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Wrinkling in Buckling and a Thin Sheet

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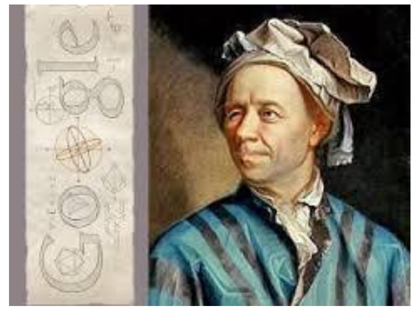
Fundamentals of pattern formation

MA Science frameworks:

5.3 Explain how the forces of tension, compression, torsion, bending, and shear affect the performance of bridges.

Module 1: Euler Buckling

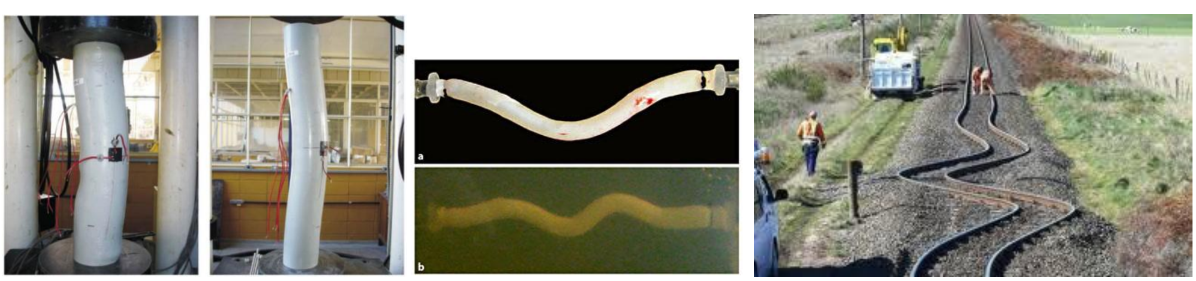
Learning Objectives:



- What is an instability? A sudden change in behaviour in response to a small change in conditions.
- Instabilities usually involve a change in symmetry from a more symmetric situation to a less symmetric one
- The mechanism for an instability usually involves two competing forces (one force stabilizing the symmetric state, and the other one destabilizing it), with one suddenly winning the contest
- These competing forces in thin objects are often the forces of compression (destabilizing force - favors buckling or wrinkling) and of bending (stabilizing force).
- Understanding by data collapse, the power of using dimensionless (unit-less) numbers, rather than dimensional parameters (measured in units of length, time, force, energy, etc).

Introduction:

Buckling is an instability in which an object first compresses when squeezed by a force from both ends, but then, after a certain threshold value of force, suddenly bends into a curved shape. This concept was first fleshed out by the Swiss mathematician Leonhard Euler around 300 years ago. This phenomenon is of obvious importance when building structures held up by pillars and columns. We clearly want the pillars designed so that the threshold force for buckling them is bigger than the weight they need to support. Here are some examples:



Buckling of (a) columns http://www.civildb.com/, (b) arteries http://engineering.utsa.edu/~hhan (c) railways lines wiki.iricen.gov.in,

Here we will study the buckling of plastic and metal rulers rather than architectural elements. However, the questions we will try to answer are the same: What force can a ruler support before it buckles? How does this force depend on the length (L) of the ruler? On its width (W)? On its thickness (t)?

Variables:

The plastic and metal ruler will be loaded with the weight of the coins. The goal will be to measure the minimum force ($F_{threshold}$) required to make the ruler buckle as a function of the length (L), width (W), thickness (t) and material of which it is made. To complete this module in a timely way, different groups work with different values of L, W, t. We'll change L by using 6" and 12" rulers, and vary thickness by using two rulers stuck together with double-stick tape.

