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# DEPARTMENT OF ECONOMICS

## Working Paper

**Relative advantage, queue jumping, and welfare  
maximizing wealth distribution**

by

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**UNIVERSITY OF MASSACHUSETTS  
AMHERST**

# Relative advantage, queue jumping, and welfare maximizing wealth distribution

## Abstract

*Suppose individuals get utilities from the total amount of wealth they hold and from their wealth relative to those immediately below them. This paper studies the distribution of wealth that maximizes an additive welfare function made up of these utilities. It interprets wealth distribution in a control theory framework to show that the welfare maximizing distribution may have unexpected properties. In some circumstances it requires that inequality be maximized at the poorest and richest ends of the distribution. In other circumstances it requires that all wealth be given to a single individual.*

*Key words: wealth distribution, positional goods, status, inequality, relative advantage.*

*JEL classification: C61, D60*

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# Relative advantage, queue jumping, and welfare maximizing wealth distribution

## 1 Introduction

Social scientists often study the question of the optimal distribution of wealth in terms of an aggregate welfare function in which individuals derive satisfaction from their own holdings of wealth independently of the holdings of others. There is an increasing body of literature, particularly that concerned with happiness and feelings of well being, that argues it is a mistake to construct models on the assumption that the utilities, or satisfaction, of individuals are completely independent. It has been argued, for example, that the satisfaction individuals derive from things like wealth, income and what they purchase may depend on relative rather than absolute holdings. This paper explores some implications of this argument.

The argument that satisfaction is relative has been made by Marx and Keynes among others and was developed at length by Veblen [7] and, more recently by Hirsch [3]. Frey and Stutzer find, for example, that there is little correlation between happiness and increases in absolute income and that 'it is not the absolute level of income that matters most, but rather one's position relative to other people' ([2], 9). Easterling [1] argues that happiness increases with relative income but not with absolute income. The same argument has been made for the satisfaction derived from the consumption of positional goods like housing, education and luxury cars. Since happiness is relative to the holdings of others it is also possible, of course, that a poorer person could get greater total happiness than a richer person, in some circumstances. This seems to be one of the points that the 'wealth does not buy happiness' argument is attempting to make. What happens if we try to build some notion of relative satisfaction into our wealth distribution models? What sort of distribution would maximize welfare? In what way would the well known distribution with independent utilities change?

The first question that might be asked about the argument that the satisfaction from wealth is relative to the holdings of others is, relative to whose holdings? Frey and Stutzer give evidence to show that the reference group is people with the same labour market characteristics and that the better this group is doing relatively to an individual, the less happy this individual becomes ([2], p.9). One interesting possible explanation is that, although people may look up and feel envious, they also look down and wish to retain their traditional privileges and position. Individuals may care more about how much better they are doing relative to those immediately below than they care how they are doing relative to those above. There are a number of plausible reasons for this. Hirsch, for example, sees the problem of relative satisfaction in terms of the conditions of use. The greater the number of people who have access to some

goods the less the satisfaction there is from consuming them ([3], p.2). He says that it is the ability of an individual to improve its position by moving to a higher place in the queue that counts ([3], p.7). This is persuasive. It may also be the case, however, that individuals are even more worried about retaining the advantage they already have in access to positional goods like housing, travel and prestigious items of consumption. This advantage may be defined, or at least most felt, in comparison to their immediate reference group. That is, individuals may want to feel that their status and advantage, compared to their group is secure. As an extension of this, they may get more pain from slipping down the scale relative to those who have been below than they get from an improvement in the position of those who have been traditionally better-off. Unlike envy, which depends on wanting what others have, this is a desire not to lose what one already has, in a relative sense.<sup>1</sup> In other words if not being able to move up in the queue is bad, having someone push in front may be even worse.

