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Abstract

This paper derives the balance of payments-constrained growth (BPCG) model as a special case of a three good framework that incorporates exportables, importables, and non-tradables. The conditions under which the canonical form of the BPCG rate can be derived are made explicit and the assumptions scrutinized. It is shown that the presence of non-tradables, substitutability between exportables and importables, and incomplete specialization in expenditure generally dampen the externally-constrained growth rate. These findings help explain why empirical estimates tend to overestimate the BPCG rate. Overall our findings underscore the observation that tests of the BPCG hypothesis are as much a test of the *internal* structure of the economy under consideration.

JEL Codes: F41, F43, O41

Keywords: Balance of payments-constrained growth model, non-tradables, demand-led growth, real exchange rates, terms of trade.

1 Introduction and Motivation

The idea of a balance of payments constraint on growth has been a staple of much demand side-oriented growth theory, especially since Thirlwall (1979). This paper re-examines the balance of payments-constrained growth (BPCG) model using an extended setup that incorporates both supply and demand side considerations and yields Thirlwall's BPCG rate as a special case.

Thirlwall's and subsequent work by others in the BPCG tradition has interpreted the balance of payments constraint as originating from the demand

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side. In this sense, growth according to this tradition can be seen as external demand-led.[1] The BPCG growth rate, in its most general form, call it "BPCG1," includes both a relative price term and a term specified as the ratio of the income elasticity of demand for a country's exports to the income elasticity of the country's demand for imports times the rate of world income growth. Assuming the satisfaction of the Marshall-Lerner (or ML) condition, the constrained growth rate then becomes a negative function of a country's terms of trade. We show that this version of the BPCG hypothesis can only be derived if the terms of trade are considered exogenous and the exportable and non-tradable sector clearing conditions ignored. As we discuss below, these assumptions are problematic. A more restrictive form of the hypothesis, "BPCG2," ignores the relative price term. This is the definition of the BPCG rate that we term the "canonical" version in the following sections. The BPCG growth rate in its most concise version, call it "BPCG3," equals a country's rate of growth of exports divided by its income elasticity of demand for imports. This assumes that the rate of growth of exports equals the income elasticity of world demand for Home exports times the rate of world income growth. As shown below, however, in the presence of an independent exportable sector clearing condition based on the supply of and demand for exportables, the BPCG hypothesis cannot be stated in its most succinct form. This is because, in this case, the rate of growth of exports does not generally equal the income elasticity of world demand for a country's exports times the rate of growth of world income.

Its parsimonious nature and sharp predictions make the BPCG framework an interesting point of departure for studying economic growth in open economies. However, like all interesting models, the BPCG model makes some sweeping assumptions. For example, foreign and domestic goods are supposed to be imperfect substitutes through a pair of constant elasticity of substitution functions, one each for exports and imports. As discussed below, importables (exportables) are implicitly assumed not to be produced (consumed) domestically. The distinction between exportables and exports, importables and imports, and tradables and non-tradables are thus ignored. Since non-tradables are excluded, real exchange rate changes, defined as changes in the relative price of non-tradables to tradables, cannot be factored into the analysis. This leaves important aspects pertaining to internal economic structure unexplored. The sectoral composition of the demand side boost to domestic income is ignored. The income and price elasticities of demand and supply are assumed given and the evolution of technology and preferences ignored. Moreover, supply side constraints are abstracted out of the framework, although it has been argued that the parameters of the model (i.e., the demand elasticities) partially capture supply side factors.[2]

We examine the logical consequences of introducing these complications. Conceptually, the BPCG hypothesis can be understood as incorporating two

[1] The assumption of balanced trade means that growth in the BPCG framework cannot be seen simply as net export-led. An expansion of exports, however, creates room for income (and thus imports) to grow.

[2] See McCombie and Roberts (2002), for example.

sub-hypotheses: (1) that growth is constrained by the need to maintain the balance of payments, and (2) that the constraint on the balance of payments originates from Home demand for imports and foreign demand for Home exports. We maintain (1) while relaxing (2) to analyze the implications of incorporating non-tradables and supply side considerations. The aim is to make explicit the assumptions underlying the BPCG framework in order to analyze its robustness under alternative scenarios. To do this, we develop and analyze a three good framework. We show that the narrower version of the BPCG hypothesis that lacks the relative price term results as a special case under certain assumptions. The broader version that does include relative prices cannot be properly derived from the assumptions implicit in Thirlwall (1979) unless the terms of trade are assumed to be exogenous. As argued below, however, the inclusion of terms of trade in BPCG1 is problematic.

On a broader note, Thirlwall (1980)[p.421] states: "The fundamental proposition I wish to make is that no country or region (for very long) can grow faster than its balance-of-payments equilibrium growth rate unless it can continually 'finance' a rate of growth of imports in excess of the rate of growth of exports." However, even with a trade balance constraint, the growth rate may not be constrained by world growth from the demand side. For example, productivity changes in the exportable, importable, or non-tradables sectors may loosen or tighten the constraint *at a given rate of world demand growth*. Under more general conditions, output growth need not necessarily be constrained by the growth of world demand, even if the trade balance condition binds. Indeed, as we show below, in the absence of substitution effects on the demand and supply sides, domestic growth becomes a *negative* function of the world growth rate in the presence of a trade balance constraint. Like Krugman and Taylor (1978), this underlines the perverse effects that can occur in the short run following an exogenous shock. Also, non-satisfaction of the Marshall-Lerner condition means that, under certain conditions, technological progress in the exportable sector can lead to "immiserizing growth," along the lines of Bhagwati (1958).

Even when domestic growth is a positive function of world growth, the constant of proportionality often involves more than the ratio of the two income elasticities. For example, a positive correlation between domestic and world income growth could either be due to the trade balance channel (perhaps due to the demand side considerations emphasized by the BPCG tradition or because of strong substitution effects on the supply side), or due to other factors such as common shocks to non-tradables at Home and abroad. An example of the latter would be the rising housing prices that led to consumption-led growth recently in many countries. Another would be a period of good weather that improves agricultural output globally in a world where trade barriers and/or phyto-sanitary requirements render agriculture largely non-tradable.

Thus, since a positive association between Home and world growth does not establish the BPCG channel, the BPCG hypothesis is really also a non-trivial test of the structure of the economy. Empirical tests of the BPCG hypothesis should ideally address these alternative hypotheses explicitly.

Thirlwall (1979)[p. 50] finds that "there is a general tendency for the estimates of the balance of payments equilibrium growth to be higher than the actual growth rate, which, if true, would produce a balance of payments surplus." More recently, Perraton (2003) finds that the "weak" form of Thirlwall's hypothesis over-predicts the actual growth rate for all but one of the countries in the sample.[3] We show that the presence of non-tradables provides an alternative explanation for this finding. In other words, the incorporation of non-tradables in the model yields a growth rate consistent with balance of payments equilibrium that is significantly lower than that yielded by the BPCG hypothesis in its traditional versions. A corollary is that countries growing slower than the BPCG rate need not be running balance of payments surpluses. Furthermore, we show that the BPCG hypothesis in its typical versions can only be derived if we assume that the Home country does not produce the importable good. Relaxing this assumption and allowing for substitution in production and expenditure too could help bridge the gap between actual growth rates and empirically estimated BPCG ones.

While we do not carry out any empirical analysis, and some of our conclusions have been arrived at by other work, we provide a compact, unified framework to explore the strengths and weaknesses of the BPCG idea, and help sharpen empirical questions. The next section introduces the three sector framework that we utilize for our analysis. Section 3 then derives the "canonical" BPCG hypothesis after imposing rather restrictive conditions on the framework. Section 4 derives the BPCG hypothesis from a more general set-up and then explores the effects of various shocks under different sets of assumptions. Section 5 then further loosens the restrictions and examines the consequences of substitutability in consumption and production between tradables and non-tradables with the help of simulation exercises. The logical progression from Section 3 to Section 5 roughly involves moving in the direction of greater generality, mainly to facilitate intuition. Finally, Section 6 concludes.

2 A three good model

In the discussion below, the *subscripts* N, X and M refer to non-traded goods, exportables, and importables, respectively. Table 1 provides concise definitions of the variables employed.