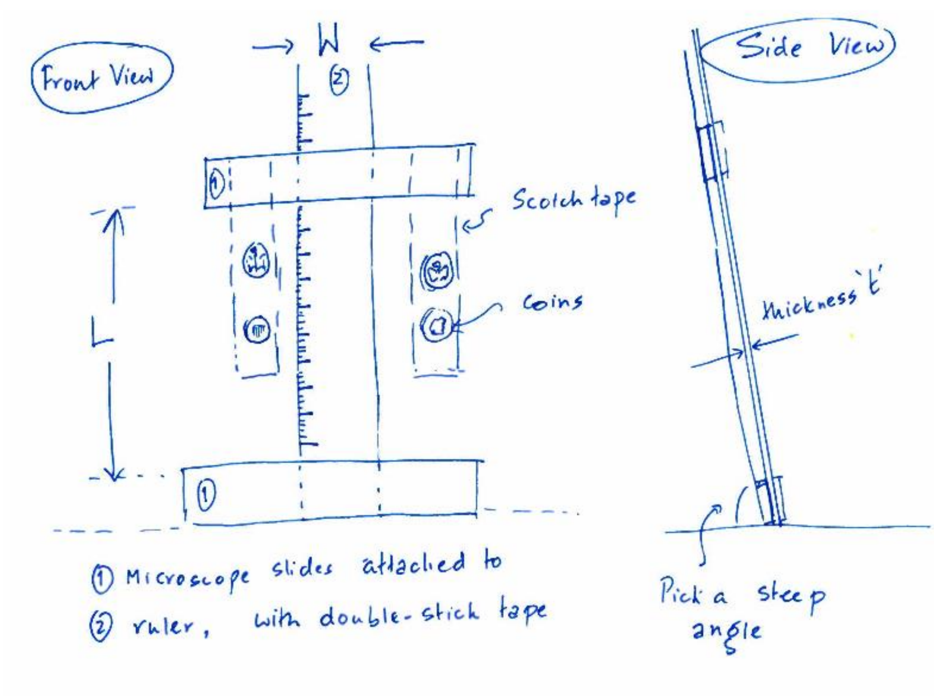
Objective:

Will changing the length (L), width (W), and thickness (t) of a material under a force affect how easily it buckles?

Materials:

- Plastic rulers (use only flat rulers, and **not** ones with beveled edges, tapers or handles)
- Microscope slides (4 per group)
- Scotch tape
- Double-stick tape
- Coins, and a balance to measure their weights

Setup and procedure:



What choice of width, length and thickness did you make:

W: ________________ L: __________ t: __________

1. Weigh the microscope slides. Call this M_{slides}. This information will be used later.

Weight of microscope slide: ______________

2. Sandwich both ends of the ruler between a pair of microscope slides using double stick tape. The ruler-and-slide assembly should look like the capital letter "I."
3. Lean the ruler assembly against a smooth surface – we did this with a wooden plank.
4. The surface should be near vertical, so that we know that any weight applied to the ruler will pull down nearly vertically.
5. Stick a piece of graph paper behind the rulers on this surface with tape. The graph paper will be used to measure the amount of buckling from the ruler.
6. To the top microscope slide, attach two long pieces of scotch tape on either side of the ruler. The sticky side should face outwards towards you.
7. Now add coins one at a time to the sticky surface of the scotch tape, and measure on the graph paper how much the top end deflects (let's call this Y) after each coin.
8. Keep track of the mass of the coins at every stage (let's call this M_{coins}).
9. Perform this demonstration 3 times in order to rule out any errors.
10. Record your results in the table below:

Results (your group):

Trial 1		**Trial 2**		**Trial 3**	
Mass of coins	Deflection, Y	Mass of coins	Deflection, Y	Mass of coins	Deflection, Y

Create a scatter plot for each trial of Y against Mass of coins on the x-axis. Determine the mass of the coins needed to push it over the threshold of the buckling instability.

