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DEPARTMENT OF ECONOMICS

Working Paper

Wage inequality and overeducation in a model with efficiency wages

by

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Wage inequality and overeducation in a model with efficiency wages*

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Abstract

This paper shows that the existence and persistence of 'overeducation' can be explained by an extension of the efficiency wage model. When calibrated to fit the amounts of overeducation found in most empirical studies, the model implies that both the relative wage and the relative employment rate of high-skill workers depend inversely on aggregate economic activity. Keeping aggregate employment constant, furthermore, low-skill unemployment rises following an increase in the relative supply of high-skill labor, and relative wages may be insensitive to changes in relative labor supplies. The model may help explain rising wage inequality in some countries since the early 1970s.

Key words: Wage inequality, overeducation, efficiency wages.

JEL classification: J31

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1 Introduction

The purpose of this paper is twofold. It is shown, first, that the existence and persistence of 'overeducation' can be explained by a simple extension of the efficiency wage model. The model is used, second, to examine the effects on wage inequality and the pattern of unemployment of 'neutral' shocks to aggregate economic activity and of shifts in the skill composition of the labor force. The presence of overeducation, it turns out, may reverse the direction of some of these effects, compared to a standard model without overeducation.

Workers are overeducated if they have education in excess of that required to do their jobs. Qualifications are not necessarily the same as formal education and the measurement of overeducation involves many difficulties, both conceptual and empirical.[1] There is strong evidence, however, that the incidence of overeducation is substantial. An influential study by Sicherman (1991) reports that 40 percent of US workers are overeducated, and Hersch (1991) finds overeducation figures ranging from 28 to 78 percent for different groups of workers in a sample from Oregon. In the UK, several studies indicate that about 30 percent of all respondents were overeducated and that the figure may be above 40 percent among those possessing more than the lowest level of qualifications (Sloane et al. (1999), Dolton and Vignoles (2000), Rigg et al (1990)). Summarizing the evidence, Green et al (1999, p.15) suggest that "overeducation is a widespread phenomenon both in Europe and the United States of America".[2]

This paper uses an efficiency wage model to account for overeducation. Efficiency wage models come in many forms. The key element of these models - the dependence of workers' productivity on wages - can be related to sociological or psychological factors, as in Akerlof (1982) and Akerlof and Yellen (1990), or it may arise in more traditional models of optimizing behavior, as in Shapiro and Stiglitz (1984) and Bowles (1985), among others.

To simplify the analysis, this paper follows the standard shirking approach of Shapiro

[1] Green et al (1999) and Hartog (2000) discuss some of the issues involved.

[2] See also Borghans and de Grip (2000) and the special issue on overeducation in Economics of Education Review (vol. 19, 2000).

Undereducation - workers who report having less education than required to get the job - also exists. Quantitatively, most studies indicate that about 10-20 percent of all workers are undereducated. The existence of undereducation on this scale could indicate 'credentialism': a change in the pool of applicants may lead employers to raise the skills required for recruitment to an otherwise unchanged job. Employers may prefer workers with the 'required education' but this level may not be needed to do the job. The formal model in this paper abstracts from both undereducation and credentialism.

and Stiglitz. The extension of the shirking model lies in the introduction of two types of workers, high and low skill, and a distinction between the skill requirements of the job and the skills of the worker. Specifically, it is assumed that there is an asymmetry between the options of high- and low-skill workers. A high-skill worker who is unable to get a high-skill job may accept a low-skill wage in a low-skill job for which she is 'overeducated'. Low-skill workers do not have the analogous option of getting high-skill jobs. We thus get three no-shirking conditions: for high-skill workers in high-skill jobs, for high-skill workers in low-skill jobs and for low-skill workers in low-skill jobs. No-shirking among workers in high-skill jobs is enforced by a combination of open unemployment and employment in low-paying low-skill jobs; open unemployment, on the other hand, is the only discipline devise for low-skill jobs.

Overeducation emerges from this analysis in a straightforward way: some high-skill workers are lucky and get well-paid jobs that utilize their skills while others are unemployed or get less-skilled jobs with a lower pay, that is, they become overeducated. This asymmetry in the fortunes of otherwise identical workers is similar to the asymmetries explained by other versions of efficiency wage models. A standard efficiency wage model explains why, in equilibrium, identical workers may have different employment status and different levels of income and utility; a multisectoral version of the model allows for the possibility that identical workers in different sectors may have different wages and utility levels. In this paper identical workers may be employed in the same firm but with different jobs and different wages and utility levels.

In order to determine the equilibrium solutions for employment and wages, the no-shirking conditions are combined with labor demand curves derived from firms' profit maximization. Thus, the position of the equilibrium may shift for a number of reasons, including shifts in the production function, changes in the degree of product market competition, changes in relative labor supplies or changes in the parameters that define the wage curves.

This paper first considers the effects of neutral shocks to aggregate economic activity. A Hicks-neutral shift in the demand for labor, which leaves the proportion of high-skill *jobs* constant if the relative wage rate is kept unchanged, represents an example of this kind of shock. In a standard model without overeducation, neutral shocks cannot generate a decline in both the relative employment and the relative wage of low-skill workers. Yet the empirical picture for both the US and the UK shows a dramatic decline in both relative employment and relative wages between the early 1970s and the mid 1990s, an observation which has been explained by a combination of skill-biases in technical progress, the effects of international competition and institutional changes in the labor markets. The model in this paper demonstrates that 'induced overeducation' may have contributed to the observed changes: a negative, neutral shock to aggregate employment will raise unemployment among both low- and high-skill workers but since both groups of unemployed workers compete for low-skill jobs, the relative wage in low-skill jobs may come under pressure. As a result, one may see an increase in the proportion of low-skill *jobs* but a decrease in the proportion of low-skill *workers* in total employment. The increase in the proportion of low-skill jobs in turn implies a relative decline in low-skill wages.

This type of (partial) explanation of the deterioration of both relative employment and relative wages for low-skill workers has been suggested by, among others, Thurow (1998) and Skott and Auerbach (2005).[3] Thurow discusses a number of reasons for increasing US wage inequality. He emphasizes the effects of intra OECD trade as a source of downward pressure on the wages of male workers in traditional industries but also notes the asymmetry arising from the ability of high-skill workers to get low-skill jobs (p.31). According to Thurow, unemployed high-skill workers "bump down the job distribution" (p. 33) but there is no attempt at a more rigorous analysis of the mechanisms involved. Skott and Auerbach examine the implications of reduced-form assumptions concerning the proportion of high-skill workers without a high-skill job that move into a low-skill job. They show that using plausible parameter values and taking as exogenous the trends in the employment rates for high and low-skill workers, this framework could explain a large substantial increase in US wage inequality.

Shifts in the composition of the labor supply represents another obvious source of movements in wage inequality and relative unemployment. If the production function exhibits constant returns to the two types of labor, the qualitative results are as one would expect: an increase in the relative supply of high-skill workers will reduce both low-skill unemployment and the average wage premium to high-skill workers. Endogenous movements in the relative supply of high-skill workers will not, however, eliminate overeducation. Extended in this way - with the supply of high-skill workers depending on the average wage premium - the model defines a long-run equilibrium with overeducation.

If there are decreasing returns to the two types of labor, the results can be more surprising. In this case, an increase in the relative supply of high-skill workers may imply

[3] Skott (2005) presents a related argument but focuses mainly on the presence of hysteresis in relative wages and employment.

that the unemployment rate for low-skill workers rises and the relative wage may also move against low-skill workers. These perverse effects on unemployment and relative wages may be empirically unlikely. They are indicative, however, of a more general feature: in the presence of overeducation, the relative wage may be quite insensitive to changes in relative labor supplies.

The remainder of this paper is organized in four sections. Section 2 presents the model. The analysis of neutral changes in aggregate labor demand is in section 3 while section 4 considers the effects of changes in the relative labor supply. Section 5 contains a few concluding remarks. Proofs and derivations are collected in appendices 1-4.

2 The model

The economy is closed and produces a single output, Y. There are only two types of jobs, high and low skill, and two types of workers, high and low skill; a worker's skill level is observable. To simplify the exposition, non-labor inputs are disregarded. Thus, if there is no shirking,

$$\begin{aligned} Y &= AF(N_H, N_L) \qquad (1) \\ A &> 0; \quad F_i > 0, F_{ii} < 0 \text{ for } i = H, L; \quad F(0, N_L) = F(N_H, 0) = 0 \end{aligned}$$

where N_H and N_L denote the number of high- and low-skill *jobs* that have been filled. All high-skill jobs are filled by high-skill workers while low-skill jobs may be filled by either low- or high-skill workers. Non-shirking high- and low-skill workers have the same productivity in low-skill jobs. To simplify the analysis it is assumed that the function F satisfies the standard assumptions of constant returns and positive but diminishing marginal productivity of both inputs. With the exception of proposition 3, the results in this paper would go through if constant returns were replaced by the weaker assumption of homotheticity; proposition 4 covers an extreme case of decreasing returns. To avoid the possibility of degenerate cases with only one type of job, it is assumed that both jobs are needed to produce a positive output. Changes in the multiplicative constant A describe Hicks-neutral technical change.

The supplies of high- and low-skill workers are H and L, respectively, and we have the following accounting relations:

$$\begin{aligned} H &= N_H + N_{HL} + U_H \\ L &= N_{LL} + U_L \\ N_L &= N_{LL} + N_{HL} \end{aligned}$$

where U_i is unemployment among workers of type i and N_{LL} and N_{HL} denote low- and high-skill workers in low-skill jobs; the number of high-skill workers in high-skill jobs is equal to the number of high-skill jobs N_H. The labour supplies H and L are taken to be fixed in section 3; section 4 considers the effects of changes in the relative supply of high-skill workers.

