wind fields for the PRINC1 and PRINC2 runs at 100 m above ground level. For both plots isotach pattern trends duplicate those given for 50 m above ground and the magnitude of the isotach curves increases accordingly using a shear coefficient equal to .25.

The adjusted wind field at 100 m above ground for PRINC1 is shown in Figure 77. One may see that this diagram closely resembles the input velocity isotach plot (Figure 75) for PRINC1. Figure 78 shows the
FIGURE 75.
PRINCI-Interpolated Velocity Field at 100. m. Above Ground Level
FIGURE 76.

PRINC2-Interpolated Velocity Field at 100. m. Above Ground Level
FIGURE 77.
PRINCI-Adjusted Velocity Field at 100. m. Above Ground Level
FIGURE 78.

PRINC2-Adjusted Velocity Field at 100 m. Above Ground Level
adjusted wind field at 100 m above ground for the PRINC2 run. One may note that the area which contained local velocity gradients at 50 m above ground (Figure 74) now shows choppy fragmented isotach lines (Figure 78) in the area surrounding site 3. At each site, the magnitude of velocity values is approximately the same for both Figures 77 and 78. Figure 78 shows that velocity adjustments occur in areas designated as having large contour gradients while Figure 77 shows that minimal velocity adjustments are made for a much larger portion of the MATHEW area.

Conclusions made from Test IV are as follow:

1. All conclusions from Test II and Test III apply to this test.

2. When using a larger grid area, the averaged topography contains less details. The smaller the grid area chosen the fewer digital terrain tape elevation values used for each segment and the more closely actual conditions are approximated.

3. Choosing a smaller grid level height value allows the conditioned topography to be divided into more grid level heights and allows a smoother topography elevation transition. The larger the delta z value, the fewer level heights defined by the conditioned contour and the larger the magnitude of topography gradients.

4. Choosing a small grid area makes local velocity gradients occur in a smaller area and appear isolated. The larger the grid area, the larger the local velocity gradients and the larger the proportion of area covered by these gradients.

5. The larger the topographic gradient, the larger the magnitude of the local velocity gradients.

6. Using a large delta z value and a small delta x, delta y value does result in adjusted velocity fields containing isolated local velocity gradients which cover a small area and have a large magnitude. Using a small delta z value and large delta x, delta y values allows the adjusted velocity field to contain local velocity gradients which cover a large area, have smaller magnitudes, and which contribute to the continuity of the isotach pattern. In this way, more segments of the MATHEW area contain velocity values which have been adjusted but the magnitude of the adjustment is less.
7. As height level above ground increases, the adjusted velocity field more closely resembles the interpolated velocity field.

8. Grid block volumes with small heights and large areas allow the MATHEW program to adjust wind velocity values in a more comprehensive and continuous manner.
The MATHEW area of interest for all runs included for this test is shown in Figure 64. In all runs compared the grid size, M, N and L values are the same. For each run used, a predetermined number of data stations are deleted from each interpolation scheme. All runs are compared against the PRINC3 run which includes all five data stations. Test V shows how MATHEW's adjusted wind fields respond to data station deletions. PRINC3 thru PRINC7 runs delete zero thru four data stations respectively; PRINC8 deletes one station and PRINC9 deletes four. All runs use delta x = 190.2 m, delta y = 317. m and delta z = 37.5 m. The M, N and L values for all runs are 25, 25 and 15.

The input contour plot used for all runs is seen on Figure 66. The conditioned contour plot is given by Figure 79. For the PRINC3 run, the interpolated wind fields using all five data stations are shown at 37.5 and 75 m in Figures 80 and 81 respectively. The output velocity field for levels 1 and 2 are given by Figures 82 and 83 respectively. For level 1 in Figure 82 local velocity gradients exist near contour gradients and for level 2 (Figure 83) positions for which local velocity gradients did exist show that the wind field has been smoothed out but topography effects are still apparent.

The PRINC4 run conditions terrain in the same way as did PRINC3 (Figure 79), but deletes station 5 from the input field. Figures 84 and 85 display the interpolated field for WIND4 at 37.5 and 75 m respectively. Adjusted wind fields for the first two levels above ground are given by Figures 86 and 87. When compared with Figures 82 and 83, one finds that
FIGURE 79.

PRINC3-Conditioned Contour
FIGURE 80.
PRINC3-Interpolated Velocity Field at 37.5 m. Above Ground Level
FIGURE 81.
PRINC3-Interplolated Velocity Field at 75 m. Above Ground Level
FIGURE 82.

PRING3-Adjusted Velocity Field at 37.5 m. Above Ground Level
FIGURE 83.
PRINC3-Adjusted Velocity Field at 75. m. Above Ground Level
FIGURE 84.
PRINCO-Interpolated Velocity Field at 37.5 m. Above Ground Level
FIGURE 85.
PRINCA-Interpolated Velocity Field at 75 m. Above Ground Level
FIGURE 8G.
PRINC4-Adjusted Velocity Field at 37.5 m. Above Ground Level
FIGURE 87.
PRINC4-Adjusted Velocity Field at 75. m. Above Ground Level
the corresponding adjusted isotach patterns duplicate themselves almost precisely. Stations 1 and 5 are surrounded by the position for which discrepancies occur. The location and shapes of all local velocity gradient patterns is the same in both Figures 82 and 86. Since interpolated velocity values surrounding station 1 and 5 (Figures 80 and 84) differ, the adjusted velocity values (Figures 82 and 86) also differ, especially in the domain where the area is assumed flat (Figure 79). Figure 88 shows an isotach plot of the absolute value of the differences between adjusted velocity values output from PRINC3 and PRINC4 runs. This plot shows that although the pattern displayed in Figures 32 and 86 are quite similar, the absence of the site 5 velocity value from the PRINC4 run causes the magnitude of the output velocity values to differ greatly from that output by PRINC3. As seen from Figure 88, the maximum value of velocity difference is 3.5 mph located slightly to the west of site 5. At 75 m above ground, adjusted isotach patterns tend to more closely agree with corresponding input patterns than at 37.5 m above ground. In the area surrounding site 5, PRINC4 isotach lines (Figure 87) correspond to those in Figure 85 and for PRINC3 lines in Figure 83 correspond to those in Figure 81. Adjusted velocity values at 75 m above ground for PRINC4 are approximately 11 mph as compared to 15 mph for PRINC3 at site 5.

Site 2 is deleted in the PRINC8 run, as may be seen in Figures 89 and 90. At 37.5 m above ground, the input velocity value of 13.8 mph (Figure 81) for the PRINC3 run now becomes 10.2 mph (Figure 89) for the PRINC8 run at site 2. Adjusted velocity fields given by PRINC8 at 37.5 and 75 m above ground are shown in Figures 91 and 92. The plot showing the
FIGURE 88.

Isotach Difference Between Adjusted Velocity Values Output from PRINC3 and PRINC4 runs at 37.5 m. Above Ground Level
FIGURE 89.

PRINCS-Interpolated Velocity Field at 37.5 m. Above Ground Level
FIGURE 90.
PRINCS-Interpolated Velocity Field at 75 m. Above Ground Level
FIGURE 91.
PRING8-Adjusted Velocity Field at 37.5 m. Above Ground Level
FIGURE 92.
PRINCS-Adjusted Velocity Field at 75. m. Above Ground Level
velocity difference between adjusted velocity values output from PRINC8 and PRINC3 (Figure 93) show that the largest velocity discrepancies occur in the area surrounding station 2. A maximum velocity difference of 2.2 mph is given. As the distance away from site 2 increases, the magnitude of velocity difference decreases. Velocity differences also exist in the northeast quadrant of the MATHEW area. Since interpolated velocity values in the northeast for the PRINC3 run are weighted most strongly by the value assigned to station 2 (Figure 80), the absence of this station does indeed induce changes in the velocity values assigned to the northeast (Figure 89). Again at 75 m the most prominent differences in isotach trends for adjusted velocity fields (Figures 83 and 92) are seen to exist because of differences in pattern trends for input fields (Figures 81 and 90).

The PRINC5 run neglects not only values given for station 5 but also for those given for station 1. At 37.5 and 75 m above ground level, interpolated velocity fields are shown in Figures 94 and 95 respectively. Figures 96 and 97 show adjusted velocity fields for the PRINC5 run at 37.5 and 75 m respectively. As seen in Figure 98, the maximum magnitude difference between output values of PRINC3 and PRINC5 runs is seen to be 3.6 mph. The two positions where outstanding magnitude differences occur are at the centers of the concentric isotach lines, where site 1 and site 5 are located.

The PRINC6 run uses only two data station, as may be seen in Figure 99 and 100. Stations 2 and 3 are seen as the centers of the concentric isotach lines which define the input velocity field for the PRINC6 run. Sites 5, 2 and 4 have been deleted from this run. At 37.5 m above ground
FIGURE 93.
Isotach Difference Between Adjusted Velocity Values Output from PRINC3 and PRINC8 at 37.5 m. Above Ground Level
FIGURE 95.
PRINC5-Interpolated Velocity Field at 75 m. Above Ground Level
FIGURE 96.

PRINC5-Adjusted Velocity Field at 37.5 m. Above Ground Level
FIGURE 97.

PRINCS-Adjusted Velocity Field at 75 m. Above Ground Level
the adjusted velocity field is shown in Figure 101 and at 75 m in Figure 102. As may be expected, Figure 103 shows that maximum magnitude differences occur at the location where wind data has been deleted.

The PRINC7 run uses only a value of 12.46 mph at 37.5 m as input data. Since all grid positions are then assigned this value, the input velocity field plots is not required. Figure 104 shows the adjusted velocity field at 37.5 m above ground level and Figure 105 shows velocity at 75 m elevation. It is seen from both these plots that only local velocity gradients are recognized. Isotach differences between output values obtained from PRINC3 and PRINC7 are plotted in Figure 106 for 37.5 m above ground. On this figure, the area surrounding site 2 shows a decreasing velocity difference with a minimum difference equal to zero mph. Velocity differences at the other four stations range from .4 mph at site 5 to 5.5 mph at site 4.

Similarly, PRINC9 uses only data taken from site 4, so that initial wind speed for all locations is assumed equal to 7.43 mph at 37.5 m above ground. For PRINC9 adjusted velocity fields at 37.5 m and 75 m are plotted in Figures 107 and 108 respectively. The velocity difference at grid locations between PRINC3 and PRINC9 outputs is shown on Figure 109. The output velocity plots of PRINC9 (Figures 94 and 95) closely resemble those for PRINC7 (Figures 107 and 108). The runs are essentially the same in that only local velocity gradients are recorded but they are different because the magnitude of the velocity gradients (which is a function of the input velocity at the area of interest) are different.

Figure 110 shows an isotach plot which records the difference between adjusted velocity values output from PRINC4 and PRINC8 runs. Since the
FIGURE 98.

Isotach Difference Between Adjusted Velocity Values Output from PRINC3 and PRINC5 at 37.5 m. Above Ground Level
FIGURE 99.
PRINC6-Interpolated Velocity Field at 37.5 m. Above Ground Level
FIGURE 100.

PRINC6-Interpolated Velocity Field at 75 m. Above Ground Level
FIGURE 101.

PRINC6-Adjusted Velocity Field at 37.5 m. Above Ground Level
FIGURE 102.
PRINC6-Adjusted Velocity Field at 75. m. Above Ground Level
FIGURE 103.
Isoach Difference Between Adjusted Velocity Values Output from PRINC3 and PRINC6 at 37.5 m Above Ground Level.
FIGURE 104.
PRINC7-Adjusted Velocity Field at 37.5 m. Above Ground Level
FIGURE 105.
PRINC7-Adjusted Velocity Fields at 75. m. Above Ground Level
FIGURE 106.
Isotach Difference Between Adjusted Velocity Values Output from PRINC3 and PRINC7 at 37.5 m. Above Ground Level
FIGURE 107.
PRINC9-Adjusted Velocity Field at 37.5 m. Above Ground Level
FIGURE 108.
PRINC9-Adjusted Velocity Field at 75. m. Above Ground Level
velocity value at site 5 is not recognized by PRINC4 and the velocity value at site 2 is not recognized by PRINC8, they are the locations where maximum velocity difference occurs.

Figure 111 shows the velocity difference between outputs from PRINC7 and PRINC9 runs. Since PRINC7 includes only site 2 and PRINC9 considers only site 4, the velocity difference for all station is quite large ranging from 3.5 mph at station 4 to 5.6 mph at station 5. The root-mean-square difference at 37.5 m is 5.09 mph.