Threshold mass of coins		
Trial 1	Trial 2	Trial 3

Average value of threshold mass of coins from three trials=

$F_{threshold}$ = (Mass of Coins + Average value of threshold Mass of microscope) x g=

Here, the acceleration due to gravity, g= 1000 cm/s^2

Now add your data to those of the entire class so that we can look for trends. Do this online at a link we will provide. The table will look like:

Length (L)	**Thickness (t)**	**Width (W)**	**$F_{threshold}$**	**Material**

A quick look at trends: Do you need more or less force for buckling a

Longer ruler:

Wider ruler:

Thinner ruler:

Metal vs. plastic ruler:

Dimensionless variables and scaling laws:

An aside: When we make measurements of physical quantities, they usually have units, such as centimeters or kilograms. The values of these numbers typically depend on the choices you have made in setting up your study. Presenting the results if your study with appropriate dimensionless (i.e. unit-less) values makes it easier to extract their underlying meaning, and to check whether the same principles are at work in two different experiments. For example, in a study on how much children have grown in a school year, you may want to measure the change in height of children in inches. However, this choice makes it difficult to compare first-graders and sixth-graders. Worse still, you may be trying to compare with a different school that chooses to measure heights in finger widths. A more useful choice would be measure the fractional change in height i.e. the change in height divided by the original height. Here you will have no problem in making comparisons across ages, or systems of measurement.

Back to buckling: Our buckling study is an example of this concept. We anticipate that the buckling instability reflects the competition of two forces: the force compressing down (which tends to buckle the sheet), and the resistance of the sheet to being bent (which favors a flat, unbuckled state). The threshold force at which buckling occurs should be some combination of the width (W), length (L) and thickness (t) of the sheet, as well its material stiffness (this material parameter is called the Young's modulus of the solid, often denoted by the letter E, and its units are [force]/[length^2] . The answer that Euler came up with a few hundred years ago was

$$\mathbf{F}_{\mathbf{threshold}} = c\ E\ W\ t^3/L^2$$

where c is a numerical constant.

How can you use your data to test this law? Instead of plotting dimensional quantities i.e. $\mathbf{F}_{\mathbf{threshold}}$ on the vertical axis and W,t, or L on the horizontal axis, let's think of appropriate dimensionless quantities. One natural choice is to plot the "dimensionless force": $\mathbf{F}_{\mathbf{threshold}}$ /(EWt) versus the "dimensionless thickness": t/ L. Plot these quantities for all the pooled data:

Dimensionless force	Dimensionless thickness
$\mathbf{F}_{\mathbf{threshold}}$ /(EWt)	t/ L

Conclusion:

1. Looking back at your data, does buckling occur slowly or does it reach a threshold and collapse? In other words, does instability happen gradually or does happen all at once?

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2. If a wall in a building collapsed and engineers wanted to rebuild it, what changes would you recommend? Higher Ceilings? Longer Wall? Thicker Wall? WHY?

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3. If you were going to use posts to support a bridge. Which shape post would you use, a 3" x 12" board or a 6" x 6" board? NOTE: They both have the same cross sectional area. Explain your answer.

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4. If you were designing concrete supports for a bridge, what shape would you use in order to use the least amount of concrete or save money? Google-search images to assess whether your shape is widely used in bridge pillar construction.

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5. Based on what you know from our findings at the end with dimensionless numbers, can you predict what force it would take to buckle a piece of steel with length = 200 cm, width 50 cm, and thickness = 2cm.

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Module 2: Wrinkling

Learning Objectives:

- What is the wavelength of a pattern? What decides the wavelength: reinforcing the idea of competition between many forces.
- Expanding and reinforcing concepts: What symmetry is broken in the wrinkling instability? Data collapse and dimensionless numbers

You know what wrinkling is – it's what you see when you look in mirror, or pinch the flesh on your arm. What we're trying to do in this experiment, is to have you realize that this is an example of buckling but one that involves a <u>repeating pattern caused by buckling</u>.



