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Evaluation of New Methods for Determining the Minimum and Maximum Index Densities of Sand

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EVALUATION OF NEW METHODS FOR DETERMINING THE MINIMUM AND MAXIMUM INDEX DENSITIES OF SAND

A Master Project Presented

By

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ABSTRACT

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This thesis presents the results of a laboratory testing program that evaluated the applicability of a new-established method for determining minimum and maximum dry density values by Norwegian Geotechnical Institute (NGI). Seven fine to medium grained cohesionless sand batches with different inclusion of fines content were selected for a series of testing programs in compliance with American Society for Testing and Materials (ASTM) and NGI recommendations. All three methods of ASTM D4254 – 16 and method 1B of ASTM D4253 – 16 were applied for minimum and maximum dry density determination, respectively. The results compared with the NGI minimum and maximum dry density outcomes.

The minimum density values obtained from the testing program revealed that the NGI procedure produces a lower minimum dry density value compared to the ASTM methods. It is also worth mentioning that the NGI mold dimensions likely play a role in generating lower minimum density values. Due to the lower ratio of diameter to height for the NGI mold than

the ASTM mold, already deposited sand particles tend to form an arch. This arching reduces extra movement of the particle which resulted in lower minimum density values.

The maximum dry density results showed that the NGI method generates equal values to the ASTM method for soils with less than 2 %, by dry mass, fines content. For sands with composition of fine content more than 9 %, by dry mass, the ASTM method results were a little higher than the NGI ones. Additionally, grain size distribution analysis were performed in compliance with ASTM D60913 – 04 in order to investigate any evidence of particle crushing during each maximum density approach. The results indicated no evidence of particle degradation during each maximum dry density method test.

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CHAPTER 1. INTRODUCTION

1.1 Introduction

Municipal areas have been growing significantly in the last decade which has required a design engineer to meet a set of building code requirements along with project budget. Acquiring a good understanding of subsurface soil characteristics leads to reasonably lowering the cost of construction and comprehensive notion of engineering behavior of soil. Index densities represent the loosest and densest condition of a granular soil layer, and relative density is used to determine the in-situ compactness of the known soil layer regarding the densest condition.

Many standards have been developed to measure and determine the upper and lower boundaries of granular soil density. Such standards are based on the soil fabrics, grain size of the soil particles, and available amount of the soil sample. Norwegian Geotechnical Institute (NGI) recommends a test procedure that applies to a fine to medium cohesionless sand sample with grain size less than 4 mm.

The purpose of this thesis was to evaluate a new NGI test procedure for minimum and maximum index densities of sands and verify the dry density values in terms of repeatability, operator dependency, applicability and consistency. The American Society for Testing and Materials (ASTM) recommendation for determining limit dry densities were considered as a conventional method and results using the ASTM methods were obtained for the purpose of comparison.

1.2 Objective and Scope of Research

The primary scope of this thesis was to study and evaluate the results of the new approach to determine minimum and maximum dry density values in accordance with

the NGI recommendations. This thesis encompasses a lab testing program to determine the limit density values and grain size distribution analysis to study the specimens for any evidence of particle segregation. Seven different fine to medium grain size cohesionless sands with inclusion of different fines content (percent less than 0.075 mm) through the lab testing program. This program contained four distinct minimum dry density methods, three of them based on ASTM D4254-16 and one using the new NGI recommendations. For the maximum dry density the ASTM D4253-16 method 1B and NGI recommendations were used in the testing program.

1.3 Thesis Organization

Chapter 2 provides a brief review of the literature on minimum and maximum density determination and effect of mold size on final values. Chapter 3 describes briefly the seven soil samples employed for the testing program. Chapter 4 presents the methods of investigation used to test the soils characteristics such as grain size distribution from sieve analysis and specific gravity. The chapter also describes the process of each method for determining minimum and maximum density. Chapter 5 presents all the results of the research including minimum and maximum density values along with statistical analysis results for each method, grain size distribution curves, and minimum and maximum void ratios. Chapter 6 presents an interpretation of the results and conclusions derived from the test results.

CHAPTER 2. BACKGROUND

2.1 Introduction

This chapter briefly presents information from prior research on determination of the maximum and minimum dry densities of cohesionless sandy soils together with associated challenges. This chapter focuses on reviewing different testing methods that has been employed by other researchers in order to determine maximum and minimum dry densities.

Investigating the natural soil condition containing sand particles requires evaluation of in-place deformation and density parameters. One of the parameters describing the engineering behavior of a granular soil layer is relative density. In the other words, relative density (D_r) presents the condition of compactness of a soil body as a function of the loosest and densest form that the soil can attain Walter et al. (1982). Retrieving an intact or high-quality sample with sufficient amount of granular soil in to assess a precise relative density can result in increased cost and specific sampling procedures such a ground freezing. Due to these difficulties, the common approach is to estimate relative density (D_r) of a soil matrix from the conventional subsurface investigation methods such as Standard Penetration Test (SPT) and Cone Penetration Test (CPT) data. Gibbs and Holtz (1957) and Terzaghi et al (1996) present a correlation between the blow count obtained from Standard Penetration Test and the density of sands. Schmertmann (1976) correlated the cone resistance (q_c) to relative density (D_r) based on Cone Penetration Test executed in calibration chambers. On the other hand, it is necessary to estimate relative density before performing any laboratory testing over the reconstituted sandy soil. To do so, limit dry densities of a soil specimen, expressed

as minimum and maximum dry density, need to be determined. Depending on the project location and the institute performing the minimum and maximum densities testing, the estimated values may vary significantly, e.g. Tavenas (1973). The following sections will discuss further the methods of measuring limit densities used by researchers.

2.2 Minimum dry density

Many alternate methods have been employed to measure the minimum dry density of the cohesionless sandy soils. These methods tend to place a soil sample in the loosest state that it can attain. Yoshimi and Tohno (1973) used two different methods called the “tilting” and “spoon” test. In the spoon method, oven-dry soil sample was poured in a mold of known volume. Then the excess soil was removed and the weight of the mold plus deposited sand obtained. The minimum dry density was then calculated. On the other hand, the tilting method includes placing oven-dry sand in a graduate cylinder and tilting the cylinder multiple times. The occupied volume by sand obtained directly from the numbers on the cylinder. Holubec and D’ Appolonia (1973) used a scheme that involved placing a tube inside a Standard Proctor container. A sufficient amount of sand specimen was place inside the tube, and then, the tube rose upright. After leveling off the specimen surface, the mass of the soil obtained for minimum dry density calculation. The techniques utilized by Brand (1973), Oda (1976), and Kolbuszewski (1948) mostly included tilting a sand specimen several times either in the end-to-end contacted mold or in a container of known volume. Afterward the sand leveled and weighed for minimum dry density calculation.

In practice, ASTM 4254-16 Standard Test Methods for Minimum Index Density and Unit Weight of Soils and Calculation of Relative Density is commonly used. The

standard provides for three options known as Methods A, B and C, which consist of using a funnel (Method A), or tube (Method B) or cylinder (Method C). All three of these methods were used in the test program presented in this thesis.

2.3 Maximum dry density

Similar to minimum density method, several methods of determining maximum density tested previously by researchers. All these tests accomplished by imparting energy to the specimen in order to reorient the sand grains and form a denser condition. The imparted energy is generated either by vibrating the container plus the applied surcharge on top of the specimen surface over a vibrating table or using a vibrating hammer. Number of tests implemented rubber-tipped hammer and hit the side of the container as a source of energy. The test used by Holubec and D'Appolonia (1973) includes a tube filled with sand and tapped horizontally until no further settlement was observed. Oda (1976) considered placing sand lift in a mold with known volume. After putting the surcharge mass on the sand, the mold hit by a rubber-tipped hammer on the side in order to densify the thin lift of the sample. The order repeated with the rest of the lifts. Then the mold weighed, and the maximum dry density calculated. Youd (1973) compacted the sand by applying simple shear method repeatedly. Yoshimi et al. (1973) performed maximum dry density on two different soils by considering a vertical vibration without surcharge mass on the specimens. Oven-dry sand placed in a mold with known volume and the whole set up subjected to a specific acceleration and frequency for 10 minutes. Brand (1973) measured the maximum density by densifying the sand sample with an electric vibrating hammer for each of three lift of the sand under water

in a standard proctor compaction container. He also evaluated the effect of the time, acceleration and container dimensions upon the compactness of the specimen.

In practice, ASTM 4253-16 *Standard Test Methods for Maximum Index Density and Unit Weight of Soils Using a Vibratory Table* is commonly used. The standard provides for the option of testing an oven-dried sample or wet soil and using either an electromagnetic or cam-driven vertically vibrating table. The wet soil and electromagnetic vibrating table method (1B) was used in the test program presented in this thesis.

CHAPTER 3. DESCRIPTION OF TEST SOILS

3.1 Introduction

Seven granular and cohesionless soil specimens were gathered for this research. BKRO2, Taiwan, Cuxhaven, and Øysand samples area all natural sands that were received from the Norwegian Geotechnical Institute (NGI). Ottawa 20-30, Ottawa 50-70 and RS – 104 samples are commercially processed sands that were available at UMass Amherst. In this chapter a brief description of the grain size distribution of each soil is given. Figure 0 presents the results of the sieve analysis conducted in general accordance with the ASTM D6913 recommendations and the particle properties of all soils are tabulated in **Error! Reference source not found.**. All of the soils classify as poorly g raded SAND using the Unified Soil Classification System (USCS, ASTM D2487). Figure 3-2 through 3-8 present images of the particles for which the shapes which are described based on the Youd (1973) recommendations.

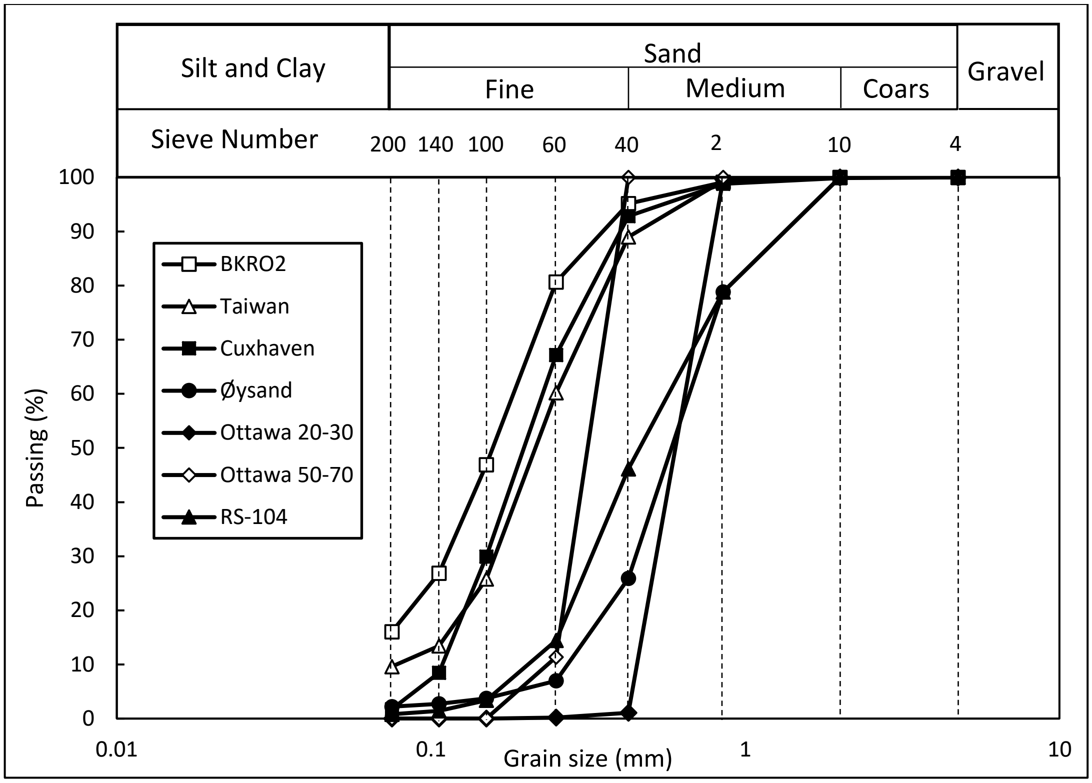

Figure 3. 1 Grain size distribution for BKRO2, Taiwan, Cuxhaven, Øysand, Ottawa 20-30, Ottawa 50-70 and RS – 104

Table 3. 1 Soil particles properties obtained from sieve analysis

Soil Name	Fine Particle[a] (%)	d_{10} (mm)	d_{30} (mm)	d_{50} (mm)	d_{60} (mm)	C_C[b]	C_U[c]	Specific Gravity (G_S)	USCS[d]
BKRO2	16.0	-	0.110	0.164	0.186	-	-	2.656	SP
Taiwan	9.6	0.075	0.167	0.220	0.227	1.6	3.0	2.707	SP
Cuxhaven	1.7	0.106	0.158	0.198	0.215	1.1	2.0	2.649	SP
Øysand	2.2	0.275	0.450	0.572	0.667	1.1	2.4	2.731	SP
Ottawa 20-30	0.0	0.455	0.524	0.600	0.644	0.9	1.4	2.650	SP
Ottawa 50-70	0.0	0.227	0.283	0.318	0.338	1.0	1.5	2.650	SP
RS-104	0.8	0.200	0.327	0.464	0.567	0.9	2.8	2.659	SP

[a]Percentage by dry mass passing the number 200 sieve

[b]$C^c = D_{30}^2/ (D_{60}\ D_{10})$

[c]$C_U = D_{60}/D_{10}$

[d]SP = poorly graded sand in Unified Soil Classification System (USCS)

3.2 BKRO2

The BKRO2 sand is a dark brown fine to medium and rounded to subangular sand with few angular particles (Figure 3-2). The sand has a fines content of about 16 %, by dry mass. The soil batch contains approximately 75 %, by dry mass, fine sand particles with medium grain size (d_{50}) equal to 0.164 mm as well as a trace of crushed shells, and it categorizes as poorly graded sand (SP). BKRO2 has the largest fines content for all the samples studied in this research.

3.3 Taiwan

The Taiwan sand is a grayish brown fine to medium and subangular to angular sand (Figure 3-3). Taiwan is mainly a fine sand with a percentage of about 78 % together with 9 % fine content smaller than 0.075 mm and 12 % medium sand particles. The medium grain size (d_{50}) for Taiwan is 0.22 mm, and it classifies as poorly graded sand (SP).

3.4 Cuxhaven

The Cuxhaven sand is a light brown fine and rounded to subrounded sand (Figure 3-4). The Cuxhaven has approximately 90 %, by dry mass, fine sand particles accompany by less than 2 % particles smaller than 0.075 mm and about 6 % medium sand grains. Cuxhaven's medium grain size (d_{50}) is 0.198 mm and the available batch classified as poorly graded sand (SP).

3.5 Øysand

The Øysand sand is a gray medium and subrounded to subangular sand (Figure 3-5). Almost three-quarter mass of the Øysand is medium sand particle and the other quarter is an inclusion of fine sand fraction and particles smaller than 0.075 mm by 23 %

and 2 %, respectively. The Øysand's medium grain size (d_{50}) and classification are 0.572 mm and poorly graded sand (SP), respectively.

3.6 Ottawa 20-30

The Ottawa 20-30 sand used in this testing is rounded silica (Figure 3-6) and poorly graded quartz sand (SP) which is mined from Ottawa, Illinois and manufactured by the U.S. Silica Company. The medium grain size (d_{50}) for Ottawa 20-30 is 0.6 mm and is classified as medium sand with no particles smaller than 0.075 mm.