Conclusions made from Test V are as follows:

1. Changing the input velocity field does dramatically change the adjusted velocity field.

2. The pattern of isotach curves output from a MATHEW run depends on two things: the pattern of the input velocity field and the pattern of conditioned contour lines. For a specified pattern of conditioned contour lines (this implies specified delta x, delta y, delta z, etc.), the local velocity gradients will always occur in the same positions of the MATHEW area and will always occupy the same amount of space. The magnitude of the velocity gradient is dependent on the assigned velocity for the specified location. Where the MATHEW program assumes flat terrain, velocity adjustment is negligible and the adjusted wind field duplicates the input wind field.

3. The MATHEW program is not able to predict the wind speed values at a deleted site. The wind data taken in the field reflects not only conservation of mass, but also considers the conservation of energy and the conservation of momentum.

4. The more data stations used to provide velocity values for the interpolation scheme, the more realistic is the interpretation of actual conditions. It appears that the optimal number of data stations is greater than five. It is highly suggested that a detailed tour of the area be conducted so that the most appropriate positions of data station locations may be determined. In this way, the interpolated wind field may provide an accurate representation of actual conditions. Possibly the user of MATHEW will desire to record wind speed values at the highest and lowest terrain levels in the MATHEW area.
FIGURE 109.
Isotach Difference Between Adjusted Velocity Values Output from PRIN3 and PRINC9 at 37.5 m. Above Ground Level
FIGURE 110.

Isotach Difference Between Adjusted Velocity Values Output from PRINC4 and PRINC8 at 37.5 m. Above Ground Level
FIGURE 111.
Isotach Difference Between Adjusted Velocity Values Output from PRINC7 and PRINC9 at 37.5 m. Above Ground Level
CHAPTER V

RESULTS, CONCLUSIONS AND RECOMMENDATIONS

5.1 RESULTS

5.1.1 Mathematical Analysis. For each MATHEW run, the difference between input and adjusted velocity values at each corresponding grid position is calculated for specified levels above ground. It has been found that for all runs, certain trends pertaining to velocity adjustment and above ground level exist. As shown in Figure 112, velocity difference calculations for the PRINC2 run are used to demonstrate these trends. UDIF, UMAX, and URMS are defined in section 3.3. For all levels, the minimum velocity difference (UMIN) equals zero. As level above ground increases, not only the magnitude of the maximum velocity difference (UMAX) but also the root-mean-square velocity difference (URMS) decreases exponentially.

It is also seen from Figure 112 that the number of grid positions which have a small velocity difference (UDIF is greater than zero but less than .1 mph) increases as the level above ground increases. In this way, it is seen that as the level increases, the velocity adjustment decreases.

Since the magnitude of velocity adjustment decreases as elevation increases, an examination of the adjusted wind shear coefficient is
FIGURE 112.

PRINCI2: Velocity Difference Between Interpolated and Adjusted Values vs. Level Above Ground
### TABLE 5. Mathematical Analysis Summary Sheet

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<th>Input $\alpha$</th>
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<th>MIN $\alpha$</th>
<th>AVE $\alpha$</th>
<th>URMS AVE (mph)</th>
<th>URMS MIN (mph)</th>
<th>URMS MAX (mph)</th>
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performed. For each grid position in the MATHEW area, all above ground velocity values are used to curve fit a shear coefficient value. In this way, the product of M times N shear coefficient values are determined for each MATHEW run. On Table 5 may be found the input shear coefficient and the maximum $\alpha$, minimum $\alpha$ and average value of $\alpha$, the adjusted shear coefficient. It may be seen from this table that the average adjusted shear coefficient consistently agrees with the input shear value. This fact implies that conservation of momentum and conservation of energy are are maintained.

Table 5 also gives for each run the average URMS, the minimum URMS, and the maximum URMS over the first five levels above ground. These values are defined in section 3.3. Note that for each site (Princeton and Windsor), the average root-mean-square velocity difference value has little deviation.

The total number of iterations (ITER) required to calculate Lagrangian multiplier values for internal grid position (i.e. solve the partial differential equation) is also recorded. It may be seen that all ITER values required are approximately the same for all runs except for for those which require a small delta $z$ value. The total number of grid positions is also recorded for all runs.

5.1.2 Algorithm Inspection. The most obvious trend which exists for all MATHEW runs is that where topographical gradients are large, the velocity flow is adjusted a great deal and where the topography is assumed level (or flat) adjustment of velocity values is negligible.
Velocity adjustments for the first level above ground have the highest magnitude and local velocity gradients exist there. Since MATHEW adjusts velocity values using as a constraint the conservation of mass equation, the trend is easily accepted. An investigation of the numerical cause of this trend and the existence of local velocity gradients at the first level above ground is performed by examining the numerical technique procedure used in MATHEW. In the investigation values calculated for all interior grid points are observed. It is noted here that to calculated adjusted velocity values equation 21 is used.

\[ u = u^0 + \frac{1}{2\alpha_x^2} \frac{\partial \lambda}{\partial x} \]

In MATHEW, the adjusted wind fields are calculated using a difference formalism for position j in the y direction and k in the z direction.

\[ u_i = \frac{1}{4} (u_{i+1}^0 + 2u_i^0 + u_{i-1}^0) + \left( \frac{\lambda_{i+1} - \lambda_{i-1}}{\Delta x} \right) \frac{1}{2\alpha_x^2} \]  \hspace{1cm} (43)

The first term in the right-hand side of the above equation may be referred to as the averaged initial velocity. The adjustment of the initial velocity, \( u_i^0 \), may be due to either (1) a large \( \Delta \lambda \); or (2) a large velocity difference in the neighboring grid positions at \( i-1 \) or \( i+1 \), which will give a significantly different value for the averaged initial velocity.

For case (2), it may be noted here that when a large topographical gradient exists, there exists a large velocity difference between
neighboring grid points. For example, if the MATHEW program sees the topography illustrated here, calculation of the averaged initial velocity occurs in the following way. If at position $i$ the initial velocity equals 2.65 m/s and at position $i-1$ $u_o$ also equals 2.65 m/s, the averaged velocity equals

$$u_i^o = \frac{1}{4} (2.55 + 2(2.55) + 0)$$

The velocity is assigned equal to zero at position $i+1$ since it is located below ground level and the average initial velocity value equals 1.9875 m/s. In this way,

$$u_i = 1.9875 + \frac{\sigma^2}{2} \frac{1}{\Delta x} (\lambda_{i+1} - \lambda_{i-1})$$

At the boundaries, the Lagrangian multiplier difference is at its largest since its value is large at boundaries and small for neighboring flat terrain positions. The multiplier value is directly related to the first derivative of initial velocity values (i.e. $u_{i+1} - u_{i-1}$) which is largest at boundaries and smallest for flat terrain conditions. The Lagrangian multiplier gradient, however, contributes little to velocity adjustment on the first level. At boundaries, for any given velocity difference between initial and adjusted values, the inclusion of the
Lagrangian derivative contributes about $4\%$ to the change and the averaging algorithm contributes $96\%$. For the above example (taken from PRINC9 run) the Lagrangian multiplier affect a maximum change equal to $0.02 \text{ m/s}$ and the averaging technique affects a change equal to $0.8 \text{ m/s}$.

It is then shown that the primary and most effective cause for adjusting velocity values is the averaging procedure which outweighs the effects of the Lagrangian multiplier by a factor of $10$. This observation demands much attention since it is responsible for establishing the adjustment trend apparent in all MATHEW runs.

It has already been noted that isotach curves on MATHEW plots in two dimensions look almost exactly the same as plots in three dimensions. This is because the vertical velocity component is initially assumed equal to zero. The adjustment of the vertical velocity component is due solely to the first derivative of the Lagrangian multiplier (and not to averaged initial velocity values) and is very small. Its contribution to adjusting the magnitude of velocity vectors is negligible.
5.2 **CONCLUSIONS**

By summarizing observations made in Tests I thru V, conclusions on the geometric, data and storage space limitation of MATHEW are made. Observations which pertain to all MATHEW runs and those specific to individual tests are discussed. Table 6 summarizes all conclusions obtained from observations of test results. The thirteen conclusions listed here define the relationship between input parameters and MATHEW's interpretation of input data.

Since where MATHEW assumes the topography to be flat velocity adjustment is negligible, and because for potential areas of interest the topography may rarely be considered flat, the optimum conditioned topography is one which shows gradual contour gradients and which closely approximates actual conditions. Ideally, a small delta z value is desired (i.e. approximately equal to 10 m). In this way, gradients of 10 m or more would be recognized by the MATHEW program. It is obvious from all analyses that the smaller the grid blocks used the more closely actual topographic conditions are represented, since the amount and geometry of the grid block used determines how topography is conditioned.

Since the MATHEW model is limited in that the number of grid block allowed in the MATHEW box is maximized by computer storage space available, the means by which topography is massaged is also limited. The memory storage space available for the University of Massachusetts Cyber system can not viably contain arrays with 13,750 positions. An array of this size must be used for a MATHEW run with $M = 25$, $N = 25$ and $L = 22$. As may be seen from Table 5 all MATHEW runs performed use far fewer
Table 6. Conclusion Summary Table

1. The MATHEW area should be larger than the actual area for which velocity information is desired.

2. A small delta z allows MATHEW to interpret terrain conditions in a detailed way. Small delta x and delta y give more detail as to where topographical gradients exist.

3. Delta x and delta y should be equal to insure an even dispersion of interpolated velocity values.

4. $M = 11$ and $N = 11$ should be the lowest limits chosen for a particular run.

5. A grid block volume with a small height and a large area allows MATHEW to adjust wind velocity values in a comprehensive and continuous manner.

6. The manner by which MATHEW adjusts velocity is a function of topography, but the amount to be adjusted and the final adjusted value is a function of the input velocity assigned.

7. The number of data stations used should not be less than five.

8. Locations where highest and lowest wind speeds are anticipated should be chosen as data collection sites.

9. A maximum number of grid blocks should be equal to 12,000.

10. Where topography is assumed flat by MATHEW, velocity adjustment is negligible.

11. As the level above ground increases, adjusted velocities begin to duplicate interpolated velocities.

12. The first level above ground contains local velocity gradients.

13. The height above ground for which adjusted wind fields are desired should be between the second and fourth level.
grid blocks.

Observations of Test II note that a small delta $z$ is important since it allows the MATHEW program in interpret terrain conditions in a more detailed way (i.e. giving a more detailed picture of contour trends) while Test III shows that a small delta $x$ and delta $y$ value gives more detail as to exactly where topographical gradients exist. A trade off then exists between the grid block area and grid block height. Test IV confronts this dilemma by comparing MATHEW runs with geometrically differing grid block sizes. It is found that maintaining a grid block volume with small height and a large area allows the MATHEW program to adjust wind velocity values in a more comprehensive and continuous manner than maintaining a grid block volume with a large height and small area.

Another important observation derived from analyses of tests performed is that as the level above ground increases, adjusted velocity fields begin to duplicate interpolated velocity fields. It is then important to realize, before running MATHEW, the approximate height above ground for which the user desires wind velocity information. Given this height value, the delta $z$ value may be determined so that the desired height level is located between the second and fourth levels. Serious consideration of the adjusted velocity field at the first level above ground, because it houses many local velocity gradients, is discouraged.

From Test III it was found that delta $x$ and delta $y$ should be equal to insure an even dispersion of input velocity values. Conclusions from Test I imply that the MATHEW area of interest should be larger than the actual area for which velocity information is desired. In all cases, the topography located on the north, south, east and west borders of the
MATHEW area of interest is not genuinely described because of terrain averaging.

Having chosen a MATHEW area of interest, a delta z value and assuming the first temperature inversion layer is equal to approximately 550 m, a delta x and delta y value (which are equal) may be chosen which abide by storage space limitations. The MATHEW run, PRINC1, uses $M = 11$ and $N = 11$ and gives acceptable results. It is suggested that with due consideration of Test II results, neither M nor N values should be less than eleven.