In order to capture the notion that individuals are concerned with maintaining status and offsetting insecurity in this manner it will be assumed that the difference between the wealth they hold relative to those below them is more important than the amount they hold relative to those above them. This means that for a fixed amount of wealth the utility of an individual will decrease as the distance between its wealth and those immediately below decreases. This might be thought of as a neighbourhood affect. As more individuals move into the wealth neighbourhood from below the greater becomes the competition for scarce goods. This is not exactly the same as someone pushing in front in the queue but the outcome will be similar. Its utility will also decrease as those immediately above move further away. It is assumed, however that the loss of security from having its neighbourhood crowded from below is greater than the loss of aspirations resulting from an increase in the relative wealth of those who are better off. In this respect concerns about security are more important than aspirations.

It should be noted that these effects only occur in the immediate neighbourhood of an individual's position. It is not assumed that the individuals have a preference for inequality or wish to be globally better off than everyone else. What they are driven by is a sense of seeking local security.<sup>2</sup>

The specific purpose of this paper is to attempt to understand some of the consequences of this neighbourhood effect for the distribution of wealth. In order to do this we explore the distribution that would be chosen by a welfare maximizing despot.

We want to study this question in the most straightforward manner possible in order to highlight the effect of these assumptions about utilities on distribution. To this end we assume that the welfare function is additive and that the only constraint is that it should respect a smoothness condition that prevents arbitrary jumps between individuals.

Among the most interesting findings of the study is that a desire for relative wealth may require a distribution that maximizes inequality at the bottom and the top end of wealth holdings. Although an inegalitarian outcome is not a surprise, this particular pattern is not what might be expected. It is also found that, under less plausible but also fairly mild assumptions, the smoothness condition cannot be met at all and the optimal distribution requires all wealth to be given to one individual.

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<sup>1</sup>See [6] p.239-46 for a discussion of envy. Not wanting to lose a position may be a less objectionable normative claim than one based on envy.

<sup>2</sup>This idea is similar in spirit to Schelling's work on micro-motives [4].

We set out the paper as follows. The model is developed in §2. The main theory is presented in §3. In §4 we briefly discuss the results.

## 2 The distribution problem

Suppose a benevolent despot wishes to distribute a fixed amount of wealth among a large number of individuals in order to maximize an additive welfare function. The distribution is anonymous in the sense that the despot does not discriminate between individuals. The despot would also like this distribution to satisfy some ideal smoothness condition that limits the rate of change in wealth holding across individuals. The motivation for this condition depends on intuitions we might have about treatment of individuals. These are that abrupt changes between individuals seem to be arbitrary, in some sense. If an unequal distribution is required it should, ideally, respect individuals by not making a jump between those that are adjacent in the distribution. Such a jump would seem to introduce an extreme inegalitarianism that, somehow, treats similar cases in a different manner. This smoothness condition is specified more precisely below. It is an ideal constraint in the sense that it will be met if possible, but it can be violated if this is necessary to make an optimal distribution.

The utility function for each individual depends on the amount of wealth it holds and the distance from those above and below in the immediate neighbourhood. It is assumed that utility increases with an increase in the distance between the wealth holdings of an individual and those in the neighbourhood below and also with a decrease in the distance from those in the neighbourhood above. To get some intuition on this we might imagine, for example, that there is a pre-existing distribution and that utility will decrease for individuals if those immediately below begin to catch up.

The way in which utility changes as differences in wealth between an individual and those in a near neighbourhood change might be thought of in the context of a large number of individuals in terms of the rate of change of wealth in a neighbourhood. This means that the faster the rate of change from those below the higher the utility. Peter Skott has pointed out that this means a poorer person could get more utility than a richer person in some circumstances.<sup>3</sup> A similar situation occurs in any model that constructs a utility function with wealth and relative position as separate arguments. Although this could possibly be avoided by a more complicated model it is taken here as reflecting the logic of the position, mentioned in the previous section, that wealth is not the only determinant of happiness. If this outcome were to be seen as unreasonable the argument would have to be modified to say that a poorer person could never be happier than a richer person. This seems to be inconsistent with what we are trying to capture.

