Astronomy Department Faculty Publication Series   Astronomy

1997

A rigorous reanalysis of the IRAS variable population: Scale lengths, asymmetries, and microlensing

S Nikolaev

MD Weinberg

Follow this and additional works at: https://scholarworks.umass.edu/astro_faculty_pubs

Part of the Astrophysics and Astronomy Commons

Recommended Citation

Nikolaev, S and Weinberg, MD, "A rigorous reanalysis of the IRAS variable population: Scale lengths, asymmetries, and microlensing" (1997). *ASTROPHYSICAL JOURNAL*. 74.
10.1086/304639

A rigorous reanalysis of the IRAS variable population: scale lengths, asymmetries, and microlensing

Sergei Nikolaev & Martin D. Weinberg[1]

Department of Physics & Astronomy

University of Massachusetts, Amherst, MA 01003-4535

---

[1]Alfred P. Sloan Fellow.

ABSTRACT

Previous work reported a bar signature in color-selected IRAS variable stars. Here, we estimate the source density of these variables while consistently accounting for spatial incompleteness in data using a likelihood approach. The existence of the bar is confirmed with shoulder at $a \approx 4$ kpc, axis ratio $a : b = 2.2$ – 2.7 and position angle of $19° \pm 1°$ degrees. The ratio of non-axisymmetric to axisymmetric components gives similar estimate for the bar size $a = 3.3 \pm 0.1$ kpc and position angle $\phi_0 = 24° \pm 2°$. We estimate a scale length 4.00 ± 0.55 kpc for the IRAS variable population, suggesting that these stars represent the old disk population.

We use this density reconstruction to estimate the optical depth to microlensing for the large-scale bar in the Galactic disk. We find an enhancement over an equivalent axisymmetric disk by 30% but still too small to account for the MACHO result. In addition, we predict a significant asymmetry at positive and negative longitudes along lines of sight through the end of the bar ($|l| \approx 30°$) with optical depths comparable to that in Baade's window. An infrared microlensing survey may be a sensitive tool for detecting or constraining structural asymmetries.

More generally, this is a pilot study for Bayesian star count analyses. Bayesian approach allows the assessment of prior probabilities to the unknown parameters of the model; the resulting likelihood function is straightforwardly modified to incorporate all available data. However, this method requires the evaluation of multidimensional density functions over the data and optimization of the function over a parameter space. We address the resulting computational extremization problem with a hybrid use of a directed search algorithm which locates the global maximum and the conjugate gradient method which converges

quickly near a likelihood maximum. Both methods are parallelizable and therefore of potential use with very large databases.

Subject headings: Galaxy: structure — stars: variables: other — stars: AGB and post-AGB — gravitational lensing — methods: statistical

1. Introduction

Weinberg (1992, Paper I) identified color-selected variables in the IRAS Point Source Catalog (PSC) with AGB stars based on color consistency and the circumstantial sensitivity of the IRAS survey to long-period variables (cf. Harmon & Gilmore 1988). These were then used as rough standard candles to infer a large-scale asymmetry in the stellar distribution. The identification of IRAS variables with AGB stars was strengthened by an in-depth study of a bright subset (Allen, Kleinmann & Weinberg 1993). Carbon-selected AGB stars (carbon stars) have also proven to be effective tracers (see e.g. Metzger & Schechter 1994). Advantages of AGB tracers are reviewed in Weinberg (1994). In general, standard candle analyses have the advantage over flux or star count analyses in providing direct information about the three-dimensional structure of the Galaxy. However, uncertainties in their selection and intrinsic properties may bias any inference and, especially for the IRAS-selected sample, the census is incomplete.

Paper I described an approach to large-scale Galactic structure using a star count analysis which allows the information to be reconstructed and possibly corrected in the observer's coordinate system before translating to a Galactocentric system. Unfortunately, this translation approach is only natural if the coverage is complete and suffered in application to the IRAS sample because of spatial gaps due to an incomplete second full-sky epoch. Here, we present the results of a different approach to the problem: the direct density estimation by maximum likelihood. A Bayesian density estimation has the advantage of directly incorporating selection effects and missing data.

The number of ongoing surveys that bear on Galactic structure—SDSS, 2MASS, DENIS—which at various stages will have surveyed parts of the sky is a

second motivation for this study; there is a need for a systematic method suited to inferential studies using possibly incomplete data from many wave bands. Recent analyses (e.g. Bahcall & Soneira 1980 in the optical; Wainscoat et al. 1992 in the infrared) have modeled the Galactic components with standard profiles and structural parameters chosen to provide a match to star count data. To explore the structural parameters themselves, we propose a Bayesian density estimation technique to treat data from scattered fields during the survey and to easily incorporate data from wave bands. Conceptually, this approach is midway between a classical inversion and modeling.

The first part of the paper describes and characterizes the method. More specifically, §2 reviews the IRAS selection procedure described in Paper I and motivates the approach. The new analysis based on statistical density estimation is presented in §3 and precisely defined in §4. The second part of the paper describes Monte-Carlo tests and the results of applying the method to the IRAS data (§5). We conclude in §6 with a summary and discussion.