3.7 Ottawa 50-70

Like Ottawa 20-30, this sand is also rounded silica (Figure 3-7) and poorly graded quartz sand (SP) but is classified as fine sand by 89 % and 11 %, by dry mass, remaining on sieves number 60 and 100, respectively. The U.S. Silica Company mines and manufactures it in Ottawa, Illinois. the medium grain size (d_{50}) obtained for this sand is equal to 0.318 mm.

3.8 RS-104

The RS-104 is a light brown fine to medium sand with fine content less than 1 %, by dry mass. The original batch of the RS-104 sand contained particles larger than 2 mm, however, in order to prepare samples in compliance with NGI method described in Chapter 4, the particles retained on sieve number 10 were removed. This sand is classified as poorly graded (SP). It can be seen from Figure 3-8 that many of the particles are rounded to subangular with little angular grains.

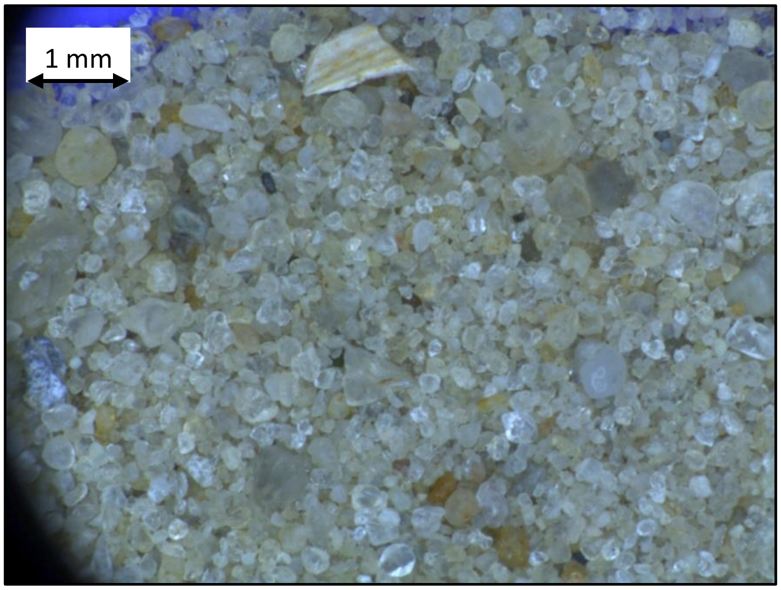

Figure 3. 2 BKRO2 particle shape – washed on the sieve # 200

Figure 3. 3 Taiwan particle shape – washed on the sieve # 200

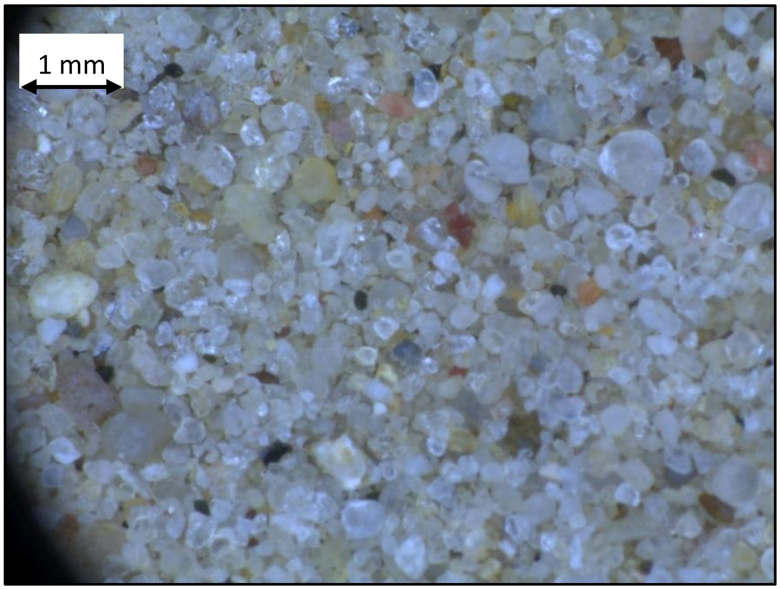

Figure 3. 4 Cuxhaven particle shape – washed on the sieve # 200

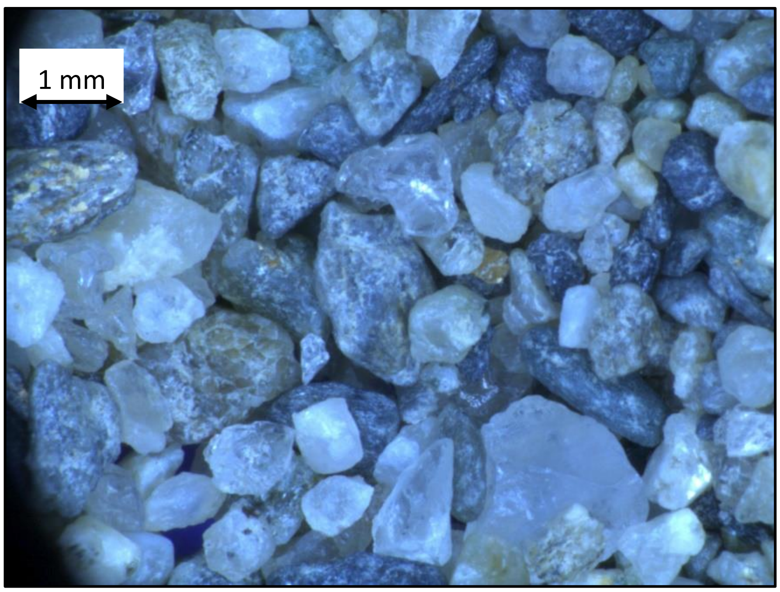

Figure 3. 5 Øysand particle shape – washed on the sieve # 200

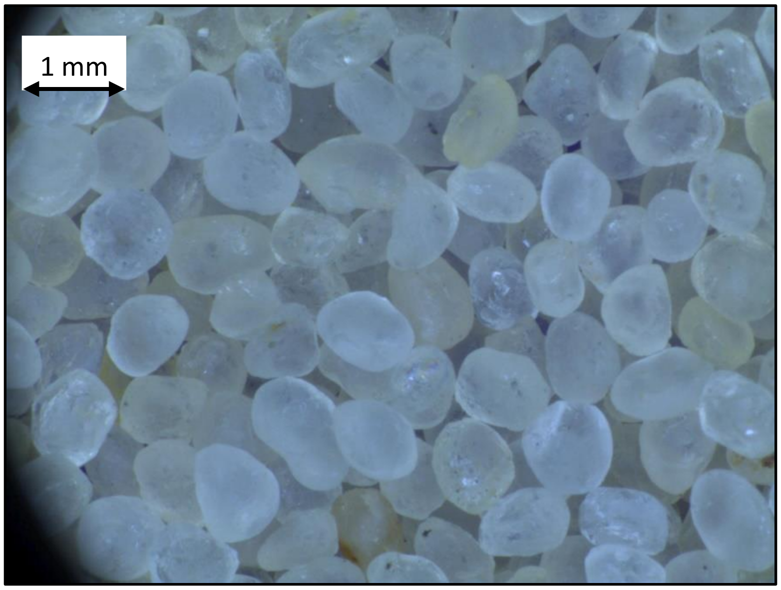

Figure 3. 6 Ottawa 20-30 particle shape – washed on the sieve # 200

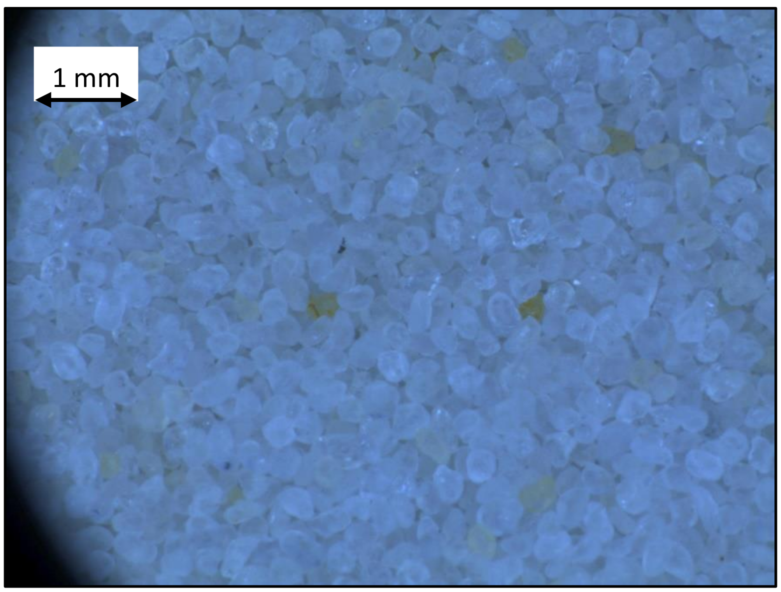

Figure 3. 7 Ottawa 50-70 particle shape – washed on the sieve # 200

Figure 3. 8 RS-104 particle shape – washed on the sieve # 200

CHAPTER 4. METHODS OF INVESTIGATION

4.1 Introduction

The following chapter describes sample preparation and main sample evaluation procedures that were employed in this research such as Water Content, Grain Size Distribution, and Specific Gravity. Later in this chapter, the Minimum and Maximum Density methods, employed to obtain limit densities is described.

4.2 Basic Index and Classification Tests

4.2.1 Water Content

Water content was determined in general accordance to ASTM D2216 – 10 Standard Test Methods for Laboratory Determination of Water (Moisture) Content of Soil and Rock by Mass. Two samples were selected from the moist representative soil sample, oven dried overnight at 110 ± 5°C and then cooled down to room temperature in a desiccator. Wet and dry mass of specimens with their associated containers' weight was measured and recorded before and after being placed in the oven. The water content of the soil samples was determined as follows:

$$w = [(M_{cms} - M_{cds})/(M_{cds} - M_c)] \times 100 \qquad \text{(Eq.0.1)}$$

Where:

w = water content, %,

M_{cms} = mass of container and moist specimen, g,

M_c = mass of container, g,

M_{cds} = mass of container and oven dry specimen, g.

4.2.2 Grain Size Distribution

The grain-size analysis was performed in general accordance with ASTM D6913–04 Standard Test Method for Particle-Size Distribution (Gradation) of Soils Using Sieve Analysis. Each Sieve analysis was carried out based on method B as recommended by ASTM for soil samples with particle size equal or less than a sieve No.4 (4.74 mm), and subsequently, a single-set sieving, containing seven sieves was used for the test procedure. Table 4.1 shows the order of sieves for the single-set sieving used in this practice. A test specimen, about 60 g, was obtained from the main soil sample using a splitter and oven dried overnight at 110 ± 5°C and brought to the room temperature in a desiccator the next day. Then, the sample was poured into the stack of sieves and shacked using a mechanical sieve shaker for 10 minutes. Afterward, the soil solid mass remaining on each sieve and the pan were measured and recorded.

Table 4. 1 Single-Set Sieving Order

Sieve Number	Sieve Opening Size
No. 10	2.00 mm
No. 20	850 µm
No. 40	425 µm
No. 60	250 µm
No. 100	150 µm
No. 140	106 µm
No. 200	75 µm

4.2.3 Specific Gravity

The specific gravity test was performed in compliance with ASTM D854 – 14 Standard Test Methods for Specific Gravity of Soil Solids by Water Pycnometer, test method B for oven-dried specimens. Approximately 50 g specimen was chosen from each batch of soil using a splitter, mixed with deaired water in a mixer for one minute.

Afterwards, the solution was poured into a calibrated pycnometer, and then agitated at least 2 hours under vacuum to release entrapped air in the soil slurry. After that, deaired water was added to the slurry in order to develop a clear water layer on top of the soil slurry from the soil slurry surface to the calibration mark on the pycnometer stem. The mass of pycnometer including soil slurry and deaired water were measured and recorded. The soil slurry was transferred into a steel bowl and oven-dried overnight at 110 ± 5°C. Next day, the specimen cooled down to the room temperature in a desiccator and weighed. The specific gravity at 20°C was calculated using the following equations:

$$G_{20°C} = C \times G_t \qquad \text{(Eq.4.2)}$$

$$G_t = M_s/(M_{p,w} + M_s - M_{p,w,s}) \qquad \text{(Eq.0.3)}$$

Where:

G_t = specific gravity of soil solids at the test temperature

M_s = mass of the oven-dried soil specimen, (g)

$M_{p,w}$ = mass of the pycnometer and water, (g)

$M_{p,w,s}$ = mass of the pycnometer, water, and soil slurry, (g)

C = temperature coefficient at the test temperature obtain from ASTM D854 - 14

4.3 Minimum Index Dry Density

The minimum density of a soil expresses the loosest form or highest porosity condition of a soil mass. Several procedures have been recommended by many institutes to obtain the minimum index density with the term index added to indicate that the measurement is based on a specific laboratory test procedure which may or may not

result in the smallest possible density. This research focused on measuring minimum index dry density of soil batches in compliance with ASTM D4254 – 14 and the newly developing NGI minimum index density method. ASTM outlines three alternative minimum density measurement procedures to cover more soil in terms of particle size, while NGI presents one procedure with a limited range of soil particle size. To evaluate all the methods from the perspective of providing repeatable results, three representative samples were tested for each procedure. Furthermore, each procedure was performed in a set containing five runs, and also, to simulate real testing, each set was done on a different day. For both the ASTM and NGI methods the molds were calibrated using the water calibration method as described in ASTM D4254. The ASTM mold had a volume of 2848.3 cm^3 and the NGI mold of 783.7 cm^3.

4.3.1 ASTM D4254 Minimum Index Density

ASTM D4254 recommends three distinct methods to determine the minimum index density. Each method requires different apparatuses and soil sample mass. Representative samples were oven dried overnight and cooled down into room temperature prior to performing each method. These methods are described in the following sections.

[redacted]Method *A*

This method, which is also referred to as the Funnel method in this study, is applicable to soils that 100%, by dry mass, of the particles pass through a 3-inch (75 mm) sieve and up to a maximum of 30%, by dry mass, of soil particles remain on a sieve 1.5-inch (37.5 mm). All the available soil samples in this research meet that soil-particle size range criteria. A funnel, mold, a straight-edge knife, and a brush with plastic bristles

were the devices used in this method (Figure 4-1). The mold was filled by pouring an oven-dried soil sample (approx. 4000 g) using the funnel with about 0.5-inch above the top of the specimen inside the mold. The soil was poured carefully into the mold in such a way that provided a continuous flow rate and the spout tip did not touch the surface of the already deposited soil. The excess soil, above the top of the mold rim, was trimmed away using the straight-edge knife in order to level the soil surface with the rim of the mold, and any extra soil on the exterior surface of the mold was removed using the brush. The weight of the mold with the deposited soil was determined and recorded. The empty mold mass subtracted from the mold plus soil mass is used to calculate the dry mass of the soil for minimum dry density calculation. These calculations are as follows:

$$\rho_{d,min} = (M_{m,s} - M_m)/V_m \qquad \text{(Eq.0.4)}$$

Where,

$\rho_{d,min}$, is minimum index dry density, g/cm^3 or Mg/m^3,

$M_{m,s}$, is mass of the mold with deposited soil, g,

M_m, is mass of the empty mold, g,

V_m, is average-water-filling volume of the mold, cm^3.