Test V demonstrates the importance of determining a realistic interpolated velocity field. Although the manner by which MATHEW adjusts velocity is a function of topography, the amount of adjustment and the amount to be adjusted are determined by the input velocity assigned. The positions in the MATHEW area where the highest and lowest windspeeds are anticipated should then be the locations of velocity data stations. Hilltops and lowlands might be acceptable positions. For any given area, at least five data stations are suggested. It appears from Test V that the amount of input velocity data values has no upper limit and that the representativeness of the output wind field is directly related to the number of input data values. Also important is an even dispersion of data stations so that no grid position is located a large distance from the data station.
5.3 **RECOMMENDATIONS**

The analytic method used to derive MATHEW (calculus of variations used to adjust the initial velocity values) and a resulting dependence of output velocity values on subjective initialization are limitations of the model. Since the model assumes that initial wind fields need only to be minimally adjusted, the importance of having a truly representative input wind field is established.

Initialization of input data is partially subjective due to input parameter variability. Because all input parameters are indirectly related and values are limited by computer storage space, the optimal value of all input parameters values should be determined. The criteria, the attainment of truly representative physical input information, may be used for optimizing parameter values. A demonstration of the relationship between input parameters and MATHEW's interpretation of physical data serves as an aid for realizing the above constraint.

Since the results and conclusions offered in the previous section allow an understanding of this relationship, the attainment of a truly representative wind field is facilitated. As an additional aid to the user of MATHEW, the following table is given (Table 7). It recommends a schedule procedure to be used for input parameter determination. Because this schedule procedure is derived from the conclusions offered in the previous section, use of this schedule allows an accurate interpretation of wind conditions to be achieved.

The use of this procedure is recommended for all applications of the MATHEW program. However, because of MATHEW's limitations which require a
Table 7. Suggested Schedule Procedure for Input Parameter Determination

GIVEN: Area for which velocity information is desired.

CHOOSE: MATHEW area.

GIVEN: Height at which velocity information is desired.

CHOOSE: Delta z.

CALCULATE: L using an approximate value of 550 m equal to the first temperature inversion layer height.

GIVEN: M x N x L is less than 12,000.

CALCULATE: M and N remembering that delta x should equal delta z.

COLLECT DATA: Using no less than five stations, choosing positions where highest and lowest windspeeds are assumed to exist and by evenly dispersing the data stations so that no location is a large distance from a data station.
truly representative input wind field, the application of the model should be carefully considered. For the potential user of MATHEW, a trade off exists between the desired accuracy of described wind conditions and the energy and cost required to run the MATHEW program.

With due consideration of the labor and computer time required to produce the adjusted wind flow fields output from the MATHEW program, the wind prospector may decide that a representative, but less accurate interpolated wind field is sufficient.
BIBLIOGRAPHY


APPENDIX A - DOCUMENTATION OF MATHEW

SUBROUTINES

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It is noted that all references to Christine Sherman's text, 1978:
"MATHEW - A Mass-Consistent Wind Field Model," Lawrence Livermore Laboratory Report UCRL - 52479.
A1. **SUBROUTINE TOPOG:**

Transfers topography values from Tape 47 and changes heights to levels; expands point to fit MATHEW array. Levels are assigned to MATHEW grid cells.

1. Reads averaged topography from Tape 47 and puts data onto CHEIGHT(i,j).

2. Translates heights from CHEIGHT(i,j) into levels. Finds lowest surface level and sets that equal to MINLVL.

3. Adjusts heights so that lowest surface height now corresponds to zero level (i.e. what had been a range from 3 - 10 levels is now a range from 0 - 7 levels). MINLVL = 0.

4. Prints averaged surface level.

5. Expands MATHEW array such that the number of x and y coordinates are doubled. It assigns equal surface height values to the newly created to right and top right neighbors (see diagram). The LVL2DLX array becomes the LVL1DLX array. The surface level height for the LVL2DLX array at (1,1) becomes the level height at (2,2), (1,2), (2,1) and (1,1) for the LVL1DLX array; the surface level height for the LVL2DLX array at (2,2) becomes the level height at (4,2), (3,2), (4,1), and (3,1) for the LVL1DLX array. In the diagram to the right, the former LVL2DLX array is marked by o and the new LVL1DLX array is marked by x. The arrows represent changes that occur by transfering surface height levels from LVL2DLX to LVL1DLX. One may note that since LVL1DLX has dimensions twice as large as LVL2DLX, the number of point required by LVL1DLX is four times as large as the number of points required by LVL2DLX.
A2. **SUBROUTINE MATTOP:**

The value of the topography grid points of MATHEW are stored in the KMATTOP array. The surface level value of each grid point is set equal to the largest surface level of the four grid cells surrounding it. Values of grid cells are taken from the LVL1DLX array created in Subroutine Topog. In the diagrams to the right the former LVL1DLX array is marked by x and the new KMATTOP array is marked by o. The arrows represent changes that occur by transferring surface level heights from LVL1DLX to KMATTOP. The points of the four corners of the MATHEW square are adjacent to only one grid cell so it takes on those values. In other words,

\[
\begin{align*}
\text{KMATTOP}(1,1) &= \text{LVL1DLX}(1,1), \\
\text{KMATTOP}(M,1) &= \text{LVL1DLX}(M-1,1), \\
\text{KMATTOP}(1,N) &= \text{LVL1DLX}(1,N-1), \\
\text{KMATTOP}(M,N) &= \text{LVL1DLX}(M-1,N-1).
\end{align*}
\]

1. The grid points along the outer boundaries are adjacent to two points. The values of KMATTOP at the top and bottom edges of the MATHEW square are found. For example, KMATTOP(2,1) = LVL1DLX(1,1). But if LVL1DLX(2,1) > LVL1DLX(1,1), then KMATTOP(2,1) = LVL1DLX(2,2), and the same follows for all positions.
2. Values for left and right boundary grid points are determined as above.

3. Interior points: now grid points are chosen equal to the largest of four surrounding grid cells. The sequence is as follows:
   1. Compare height at positions (1,1) and (2,2).
   2. Choose largest value and call this winb.
   3. Compare heights at position (1,2) and (2,2).
   4. Choose largest value and call this wint.
   5. Compare winb and wint.
   6. Choose largest value and call this KMATTOP.

This sequence continues until all positions have been compared. The routine is shown in the figure below. Where

\[ \text{winb} = \text{the largest of these two values chosen.} \]

\[ \text{wint} = \text{largest of these two values chosen.} \]

\[ \text{KMATTOP} = \text{largest of \text{winb} and \text{wint}.} \]
4. Add one to each level.
5. Print the KMATTOP array.
A3. **SUBROUTINE GRIDES:**

Grid descriptors in each grid block are assigned to the MATHEW box using the array IGRIDS.

1. Assigns all IGRIDS blocks a value equal to -1.

2. Sets up interior indices. The value at the block defines its position in relation to the surrounding surface area. Surface area levels are given by KMATTOP. IGRIDS(i,j,k) for all interior indices has an assigned value greater than or equal to 0 and less than or equal to 31. See Christines Sherman's thesis, p. 23, for the table which correlates boundary assignment for interior grid points with grid descriptors. Note that when another IGRIDS value assigned is greater than or equal to 0 and less than or equal to 15, K = k-1 where k has an assigned range from KMATTOP+1 to L-1 and K = KMATTOP so that K = KMATTOP = (KMATTOP+1) - 1 = k-1 applies. In this way the k position is exactly one level above the terrain height at the (i,j) position. The diagram on the left shows a top view of grid position (i,j) and its neighbors. In this way we compare the level k (which ranges from KMATTOP+1 TO L-1) to the surface heights (KMATTOP) at surrounding grid points and at that grid point itself.

For the diagram above, if

\[
\begin{align*}
\text{KMATTOP}(i,j) &= k-1 \\
\text{KMATTOP}(i+1,j) &> k \\
\text{KMATTOP}(i-1,j) &> k \\
\text{KMATTOP}(i,j+1) &> k
\end{align*}
\]
One may refer to Sherman, p. 23, to find that $\text{IGRIDS}(i,j) = 31$. It should be noted that for $\text{KMATTOP}(i-1,j) > k(i,j)$, the boundary between these two positions is closed and for $\text{KMATTOP}(i+1,j) < k(i,j)$ the boundary between the two positions is considered open.

3. Set up indices for $x = 1$. If $\text{KMATTOP}(i-1,j) < k(i,j)$, then $\text{IGRIDS}(i-1,j) = 101$ or 102. In this way, the left hand boundaries have grid values equal to -1 if closed and 101 or 102 if open.

4. Set boundary indices at $x = N$. The same procedure as above except that $\text{IGRIDS}(i,j,k)$ equals -1 if closed and 103 or 104 if open.

5. Set boundary indices at $y = 1$. The same procedure as above except that $\text{IGRIDS}(i,j,k) = -1$. If a closed boundary exists and equals 105 or 106 if an open boundary exists.

6. Set boundary values at $y = M$. This is the same as above except indices assigned at $\text{IGRIDS}(i,j,k)$ equal -1 if closed and 107 or 108 if open.

7. Set boundary indices at $z = L$ and at $z = \text{KMATTOP}$. For $z = L$, a value of 109 is given. For $z = \text{KMATTOP}$, an IGRID value of 200 is initially assigned and then the appropriate number is added on which depends on the positions of its neighbors and the IGRIDS value at $\text{KMATTOP}+1$. 
A summary of steps 4-7 is given.

<table>
<thead>
<tr>
<th>GRID VALUE</th>
<th>BOUNDARY</th>
<th>CRITERION</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-31</td>
<td></td>
<td></td>
</tr>
<tr>
<td>101</td>
<td></td>
<td></td>
</tr>
<tr>
<td>102</td>
<td>x = 1</td>
<td>KMATTOP(i-1,j) &lt; k</td>
</tr>
<tr>
<td>103</td>
<td>x = M</td>
<td>KMATTOP(i+1,j) &lt; k</td>
</tr>
<tr>
<td>104</td>
<td></td>
<td></td>
</tr>
<tr>
<td>105</td>
<td></td>
<td></td>
</tr>
<tr>
<td>106</td>
<td>y = 1</td>
<td>KMATTOP(i,j-1) &lt; k</td>
</tr>
<tr>
<td>107</td>
<td></td>
<td></td>
</tr>
<tr>
<td>108</td>
<td>y = N</td>
<td>KMATTOP(i,j+1) &lt; k</td>
</tr>
<tr>
<td>109</td>
<td>z = L</td>
<td></td>
</tr>
<tr>
<td>200+x</td>
<td>z = KMATTOP</td>
<td>MANY</td>
</tr>
<tr>
<td>-1</td>
<td>ALL POINTS</td>
<td>BELOW</td>
</tr>
<tr>
<td></td>
<td></td>
<td>GROUND</td>
</tr>
</tbody>
</table>
A4. **SUBROUTINE SETUP:**

Sets up coefficients for the differential equation (equation 14, p. 22, Sherman). It defines, by using data statements, the constants $A1(1)$ thru $A1(32)$, $A2(1)$ thru $A2(32)$, etc. to $A9(1)$ thru $A9(32)$. $A1$, $A2$, etc. refer $A_m$, $B_m x \left(\frac{\Delta x^2}{\Delta y}\right)$, $C_m x \left(\frac{\Delta x^2}{\Delta z}\right)$ etc. in Sherman's equations. The array descriptor numbers, 1 to 32 refer to grid descriptor values, IGRIDS(i,j), assigned for specified boundary conditions in Subroutine Grides. Combining equations 27, 28 and 29 one obtains equation 39 for the $x$ direction

$$
\lambda_{i-1,j,k} - 2\lambda_{i,j,k} + \lambda_{i+1,j,k} = D_m \lambda_{i-1,j,k} - A_m \lambda_{i,j,k} + E_m \lambda_{i+1,j,k}
$$

We may look at only the $x$ direction since the $y$ and $z$ direction coefficients are derived in a similar way. Using Tables 2 and 3 of Sherman, we may develop the following table which gives us boundary condition cases verses coefficient. Following this table is shown for the $x$ direction how these coefficients are attained.
<table>
<thead>
<tr>
<th>CASE LETTER</th>
<th>D&lt;sub&gt;m&lt;/sub&gt;</th>
<th>A&lt;sub&gt;m&lt;/sub&gt;</th>
<th>B&lt;sub&gt;m&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>C3</td>
<td>2/3</td>
<td>2/3</td>
<td>0</td>
</tr>
<tr>
<td>C4</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C5</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>C6</td>
<td>2/3</td>
<td>2/3</td>
<td>0</td>
</tr>
<tr>
<td>C7</td>
<td>0</td>
<td>2/3</td>
<td>2/3</td>
</tr>
<tr>
<td>C8</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C9</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>C10</td>
<td>2/3</td>
<td>2/3</td>
<td>0</td>
</tr>
</tbody>
</table>
For the following examples, one should note that since only the \( x \) direction is of concern, only the \( i \) subscript is given. To calculate the constants, we know

\[
\frac{\partial^2 \lambda}{\partial x^2} = \frac{1}{\Delta x} \left( \frac{\partial \lambda}{\partial x_{i+\frac{1}{2}}} - \frac{\partial \lambda}{\partial x_{i-\frac{1}{2}}} \right)
\]