The model is constructed by representing each individual as a point  $\frac{i}{n+1}$  in  $(0, 1)$ , where  $i \in \{1, \dots, n\}$ . Individuals are placed in sequence in increasing order of wealth and the wealth given to the individual at point  $\frac{i}{n+1}$  is  $\tilde{x}(\frac{i}{n+1})$ . Let  $x(t)$  be a piecewise-continuously differentiable function interpolating  $\tilde{x}(\frac{i}{n+1})$ .

The utility that the individual at  $t = \frac{i}{n+1}$  gets from wealth, without reference to relative holdings, is

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<sup>3</sup>Private correspondence with one of the authors.

$f(x(t))$  where  $f : \mathbb{R} \rightarrow \mathbb{R}$  is twice continuously differentiable. Assume that  $\frac{df}{dx} > 0$  everywhere and either  $\frac{d^2f}{dx^2} < 0$  everywhere, or  $\frac{d^2f}{dx^2} = 0$  everywhere.

The utility that the individual at  $t = \frac{i}{n+1}$  gets from the rate at which wealth holding changes, relative to those below, is written  $\delta(x(t)) \equiv (n+1)(x(\frac{i}{n+1}) - x(\frac{i-1}{n+1}))$ . In order to account for the way in which utilities change with different rates of change in wealth, the utility function is written  $\bar{b}(\delta(x(t)))^a$  where  $a, \bar{b} > 0$ . The exponent  $a$  determines the marginal value to the individual at  $t$  of changes in the rate of increase of wealth. For  $a < 1$  there are declining marginal returns and for  $a > 1$  there are increasing marginal returns. There will be an analogous expression for the change in utility as the rate of change in wealth relative to those above alters. This is written  $\bar{c}(\delta(x(t)))^a$

Since the despot is maximizing an additive welfare function, we can treat the problem as that of maximizing the Riemann sum of  $f(x(t)) + \bar{b}(\delta(x(t)))^a - \bar{c}(\delta(x(t)))^a$ , evaluated at  $t = \frac{i}{n+1}$ , over  $[0, 1]$  with a partition  $\kappa = \kappa_1, \dots, \kappa_n$  where  $\kappa_i = (\frac{2i-1}{2(n+1)}, \frac{2i+1}{2(n+1)})$ . This sum is written  $R_0^1 = \sum_{i=1}^{i=n} (f(x(\frac{i}{n+1})) + b(\delta(x(\frac{i}{n+1})))^a) \kappa_i$  where  $b = \bar{b} - \bar{c}$  and  $b > 0$ . For  $n$  sufficiently large, we can make  $\sup | \int_0^1 (f(x(t)) + b(\frac{dx(t)}{dt})^a) dt - R_0^1 |$  as small as we wish.

Then the despot's problem is that of distributing wealth to maximize

$$J = \int_0^1 (f(x(t)) + b\dot{x}(t)^a) dt \quad (1)$$

subject to

$$\int_0^1 x(t) dt = 1 \quad (2)$$

where (2) captures the assumption that wealth is fixed.

In order to meet the ideal smoothness constraint on the level of permissible inequality the rate of change in wealth distribution is restricted to give

$$0 \leq \dot{x}(t) \leq m \quad (3)$$

for some  $m > 0$  and all  $t$ . The initial level of wealth  $x(0)$  is to be determined. With constraints (2) and (3) given we want to focus on the properties of the distribution and do not prohibit negative wealth.

Note that the properties of the optimum distribution are not at all obvious. It might be instructive to try and guess the nature of the distribution on the basis of (1), (2), and (3). Would the problem actually have a solution? Would it be a convex or concave curve? Would it increase the wealth for the poorest individual as quickly as possible and then give an egalitarian distribution? Would it leave a group of individuals at zero and accelerate wealth for the rest as quickly as possible?