Figure 4. 1 Equipment used for ASTM D4254–14, Method–A (i.e., Funnel)

[redacted] Method B

Method B, also referred as the Tube method, applies to soil samples with particles size less than 19 mm (⅜-inch sieve), which all the soil samples in this study fit in this range. In contrast with method A, the funnel is substituted with a plastic tube with an inner diameter of about 0.7 times the inner diameter of the mold while all other equipment is the same as that used for Method A (Figure 4. 2). The tube was placed inside the mold and filled carefully with scoops of the oven-dried soil up to approximately 5 mm below the tube rim. Then, the tube was lifted quickly, and the excess soil leveled off with mold rim using a straight-edge knife. The mold with deposited soil weighed and the dry mass of the deposited soil calculated using (Eq.0.4).

Figure 4. 2 Equipment used for ASTM D4254–14, Method-B (i.e., Tube)

[redacted]Method C

Soil particles for method C, also referred as the Cylinder method, should be smaller than 9.5 mm (⅜-in. sieve) and up to a maximum of 10 %, by dry mass, remain on a No. 10 sieve. A soil specimen of about 1000 g was placed in a 2000 mL graduated cylinder (Figure 4. 3), and the top of the cylinder covered with a piece of plastic wrap. The cylinder was tilted upside down and returned to the beginning position. The volume of soil within the cylinder was recorded from four equidistant points on the external perimeter of the cylinder. The minimum dry density calculated by dividing the mass of the soil sample (1000 g) by the averaged volume of the soil.

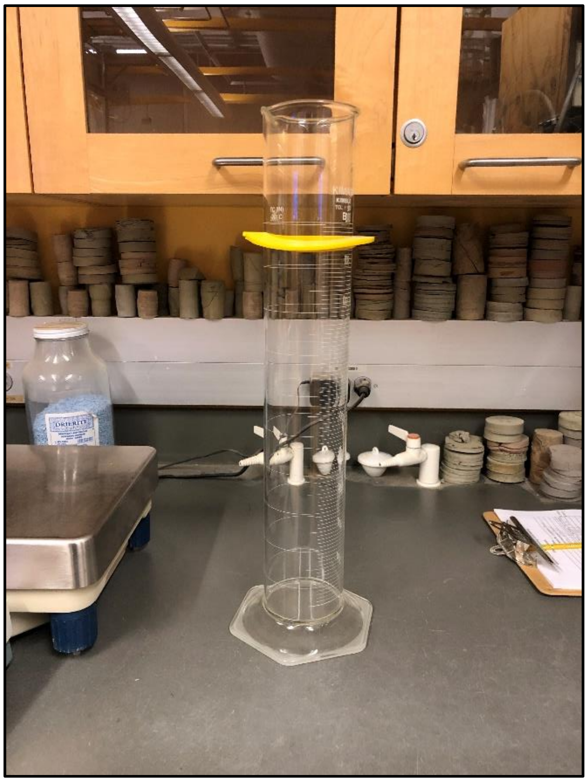

Figure 4. 3 Equipment used for ASTM D4254–14, Method–C (i.e., Cylinder)

4.3.2 NGI Minimum Index Density

The Norwegian Geotechnical Institute (NGI) has proposed established a new draft procedure in order to measure the minimum index density of sands. Applicable soils in this method are the ones with particle size less than 4 mm. The particles larger than 4 mm should be removed from the soil sample prior to testing. In addition, the percentage of the fine particles, by dry mass, is limited to 10 %. Figure 4. 4 NGI minimum density method apparatusesFigure 4. 4 presents the equipment used in the NGI minimum density method including a cylindrical-steel mold, steel funnel, plastic-bristle brush, and straight edge knife. The mold and funnel were manufactured by UMass Amherst in general accordance to the specifications provided by NGI. Approximately 700 g of soil is first used as a trial and oven dried at 110 ± 5°C. After cooling down the sample to room

temperature in a desiccator, two trial tests were conducted to estimate the amount of soil needed to shape a cone above the rim of the mold. Adjusting the sample mass eliminates the effect of the excess pouring soil upon the already deposited particles. With the objective to minimize the moisture absorption, after mass adjustment, the sample was oven dried and cooled down again. Then, the funnel was placed in the mold and the soil sample poured in the funnel. Afterward, the funnel was lifted vertically and adjusted continuously to maintain the flow rate in order to keep the tip of the funnel just above contact with the top of the cone of the already deposited soil particles. After filling, the extra soil above the mold rim was trimmed off carefully with the knife edge, and excess material on the exterior surface of the mold was removed using the brush. The deposited soil mass was determined by subtracting the mass of the empty mold from the mass of the mold plus deposited soil. Then the minimum dry density of the soil calculated using Eq. 4-4.

Figure 4. 4 NGI minimum density method apparatuses

4.4 Maximum Index dry Density

The theoretical upper boundary of a soil density is regarded as maximum dry density which represents the densest condition of a soil. In the second part of this study, two methods, ASTM D4253 - 14 and the newly developed NGI method, were used to determine the maximum index dry density of the available soils for this research. The NGI method is applicable for soils containing particles smaller than 2 mm (i.e., sieve No. 10). For the purpose of verifying NGI maximum density method, all the particles larger than 2 mm were removed from the chosen soil samples for both ASTM and NGI testing. Due to the possibility of particle degradation or crushing, each soil sample, after every test was not reused, and consequently, two discrete representative samples was used for each method.

4.4.1 ASTM Maximum Index Density

ASTM recommends four distinct methods based on the available vibrating table and the condition of a soil sample. With the intention of simulating a similar procedure to the NGI method, method 1B among those four methods was chosen. Figure 4-5 presents the apparatus used in determination of maximum dry density based on the ASTM recommendations.


Figure 4. 5 ASTM maximum dry density method apparatus

After choosing an adequate representative sample using a splitter, the soil was mixed with distilled water and left for about an hour in order to just saturate the particles. Then, the mold was attached to the vibrating table, and the saturated sample scooped into the mold while the table was on. Meanwhile, a small amount of water was added to the already scooped-in soil in order to form a small layer of free water, whenever this did not occur by itself according to the standard requirement. Afterward, the vibration table was turned off, and any excess water on top of the soil was removed. Then, the base plate, surcharge, and guide sleeve were placed, and the table gauge was set at a dial reading of 55, representing an amplitude of 0.33 ± 0.05 mm at 60 Hz, for 9 minutes. These settings were determined by calibration using a displacement transducer and a high speed PicoScope digital oscilloscope. After the shaking period, the guide sleeve and surcharge were removed, the mold was detached from the table, and the distance from the top rim of the mold to the top of the base plate was measured using a depth micrometer. The specimen was then collected and placed in an oven overnight at 110 ±

5°C and cooled down in a desiccator the following day. The mass of the oven-dried soil obtained, and the maximum dry density calculated as follows:

$$\rho_{d,max} = M_s / (H_s \times A_m) \quad \text{(Eq. 0.5)}$$

Where,

M_s is the oven-dried mass of the densified soil, g,

H_s is the thickness of the densified soil, cm,

A_m is the cross section of area of the mold, cm^2.

4.4.2 NGI Maximum Index Density

The NGI approach to determine the maximum index dry density is limited to the sands with particles smaller than 2 mm (sieve No. 10). Thus all representative samples were passed through a sieve No. 10 and the particles larger than 2 mm were removed. Unlike ASTM, the NGI maximum and minimum index density determinations use a much smaller mold. Figure 4. 6 shows the equipment and supplies used including a cylindrical plastic mold, surcharge disc, filter paper, surcharge weight, and double-sided tape used in NGI maximum index density method.

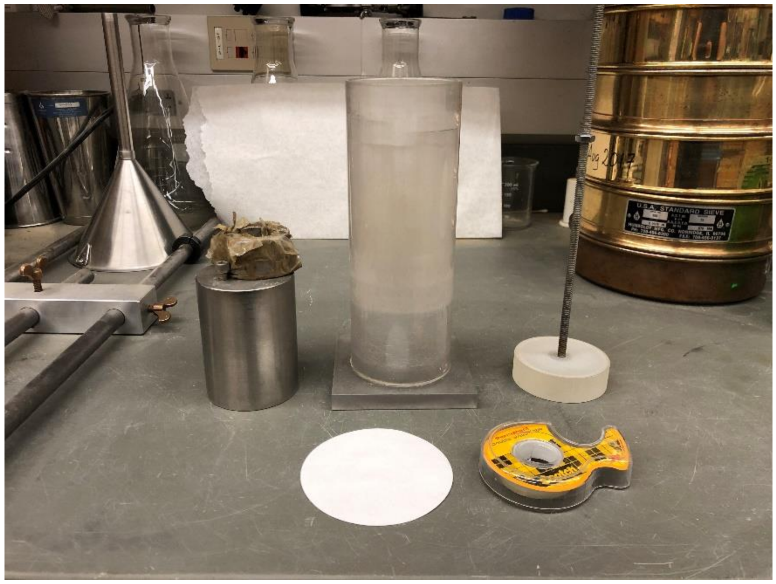

Figure 4. 6 NGI maximum dry density method apparatuses

After obtaining an approximate 500-gram-representative sample, the plastic mold was clamped down to the vibrating table (the same table as recommended by ASTM), 100-milliliter-distilled water was poured into the mold, and the vibrating table was set at 55, representing an amplitude of 0.33 ± 0.05 mm at 60 Hz. While the table was on, the soil sample was poured slowly into the mold and left under the vibration for 2 minutes. After switching off the table, the surcharge disc was pushed into the mold in such a way that the filter paper was face down to the soil surface. As soon as placing the surcharge disc, any excess water was removed from top of the disc. Afterward, the surcharge weight was placed, and the dial set again at 55, and the whole set remained under vibration for 2 minutes. After vibration period, the surcharge weight was lifted off, and any fine particles on top of the disc and internal surface of the mold were carefully rinsed off. Then, the distance from the rim of the mold to top of the disc was measured and recorded. This distance was added to the summation thickness of the disc, double-sided tape, and filter paper, and the whole figure was subtracted from the inner depth of the mold. The

soil sample washed out into a bowl and oven dried at 110 ± 5°C. Subsequently, the next day, the soil was cooled down to room temperature, and the dry mass of the tested soil obtained and recorded. The maximum dry density of the soil calculated using (Eq. 0.5.

Note that the procedure described above was the draft procedure being explored by NGI at the time this research work was being conducted. Subsequently, and after all the tests presented in this thesis were performed, NGI updated the method to use a amplitude of vibration equal to 2 mm at 50 Hz for both vibration periods, i.e., without and with the surcharge, and the duration of vibration remaining at 2 min for without the surcharge and reduced to 15 seconds for with the surcharge.

CHAPTER 5. PRESENTATION AND INTERPRETATION OF RESULTS

5.1 Introduction

The following chapter presents and evaluates the results of the minimum and maximum dry density testing and the associated grain size distribution analysis. This includes the measured minimum dry densities based on the ASTM method A, B and C, and NGI method as well as maximum dry index densities in general accordance with ASTM and NGI procedures, in sections one and two, respectively. The third section of this chapter discusses the grain size distribution curves used to identify any potential particle crushing during maximum density testing programs. Additionally, the minimum and maximum void ratios are presented based on the measured specific gravity of each soil.

5.2 Minimum Index dry Density

This section presents and discusses the minimum density values obtained using three different methods outlined by ASTM (i.e., method A, method B and method C) and the NGI method. Each of these four methods was carried on the basis of three distinct sets and each set included five repetitions of a method on a soil sample. Figures 5-1 through 5-7 show a summary of the testing outcomes for each soil, and Tables 5-1 through 5-7 provide a summary of the statistics data regarding each method for every soil.

The summary of the minimum dry density results (i.e., Figures 5-1 through 5-7) shows that NGI method generates the lowest minimum dry density for all soils compare to the other methods. It is hypothesized that this trend might be caused by some arching occurring during sample deposition from the funnel into the mold. Furthermore, given

that the NGI mold is significantly smaller than the ASTM mold, it is also possible that side friction between the soil and mold plays a greater role.

Among the ASTM recommended procedures, method A produces higher dry density than method B for soils with a very low to zero percent fine particles (i.e., silt and clay). On the contrary, method B generates greater dry density for BKRO2 and Taiwan. These soils contain higher fine particles about 16 % and 9 %, by dry mass, respectively.

It is noticeable from Figures 5-1 to 5-7 that the ASTM's method C produces more biased minimum density values than the other methods. The statistical calculation also reveals that the highest coefficient of variation (COV) for every soil belongs to method C. The statistical evaluation results are tabulated in Tables 5.1 through 5.7 for each soil separately. The COVs for BKRO2, Cuxhaven, Ottawa 20-30 and Ottawa 50-70 are about 1.1 %, and the COVs are 0.78 % and 0.87 % for Taiwan and Øysand, respectively. It was not applicable to perform the ASTM's method C on RS-104 because more than 10 %, by dry mass, of the virgin soil remained on the sieve number 10 (2 mm). Figure 5-8 illustrates the likely reason for high COVs generated by method C. This figure reveals that the sand particles and the interior surface of the cylinder became electrically charged after the cylinder with sand inside tilted once. The magnitude of the attractive forces is high enough that is capable to counteract the gravity force and the fine sand particles adhere to the interior surface of the cylinder. This phenomenon results in an error in the volume calculation of the deposited specimen inside the cylinder.

The NGI method was performed on BKRO2 and Taiwan by another operator in order to examine possible operator dependency on the measured minimum dry density

values. Figures 5-9 and 5-10 show the average of fifteen minimum dry densities obtained by each operator for every soil. The discrepancy of the averaged minimum densities between different operators for BKRO2 and Taiwan is 0.04 g/cm^3 and 0.002 g/cm^3, respectively. The difference between the standard deviations of the operators for both BKRO2 and Taiwan is 0.001. Similarly, the difference between coefficient of variations of the operators for both BKRO2 and Taiwan presented in Tables 5-1 and 5-2 are 0.1 % and 0.05 %, respectively.