The wage structure is determined by efficiency wage considerations and, using a simple shirking setup, employed workers either "shirk" or "exert effort". Workers get instantaneous utility u_{ij} given by

$$u_{ij} = \begin{array}{cc} w_{ij} - e_{ij} & \text{if employed and exerting effort} \\ w_{ij} & \text{if employed and shirking} \\ 0 & \text{if unemployed} \end{array}$$

where w and e are wages and the costs in terms of utility of exerting effort; subscripts $i = H, L$ and $j = H, L$ denote the skills of the worker and the skill requirement of the job, respectively. This specification implies that shirking raises a worker's instantaneous utility. Shirking also increases the worker's risk of losing the job (since a worker who is caught shirking will be fired), and the effort/shirking decision is based on this tradeoff. An increase in the wage raises the cost of job loss and shifts the balance against shirking. Firms set wages sufficiently high to prevent shirking.

By assumption there are no low-skill workers in high-skill jobs and the skill levels are observable. Hence, there are three sets of no-shirking conditions, one for each group of employed workers. In a long-run equilibrium the no-shirking conditions can be written:

$$\begin{aligned} \rho V_{HH} &= w_{HH} - e_{HH} - p(V_{HH} - V_{HU}) + q_{HHL}(V_{HL} - V_{HH}) \\ &= w_{HH} - (p+\delta)(V_{HH} - V_{HU}) + q_{HHL}(V_{HL} - V_{HH}) \end{aligned} \tag{2}$$

$$\begin{aligned} \rho V_{HL} &= w_{HL} - e_{HL} - p(V_{HL} - V_{HU}) + q_{HLH}(V_{HH} - V_{HL}) \\ &= w_{HL} - (p+\delta)(V_{HL} - V_{HU}) + q_{HLH}(V_{HH} - V_{HL}) \end{aligned} \tag{3}$$

$$\rho V_{LL} = w_{LL} - e_{LL} - p(V_{LL} - V_{LU}) = w_{LL} - (p+\delta)(V_{LL} - V_{LU}) \tag{4}$$

where

$$\rho V_{HU} = q_{HUH}\,(V_{HH} - V_{HU}) + q_{HUL}(V_{HL} - V_{HU}) \tag{5}$$

$$\rho V_{LU} = q_{LUL}\,(V_{LL} - V_{LU}) \tag{6}$$

The variables V_{ij} $(i = H, L; j = H, L, U)$ denote the present values of the future flows of utility for a worker of type i with job status j. The parameters ρ, p and δ are the discount rate, the exogenous rate of job terminations for non-shirking workers and the rate of detection for workers that shirk.[4] While ρ, p and δ are exogenously given, the values of the hiring rates q are determined endogenously. Low-skill workers never get high-skill jobs and their hiring rates into low-skill jobs is q_{LUL}. Unemployed high-skill workers have hiring rates q_{HUL} and q_{HUH} into low- and high-skill jobs, respectively. High-skill workers, finally, may move directly from a high- to a low-skill job (if low-skill jobs are the more attractive) or, alternatively, directly from a low- to a high-skill job (if high-skill jobs are more attractive). The transition rates for these direct moves are q_{HHL} and q_{HLH}.

Straightforward manipulation of the no-shirking conditions yields

$$V_{HH} - V_{HU} = \frac{e_{HH}}{\delta} \tag{7}$$

$$V_{HL} - V_{HU} = \frac{e_{HL}}{\delta} \tag{8}$$

$$V_{LL} - V_{LU} = \frac{e_{LL}}{\delta} \tag{9}$$

Comparing (7) and (8) it follows that $V_{HH} - V_{HL} = (e_{HH} - e_{HL})/\delta$. Thus, the relative magnitudes of the effort costs e_{HH} and e_{HL} will determine whether high-skill workers prefer low- or high-skill jobs. Theoretically, as well as empirically, the relevant scenario is one in which they prefer high-skill jobs. In the rest of this paper I shall therefore assume that $e_{HH} > e_{HL}$.[5] This assumption implies that no high-skill workers will want to move to a low-skill job. The transition rate q_{HHL} will therefore be zero. Furthermore, all high-skill workers are identical and, when filling a high-skill job, firms will be indifferent between hiring an unemployed high-skill worker or a high-skill worker who is currently in a low-skill job. Both of these groups of workers want high-skill jobs, and I shall assume that they have the same hiring rates into high-skill jobs (that is, $q_{HUH} = q_{HLH} = q_{HH}$).

With these assumptions, the steady state conditions require that the hiring rates

[4] One might expect the exogenous termination and detection rates, p and δ, to be higher for low- than for high-skill jobs, while the discount rate ρ may be higher for low- than for high-skill workers. The combined effects of these differences in the parameters on the no-shirking wages are ambiguous. Low rates of discount and exogenous termination reduce the no-shirking wage but this effect may be offset by a low detection rate. To simplify the analysis it is assumed, therefore, that the termination, detection and discount rates are the same across workers and jobs

[5] In a more disaggregate setup, high-skill workers need not have a relatively low utility cost of effort in all low-skill jobs. It is sufficient that for any particular skill there exist some low-skill jobs which a high-skill person, trained in that area, finds it relatively easy to perform.

satisfy

$$\begin{aligned} pN_H &= q_{HH}(H - N_H) \\ pN_{HL} + q_{HH}N_{HL} &= q_{HUL}(H - N_H - N_{HL}) \\ pN_{LL} &= q_{LUL}(L - N_{LL}) \end{aligned}$$

or

$$\begin{aligned} q_{HH} &= \frac{pN_H}{H - N_H} \\ q_{HUL} &= \frac{(p + q_{HH})\,N_{HL}}{H - N_H - N_{HL}} = p\frac{H}{H - N_H}\frac{N_{HL}}{H - N_H - N_{HL}} \\ q_{LUL} &= \frac{pN_{LL}}{L - N_{LL}} = p\frac{N_L - N_{HL}}{L - N_L + N_{HL}} \end{aligned}$$

Using these expressions for the hiring rates, equations (2)-(9) can be used to derive the following wage equations:

$$w_{HH} = e_{HL}\frac{\delta + \rho + p\frac{H}{H - N_H - N_{HL}}}{\delta} + (e_{HH} - e_{HL})\frac{\delta + \rho + p\frac{H}{H - N_H}}{\delta} \tag{10}$$

$$w_{HL} = e_{HL}\frac{\delta + \rho + p\frac{H}{H - N_H - N_{HL}}}{\delta} \tag{11}$$

$$w_{LL} = e_{LL}\frac{\delta + \rho + p\frac{L}{L - N_{LL}}}{\delta} \tag{12}$$

By assumption, only high-skill workers have high-skill jobs, and the productivity of high- and low-skill workers is the same in low-skill jobs. Hence, if firms maximize profits, we must have

$$w_{HL} = w_{LL} = w_L \text{ if } N_{LL} > 0 \text{ and } N_{HL} > 0 \tag{13}$$

$$w_{LL} > w_{HL} = w_L \text{ if } N_{HL} > 0 \text{ and } N_{LL} = 0 \tag{14}$$

$$w_{HL} > w_{LL} = w_L \text{ if } N_{LL} > 0 \text{ and } N_{HL} = 0 \tag{15}$$

$$w_{HH} = w_H \tag{16}$$

where w_H and w_L denote the wage rates for high and low-skill jobs.

In order to find the equilibrium solution of the model, equations (10)-(16) are combined with firms' first order conditions with respect to the number of high- and low-skill jobs. Using (1), these first order conditions are given by:

$$\begin{aligned} w_H &= mAF_1(N_H, N_L) \\ w_L &= mAF_2(N_H, N_L) \end{aligned}$$

where $m \leq 1$ is the inverse of the markup on marginal cost ($m = 1$ under prefect competition). The homogeneity of the production function (1) implies that the ratio of high- to low-skill jobs is an increasing function of the relative wage in low-skill jobs

$$\frac{w_L}{w_H} = \phi(\frac{N_H}{N_L}) \tag{17}$$

where the function $\phi(.)$ is unaffected by changes in A and m.

3 Wage effects of neutral shocks to aggregate employment

3.1 A case with induced overeducation

This section examines the implications of neutral shocks to aggregate employment. A neutral shock is defined as one that leaves unchanged (i) the relation (17) between w_L/w_H and N_H/N_L, (ii) the relation between $(u_L, u_H, \frac{N_H}{H})$ and $(\frac{w_{HH}}{w_{LL}}, \frac{w_{HL}}{w_{LL}})$ implied by (10)-(12), and (iii) the labor supply ratio H/L. In terms of the model, a Hicks-neutral shift of the production function, a change in the markup on marginal cost, or a proportional change in the three utility parameters e_{HH}, e_{HL} and e_{LL} could produce neutral shocks of this kind.[6] For present purposes, however, the underlying cause of the shift in aggregate employment is irrelevant. Moreover, changes in employment are observable (unlike shifts in underlying parameters). The analysis, therefore, will be cast in terms of the effects of neutral shocks to total employment.

At an interior solution the wage rates satisfy (10)-(13) and (16). Using (11)-(13), it follows that the unemployment rate for low-skill workers can be expressed as an

[6] It is readily seen that shifts of this kind will affect aggregate employment. Consider for example an upward shift in the production function. By assumption F is linearly homogeneous and it follows that $F_1(N_H, N_L) = F_1(\frac{N_H}{N_L}, 1)$ and $F_2(N_H, N_L) = F_2(1, \frac{N_L}{N_H})$. The first order conditions with respect to N_H and N_L therefore imply that an increase in A must produce a rise in w_H and / or w_L. A rise in w_L translates directly into an fall in unemployment among both groups of workers (use (11)-(12)) and hence a rise in total employment. Thus, in order to prove that employment will increase we just need to show that w_L must rise.