### 3 The main theorem and optimal distribution.

The main results are set out in Theorem 1 for different possibilities on the utility function for wealth and the value of the exponent  $a$ . Of these the most interesting is for the case where marginal returns

to wealth are declining and  $a < 1$ . When  $a < 1$  the utility individuals get from the rate of change in wealth in their neighbourhood is declining and this means that as the distance between their wealth and those immediately below increases the utility they get from this becomes less. This is consistent with the interpretation of this rate of change as a proxy for security. In this case wealth is either distributed at a maximum rate across all individuals, or for some values of  $a$  and  $b$ , wealth is distributed to maximize inequality at the bottom and the top end of the wealth distribution. This result is something of a surprise and difficult to explain intuitively. It is illustrated in fig. 1.

In the case where  $a > 1$  and utilities are increasing as the rate of change in the neighbourhood increases, the despot either distributes at the maximum rate across all individuals or there is no solution that maximizes welfare under the ideal constraint. In this case the despot distributes all wealth to one individual. This is something of a surprise since the marginal utility of wealth is declining. What is happening here is that small neighbourhood desires whereby everyone wants to be a little better off than those below produce an outcome where everybody is no better off than anyone else, with the exception of a single individual. In this sense the desire to be a little better off produces a form of distribution paradox.

**Theorem 1** *The optimal rate of distribution is:*

[a] for  $\frac{d^2 f}{dx^2} = 0$  everywhere,  $\dot{x}(t) = m$  for all  $t \in [0, 1]$ ;

[b] for  $\frac{d^2 f}{dx^2} < 0$  everywhere and  $0 < a < 1$ , there exists an  $\alpha, \beta$  with  $0 < \alpha \leq \beta < 1$  such that  $\dot{x}(t) = m$  for all  $t$  in the intervals  $[0, \alpha) \cup (\beta, 1]$  and  $\dot{x}(t) < m$  for all  $t$  in the interval  $(\alpha, \beta)$ ;

[c] for  $\frac{d^2 f}{dx^2} < 0$  everywhere and  $a \geq 1$  either,  $\dot{x}(t) = m$  for all  $t$ , or  $\dot{x}(t) = 0$ , for all  $t \in [0, 1]$ .

In part [c] the last result means that there is no optimal distribution that satisfies (3) and the despot can do no better than give all wealth to one individual.



Figure 1. Example of optimal wealth distribution for  $a < 1$ .

**Proof.** The proof uses the Pontryagin maximum principle ([5] p.103 - 33) and in order to get the problem in an appropriate form begin by defining

$$\dot{x}(t) = u(t)$$

where  $u(t)$  is to be chosen to solve the problem (1) - (3).  $u(t)$  is required to be measurable, for example piecewise continuous, almost everywhere. To save notation the condition almost everywhere is taken as given in what follows. Now define a new variable  $w(t)$  such that

$$\dot{w}(t) = x(t)$$

with  $w(0) = 0$  and  $w(1) = 1$  in order to satisfy (2).



From the Pontryagin principle, the necessary conditions for a measurable function  $u^*$  that solves (1) - (3) to exist are that  $u = u^*$  maximizes the Hamiltonian

$$H = f(x(t)) + \frac{u(t)^a}{a} + \lambda(t)u(t) + \zeta(t)x(t) \quad (4)$$

where  $b$  has been set at  $\frac{1}{a}$  to simplify the notation without any loss of generality and  $\lambda(t)$  and  $\zeta(t)$ , are continuous functions of  $t$ . In addition

$$\dot{\lambda}(t) = -\frac{df}{dx} \Big|_{x^*(t)} - \zeta(t) \quad (5)$$

$$\dot{\zeta}(t) = 0 \quad (6)$$

where  $x^*(t)$  is the optimal path. It is also necessary that the transversality conditions  $\lambda(0) = 0$ ,  $\lambda(1) = 0$  be satisfied. Note that  $\zeta$  is constant for all  $t$ . Differentiating  $\dot{\lambda}(t)$  gives

$$\ddot{\lambda}(t) = -\frac{d^2f}{dx^2} \Big|_{x^*(t)} \dot{x}^*(t) \geq 0 \quad (7)$$

**Remarks.**

1. It follows from (7) and the transversality conditions that  $\lambda(t) \leq 0$  always.
2. Since  $f$  is continuous and  $\zeta$  is continuous  $\dot{\lambda}$  is continuous.