Table 5. 1 Results of the statistical evaluation of BKRO2's minimum dry density

Method Name	Mean value (g/cm^3)	Standard Deviation	Coefficient of Variation (%)
Funnel (A)	1.432	0.003	0.22
Tube (B)	1.448	0.003	0.23
Cylinder (C)	1.414	0.016	1.10
NGI (HG)	1.360	0.006	0.46
NGI (SP)	1.356	0.005	0.36

Table 5. 2 Results of the statistical evaluation of Taiwan's minimum dry density

Method Name	Mean value (g/cm^3)	Standard Deviation	Coefficient of Variation (%)
Funnel (A)	1.363	0.003	0.22
Tube (B)	1.396	0.005	0.32
Cylinder (C)	1.352	0.011	0.78
NGI (HG)	1.320	0.003	0.26
NGI (SP)	1.318	0.004	0.31

Table 5. 3 Results of the statistical evaluation of Cuxhaven's minimum dry density

Method Name	Mean value (g/cm^3)	Standard Deviation	Coefficient of Variation (%)
Funnel (A)	1.501	0.009	0.58
Tube (B)	1.497	0.007	0.49
Cylinder (C)	1.474	0.016	1.11
NGI (HG)	1.454	0.005	0.32

Table 5. 4 Results of the statistical evaluation of Øysand's minimum dry density

Method Name	Mean value (g/cm^3)	Standard Deviation	Coefficient of Variation (%)
Funnel (A)	1.497	0.003	0.19
Tube (B)	1.472	0.009	0.62
Cylinder (C)	1.437	0.012	0.87
NGI (HG)	1.428	0.008	0.59

Table 5. 5 Results of the statistical evaluation of Ottawa 20-30's minimum dry density

Method Name	Mean value (g/cm^3)	Standard Deviation	Coefficient of Variation (%)
Funnel (A)	1.584	0.004	0.23
Tube (B)	1.530	0.004	0.25
Cylinder (C)	1.509	0.018	1.18
NGI (HG)	1.522	0.002	0.13

Table 5. 6 Results of the statistical evaluation of Ottawa 50-70's minimum dry density

Method Name	Mean value (g/cm^3)	Standard Deviation	Coefficient of Variation (%)
Funnel (A)	1.459	0.003	0.17
Tube (B)	1.445	0.006	0.44
Cylinder (C)	1.414	0.015	1.08
NGI (HG)	1.436	0.003	0.17

Table 5. 7 Results of the statistical evaluation of RS-104's minimum dry density

Method Name	Mean value (g/cm^3)	Standard Deviation	Coefficient of Variation (%)
Funnel (A)	1.602	0.005	0.31
Tube (B)	1.586	0.005	0.33
Cylinder (C)	-	-	-
NGI (HG)	1.506	0.007	0.44

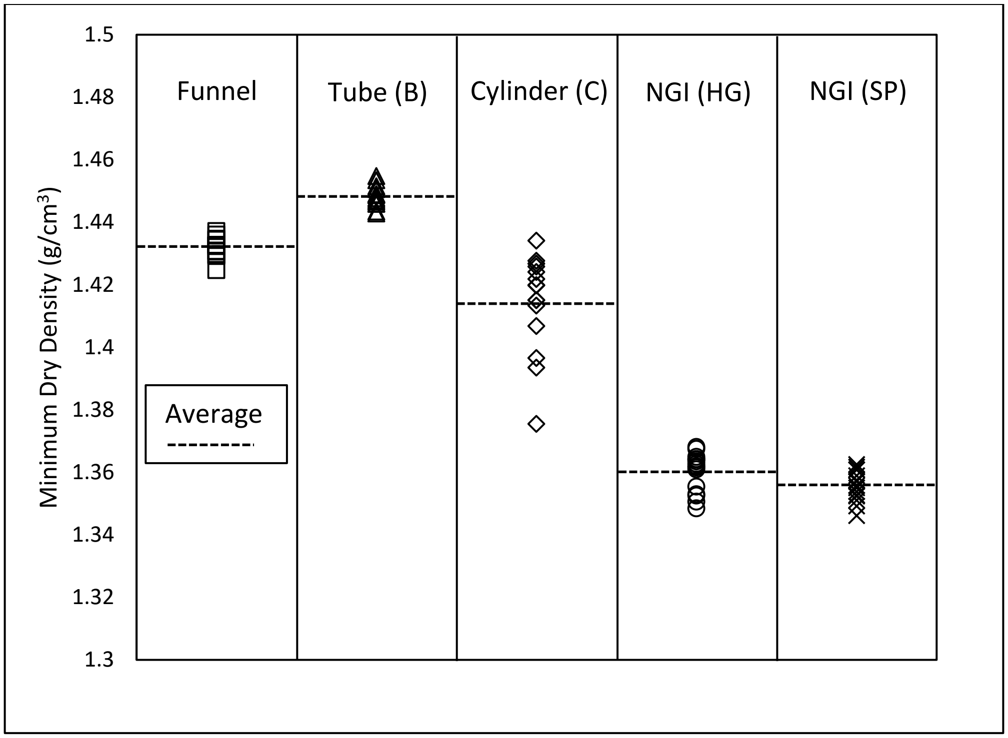

Figure 5. 1 Measured minimum dry density of BKRO2 based on ASTM and NGI methods (HG and SP for NGI method are two difference operators).

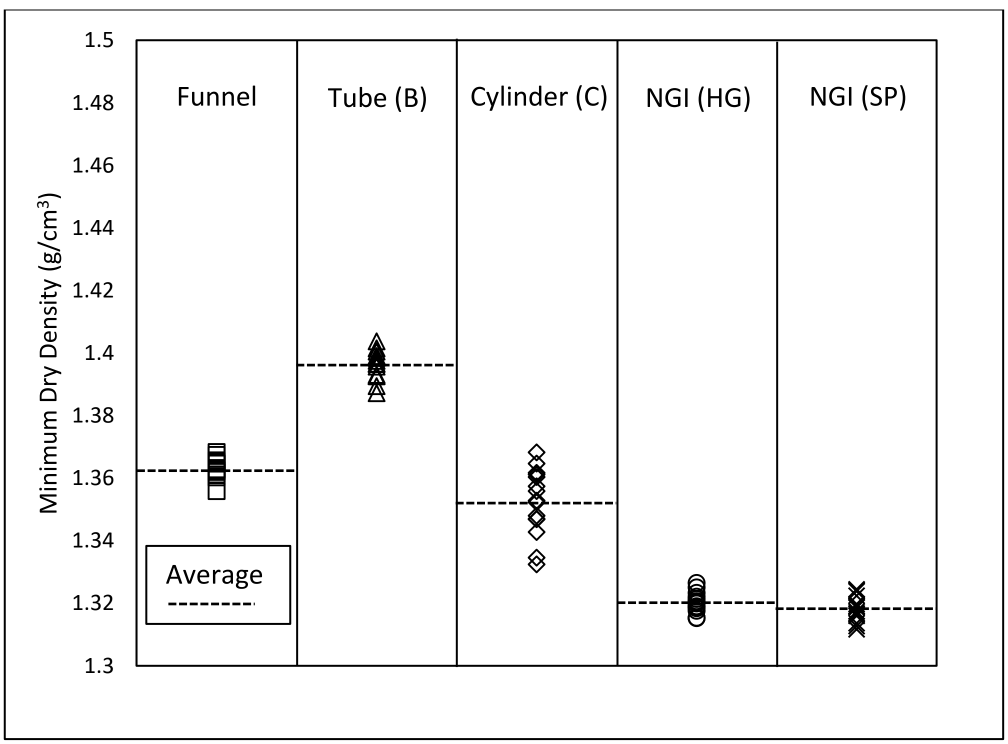

Figure 5. 2 Measured minimum dry density of Taiwan based on ASTM and NGI methods (HG and SP for NGI method are two difference operators).

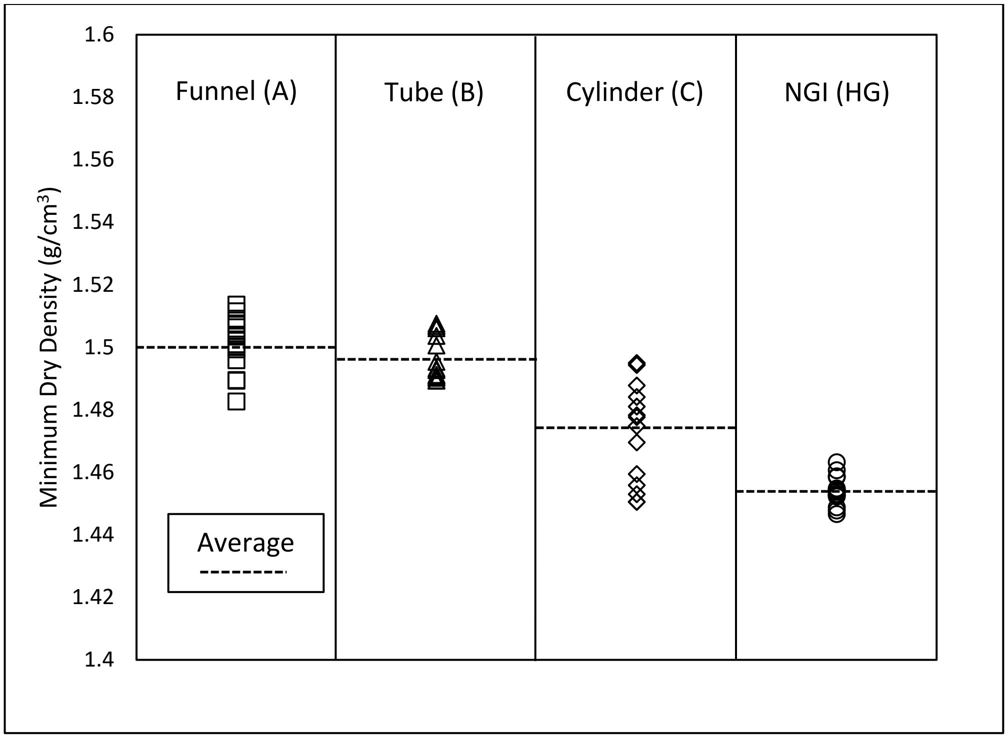

Figure 5. 3 Measured minimum dry density of Cuxhaven based on ASTM and NGI methods

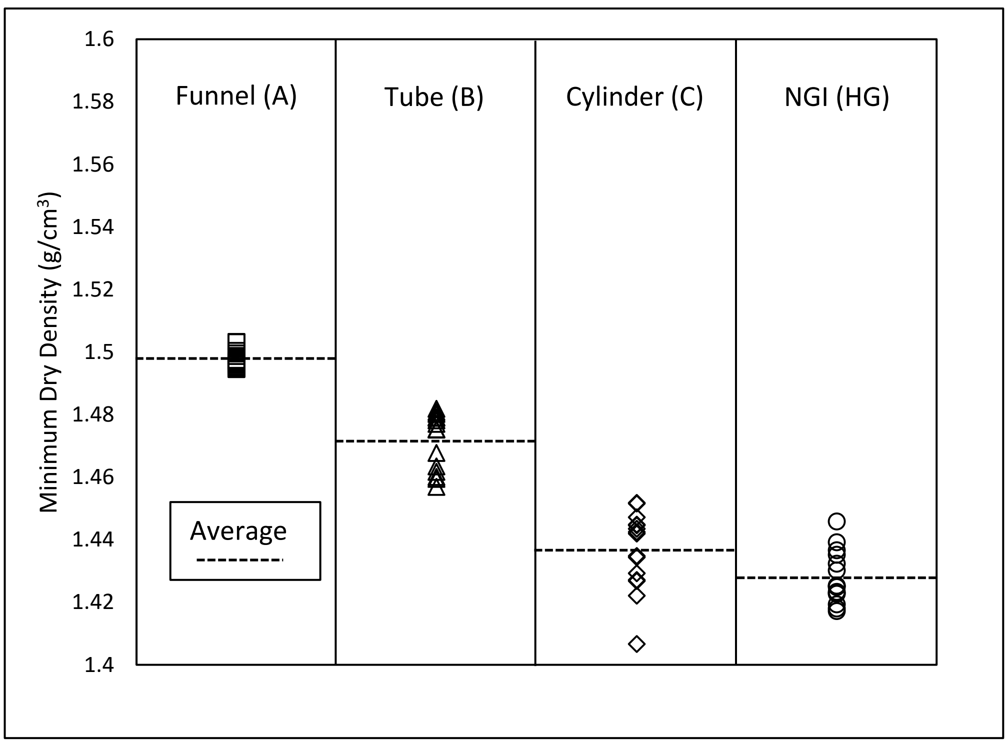

Figure 5. 4 Measured minimum dry density of Øysand based on ASTM and NGI methods

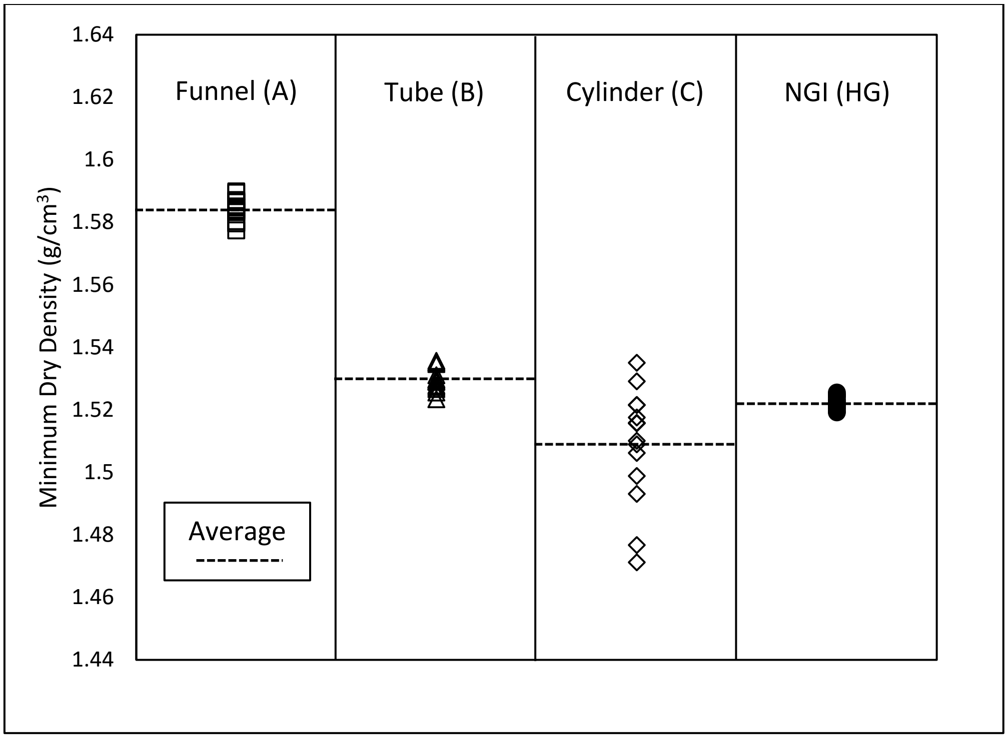

Figure 5. 5 Measured minimum dry density of Ottawa 20-30 based on ASTM and NGI methods

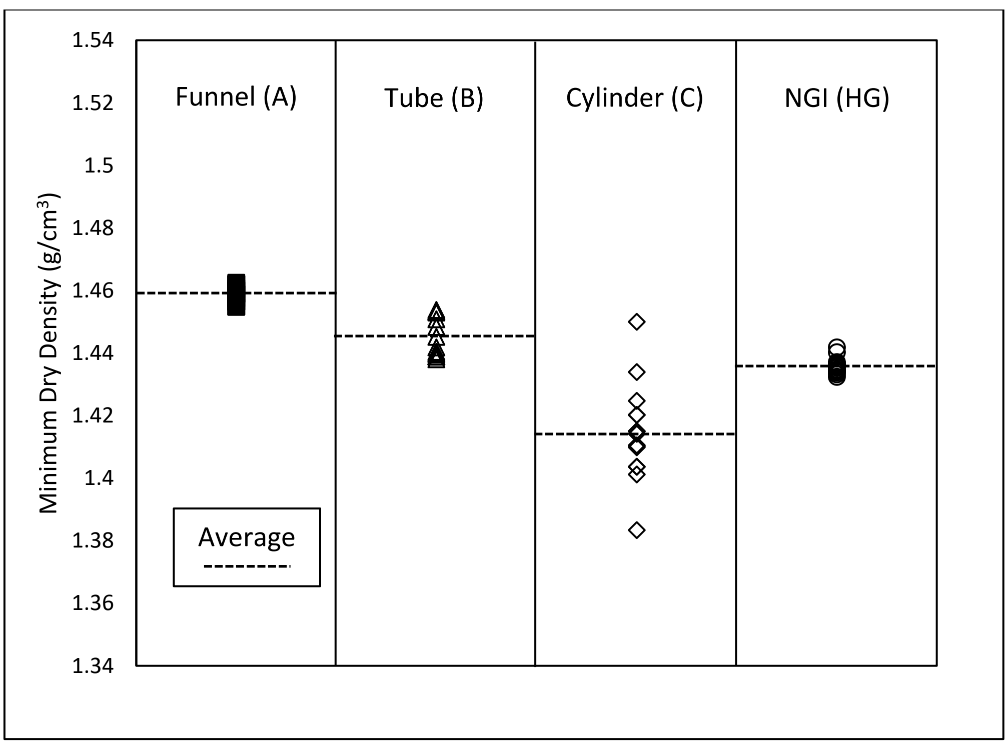

Figure 5. 6 Measured minimum dry density of Ottawa 50-70 based on ASTM and NGI methods