Assume the contrary. Since both wages cannot fall, the relative wage w_L/w_H and the employment ratio N_H/N_L must therefore fall and, using (11)-(12) the employment rate $(N_H + N_{HL})/H$ will also fall if w_L falls. Equation (10), however, implies that a decline in both N_H/N_L and $(N_H + N_{HL})/H$ (and therefore also in N_H/H) is inconsistent with a rise in the no-shirking wage w_H (since the right hand side of (10) is increasing in $(N_H + N_{HL})/H$ and N_H/H). Thus, the assumption that w_L falls has produced a contradiction.

increasing function of the unemployment rate for high-skill workers,

$$u_L = \frac{u_H}{\frac{e_{HL}}{e_{LL}} + \frac{e_{HL}-e_{LL}}{e_{LL}}\frac{\delta+\rho}{p}u_H} \tag{18}$$

where $u_H = (H - N_H - N_{HL})/H$ and $u_L = (L - N_{LL})/L$ are the unemployment rates for the two groups. Equation (18) implies that the unemployment rate for low-skill workers will exceed the unemployment rate for high-skill workers when $e_{HL} < e_{LL}$ (and that $u_L < u_H$ when $e_{HL} > e_{LL}$). The interesting case - the one that fits the empirical evidence of relatively low unemployment rates for high-skill workers - arises when the ratio e_{HL}/e_{LL} is below one, and in what follows I shall focus on this case.

If $n = N_H + N_L$ is total employment, equations (10)-(13) and (16) imply (see Appendix 1) that

$$\frac{d\log\frac{w_H}{w_L}}{d\log n} = En[B(x)\frac{N_H}{n}\frac{d\log N_H}{d\log n} - C(y)D(y,z)] \tag{19}$$

where

$$\begin{aligned}
x &= \frac{H}{H-N_H} = \frac{1}{u_H + \frac{N_{HL}}{H}} \\
y &= \frac{L}{L+N_{HL}-N_L} = \frac{1}{u_L} \\
z &= \frac{H}{H-N_H-N_{HL}} = \frac{1}{u_H} \\
B(x) &= \frac{\frac{L}{H}x^2}{\delta+\rho+px} > 0 \\
C(y) &= \frac{y^2}{\delta+\rho+py} > 0 \\
D(y,z) &= \frac{e_{HL}\frac{L}{H}z^2}{e_{HL}\frac{L}{H}z^2 + e_{LL}y^2} > 0 \\
E &= \frac{w_H - w_L}{w_H}p\frac{1}{L} > 0 \text{ for } e_{HH} > e_{HL}
\end{aligned}$$

Using equation (17) we get another relation between changes in relative wages and relative factor inputs,

$$\frac{d\log\frac{w_H}{w_L}}{d\log n} = \eta\left(\frac{d\log N_L}{d\log n} - \frac{d\log N_H}{d\log n}\right) \tag{20}$$

where η is the inverse of the elasticity of substitution of the production function. From the definition of total employment, finally, we have

$$1 = \theta\frac{d\log N_H}{d\log n} + (1-\theta)\frac{d\log N_L}{d\log n} \tag{21}$$

where $\theta = N_H/n$. Combining (19)-(21), the effects of a change in aggregate employment on relative wages can be derived. We get (see Appendix 2)

$$\frac{d\log\frac{w_H}{w_L}}{d\log n} = \eta\frac{n}{N_L}(1 - \frac{d\log N_H}{d\log n}) \tag{22}$$

where

$$\frac{d\log N_H}{d\log n} = \frac{\eta + N_L EC(y)D(y,z)}{\eta + N_L EB(x)\frac{N_H}{n}} \tag{23}$$

Using (22)-(23) we have the following result:

Proposition 1 *At an interior equilibrium with $e_{HH} > e_{HL}$ and $e_{LL} > e_{HL}$,*

- *high-skill workers will have a higher average wage and a lower unemployment rate than low-skill workers.*
- *an increase in aggregate employment reduces the unemployment rate of both low- and high-skill workers, but low-skill workers benefit disproportionately: the relative unemployment rate for low-skill workers, u_L/u_H, depends inversely on aggregate employment.*
- *if $\Omega = N_{HL}/(N_H + N_L)$ denotes the degree of overeducation, the restriction $\Omega > \Omega^{crit} = u_L/(1 + (1 - u_L)\frac{L}{H})$ is sufficient to ensure an inverse relation between the relative wage w_H/w_L and aggregate employment.*

Proof: See Appendix 3.

It follows from Proposition 1 that with an unemployment rate among low-skill workers of, say, 0.2 or lower, the wage ratio w_H/w_L varies inversely with the aggregate rate of unemployment even if the degree of overeducation is far below the figure of 30-40 percent suggested by most studies.

The wage ratio w_H/w_L does not capture wage inequality as it is usually measured. Standard measures of the skill premium focus on the ratio w_{HA}/w_L where $w_{HA} = \frac{N_H}{N_H+N_{HL}}w_H + \frac{N_{HL}}{N_H+N_{HL}}w_L$ is the average wage of high-skill workers. Furthermore, there is within-group inequality among high-skill workers. This within-group inequality can be described by

$$\begin{aligned} \sigma &= \sqrt{\frac{N_H}{N_H + N_{HL}}\left(\frac{w_H - w_{HA}}{w_{HA}}\right)^2 + \frac{N_{HL}}{N_H + N_{HL}}\left(\frac{w_L - w_{HA}}{w_{HA}}\right)^2} \\ &= \frac{w_{HA} - w_L}{w_{HA}}\sqrt{\frac{N_{HL}}{N_H}} \end{aligned}$$

These measures of between- and within-group inequality both depend on the rate of aggregate employment. We have the following result:

Proposition 2 *Assuming that the conditions in Proposition 1 for $d\log\frac{w_H}{w_L}/d\log n$ to be negative are met,*

- *the relative wage w_{HA}/w_L will vary directly with aggregate employment (that is, $d\log\frac{w_{HA}}{w_L}/d\log n > 0$) if the two types of labor are perfect substitutes in production (if $\eta \to 0$).*
- *depending on parameter values, the relation between the relative wage w_{HA}/w_L and aggregate employment may be inverse or direct when the elasticity of substitution is finite.*
- *if the relation between the relative wage w_{HA}/w_L and aggregate employment is inverse then within-group inequality σ will also be inversely related to aggregate employment.*

Proof: See Appendix 4.

3.2 Numerical examples

Proposition 2 fails to give unambiguous results for the relative wage w_{HA}/w_L. Numerical analysis, however, suggests an inverse relation between aggregate employment and inequality for plausible parameter values, unless the existing estimates of overeducation greatly exaggerate actual overeducation.

Table 1 gives the initial values of the two measures of the relative wage (w_H/w_L and w_{HA}/w_L) and the within-group dispersion among high-skill workers for different values of e_{HL}/e_{LL} and e_{HH}/e_{LL}. The table is derived using equations (10)-(13) and (16). Tables 2-3, based on equations (18) and (a10)-(a11) in Appendix 4, show the effects of changes in employment on overeducation and wage inequality. Most studies (e.g. Card et al. (1999)) suggest a relatively low elasticity of substitution between high- and low-skill jobs and the production function is assumed to be either Leontief (Table 2a) or Cobb-Douglas (Table 2b).[7] The initial degree of overeducation and the initial unemployment

[7] The parameters of the production function are calibrated so as to ensure that the initial employment and wage rates (derived from (10)-(13) and (16)) are consistent with profit maximisation. If $e_{HL} = 0$ and $e_{HH} = e_{LL} = 1$, for instance, the parameter assumptions and the initial values of overeducation

rate for low-skill workers are 0.3 and 0.1, respectively, in Tables 1-2; these initial values are changed to 0.1 and 0.2 in Table 3. All tables use $\delta = 1, \rho = 0.1, p = 0.2$ and $L = H$.[8]

Table 1: Wage inequality for different values of e_{HL}/e_{LL} and e_{HH}/e_{LL}
($\delta = 1, \rho = 0.1, p = 0.2; L = H; \Omega = 0.3, N_{LL}/L = 0.9$)

			$e_{HH}/e_{LL} = 1$		
e_{HL}/e_{LL}	w_H/w_L	w_{HA}/w_L	σ	u_H	$\frac{w_{HA}(N_H+N_{HL})}{w_H N_H + w_L N_L}$
0.0	1.47	1.20	0.19	0	0.57
0.2	1.37	1.16	0.16	0.01	0.56
0.4	1.28	1.12	0.12	0.03	0.55
0.6	1.18	1.08	0.08	0.05	0.53
0.8	1.09	1.04	0.04	0.07	0.52
1.0	1.00	1.00	0.00	0.1	0.5
			$e_{HH}/e_{LL} = 2$		
e_{HL}/e_{LL}	w_H/w_L	w_{HA}/w_L	σ	u_H	$\frac{w_{HA}(N_H+N_{HL})}{w_H N_H + w_L N_L}$
0.0	1.93	1.40	0.33	0	0.61
0.2	1.84	1.36	0.31	0.01	0.60
0.4	1.74	1.31	0.28	0.03	0.59
0.6	1.65	1.27	0.25	0.05	0.57
0.8	1.55	1.23	0.22	0.07	0.56
1.0	1.46	1.18	0.19	0.1	0.54

As indicated by Table 1, the relative wage of high-skill workers is decreasing in e_{HL}/e_{LL} but increasing in e_{HH}/e_{LL}. The intuition is straightforward. An increase in e_{HL}/e_{LL} tightens the no-shirking condition for high-skill workers in low-skill jobs and puts upward pressure on the relative wage for low-skill jobs; an increase in e_{HH}/e_{LL}, analogously, tightens the no-shirking condition for high-skill jobs and raises the high-skill wage premium.[9]

and unemployment for low-skill workers in Table 2 imply that the production functions are given by $Y = \min\{\lambda_H N_H, \lambda_L N_L\}$ with $\lambda_H/\lambda_L = 3.42$ in the Leontief case and $Y = N_H^{\alpha} N_L^{1-\alpha}$ with $\alpha = 0.30$ in the Cobb-Douglas case.