2. IRAS source selection

The analysis in Paper I was based on the variables selected in the IRAS Point Source Catalog (1988) by both color and P_{var}. Following the source selection procedure described in Paper I, we selected stars from IRAS Point Source Catalog with $F_{12} > 2$ Jy and variability flag $P_{var} \geq 98\%$. Although the flux limit reduces the confusion in source identification toward the center of the Galaxy, it also restricts the sensitivity to distant sources. The limiting distance to a star (d) is estimated using a simple exponential layer with vertical scale height h and mid-plane extinction coefficient K_{12}:

$$m = M + 5 \lg d - 5 + K_{12} h \left(1 - e^{-d \sin |b|/h}\right) / \sin |b|. \qquad (1)$$

For a typical AGB star ($L = 3500L_\odot$, see Appendix A) and $K_{12} = 0.18$ kpc^{-1}, the limiting distance in the plane is $R_{lim} = 7$ kpc. We assume that the extinction is dominated by the molecular gas, $h = 100$ pc and the extincting layer is horizontally isotropic. The true extinction toward the inner Galaxy is most likely dominated by the molecular ring and nuclear region given the molecular gas distribution. However, precise estimate of the true distribution is not available and an horizontally isotropic model will adequately represent its systematic effect on the photometric distances.

Of the more than 158,000 good flux-quality sources listed in IRAS PSC, 5,736 satisfy both flux limit and variability criteria. Their spatial distribution is shown in Figure 1. To obtain variability data, at least two epochs are needed. Unfortunately, IRAS' multiple epochs did not have complete sky coverage. Most of the coverage (77% in the galactic plane) was achieved in HCON 2 and HCON 3 separated by roughly 7.5 months on average. The rest of the galactic plane is poorly sampled (shaded regions in Figure 1). For this analysis, all the data in the poorly sampled sectors have been excised, reducing the size of the sample to 5,500 stars.

3. Method overview

All of the selection effects but especially data incompleteness greatly complicate the analysis. Bayesian techniques are ideally suited to parameter estimation over data with general but well-defined selection criteria and underlies both the maximum entropy and maximum likelihood procedures. Below, we will parameterize the source density by an exponentiated orthogonal series with unknown coefficients A_{ij} and B_{ij} (cf. eq. 14). In this context, the basic theorem

of the theory reads:

$$P\left(\{A_{ij}\},\,\{B_{ij}\}\,|\,D,\,I\,\right)=\frac{P\left(\{A_{ij}\},\,\{B_{ij}\}\,|\,I\,\right)\cdot P\left(D\,|\,\{A_{ij}\},\,\{B_{ij}\},\,I\,\right)}{P\left(D\,|\,I\,\right)}. \quad (2)$$

The probability $P\left(\{A_{ij}\},\,\{B_{ij}\}\,|\,D,\,I\,\right)$ is the conditional (or *posterior*) probability of the coefficients of the source density provided the data (D) and information (I) describing its incompleteness. The probability $P\left(\{A_{ij}\},\,\{B_{ij}\}\,|\,I\,\right)$ is the prior probability (or simply, *prior*) of the coefficients provided only the information. Following Bretthorst (1990), we assign the prior using the maximum entropy principle. In our case it is constant implying that all coefficient values are equally likely initially. The function $P\left(D\,|\,\{A_{ij}\},\,\{B_{ij}\},\,I\,\right)$ is the direct probability which describes the likelihood of data given the coefficients. Finally, $P\left(D\,|\,I\,\right)$ is a normalization constant which may be omitted provided that the posterior probability is normalized.

With these definitions, it follows that

$$P\left(\{A_{ij}\},\,\{B_{ij}\}\,|\,D,\,I\,\right)=\mathrm{Const}\cdot P\left(D\,|\,\{A_{ij}\},\,\{B_{ij}\},\,I\,\right), \quad (3)$$

or in words, the posterior probability is proportional to the likelihood function. Therefore, the best estimate of posterior probability is obtained for the set coefficients which maximize the likelihood function.

4. Likelihood function

The likelihood is the joint probability of the observed stars given a source density. We may then consider the probability of observing a star with intrinsic luminosity in the range $(L, L + dL)$ to be detected in the distance interval $(s, s + ds)$, in the azimuth interval $(l, l + dl)$, in the galactic latitude interval $(b, b + db)$ and with magnitude in the range $(m, m + dm)$. Assuming a normal

distribution of intrinsic luminosities L and a normal error distribution for the apparent magnitudes m this becomes:

$$P_n \left(s, l, b, m, L \,|\, \sigma_m, \sigma_L, K_{12}, h, R_0 \right) s^2 \, ds \, \cos b \, db \, dl \, dL \, dm =$$
$$C \cdot \Sigma(r, \phi, z) \, e^{-(L-\overline{L})^2/2\sigma_L^2} \, e^{-(m-\overline{m})^2/2\sigma_m^2} \, s^2 \, ds \, \cos b \, db \, dl \, dL \, dm. \qquad (4)$$

Here s, l, b are coordinates about the observer's position, r, ϕ, z are coordinates about the center of the Galaxy, C is the normalization constant, $\Sigma(r, \phi, z)$ is the source density at galactocentric radius R_0, $\overline{L}$ and σ_L are the mean intrinsic luminosity and the dispersion of the sample, σ_m is the measurement error in magnitudes and $\overline{m} = \overline{m}(s, b)$ is given by equation (1). Alternatively, we may replace luminosity by absolute magnitude:

$$P_n \left(s, l, b, m, M \,|\, \sigma_m, \sigma_M, K_{12}, h, R_0 \right) s^2 \, ds \, \cos b \, db \, dl \, dM \, dm =$$
$$C \cdot \Sigma(r, \phi, z) \, e^{-(M-\overline{M})^2/2\sigma_M^2} \, e^{-(m-\overline{m})^2/2\sigma_m^2} \, s^2 \, ds \, \cos b \, db \, dl \, dM \, dm, \qquad (5)$$

where $\overline{M}$ and σ_M correspond to $\overline{L}$ and σ_L. The Gaussian distributions in L or M in the above two equations can be generalized to an arbitrary luminosity function for traditional star count applications. Although we will not give the general expressions below, the development is parallel.