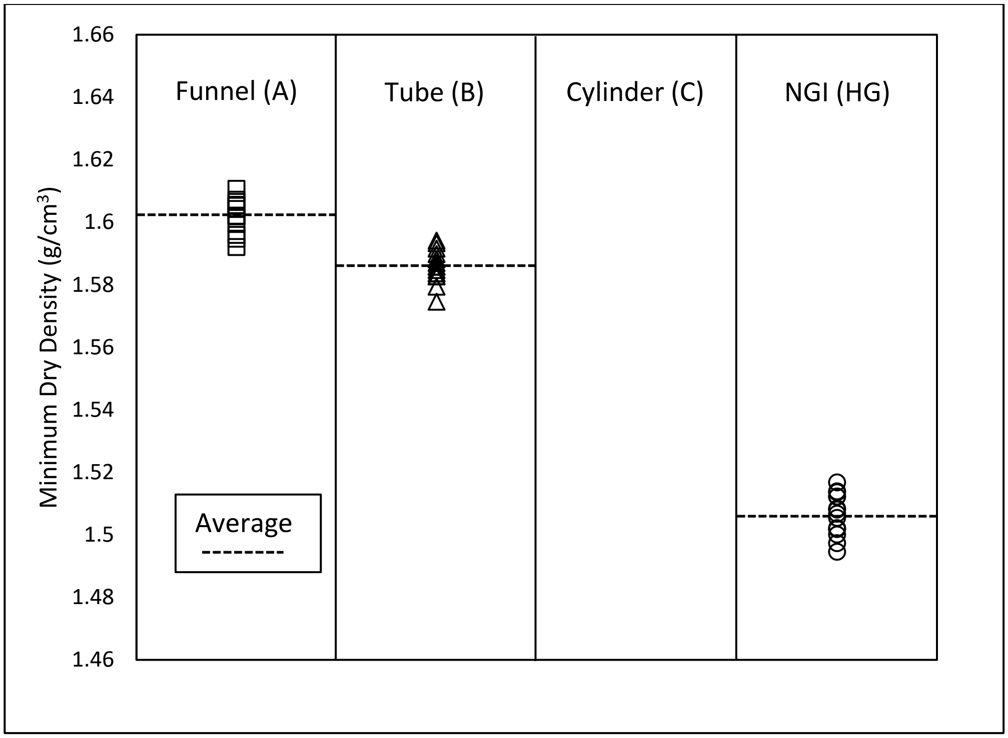

Figure 5. 7 Measured minimum dry density of RS-104 based on ASTM and NGI methods

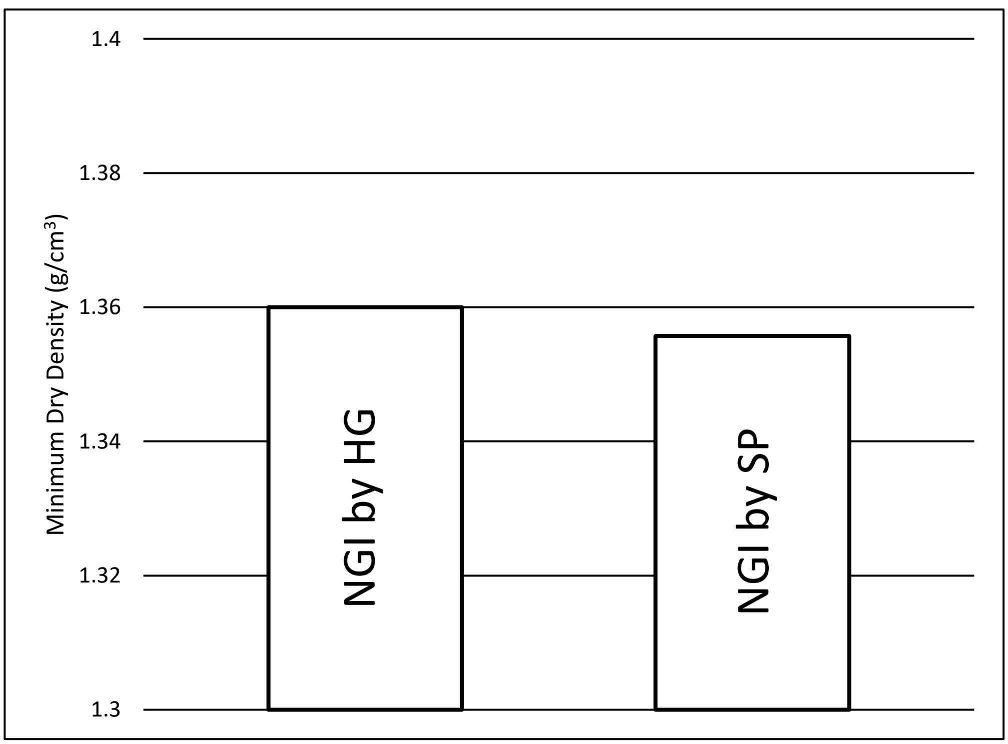

Figure 5. 8 Averaged minimum dry density of BKRO2 obtained by different operators based on NGI minimum density method (the averaged minimum dry density is a mean value of fifteen different densities)

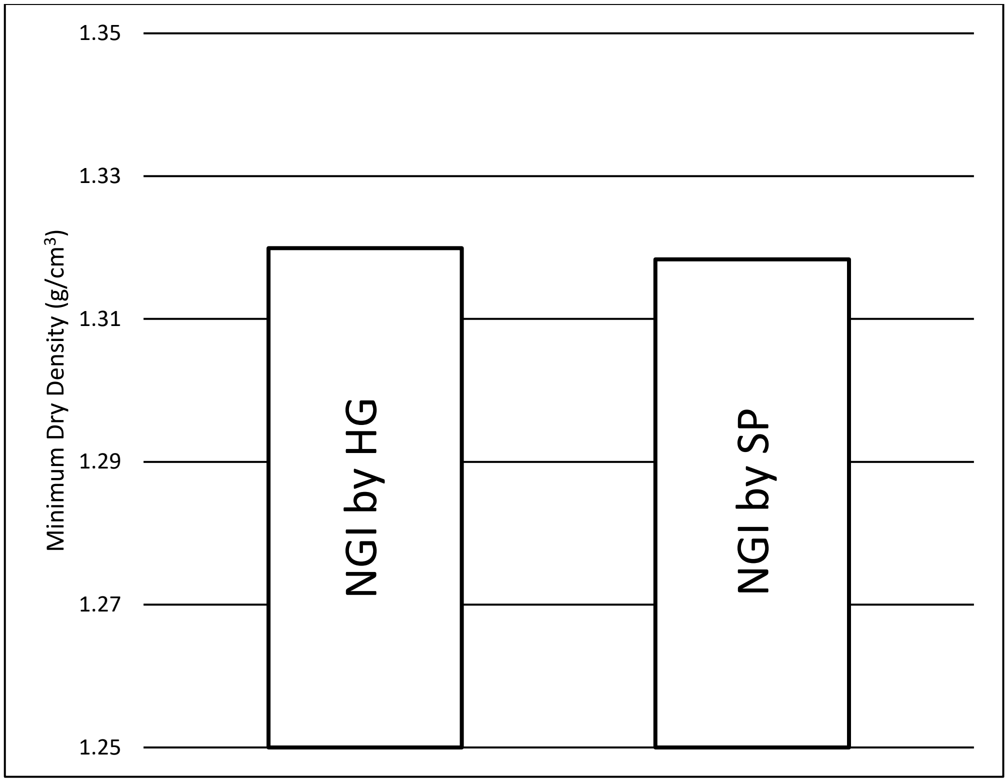

Figure 5. 9 Averaged minimum dry density of Taiwan obtained by different operators based on NGI minimum density method (the averaged minimum dry density is a mean value of fifteen different densities)

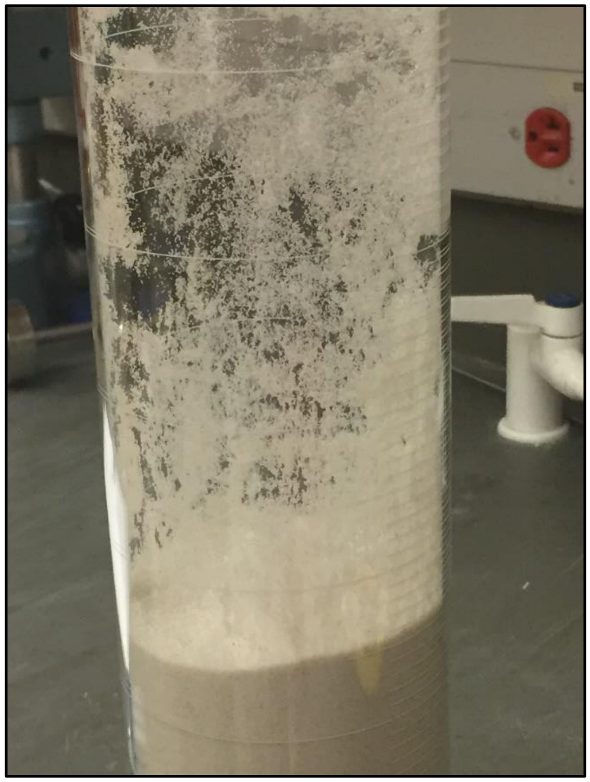

Figure 5. 10 Electrically charged particles adhered to the inner surface of the cylinder.

5.3 Maximum Dry Density

Figures 5.11 through 5.17 and table 5.8 present a summary of the maximum dry densities of each soil determined in compliance with ASTM method 1B – using wet soil and an electromagnetic vertically vibrating table - and the NGI method. The projected dash line in every figure shows a mean value of the measured maximum densities based on the corresponding method. Figures 5.13 through 5.16 indicate that both ASTM and NGI procedures create almost similar average values. The difference between the ASTM and NGI mean value for Cuxhaven, Øysand, Ottawa 20-30 and Ottawa 50-70 is 0.006, 0.013, 0.008 and 0.005 g/cm^3, respectively. The majority of these four soil samples contain fine sand particles with inclusion of less than 2 %, by dry mass, silt and clay content. On the other hand, the ASTM's maximum density procedure shows higher

numbers compare to the NGI for BKRO2, Taiwan and RS-104. The difference between the measured ASTM and NGI mean value for BKRO2, Taiwan and RS-104 is 0.098, 0.053 and 0.078 g/cm^3. BKRO2 and Taiwan have high fine content (i.e., silt and clay) equal to 16 % and 9.6 %, by dry mass, and more than 50 %, by dry mass, of the RS-104 soil sample contains medium sand particles and 0.8 %, by dry mass, particles less than 0.075 mm. The reason why ASTM produces higher values is due to the heavier surcharge weight more energy transforms to the soil body. In addition, there is a possibility that the fines content, when saturated, works as lubricate and eases the larger particles rolling over each other to form a denser arrangement. Also, the ASTM's mold has a volume equal to 2849 cm^3 and height to diameter ratio of 1.01 compare to the NGI's mold with a volume equal to 789 cm^3 and height to diameter ratio of 2.89. These numbers indicate that the ASTM's mold gives a more space to the soil particles to relocate easier than the NGI's mold.

As it can be seen from Figures 5.11 through 5.17, each method was able to produce consistent results. The difference of the first and second maximum density value and corresponding mean value for all soil samples is less than 0.01 g/cm^3 except for Øysand's first and second NGI maximum density testing, Cuxhaven's first and second NGI maximum density testing and BKRO2's third NGI maximum density testing. These values are equal or less than 0.02 g/cm^3 but greater than 0.01 g/cm^3.

Figure 5.18 indicates a summary of all maximum and minimum density values. It is evident that the range of $e_{max} - e_{min}$ for the soils with very low to zero fine content is smaller than BKRO2 and Taiwan containing higher fine content about 16 % and 9 %, by dry mass, respectively.

Table 5. 8 Summary of measured maximum density results in compliance with ASTM and NGI methods

Method	BKRO2	Taiwan	Cuxhaven	Øysand	Ottawa 20-30	Ottawa 50-70	RS-104
	$\rho_{S,max}$ (g/cm^3)	$\rho_{S,max}$ (g/cm^3)	$\rho_{S,max}$ (g/cm^3)	$\rho_{S,max}$ (g/cm^3)	$\rho_{S,max}$ (g/cm^3)	$\rho_{S,max}$ (g/cm^3)	$\rho_{S,max}$ (g/cm^3)
ASTM	1.926	1.821	1.757	1.761	1.783	1.709	1.885
	1.909	1.803	1.771	1.788	1.770	1.718	1.894
NGI	1.826	1.764	1.790	1.802	1.780	1.711	1.814
	1.828	1.753	1.751	1.773	1.789	1.726	1.810
	1.803	-	1.769	-	-	-	-

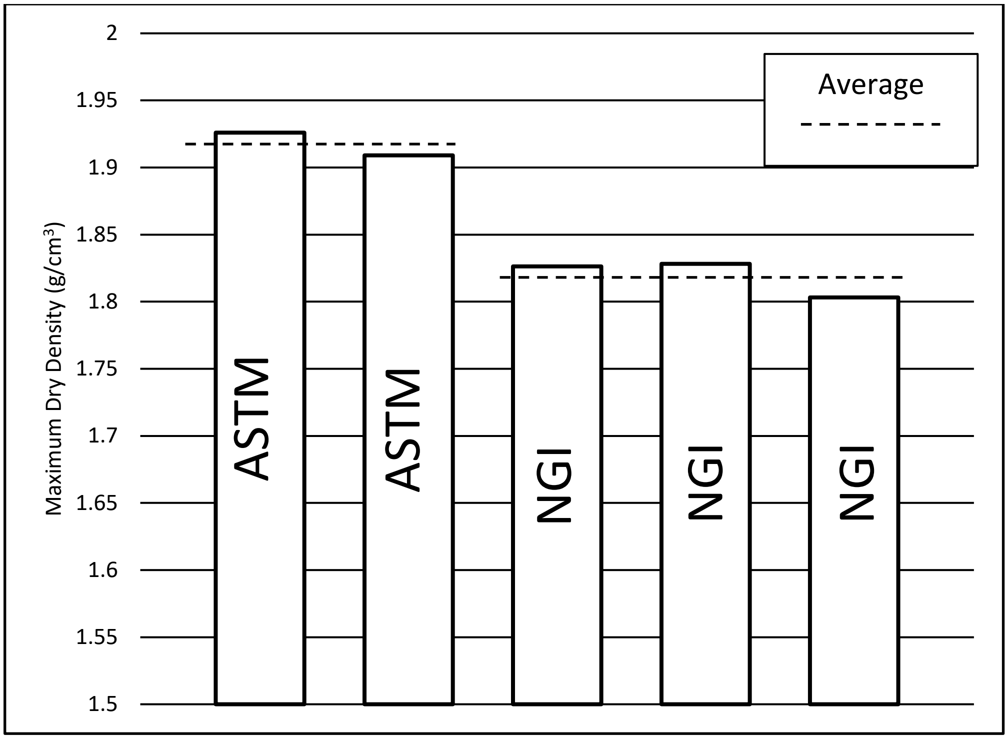

Figure 5. 11 Measured maximum dry densities of BKRO2 based on ASTM method 1B and NGI