We begin by specifying a general model in which all three goods are substitutable in consumption and production. Later sections then explore the properties of the model and analyze the assumptions under which the balance of payments constraint, as captured by world demand for our exports and our demand for imports, is valid. Consistent with the BPCG framework, throughout we assume less than full employment of resources (so that there is slack in the factor markets and output can be increased in response to higher demand without a one-to-one trade-off between the employment of labor in different

[3] The sample consists of developing countries. The paper reaches similar conclusions for the "strong form." The "strong" and "weak" forms of the hypothesis correspond to our BPCG2 and BPCG3, respectively.

Table 1: Definitions of variables

Variable	Definition ($i = N,\ X,\ M$ and $k = N, X$)
Y	Aggregate real domestic output
E	Aggregate real domestic expenditure
Y_i	Real output of sector i
E_i	Real domestic expenditure on sector i's output
P_i	Nominal price per unit of sector i's output
p_i	Price per unit of sector i's output relative to that of importables p_x=terms of trade and p_N=real exchange rate
M^*	Foreign demand for Home exports
X	Home supply of exports
M	Home demand for imports
Z	World income or expenditure
α_i	Demand shock to sector i
β_i	Supply shock to sector i
θ_X	Export share of total exportable production
θ_M	Import share of total importable consumption
η_i	Expenditure elasticity of demand for sector i's output
η_z^*	Foreign income elasticity of demand for Home exports
σ_{ii},σ_{ik}	Own- and cross-price supply elasticity of sector i's output
δ_{ii},δ_{ik}	Own- and cross-price demand elasticity for sector i's output

sectors).[4]

Consider an economy in which the Home country, or simply Home, produces a non-tradable good, an exportable good, and an importable good. Home consumers face a perfectly elastic import supply curve. The relative price of exportables is assumed to adjust, by contrast, to match supply and demand in the exportable sector. Thus, while the country is invariably small on the import side, this is not generally true on the export side. This assumption, apart from simplifying the analysis, also reflects the stylized fact that countries tend to be larger on the export side than on the import side. As we will see below, it leads to the conclusion that, in the presence of a binding external constraint, the consequence of an external demand side boost for domestic income growth is ambiguous, and could even be negative. Since P_X and P_N are both allowed to vary endogenously in the following sections, it will be notationally convenient to denominate all prices in terms of the importable good. Thus, $p_N = P_N/P_M$ and $p_X = P_X/P_M$. We use the terms "real exchange rate" and "terms of trade," respectively, for these ratios. Our general set-up can be captured with the help of the following behavioral equations and equilibrium conditions.

[4] The presence of slack in the factor markets raises a question about the degree of substitutability on the production side. In other words, why would Y_i be functions of p_i if firms can simply utilize unemployed resources to expand production in response to higher demand? However, even in the presence of slack, relative price changes lead to changes in relative sectoral profitability, encouraging firms to shift resources.

$$E_N = E_N(E, p_X, p_N; \alpha_N);\; E_{N1},\, E_{N2},\, E_{N4} > 0, E_{N3} < 0 \tag{1}$$

$$E_X = E_X(E, p_X, p_N; \alpha_X);\; E_{X1},\, E_{X3},\, E_{X4} > 0, E_{X2} < 0 \tag{2}$$

$$E_M = E_M(E, p_X, p_N; \alpha_M);\; E_{M1},\, E_{M2},\, E_{M3},\, E_{M4} > 0 \tag{3}$$

$$Y_N = Y_N(p_X, p_N; \beta_N);\; Y_{N2},\, Y_{N3} > 0,\, Y_{N1} < 0 \tag{4}$$

$$Y_X = Y_X(p_X, p_N; \beta_X);\; Y_{X1}, Y_{X3} > 0,\, Y_{X2} < 0 \tag{5}$$

$$Y_M = Y_M(p_X, p_N; \beta_M);\; Y_{M3} > 0,\, Y_{M1}, Y_{M2} < 0 \tag{6}$$

$$X = Y_X - E_X \tag{7}$$

$$M^* = M^*\left(p_X, Z\right);\; M_1^* < 0, M_2^* > 0 \tag{8}$$

$$Y_N = E_N \tag{9}$$

$$M = E_M - Y_M \tag{10}$$

$$X = M^* \tag{11}$$

$$Y_M + p_X Y_X + p_N Y_N = E_M + p_X E_X + p_N E_N \tag{12}$$

$$p_X M^* = M \tag{13}$$

where, by definition, $Y = Y_M + p_X Y_X + p_N Y_N$ and $E = E_M + p_X E_X + p_N E_N$. Equations (1)-(3) define the sectoral expenditure functions, which are functions of relative prices, aggregate expenditure, and exogenous parameters. Equations (4)-(6) specify the sectoral output functions. We implicitly assume that aggregate income Y identically equals aggregate expenditure E.

Looked at from the supply side, exports are the difference between domestic output and consumption of exportables (equation (7)) while analyzed from the demand side, exports are determined by world demand for exportables, which is a function of the terms of trade p_X and world income Z (equation (8)). Finally, eqs. (9)-(13) constitute the equilibrium conditions, respectively, for: (i) the non-tradable goods sector, (ii) the importable goods sector, (iii) the exportable goods sector, (iv) the aggregate (macro) economy, and (v) the trade balance. As shown in the appendix, only four of these conditions are independent.

Our set-up assumes that the economy is a price taker in the international market for the importable good, and that M adjusts instantaneously to clear this market domestically.[5] Further, in order to simplify the analysis and intuition, we assume that the price of non-tradables adjusts instantaneously to clear the non-tradable market in response to changes in aggregate expenditure and the terms of trade. In other words, E and p_X are determined simultaneously by the trade balance and the exportable sector equilibrium condition, while p_N adjusts in response to clear the non-traded sector at the new values of these variables.[6] This requires that p_N be a differentiable function of E and p_X. Using eqs. (1), (4), and (9) yields, after log-linearization and differentiation (in order to maintain consistency with the BPCG framework),

$$\hat{p}_N = \frac{1}{\sigma_{NN} + \delta_{NN}} [\eta_N \hat{Y} + (\sigma_{NX} + \delta_{NX})\hat{p}_X + \hat{\alpha}_N - \hat{\beta}_N] \tag{14}$$

or,

$$\hat{p}_N = \hat{p}_N(\overset{+}{\hat{Y}}, \overset{+}{\hat{p}_X}, \overset{+}{\hat{\alpha}_N}, \overset{-}{\hat{\beta}_N})$$

where hats or circumflexes denote growth rates. The intuition underlying these signs is simple. An increase in the relative price of exportables or a rise in aggregate expenditure or a positive demand shock creates excess demand for non-tradables due both to income and substitution effects. The relative price of non-tradables must rise in order to remove the excess demand. A supply side shock, on the other hand, creates an excess supply of non-tradables, putting downward pressure on their relative price.

The system of equations (1)-(13) can now be reduced to two equilibrium conditions in two variables ($\hat{Y}$ and $\hat{p}_X$). Substituting (2), (5), (7), and (8) into the exportable sector clearing condition (i.e., equation (11)), yields, after log differentiation, the following excess demand condition:

$$(1-\theta_X)\eta_X \hat{Y} - [\sigma_{XX} + (1-\theta_X)\delta_{XX} + \theta_X \delta_X^*]\hat{p}_X + [\sigma_{XN} + (1-\theta_X)\delta_{XN}]\hat{p}_N \\ + (1-\theta_X)\hat{\alpha}_X - \hat{\beta}_X + \theta_X \eta_Z^* \hat{Z} = 0 \tag{15}$$

Similarly, substituting (3), (6), (8), and (10) into the trade balance condition (i.e., equation (13)), yields, after log differentiation, the following expression for the trade deficit:

$$\eta_M \hat{Y} + [(1-\theta_M)\sigma_{MX} + \delta_{MX} - \theta_M(1-\delta_X^*)]\hat{p}_X + [(1-\theta_M)\sigma_{MN} + \delta_{MN}]\hat{p}_N \\ + \hat{\alpha}_M - (1-\theta_M)\hat{\beta}_M - \theta_M \eta_Z^* \hat{Z} = 0 \tag{16}$$

[5] Put differently, equation (10) holds continuously.