Since the convolution of two Gaussians is a new Gaussian whose variance is the sum of the two individual variances

$$\sigma_{m,eff}^2 = \sigma_m^2 + \sigma_M^2, \qquad (6)$$

equation (5) can be rewritten as

$$P_n \left(s, l, b, m \,|\, \sigma_{m,eff}, k, H, R_0 \right) s^2 \, ds \, \cos b \, db \, dl \, dm =$$
$$C \cdot \Sigma(r, \phi, z) \, e^{-(m-\overline{m})^2/2\sigma_{m,eff}^2} \, s^2 \, ds \, \cos b \, db \, dl \, dm \qquad (7)$$

after integrating over the unmeasured absolute magnitude M. For notational clarity, we will omit the subscript "eff" and write simply σ_m. The constant C is determined from the normalization condition:

$$C \int_{-\infty}^{+\infty} e^{-(m-\overline{m})^2/2\sigma_m^2} \, dm \int dl \int_0^{s_{max}(b)} s^2 \, ds \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \Sigma(r, \phi, z) \, \cos b \, db = 1. \qquad (8)$$

The integration over l runs over entire circle except missing azimuthal sectors, explicitly accounting for missing data at particular ranges in azimuth. The limiting distance s_{max} in the l, b direction incorporates the 2 Jy flux limit.

In a standard star count analysis no explicit distance information is provided and s is eliminated from analysis by integration, yielding

$$P_n(l, b, m \mid \ldots) \, \cos b \, db \, dl \, dm =$$
$$C \int_0^{s_{max}(b)} \Sigma(r, \phi, z) \, e^{-(m-\overline{m})^2/2\sigma_m^2} \, s^2 \, ds \cos b \, db \, dl \, dm. \qquad (9)$$

For our relatively small sample of IRAS stars, sensitivity to vertical structure will be poor. This motivates replacing the general unknown three-dimensional disk density with a density which depends on radial position and azimuth alone: $\Sigma(r, \phi, z) = \overline{\Sigma}(r, \phi)$.

Finally, the joint probability of observing N stars selected from the IRAS PSC is

$$L \equiv P_{total} = \prod_{n=1}^{N} P_n(l, b, m \mid \ldots). \qquad (10)$$

Expressing the likelihood function in logarithmic form, our desired solution is the set of parameters which maximize

$$\log L = \sum_{n=1}^{N} \log P_n(l, b, m \mid \ldots). \qquad (11)$$

This and nearly all star count analyses reduce to standard problem of density estimation: find the density function $f(x)$, which satisfies non-negativity

constraint

$$f(x) \geq 0 \qquad (12)$$

and integral constraint

$$\int f(x)dx = 1 \qquad (13)$$

which best describes the observed data distribution. Both parametric and non-parametric estimation techniques have been used to solve this problem (e.g. Silverman 1986; Izenman 1991). For inhomogeneous multidimensional data, the positivity constraint is cumbersome. However, searching for the unknown function $f(x)$ in the form of an exponentiated orthogonal series (Clutton-Brock 1990), guarantees positivity. A candidate stellar surface density is:

$$\overline{\Sigma}(r,\phi) = \exp\left\{ \sum_{i=1}^{i_{max}} \sum_{j=0}^{j_{max}} [A_{ij} \cos j\phi + B_{ij} \sin j\phi] \, J_j(k_i^j r) \right\}, \qquad (14)$$

where $J_j(x)$ is Bessel function of jth order and k_i^j is i^{th} root of Bessel function of jth order and are chosen to produce a complete orthogonal set over the disk of radius R_{max}. The coefficients A_{ij}, B_{ij} are the parameters to be determined. There is no loss of generality in taking the Fourier-Bessel series although the choice is arbitrary.

5. Results

5.1. Sensitivity to incompleteness

A major advantage of the approach presented here over that in Paper I is that the significance of inferred structure is robustly quantified. In particular, we can test the sensitivity of selection effects to the detection of a bar. To test the presence of the coverage gaps, we generated four sample disks of 1,000 stars each using the source density (14) with $\sqrt{A_{ij}^2 + B_{ij}^2} = 1$ for $j = 0, 2$ and zero

otherwise and the following bar position angles: $0°$, $±45°$, and $90°$. The root sum square of the coefficients A_{ij} and B_{ij} represents the strength of ith radial component for the jth polar harmonic. Figure 3 shows the restored strength of a harmonic $\sqrt{A_{ij}^2 + B_{ij}^2}$ as a function of the position angle of the bar. Insensitivity of these strengths to bar position angle suggests that missing azimuths will not obscure the inference of true bar. The computed values are consistent with the expected value of unity.