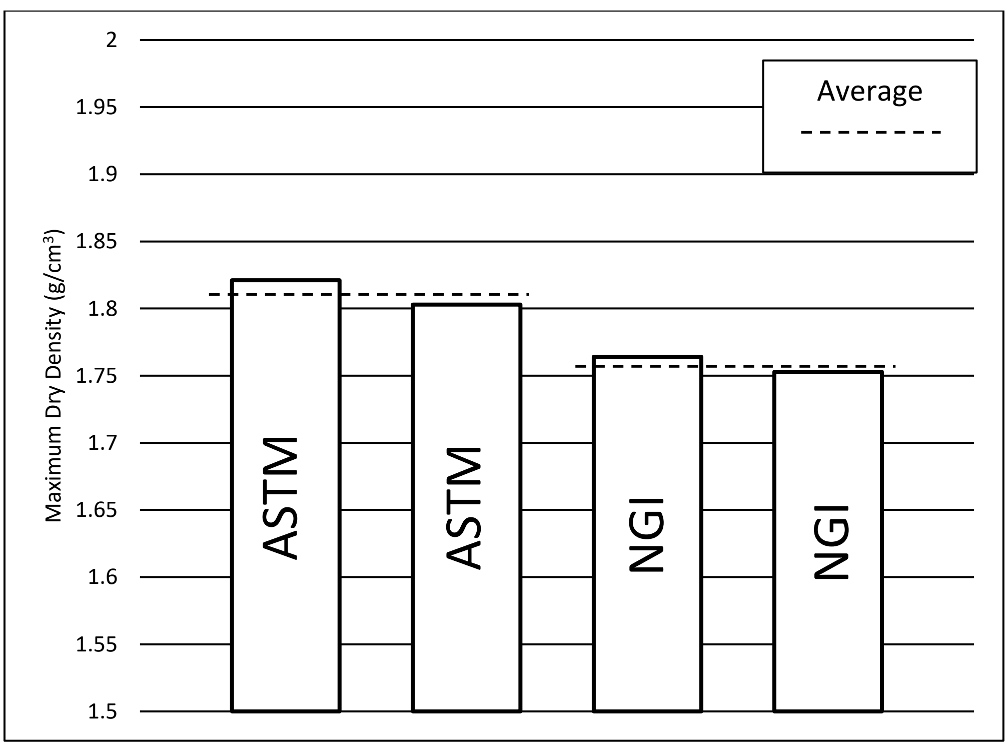

Figure 5. 12 Measured maximum dry densities of Taiwan based on ASTM method 1B and NGI

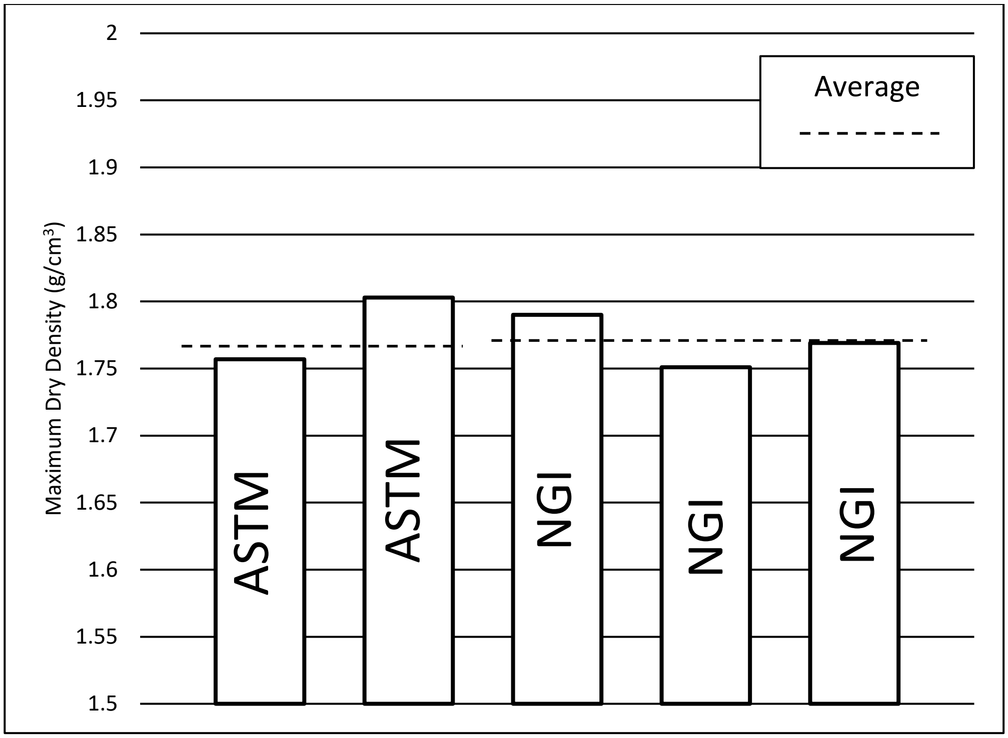

Figure 5. 13 Measured maximum dry densities of Cuxhaven based on ASTM method 1B and NGI

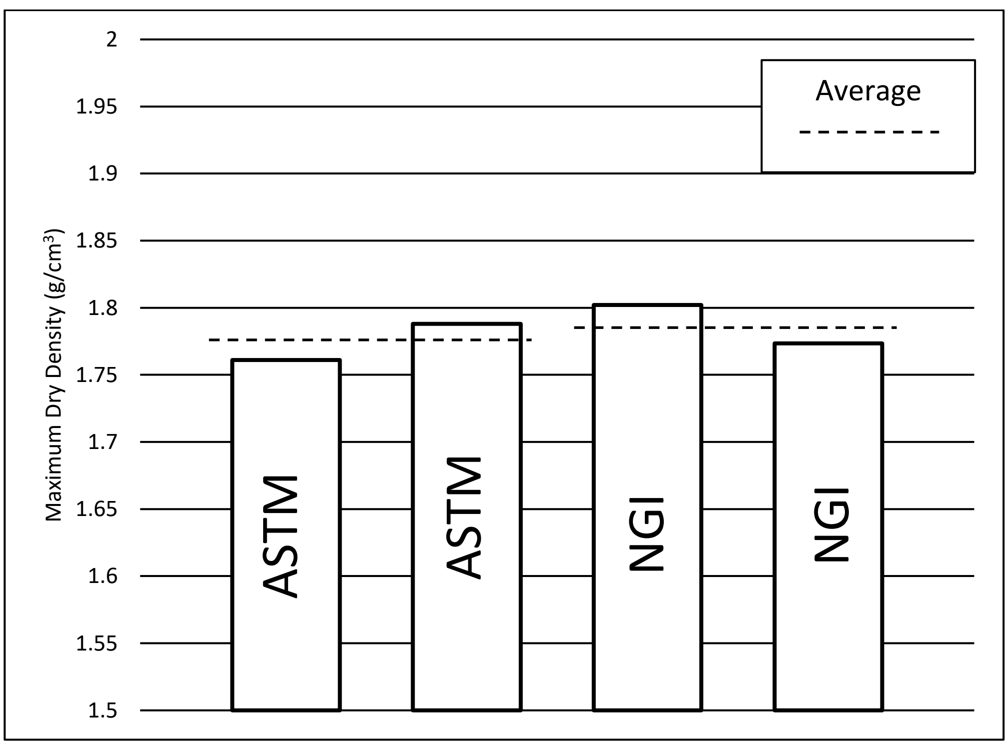

Figure 5. 14 Measured maximum dry densities of Øysand based on ASTM method 1B and NGI

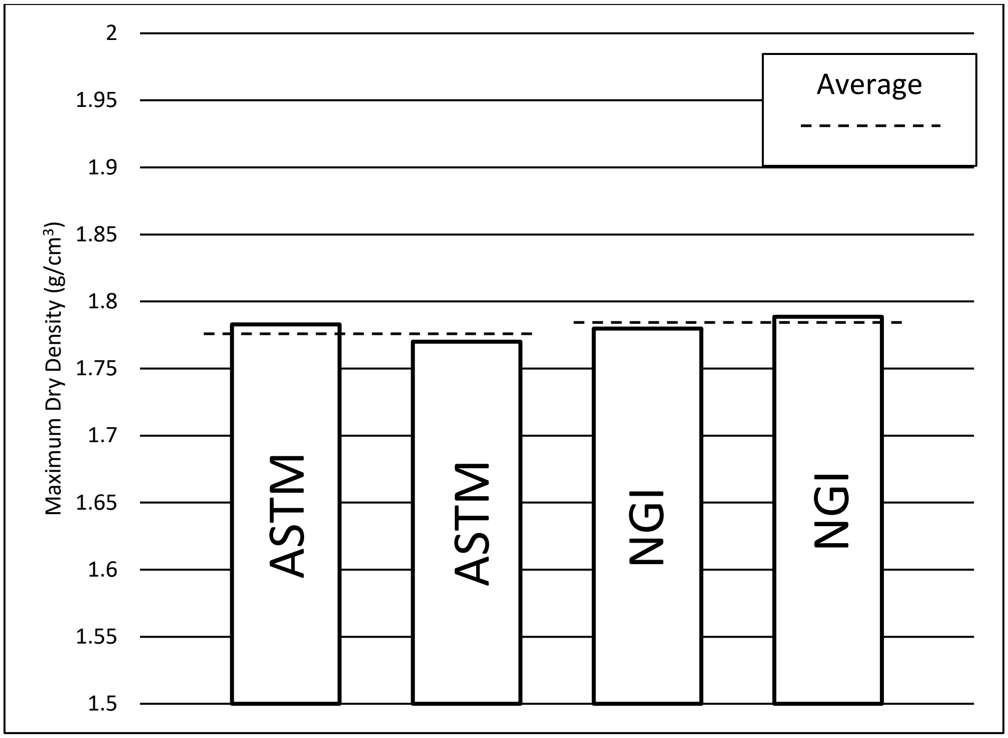

Figure 5. 15 Measured maximum dry densities of Ottawa 20-30 based on ASTM method 1B and NGI

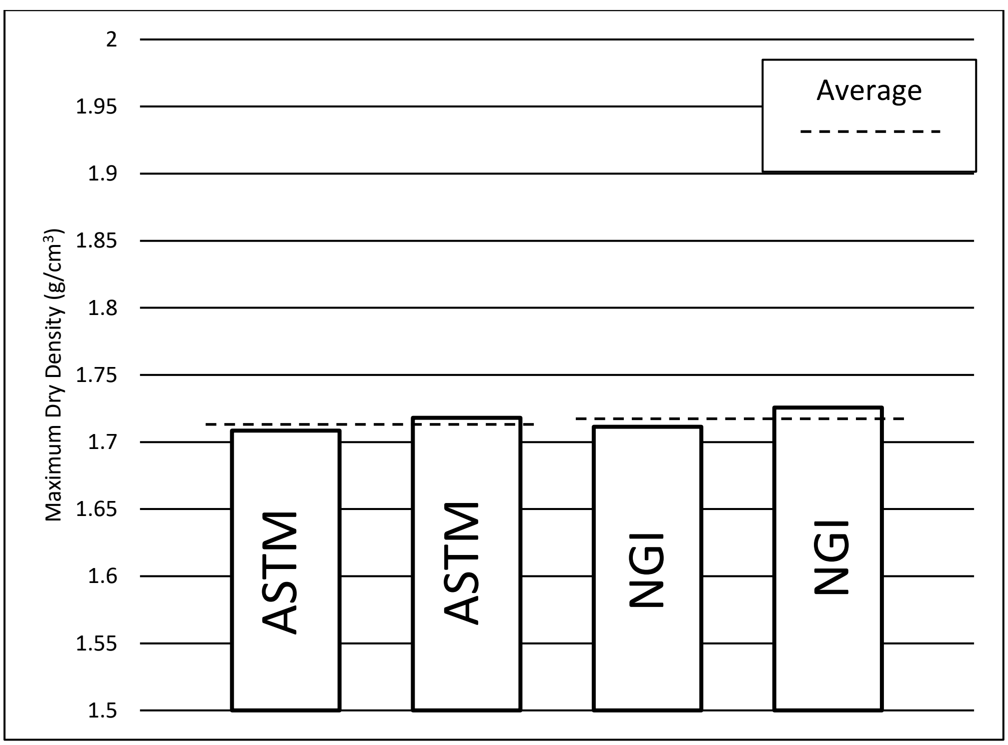

Figure 5. 16 Measured maximum dry densities of Ottawa 50-70 based on ASTM method 1B and NGI

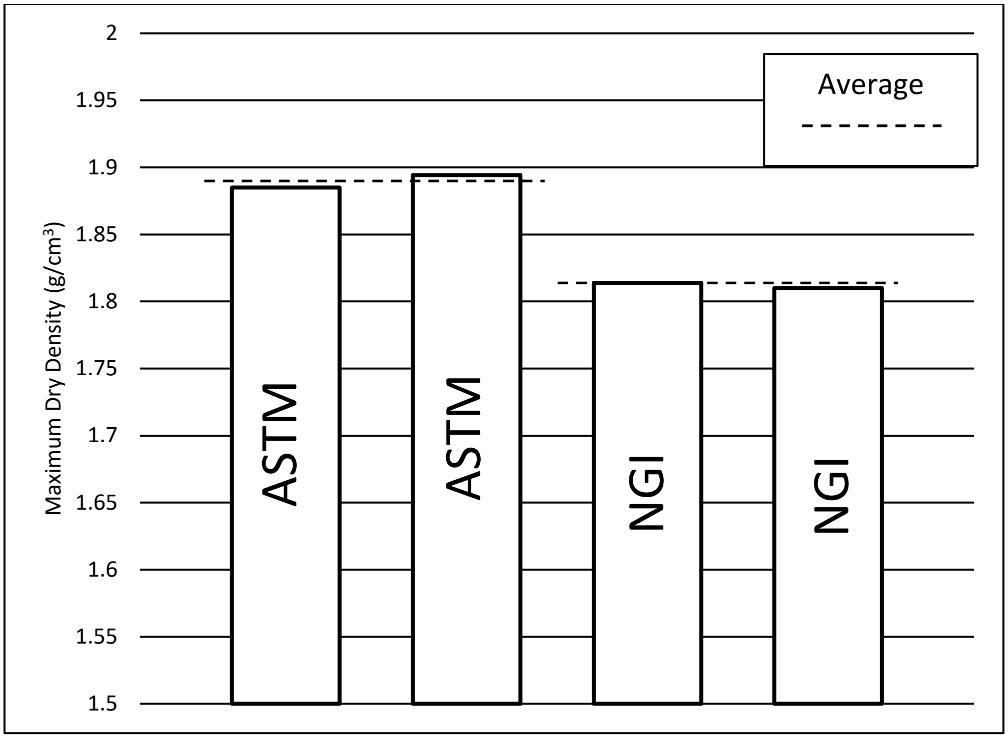

Figure 5.17 Measured maximum dry densities of RS-104 based on ASTM method 1B and NGI