1. For case C1

\[
\frac{\partial \lambda}{\partial x_{i+\frac{1}{2}}} = \frac{\lambda_{i+1} - \lambda_i}{\Delta x}
\]

\[
\frac{\partial \lambda}{\partial x_{i-\frac{1}{2}}} = \frac{\lambda_i - \lambda_{i-1}}{\Delta x}
\]

Combining

\[
\frac{\partial^2 \lambda}{\partial x^2} = \frac{1}{\Delta x^2} \left( \lambda_{i+1} - 2\lambda_i + \lambda_{i-1} \right)
\]

\( D_m = 1 \)

\( A_m = 2 \)

\( E_m = 1 \)
2. For case C2

\[
\frac{\partial^2 \lambda}{\partial x^2} = \frac{1}{\Delta x^2} (\lambda_{i+1} - 2\lambda_i + \lambda_{i-1})
\]

\[
\frac{\partial \lambda_i}{\partial x_{i+1}} = 0
\]

\[
\lambda_{i+1} = \frac{4\lambda_i - \lambda_{i+1}}{3}
\]

\[
\frac{\partial^2 \lambda}{\partial x^2} = \frac{1}{\Delta x^2} \left( \frac{2}{3} \lambda_{i+1} - \frac{2}{3} \lambda_i + 0 \right)
\]

\[
D_m = \frac{2}{3}
\]

\[
A_m = \frac{2}{3}
\]

\[
E_m = 0
\]

3. For case C3

\[
\frac{\partial^2 \lambda}{\partial x^2} = \frac{1}{\Delta x^2} (\lambda_{i+1} - 2\lambda_i + \lambda_{i-1})
\]

\[
\frac{\partial \lambda_i}{\partial x_{i+1}} = 0
\]

\[
\lambda_{i+1} = \frac{4\lambda_i - \lambda_{i+1}}{3}
\]

\[
\frac{\partial^2 \lambda}{\partial x^2} = \frac{1}{\Delta x^2} \left( 0 - \frac{2}{3} \lambda_i + \frac{2}{3} \lambda_{i+1} \right)
\]

\[
D_m = 0
\]

\[
A_m = \frac{2}{3}
\]

\[
E_m = \frac{2}{3}
\]
4. For case $C_4$

\[
\frac{\partial^2 \lambda}{\partial x^2} = \frac{1}{\Delta x^2} (\lambda_{i-1} - 2\lambda_i + \lambda_{i+1})
\]

\[
\frac{\partial \lambda}{\partial x}_{i=1} = 0 = \frac{\partial \lambda}{\partial x}_{i=1}
\]

\[
\lambda_{i-1} = \lambda_i = \lambda_{i+1}
\]

\[
\frac{\partial^2 \lambda}{\partial x^2} = 0
\]

\[
D_m = 0
\]

\[
A_m = 0
\]

\[
E_m = 0
\]
A5. **SUBROUTINE PWLAW:**

Takes initial velocity values in both the x and y directions and expands to all levels (i.e. from KMATTOP+1 to the top level, L). PWLAW use the relationship

\[ u = u_0 \left( \frac{z}{z_0} \right)^\alpha \]

where \( u_0 \) is the initial velocity value at height \( z_0 \). Note that for all MATHEW runs performed, the reference height was assigned equal to 50 m. The variable \( u \) is the calculated velocity at height \( z \) and \( \alpha \) is the shear coefficient.
A6. **SUBROUTINE SETORIG:**

Assigns the TLAM array (to be used for storing the Lagrangian multiplier values) = 0. The right-hand side of the differential equation (equation 29) is calculated using interpolated velocity values. This procedure is performed for all interior grid points. The calculated values are stored in the ORIG array.

1. Sets all points in the TLAM and ORIG arrays = 0.
2. Calculates ORIG(i,j,k) for all interior points

\[
\text{ORIG}(i,j,k) = 2 \alpha_i^2 \Delta x^2 \frac{1}{2} \left( \frac{u^0(i+1) - u^0(i-1)}{\Delta x} \right)_{jk} \\
+ \frac{v^0(j+1) - v^0(j-1)}{\Delta y} \right)_{jk}
\]

where \(u^0\) and \(v^0\) are interpolated velocity values. Since

\[
\epsilon_0 = \frac{1}{2} \left( \frac{u^0(i+1) - u^0(i-1)}{\Delta x} \right)_{jk} + \frac{v^0(j+1) - v^0(j-1)}{\Delta y} \right)_{jk}
\]

Then

\[
\text{ORIG}(i,j,k) = \Delta x^2 2 \alpha_i^2 \epsilon_0
\]
A7. **SUBROUTINE SETUW:**

Averages interpolated velocity values at each grid position. These values will be later used in calculating adjusted velocity values. It is noted that this procedure is discussed in section 5.1.2.

1. Sets interpolated velocity values for all grid cells in the MATHEW box (i.e. this includes grid cells that are located beneath surface). Velocity values for below ground positions have a value of zero.

2. Calculates

\[
\begin{align*}
    u^o_n &= \frac{1}{4} (u^o_{i-1,j,k} + 2u^o_{i,j,k} + u^o_{i+1,j,k}) \\
    v^o_n &= \frac{1}{4} (v^o_{i,j-1,k} + 2v^o_{i,j,k} + v^o_{i,j+1,k})
\end{align*}
\]

where \( u^o \) and \( v^o \) are the average interpolated velocity values in the \( x \) and \( y \) direction respectively.
A8. SUBROUTINE ADJUV:

Calculates adjusted velocity values by using equations 32, 33 and 34. The appropriate numerical equations for \( \frac{\partial \lambda}{\partial n} \) calculations for given boundary conditions. Boundary conditions have been defined by grid descriptor values and \( u^0(i,j,k) \) is the average interpolated velocity value from Subroutine Setuvw. Note that the averaged initial velocity is calculated for all points except at KMATTOP. The Lagrangian multiplier values have been solved for in the main program by solving the partial differential equation using an iteration procedure.

1. For the time being, we shall restrict our attention to numerical equations specific only to interior grid points. The table on the following page compares grid descriptors and boundary conditions for the x and y directions. (See Sherman, p. 23).
<table>
<thead>
<tr>
<th>GRID DESCRIPTOR</th>
<th>CASE</th>
<th>BOUNDARY CONDITIONS</th>
</tr>
</thead>
</table>
| 0, 3, 4, 9, 16, 19, 20, 25 | C1 | \( \text{KMATTOP}(i-1,j) < k(i,j) \)  
|                      |     | \( \text{KMATTOP}(i+1,j) < k(i,j) \) |
| 1, 5, 6, 11, 17, 21, 22, 27 | C2 | \( \text{KMATTOP}(i-1,j) > k(i,j) \)  
|                      |     | \( \text{KMATTOP}(i+1,j) < k(i,j) \) |
| 2, 7, 8, 12, 18, 21, 23, 24 | C3 | \( \text{KMATTOP}(i-1,j) < k(i,j) \)  
|                      |     | \( \text{KMATTOP}(i+1,j) > k(i,j) \) |
| 10, 13, 14, 15, 26, 29, 30, 31 | C4 | \( \text{KMATTOP}(i-1,j) > k(i,j) \)  
|                      |     | \( \text{KMATTOP}(i+1,j) > k(i,j) \) |
| 0, 1, 2, 10, 16, 17, 18, 26 | C5 | \( \text{KMATTOP}(i,j-1) < k(i,j) \)  
|                      |     | \( \text{KMATTOP}(i,j+1) < k(i,j) \) |
| 3, 5, 7, 13, 19, 21, 23, 29 | C6 | \( \text{KMATTOP}(i,j-1) > k(i,j) \)  
|                      |     | \( \text{KMATTOP}(i,j+1) < k(i,j) \) |
| 4, 6, 8, 14, 20, 22, 24, 30 | C7 | \( \text{KMATTOP}(i,j-1) < k(i,j) \)  
|                      |     | \( \text{KMATTOP}(i,j+1) > k(i,j) \) |
| 9, 11, 12, 15, 25, 27, 28, 29 | C8 | \( \text{KMATTOP}(i,j-1) > k(i,j) \)  
|                      |     | \( \text{KMATTOP}(i,j) > k(i,j) \) |
The boundary conditions are schematically described below.

CASE C1

CASE C2

CASE C3

CASE C4

CASE C5

CASE C6

CASE C7

CASE C8
2. In Subroutine Adjuvw, $\frac{\partial \lambda}{\partial n}$ is required to calculate the adjusted velocity values. The equations used (specific to boundary conditions at each grid point) are given below. It is noted that we are only viewing interior grid points. We may look only at equations used in calculation of $u$ in the $x$ direction since the other directions essentially follow the same pattern. To calculate the adjusted velocity in the $x$ direction:

$$u = u_0 + \frac{\sigma h^2}{2} \left( \frac{\lambda_{i+1} - \lambda_{i-1}}{\Delta x} \right)$$

For the cases C1 thru C4 listed in the table above given values of the Lagrangian multiplier, $\lambda$, must be calculated before the above equation may be used.

1. Case C2

Calculate $\lambda_{i+1}$. Since at $i-1$ a closed boundary exists, one may use a first backward difference equation.

$$\frac{\partial \lambda}{\partial x}_{i-1} = 0 = \frac{3\lambda_{i+1} - 4\lambda_{i} + \lambda_{i-1}}{2\Delta x}$$

$$\lambda_{i+1} = \frac{4\lambda_{i} - \lambda_{i+1}}{3}$$

2. Case C3

Calculate $\lambda_{i+1}$. One may use first forward difference and since there is a closed boundary at $i+1$,

$$\frac{\partial \lambda}{\partial x}_{i+1} = 0 = \frac{-3\lambda_{i+1} + 4\lambda_{i} - \lambda_{i-1}}{2\Delta x}$$

$$\lambda_{i+1} = \frac{4\lambda_{i} - \lambda_{i+1}}{3}$$
3. Case C4

Calculate \( \lambda_{i+1} \) and \( \lambda_{i-1} \). We know that the boundary is closed at \( i+1 \) and \( i-1 \) so that

\[
\frac{\partial \lambda}{\partial x_{i+1}} = 0 = \frac{\partial \lambda}{\partial x_{i-1}}
\]

\[3\lambda_{i-1} = 4\lambda_i - \lambda_{i+1}\]

\[3\lambda_{i+1} = 4\lambda_i - \lambda_{i-1}\]

We have

\[3\lambda_{i-1} = 4\lambda_i - \left(\frac{4\lambda_i - \lambda_{i+1}}{3}\right)\]

Substituting

\[\lambda_{i-1} = \lambda_i\]

\[\lambda_{i+1} = \lambda_i\]

4. Case C1

Calculates nothing since neighboring boundaries are open.

\[\lambda_{i-1} = \lambda_{i+1}\]

\[\lambda_{i+1} = \lambda_{i-1}\]
3. Open Boundary points are examined. This includes grid cells on 
\( x = 1, x = M, y = 1, y = N \) and \( z = L \). It is only necessary (as 
for interior grid cells) to derive constants for values in 
one direction since in the \( y \) and \( z \) direction the same technique 
may be used. We will again derive constants applied to the \( x 
\) direction for \( x = 1 \) and \( \text{IGRIDS}(i,j) = 101 \) or \( 102 \).

1. \( \text{IGRIDS}(i,j) = 101 \) 

The terrain appears as is seen on 
the left. At \( x = 1 \) an open 
boundary exists. In this way,

\[ \lambda_1 = 0 \]

Using a backward first difference

\[ \frac{\partial \lambda}{\partial x} = \frac{3\lambda_i - 4\lambda_{i+1} + \lambda_{i+2}}{\Delta x} \]

\[ \frac{\partial \lambda}{\partial x} = \frac{4\lambda_{i+1} - 3\lambda_{i+2}}{\Delta x} \]

2. \( \text{IGRIDS}(i,j) = 102 \) 

As above an open boundary at \( x = 1 \) 
exists. But at \( x = 2 \) or at \( i+1 \) a 
closed boundary exists.