Conversely, regions of missing data can produce non-axisymmetric distortions, and in principle, suggest the existence of a bar in initially axisymmetric sample. However, analysis of a simulated axisymmetric disk ($A_{10} = A_{20} = 1$; all others $= 0$) and the same azimuthal incompleteness as in the real sample shows that the power in the non-axisymmetric harmonics is about 3% of the axisymmetric contribution. Together these tests suggest that the misidentification of a bar relative due to missing azimuthal sectors alone is unlikely.

5.2.   Application to IRAS data

The formalism developed in §4 requires the distance to galactic center R_0, extinction in the plane K_{12} and average luminosity of the AGB stars $\overline{L}$. We adopted $R_0 = 8.0$ kpc, $K_{12} = 0.18$ mag/kpc and $\overline{L} = 3500 L_\odot$. The method can be straightforwardly modified for complex models (e.g. patchy or non-uniform extinction), the only limitation here is the CPU available and sufficient data to attain a satisfactory measure of confidence.

Choosing the truncation of the series in equation (14) poses a problem common to many non-parametric density estimations: because too few terms result in large bias and too many terms increase variance, i_{max}, j_{max} would

be best determined by jointly minimizing the bias and the variance. However, this approach is computationally prohibitive due to the integral in equation (9) and the normalization (8). Therefore, a heuristic approach was adapted in selecting i_{max}, j_{max} based on the increase in the likelihood function when a particular term or set of terms is added. Significance could be quantified in terms of the likelihood ratio (Wilks 1962) but we have not done this here. In addition, the hardware available to us makes it impossible to sample the parameter space beyond $i_{max} = 4$, $j_{max} = 4$. Nevertheless, up to that limit, the space was sampled thoroughly, with some of the solutions shown in Figure 4 along with the corresponding offsets of the likelihood function (the lowest value of likelihood is set to 0 for ease in comparison). Some of the figures feature the ghost peaks due to the absence of data beyond the galactic center or in missing azimuthal sectors (see Figs. 1 and 2). The likelihood analysis may attempt to place a non-existing source density peak in that region, provided it will increase the overall score. We will pursue penalizing the likelihood function and other procedures for choosing an alternative prior (dropping the assumption that all coefficients in (14) are equally likely initially) in future work.

More importantly, all reconstructions in Figure 4 imply a jet-like feature in the first quadrant. As in Paper I, the depth of our sample (estimated to correspond to a mean distance of 7 kpc in the plane) prevents ascertaining whether this feature corresponds to a bisymmetric bar or is a lopsided distortion. However, decreasing the flux limit to 1 Jy leads to detection of similar feature on the far side of the Galaxy, suggesting a real bar. This motivates a reconstruction with enforced bisymmetry, shown in Figure 5. Here the corresponding prior assigns zero values to coefficients of odd azimuthal order. The likelihood value (the origin is the same as in Figure 4) has dropped substantially, because the

resulting density lacks data support beyond the Galactic center. In both figures, the bar is well defined and has a similar length and position angle.

To quantify the strength and position angle of the bar, we fitted the isodensity contours ($i_{max} = j_{max} = 4$) by ellipses. The logarithm of a suitable likelihood function for estimating the semi-major axes, eccentricity and position angle is

$$\log L = \sum_{i=1}^{M} \left[\Sigma_{rec}(r_i, \phi_i) - C \right]^2, \tag{15}$$

where $\Sigma_{rec}(r, \phi)$ is the reconstructed density function and C is isodensity level. The summation runs over equally spaced points on ellipse. For a given ellipse, a grid of semimajor axis values are specified and the surface density C, position angle ϕ_0 and eccentricity e which maximizes $\log L$ are found. The results are presented in Figures 6 and 7.

Figure 6 indicates that the density profile drops to half of its central value at about 4 kpc. The half-length would then be about 4 kpc, in good agreement with the value obtained in Paper I. If we take this value as the size of the major axis of the bar, then the axis ratio varies from 2.2 in the central regions to 2.7 in the outer regions of the bar. The value of the position angle for the entire extent of the bar (out to 4 kpc) is $\approx 19°$. The accuracy of the position angle determination can be quantified in terms of confidence interval, making use of the fact that in the limit of large number of sources N, the likelihood in n dimensions is distributed as $\chi^2/2$ with n degrees of freedom (e.g. Lehmann 1959). We analyzed the likelihood as the function of a single variable – orientation angle of the bar in the plane. The analysis gives the uncertainty of $1°$ at 3σ level.

Another way to determine the parameters of the bar is to look at the map of the ratio of non-axisymmetric to axisymmetric components of the density. The

ratio displays two peaks at 3.3 ± 0.1 kpc located on the opposite sides from the center, the line connecting them has the position angle of $\sim 24° \pm 2°$. The peak ratio, the relative strength of the bar, is 0.73. This implies the existence of a strong bar in the intermediate age population responsible for the AGB stars.