[6] Relaxing this assumption would involve solving our system as a 3×3 system of simultaneous equations, which would yield the same results but the intuition would be harder to convey.

Finally, utilizing the assumption that the non-traded sector clears continuously allows us to substitute from equation (14) into eqs. (15) and (16) and collect terms to yield:

$$\left[(1-\theta_X)\eta_X + \frac{\sigma_{XN}+(1-\theta_X)\delta_{XN}}{\sigma_{NN}+\delta_{NN}}\eta_N\right]\hat{Y}$$
$$-\left\{\sigma_{XX}+(1-\theta_X)\delta_{XX}+\theta_X\delta_X^* - \frac{\sigma_{NX}+\delta_{NX}}{\sigma_{NN}+\delta_{NN}}[\sigma_{XN}+(1-\theta_X)\delta_{XN}]\right\}\hat{p}_X$$
$$+\frac{\sigma_{XN}+(1-\theta_X)\delta_{XN}}{\sigma_{NN}+\delta_{NN}}(\hat{\alpha}_N-\hat{\beta}_N)+(1-\theta_X)\hat{\alpha}_X-\hat{\beta}_X+\theta_X\eta_Z^*\hat{Z}=0 \quad (17)$$

$$\left[\eta_M + \frac{(1-\theta_M)\sigma_{MN}+\delta_{MN}}{\sigma_{NN}+\delta_{NN}}\eta_N\right]\hat{Y}$$
$$+\left\{(1-\theta_M)\sigma_{MX}+\delta_{MX}-\theta_M(1-\delta_X^*)+\frac{\sigma_{NX}+\delta_{NX}}{\sigma_{NN}+\delta_{NN}}[(1-\theta_M)\sigma_{MN}+\delta_{MN}]\right\}\hat{p}_X$$
$$+\frac{(1-\theta_M)\sigma_{MN}+\delta_{MN}}{\sigma_{NN}+\delta_{NN}}(\hat{\alpha}_N-\hat{\beta}_N)+\hat{\alpha}_M-(1-\theta_M)\hat{\beta}_M-\theta_M\eta_Z^*\hat{Z}=0 \quad (18)$$

Equations (17) and (18) constitute our general system. The following sections present the solutions under specific assumptions and often in implicit form. The Appendix presents the general solutions. Table 2 summarizes the key comparative static results.

3 The "canonical" BPCG case

Before we solve our system under various scenarios, notice that the BPCG model, as derived originally by Thirlwall (1979), ignores equation (17). In other words, specification only of an export *demand* function implies the absence of an independent exportable sector clearing condition. We discuss this assumption shortly. For now let us assume that, for one reason or another, equation (17) can be ignored. This requires that the terms of trade be exogenous, so that we have one equilibrium condition (the trade balance one) in one variable ($\hat{Y}$). Furthermore, assume that (1) either there is no substitutability in production or expenditure between tradables and non-tradables (i.e., $\sigma_{MN}=\sigma_{XN}=\sigma_{NM}=\sigma_{NX}=\delta_{MN}=\delta_{XN}=\delta_{NM}=\delta_{NX}=0$) or the price of non-tradables is fixed ($\hat{p}_N=0$), (2) importables are not produced at Home ($\theta_M=1$), and (3) there are no demand side shocks in the importable sector ($\hat{\alpha}_M=\hat{\beta}_M=0$). Under these conditions our system, now consisting only of equation (16) minus the term containing $\hat{p}_N$, reduces to the BPCG1 solution:

$$\hat{Y}_{BPCG1}=\frac{-(\delta_{MX}+\delta_X^*-1)\hat{p}_X+\eta_Z^*\hat{Z}}{\eta_M} \quad (19)$$

The term inside brackets on the right hand side is simply the ML condition as long as we retain assumption (2).[7] Assuming further, following Thirlwall (1979), either constant terms of trade or that the Marshall-Lerner condition is *exactly* satisfied (so that $\delta_{MX} + \delta^*_X = 1$) yields the BPCG2 solution. That is,

$$\hat{Y}_{BPCG2} = \frac{\eta^*_Z}{\eta_M}\hat{Z} \tag{20}$$

Thus, the growth rate of our economy is a function of the growth rate of world income, the constant of proportionality being given by the ratio of the foreign elasticity of demand for Home goods divided by the Home elasticity of demand for foreign goods.[8] This, incidentally, is what Perraton (2003) terms the "strong" form of the BPCG hypothesis.

If assumption (3) is relaxed, supply and demand side shocks in the importable sector have a positive and negative impact on the domestic growth rate, respectively.[9] This reflects the trade balance constraint. The only difference from the canonical framework is that our set-up allows us to analyze the impact of changes in the evolution of preferences and technology explicitly, whereas the former only incorporates the levels of these parameters.

Given that the relative price of exportables is exogenous, we can, therefore, derive the BPCG hypothesis in its most concise form, BPCG3, which Perraton (2003) terms the "weak" form of the BPCG hypothesis.[10]

$$\hat{Y}_{BPCG3} = \frac{1}{\eta_M}\hat{X} \tag{21}$$

A few words about this set-up are in order here. Ignoring equation (17) while deriving BPCG1 can only be valid if the terms of trade are assumed to be exogenous.[11] This is due to the fact that, as long as the terms of trade are endogenous, relative prices and quantities are determined simultaneously, and, therefore, BPCG1 cannot be derived (although BPCG2 can, under the

[7] The ML condition pertains to the sum of the Home elasticity of demand for imports and foreign elasticity of demand for Home exports. Without assumption (2), the expression above can no longer be termed the ML condition since δ_{MX} is now the elasticity of substitution between exportables and importables at Home rather than the price elasticity of demand for imports.

[8] Alternatively we would get the same result if we continue to assume (2) and (3) in addition to assuming that the sum of thr own-price supply elasticities of supply and expenditure in the non-tradable sector is infinitely high.

[9] Such shocks in the exportable sector would have no impact on output growth, since the canonical framework rules out an indepedent exportable sector clearing condition. In other words, the export market clearing condition is ruled out as exports are determined solely by international demand. This rules out, among other things, "immiserizing growth" caused by technological growth in the exportable sector.

[10] Thirlwall (1979) derived this by assuming that either the ML condition is *exactly* satisfied or $\hat{p}_X = 0$. However, McGregor and Swales (1985) correctly point out that it is only the satisfaction of the latter condition that enables us to derive BPCG3. If only the former condition is satisfied, then BPCG2 can be derived from BPCG1 but BPCG3 cannot, since in this case $\hat{X} = -\delta^*_X\hat{p}_X + \eta^*_Z\hat{Z}$.

[11] In other words, $\hat{p}_x = 0$ barring exogenous shocks.

assumption that the own-price elasticity of exports is infinite; more on this in Section 4.1). Exogenous terms of trade typically imply that Home is a small country, and that, therefore, the price elasticity of foreign demand for Home exportables is infinite. But this does not sit well with the assumption typically made while deriving the BPCG hypothesis that exports and imports are imperfect substitutes (and hence domestic producers face a downward sloping demand curve). If, on the other hand, Home exporters are price-setters in a setting where Home's own-price supply elasticity of exportables is infinite, then the exogeneity of the terms of trade implies that Home producers set prices in *foreign* currency, letting domestic prices vary in response to nominal exchange rate changes. In other words, Home firms practice local currency pricing (LCP). The underlying pricing mechanism appears to involve mark-up pricing with zero pass-through into foreign prices. However, exchange rate pass-through into export prices tends to be far from zero, especially in the long run.[12] A pricing to market (PTM) assumption cannot be made either since it is being assumed that the Home country is specialized in consumption and production. We postpone derivation of the canonical BPCG hypothesis in the presence of an independent exportable clearing condition to Section 4.1, where we show that BPCG1 cannot be properly derived from our system of equations *even* if the own-price supply elasticity of exportables is infinite.