It is now necessary to consider the possibilities for  $f(x)$  and different values of  $a$ . In what follows the constraint (3) is assumed in force.

**Proof of [a].**  $\frac{d^2f}{dx^2}$  is identically 0.

In this case the Hamiltonian gives  $\frac{\partial H}{\partial u} > 0$  for all  $u(t) > 0$  and all  $a > 0$  since  $\lambda(t) = 0$  for all  $t$  from (7). Hence  $u^*(t) = m$  for all  $a > 0$  and all  $t \in [0, 1]$ .

**Proof of [b].**  $\frac{d^2f}{dx^2} < 0$  for all  $x$  and  $0 < a < 1$ .

In this case  $\frac{\partial H}{\partial u^2} < 0$ . Hence the necessary and sufficient condition for  $u(t)$  to maximize (4) is

$$u^*(t) = \min\{m, (-\lambda(t))^{1/(a-1)}\} \quad (8)$$

Note that  $(-\lambda(t))^{1/(a-1)} \rightarrow \infty$  as  $\lambda(t) \rightarrow 0$ . From the transversality conditions and the continuity of  $\dot{\lambda}(t)$  we have  $(-\lambda(t))^{1/(a-1)} > m$  in intervals  $[0, \alpha)$  and  $(\beta, 1]$  for some  $0 < \alpha \leq \beta < 1$  with  $\alpha$  sufficiently small and  $\beta$  sufficiently near 1. This is illustrated in fig. 2. Suppose that  $T \equiv \{t : (-\lambda(t))^{1/(a-1)} < m\} \neq \emptyset$  and define  $\alpha \equiv \inf T$  and  $\beta \equiv \sup T$ . We show that  $u^*(t) < m$  for  $t \in (\alpha, \beta)$  by assuming that  $u(t) = m$  for some  $t = t_k \in (\alpha, \beta)$  and arguing by contradiction. Since  $u(t) > 0$  for all  $t$ ,  $\lambda(t)$  has a single turning point by (7), and this means that  $\sup T \leq t_k$ . This contradicts the definition of  $\beta$ . This means that either:

(i).  $u^*(t) = m$  for all  $t \in [0, 1]$

or

(ii).  $u^*(t) = m$  for all  $t \in [0, c) \cup (d, 1]$  and  $u^*(t) < m$  for  $t \in (\alpha, \beta)$ .

**Proof of [c].**  $\frac{d^2f}{dx^2} < 0$  for all  $x$  and  $a \geq 1$ .

The term to be maximized in (4) can be written

$$u(t)\left(\frac{u(t)^{a-1}}{a} + \lambda(t)\right)$$

and, since  $\frac{\partial^2 H}{\partial u^2} \geq 0$ , the necessary and sufficient condition for  $u = u^*$  to maximize (4) is

$$u^*(t) = \begin{cases} m & \text{for } \lambda(t) > -p \\ 0 & \text{for } \lambda(t) < -p \end{cases} \quad (9)$$

where  $p = \frac{m^{a-1}}{a}$ .

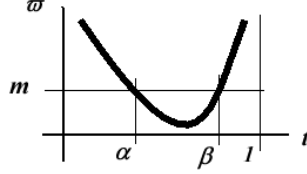


Figure 2. Examples of  $\varpi = (-\lambda(t))^{\frac{1}{a-1}}$

From the transversality conditions and the continuity of  $\lambda(t)$  we have  $\lambda(t) > -p$  for  $t \in [0, r) \cup (s, 1]$  for  $0 < r \leq s < 1$  and  $r$  sufficiently small and  $s$  sufficiently close to 1. Hence there are two possibilities:

1.  $\lambda(t) > -p$  for all  $t$ .