Substituting

\[ \lambda_i = 0 \]

\[ \frac{\partial \lambda}{\partial x} = \frac{3\lambda_{i+2} - 4\lambda_{i+1} + \lambda_i}{\Delta x} \]

\[ \frac{\partial \lambda}{\partial x} = 0 = 3\lambda_{i+2} - 4\lambda_{i+1} + \lambda_i \]
\[ \lambda_{i+2} = \frac{4\lambda_{i+1} - \lambda_i}{3} \]

\[ \frac{\partial \lambda}{\partial x_i} = \frac{-3\lambda_i + 4\lambda_{i+1} - \frac{4}{3}\lambda_{i+2} - \frac{1}{3}\lambda_i}{\Delta x} \]

\[ \frac{\partial \lambda}{\partial x_i} = \frac{8\lambda_{i+1}}{3\Delta x} \]

3. The first derivative of \( \lambda \) with respect to \( x \) for \( \text{IGRIDS}(i,j) = 103 \) or 104 are derived in a similar manner.

4. All grid points on the topography have grid descriptor values greater than 200. Grid descriptors for the \( x \) direction boundary conditions are given values of 10, 20, 30 to 90 and corresponding \( y \) direction descriptors are given values of 1, 2, 3 to 9. Since there are nine cases for the \( x \) and \( y \) direction, both of these values are added to the initial 200 value to give any number between 211 and 299. Since \( u \) values are only dependent on the \( x \) direction and \( v \) only on the \( y \), we may note that the technique for calculating Lagrangian multiplier values is the same as for interior and edge points and need not be documented here. For all cases, the surrounding terrain conditions are categorized and defined by assigned grid descriptor values. Boundary conditions are applied accordingly. These boundary conditions, used in conjunction with the numerical difference equations permits the calculation of Lagrangian multiplier values and the first derivative of the Lagrangian values at any grid position. The velocity adjustment equations are then used. The velocity component, \( u \), is calculated for \( x = \) constant grid edges, \( v \) is calculated for \( y = \) constant grid values and \( w \) for \( z = \) constant.
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B.1 IMPLEMENTATION FLOW CHART

Digital Terrain
Tape

Plot and Repeat
to Get
Correct Area

PICKEL2
RUNPICK

TAPE45

READSFT

TAPE47

MATLLL

RUNMATL

TAPE86
TAPE21

SUBTRAC

TAPE18

TAPE14
TAPE16

TAPE49

PLOTTOP2

TAPE50

PLOT, PARAM

CONTOUR PLOT OF LVLDLX

INPUT CONTOUR PLOT

ISOTACH PLOT OF ADJUSTED VELOCITY FIELD

ISOTACH PLOT OF INPUT VELOCITY FIELD

PLOT, PARAM

PLOT, PARAM

PLOT, PARAM

PLOT, PARAM
B2. LISTING OF PROGRAMS

B2.1 PICKEL2, RUNPICK

PROGRAM P3X30 ( INPUT, OUTPUT, TAPE1, TAPE45 )
C
DIMENSION INP( 4608 )
DIMENSION ISTORE( 5, 3 )
COMMON IBYT( 17230 )
C
DATA MASK/0"177777" /
DATA IBLK, IBGNX, IBGNY / 0, 0, 0 /
DATA IPTSOUT / 0 / 
DATA MAXX, MINX, MAXY, MINY / 0, 99999, 0, 99999 / 
C
DATA ISTRTX, ISTOPY, IDISTX, ISTRTY, ISTOPY, IDISTY/1186, 1259, 1, 
1 840, 849, 1 /
PRINT 7000, ISTRTX
PRINT 7010, ISTOPY
PRINT 7020, IDISTX
PRINT 7030, IDISTY
7000 FORMAT(1X, "THE STARTING X COORDINATE IS ", I10)
7010 FORMAT(1X, "THE STARTING Y COORDINATE IS ", I10)
7020 FORMAT(1X, "THE ENDING X COORDINATE IS ", I10)
7030 FORMAT(1X, "THE ENDING Y COORDINATE IS ", I10)
PRINT 7100, IDISTX, IDISTY
7100 FORMAT(1X, "EVERY ", I5, " POINT IN THE X DIRECTION IS SELECTED");
C
INCR=0
IBGNX=39
IBGNY=45
10 READ( 1 ) LEN, ( INP( I ), I = 1, LEN )
IF( EOF( 1 ) .NE. 0 ) GO TO 500
C
20 IBLK = IBLK + 1
IPT = 0
C
DC 46 I = 1, LEN, 4
IPT = IPT + 1
IPT( IPT ) = SHIFT( INP( I ), -44 ) .AND. MASK
IPT = IPT + 1
IPT( IPT ) = SHIFT( INP( I ), -28 ) .AND. MASK
IPT = IPT + 1
IPT( IPT ) = SHIFT( INP( I ), -12 ) .AND. MASK
C
INT = SHIFT( INP( I ) * 4 ) .AND. 0"177760"
IPT = IPT + 1
IYTE( IPT ) = INT .OR. INU
C
IPT = IPT + 1
IYTE( IPT ) = SHIFT( INP( I + 1 ), -40 ) .AND. MASK
IPT = IPT + 1
IYTE( IPT ) = SHIFT( INP( I + 1 ), -24 ) .AND. MASK
IPT = IPT + 1
IYTE( IPT ) = SHIFT( INP( I + 1 ), -8 ) .AND. MASK
C
INT = SHIFT( INP( I + 1 ) * 8 ) .AND. 0"177480"
IPT = IPT + 1
IYTE( IPT ) = INT .OR. INU
C
IPT = IPT + 1
IYTE( IPT ) = SHIFT( INP( I + 2 ), -36 ) .AND. MASK
IPT = IPT + 1
IYTE( IPT ) = SHIFT( INP( I + 2 ), -20 ) .AND. MASK
IPT = IPT + 1
C

\[
\text{IBYT( IPT )} = \text{SHIFT( INP( I + 2 ), -4 ) .AND. MASK}
\]

C

\[
\text{INT} = \text{SHIFT( INP( I + 2 ), 12 ) .AND. 0"7777777777"}
\]

C

\[
\text{INU} = \text{SHIFT( INP( I + 3 ), -43 ) .AND. 0"7777777777"}
\]

C

\[
\text{ITDT} = \text{IPT + 1}
\]

C

\[
\text{IBYT( IPT )} = \text{INT .OR. INU}
\]

C

\[
\text{ITDT} = \text{IPT + 1}
\]

C

\[
\text{IBYT( IPT )} = \text{SHIFT( INP( I + 3 ), -32 ) .AND. MASK}
\]

C

\[
\text{ITDT} = \text{IPT + 1}
\]

C

\[
\text{IBYT( IPT )} = \text{SHIFT( INP( I + 3 ), -16 ) .AND. MASK}
\]

C

\[
\text{ITDT} = \text{IPT + 1}
\]

C

\[
\text{IBYT( IPT )} = \text{INP( I + 3 ) .AND. MASK}
\]

C

40 CONTINUE

C

\[
\text{THE FIRST 32 BITS OF THE PECCORD CONTAIN THE BLOCK DESCRIPTOR WORD}
\]

C

\[
\text{FIRST 16 BITS CONTAIN THE LENGTH OF THE PHYSICAL RECORD}
\]

C

\[
\text{LENPHYS} = \text{IBYT( 1 ) / 2}
\]

C

\[
\text{THE NEXT 32 BITS CONTAIN THE SEGMENT DESCRIPTOR WORD OF THE FIRST}
\]

C

\[
\text{THE UPPER 16 BITS CONTAIN THE SEGMENT LENGTH}
\]

C

\[
\text{SEGMENT}
\]

C

\[
\text{IBEGSEG} = 3
\]

C

\[
\text{60 LENSEGT} = \text{IBYT( IBEGSEG ) / 2}
\]

C

\[
\text{NEWSEG} = \text{IBEGSEG + LENSEGT}
\]

C

\[
\text{HERE, IF THE LENGTH OF THE SEGMENT IS LESS THAN 100 BYTES}
\]

C

\[
\text{ASSUME IT IS TYPE A OR TYPE C LOGICAL RECORDS}
\]

C

\[
\text{IF( LENSEGT .LT. 100 ) GO TO 100}
\]

C

\[
\text{MAP X-COORD IN BYTE FOLLOWING SEGMENT DESCRIPTOR WORD}
\]

C

\[
\text{MAP Y-COORD IN BYTE FOLLOWING X-COORDINATE}
\]

C

\[
\text{MAPXCRO} = \text{IBYT( IBEGSEG + 2 ) + INCX - IBGNX}
\]

C

\[
\text{MAPYCRD} = \text{IBYT( IBEGSEG + 3 ) - IBGNX}
\]

C

\[
\text{NUMEPT} = \text{IBYT( IBEGSEG + 6 )}
\]

C

\[
\text{IF( MAPXCRO .LT. ISTRTX ) GO TO 90}
\]

C

\[
\text{IF( MAPYCRD .GE. ISTOPY ) GO TO 500}
\]

C

\[
\text{IRECTTY = MAPXCRO - ISTRTY}
\]

C

\[
\text{IF( IRECTTY .GE. ( IRECTTY / IDISTX ) * IDISTX ) GO TO 90}
\]

C

\[
\text{IF( MAPYCRD .GT. MAXY ) MAXY = MAPYCRD}
\]

C

\[
\text{IF( MAPYCRD .LT. MINX ) MINX = MAPYCRD}
\]

C

\[
\text{ELEVATION POINTS BEGIN IN IBYT( IBEGSEG + 7 )}
\]

C

\[
\text{SE 10 SELECT THE POINTS WANTED}
\]

C

\[
\text{IBEG} = \text{IBEGSEG + 7}
\]

C

\[
\text{IENO = ISEG + NUMEPT - 1}
\]

C

\[
\text{ILOW = ISEG + ISTOPY - MAPYCRD}
\]

C

\[
\text{IHI = ISEG + ISTOPY - 1 - MAPYCRD}
\]

C

\[
\text{IF( ILOW .LT. ISEG ) ILOW = ISEG}
\]

C

\[
\text{IF( IHI .GT. IENO ) IHI = IENO}
\]

C

\[
\text{IF( IHI .LT. ILOW ) GO TO 30}
\]

C

\[
\text{OC 70 I = ILOW, IHI, IDISTY}
\]

C

\[
\text{IPTSOUT} = \text{IPTSOUT + 1}
\]

C

\[
\text{J=J+1}
\]
EN000
END

2000 FRAMAT (X1, YI)

C 60 TO 90

2000 FRAMAT (X2, Y2)

C 90 TO 180

END
/JCS
PICK50.
USER: A 338 300,4WLM.
ATTACH, TAPE45=TWISOM/M=M.
GET, PICK50.
FIN3(I=PICK50)
VSN(TAPE1=AC0479)
LABEL(TAPE1, N1, 0=1600, F=I, P0=9, LB=KL)
LG0.
GOTO, 1.
EXIT.
1, DAYFTLE, DAY.
REPLACE, DAY.
/EOF
B2.2 READSFT

00100C PROGRAM SURFACE(INPUT,OUTPUT,TAPE45,TAPE47)
00110C DIMENSION SFCHT(70,106),CHEIGHT(11,17)
00130C 00140C SMALLP=0.00000001
00150C DELXTOP=53.4
00160C DELYTOP=53.4
00170C ILIMIT=(22)/2
00180C JLIMIT=(34)/2
00190C DELX=63.4*3
00200C DELY=63.4*3
00210C M=23
00220C N=35
00230C L=11
00240C ISTEPT=6
00250C JSTEPT=6
00260C ITOP=1
00270C JTOP=1
00280C SUMHGT=0.0
00290C AREAINV=1.0/FLOAT(ISTEPT*JSTEPT)
00300C 00310C DO 10 I=1,70
00320C DO 15 J=1,106
00330C READ(45,150) SFCHT(I,J)
00340C FORMAT(12X,F6.0) SFCHT(I,J)=.3048*SFCHT(I,J)
00350C 15 CONTINUE
00360C 10 CONTINUE
00370C 00380C SUMHGT=0.0
00390C DO 20 I=1,ILIMIT
00400C DO 30 J=1,ILIMIT
00410C DO 55 IT=1,ISTEPT
00420C DO 55 JT=1,JSTEPT
00430C SUMHGT=SUMHGT+SFCHT(ITOP+IT-1,JTOP+JT-1)
00440C 35 CONTINUE
00450C 30 CONTINUE
00460C 00470C CHEIGHT(I,J)=SUMHGT*AREAINV
00480C WRITE(47,100) CHEIGHT(I,J)
00490C 100 FORMAT(F8.2)
00500C 00510C SUMHGT=0.0
00520C JTOP=JTOP+JSTEPT
00530C CONTINUE
00540C 25 CONTINUE
00550C 00560C ITOP=ITOP+ISTEPT
00570C JTOP=1
00580C 20 CONTINUE
00590C 00560C STOP
00570C END
PROGRAM TOPOG2 (INPUT, OUTPUT, TAPE47, TAPE51)

DIMENSION CHEIGHT(11, 17), LVL2DLX(11, 17), LVL1DLX(23, 35)

DIMENSION X(23), Y(35), XLVL(23, 35)

DELX = 63.4
DELY = 63.4
DELZ = 50.