If assumption (2) is relaxed so that importables are consumed at Home, then specifying constant terms of trade yields the following modified form of equation (20):

$$\hat{Y} = \theta_M \frac{\eta_Z^*}{\eta_M} \hat{Z} \tag{22}$$

Thus, the balance of payments constrained growth rate now becomes a *positive* function of the proportion of the expenditure on importables that falls on goods produced abroad. In a mercantilist utopia where all importables are produced at Home (so that $\theta_M = 0$), an increase in world expenditures on Home goods has no impact on domestic growth. This somewhat counterintuitive result provides a nice illustration of the BPCG logic: an increase in world demand for

[12] See, for example, Campa and Goldberg (2005). Obstfeld and Rogoff (2000) find that, contrary to what one would expect in the presence of LCP pricing, nominal depreciation tends to be associated with deteriorating terms of trade.

A formal way to explore the issue is to utilize the typical specification of price setting in a monopolistic environment. Suppose that labor is the only factor of production and a denotes the unit labor coefficient. Then, letting the price elasticity of demand be represented by δ, a price setting equation would typically assume the form

$$P = (Wa)^{\gamma}(EP^*)^{1-\gamma}$$

where P and P^* denote the domestic and foreign price, respectively, and γ and $1-\gamma$ represent the weights placed on domestic costs and foreign competition, respectively. The case where $\gamma = 0$ represents the small country scenario, in which case exports are determined from the supply side. If $\gamma = 1$, by contrast, foreign competition does not matter. This assumption is not consistent with the imperfect substitutes specification used for the export and import equations while deriving the BPCG model. Given a fixed unit labor cost, $\hat{P} - \hat{E} - \hat{P}^*$ equals zero only when $\gamma = 0$. When this is not the case, $\hat{P} = (1-\gamma)(\hat{E} + \hat{P}^*)$.

Home exports creates room for Home to spend more on imports while maintaining balanced trade. The lower the initial proportion of importables that is purchased from abroad, the lower the proportional expansion in domestic expenditures on importables required to maintain balanced trade. With a given η_M, the latter, in turn, translates into a smaller required expansion of aggregate expenditure and income. Notice also that, to the best of our knowledge, all existing estimates of the BPCG hypothesis implicitly assume that $\theta_M = 1$. If $\theta_M < 1$, which is almost always true, empirical estimates would deliver a BPCG rate higher than the actual growth rate of the economy that is consistent with external balance.[13]

Finally, if assumption (1) is relaxed, equation (18) tells us that a concurrent productivity shock that occurs in the non-tradable sector at Home raises the constrained growth rate, but this is not due to an increase in world demand. Rather, the excess supply of non-tradables created leads to a depreciation of the real exchange rate (that is, a decline in the relative price of non-tradables via equation (14)), which shifts domestic demand towards non-tradables, thus lowering net Home demand for importables. This in turn helps loosen the external constraint.

4 When non-tradables and tradables are not substitutes

Now let us reintroduce the exportable sector clearing condition by reverting to our system of equations (17) and (18). Consider a simple economy where the elasticity of substitution between non-tradables and exportables on the one hand, and between non-tradables and importables on the other is zero, both on the demand and supply sides.[14] These assumptions simplify the system by essentially reducing it to equations (15) and (16) minus the terms that include $\hat{p}_N$. In this case, the effect of world income growth on domestic (external account-constrained) growth is ambiguous. If, for example, foreign demand for Home products is relatively price elastic ($\delta^*_X > 1$), then it can be shown that:

$$\hat{Y} = \hat{Y}(\overset{+/-}{\hat{Z}}, \overset{-}{\hat{\alpha}_M}, \overset{+}{\hat{\beta}_M}, \overset{-}{\hat{\alpha}_X}, \overset{+}{\hat{\beta}_X})$$

and,

$$\hat{p}_X = \hat{p}_X(\overset{+}{\hat{Z}}, \overset{-}{\hat{\alpha}_M}, \overset{+}{\hat{\beta}_M}, \overset{+}{\hat{\alpha}_X}, \overset{-}{\hat{\beta}_X})$$

Further simplifying assumptions are required to derive the BPCG result.

[13] This result is nicely illustrated later for the general case by Figure 3.

[14] Alternatively, one could assume that the price of non-tradables remains constant, or equivalently, given equation (14), and in the absence of demand or supply side shocks in the non-tradable sector, that non-tradables and exportables are not substitutes on the demand or supply sides, and that the income elasticity of demand for non-tradables is zero. Finally, one would get the same results if one assumes that the sum of own price elasticities of demand for and supply of non-tradables is infinite.

4.1 Assuming specialization and imposing restrictions on price elasticities

Assuming that: (i) *initially* exportables are not consumed at Home and importables are not produced at Home,[15] and (ii) the own-price exportable supply elasticity is infinite,

$$\hat{Y} = \frac{\eta^*_Z \hat{Z} - \hat{\alpha}_M}{\eta_M}$$

$$\hat{p}_X = 0$$

so that, ignoring demand side shocks in the importable sector yields the canonical BPCG solution (i.e., BPCG2). The intuition is simple. An increase in world income raises demand for our goods, creating an excess supply of exportables via equation (7) and a trade surplus via equation (13). Given the infinite own-price elasticity of exportable supply, the volume of exports rises without a change in the relative price of exportables to clear the exportable market. The trade surplus, on the other hand, is removed through an increase in income and expenditure.

A shift in preferences towards the importable good (i.e., a rise in $\hat{\alpha}_M$) reduces income without affecting the terms of trade. The traditional BPCG framework treats the income elasticity of imports as a parameter. However, even if it is given at a point in time, the result above explicitly shows that the evolution over time matters.

Thus, it is clear that the BPCG model as typically specified (see equation (20)) assumes an infinite elasticity of export supply. However, making this assumption renders it impossible to derive BPCG1. Unless the terms of trade are treated as exogenous, therefore, deriving the hypothesis correctly yields the version that lacks the relative price term. Intuitively, with the own-price elasticity of exports approaching infinity, an infinitesimally small change in relative prices suffices to generate the adjustment in the volume of exports required for these to equal foreign demand. Moreover, combined with equation (8), unchanged terms of trade imply that, in equilibrium, $\hat{X} = \eta^*_Z \hat{Z}$, so that BPCG3 can indeed be derived from BPCG2.

In sum, of the three versions of the BPCG hypothesis mentioned in Section 3, only BPCG2 and BPCG3 can be properly derived unless we assume that the terms of trade are exogenous. For reasons discussed earlier, this assumption is problematic. Since a lot of recent empirical work has focused on estimating BPCG1 while treating the relative price variable as endogenous, often in a vector error correction framework, this point is far from trivial.

On a related note, notice that under the set of assumptions that yields BPCG2 from our general framework, supply side factors (i.e., the β's) do not

[15] If we exclude even *potential* Home consumption of exportables and Home production of importables, then δ_{MX} and σ_{MX} would both be zero. We do not make this assumption, leaving open the possibility that Home could become less specialized in the future.

make an appearance in the solution for the domestic growth rate. This is not surprising since, with an infinitely elastic supply of exportables, no effective presence of non-tradables, and no production of importables at Home, supply side shocks become irrelevant. Income growth is now fully demand-determined. It is helped by growth in foreign demand (as in the BPCG framework) but hurt by growth in domestic demand (as captured by $\hat{\alpha}_M$ in our extended framework).

Continuing to assume for the rest of this section that initially exportables are not consumed at Home and importables are not produced at Home, but now moving to the other extreme in assuming that the own-price supply elasticity of exportables equals zero yields,

$$\hat{Y} = \hat{Y}(\overset{+/-}{\hat{Z}}, \overset{-}{\hat{\alpha}_M}, \overset{+/-}{\hat{\beta}_X})$$

and,

$$\hat{p}_X = \hat{p}_X(\overset{+}{\hat{Z}}, \overset{-}{\hat{\beta}_X})$$

Again, given (11), we know that the increase in demand for Home exports caused by faster growth of world income requires that either domestic export or terms of trade growth rise. Given that the domestic economy does not consume the exportable good and that the supply response is zero, only the latter mechanism occurs to clear the market for exportables. The resulting higher terms of trade have valuation effects (on exports) and substitution effects (on imports) via equation (13). If $\delta_{MX} < 1$, the valuation effect dominates so that a trade surplus is created. Imports have to rise via faster Home income growth. If, by contrast, $\delta_{MX} > 1$ so that the substitution effect dominates, then domestic demand for importables increases adequately following the improvement in Home terms of trade to create a trade deficit. Home has to adjust through lower income growth.