Then  $u^*(t) = m$  for all  $t \in [0, 1]$ .

2.  $\lambda(t) \leq -p$  for some  $0 < t < 1$ .

Then  $M \equiv \{t : \lambda(t) \leq -p\} \neq \emptyset$ . Set  $r \equiv \inf M$  and note that, since  $\lim_{t \rightarrow r^-} \lambda(t) \geq -p$ , we have  $\lambda(r) = -p$ . From (9)  $u^*(t) = m$  in  $[0, r)$ , if there is a solution that satisfies (3). We want to show that  $\dot{\lambda} < 0$  in  $[0, r)$ . Since  $\lambda(0) = 0$  and  $\lambda(r) = -p$ , and  $\ddot{\lambda}(t) > 0$  for all  $t < r$  this will be true if there no turning point in  $[0, r)$ . Suppose there is a turning point at  $t_q < r$ . Since the turning point is unique in  $[0, r)$ ,  $\dot{\lambda}(t_k) > 0$  whenever  $t_q < t_k < r$ . Since  $\lambda(t_q) > -p$ ,  $\lambda(t_k) > -p$  and  $\lambda(r) > -p$ . This contradicts  $\lambda(r) = -p$ . This gives us two cases to consider:

2.i.  $\dot{\lambda}(r) = 0$ .

The aim here is to show that  $u^*(t) = m$ , for all  $t$ , by comparing values of  $H$ . Start by supposing  $\lambda(t) > -p$  for  $t \in (r, r + \epsilon)$  for some small  $\epsilon > 0$ . Then  $u^*(t) = m$ . This means that the value of the Hamiltonian at  $r + \delta$  for any  $0 < \delta \leq \epsilon$  is

$$\varphi = f(x(r + \delta)) + m(p + \lambda(r + \delta)) + \zeta x(r + \delta)$$

where  $x(r + \delta) > x(r)$  and  $p + \lambda(r + \delta) > 0$ .

Suppose now  $\lambda(t) < -p$  for  $t \in (r, r + \epsilon)$  where  $\epsilon > 0$ . Then  $u^*(t) = 0$ . This means that  $\dot{x} = 0$  for all  $t \in (r, r + \epsilon)$  and the value of the Hamiltonian is

$$\tilde{\varphi} = f(x(r)) + \zeta x(r)$$

From (6)  $\zeta$  is a constant. Since  $x(r + \delta) > x(r)$  and  $df/dx > 0$ , we have  $\varphi > \tilde{\varphi}$ . Therefore  $u(t) = m$  maximizes (4) for all  $t \in (r, r + \epsilon)$ , which is a contradiction.

Now consider  $t > r + \epsilon$ . The fact that  $\dot{\lambda}(t) > 0$  for all  $t \in (r, r + \epsilon)$  means that  $\lambda(r + \epsilon) > \lambda(r) = -p$ . Because  $\ddot{\lambda}(t) \geq 0$  from (7), and  $r$  is the unique turning point,  $\dot{\lambda}(t) > 0$  for all  $t > r + \epsilon$ . Hence  $\lambda(t) > -p$  for all  $t > r + \epsilon$ . It follows that  $u(t) = m$  for all  $t \in (r + \epsilon, 1]$ . Hence  $u^*(t) = m$  for all  $t > r$ .

2.ii.  $\dot{\lambda}(r) < 0$ .