What leads to this difference between wrinkling (multiple buckles) and Euler buckling (single buckle), is that there is a new factor here – apart from the slender object (skin, foil, film) feeling a compressive force there is also a substrate (flesh, water) or a force of tension pulling at the material and trying to reduce its distortion from a flat state.

In this experiment, we study the effect of the shape and dimensions of the sheet and the tension applied to the sheet on the wavelength of the pattern. The wavelength, λ is the separation between adjacent peaks or valleys in the wrinkle pattern.

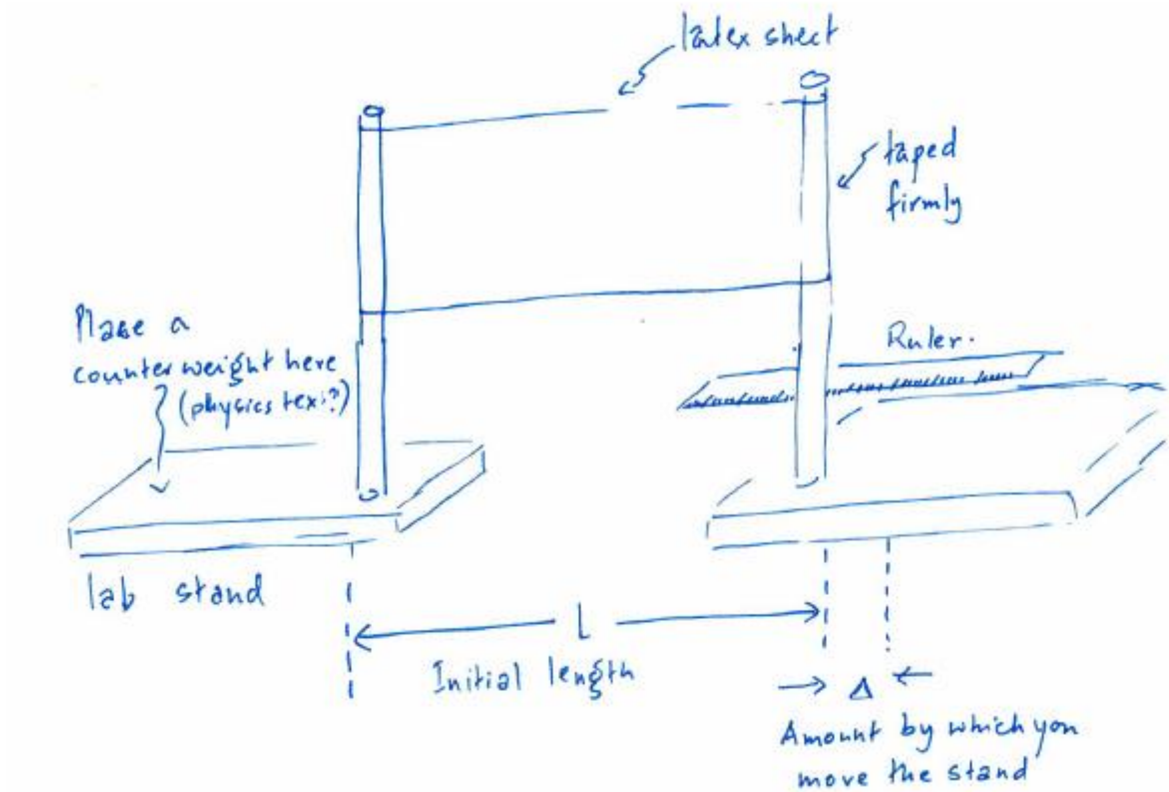
<u>Materials:</u>

1. Latex sheet (same material as laboratory or hospital gloves)
2. Scotch tape
3. Two lab chemistry stands
4. Ruler
5. Optional: Camera – a phone camera will suffice

<u>Relevant variables:</u>

The latex sheet has length, L, width, W, and thickness t. (The material also has some "stretchiness" or elasticity which is quantified by the Young's modulus, and this will also affect the pattern). We will stretch it out by a length Δ.