5.3. Disk scale length

Having calculated the source density, we are in a position to characterize the parent population of the IRAS variables. In Paper I, we assumed that these variables represented a disk population based on their flux distribution but several colleagues have suggested in discussion that the IRAS variables are more likely to be bulge stars. Here, we determine the scale length of the population in the Galactic plane. For comparison, we fit our reconstruction by an oblate spheroid model (G0 bulge model from the DIRBE study by Dwek et al. 1995):

$$\Sigma_{G0}(x, y) = \Sigma_0 e^{-0.5r^2}, \tag{16}$$

with $r^2 = (x^2 + y^2)/r_0^2$. The scale length r_0 is found by minimizing the following cost function while simultaneously satisfying the overall normalization constraint for Σ_{G0} (eq. 13):

$$\text{cost} = \int d^2r \left[\Sigma_{rec} - \Sigma_{G0} \right]^2. \tag{17}$$

To estimate the value of r_0, we used the covariance matrix from the likelihood analysis used to determine Σ_{rec} to make 5000 Monte Carlo realizations of the source density. The ensemble of realizations, then, have Σ_{rec} as their mean. For each realization, we found r_0 by minimizing the cost function (17) and the resulting distribution of scale lengths is shown in Figure 8. Our result $r_0 = 4.00 \pm 0.55$ kpc indicates that the IRAS variables have the scale length of the old disk population. This value is in good agreement with the scale length 4.5

kpc reported by Habing (1988), derived from analysis of a color-selected IRAS sample. Dwek's value obtained by analyzing bulge emission was $r_0 = 0.91 \pm 0.01$ kpc. The factor of 4 difference between the scale lengths suggests that the IRAS bar and the bulge-bar belong to distinct populations.

5.4. Optical depth due to microlensing

Originally proposed as a test for dark matter in the Milky Way halo (Paczyński 1986), gravitational microlensing was later shown (Griest et al. 1991; Paczyński 1991) to be potentially useful for extracting information about the inner regions of our Galaxy. Three groups (OGLE, MACHO and EROS) are monitoring stars in the Galactic bulge for gravitational microlensing and have found higher event rates than most theoretical estimates. Udalski et al. (1994) derived lensing optical depth $\tau = (3.3 \pm 1.2) \times 10^{-6}$ toward the Baade's window ($l = 1°, b = -3.9°$) based on the analysis of the OGLE data, and MACHO group reported $\tau = 3.9^{+1.8}_{-1.2} \times 10^{-6}$ (Alcock et al. 1995a) estimated from the sample of clump giants, while theoretical estimates give optical depths in the range $0.5 - 2.0 \times 10^{-6}$ (e.g. Alcock et al. 1995a; Evans 1994). Following Paczyński's et al. (1994) suggestion that a bar with a small inclination angle could enhance the optical depth, Zhao et al. (1995) have developed a detailed bar model and found $\tau = (2.2 \pm 0.5) \times 10^{-6}$. Here, we estimate the optical depth using our density reconstruction, Σ_{rec}, assuming that our AGB sample represents the entire stellar disk.

The lensing optical depth is defined as the probability of any of the sources being lensed with magnification factor $A > 1.34$, with

$$A = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}, \qquad u \equiv \frac{r}{R_E} \tag{18}$$

(Refsdal 1964), where r is the distance between the projected position of the source and the lensing mass, R_E is the radius of Einstein ring. Kiraga & Paczyński (1994) derived

$$\tau = \frac{4\pi G}{c^2} \frac{\int_0^\infty \left[\int_0^{D_s} \rho \, \frac{D_d(D_s - D_d)}{D_s} \, dD_d \right] \rho \, D_s^{2+2\beta} \, dD_s}{\int_0^\infty \rho \, D_s^{2+2\beta} \, dD_s}, \tag{19}$$

where D_s is the distance to the sources, D_d is the distance to the deflectors and the free parameter β accounts for detectablity of sources in a flux-limited survey. The reasonable range is $-3 \le \beta \le -1$ and we take $\beta = -1$ following Evans (1994) and Kiraga & Paczyński (1994). The density $\rho = \rho_{bulge} + \rho_{disk}$, with ρ_{bulge} given by equation (1) of Kent (1992), and

$$\rho_{disk} = C \, \Sigma_{44}(r, \phi) \, e^{-|z|/h}, \tag{20}$$

where Σ_{44} is the surface density of our $i = 4, j = 4$ model (14) and $h = 0.325$ kpc is the scale height. We explored two possible normalization prescriptions: (1) Assign a local column density of $\sim 50 \, M_\odot \, pc^{-2}$ ("canonical disk" following Kuijken & Gilmore 1989; Gould 1990). The mass of the disk in this case is $M_{disk} = 1.95 \times 10^{10} M_\odot$. (2) Assign the total disk mass of $M = 6 \times 10^{10} M_\odot$ (Bahcall & Soneira 1980). The second normalization gives local column density of approximately $100 \, M_\odot \, pc^{-2}$ ("maximal disk" of Alcock et al. 1995b). We prefer the latter here because the optical depth estimate depends on the global mass distribution rather than the local density. In addition, there are some indications that the variation of the column density with galactic longitude may be quite significant – a factor of $2 - 3$ (Rix & Zaritsky 1995; Gnedin, Goodman & Frei 1995). The mass of the bulge is $M_{bulge} = 1.65 \times 10^{10} M_\odot$.