Growth is now no longer solely demand-determined. Faster productivity growth in the export sector creates an excess supply of exportables and thus lowers their relative price. If the ML condition is satisfied, the resulting decrease in Home import growth and increase in world demand for Home exports more than offsets the negative valuation effect on Home exports (equation (13)), thus creating a trade surplus. Income growth must accelerate to restore balanced trade. If, on the other hand, rapid technological progress in the export sector is accompanied by non-satisfaction of the ML condition, then income growth must decelerate to restore the trade balance. This latter case can be interpreted as an analog of "immiserizing growth" a la' Bhagwati (1958). In this case, BPCG3 cannot be derived from BPCG2, since the terms of trade are not constant.

Finally, consider the case where Home is small not only on the import side but also on the export side. In other words, Home is a small, open economy with δ_X^* approaching infinity. Income growth in this case is independent of world income growth. To understand why, consider equation (11). An increase in world demand for Home exports due to faster world income growth requires that either a change in the terms of trade neutralize the greater demand for Home exports through substitution effects or that export growth accelerate. Given that δ_X^* approaches infinity, an infinitesimally small rise in the terms of trade suffices to neutralize the initial increase in world demand without the volume of exports changing. Since world demand for Home exports is unchanged, equation (13) implies that income growth is also unchanged.[16]

4.2 Assuming no specialization

Next, consider another interesting case where we relax the assumption of specialization in tradable consumption and production. For simplicity, consider the case where $\delta_{ii} = \delta_{ij} = 0$ and $\sigma_{ii} = \sigma_{ij} = 0$. This means that producers (consumers) cannot change output (expenditure) in response to relative price changes. The solutions take the following form:

$$\hat{Y} = \hat{Y}(\overset{-}{\hat{Z}}, \overset{-}{\hat{\alpha}_X}, \overset{+}{\hat{\beta}_X})$$

and,

$$\hat{p}_X = \hat{p}_X(\overset{-}{\hat{Z}}, \overset{+}{\hat{\alpha}_M}, \overset{-}{\hat{\beta}_M}, \overset{-}{\hat{\alpha}_X}, \overset{+}{\hat{\beta}_X})$$

Now the effect of world growth on (external account-constrained) domestic growth is negative! To understand why, again consider eqs. (11) and (13). From equation eqs. (11) and (13) we know that higher global demand raises demand for Home exports. Since producers cannot react to changed relative prices, the supply of exports can only rise if the domestic demand for the exportable good declines relative to the (fixed) supply. This, in turn, is only possible if domestic income declines (see equation (7)). Indeed, its not just domestic income that declines. The terms of trade fall as well. To see why, consider equation (13). A rise in world income increases demand for our exports just as the exportable market clearing condition requires that our exports decline. Given the absence of substitution on both the demand and supply side, the resulting upward pressure on the trade balance can only be removed if the terms of trade decline (creating a negative *valuation* effect on our exports). The absence of substitution effects can lead to perverse effects on output in the short run, as demonstrated by Krugman and Taylor (1978) for the case of nominal devaluations.

[16] This result, which holds regardless of the degree of substitutability between tradables and non-tradables, reflects the case discussed by McGregor and Swales (1985, p. 29). They postulate a neoclassical model of a small open economy albeit with a chain of causation flowing from income growth to exports. The paper, however, does not specify an explicit model making it difficult to pin down the exact structure of the argument.

5 Allowing for substitution between tradables and non-tradables

Consider again the general system constituted by equations (17) and (18), now allowing for an effective presence of non-tradables. In the unrestricted case, the solutions are generally ambiguous in terms of direction (see Section 5.2). A few simplifying assumptions in line with the spirit of the BPCG framework, however, yield interesting results.

5.1 Assuming specialization and imposing restrictions on price elasticities

Again, we begin by assuming that initially Home producers do not produce importables and Home consumers do not consume exportables. Further, we assume that the own-price elasticity of supply of exportables is infinite. It can be shown that,

$$\hat{Y} = \hat{Y}(\overset{+}{\hat{Z}}, \overset{-}{\hat{\alpha}_N}, \overset{+}{\hat{\beta}_N}, \overset{-}{\hat{\alpha}_M},)$$

and,

$$\hat{p}_X = 0$$

The former expression reduces to BPCG2, and further to BPCG3, in the absence of non-tradables (see below). However, BPCG1 cannot be derived. To understand the intuition underlying the positive impact of world growth on domestic growth, consider eqs. (11) and (13), starting with the former. Higher global income growth means faster demand growth for Home exports. Since the own-price elasticity is infinite, an infinitesimally small increase in the relevant relative price motivates Home producers to sharply increase the production of exportables to match the rise in demand (equation (11)). This means, given equation (13), that Home import growth must be higher to maintain the trade balance. Given the negligible change in the terms of trade, this is only possible if income growth accelerates, which is consistent with the canonical BPCG result. Thus, more rapid world growth translates into higher Home output growth.

We know from equation (14), that change in the relative price of non-tradables is a positive function of income growth. Thus, the real exchange rate appreciates as a consequence of world income growth. The inclusion of non-tradables has another interesting consequence. To see this, consider the comparative static solution for the effect of world demand growth:

$$\hat{Y} = \frac{\eta_Z^*}{\eta_M + \frac{\delta_{MN}}{\sigma_{NN}+\delta_{NN}}}\hat{Z}$$

The expression above demonstrates that the the presence of non-tradables dampens the effect of world growth on Home. Put differently, the presence of non-tradables tends to dampen the external account-constrained growth rate. This happens because the substitution between importables and non-tradables that occurs as a consequence of the real appreciation arising from greater world demand switches domestic expenditure towards non-tradables, thus tightening the balance of payments constraint. The implication is that empirical estimates that ignore the non-tradable sector (as all existing estimates of the BPCG rate do) would tend to over-estimate the BPCG rate. This may at least partially explain why, as noted by Thirlwall (1979) and Perraton (2003), countries are often found to grow slower than the BPCG2 and BPCG3 rates.

The other comparative static results follow from eqs. (11) and (13) in a similar manner. Consider faster growth of expenditures on non-tradables (i.e., a rise in $\hat{\alpha}_N$). At a given level of world income, and due to the negligible change in the terms of trade, foreign demand for Home exports does not change. However, the excess demand for non-tradables raises their relative price, which shifts domestic demand towards importables. Equation (13) implies that the resulting trade deficit must be removed via a decline in domestic income growth. The effect of technological progress in the non-tradable sector has mirror image consequences. Notice, on an interesting note, that such technological progress causes a real depreciation via equation (14), and thus boosts the production of exportables. However, since the terms of trade are unchanged, foreign demand for Home exports is not affected: equation (11) implies that the growth of export supply must therefore be unchanged too. Thus, the increase in domestic consumption of exportables (due to increased income) must be *exactly* offset by the increase in exportable production.

Increased preference for importables (i.e., a rise in $\hat{\alpha}_M$) creates a trade deficit. Domestic income growth must decline to restore the trade balance.

Following our strategy in Section 4, consider the opposite extreme where all supply elasticities are negligible. In this case,

$$\hat{Y} = \hat{Y}(\overset{+/-}{\hat{Z}}, \overset{-}{\hat{\alpha}_N}, \overset{+}{\hat{\beta}_N}, \overset{-}{\hat{\alpha}_M}, \overset{+}{\hat{\beta}_X})$$

where the sign on $\hat{\beta}_X$ assumes that the ML condition is satisfied.

$$\hat{p}_X = \hat{p}_X(\overset{+}{\hat{Z}}, \overset{-}{\hat{\beta}_X})$$

The detailed expression for the effect of world income growth on domestic growth now becomes

$$\hat{Y} = -\frac{\left(\delta_{MX} + \frac{\delta_{NX}\delta_{MX}}{\delta_{NN}} - 1\right)\eta_Z^*}{\left(\eta_M + \frac{\delta_{MN}}{\delta_{NN}}\right)\delta_X^*}\hat{Z}$$

which reduces to BPCG2 if there is no substitution between tradables and non-tradables on the one hand, and exportables and importables on the other. Since the terms of trade are not constant, BPCG2 does not reduce to BPCG3.