[8] The value $p = 0.2$ implies that just over 18 percent of all workers will lose (or choose to leave) their jobs within one period; $\delta = 1$ implies that a shirking worker has a 63 percent probability of detection within one period.

[9] There is full employment for high-skill workers if $e_{HL} = 0$. If their cost of effort is zero, high-skill workers in low-skill jobs will never shirk. Hence, the 'wage curve' for workers in low-skill jobs becomes horizontal at $w_L = 0$ until all high-skill workers have a job; further increases in output requires the use

The wage premium obtained by high-skill workers in Table 1 is not due to a skill shortage. The high-skill wage is high because many high-skill workers get low-skill jobs. Thus, suppose that high-skill workers were precluded from low-skill jobs. Using the same parameter values, relative labor supplies and initial employment rate for low-skill workers as in Table 1, this preclusion implies that if, for example, $e_{HH} = e_{LL}$ and $e_{HL} = 0$, the initial employment rate and relative wage for high-skill workers would be $N_H/H = 0.26$ and $w_H/w_L = 0.44$ in the Leontief case and $N_H/H = 0.69$ and $w_H/w_L = 0.56$ in the Cobb-Douglas case. Thus, without overeducation high-skill workers would have experienced lower employment and lower wage rates than low-skill workers.

Table 2a: Effects of changes in aggregate employment on overeducation and wage inequality: The Leontief case.

$(\delta = 1, \rho = 0.1, p = 0.2; L = H; \Omega = 0.3, N_{LL}/L = 0.9; \eta \to \infty)$

$e_{HH}/e_{LL} = 1$

$\frac{e_{HL}}{e_{LL}}$	$\frac{d\Omega}{d\log n}$	$\frac{d\log(w_H/w_L)}{d\log n}$	$\frac{d\log(w_{HA}/w_L)}{d\log n}$	$\frac{d\sigma}{d\log n}$
0.0	−0.53	−3.85	−1.86	−1.95
0.2	−0.43	−2.97	−1.38	−1.50
0.4	−0.33	−2.11	−0.95	−1.06
0.6	−0.23	−1.30	−0.56	−0.65
0.8	−0.11	−0.59	−0.24	−0.29
1.0	0.00	0.00	0.00	0.00

$e_{HH}/e_{LL} = 2$

$\frac{e_{HL}}{e_{LL}}$	$\frac{d\Omega}{d\log n}$	$\frac{d\log(w_H/w_L)}{d\log n}$	$\frac{d\log(w_{HA}/w_L)}{d\log n}$	$\frac{d\sigma}{d\log n}$
0.0	−0.53	−5.84	−3.18	−2.90
0.2	−0.43	−4.98	−2.66	−2.49
0.4	−0.33	−4.12	−2.15	−2.08
0.6	−0.23	−3.28	−1.68	−1.66
0.8	−0.11	−2.49	−1.25	−1.27
1.0	0.00	−1.78	−0.88	−0.91

of low–skill workers and their no-shirking condition now determines the wage. It follows that when w_L is positive, there is no open unemployment among high-skill workers if $e_{HL} = 0$. Those that fail to get a high-skill job get a low-skill job instead, bumping out low-skill workers in the process.

Table 2b: Effects of changes in aggregate employment on overeducation and wage inequality: The Cobb-Douglas case.

($\delta = 1, \rho = 0.1, p = 0.2; L = H; \Omega = 0.3, N_{LL}/L = 0.9; \eta = 1$)

$e_{HH}/e_{LL} = 1$

$\frac{e_{HL}}{e_{LL}}$	$\frac{d\Omega}{d\log n}$	$\frac{d\log(w_H/w_L)}{d\log n}$	$\frac{d\log(w_{HA}/w_L)}{d\log n}$	$\frac{d\sigma}{d\log n}$
0.0	−1.17	−3.68	−1.29	−1.89
0.2	−0.93	−2.86	−1.03	−1.45
0.4	−0.68	−2.05	−0.75	−1.03
0.6	−0.44	−1.28	−0.48	−0.64
0.8	−0.21	−0.58	−0.23	−0.29
1.0	0.00	0.00	0.00	0.00

$e_{HH}/e_{LL} = 2$

$\frac{e_{HL}}{e_{LL}}$	$\frac{d\Omega}{d\log n}$	$\frac{d\log(w_H/w_L)}{d\log n}$	$\frac{d\log(w_{HA}/w_L)}{d\log n}$	$\frac{d\sigma}{d\log n}$
0.0	−1.48	−5.47	−1.74	−2.95
0.2	−1.25	−4.70	−1.53	−2.50
0.4	−1.00	−3.91	−1.31	−2.06
0.6	−0.75	−3.13	−1.08	−1.63
0.8	−0.51	−2.40	−0.85	−1.23
1.0	−0.28	−1.72	−0.64	−0.88

Turning to Tables 2a-2b, an increase in aggregate employment reduces both measures of the relative wage as long as e_{HL}/e_{LL} is less than one. Thus, the numerical analysis suggests that when there is a substantial overeducation, the elasticity of the relative wage w_{HA}/w_L with respect to aggregate employment will be negative for a wide range of parameter values. From Proposition 1 it now follows that a rise in aggregate employment will benefit low-skill workers in terms of both relative wages and relative employment.

As shown by Proposition 1, however, the degree of overeducation in combination with the unemployment rate for low-skill workers can be critical for the relation between relative wages and aggregate employment, and measures of overeducation, in particular, are subject to considerable uncertainty. Table 3 presents the implications of assuming initial values of 0.1 and 0.2 for overeducation and low-skill unemployment, respectively. Even with these less favorable assumptions, an increase in aggregate employment will lead to a reduction in the average skill premium in almost all cases. The only exceptions arise when the production function is Cobb-Douglas and the ratio e_{HL}/e_{LL} is below 0.4.

Table 3: Effects of changes in aggregate employment on overeducation and wage inequality when initial rates of overeducation and low-skill employment are low ($\delta = 1, \rho = 0.1, p = 0.2; L = H; e_{HH}/e_{LL} = 2; \Omega = 0.1, N_{LL}/L = 0.8$)

Leontief

$\frac{e_{HL}}{e_{LL}}$	$\frac{d\Omega}{d\log n}$	$\frac{d\log(w_H/w_L)}{d\log n}$	$\frac{d\log(w_{HA}/w_L)}{d\log n}$	$\frac{d\sigma}{d\log n}$
0.0	−0.56	−1.35	−0.63	−0.93
0.2	−0.50	−1.34	−0.71	−0.83
0.4	−0.42	−1.26	−0.73	−0.70
0.6	−0.31	−1.09	−0.69	−0.54
0.8	−0.17	−0.85	−0.61	−0.35
1.0	0.00	−0.58	−0.50	−0.16

Cobb-Douglas

$\frac{e_{HL}}{e_{LL}}$	$\frac{d\Omega}{d\log n}$	$\frac{d\log(w_H/w_L)}{d\log n}$	$\frac{d\log(w_{HA}/w_L)}{d\log n}$	$\frac{d\sigma}{d\log n}$
0.0	−0.74	−0.73	0.20	−1.12
0.2	−0.69	−0.80	0.06	−1.02
0.4	−0.62	−0.82	−0.08	−0.89
0.6	−0.50	−0.79	−0.20	−0.71
0.8	−0.34	−0.68	−0.28	−0.49
1.0	−0.12	−0.50	−0.32	−0.24

The key empirical question is not so much the precise calibration of the model to match the levels and changes of relative wages to real-world data. The model is highly stylized and empirical counterparts for relative wages, for instance, depend on the delineation of high and low skill. More importantly, the model deliberately leaves out many aspects that may have influenced real-world developments. The mechanism of induced overeducation, however, is at the center of the model. The key question therefore concerns the robustness and likely magnitude of the effects of induced overeducation. Numerical exercises, like the ones above, may shed some light on this issue but the implications of induced overeducation described in Propositions 1-2 clearly invite further empirical testing. Unfortunately, most empirical studies of overeducation rely on surveys for a particular year and, to my knowledge, the direct evidence on induced changes of this kind is limited and inconclusive. Short-run data on movements in overeducation, moreover, may be hard to interpret.

The steady-state focus of the theoretical model in section 2 makes the model ill-suited to the analysis of short-term fluctuations but disregarding this problem, there are additional complications: the short-run effects of induced overeducation may be offset by the effects of differential labor hoarding. Like induced overeducation, differential labour hoarding implies that low-skill workers are affected disproportionately by unemployment, but the underlying mechanism and the effects on measured overeducation are different. Induced overeducation focuses on the effects on different groups of workers of proportional changes in the number of high- and low-skill jobs; differential labour hoarding, on the other hand, suggests that temporary changes in demand will lead to non-proportional changes in the number of jobs, and when high-skill workers in low-skill jobs are laid off as a result of differential labour hoarding, there is a tendency for overeducation to decrease.[10] A priori it is difficult to say which of these effects will dominate in the short run.[11] In the medium term, however, differential hoarding ceases to be important and we would expect a negative correlation between employment and overeducation.