M = 23
N = 35

SMALERR = .0000001
DELXTOP = 63.4
DELYTOP = 63.4
LIMIT = (M) / 2

ISTEP = (2 * DELX) / DELXTOP
JSTEP = (2 * DELY) / DELYTOP
ITOP = 1
JTOP = 1

SUMHGT = 0.0
A = 1.0 / FLOAT (ISTEP * JSTEP)

XSTP2 = 3.
YSTP2 = 3.

DO 10 I = 1, M
X(I) = 1187.0 + (I - 1) * XSTP2
10 CONTINUE

DO 20 J = 1, N
Y(J) = 841.0 + (J - 1) * YSTP2
20 CONTINUE

DO 31 I = 1, LIMIT
DO 32 J = 1, LIMIT
READ (47, 15) CHEIGHT(I, J)
32 CONTINUE

PRINT 6, I, J, CHEIGHT(I, J)
6 FORMAT (13, 13, F4.2)

DO 50 I = 1, LIMIT
DO 40 J = 1, LIMIT
LVL2DLX(I, J) = INT ((CHEIGHT(I, J) + (DELZ / 2.0) + SMALERR) / DELXTOP)
40 CONTINUE
50 CONTINUE

DO 55 I = 1, LIMIT
55 CONTINUE

DO 59 J = 1, LIMIT
59 CONTINUE

DO 55 I = 1, LIMIT
DO 59 J = 1, LIMIT
ITWOM1 = ITWO + 1
45 CONTINUE
55 CONTINUE
59 CONTINUE

ILMT2 = 2 * LIMIT
JLMT2 = 2 * LIMIT
00750    DD 101 I=1,ILMTZ
00760    DD 102 J=1,ILMTZ
00770    XLVL(I,J) = (LVL13LY(I,J) * DELZ) / .3043
00780    WRITE (51,103), X(I), Y(J), YLVL(I,J)
00790 103 FORMAT(2F6.0, F5.2)
00800    102 CONTINUE
00810    101 CONTINUE
00820    IET5=99
00821    WRITE(51,115) IETP
00822    115 FORMAT(20X,I2)
00830    END
B2.4 PLOTTOP, PLOTP2

00100      PROGRAM PLOTTOP(INPUT,OUTPUT,TAPE47,TAPE48)
00110  THIS PROGRAM IS DESIGNED TO READ THE INPUT TOPOGRAPHY FILE
00120 FOR MATHEW (LLL VERSION) AND OUTPUT ANOTHER FILE WHICH WILL
00130 ALLOW THE TOPOGRAPHY TO BE PLOTTED
00140
00150      DIMENSION X(11),Y(17),CHEIGHT(11,17)
00155      MH=11
00160      NH=17
00165  DO 10 I=1,MH
00170      X(I)=1131. + 6.*I
00180  10   CONTINUE
00190  DO 20 J=1,NH
00200      Y(J)=135. + 6.*J
00210  20   CONTINUE
00220  DO 30 I=1,MH
00230      DO 30 J=1,NH
00240      READ(47,100) CHEIGHT(I,J)
00250  100  FORMAT(F3,2)
00260      CHEIGHT(I,J)=CHEIGHT(I,J)/3048
00270  30   CONTINUE
00280
00290  END OF FILE MARKER FOR PLOTTING ROUTINE - 99
00300      IETP=99
00310  WRITE(48,110) ((X(I),Y(J),CHEIGHT(I,J),J=1,NH),I=1,MH))
00320  110  FORMAT(2F5.0,F8.2)
00330  WRITE(48,115) IETP
00340  115  FORMAT(20X,12)
00350
00360      STOP
00370      END
PROGRAM PLOTOPZ (INPUT, OUTPUT, TAPE49, TAPE50)

THIS PROGRAM IS DESIGNED TO READ MATLAB'S KMATTOP GRID ARRAY
(I.E. THE ACTUAL TOPOGRAPHY USED IN MATLAB SOLUTION
TECHNIQUE), CALLED TAPE49, AND OUTPUTS TAPE50 WHICH
ALLOWS THE TOPOGRAPHY TO BE PLOTTED.

DIMENSION X(23,35), Y(35), ZHEIGHT(23,35), CHEIGHT (23,35)

DATA DELX, DELY, M, N, 3, 3, 23, 35

GIVE THE DELTA Z VALUE IN METERS

DELZ=50.

MINLVL=11

INPUT MINLVL VALUE FROM MATLAB OUTPUT

READ (44, 44) MINLVL

44 FORMAT (I3)

DO 10 I=1, M

10 X(I) = 137. + (I-1)*DELX

CONTINUE

DO 20 J=1, N

Y(J) = 841. + (J-1)*DELY

CONTINUE

DO 30 J=1, N

READ (49, 100) ZHEIGHT(I, J)

30 CONTINUE

CHEIGHT(I, J) = CHEIGHT(I, J) + MINLVL - 1) / 3.948

31 CONTINUE

END OF FILE MARKER FOR PLOTTING ROUTINE - 99

WRITE (50, 111) ((X(I), Y(J), CHEIGHT(I, J), J=1, N), I=1, M))

WRITE (50, 115) IETP

FORMAT (20X, I2)

STOP

END
PROGRAM INTERPA (INPUT, OUTPUT, TAPE?, TAPE20)

INTERPOLATES U AND V AT REFERENCE HEIGHT SPECIFIED IN TAPE10

DIMENSION XS(5), YS(5), SFCHGT(5)

DIMENSION SPEEDSF(5), SFCDIR(5), SPOINTR(23, 35), SFCSPD(5)

DIMENSION SINDRSF(5), COSDRSF(5), SININTR(23, 35)

DIMENSION COSINTR(23, 35), UREF (23, 35), VREF (23, 35)

READ IN SURFACE OBSERVATIONS: NUMBER OF DATA STATIONS (NS)

X-COORDINATES (XS), Y-COORDINATES (YS), HEIGHT ABOVE GROUND

IN METERS (SFCHGT), WINDSPEED OBSERVED IN M/SEC (SPEEDSF)

DIRECTION IN DEGREES MEASURED FROM NORTH- DIRECTION WIND

COMING FROM (SFCDIR)

DEGTPAD = (3.14159265358979) / 180.

READ (9, 10) NS

DO 100 I = 1, NS

READ (9, 11) XS(I), YS(I), SFCHGT(I), SFCSPD(I), SFCDIR(I)

PRINT *, XS(I), YS(I), SFCHGT(I), SFCSPD(I), SFCDIR(I)

100 CONTINUE

READ (9, 12) REF, PWRSL

DO 110 I = 1, NS

SPEEDSF(I) = SFCSPD(I) * (REF / SFCHGT(I)) ** PWRSL

110 CONTINUE

DC 140 I = 1, 23

DO 140 J = 1, 35

SPOINTR(I, J) = 0.0

SININTR(I, J) = 0.0

COSINTR(I, J) = 0.0

140 CONTINUE

INTERPOLATE THE SPEED

CALL INTER (SPEEDSF, XS, YS, NS, SPOINTR)

DO 120 I = 1, NS

ANGLE = SFCDIR(I) * DEGTPAD

SINDRSF(I) = SIN(ANGLE)

COSDRSF(I) = COS(ANGLE)

120 CONTINUE

CALL INTER (SINDRSF, XS, YS, NS, SININTR)

CALL INTER (COSDRSF, XS, YS, NS, COSINTR)

DO 130 I = 1, 23

DO 130 J = 1, 35

UREF(I, J) = SPOINTR(I, J) * SININTR(I, J)

VREF(I, J) = SPOINTR(I, J) * COSINTR(I, J)

WRITE (20, 13) I, J, UREF(I, J), VREF(I, J)

130 CONTINUE

150 CONTINUE

130 CONTINUE

STOP

END
**SUBROUTINE TO DO INTERPOLATION USING 1/R**\(^2\) FORMUL**

**SUBROUTINE INTER(VAL, RX, RY, NS, OVAL)**

**DIMENSION VAL(5), RX(5), RY(5), OVAL(23, 35), R(3), RV(5)**

**IF(NS .NE. 1) GO TO 300**

**DO 200 J = 1, 35**

**OVAL(I, J) = VAL(5)**

**200 CONTINUE**

**RETURN**

**GO TO 300**

**DO 220 I = 1, 23**

**DO 220 J = 1, 35**

**30 230 K = 1, NS**

**RR = (RX(K) - I)**\(^2\) \(+ (RY(K) - J)**\(^2\)**

**RV(K) = RV(K)**

**CONTINUE**

**230 CONTINUE**

**RR = 0.**

**DO 240 K = 1, NS**

**IF (R(K), EQ. 0.) GO TO 250**

**240 CONTINUE**

**RR = RR + RV(K)**

**GO TO 220**

**250 OVAL(I, J) = RV(1)**

**GO TO 220**

**CONTINUE**

**RETURN**

**END**
B2.6 RUNMATH

?JC
THINC(CY32700),TL024
USER,ASDP004,ALWLM.
GET,TAP47=M95P47.
GET,TAP39=M95P39.
GET,TAP95=M95P95.
GET,TL95.
ATTACH,TAP21=M95P21/M=1.
ATTACH,TAP96=M95P96/M=1.
ATTACH,TAP49=M95P49.
FTN(I=MATL95,SEQ,TS).
LG0.
REPLACE,TAP49=M95P49.
REPLACE,TAP39=M95P39.
GOTO1.
EXIT.
1,DAYFILE,DAY.
REPLACE,DAY.
/EOF
PROGRAM RESL \$(INPUT, OUTPUT, TAPE36, TAPE14)

THIS PROGRAM COMPUTES TOTAL VELOCITY IN TWO
DIMENSIONS USING THE INTERPOLATED VELOCITY COMPONENTS COMPUTE
BY THE MATHEM PROGRAM (ENTITLED MATELL). THE OUTPUT TAPE
\$(TAPE14) IS IN A FORMAT SUITABLE FOR PLOTTING.

VELOCITIES ARE IN MPH.

ALL ARRAYS MUST BE DIMENSIONED.

DIMENSION U(11,11), V(11,11), W(11,11), U2(11,11), U3(11,11)

DIMENSION X(11), Y(11)

MATHEM GRID STRUCTURE MUST BE SPECIFIED.

DATA M, N, L/11, 11, 2/7

THE LEVEL TO BE PLOTTED MUST BE SPECIFIED.

LEVEL=4

DO 9 J=1,2

DO 14 J=1,3

GET TO CORRECT POINT ON INPUT TAPE.

READ (86,15), KWORD

FORMAT (A9)

CONTINUE

READ (86,16), KWORD

FORMAT (A9)

DATA IS TAKEN AT EACH HORIZONTAL GRID POINT.

DO 35 J1=1, M

DO 30 J2=1, N

IF(J1+J2.EQ.1)GC TO 5

GET TO CORRECT POINT ON INPUT TAPE.

DO 40 J3=1, ISKIP

READ (86,10), D

CONTINUE

READ (86,11), L1

ISKIP=L-LEVEL-L1

DO 50 J4=1,LEVEL

READ (86,12), U(J1,J2), V(J1,J2), W(J1,J2)

GO TO 50

CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

COMBINE THE U, V, AND W COMPONENTS

DO 60 I=1, M

DO 60 J=1, N

U2(I,J)=SQR(T(U(I,J)**2+V(I,J)**2))

U3(I,J)=SQR(T(U(I,J)**2+V(I,J)**2+W(I,J)**2))

U2(I,J)=2.37*U2(I,J)

U3(I,J)=2.37*U3(I,J)

CONTINUE

SET UP COORDINATE GRID POINTS.