Experimental Setup:



1. Cut a rectangular piece of latex out. So that we can explore many parameters in the class, different groups pick different dimensions of length and width.
2. Set the two lab stands on the table, spaced by about the length of the rectangles.
3. Stick with scotch tape two opposite edges of the rectangle to the upright posts of the two lab stands. Be careful in doing so to make sure that the two sides are parallel, and that the edges are smooth.
4. Place a weight (a heavy book is fine) on the base of the lab stands so that they don't wobble when you move them. (For our lab, we have table clamps).

Measurements:

Measure carefully the length L and width W of the rectangle of latex. Now, adjust the two lab stands so that the two posts are exactly the right space apart to hold up the latex sheet flat. This is the position of zero extension Δ=0.

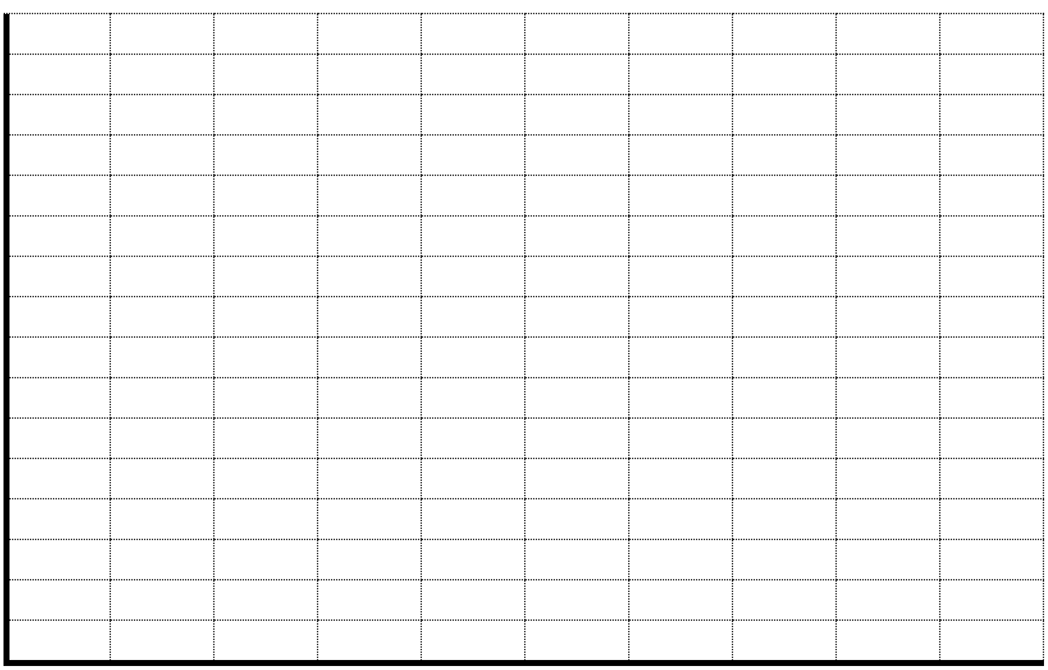
Now gently move one of the two lab stands away from the other by a distance that we will call the extension, Δ. The rectangle is now stretched. Note the threshold value of extension when the smoothly stretched piece of latex starts showing wrinkles running between the two posts. Continue extending beyond this value of extension. Count the number of wrinkles (N) at every value of the extension Δ and fill it in the table below.