With respect to medium and long-run trends, UK evidence suggests that the incidence of overeducation increased strongly between the 1970s and 1980s (a period of rising unemployment) but may have stabilized since the late 1980s (Green et al (1999)). Robinson and Manacorda (1997, p. 3) find that in the UK between 1984 and 1994 "the increase in the supply of better educated labour has allowed firms to indulge in 'credentialism', employing more highly qualified staff to do jobs which previously were done by less qualified staff". Furthermore, in the UK an index of required qualifications rose between 1986 and 1992, but then fell slightly during the period of falling unemployment from 1992 to 1997 (Green et al (2000)). In the US, the evidence is ambiguous. Wolff (2000, p. 27) concludes that between 1950 and 1990 there has been a growing mismatch "between skill requirements of the workplace and the educational attainment of the workforce, with the latter increasing much more rapidly than the former". Daly et al. (2000), on the other hand, find a decline in overeducation between 1976 and 1985. With a rapid rise in average years of schooling, however, overeducation may increasingly take the form of a discrepancy between actual and required quality of education, and a focus on years of schooling will fail to register any overeducation if, for instance, MIT graduates accept jobs which otherwise could and would have been filled by graduates

[10] The measure of overeducation may be biassed in a downturn, however. According to Doeringer and Piore (1971) large American firms with well-developed internal labor markets respond to a temporary decline in demand by laying off unskilled workers and letting their skilled workers take over unskilled tasks.

[11] There is some evidence that differential labour hoarding may dominate in the short run. Thus, using Dutch data from the 1990s, Gautier (2000) reports that the proportion of high-skill workers in low-skill positions falls in a recession.

from less prestigious institutions.

3.3 The case without overeducation

A corner solution without overeducation ($N_{HL} = 0$) can be obtained if the value of e_{HL}/e_{LL} is sufficiently high.[12] In this case high-skill workers will exert no effort when the wage rate satisfies the no-shirking condition for low-skill workers. Algebraically, we may have $e_{HL}\left(\delta+\rho+p\frac{H}{H-N_H-N_{HL}}\right) > e_{LL}\left(\delta+\rho+p\frac{L}{L-N_{LL}}\right)$, when $N_{HL} = 0$ and (N_H, N_L) satisfy firms' first order conditions. That is, we may have

$$e_{HH}\frac{\delta+\rho+p\frac{H}{H-N_H}}{\delta} = mAF_1(N_H, N_L) = w_H \tag{24}$$

$$e_{HL}\left(\delta+\rho+p\frac{H}{H-N_H}\right) > e_{LL}\frac{\delta+\rho+p\frac{L}{L-N_L}}{\delta} = mAF_2(N_H, N_L) = w_L \tag{25}$$

Now consider the effects of a change in aggregate employment in this "standard case" where all workers have the exact skills to match the requirements of their jobs. Using (20)-(21) and (24)-(25), tedious but straightforward calculations imply that

$$\begin{aligned}\frac{d\log\frac{w_H}{w_L}}{d\log n} &= \eta\frac{n}{N_L}\left[1-\frac{d\log N_H}{d\log n}\right]\\ &= \eta\frac{n}{N_L}\frac{(1-\theta)\left(h(\frac{H}{H-N_H})-h(\frac{L}{L-N_L})\right)}{\eta+(1-\theta)\,h(\frac{H}{H-N_H})+\theta h(\frac{L}{L-N_L})}\end{aligned} \tag{26}$$

where the function $h(x)$, $x > 1$, is defined by

$$h(x) = \frac{x^2}{\delta+\rho+px}\frac{x-1}{x}; h > 0, h' > 0 \text{ for } x > 1$$

The function h is increasing in the relevant range and it follows that the numerator of (26) is positive if and only if the employment rate for high-skill workers exceeds that

[12] Analogously, if e_{HL}/e_{LL} is sufficiently low and the supply of high-skill workers is large (relative to total employment, $N_H + N_L$), we get a corner solution with $N_{LL} = 0$.

A solution with $N_{HL} = 0$ may also obtain in the case where $e_{HL}/e_{HH} \geq 1$ if the supply of low-skill workers is sufficiently large. If high-skill workers find the effort associated with low-skill jobs more onerous than high-skill jobs then the no-shirking condition will require that they are paid a relatively high wage in low-skill jobs. If they were to be offered a low-skill job at this wage, they would be better off than in a high-skill job. If there are enough low-skill workers, however, the high value of the no-shirking wage w_{HL} will mean simply that firms fill all low-skill jobs with low-skill workers, paying $w_{LL} < w_{HL}$.

for low-skill workers ($N_H/H > N_L/L$). An increase in aggregate employment, in other words, will raise the relative wage of high-skill workers in the empirically relevant case where $N_H/H > N_L/L$. The reason is straightforward. With an unchanged relative wage, firms would choose the same proportional increase in employment for the two groups. This proportional increase in employment would cause a disproportionate decline in the unemployment rate for high-skill workers, who initially have the lowest unemployment rate, and, given the non-linearity of the wage equations (24)-(25), the relative wage for high-skill workers would have to rise. An increase in output, therefore, must lead to a rise in the relative employment of low-skill workers and a decline in their relative wage.

4 Shifts in relative labor supply

The existence of a wage premium serves as an incentive for workers to invest in skills. As a simple, reduced-form specification, we may assume that the relative labor supply H/L adjusts towards an equilibrium level $(H/L)^*$ determined by the relative wage w_{HA}/w_L :

$$\left(\frac{H}{L}\right)^* = \psi(\frac{w_{HA}}{w_L}), \psi' > 0 \tag{27}$$

The endogenization of relative skill supplies along these lines does not automatically eliminate overeducation since, as pointed out in section 3, large amounts of overeducation can be consistent with a relative wage w_{HA}/w_L that is significantly above one. Using the benchmark values in table 1 and assuming $e_{HH}/e_{LL} = 1$, for instance, the relative supply of high-skill workers would tend to increase - despite 30 percent overeducation - if $e_{HL}/e_{LL} \leq 0.6$ and $\psi(1.25) = 1$. Using equation (27) instead of an exogenous factor ratio merely generates long-run solutions for relative factor supplies as well as for employment rates, overeducation and wages.

Assuming that the model in section 2 describes the determination of relative wages for given factor supplies, one may ask whether slow adjustments in factor supplies will take the economy to a stationary solution satisfying (27). Proposition 3 suggests an affirmative answer.[13]

[13] The model in section 2 describes steady state solutions associated with given factor supplies. Even if the changes in relative supplies are slow, these steady state solutions for the relative wages may be misleading. Moreover, the adjustment process for relative factor supplies should take into account expected future changes in relative wages. The inverse relation between w_{HA}/w_L and H/L in proposition 3, therefore, is merely suggestive of a stable adjustment process.

Proposition 3 *Assume that the production function (1) exhibits constant returns. At an interior equilibrium with $e_{HH} > e_{HL}$ and $e_{LL} > e_{HL}$, an increase in the factor ratio H/L generates*

- *a decline in the wage ratios w_H/w_L and w_{HA}/w_L,*
- *a decline in the unemployment rates for high- and low-skill workers, u_H and u_L, as well as in the average unemployment rate u, and*
- *an increase in overeducation Ω.*

Proof: See Appendix 5.

The results in proposition 3 are quite intuitive. The increase in the supply of high-skill workers puts downward pressure on high-skill wages and sends more high-skill workers into jobs for which they are overeducated. Overeducation in turn represents a kind of hidden underemployment. It acts as a disciplining device and, as a result, less open unemployment is needed to prevent shirking. In the case of constant returns to (N_H, N_L) the downward pressure an average unemployment will be associated with declines in the unemployment rates for high- and low-skill workers as well as in the two wage ratios.

These conclusions with respect to u_H, u_L, w_H/w_L and w_{HA}/w_L need not hold under decreasing returns. Decreasing returns implies that the response of total employment to changes in the skill composition of the labor force may be quite small.[14] Since the weights in the expression for average unemployment ($u = \frac{H}{H+L}u_H + \frac{L}{H+L}u_L$) change when the composition of the labor force changes, a small decline in u can be associated with an increase in both u_H and u_L. Moreover, if both of these unemployment rates increase, the wage ratios may also go up. Thus, it may be useful to analyze the effects of changes in relative supplies from a slightly different angle.

The observable variables are employment and factor supplies, and section 3 analyzed the pure case of neutral changes in employment with factor supplies kept constant. I now examine the other pure case in which factor supplies change but aggregate employment is kept constant. This constancy of aggregate employment could be the result

[14] This is seen most clearly, perhaps, by considering a simple limiting case in which

$$Y = \min\{N_H, N_L, M\}$$

where M can be interpreted as some fixed resource constraint (land, for instance). In this limiting case, an increase in H has no effect on employment if the resource constraint is binding at the initial position.

of a combination of changes in relative labor supplies and neutral shocks to aggregate employment. However, the case of constant aggregate employment also covers a scenario without employment shocks but with extreme decreasing returns, as in footnote 14.

Consider first the effects on the pattern of employment of an increase in the proportion of high-skill workers, assuming that the economy is initially at an interior equilibrium and that $e_{HL} < e_{LL}$. Using equation (18) we know that $u_L > u_H$ and that u_L is a increasing function of u_H. Since

$$u = \frac{H}{H+L} u_H + \frac{L}{H+L} u_L$$

it therefore follows that if aggregate unemployment is kept constant, an increase in $H/(H+L)$ implies a rise in the unemployment rates of both low- and high-skill workers: as the composition of the labor force shifts toward to the group with a relatively low unemployment rate, the unemployment rates of both groups rise to compensate and keep aggregate unemployment constant. But equation (18) also implies that as u_H rises, the ratio u_L/u_H must go up. Thus, the increases in the group-specific unemployment rates are skewed toward the low-skill workers. In terms of employment prospects, low-skill workers are hurt *more* than high-skill workers by an increase in the relative supply of high-skill workers.