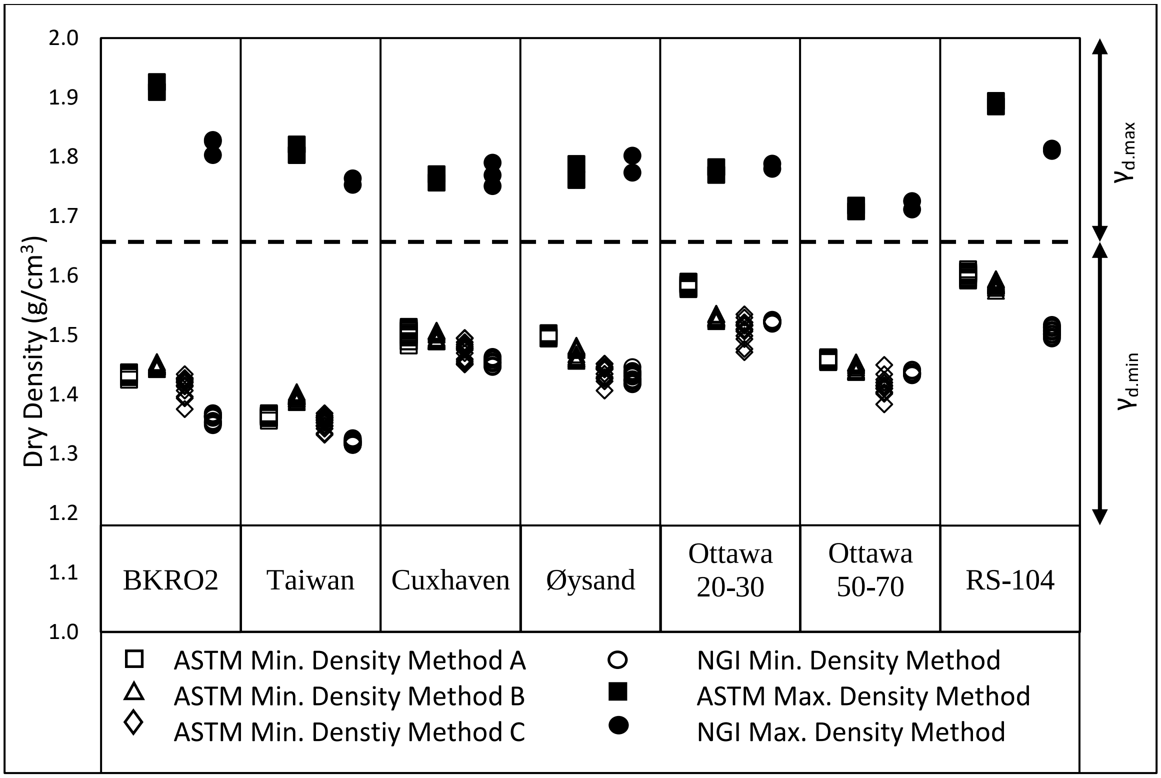

Figure 5.18 Summary of maximum and minimum dry density determinations based on the ASTM and NGI recommendations.

5.4 Grain Size Distribution

Grain size distribution analysis was performed in order to evaluate and identify any potential particle degradation during the ASTM and NGI maximum density testing. The amount of the applied force and corresponding energy are the most significant factor to crush the soil particles in such a way that the whole soil body forms the most compacted condition. The ASTM method applies a vertical stress of 14 kPa versus the NGI method at 7 kPa. Sieve analysis in compliance with ASTM D6913 - 04 recommendation was performed on each soil sample before and after determination of the maximum dry density. Then, both grain distribution curves obtained after the ASTM and NGI testing were plotted in the same chart as the virgin grain distribution curve of the same soil to identify any evidence of particle crushing. Figures 5-19 through 5-25

present the sieve analysis results on all soils. As an overall evaluation, the represented figures show no particle crushing during ASTM and NGI because the grain size distribution curves obtained after these two procedures were nearly identical to the virgin grain size distribution curve of the corresponding soil. The only discrepancy appears in RS-104 (Figure 5.25) between the virgin and after the ASTM 1B method grain size distribution curves. Thus it is possible crushing may have occurred for this sample although no difference is seem in the fines fraction and only some difference for the coarse fraction; it is possible that the subsamples used for the sieves were not the same. The soils tested in this research were largely silica based sands and therefore the finding of little to no crushing is limited to such soils and not for other sands with weaker grains such as carbonate sands.

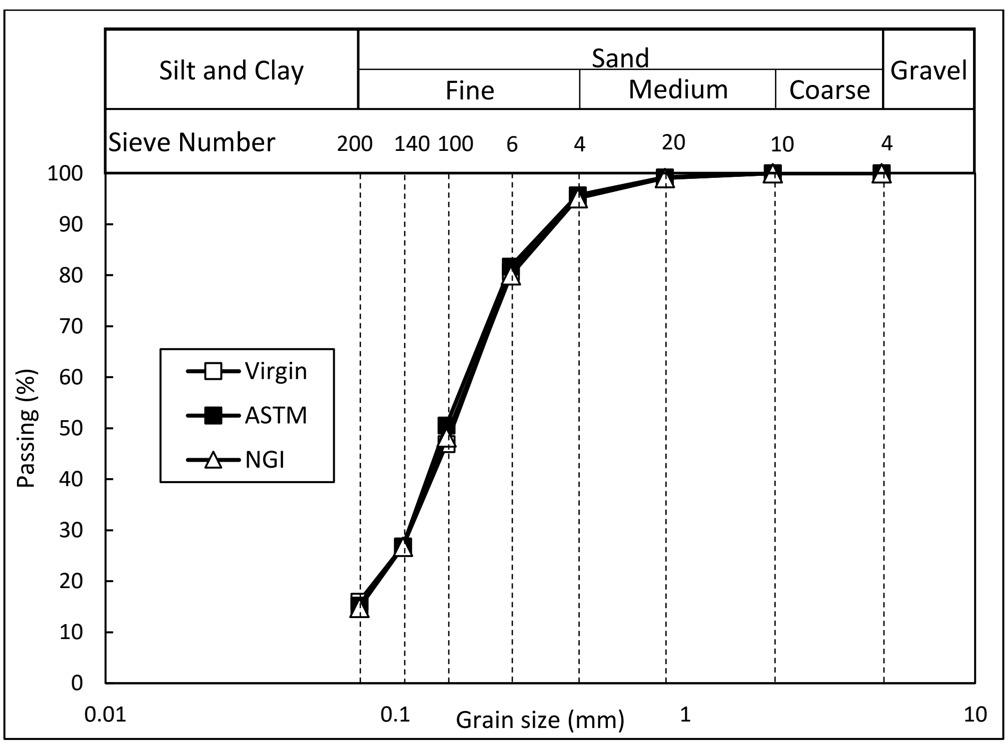

Figure 5.19 Grain size distribution curves prior to and after determination of maximum dry densities of BKRO2

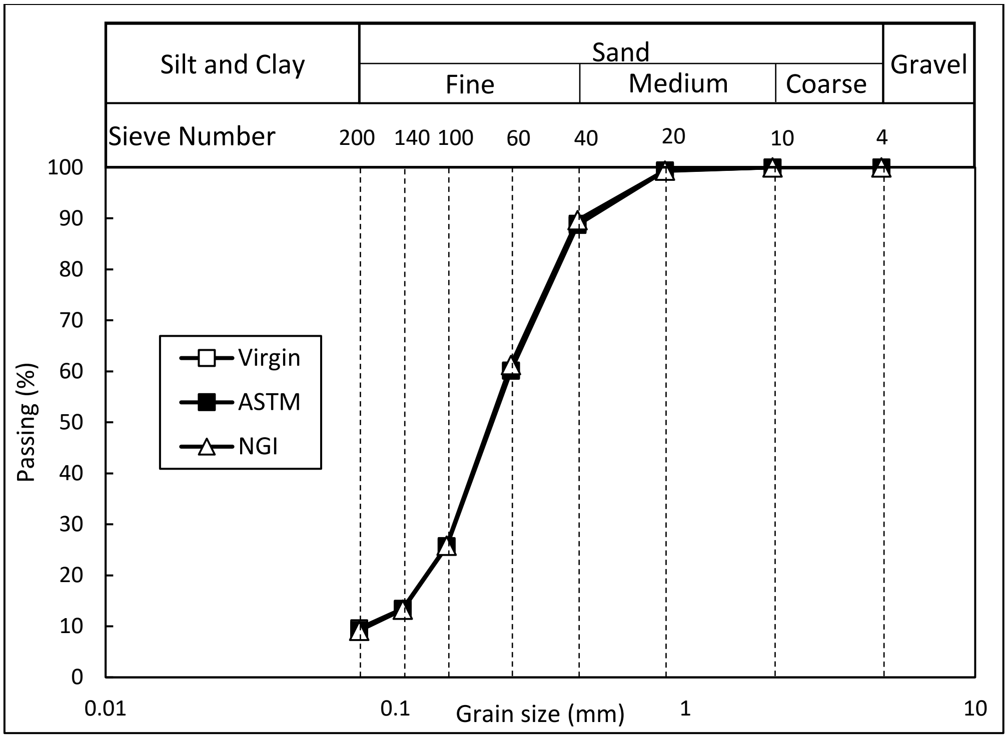

Figure 5.20 Grain size distribution curves prior to and after determination of maximum dry densities of Taiwan

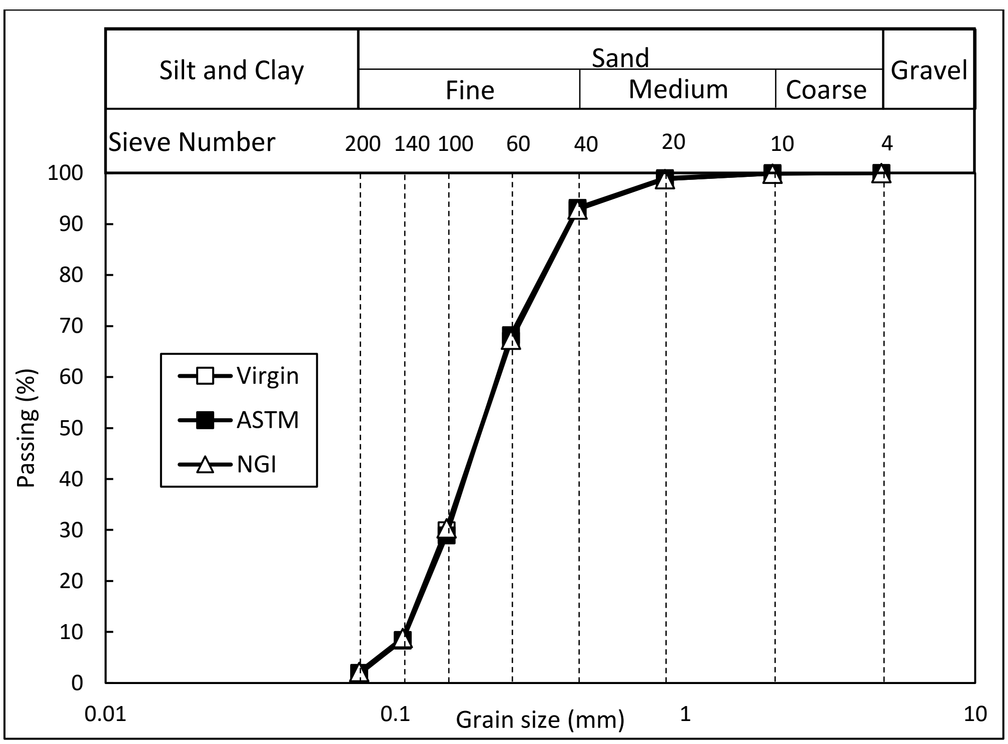

Figure 5.21 Grain size distribution curves prior to and after determination of maximum dry densities of Cuxhaven

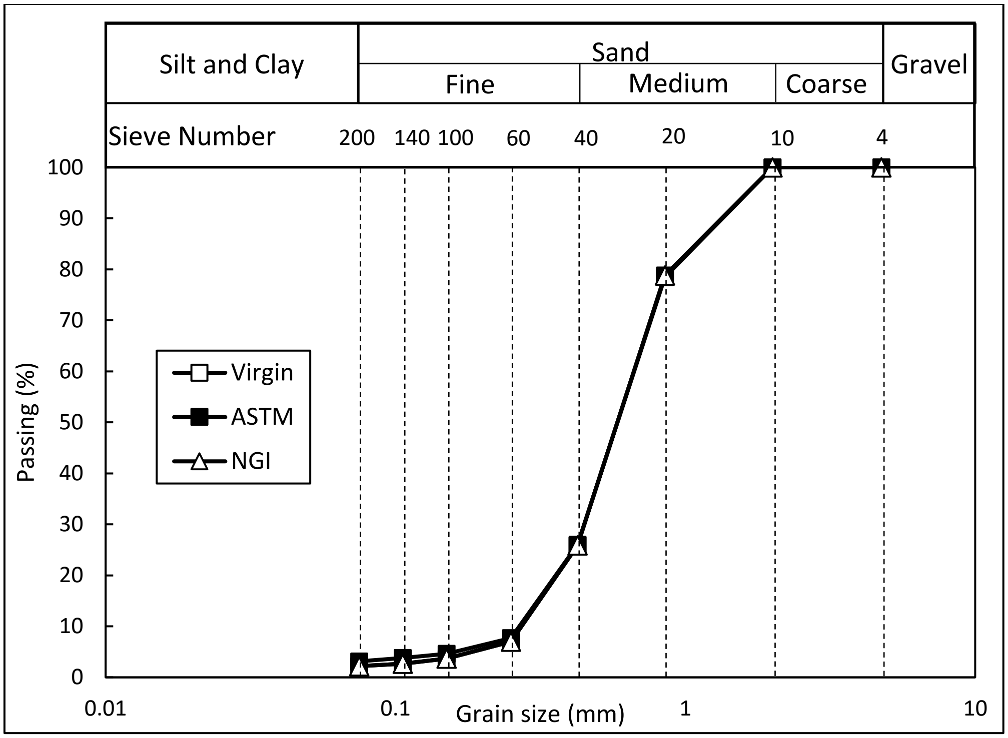

Figure 5.22 Grain size distribution curves prior to and after determination of maximum dry densities of Øysand