To intuitively understand the effect of world income growth on domestic growth, consider that the former raises world demand for Home exportables, creating an excess demand for them. Since a supply side response is ruled out by assumption, equation (11) tells us that the terms of trade must rise to neutralize the initial rise in world demand. The rise in the terms of trade means substitution in consumption towards non-tradables and importables. Given equation (13), this means that, if the substitution effects dominate,[17] then a trade deficit is created and income growth must decline to remove the this deficit through fewer imports. If, by contrast, the positive valuation effect of the terms of trade on exports dominates, then a trade surplus is created. Income growth must then accelerate to counter the surplus. Notice again that the presence of non-tradables has a dampening effect on the BPCG rate.

5.2 The Most General Case

We are now ready to consider – in the most general case – eqs. (17) and (18) without any infinity or null assumptions. Not surprisingly, unambiguous solutions cannot be found analytically in this case. A resort to numerical simulations, presented in Table 3, yields interesting results. A value of unity is assigned to all the income and price elasticities, except for the foreign income elasticity of demand for Home products which is assumed to be 1.5 for illustrative purposes. We plausibly assume that, for any two sectors, $\delta_{ij} = \delta_{ji}$ and $\sigma_{ij} = \sigma_{ji}$.[18]

Begin by assuming, in the most general case, that one-fifths of the exportables produced are consumed and one-fifths of the importables consumed are produced at Home. The results are presented in the second column. Home income growth in this case turns out to be a negative function of world income growth. As we see below, the sign is sensitive to parameter values.

Next, we extend our numerical simulations to highlight some of the lessons learned from earlier sections. Not surprisingly, "autarky," that is, the assumption that all exportables are consumed at Home and all importables are produced at Home renders domestic income independent of foreign income growth. Complete specialization leaves the externally-constrained growth rate unchanged (at $-0.6\hat{Z}$). As we show later, when we discuss Figure 3, this is due to the offsetting effects of changes in θ_X and θ_M. The BPCG case, in which non-tradables are rendered irrelevant through the assumption that $\sigma_{NX} = \delta_{NX} = 0$, and complete specialization is assumed along with a very high own-price supply elasticity of exports, yields BPCG2, as we already know from Section 4.1.[19] Here the growth rate equals $1.5\hat{Z}$ $(= \eta^*_Z \hat{Z}/\eta_M)$. Maintaining the assumptions of specialization and a very high own-price supply elasticity while introducing non-tradables lowers the constrained growth rate, as again we already know from Section 5.1. In

[17] That is, $\delta_{MX} + \frac{\delta_{NX}\delta_{MX}}{\delta_{NN}} - 1 > 0$.

[18] For example, $\sigma_{NX} = \sigma_{XN}$.

[19] We do not need to make any special assumptions about $\sigma_{NM} = \delta_{NM}$ in order to render non-tradables moot here because we are assuming that Home does not initially produce any importables ($\theta_M = 1$).

this case the growth rate declines from $1.5\hat{Z}$ to $\hat{Z}$. Assigning high values to non-tradable own- or cross-price elasticities relative to exportables while returning to partial specialization turns the constrained growth rate from $-0.6\hat{Z}$ (in the most general case) to $0.429\hat{Z}$ and $1.2\hat{Z}$, respectively, although it is still lower than the growth rate derived in the absence of non-tradables. The intuition is relatively straightforward. Equation (11) tells us that an increase in world demand tends to create excess demand for exportables. The terms of trade must rise in order to dampen foreign demand and boost domestic supply of exportables. However, the rise in the terms of trade would tend to create a trade deficit (equation (13)).[20] Demand must be diverted away from importables. This is where the supply elasticities for non-tradables play a role. With very high elasticities, a negligibly small decline in the real exchange rate is required to achieve the required shift of resources towards the exportable sector. Substitution from importables to non-tradables is also small, as a result. Thus, the balance of payments-constrained growth rate can be higher.

To highlight the role played by non-tradables, Figure 1 illustrates the effect of substitution between importables and non-tradables. It is assumed in this and the following figures that Home consumes 20 percent of the exportables produced and produces 20 percent of the importables consumed. The continuous line shows the effect of world growth on domestic growth as the elasticity of substitution on the *supply* side (σ_{NN}) increases from 0 to 10^{100} (along the horizontal axis). The dash line marked presents the same effect as the elasticity of substitution on the side *demand (*δ_{NN}) increases from 0 to 10^{100}. In both cases, the external account-constrained growth rate declines as the two goods become more substitutable.

Relaxing the assumption of non-substitutability between exportables and importables on the supply and demand sides too tends to lower the external account-constrained growth rate. Figure 2 illustrates this effect. The values of δ_{XM} and σ_{XM} are varied from 0 to 10^{100}. The continuous line illustrates the impact of world growth on domestic growth for changes in σ_{XM} while the dash line illustrates the same impact for changes in δ_{XM}.

Finally, relaxing the assumption of complete specialization in tradable production and consumption has interesting effects. Figure 3 illustrates changes in the effect of world growth on domestic growth for a range of values of θ_X and θ_M. We fix θ_X at 0.8 while varying θ_M from 1 (complete specialization) to 0.1 (almost no specialization), and vice versa. As shown earlier in Section 3, the effect of declining specialization on the import side is to dampen the constrained growth rate (represented by the continuous line). The effect of declining specialization on the export side, on the other hand, is to amplify this growth rate. The presence of some domestic production of importables in the real world, therefore, provides another reason why the trade balance may impose a tighter constraint on actual economies than that suggested by the BPCG hypothesis in its traditional versions, although domestic consumption of exportables counteracts this effect.

[20] Notice that the Marshall-Lerner condition is satisfied by construction in these simulations.

The right most column gives an idea of the *cumulative* magnitude of effects involved when we incorporate non-tradables, incomplete specialization in consumption, and intra-tradable substitution into the BPCG2 version of the BPCG model. The growth rates declines by 40 percent to $0.87\hat{Z}$. For the sake of comparison, consider Figure 4, which simulates the actual growth rates and those predicted by the BPCG hypothesis in its weak form (our BPCG1). The predicted BPCG rates are based on the estimated equation reported in Perraton (2003, p. 9).[21] The degree of overprediction ranges between 31 percent and 40 percent as we go from actual growth rates of 0.5 percent to 10 percent.

Taken together, Table 3 and Figures (1)-(3) underscore at least three reasons to expect actual growth rates to be lower – for an economy that is constrained by the trade balance – than those predicted by the BPCG hypothesis. These effects are likely to be significant in real economies.

6 Concluding Remarks

External imbalances (or imbalances in the tradable goods sector) are an important consideration, especially for developing countries with relatively shallow financial markets. The non-tradable sector typically constitutes a major part of the economy, both on the production and expenditure sides. Moreover, countries typically consume a portion of their exportable sector output and satisfy a portion of their importable demand through domestic production. Introducing non-tradables and endogenous terms of trade in a multi-sectoral framework, therefore, allows for a more complete analysis of growth in an open economy.

Our analysis specifies a three-sector model with an exportable good, an importable good, and a non-tradable good. In addition to the levels of elasticities, our model also incorporates sectoral changes in the rate of technological progress and evolution of preferences, reflecting the fact that elasticities change over time. We are able to analyze movements in both the terms of trade and the real exchange rate.

We first demonstrated that the BPCG model in its most complete version (BPCG1) (that includes relative price changes) cannot be derived from within our framework. In order to derive this version of the hypothesis, we have to ignore the exportable sector clearing condition and assume that the terms of trade are exogenously given. These assumptions are problematic in a demand-led growth framework. We then made explicit the conditions under which the versions of the BPCG hypothesis that ignore relative price changes can be derived. A necessary condition essentially boils down to eliminating the role of non-tradables. Furthermore, even in the absence of non-tradables, the BPCG hypothesis, interpreted broadly as the constraint imposed by the relative growth of external demand, cannot always be derived since world income growth may have ambiguous effects on domestic growth depending on the initial structure and evolution of supply and demand in the economy. The BPCG hypothesis

[21] The equation can be written as follows:
Predicted Growth Rate = $-0.11+(1.67\times$ Actual Growth Rate$)$

is, therefore, as much about the balance of payments constraint as it is about the internal structure of the economy.