The presence of overeducation also affects the sensitivity of the relative wage to changes in the relative labor supply. Consider, for example, the implications of a change in the relative labor supply from $(H, L) = (1, 1)$ to $(H, L) = (0.5, 1.5)$, $(H, L) = (1.5, 0.5)$, $(H, L) = (1.9, 0.1)$ or $(H, L) = (1.95, 0.05)$. Using the parameter values in Tables 1 and 2 and assuming that the aggregate employment rate is kept constant, these massive changes in the skill composition of the labor force are reflected in the degree of overeducation and, to a lesser extent, within-group inequality. But, as indicated in Table 4, the wage ratio w_H/w_L need not decline monotonically as the proportion of high-skill workers increases. In fact, a 57-fold increase in H/L from $H/L = 1/3$ to $H/L = 19$, is associated with a rise in this wage ratio. The average wage premium w_{HA}/w_L, and hence the incentives to enter a training programme, also changes in a non-monotonic way, rising slightly as the relative supply increases from $H/L = 19$ to $H/L = 39$. Considering the magnitude of the changes in relative supply, moreover, the movements in in the wage ratios are quite modest. It should be noted, finally, that in the numerical examples the implications are strikingly similar for the Leontief and Cobb-Douglas specifications. This similarity is closely related to the other findings: if the wage ratio w_H/w_L (as determined by the non-shirking conditions) does not change much in the

Leontief case where N_H/N_L is fixed, firms will have little incentive to change their input proportions in the Cobb-Douglas case.

Proposition 4 summarizes these results:

Proposition 4 *Consider a combination of neutral employment shocks and changes in the relative labor supply that leave aggregate employment unchanged. At an interior equilibrium with $e_{HH} > e_{HL}$ and $e_{LL} > e_{HL}$, an increase in the relative supply of high-skill workers*

- *leads to a rise in the unemployment rates of both high- and low-skill workers.*
- *leads to an increase in the relative unemployment rate for low-skill workers, u_L/u_H.*
- *can in some cases lead to a rise in all three measures of wage inequality: $w_H/w_L, w_{HA}/w_L$ and σ.*

Table 4: Wage inequality for different values of the relative supply of labour
($\delta = 1, \rho = 0.1, p = 0.2; e_{HH}/e_{LL} = 2, e_{HL} = 0.4; n = 0.95$)

Leontief case ($Y = A\min\{3.684N_H, N_L\}$)

	Ω	w_H/w_L	w_{HA}/w_L	σ
$H = 0.5, L = 1.5$	0.04	1.78	1.645	0.18
$H = 1, L = 1$	0.30	1.61	1.255	0.24
$H = 1.5, L = 0.5$	0.55	1.74	1.205	0.27
$H = 1.9, L = 0.1$	0.74	1.95	1.213	0.33

Cobb-Douglas case ($Y = AN_H^{\alpha}N_L^{1-\alpha}; \alpha = 0.3048$)

	Ω	w_H/w_L	w_{HA}/w_L	σ
$H = 0.5, L = 1.5$	0.05	1.72	1.565	0.19
$H = 1, L = 1$	0.30	1.61	1.256	0.24
$H = 1.5, L = 0.5$	0.56	1.73	1.193	0.27
$H = 1.9, L = 0.1$	0.77	1.95	1.182	0.32
$H = 1.95, L = 0.05$	0.80	1.99	1.183	0.33

5 Concluding remarks

This paper has demonstrated, first, that the existence and persistence of overeducation can be explained by efficiency wage considerations. By construction, the relative supply of high-skill workers in the numerical examples in section 3.2 was such that had high-skill workers been precluded from low-skill jobs, their wage would have fallen below that of low-skill workers. High-skill workers therefore had an incentive to seek low-skill employment, employers had an incentive to hire them, and the efficiency-wage equilibrium was characterized by a wage premium to workers in high-skill jobs. This wage premium provides an incentive for workers to acquire the high skill, even though they face a risk of spending at least part of their working life in low-skill jobs. Thus, there is no reason to expect overeducation to be eliminated by endogenous changes in the relative supply of high-skill labor.

It has been shown, second, that the presence of overeducation may have profound effects on the reaction of the economy to different shocks. When calibrated to fit the amounts of overeducation found in most empirical studies, the model predicts that *both* the relative wage and the relative employment rate of high-skill workers will depend inversely on the aggregate rate of employment.[15] Induced changes in the degree of overeducation lie behind these results: an increase in aggregate employment pulls high-skill workers out of low-skill jobs and leads to a disproportionate increase in the employment rate for low-skill workers. The presence of overeducation also produces paradoxical effects following a change in relative labor supplies. Holding constant the average employment rate, an increase in the supply of high-skill labor hurts the employment prospects of low-skill workers, and in extreme cases the skill premium also increases.

The analysis, needless to say, has been based on a simplified model. Efficiency wage models are not the only explanations of structural unemployment, and it remains an open question whether induced changes in overeducation could play a similar role within alternative theoretical frameworks, including search and matching theories and insider/outsider models. The Shapiro-Stiglitz approach to efficiency wages, moreover, has been criticized by, among others, Carmichael (1985) for its exclusion of bonding or job selling. Other efficiency-wage theories, including Akerlof-type models of gift exchange or 'fair wages', are immune to the bonding critique. Without restrictions on the specification of the norms of fairness, however, it is almost too easy to generate

[15] Even if the degree of overeducation is insufficient to reverse the sign of $d\log\frac{w_H}{w_L}/d\log n$ and ensure that low-skill workers benefit form a rise in output, both in terms of relative employment and relative wages, the presence of induced overeducation will reduce the value of $d\log\frac{w_H}{w_L}/d\log n$.

induced overeducation in models of this kind.[16] Despite its potential vulnerability to the bonding critique, the Shapiro-Stiglitz setting therefore provides a more stringent test of the induced-overeducation hypothesis. Accepting a Shapiro-Stiglitz setup, however, one might question the restrictions $e_{HL} < e_{HH}$ and $e_{HL} < e_{LL}$ on the relative magnitudes of the direct utility costs associated with non-shirking. These restrictions - which are not, I believe, *a priori* implausible - were imposed because without them the efficiency wage model cannot capture the key stylized facts that (i) high-skill workers prefer high-skill jobs and (ii) low-skill workers have a relatively high unemployment rate. Thus, the two restrictions define the interesting, empirically relevant case.

It should be noted, finally, that the derivation of the non-shirking conditions assumed a steady state. The steady-state assumption could be relaxed along the lines of Kimball (1994), but the analysis would then need to consider the complications arising from different adjustment speeds for high- and low-skill employment in response to shocks. These complications associated with fluctuations in employment and differential labor hoarding become less important as the time frame is extended. Although it is ill-suited in its present form for the analysis of short term fluctuations, the model may therefore be relevant for medium and long-term changes in unemployment and wage inequality. Induced changes in the degree of overeducation may have contributed to the pattern of rising relative unemployment for low-skill workers and rising wage inequality, both within and between groups, that has been observed in a number of countries.

6 Appendices

6.1 Appendix 1

Using (10), (11), (13) and (16), we get

$$\frac{w_H}{w_L} = 1 + \frac{e_{HH} - e_{HL}}{e_{LL}} \frac{\delta + \rho + p\frac{H}{H-N_H}}{\delta + \rho + p\frac{L}{L-N_{LL}}}$$

[16] Skott (2005) introduces overeducation in a model in which workers' effort is related to the perceived fairness of wages. The main focus of this paper, however, is on the implications of endogenous changes in the norms of fairness.

Hence,

$$
\begin{aligned}
\frac{d\log\frac{w_H}{w_L}}{d\log n} &= \frac{d\log\left(1+\frac{e_{HH}-e_{HL}}{e_{LL}}\frac{\delta+\rho+p\frac{H}{H-N_H}}{\delta+\rho+p\frac{L}{L-N_{LL}}}\right)}{d\log n} \\
&= \frac{w_L}{w_H}\frac{e_{HH}-e_{HL}}{e_{LL}}\frac{\delta+\rho+p\frac{H}{H-N_H}}{\delta+\rho+p\frac{L}{L-N_{LL}}}\left(\begin{array}{c}\frac{d\log\left(\delta+\rho+p\frac{H}{H-N_H}\right)}{d\log n} \\ -\frac{d\log\left(\delta+\rho+p\frac{L}{L-N_{LL}}\right)}{d\log n}\end{array}\right) \\
&= \frac{w_L}{w_H}\frac{w_H-w_L}{w_L}\left(\begin{array}{c}\frac{p\frac{H}{(H-N_H)^2}N_H}{\delta+\rho+p\frac{H}{H-N_H}}\frac{d\log N_H}{d\log n} \\ -\frac{p\frac{L}{(L-N_{LL})^2}}{\delta+\rho+p\frac{L}{L-N_{LL}}}\frac{d(N_L-N_{HL})}{d\log n}\end{array}\right) \\
&= \frac{w_H-w_L}{w_H}\left(\begin{array}{c}\frac{p\frac{H}{(H-N_H)^2}N_H}{\delta+\rho+p\frac{H}{H-N_H}}\frac{d\log N_H}{d\log n}- \\ \frac{p\frac{L}{(L-N_{LL})^2}N_L}{\delta+\rho+p\frac{L}{L-N_{LL}}}\frac{d\log N_L}{d\log n}+\frac{p\frac{L}{(L-N_{LL})^2}N_{HL}}{\delta+\rho+p\frac{L}{L-N_{LL}}}\frac{d\log N_{HL}}{d\log n}\end{array}\right) \qquad \text{(a1)}
\end{aligned}
$$