XA=1.54.

YA=6.84.

DO 70 I=1, M
00640  70  X(I)=X(I)+7*J
00650  DO  90  J=1,N
00660  80  Y(J)=YA+11*J
00670C  SET UP COORDINATE GRID POINTS.
00680C  IEIP=99
00690C  WRITE OUT COMBINED VELOCITY ONTO TAPE.
00700C  WRITE (14,13),(X(I),Y(J),U2(I,J),J=1,N),I=1,M)
00710C  WRITE (14,14),IEIP
00720C  WRITE (14,13),(X(I),Y(J),U3(I,J),J=1,N),I=1,M)
00730C  WRITE (14,14),IEIP
00740C  FORMAT(12X,F6.2)
00750C  FORMAT(6X,13)
00760C  FORMAT(3(2X,F6.2))
00770C  FORMAT(2F5.0,F8.2)
00780C  FORMAT(20X,12)
00790C  END

PROGRAM PESL2(INPUT,OUTPUT,TAPE21,TAPE16)

THIS PROGRAM COMPUTES TOTAL VELOCITY IN TWO DIMENSIONS USING THE INTERPOLATED VELOCITY COMPONENTS COMPUTED BY THE MATHEW PROGRAM (ENTITLED MATHLLL). THE OUTPUT TAPE (TAPE16) IS IN A FORMAT SUITABLE FOR PLOTTING. VELOCITIES ARE IN MPH.

ALL ARRAYS MUST BE DIMENSIONED.

DIMENSION U(11,11),V(11,11),W(11,11),U2(11,11),U3(11,11)

MATHEW GRID STRUCTURE MUST BE SPECIFIED.

DATA M,N,L/11,11,4/

THE LEVEL TO BE PLOTTED MUST BE SPECIFIED.

LEVEL=4

DATA IS TAKEN AT EACH HORIZONTAL GRID POINT.

DO 35  J1=1,M
  DO 30  J2=1,N
  IF(J1*J2.EQ.1)GO TO 5

GET TO CORRECT POINT ON INPUT TAPE.

DO 40  J3=1,ISKIP
  READ (21,10),D
  CONTINUE

READ (21,11),L1

ISKIP=L-LEVEL-L1

DO 30  J4=1,LEVEL
  READ (21,12),U(J1,J2),V(J1,J2)
  CONTINUE

GO TO 50

50  CONTINUE

GET TO CORRECT POINT ON INPUT TAPE.

DO 40  I=1,M

COMBINE THE U,V,W COMPONENTS
DO 50 J=1,N
U2(I,J)=SQRT(U(I,J)**2+V(I,J)**2)

DO 50 CONTINUE

SET UP COORDINATE GRID POINTS.

X=154.
YA=694.

DO 70 I=1,N
X(I)=XA+7*I
DO 80 J=1,N
Y(J)=YA+11*J

SET UP COORDINATE GRID POINTS.

IEIP=99
WRITE OUT COMBINED VELOCITY ONTO TAPE.
WRITE (16,13), (X(I),Y(J),U2(I,J),J=1,N),I=1,M
WRITE (16,14),IEIP

10 FORMAT(12X,F6.2)
11 FORMAT(6X,I3)
12 FORMAT(3(2X,F6.2))
13 FORMAT(2F6.0,F8.2)
14 FORMAT(20X,I2)
END
00100 PROGRAM SUBTRAC(INPUT, OUTPUT, TAPE86, TAPE21, TAPE48)
00110 THIS PROGRAM DETERMINES THE ROOT MEAN SQUARE DIFFERENCE
00120 BETWEEN THE ADJUSTED VELOCITY AND THE INTERPOLATED VE-
00130 LOCITY VALUES IN THE MATHEW PROGRAM (MALLLL) AT A SPECIFIED
00140 DISTANCE ABOVE GROUND LEVEL. IT DETERMINES THE AVERAGE
00150 RMS VELOCITY DIFFERENCE, THE MAXIMUM, AND THE MINIMUM
00160 RMS VALUES. TAPE 15 GIVES RMS VALUES AT EACH GRID POINT
00170 ON THE SPECIFIED LEVEL.

00180 60 LEVEL=LEVEL+1
00210 61 FORMAT(A9)
00230 CONTINUE

00240 DIMENSION UAD(23,35), VAD(23,35), WAD(23,35)
00250 DIMENSION UIN(23,35), VIN(23,35), U0(23,35)
00260 DIMENSION UX(23), MY(35)
00270 MXLVL=5
00280 LEVEL=0
00290 REMIND 99
00300 READ (86,21), KWORD
00310 99 CONTINUE

00320 READ(86,61), KWORD2
00330 61 FORMAT(A9)
00340 90 CONTINUE

00350 DO 20 J1=1,M
00360 DO 30 J2=1,N
00370 IF(J1*J2.EQ.1) GO TO 1
00380 IF(ISKIP.EQ.0) GO TO 1
00390 CONTINUE
00400 READ (86,10), D
00410 1 CONTINUE

00420 READ (86,24), L1
00430 READ (21,24), L1
00440 ISKIP=L-LEVEL-L1
00450 CONTINUE

00460 DO 40 J4=1,LEVEL
00470 READ (86,22)UAD(J1,J2), VAD(J1,J2), WAD(J1,J2)
00480 READ (21,23) UIN(J1,J2), VIN(J1,J2)
00490 40 CONTINUE

00500 UI(J1,J2)=SQRT(UIN(J1,J2)**2.+VIN(J1,J2)**2.)
00510 U0(J1,J2)=SQRT(UAD(J1,J2)**2.+VAD(J1,J2)**2.)
WAQ(J1,J2)**2.}
00570C  U1(J1,J2)=2.237*U1(J1,J2)
00580C  UD1F(J1,J2)=2.237*UD1F(J1,J2)
00590C  PRINT*1,J1,J2,U1(J1,J2),UD1F(J1,J2)
00600C  U2(J1,J2)=ABS(U1(J1,J2)-U0(J1,J2))
00610C  UD1F(J1,J2)=ABS(U1(J1,J2)-U0(J1,J2))
00620C  USUM=UD1F(J1,J2)+USUM
00630C  PRINT*,UD1F(J1,J2),UD1F(J1,J2)
00640C  PRINT*,J1,J2,URMS(J1,J2)
00650C  IF(UD1F(J1,J2).*GT.UMAX)UMAX=UD1F(J1,J2)
00660C  IF(UD1F(J1,J2).LT.*1) COUNTA=COUNTA+1
00670C  IF(UD1F(J1,J2).LT.*2) COUNTB=COUNTB+1
00680C  IF(UD1F(J1,J2).LT.*3) COUTNC=COUTNC+1
00690C  IF(UD1F(J1,J2).LT.*4) COUNTD=COUNTD+1
00700C  IF(UD1F(J1,J2).LT.*5) COUNTE=COUNTE+1
00710C  IF(UD1F(J1,J2).LT.*6) COUNTF=COUNTF+1
00720C  IF(UD1F(J1,J2).LT.*7) COUNTG=COUNTG+1
00730C  IF(UD1F(J1,J2).LT.*8) COUNTH=COUNTH+1
00740C  IF(UD1F(J1,J2).LT.*9) COUNTI=COUNTI+1
00750C  IF(J1*J2.EQ.*1) GO TO 2
00760C  GO TO 30
00770C  2 UMIN=UD1F(J1,J2)
00780C  30 CONTINUE
00790C  20 CONTINUE
00800C  URMS=SQRT(USUM/FLOAT(M*N))
00810C  45 FORMAT(*AT LEVEL = *,1X,I2,1X,*THE RMS VALUE IS *,1X,F6.2,1X,*MPH*)
00820C  46 FORMAT(*THE RANGE OF DIFFERENCE VALUES LIES BETWEEN A MAXIMUM OF*,1X,F6.2,1X,*AND A MINIMUM OF*,1X,F6.2,1X,*MPH*)
00830C  31 FORMAT(*THE NUMBER OF STATIONS BTWN 0 & 1 = *,F5.0)
00840C  32 FORMAT(*THE NUMBER OF STATIONS BTWN .1 & .2 = *,F5.0)
00850C  33 FORMAT(*THE NUMBER OF STATIONS BTWN .2 & .3 = *,F5.0)
00860C  34 FORMAT(*THE NUMBER OF STATIONS BTWN .3 & .4 = *,F5.0)
00870C  35 FORMAT(*THE NUMBER OF STATIONS BTWN .4 & .6 = *,F5.0)
00880C  36 FORMAT(*THE NUMBER OF STATIONS BTWN .6 & 1. = *,F5.0)
00890C  37 FORMAT(*THE NUMBER OF STATIONS BTWN 1. & 2. = *,F5.0)
00900C  38 FORMAT(*THE NUMBER OF STATIONS BTWN 2. & 3. = *,F5.0)
00910C  39 FORMAT(*THE NUMBER OF STATIONS BTWN 3. & 10. = *,F5.0)
00920C  40 FORMAT(*WRITE OUT MAXIMUM RMS VELOCITY DIFFERENCE*)
00930C  WRITE(18,31),COUNTA
00940C  WRITE(18,32),COUNTB
00950C  WRITE(18,33),COUTNC
00960C  WRITE(18,34),COUNTD
00970C  WRITE(18,35),COUNTE
00980C  WRITE(18,36),COUNTF
00990C  WRITE(18,37),COUNTG
01000C  WRITE(18,38),COUNTH
01010C  WRITE(18,39),COUNTI
01020C  WRITE(18,310),COUNTJ
01030C  WRITE(18,27),LEVEL,UMIN,UMAX
01190 27 FORMAT(*LEVEL=*,I3,1X,*UMIN=*,F6.2,1X,*UMAX=*,F6.2)
01200C WRITE(18,28),URMS
01220 28 FORMAT(*URMS=*,F6.2)
01230C WRITE(18,29)
01250 29 FORMAT(*ALL RMS VELOCITY DIFFERENCE VALUES ARE IN MPH*)
01260C DO 30 J=1,N
01280C JC=N+1-J
01300 WRITE(18,25),(UOIF2(I,JC),I=1,23)
01320C 30 CONTINUE
01330C IF(LEVEL.LT.MXLVL) GO TO 60
01340C END
CONTOUR.
USER, A93R100, AWLM.
FINOLIB, CONTOUR.
ATTACH (TAPE9=3RTPE3)
REWIND (TAPE9)
GET (PARAM82)
REWIND (PARAM82)
FINOLIB (CAL151)
LOSE (LIB=CAL151)
CONTOUR (PARAM82)
PLOTII (TAPE20)
goto 1.
exit.
1, DAYFILE, DAY1.
replace, DAY1.
/EOF

1 CONTOUR PLOT OF PRINCETON-TRIAL 2
2 3 695 3 6.95 6.95
3 200 1 (3F6.0*A2)
4 3 3 1 -3.0 3 -3.0
5 111 4 21.01
6 ENDMAP
9
B3. **PROGRAM IMPLEMENTATION**

**B3.1 PICKEL2, RUNPICK**

**Input:** Digital Terrain Tape  
**Program:** PICKEL2  
**Procedure File:** RUNPICK  
**Output:** TAPE45

1. **Description:**

The Fortran program, PICKEL2, selects points from the digital terrain tape, which gives topographic elevation above sea level height in ft. The user specifies these points from the digital terrain tape by supplying the beginning and ending x and y coordinates and the sampling frequency. One unit from the digital terrain tape corresponds to .1 in on a 1:25000 scale map as measured from the southwest corner of a state map. TAPE45 output gives the x coordinate, the y coordinate and height (in ft) on each output line. Massachusetts has two digital terrain tapes, one for eastern and one for western Mass.

2. **Edit Changes:**

Specify variables

ISTRTX - starting x coordinate.  
ISTOPX - stopping x coordinate.  
IDISTX - sampling frequency (usually equal to one).  
ISTRTY - starting y coordinate.  
ISTOPY - stopping y coordinate.  
IDISTY - sampling frequency (usually equal to one).