In this case we show that there is no solution that satisfies the transversality conditions. Since  $\dot{\lambda}(r) < 0$ , for some  $\epsilon > 0$ ,  $\dot{\lambda}(t) < 0$  for  $t \in [r, r + \epsilon)$  so  $W \equiv \{q > r : \dot{\lambda}(t) < 0 \text{ for all } t \in [r, q]\} \neq \emptyset$ . The mean value theorem gives  $\dot{\lambda}(s) - \dot{\lambda}(r) = \ddot{\lambda}(\tilde{t})(s - r)$  for some  $r < \tilde{t} < s$ . Since  $s = \sup W$ ,  $\dot{\lambda}(t) < 0$  for all  $t \in [r, s)$  and  $\lambda(t_k) < \lambda(r) = -p$ . Therefore  $u(t) = 0$  and  $\ddot{\lambda}(t) = 0$  for all  $t \in [r, s)$ . Hence  $\dot{\lambda}(s) - \dot{\lambda}(r) = 0$  and  $\dot{\lambda}(s) < 0$ . So  $\dot{\lambda}(q) < 0$  for  $q \in [s, s + \bar{\epsilon})$  for some  $\bar{\epsilon} > 0$ , hence for all  $q \in [r, s + \bar{\epsilon})$ . Therefore  $s = \sup W \geq s + \bar{\epsilon} > s$  and this contradiction proves  $s = 1$ . This means  $\dot{\lambda}(t) \leq 0$  for all  $t \in [r, 1)$  and, since  $\lambda(r) = -p$  we must have  $\lambda(1) \leq -p$ . This contradicts  $\lambda(1) = 0$ .

For the case where there is no measurable  $u$  that satisfies (3), consider  $J = \int_0^1 f(x(t))dt + \int_0^1 \frac{\dot{x}(t)^a}{a} dt$ . Observe that, for  $a \geq 1$  and  $\dot{x} > K$  sufficiently large, we have  $\int_0^1 \frac{\dot{x}(t)^a}{a} dt \geq \int_0^1 \dot{x}(t) dt = x(1) - x(0)$ . Since  $f(0) = 0$ ,  $J$  can be made arbitrarily large by setting  $x(0) = 0$  and  $\dot{x} > K$  arbitrarily large to give  $x(1)$  arbitrarily large. Consider the control

$$u(t) = \begin{cases} 0 & \text{for } t \in [0, 1 - \epsilon) \\ k & \text{for } t \in (1 - \epsilon, 1] \end{cases} \quad (10)$$

where  $\epsilon > 0$  and  $k > K$ . For  $a = 1$  we must have  $\epsilon k = 1$  in order to meet (2) and we can consider  $k$  as an implicit function of  $\epsilon$ . Because  $\epsilon$  is not bounded away from 0, and  $\frac{dk(\epsilon)}{d\epsilon} < 0$ , there is no upper bound for  $k$ . This means that there is no measurable  $u(t)$  that maximizes (4) and the despot must violate (3) and give all wealth to the individual at 1.

## 4 Conclusion.

This paper has attempted to explore some of the consequences of the idea that the happiness an individual gets from wealth is relative to the holdings of other individuals. In particular it has been assumed that individuals are mostly concerned with their security and status relative to those immediately below. This concern has been modelled in terms of rates of change in wealth in the neighbourhood of each individual.

It was shown that if the marginal utility of wealth is decreasing and the exponent  $a < 1$  there are some parameters for which the welfare maximizing distribution will have the property that the rate of change in inequality is maximized at the richest and poorest ends but tends to be egalitarian in the middle. This is the most interesting case since it has the most plausible story about the nature of individual utilities. It roughly give a profile in which there are some very poor, a flat middle, and some very rich.

In other cases it was shown that either the rate of change in wealth is at the maximum rate or all the wealth goes to a single individual. In the latter case we have a situation in which a desire amongst all to be slightly better-off locally may result in a situation where all, but one, individual is not to better-off in the optimal allocation. This might be thought of as a paradox of individual desires and collective outcomes. It may also be pointing at some form of collective inconsistency in individual desires for inequality, no matter how local.

In order to highlight the effects of a desire to secure position relative to those below no attempt has been made to soften the analysis by introducing other factors. Nonetheless, to the extent that this desire exists, these findings may help explain why there has not been a stronger demand for wealth equalizing programmes in affluent societies.

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