To get an expression for $d\log N_{HL}/d\log n$, we combine equations (11)-(13) to get

$$
e_{HL}\frac{\delta+\rho+p\frac{H}{H-N_H-N_{HL}}}{\delta}=e_{LL}\frac{\delta+\rho+p\frac{L}{L-N_{LL}}}{\delta}
$$

Hence,

$$
\begin{aligned}
e_{HL}\frac{H}{H-N_H-N_{HL}}\frac{d\log(H-N_H-N_{HL})}{d\log n} &= e_{LL}\frac{L}{L-N_{LL}}\frac{d\log(L-N_{LL})}{d\log n} \\
&= e_{LL}\frac{L}{L+N_{HL}-N_L}\frac{d\log(L+N_{HL}-N_L)}{d\log n}
\end{aligned}
$$

or

$$
\begin{aligned}
& e_{HL}\frac{H}{(H-N_H-N_{HL})^2}(N_H\frac{d\log N_H}{d\log n}+N_{HL}\frac{d\log N_{HL}}{d\log n}) \\
= \; & e_{LL}\frac{L}{(L+N_{HL}-N_L)^2}(N_L\frac{d\log N_L}{d\log n}-N_{HL}\frac{d\log N_{HL}}{d\log n})
\end{aligned}
$$

Rearranging this equation, we get

$$
\frac{d\log N_{HL}}{d\log n}=\frac{1}{N_{HL}}\left(\begin{array}{c}\frac{e_{LL}\frac{L}{(L+N_{HL}-N_L)^2}}{e_{HL}\frac{H}{(H-N_H-N_{HL})^2}+e_{LL}\frac{L}{(L+N_{HL}-N_L)^2}}N_L\frac{d\log N_L}{d\log n} \\ -\frac{e_{HL}\frac{H}{(H-N_H-N_{HL})^2}}{e_{HL}\frac{H}{(H-N_H-N_{HL})^2}+e_{LL}\frac{L}{(L+N_{HL}-N_L)^2}}N_H\frac{d\log N_H}{d\log n}\end{array}\right) \qquad \text{(a2)}
$$

Substituting this expression into (a1) gives

$$\begin{aligned}
\frac{d\log\frac{w_H}{w_L}}{d\log n} &= \frac{w_H-w_L}{w_H}p\frac{1}{L}\left[\begin{array}{c} \left(\frac{\frac{L}{H}x^2}{\delta+\rho+px}-\frac{y^2}{\delta+\rho+py}\frac{e_{HL}\frac{L}{H}z^2}{e_{HL}\frac{L}{H}z^2+e_{LL}y^2}\right)N_H\frac{d\log N_H}{d\log n} \\ -\frac{y^2}{\delta+\rho+py}\left(1-\frac{e_{LL}y^2}{e_{HL}\frac{L}{H}z^2+e_{LL}y^2}\right)N_L\frac{d\log N_L}{d\log n}\end{array}\right] \\
&= \frac{w_H-w_L}{w_H}p\frac{1}{L}\left[\frac{\frac{L}{H}x^2}{\delta+\rho+px}N_H\frac{d\log N_H}{d\log n}-\frac{y^2}{\delta+\rho+py}\frac{e_{HL}\frac{L}{H}z^2}{e_{HL}\frac{L}{H}z^2+e_{LL}y^2}n\right] \\
&= En\left[B(x)\frac{N_H}{n}\frac{d\log N_H}{d\log n}-C(y)D(y,z)\right] \qquad \text{(a3)}
\end{aligned}$$

where

$$\begin{aligned}
x &= \frac{H}{H-N_H}=\frac{1}{u_H+\frac{N_{HL}}{H}} \\
y &= \frac{L}{L+N_{HL}-N_L}=\frac{1}{u_L} \\
z &= \frac{H}{H-N_H-N_{HL}}=\frac{1}{u_H} \\
E &= \frac{w_H-w_L}{w_H}p\frac{1}{L}>0 \text{ for } e_{HH}>e_{HL} \\
B(x) &= \frac{\frac{L}{H}x^2}{\delta+\rho+px}>0 \\
C(y) &= \frac{y^2}{\delta+\rho+py}>0 \\
D(y,z) &= \frac{e_{HL}\frac{L}{H}z^2}{e_{HL}\frac{L}{H}z^2+e_{LL}y^2}>0
\end{aligned}$$

6.2 Appendix 2

Profit maximization implies that

$$\frac{d\log\frac{w_H}{w_L}}{d\log\frac{N_L}{N_H}}=\eta$$

Using $n=N_H+N_L$ and thus $1=\frac{N_H}{n}\frac{d\log N_H}{d\log n}+(1-\frac{N_H}{n})\frac{d\log N_L}{d\log n}$, it follows that

$$\frac{d\log\frac{w_H}{w_L}}{d\log n}=\eta\frac{n}{N_L}(1-\frac{d\log N_H}{d\log n}) \qquad \text{(a4)}$$

Combining (a3)-(a4), we get

$$\eta\frac{n}{N_L}(1-\frac{d\log N_H}{d\log n})=En\left[B(x)\frac{N_H}{n}\frac{d\log N_H}{d\log n}-C(y)D(y,z)\right]$$

or

$$\frac{d\log N_H}{d\log n} = \frac{\frac{\eta}{N_L} + EC(y)D(y,z)}{\frac{\eta}{N_L} + EB(x)\frac{N_H}{n}} \tag{a5}$$

6.3 Appendix 3

Proof of proposition 1:

The results about the levels of the relative wage and relative unemployment and about the effects on changes in aggregate employment on relative unemployment follow directly from equations (10)-(13), (16) and (18).

To derive the relative-wage effects of changes in aggregate employment, note first that from (22)-(23) it follows that a general rise in economic activity will reduce wage inequality if and only if

$$EC(y)D(y,z) > EB(x)\frac{N_H}{n} \tag{a6}$$

The value of $D(y,z)$ is decreasing as a function of the ratio e_{HL}/e_{LL}, and we have $D(y,z) = \frac{L}{L+H}$ for $e_{HL} = e_{LL}$. To see this, note that at an interior solution we have

$$e_{HL}(\delta + \rho + pz) = e_{LL}(\delta + \rho + py) \tag{a7}$$

and

$$z = \frac{e_{LL}(\delta + \rho + py) - e_{HL}(\delta + \rho)}{e_{HL}p}$$

Hence,

$$e_{HL}\frac{L}{H}z^2 = \frac{L}{H}\frac{1}{p^2}\frac{1}{e_{HL}}[e_{LL}(\delta + \rho + py) - e_{HL}(\delta + \rho)]^2 \tag{a8}$$

For $e_{HL} = e_{LL}$ we get $z = y$ (using (a7)), and from the definition of $D(y,z)$ in combination with it follows (using (a8)) that D is decreasing in e_{HL} and (using (a7)) that $D(y,z) = L/(L+H)$ if $e_{HL} = e_{LL}$.

B, C and D are all positive and E is positive when $e_{HH} > e_{HL}$. Thus, using (a6) and given the assumptions $e_{HH} > e_{HL}$ and $e_{LL} > e_{HL}$, we have the following sufficient condition for an inverse relation between the relative wage w_H/w_L and aggregate employment:

$$C(y)\frac{L}{L+H} > B(x)\frac{N_H}{n}$$

or

$$\frac{\frac{N_H}{H}\left(1 - \frac{N_H}{H}\right)^{-2}}{\delta + \rho + p\left(1 - \frac{N_H}{H}\right)^{-1}} < \frac{\frac{N_{LL}}{L}\left(1 - \frac{N_{LL}}{L}\right)^{-2}}{\delta + \rho + p\left(1 - \frac{N_{LL}}{L}\right)^{-1}}\frac{n}{H+L}\frac{L}{N_{LL}} \tag{a9}$$

The function $f(x) = x(1-x)^{-2}/(\delta + \rho + p(1-x)^{-1})$ is increasing in x for $x < 1$. It follows, therefore, from (a9) that $N_H/H < N_{LL}/L < n/(H+L)$ is a sufficient condition for $d \log \frac{w_H}{w_L} / d \log n$ to be negative.