Some of our main results can be summarized as follows:

- The presence of non-tradables tends to lower the external account-constrained growth rate due to inter-sectoral substitution effects. This is consistent with the result, due originally to Thirlwall (1979), that the actual growth rate tends to be lower than the estimated one. Thus, unlike Thirlwall (1979), one need not resort to trade imbalances to explain this finding.

- The presence of incomplete specialization in the consumption of tradables is another potential dampening force on the external account-constrained growth rate, as is the presence of substitution in production and expenditure between exportables and importables.

- Positive trends in technological progress may hurt or help an economy that faces balance of payments constraints. The impact depends partly on the sector that the progress takes place in and partly on the structure of the economy. In other words, it is not just the direction of shocks to supply (or demand), but also the sectoral composition of these shocks that matters.

- Increased demand for importables or non-tradables generally has a negative impact on the domestic growth rate.

- Under certain conditions it is possible for income to *decline* following a rise in external demand for a country's products. This happens, for example, if the substitution elasticities are very low both on the supply and demand sides.

The relevance of these conclusions depends, among other things, on the time frame under consideration. For example, while valuation effects are a one shot phenomenon, substitution elasticities tend to rise over time (the famous J-curve effect). Moreover, some of these conclusions could conceivably be reversed in a dynamic framework where the external constraint binds only in the long run, and technological progress or demand side shocks introduce structural changes during the transition. Development economists, particularly those working in the structuralist tradition, have often argued that the response to relative price signals is muted by structural inertia in developing countries. If we are correct in concluding that the presence of non-tradables and inter-sectoral substitution contribute to actual growth rates being lower than the empirically estimated ones, one would therefore expect this growth rate gap to be systematically smaller for developing countries. Finally, whether a country experiences technological progress in the exportable, importable, or non-tradable sectors should affect the long-run growth rate that is sustainable. As more country-level data on the sectoral composition of production, consumption, and trade become available from input-output tables, these propositions become empirically testable. We leave these questions to future research.

Table 2: Comparative Statics ("n.a" denotes "not applicable.")

Case		$\widehat{Z}$	$\widehat{\alpha}_N$	$\widehat{\alpha}_X$	$\widehat{\alpha}_M$	$\widehat{\beta}_N$	$\widehat{\beta}_X$	$\widehat{\beta}_M$
Canonical	$\widehat{Y}$	$+$	0	0	$-$	0	0	0
	$\widehat{p}_X$	n.a	n.a	n.a	n.a	n.a	n.a	n.a
	$\widehat{p}_N$	n.a	n.a	n.a	n.a	n.a	n.a	n.a
Section 4.1	$\widehat{Y}$	$+/-$	0	0	0	0	$+$	$-$
	$\widehat{p}_X$	$+$	0	0	0	0	$-$	0
	$\widehat{p}_N$	n.a	n.a	n.a	n.a	n.a	n.a	n.a
Section 4.1, $\sigma_{XX} \to \infty$	$\widehat{Y}$	$+$	0	0	$-$	0	0	0
	$\widehat{p}_X$	0	0	0	0	0	0	0
	$\widehat{p}_N$	n.a	n.a	n.a	n.a	n.a	n.a	n.a
Section 4.1, $\sigma_{XX} = 0$	$\widehat{Y}$	$+/-$	0	0	$-$	0	$+/-$	0
	$\widehat{p}_X$	$+$	0	0	0	0	$-$	0
	$\widehat{p}_N$	n.a	n.a	n.a	n.a	n.a	n.a	n.a
Section 4.2	$\widehat{Y}$	$-$	0	$-$	0	0	$+$	0
	$\widehat{p}_X$	$-$	0	$-$	$+$	0	$+$	$-$
	$\widehat{p}_N$	n.a	n.a	n.a	n.a	n.a	n.a	n.a
Section 5.1, $\sigma_{XX} \to \infty$	$\widehat{Y}$	$+$	$-$	0	$-$	$+$	0	0
	$\widehat{p}_X$	0	0	0	0	0	0	0
	$\widehat{p}_N$	$+$	$+$	0	0	$-$	0	0
Section 5.1, $\sigma_{ii} = \sigma_{ij} = 0$	$\widehat{Y}$	$+/-$	$-$	0	$-$	$+$	$+$	0
	$\widehat{p}_X$	$+$	0	0	0	$-$	0	0
	$\widehat{p}_N$	$+/-$	$+/-$	0	0	$-$	0	0

Table 3: Simulations based on the most general case

	Most general case	Autarky	Complete specialization	BPCG Case without non-tradables	BPCG case with non-tradables	High σ_{NN}	High σ_{NX} (or high δ_{NX})	BPCG case with more relaxed assumptions
θ_X	0.8	0	1	1	1	0.8	0.8	0.9
θ_M	0.8	0	1	1	1	0.8	0.8	0.9
σ_{NX}	1	1	1	0	1	1	10^{100}	1
σ_{NM}	1	1	1	1	1	1	1	1
σ_{NN}	1	1	1	1	1	10^{100}	1	1
σ_{XM}	1	1	1	1	1	1	1	1
σ_{XX}	1	1	1	10^{100}	10^{100}	1	1	10^{100}
σ_{MM}	1	1	1	1	1	1	1	1
δ_{NX}	1	1	1	0	1	1	1	1
δ_{NM}	1	1	1	1	1	1	1	1
δ_{NN}	1	1	1	1	1	1	1	1
δ_{XM}	1	1	1	1	1	1	1	1
δ_{XX}	1	1	1	1	1	1	1	1
δ_{MM}	1	1	1	1	1	1	1	1
δ^*_X	1	1	1	1	1	1	1	1
η_N	1	1	1	0	1	1	1	1
η_M	1	1	1	1	1	1	1	1
η_X	1	1	1	1	1	1	1	1
η^*_Z	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
$\hat{Y}/\hat{Z}$	−0.600	0.000	−0.600	1.500	1.000	0.429	1.200	0.871
$\hat{p}_X/\hat{Z}$	0.900	0.000	1.200	0.000	0.000	0.643	0.000	0.000
$\hat{p}_N/\hat{Z}$	0.600	0.000	0.900	0.000	0.500	0.000	0.000	0.435

7 Appendix: Mathematical Solutions

Only four of the equations (9)-(13) are independent. To see this, substitute eqs. (9) and (10) into equation (12) to get:

$$p_X(Y_X - E_X) = M$$

Substituting further from eqs. (7), and (11) yields equation (13).

Section 3: In this case, the detailed solution is given by:

$$\hat{Y} = \frac{-(\delta_{MX} + \delta_X^* - 1)\hat{p}_X - \hat{\alpha}_M + (1-\theta_M)\hat{\beta}_M + \theta_M \eta_Z^* \hat{Z}}{\eta_M}$$

Section 4: The general solutions take the form:

$$\hat{Y} = \frac{\begin{vmatrix} -(1-\theta_X)\hat{\alpha}_X + \hat{\beta}_X - \theta_X \eta_Z^* \hat{Z} & -[\sigma_{XX} + (1-\theta_X)\delta_{XX} + \theta_X \delta_X^*] \\ -\hat{\alpha}_M + (1-\theta_M)\hat{\beta}_M + \theta_M \eta_Z^* \hat{Z} & (1-\theta_M)\sigma_{MX} + \delta_{MX} - \theta_M(1-\delta_X^*) \end{vmatrix}}{\Delta_1}$$

$$\hat{p}_X = \frac{\begin{vmatrix} (1-\theta_X)\eta_X & -(1-\theta_X)\hat{\alpha}_X + \hat{\beta}_X - \theta_X \eta_Z^* \hat{Z} \\ \eta_M & -\hat{\alpha}_M + (1-\theta_M)\hat{\beta}_M + \theta_M \eta_Z^* \hat{Z} \end{vmatrix}}{\Delta_1}$$

where $\Delta_1 = \begin{vmatrix} (1-\theta_X)\eta_X & -[\sigma_{XX} + (1-\theta_X)\delta_{XX} + \theta_X \delta_X^*] \\ \eta_M & (1-\theta_M)\sigma_{MX} + \delta_{MX} - \theta_M(1-\delta_X^*) \end{vmatrix}$

The other solutions in Section 4 follow from applying the restrictions listed in the text.