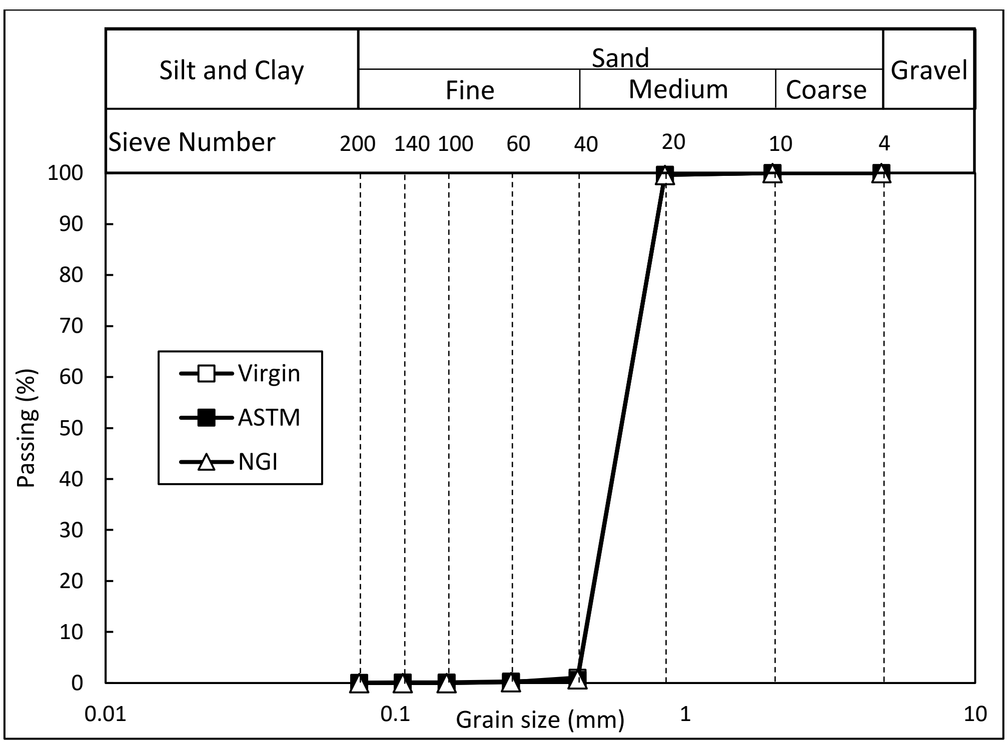

Figure 5.23 Grain size distribution curves prior to and after determination of maximum dry densities of Ottawa 20-30

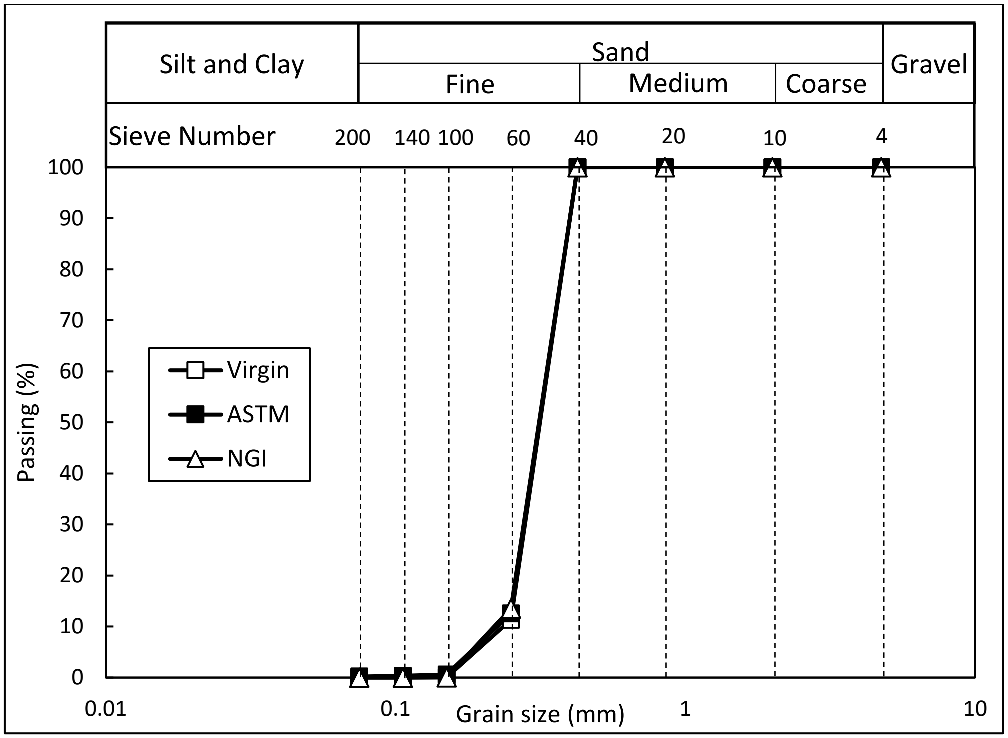

Figure 5.24 Grain size distribution curves prior to and after determination of maximum dry densities of Ottawa 50-70

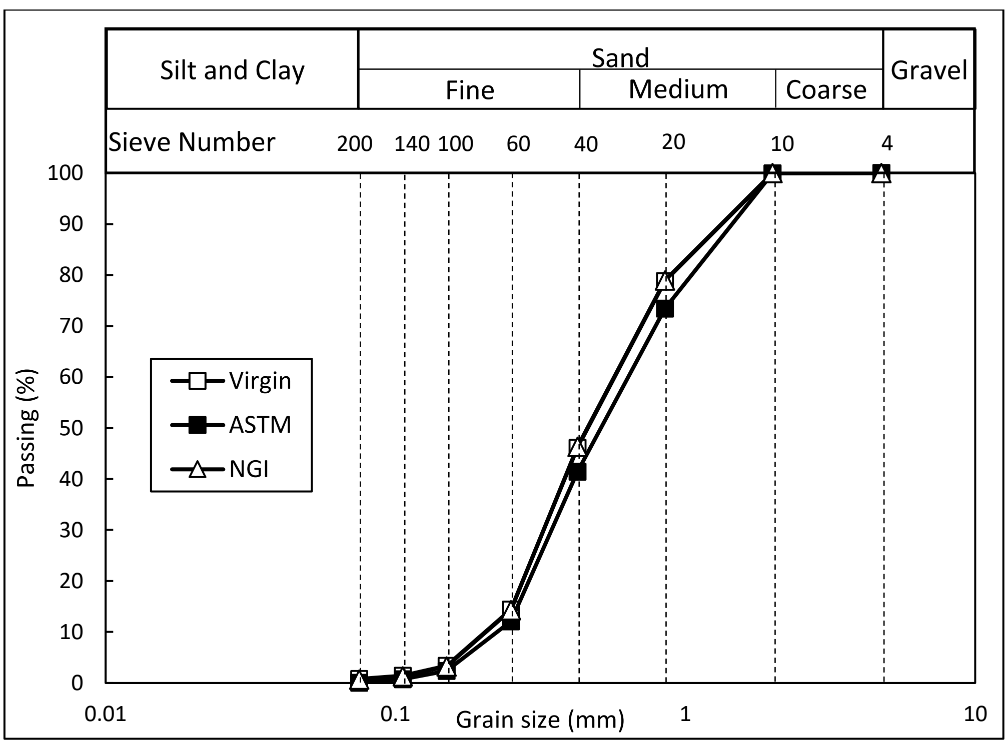

Figure 5.25 Grain size distribution curves prior to and after determination of maximum dry densities of RS-104

5.5 Minimum and maximum void ratios

Figure 5.26 plots a summary of the maximum and minimum void ratios of all the tested soils. It can be seen that the value of the e_{max}-e_{min} for soils with a small fraction and/or zero fine content is less than the soil samples with fine content higher than 9 %, by dry mass.

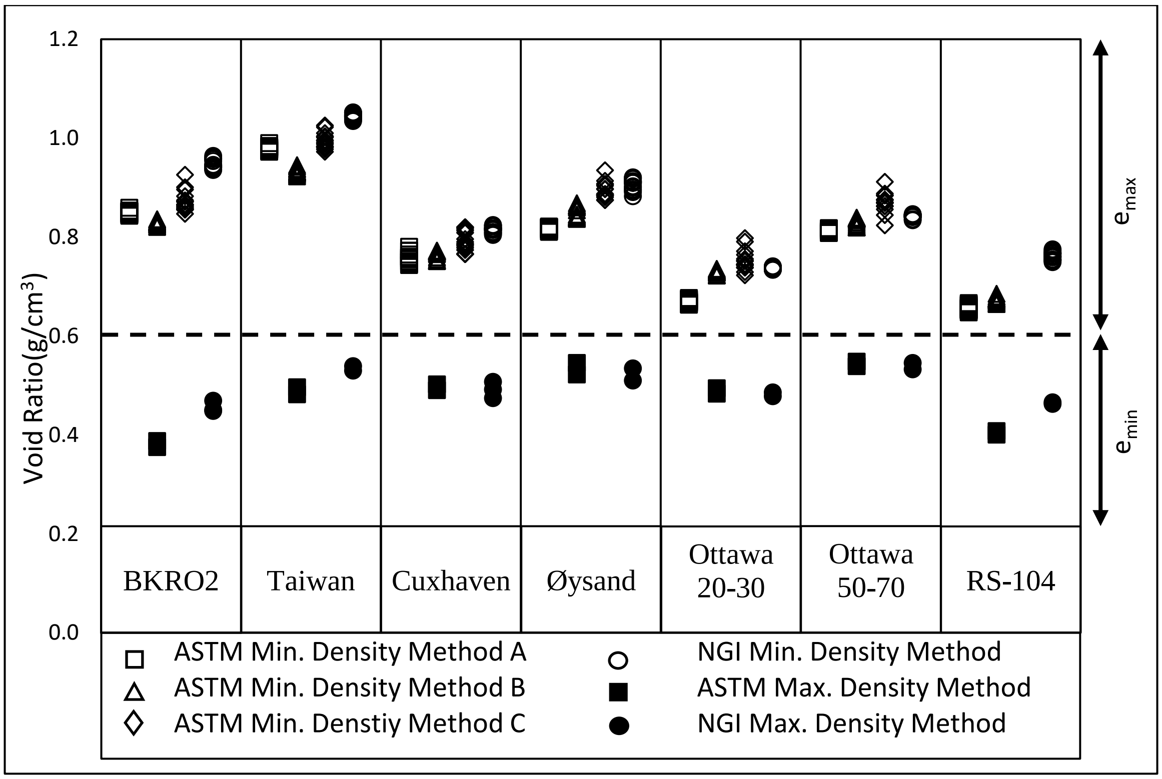

Figure 5.26 Summary of the all calculated maximum and minimum void ratios

CHAPTER 6. SUMMARY AND CONCLUSIONS

6.1 Summary

The in-situ condition of granular soils – in this study sands - represents the deformation and strength characteristics of the soil, which can be described by obtaining the effective stress state and relative density. There are several empirical and laboratory methods to facilitate the determination or estimation of the relative density as an indication of the existing soil state. Determining the maximum and minimum dry densities of a soil identifies the upper and lower limits of a relative density for a given soil and test method. This study has examined the new-established method by the Norwegian Geotechnical Institute (NGI) to measure the limit densities in the laboratory. The NGI approach for the limit densities was performed on seven different fine to medium grain size cohesionless sands, and the outcomes compared with the measured limit dry densities using the American Society for Testing and Materials (ASTM) methods.

6.2 Interpretation and Conclusions

The first part of this study evaluated the minimum dry densities determined using the ASTM and NGI recommendations and provided a means for comparing the four methods. Overall, the NGI method produced the lowest minimum dry density values compared to the three ASTM methods. This can happen because of the difference in dimensions of the NGI and ASTM's mold. It is possible that pouring soil particles through a funnel into the NGI mold with diameter equal to 70.3 mm may have resulted in some arching preventing free movement of the particles. Conversely, the diameter of

the ASTM's mold is 152.6 mm, which is 2.2 times larger than the NGI mold, gives an enough space to the soil particles to move freely without any confining resistance and shape a denser configuration.

On the other hand, ASTM's method A, which also uses a funnel, generates the highest minimum density values for soil with fine content less than 2 % to zero. This may cause due to the fact that the fine content of a soil sample partially participates in formation of a larger void ratio. Figure 6-1 shows a schematic sketch of a soil particles deposition during the ASTM's method A indicating how fine particles less than 0.075 mm can locate between larger particles, and then, they partially increase the final volume of the specimen within the mold. In addition to that, depositing sand particles through a funnel conveys much less energy to the already deposited sand compare to the ASTM method B. On the contrary, for Taiwan and BKRO2 with high fines content of about 9 % and 16 %, respectively, method B generates the highest minimum dry density values due to the function of the fine particle in filling the void spaces between the larger particles.

The statistical evaluation of all results revealed that the minimum dry densities obtained using NGI recommendations by single operator has a very low coefficient of variation (COV) value and they are reproducible. The consistent results and the low COV values obtained from another operator for BKRO2 and Taiwan showed that the recommendations outlined by the NGI to measure the minimum dry density were not affected by two different operators in this case. On the other hand, ASTM method C produced results with higher COV due to episodic charging particles and interparticle forces which resulted in variations in the volume of the deposited sand.

Considering time to complete a single test from commencing a testing procedure to the end of calculation, the NGI and ASTM method A, B and C can be done in approximately 1.5, 4, 2 and 4 minutes, respectively. The amount of the required soil for each set of the NGI tests is less than 100 grams, which is considerably less than 4000 grams sample using in the ASTM A and B methods. This is an advantage of the NGI method in cases where collecting samples is timing consuming and costly such as for example in offshore projects. However, despite requiring a larger amount of soil, the ASTM A and B method are applicable for a very wide range of soil particle size including sand and gravel content.

It is important to mention the need to minimize any moisture absorption by the soil particles prior to testing. Air moisture can flocculate soil particles and form small clumps which result in generating a higher minimum dry density value. To prevent that, an oven-dry soil sample should be cooled down to the temperature room in a desiccator and/or preserve the sample in a bowl wrapped with a plastic wrap.

The second part of this study evaluated and compared outcomes of the NGI and ASTM maximum dry density approaches. The ASTM method generated the highest maximum dry density values over soils with higher fine content than the NGI method. This may happen because in the ASTM testing higher energy is transmitted to the soil body than NGI method (14 kPa confining stress versus 7 kPa), and also, soils with higher fine content like BKRO2 and Taiwan tend to form denser condition due to the lubrication effect of the fine content. Additionally, the mold used in the ASTM method has a volume equal to 2849 cm^3 and height to diameter ratio of 1.01 compare to the NGI mold with a volume equal to 789 cm^3 and height to diameter ratio of 2.89. The large ASTM mold

provides an opportunity for soil particles to be relocated under higher energy in order to shape the densest condition compare to the NGI mold.

As a matter of time, the ASTM method 1B takes half an hour longer than the NGI maximum density method. This additional time is because the ASTM recommends about half an hour to let soil particle become saturated. The required quantity of soil needed for the NGI method is 1000 grams which can eliminate any shortage in sampling compare to the 4000 grams required sample based on the ASTM recommendations. However, ASTM approach is applicable to a larger range of soil particle size than the NGI method. ASTM method can be applied to soils with particles less than 75 mm and inclusion of fine content up to 15 %, by dry mass, and NGI is suitable for soils with particles less than 2 mm.

To determine any possible degradation of the soil particles, a grain size distribution performed after determination of each maximum density test based on the ASTM D6913–04 recommendations. The wet sieve analysis results indicated no evidence of soil particles degradation after the ASTM and NGI maximum density testing procedure, and the grain size distribution curves remain the same as the virgin soil curves.

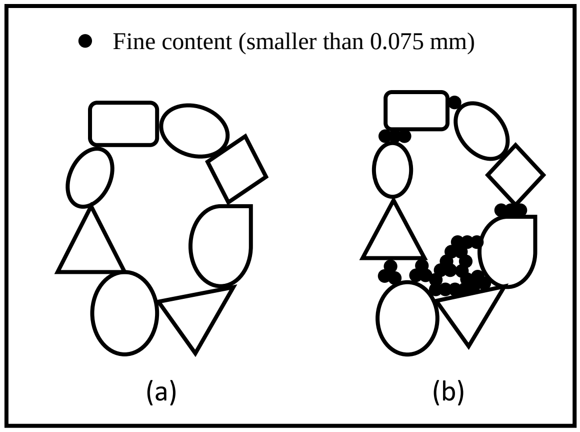

Figure 6.1 An example of soil particles deposition during the ASTM's method A (the left sketch is soil without fine content and the right sketch is with fine content).

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