For the canonical disk case, the total lensing optical depth at Baade's window is 1.1×10^{-6}, and both bulge and disk lenses contribute 50% to that number. Most of the optical depth (76%) is due to lensing of bulge sources. If

the disk is maximal, optical depth is 1.6×10^{-6}. Disk lenses now account for 1.1×10^{-6} (68% of the total optical depth) and the contribution by bulge sources still dominates (59%). For both scenarios, optical depth is a function of the orientation of the bar. We investigate the enhancement produced by the bar over axisymmetric models of the disk $\rho \propto e^{-r/R} e^{-|z|/h}$, where $R = 3.5$ kpc for fixed disk mass. Figure 9 displays the ratio of optical depths of non-axisymmetric to axisymmetric disk models as a function of the position angle of the bar for both normalization scenarios. The difference between the two curves illustrates the role of the disk in lensing. The largest enhancement of approximately 30% obtains when the bar is aligned along the line of sight as expected. The ratio of optical depths decreases gradually when the bar is in the first Galactic quadrant, with $\geq 20\%$ enhancement out to $\phi_0 = 50°$.

Current generation optical-band lensing surveys have concentrated on low-extinction bulge-centered windows to maximize the lensing event rate. An infrared-band lensing microlensing survey would be less constrained by extinction and therefore more efficient probe of the overall structure of the Galaxy. In particular, any bar which is not perfectly aligned along the Sun–Galactic Center axis will produce an asymmetry in the optical depth. We describe this asymmetry by the ratio of the difference in optical depths at positive and negative longitude to their arithmetic mean. This ratio is shown in Figure 10 for our model (cf. eqns. 19 and 20). Comparison with the Bahcall & Soneira model (1980) suggests that $\beta \approx -1$ is a fair approximation of the high-luminosity end of the disk luminosity function. Therefore, equation (19) also applies at large $|l|$ where both lenses and sources are disk members. The large 40% asymmetry about $|l| \approx 30°$ is due to a local increase in the surface density at negative longitudes close to the observer (Figure 5). More important

than the details of asymmetry is the suggestion that a pencil-beam microlensing survey in the infrared would be sensitive to global asymmetries in the stellar disk component. Confusion is not a limitation at $b = 0°$ for larger values of $|l|$ and the optical depth has a magnitude similar to Baade's window.

6. Summary and discussion

This paper explores a model-independent Bayesian estimation of the stellar density from star counts, rigorously accounting for incomplete data. The general approach can incorporate multiple colors and even different databases. The usual high dimensionality and topological complexity of the posterior distribution, however, complicates both optimization algorithms and subsequent moment analyses. We propose here a hybrid downhill plus directed-search Monte Carlo algorithm; the former speeds convergence and the latter facilitates the location of the global extremum. Other similar and potentially more efficient techniques which can bypass the extremization step altogether (such as general Markov Chain Monte Carlo) are worth careful consideration.

Application of the technique to the variability-selected sample described in Weinberg (1992), assumed to be AGB stars, confirms the presence of a strong non-axisymmetric feature in the first Galactic quadrant. By imposing bisymmetry on the source density, clear signature of a bar is obtained. The size and shape of density isophotes suggests a bar semi-major axis of approximately 4 kpc and position angle of $\phi_0 = 18° \pm 2°$ at the outer edge of the bar. The analysis of the scale length for the AGB candidate distribution gives $r_0 = 4.00 \pm 0.55$ kpc, indicating that these objects are part of the old disk population.

Finally, we use our estimate for non-axisymmetric Galactic disk to explore the dependence of optical depth to gravitational microlensing by bulge and disk

stars. The disk bar does enhance the optical depth τ towards Baade's window by roughly 30% but the overall value is still roughly a factor of two below the MACHO result $\tau = 3.9^{+1.8}_{-1.2} \times 10^{-6}$. Of interest for future microlensing surveys is the finding that our inferred large-scale bar will produce a significant asymmetry in τ at positive and negative longitudes beyond the bulge. The peak asymmetry for our model occurs at $|l| = 30°$ and at $b = 0$ we predict similar values of τ to the Baade's window field. Such a survey might best be carried out in the infrared to take advantage of the low interstellar extinction and colors of the late-type giants. At $|l| \gtrsim 30°$, confusion should not be a limitation at $b = 0°$.

A. Luminosities of AGB stars

The luminosities of AGB variables and the inference of their progenitor masses plays a role in constraining the stellar evolution history of the Galaxy and has received some attention. Investigations based on theoretical approach (Iben & Renzini 1983) and observations of sources close to the Galactic center (Jones & Hyland 1986) placed the luminosities somewhere between a few $\times 10^3 L_\odot$ and $6 \times 10^4 L_\odot$. Van der Veen & Habing (1990) revised the results of Jones & Hyland based on the analysis of a larger sample of OH/IR stars and found that the luminosities are in the range $10^3 - 10^4 L_\odot$ with the peak of the distribution about $5,000 - 5,500 L_\odot$. They suggested the variability of the sources ($\Delta m \leq 2^m$) and possible selection effects as main reasons for higher limits of Jones & Hyland. They also noted that as many as 20% of the stars