Section 5: The general solutions take the form:

$$\hat{Y} = \frac{\begin{vmatrix} B_{11} & A_{12} \\ B_{21} & A_{22} \end{vmatrix}}{\Delta_2}$$

$$\hat{p}_X = \frac{\begin{vmatrix} A_{11} & B_{11} \\ A_{21} & B_{21} \end{vmatrix}}{\Delta_2}$$

where

$A_{11} = (1-\theta_X)\eta_X + \frac{\sigma_{XN} + (1-\theta_X)\delta_{XN}}{\sigma_{NN} + \delta_{NN}} \eta_N$

$A_{12} = -\left\{\sigma_{XX} + (1-\theta_X)\delta_{XX} + \theta_X \delta_X^* - \frac{\sigma_{NX} + \delta_{NX}}{\sigma_{NN} + \delta_{NN}} [\sigma_{XN} + (1-\theta_X)\delta_{XN}]\right\}$

$A_{21} = \eta_M + \frac{(1-\theta_M)\sigma_{MN} + \delta_{MN}}{\sigma_{NN} + \delta_{NN}} \eta_N$

$A_{22} = (1-\theta_M)\sigma_{MX} + \delta_{MX} - \theta_M(1-\delta_X^*) + \frac{\sigma_{NX} + \delta_{NX}}{\sigma_{NN} + \delta_{NN}} [(1-\theta_M)\sigma_{MN} + \delta_{MN}]$

$B_{11} = -\frac{\sigma_{XN} + (1-\theta_X)\delta_{XN}}{\sigma_{NN} + \delta_{NN}} (\hat{\alpha}_N - \hat{\beta}_N) - (1-\theta_X)\hat{\alpha}_X + \hat{\beta}_X - \theta_X \eta_Z^* \hat{Z}$

$B_{21} = -\frac{(1-\theta_M)\sigma_{MN}+\delta_{MN}}{\sigma_{NN}+\delta_{NN}}(\hat{\alpha}_N - \hat{\beta}_N) - \hat{\alpha}_M + (1-\theta_M)\hat{\beta}_M + \theta_M \eta^*_Z \hat{Z}$

and $\Delta_2 = \begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix}$

The other solutions in Section 5 follow from applying the restrictions listed in the text.

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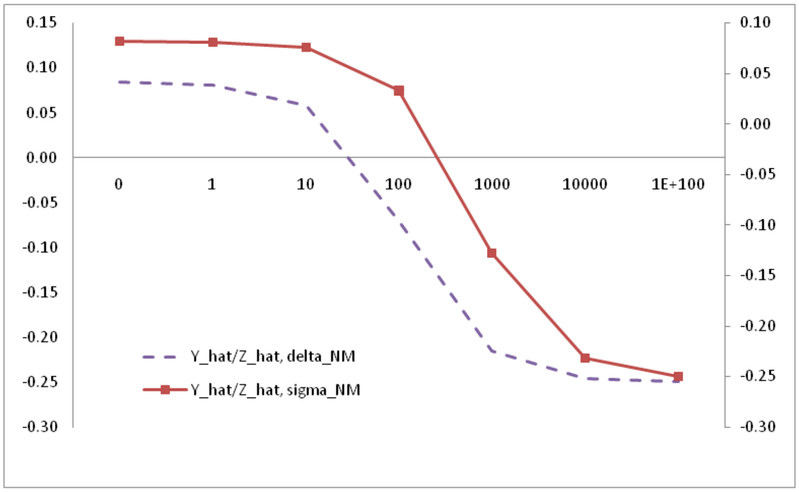

Figure 1: Numerical simulations of the effect of changes in δ_{NM} (left hand scale) and σ_{NM} (right hand scale) on $\hat{Y}/\hat{Z}$.

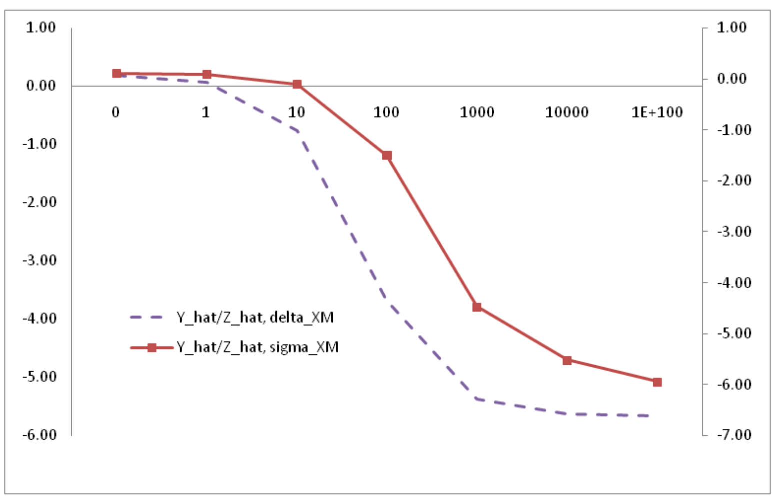

Figure 2: Numerical simulations of the effect of changes in δ_{XM} (left hand scale) and σ_{XM} (right hand scale) on $\hat{Y}/\hat{Z}$.

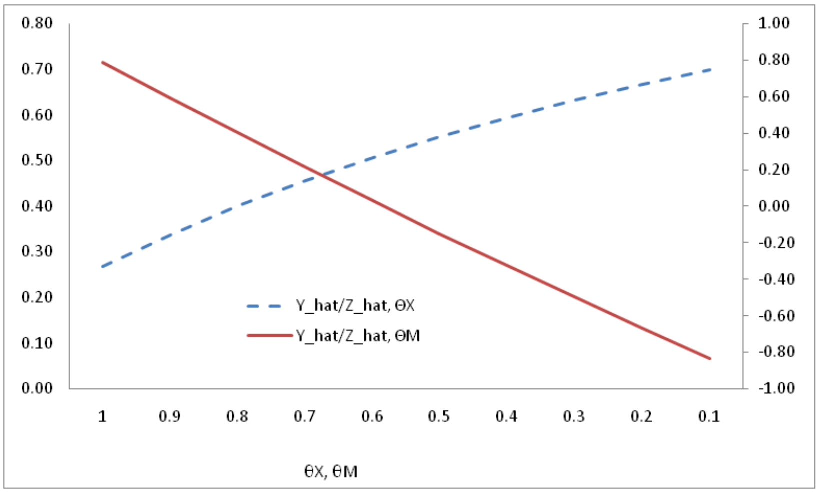

Figure 3: Numerical simulations of the effect of changes in θ_X (left hand scale) and θ_M (right hand scale) on $\hat{Y}/\hat{Z}$.

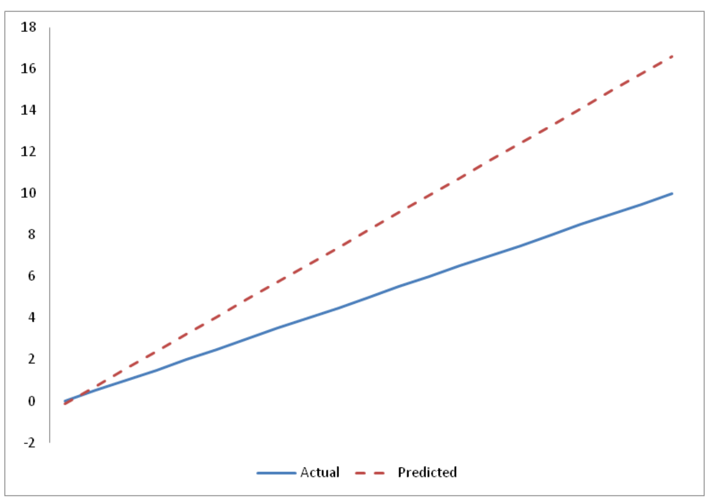

Figure 4: Actual and predicted growth rates based on Perraton (2003)