3. **To Run:**

Type: OLD, RUNPICK  
Type: BATCH  
Type: DEFINE, TAPE45  
Type: SUBMIT, RUNPICK, B
B3.2 **READSFT**

Input: TAPE45  
Program: READSFT  
Output: TAPE47  

1. Description:  

This program creates the input topography tape for MATHEW. The averaged two delta x by two delta y topography is calculated.

2. Edit Changes:  

1. Specify A and B of SFCHT(A,B) where A = number of points in the x direction (of TAPE45) plus one and B = number of points in the y direction (of TAPE45) plus one.  
2. Specify C, D of CHEIGHT(C,D) where C = (M-1)/2 AND D = (N-1)/2.  
3. Specify E of ILIMIT(E/2) where E = M-1.  
4. Specify F of JLIMIT(F/2) where F = N-1.  
5. Specify DELX = 63.4 x delta x and DELY = 63.4 x delta y.  
6. Specify M, N AND L.  
7. Specify A, B of I = 1,A and J = 1,B where A and B are defined as above.

3. To Run:

Type: OLD,READSFT  
Type: ATTACH,TAPE45  
Type: FTNTS  
Type: RUN  
Type: SAVE,(REPLACE,)TAPE47
1. Description:

This program takes MATHEW's input topography tape and levels off heights to correspond with the delta z value assigned to the MATHEW box. This process is also performed in the MATHEW program (subroutine Topog) and need not be run. It is presented here as an informative addition so that the user can quickly determine what the leveled off contour will look like.

2. Edit Changes:

1. Specify A, B of CHEIGHT(A,B) and LVL2DLX(A,B) where A = (M-1)/2 and B = (N-1)/2.

2. Specify LVL2DLX(M,N).

3. Specify DELX = 63.4 \times \text{delta x}, DELY = 63.4 \times \text{delta y and DELZ} = \text{delta z (in m)}.

4. Specify M, N.

5. Specify XSTP2 = \text{delta x} and YSTP2 = \text{delta y}.

6. Specify C and D of X(I) = C + (I-1) and Y(J) = D + (J-1) where C = \text{x starting point} and D = \text{y starting point}.

To Run:

Type: OLD, TOPOG2
Type: GET, TAPE47
Type: FTNTS
Type: RUN
Type: SAVE, (REPLACE,) TAPE51
B3.4 PLOTTOP (PLOTOP2)

Input: TAPE47 (TAPE49)
Program: PLOTTOP (PLOTOP2)
Output: TAPE48 (TAPE50)

1. Description:

This program reformats the topography tape. PLOTTOP reformats the input topography tape (TAPE47) and PLOTOP2 reformats the conditioned topography output from the MATHEW program (TAPE50). Both programs are essentially the same.

2. Edit Changes:

1. Specify \(X((M-1)/2), Y((N-1)/2)\).
2. Specify \(CHEIGHT((M-1)/2,(N-1)/2)\).
3. Let \(MH = (M-1)/2\) and \(NH = (N-1)/2\).
4. Specify \(A, B, C\) and \(D\) of \(X(I) = A + B*I\) and \(Y(J) = C + D*J\) where \(A = x\) starting point \(- 2x\) delta \(x\), \(B = 2x\) delta \(x\), \(C = y\) starting point \(- 2y\) delta \(y\) and \(D = 2y\) delta \(y\).

3. To Run:

Type: OLD,PLOTTOP (PLOTOP2)
Type: GET,TAPE47 (TAPE49)
Type: FTNTS
Type: RUN
Type: SAVE,(REPLACE,)TAPE48 (TAPE50)
**B3.5 INTERPL**

Input: TAPE9  
Program: INTERPL  
Output: TAPE20

1. **Description:**

This program interpolates observed velocity values using a $1/(R \times R)$ interpolation scheme. TAPE20 is the input velocity tape into MATHEW.

**Edit Changes:**

1. Create TAPE9 using the following format:

```
<table>
<thead>
<tr>
<th>NS</th>
<th>XS</th>
<th>YS</th>
<th>SFCHGT</th>
<th>SPEEDSF</th>
<th>SFCDIR</th>
<th>REF</th>
<th>PWRSL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>
```

- **NS** = number of data stations
- **XS** = x coordinate relative to the MATHEW grid.
- **YS** = y coordinate relative to the MATHEW grid.
- **SFCHGT** = height of data collected above ground (m).
- **SPEEDSF** = wind speed (m/s).
- **SFCDIR** = degrees from north direction (using a clockwise rotation). This is the direction that the wind is coming from.
- **REF** = Reference height.
- **PWRSL** = Shear coefficient.

2. Change all one dimensional arrays to NS dimension where NS is given above.

3. Change all two dimensional arrays to $(M,N)$ dimensions.

4. Let $I = 1,M$ and $J = 1,N$. 
To Run:

Type: OLD, INTERPL
Type: GET, TAPE9
Type: FTNTS
Type: RUN
Type: SAVE, (REPLACE,) TAPE20
B3.6 MATLLL, RUNMATL

Input: TAPE20, TAPE47
Program: MATLLL
Procedure File: RUNMATL
Output: TAPE21, TAPE86, TAPE49

1. Description:

This is the MATHEW program. The input tapes have already been created and grid dimensions already determined. Output are the adjusted wind velocity field (TAPE86), the input wind velocity field (TAPE21) and the conditioned topography tape (TAPE49).

2. Edit Changes:

1. Specify A, B of CHEIGHT(A,B) and MINLVL2(A,B) where A = (M-1)/2 and B = (N-1)/2.

2. Change all two dimensional arrays to (M,N) dimensions.

3. Change all three dimensional arrays to (M,N,L) dimensions.

4. Specify M, N and L.

5. Specify SIGH, SIGV where (SIGH/SIGV) = (vertical area)/(horizontal area). Areas refer to MATHEW box sides.

6. Specify REF = Reference height, DELZ = delta z (in m) and PHIRSL = shear coefficient.

3. To Run:

Type: OLD, RUNMATL
Type: BATCH
Type: SUBMIT, RUNMATL, B
1. Description:

TAPE86 contains the adjusted u, v and w velocity components for each grid point. Similarly, TAPE21 contains the interpolated u and v components. RESLL computes the resultant two and three dimensional adjusted velocity vectors and creates a file (TAPE14) to be plotted. RESL2 computes the two dimensional input velocity vectors and creates a file (TAPE16) to be plotted.

2. Edit Changes:

1. Change all two dimensional arrays to (M,N) dimensions.
2. Specify X(M) and Y(N).
3. Specify M, N and L.
4. Specify LEVEL = level to be plotted.
5. Specify XA, XB, C and D of X(I) = XA + C*I and Y(J) = YA + D*J
   where XA = x starting point - delta x, YA = y starting point - delta y, C = delta x and D = delta y.

3. To Run:

Type: OLD,RESLL (RESL2)
Type: ATTACH,TAPE86 (TAPE21)
Type: FTNTS
Type: RUN
Type: SAVE,(REPLACE,)TAPE14 (TAPE16)
B3.8 SUBTRAC

Input: TAPE86, TAPE21
Program: SUBTRAC
Output: TAPE18

1. Description:

The program SUBTRAC computes the root-mean-square difference between the adjusted velocity (TAPE86) and the interpolated velocity values at a specified distance above ground level. It determines the average RMS velocity, the maximum and minimum RMS values. TAPE18 also gives velocity difference output at each grid point above ground level.

2. Edit Changes:

1. Change all two dimensional arrays to \((M, N)\) dimensions.
2. Specify \(MX(M)\) AND \(MY(N)\).
3. Specify \(MXLVL\) where \(MXLVL = \) the maximum above ground level for the highest topographic point in the MATHFA box.
4. Specify \(M, N\) and \(L\).
5. Specify \(I = 1, M\) and \(J = 1, N\).

To Run:

Type: OLD,SUBTRAC
Type: ATTACH,TAPE21
ATTACH,TAPE86
Type: FTNTS
Type: RUN
Type: SAVE,(REPLACE,)TAPE18
B3.9 PLOT, PARAM

Input: Tape to be plotted
Procedure File: PLOT
Output: Contour (Isotach) Plot

1. Description:

In order to use the UCC CONTOUR plotting routine, the user must submit the procedure file called PLOT. The file contains the necessary job control cards and calls the appropriate tapes and parameter files. The user must specify the tape to be plotted on and the parameter file (PARAM) must be indicated. The parameter file is tailored for each particular job. There are five parameter cards which must be specified for each plot. The details of the format and content are given in the UCC manual for the CONTOUR software package.

2. To Run:

Type: OLD,PLOT
Type: BATCH
Type: SUBMIT,PLOT,B
APPENDIX C - INPUT DATA SUMMARY

C1. PRINCETON

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2. PRINCETON - DATA STATION SUMMARY SHEET .................. 256

C2. WINDSOR

1. WINDSOR - U.S. GEOLOGICAL SURVEY MAP ..................... 257
2. WINDSOR - DATA STATION SUMMARY SHEET ..................... 258
## C1.2 PRINCETON - DATA STATION SUMMARY SHEET

<table>
<thead>
<tr>
<th>Site Number</th>
<th>Wind Speed (mph)/(m/s)</th>
<th>Height Above Ground (m)/(ft)</th>
<th>Direction (degrees)</th>
<th>Location on MATHEN Grid (x,y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.25/5.48</td>
<td>110.27/361.8</td>
<td>303</td>
<td>188,775</td>
</tr>
<tr>
<td>2</td>
<td>14.50/6.48</td>
<td>64.55/211.8</td>
<td>285</td>
<td>197,755</td>
</tr>
<tr>
<td>3</td>
<td>5.75/7.04</td>
<td>108.12/354.7</td>
<td>340</td>
<td>200,730</td>
</tr>
<tr>
<td>4</td>
<td>8.75/3.91</td>
<td>71.88/235.8</td>
<td>307</td>
<td>167,740</td>
</tr>
<tr>
<td>5</td>
<td>17.50/7.82</td>
<td>111.45/365.7</td>
<td>310</td>
<td>173,795</td>
</tr>
</tbody>
</table>

**Notes:**

1. The above data is taken from the summary sheet dated June 12, 1980.

2. The direction the wind is coming from is measured clockwise from the north which is set equal to zero degrees.

3. The shear coefficient measured for this day is = .25.

4. The site locations given above use digital terrain tape coordinates.

5. All velocity measurements given above are one minute averages over four 15 sec intervals. The velocity values are measured concurrently using synchronized watches. The values used are chosen from a series of data to best represent the wind flow field.
### C2.2 WINDSOR - DATA STATION SUMMARY SHEET

<table>
<thead>
<tr>
<th>Site Number</th>
<th>Wind Speed (mph)/(m/s)</th>
<th>Height Above Ground (m)/(ft)</th>
<th>Direction (degrees)</th>
<th>Location on MATHEW Grid (x,y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.75/4.81</td>
<td>18.29/60.0</td>
<td>300</td>
<td>1221,872</td>
</tr>
<tr>
<td>2</td>
<td>13.85/6.19</td>
<td>151.33/496.5</td>
<td>270</td>
<td>1230,878</td>
</tr>
<tr>
<td>3</td>
<td>15.40/6.88</td>
<td>281.10/425.8</td>
<td>292</td>
<td>1234,893</td>
</tr>
<tr>
<td>4</td>
<td>13.00/5.81</td>
<td>99.97/328.0</td>
<td>298</td>
<td>1222,915</td>
</tr>
<tr>
<td>5</td>
<td>10.70/4.78</td>
<td>106.40/349.2</td>
<td>278</td>
<td>1208,887</td>
</tr>
</tbody>
</table>

**Notes:**

1. The above data is taken from the summary sheet dated October 18, 1982.

2. The direction the wind is coming from is measured clockwise from the north which is set equal to zero degrees.

3. The shear coefficient measured for this day is = .40.

4. The site locations given above use digital terrain tape coordinates.

5. All velocity measurements given above are five minute averages over twenty 15 sec intervals. The velocity values are measured concurrently using synchronized watches. The values used are chosen from a series of data to best represent the wind flow field.
APPENDIX D - PLOTS OF MATHEW RUNS

D1. PRINCETON

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2. PRINC1 - TWO DIMENSIONSAL ADJUSTED VELOCITY FIELD .... 261

D2. WINDSOR

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D1.2 PRINC1 - TWO DIMENSIONAL ADJUSTED VELOCITY FIELD

PRINC1 - Two Dimensional Adjusted Velocity Field
WIN1 - Two Dimensional Adjusted Velocity Field