The second of these inequalities is always satisfied when $e_{HL} < e_{LL}$ since in this case the employment rate for high-skill workers $((N_H + N_{HL})/H)$ will exceed that for low-skill workers (cf. equation 18), and the average employment rate $(n/(H+L)$ is a weighted average of the employment rates for the two groups. The first inequality will be satisfied as long as the degree of overeducation exceeds a critical value given by $\Omega > \Omega^{crit} = (1 - N_{LL}/L)/(1 + N_{LL}/H)$ where $\Omega = N_{HL}/(N_H + N_L)$ is the degree of overeducation. To see this, observe that

$$N_{HL} = \Omega(N_H + N_L) = \Omega\,(N_H + N_{HL} + N_{LL})$$

and

$$(1-\Omega)(H - N_H) \geq (1-\Omega)N_{HL} = \Omega(N_H + N_{LL})$$

Hence,

$$\frac{N_H}{H} \leq 1 - \Omega(1 + \frac{N_{LL}}{H})$$

and

$$\frac{N_{LL}}{L} > \frac{N_H}{H}$$

will be satisfied for

$$\Omega > \frac{1 - \frac{N_{LL}}{L}}{1 + \frac{N_{LL}}{H}}$$

6.4 Appendix 4

Proof of Proposition 2:

By definition,

$$w_{HA} = \frac{N_H}{N_H + N_{HL}} w_H + \frac{N_{HL}}{N_H + N_{HL}} w_L$$

Hence,

$$\frac{w_{HA}}{w_L} = 1 + \frac{N_H}{N_H + N_{HL}} (\frac{w_H}{w_L} - 1)$$

and

$$\begin{aligned}
\frac{d\log\frac{w_{HA}}{w_L}}{d\log n} &= \frac{w_L}{w_{HA}}\frac{N_H}{N_H+N_{HL}}(\frac{w_H}{w_L}-1)\left(\frac{d\log\left(\frac{N_H}{N_H+N_{HL}}\right)}{d\log n}+\frac{d\log(\frac{w_H}{w_L}-1)}{d\log n}\right) \\
&= \frac{w_L}{w_{HA}}\frac{N_H}{N_H+N_{HL}}\left[\frac{w_H-w_L}{w_L}\left(\begin{array}{c}\frac{d\log N_H}{d\log n}-\frac{N_H}{N_H+N_{HL}}\frac{d\log N_H}{d\log n} \\ -\frac{N_{HL}}{N_H+N_{HL}}\frac{d\log N_{HL}}{d\log n}\end{array}\right)+\frac{w_H}{w_L}\frac{d\log\frac{w_H}{w_L}}{d\log n}\right] \\
&= \frac{w_L}{w_{HA}}\frac{N_H}{N_H+N_{HL}}\left[\frac{w_H-w_L}{w_L}\frac{N_{HL}}{N_H+N_{HL}}\left(\frac{d\log N_H}{d\log n}-\frac{d\log N_{HL}}{d\log n}\right)+\frac{w_H}{w_L}\frac{d\log\frac{w_H}{w_L}}{d\log n}\right]
\end{aligned}$$

Substituting from (a2), we get

$$\frac{d\log\frac{w_{HA}}{w_L}}{d\log n}=\frac{w_L}{w_{HA}}\frac{N_H}{N_H+N_{HL}}\left[\begin{array}{c}\frac{w_H-w_L}{w_L}\frac{N_{HL}}{N_H+N_{HL}}\left(\begin{array}{c}\left(1+D(y,z)\frac{N_H}{N_{HL}}\right)\frac{d\log N_H}{d\log n} \\ -(1-D(y,z))\frac{N_L}{N_{HL}}\frac{d\log N_L}{d\log n}\end{array}\right) \\ +\frac{w_H}{w_L}\frac{d\log\frac{w_H}{w_L}}{d\log n}\end{array}\right]$$

and, using $\frac{d\log N_L}{d\log n}=\frac{n}{N_L}1-\frac{N_H}{N_L}\frac{d\log N_H}{d\log n}$,

$$\begin{aligned}
\frac{d\log\frac{w_{HA}}{w_L}}{d\log n} &= \frac{w_L}{w_{HA}}\frac{N_H}{N_H+N_{HL}} \\
&\quad \left\{\begin{array}{c}\frac{w_H-w_L}{w_L}\frac{N_{HL}}{N_H+N_{HL}}\left[\begin{array}{c}\left(1+D(y,z)\frac{N_H}{N_{HL}}+(1-D(y,z))\frac{N_H}{N_{HL}}\right)\frac{d\log N_H}{d\log n} \\ -(1-D(y,z))\frac{N_L}{N_{HL}}\frac{n}{N_L}\end{array}\right] \\ +\frac{w_H}{w_L}\frac{d\log\frac{w_H}{w_L}}{d\log n}\end{array}\right\} \\
&= \frac{w_L}{w_{HA}}\frac{N_H}{N_H+N_{HL}}\left\{\begin{array}{c}\frac{w_H-w_L}{w_L}\left[\frac{d\log N_H}{d\log n}-(1-D(y,z))\frac{n}{N_H+N_{HL}}\right] \\ +\frac{w_H}{w_L}\frac{d\log\frac{w_H}{w_L}}{d\log n}\end{array}\right\} \qquad \text{(a10)}
\end{aligned}$$

The term in square brackets is positive if the conditions for $\frac{d\log\frac{w_H}{w_L}}{d\log n}$ to be negative are met. To see this, note first that $\frac{d\log N_H}{d\log n}>1$ when $\frac{d\log\frac{w_H}{w_L}}{d\log n}<0$. The term $D(y,z)$, second, is greater than or equal to $\frac{L}{L+H}$ (cf. Appendix 3) and hence

$$(1-D(y,z))\frac{n}{N_H+N_{HL}}<\frac{H}{L+H}\frac{n}{N_H+N_{HL}}<1$$

where the last inequality follows from the fact that the overall employment rate $(\frac{n}{L+H})$ is a weighted average of the employment rates for the two groups. Since, by assumption, the utility costs of effort are such that high-skill workers have the higher employment rate, it follows that $\frac{N_H+N_{HL}}{H}>\frac{n}{L+H}$.

Using (a4)-(a5) it is readily seen that $\frac{d\log\frac{w_H}{w_L}}{d\log n} \to 0$ for $\eta \to 0$. The term in square brackets on the right hand side of (a10), on the other hand is bounded above zero. It follows that $\frac{d\log\frac{w_{HA}}{w_L}}{d\log n}$ will be positive for sufficiently small values of η. But both $\frac{d\log N_H}{d\log n}$ and $\frac{d\log\frac{w_H}{w_L}}{d\log n}$ are increasing in η - use (a4)-(a5) - and the numerical examples in section 3.2 demonstrate that, depending on parameter values and initial employment values, $\frac{d\log\frac{w_{HA}}{w_L}}{d\log n}$ may be either positive or negative for positive values of η.

With respect to within-group inequality we have

$$\frac{d\sigma}{d\log n} = \sigma[\frac{d\log\frac{w_{HA}-w_L}{w_{HA}}}{d\log n} + 0.5(\frac{d\log N_{HL}}{d\log n} - \frac{d\log N_H}{d\log n})]$$

Using equation (a2) this can be rewritten

$$\begin{aligned}\frac{d\sigma}{d\log n} &= \sigma[\frac{d\log\frac{w_{HA}-w_L}{w_{HA}}}{d\log n} + 0.5(\frac{1}{N_{HL}}((1-D(y,z))n - N_H\frac{d\log N_H}{d\log n}) - \frac{d\log N_H}{d\log n})] \\ &= \sigma\left[\begin{array}{c} \frac{w_L}{w_{HA}-w_L}\frac{d\log\frac{w_{HA}}{w_L}}{d\log n} + 0.5(1-D(y,z))\frac{n}{N_{HL}} - \\ 0.5\frac{N_H+N_{HL}}{N_{HL}}\frac{d\log N_H}{d\log n}\end{array}\right] \qquad \text{(a11)}\end{aligned}$$

Since $D \geq L/(L+H)$ and $(N_H+N_{HL})/H \geq n/(L+H)$ (cf. above) we then get

$$\frac{d\sigma}{d\log n} \leq \sigma[\frac{w_L}{w_{HA}-w_L}\frac{d\log\frac{w_{HA}}{w_L}}{d\log n} + 0.5\frac{N_H+N_{HL}}{N_{HL}}(1-\frac{d\log N_H}{d\log n})]$$

Now, $d\log N_H/d\log n > 1$ if $d\log(w_{HA}/w_L)/d\log n < 0$ and it follows that if between-group inequality is inversely related to aggregate employment, the relation between within-group inequality and employment will also be inverse.

6.5 Appendix 5

Proof of proposition 3:

An increase in H/L will generate an increase in w_L. To see this, assume the contrary, that is, assume that w_L falls. By assumption there are constant returns to labor and the high-skill wage w_H therefore must increase if w_L falls. It now follows that we get an increase in N_L/N_H (using (17)), an increase in u_H and u_L (using (12) and (18)), and an increase in N_H/H (using (10)-(11) and the rise in $w_H - w_L$). An increase in $N_L/N_H, N_H/H, u_L$ and H/L, however, implies that $\frac{N_{LL}}{H} = \frac{N_{LL}}{L}\frac{L}{H}$ will fall and that $\frac{N_L}{H} = \frac{N_L}{N_H}\frac{N_H}{H}$ will rise. Hence, $\frac{N_{HL}}{H} = \frac{N_L}{H} - \frac{N_{LL}}{H}$ will rise. But a simultaneous increase in both N_H/H and N_{HL}/H is inconsistent with a rise in high-skill unemployment u_H.

Thus, we have a contradiction and it follows that w_L must rise following an increase in H/L.

Using the result that w_L must increase, the statements in the proposition now follow:

- the fall in w_H and w_H/w_L follows from firms' first order conditions and the linear homogeneity of the production function
- the fall in u_H and u_L follows from the no-shirking conditions (11)-(12). The fall in average unemployment, $u = \frac{H}{H+L}u_H + \frac{L}{H+L}u_L$, follows from the fall in both u_H and u_L and the result that $u_L > u_H$ (from equation (18)).
- the fall in $\frac{w_{HA}}{w_L} = \frac{N_H}{N_H+N_{HL}}\frac{w_H}{w_L} + 1$ follows from the decline in w_H/w_L and an increase in N_{HL}/N_H. To get the latter result, note that $\frac{N_{HL}}{N_H} = \frac{N_{HL}}{H}\frac{H}{N_H}$ and that N_H/H must fall (using (12) and the decline in $w_H - w_L$) while N_{HL}/H must rise (since both u_H and N_H/H fall).
- the rise in overeducation $\Omega = \frac{N_{HL}}{N_H+N_L} = \frac{N_{HL}/N_H}{1+N_L/N_H}$ follows from the rise in N_{HL}/N_H and fall in N_L/N_H.

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