may be in the low-luminosity tail of the distribution but only 2% or fewer can exceed the upper limit. Kastner et al. (1993) obtained kinematic luminosities based on the radial velocities of circumstellar envelopes with respect to the LSR and distances derived from the Galactic rotation curve. They found the range of $1.3 \times 10^4 - 2 \times 10^4 L_\odot$ with average uncertainty of factor of 2. The theoretical estimate was recently revised by Groenewegen et al. (1995) who obtained luminosity functions for carbon and oxygen-rich stars based on the synthetic evolution. They found a mean luminosity for Galactic carbon and oxygen-rich AGB stars to be $7050 L_\odot$ and $3450 L_\odot$, respectively. They stated "the luminosity of a typical Galactic AGB star is in any case less than the $10^4 L_\odot$ often assumed". Habing (1988) reported the average luminosity of $4000 L_\odot$ for a color selected sample from IRAS PSC catalog. Finally, analysis of a sample of oxygen Miras using P–L relation established on the observations of LMC Miras (Feast et al. 1989), places their average luminosity at $\overline{L} = 3900 \pm 450 L_\odot$. Unfortunately, we can not use the P–L relation, since IRAS had insufficient temporal coverage to reliably constrain periods. Rather, we approximate the source density by an axisymmetric distribution at $R_0 = 8$ kpc and choose the average luminosity which maximizes the likelihood function. The results for different number of radial terms are shown in Figure 12. For ten terms, the maximum likelihood of this axisymmetric density is achieved when $L \approx 3000 L_\odot$. We adopt $\overline{L} = 3500 L_\odot$ which is the low end of published results and interpret our statistical analysis as a consistency check.

B.   Computational Notes

Likelihood maximization is the rate limiting step in inferring the surface density from a source catalog. The cost of computing the likelihood is

proportional to the sample size so analyses of very large data sets will be technically challenging. Our "workhorse" algorithm for locating the maximum of the likelihood function is the conjugate gradient method which is thoroughly discussed in the literature (e.g. Press et al. 1988). We have adopted an implementation by Shanno & Phua (1976, CONMIN). The algorithm has good convergence properties, but requires a good initial approximation. Near the expected quadratic maximum the convergence should be extremely rapid.

However, the likelihood function may have a large number of extrema, limiting the use of the standard downhill technique. In such cases, the Simulated Annealing (SA) algorithm (Metropolis et al. 1953; Otten & van Ginneken 1989) has the advantage. It places no restrictions on continuity and easily incorporates arbitrary boundary conditions and constraints. Adaptive Simulated Annealing (ASA, Ingber 1989)—a faster version of the SA algorithm—proved to be effective in narrowing the domain of the search to the comparatively small region in parameter space. However, in the vicinity of the extremum it converges slowly.

The complementary features of the two techniques, suggest the following two-step hybrid scheme:

1. Use a directed search algorithm (ASA) to isolate the global maximum. Although SA class of algorithms converge slowly, there is a probabilistic guarantee of convergence: the probability of finding the maximum is inversely proportional to the total number of iterations to some power (e.g. Shu & Hartley 1987; Ingber 1993).

2. After either a limiting number of steps or a significant drop off in convergence, use the current ASA solution as input to conjugate gradient scheme. This is motivated by our expectation that the true maximum of the likelihood function will be a quadratic form in the unknown variables.

This sequence can be repeated again, in case if Step 2 fails to find a well-defined maximum. The scheme is difficult to analyze but appears to work well in practice and is potentially useful for large parameter space and complex geometry (boundary conditions, irregular likelihood function) cases.

The entire computation time scales as the number of coefficients M (total number of A_{ij} and B_{ij} in the sum in eq. 14) and the sample size N: $N(2M + 1)$. Computation of the Hessian matrix requires CPU time proportional to $M^3 N$. For a large M, this is the bottleneck. However, the algorithm is straightforwardly parallelized by partitioning the data.

REFERENCES

Alcock, C., et al. 1995a, preprint

Alcock, C., et al. 1995b, ApJ, 445, 133

Allen, L. E., Kleinmann, S. G. & Weinberg, M. D. 1993, ApJ, 411, 188

Bahcall, J. N. & Soneira, R. M. 1980, ApJS, 44, 73

Bretthorst, G. L. 1990, in Maximum Entropy and Bayesian Methods, ed. P.F.Fougere (Dordrecht: Kluwer), 53

Clutton-Brock, M. 1990, Journal of the Amer. Stat. Assoc., 85, 411, 760

Dwek, E., et al. 1995, ApJ, 445, 716

Evans, N. W. 1994, ApJ, 437, L31

Feast, M. W., Glass, I. S., Whitelock, P. A. & Catchpole, R. M. 1989, MNRAS, 241, 375

Gnedin, O. Y., Goodman, J., & Frei, Z. 1995, AJ, 110, 1105

Gould, A. 1990, MNRAS, 244, 25

Griest, K., et al. 1991, ApJ, 372, L79

Groenewegen, M. A. T., van den Hoek, L. B. & de Jong, T. 1995, A&A, 293, 381

Habing, H. J. 1988, A&A, 200, 40

Harmon, R. & Gilmore, G. 1988, MNRAS, 235, 1025

Iben, I., Jr., & Renzini, A. 1983, ARA&A, 21, 271

Ingber, L. 1989, J. Mathl. Comput. Modelling, 12, 967

Ingber, L. 1993, Math. and Comput. Modelling, 18, 11, 29

IRAS, Explanatory Supplement. 1988, Joint IRAS Science Working Group (Washington DC: GPO)