- Data and preliminary analysis

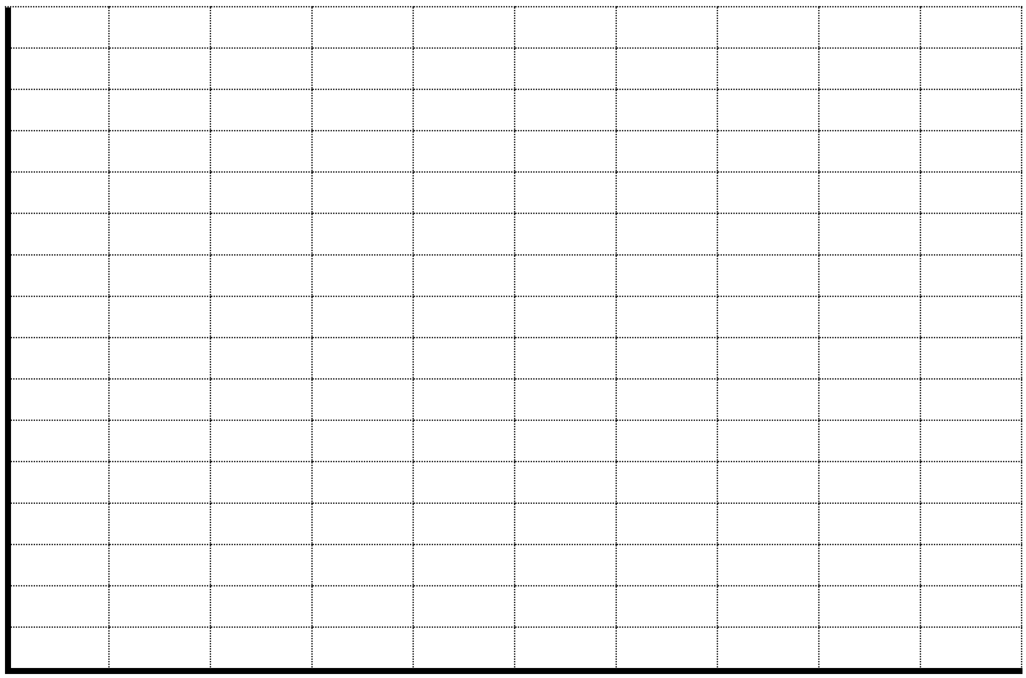
Fill in the table below of the number of wrinkles versus the extension.

	Extension Δ (cm)	**Number of wrinkles, N**	**Fractional extension, Δ/L**	**Wavelength, λ = W/N**	**Comments**
1					
2					
3					

Make a graph of wrinkle number, N versus extension Δ



Make a graph of wavelength λ versus fractional extension Δ/L



Data reduction and scaling

Now let's think about how to pool data from various lengths and widths from different groups.

A scaling law for the wavelength (λ) of wrinkles: We used dimensionless variables and data collapse to test the physical law of Euler buckling that tells us how $\mathbf{F_{threshold}}$ varies with thickness, width, and length of the sheet. We need to develop similar ideas to study change in wavelength λ of the pattern with the tension in the sheet (T), its thickness (t), and its length (L).

The new ingredient in wrinkling which was not there in Euler buckling is the tension (T) stretching the sheet. The tension acts to reduce the height of the wrinkles, just as a guitar string or a drum head will want to spring back when you pull it up. This is also the role played by the flesh beneath the skin in facial wrinkles.

Again, we will state just state the law for the wrinkle wavelength and try to confirm it by using dimensionless parameters and data collapse:

$\lambda = c * L^{3/4} * t^{1/2} / \Delta^{1/4}$

where c is again a constant number. In order to test this law, it is again much more useful to plot the data in a dimensionless form.

Propose a “dimensionless wavelength” and “dimensionless thickness” of the stretched sheet, plot them in the vertical and horizontal axes, and see whether the results of experiments from sheets with various control parameters (L, t, and Δ) collapse on a single curve.

Conclusion:

Do you get more or less wrinkles on thinner sheets for the same length and extension?

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Does the wavelength of wrinkles increase or decrease when you increase the tension? Does this agree with your experience regarding wrinkles on taut skin versus slack skin?

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Here’s a harder follow-up to the previous question: When a surgeon makes an incision on a patient, she tries to pick a direction to cut where the tension of the skin will be pulling less hard on the sutures. Can you figure out the scheme they use to determine this direction? Hint: Think wrinkle wavelength.