Izenman, A. J. 1991, Journal of the Amer. Stat. Assoc., 86, 413, 205

Jones, T. J. & Hyland, A. R. 1986, AJ, 92, 805

Kastner, J. H., Forveille, T., Zuckerman, B. & Omont, A. 1993, A&A, 275, 163

Kent, S. M. 1992, ApJ, 387, 181

Kiraga, M. & Paczyński, B. 1994, ApJ, 430, L101

Kuijken, K. & Gilmore, G. 1989, MNRAS, 239, 571

Lehmann, E. L. 1959, Testing Statistical Hypotheses (New York: Wiley)

Metropolis, N., Rosenbluth, A., Rosenbluth, M., Teller, A. and Teller, E. 1953, Journal of Chem. Phys., 21, 1087

Metzger, M. R. & Schechter, P. L. 1994, ApJ, 420, 177

Otten, R. H. J. M. & van Ginneken, L. P. P. P. 1989, The Annealing Algorithm (Boston: Kluwer)

Paczyński, B. 1986, ApJ, 304, 1

Paczyński, B. 1991, ApJ, 371, L63

Paczyński, B., Stanek, K., Udalski, A., Szymanski, M., Kałuzny, J., Kubiak, M., Mateo, M. & Krzeminski, W. 1994, ApJ, 435, L113

Press, W. H., Flannery, B. P., Teukolsky, S. A., Vetterling, W. T. 1988, Numerical Recipes in C (New York: Cambridge University Press)

Refsdal, S. 1964, MNRAS, 128, 295

Rix, H.-W. & Zaritsky, D. 1995, ApJ, 447, 82

Shanno, D. F. & Phua, K. H. 1976, ACM Trans. Math. Softw., 2, 1, 87

Shu, H. and Hartley, R. 1987, Phys. Lett. A 122(3-4), 157

Silverman, B. W. 1986, Density Estimation for Statistics and Data Analysis (New York: Chapman and Hall)

Udalski, A., et al. 1994, Acta Astron., 44, 165

van der Veen, W. E. C. J. & Habing, H. J. 1990, A&A, 231, 404

Wainscoat, R. J., Cohen, M., Volk, K., Walker, H. J., Schwarz, D. E. 1992, ApJS, 83, 111

Weinberg, M. D. 1992, ApJ, 384, 81 (Paper I)

Weinberg, M. D. 1994, in Unsolved Problems of the Milky Way, IAU Symposium No.169, The Hague

Wilks, S. S. 1962, Mathematical Statistics (New York: Wiley)

Zhao, H. S., Spergel, D. N. & Rich, R. M. 1995, ApJ, 440, L13

Fig. 1.— The sample of 5,500 IRAS PSC variables (dots). The Sun is located at $X = -8$, $Y = 0$. The data from the shaded sectors are eliminated from the analysis. The circle shows the distance in the plane where an AGB star ($L = 3500L_\odot$) can be detected.

Fig. 2.— The same sample projected on the X-Z plane. All the data are inside the region bounded by two solid lines which are solutions of the equation (1).

Fig. 3.— The amplitude of harmonic coefficients as functions of the position angle of the bar. Open triangles: $i = 1, j = 0$; open squares: $i = 1, j = 2$; filled triangles: $i = 2, j = 0$; filled squares: $i = 2, j = 2$. The symbols are slightly offset along the x-axis for clarity.

Fig. 4.— The reconstructed density profiles. Ten equally spaced contours between 10% and 100% of peak value are shown in each panel.

Fig. 5.— The reconstructed density profile obtained with assumption of bisymmetric source density. There are 10 contours between 10% and 100% of peak value in each panel.

Fig. 6.— Isophotal fits to the reconstructed source density: surface density C normalized to its central value (left scale, solid line) and axis ratio $a : b$ (right scale, dashed line) versus semimajor axis.

Fig. 7.— The position angle ϕ in degrees (left scale, solid line) and eccentricity of ellipses (right scale, dashed line) versus semimajor axis.

Fig. 8.— The distribution of the scale lengths r_0 in 5000 realizations of the source density (histogram). The best fit normal distribution is shown (solid curve) with mean and rms value as labeled.

Fig. 9.— The ratio of optical depths toward Baade's window obtained with non-axisymmetric (bar) and axisymmetric disk models as the function of the position angle of the bar, ϕ_0. Solid line – "maximal disk", dashed line – "canonical disk" (see text).

Fig. 10.— Asymmetry in the microlensing optical depth. The disk is "maximal". Solid line, dashed line and dotted line represent cuts with $b = 0°, 2°$ and $4°$, correspondingly.

Fig. 11.— Average optical depth as the function of the galactic longitude. The lines represent the same latitudes as in Fig. 10

Fig. 12.— The values of the likelihood function with varying luminosity of the sources. The center of the Galaxy is fixed at 8 kpc. Solid line — $i_{max} = 2$, dotted line — $i_{max} = 6$, dashed line — $i_{max} = 10$.

i$_{max}$=3

j$_{max}$=2

L=0

$i_{max}=4$

$j_{max}=2$

$L=48$

X (kpc)

Y (kpc)

$i_{max}=4$

$j_{max}=4$

$L=-217$

X (kpc)

Y (kpc)

$r_0 = 4.00 \pm 0.55$ kpc