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# **Risk Aversion, Insurance, and the Efficiency-Equality Tradeoff\***

Samuel Bowles

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March 15, 2000

## **Abstract**

Under conditions of informational asymmetry, redistributing the property rights may improve work incentives but lead to an inefficient choice of entrepreneurial risk. We present a model in which reassignment of property rights does not affect factor prices and we show that there exist egalitarian asset redistributions that enhance allocative efficiency. The scope for such redistributions can be broadened by offering fair insurance protecting the independent entrepreneur against risk unassociated with the production process and against production uncertainties that are unrelated to the quality of their individual decisions. The market will generally supply insurance of this type suboptimally.

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## 1 Introduction

Redistributing economic resources in favor of the nonwealthy often entail considerable allocative inefficiencies by distorting incentives facing economic decision makers. One often proposed, potentially efficiency-enhancing, form of redistribution is to turn employees into owners and entrepreneurs, thereby improving work incentives while at the same time reducing wealth inequality.<sup>1</sup> A weighty impediment to such policies is that nonwealthy entrepreneurs tend to be more risk averse than wealthy and/or highly diversified owners—for instance stockholders.<sup>2</sup> As a consequence, there is generally a tradeoff between effective work incentives and socially optimal risk choices.<sup>3</sup> We explore ways of attenuating this tradeoff, extending an approach suggested by Domar and Musgrave (1944) and Sinn (1995).

A number of empirical investigations document a high level of risk aversion on the part of the nonwealthy. Low wealth entails lower return to independent agricultural production, for instance, because farmers sacrifice expected returns for more secure returns. Rosenzweig and Wolpin (1993) find that low-wealth Indian farmers seeking a means to secure more stable consumption streams, hold bullocks,

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<sup>1</sup>See Bardhan, Bowles and Gintis (2000) and the works cited there, especially Laffont and Matoussi (1995), Legros and Newman (1996), Banerjee and Ghatak (1996), Mookherjee (1997) and Hoff (1996b).

<sup>2</sup>See Saha, Shumway and Talpaz (1994) and the many studies cite therein.

<sup>3</sup>We assume in this paper that the socially optimal risk level for a project is that which maximizes expected return; i.e., society is risk neutral. The Capital Assets Pricing Model asserts that this is true only if ‘market risk’ affecting the entire economy is zero. To simplify our analysis, we assume that this is the case.

which are a highly liquid form of capital, instead of buying pumps, which are illiquid but have high expected return. The relevant effects are not small. Rosenzweig and Binswanger (1993) find, for example, that a one standard deviation reduction in weather risk would raise average profits by about a third among farmers in the lowest wealth quartile (p. 75), and virtually not at all for the top wealth-holders. Moreover, they conclude that the demand for weather insurance would come primarily, if not exclusively, from poor farmers. Nerlove and Soedjiana (1996) find a similar effect in Indonesia with respect to sheep.<sup>4</sup>

Thus because of risk aversion, a reassignment of property rights to low-wealth entrepreneurs might be unsustainable if as a result entrepreneurs' income streams are subject to high levels of stochastic variation. Carter, Barham and Mesbah (1996) and Jarvis (1989) provide a vivid example: in the Central Valley of Chile three quarters of those families who received individual assignment of land rights under a land redistribution program in the 1970's sold their assets within a decade.

However, as Musgrave, Domar, and Sinn suggest, the availability of insurance can lead to increased risk-taking and willingness to hold risky assets.<sup>5</sup> But the market for forms of insurance that promote entrepreneurial risk-taking may be imperfect (Atkinson and Stiglitz 1980). Shiller (1993) provides several contemporary applications, arguing that capital market imperfections even in the most advanced

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<sup>4</sup>See Hoff (1996a) for a discussion of this and related studies.

<sup>5</sup>More recently, Black and de Meza (1997) argue that public insurance may improve efficiency when there heterogeneous occupational risk, but their model works through price changes rather than wealth changes. For more on mixed public/private insurance, see Blomqvist and Johansson (1997) and Selden (1997).

economies lead to the absence of insurance markets for major sources of individual insecurity and inequality. For instance, a major form of wealth insecurity in many families is the capital value of the family home, due to medium- to long-term fluctuations in average housing prices in a region. No insurance for such fluctuations is available, but Shiller suggests that this and other similar insurance markets can be activated through proper financial interventions. Along these same lines, Sinn (1995) argues that the welfare state in the advanced economies can be understood in part as a successful set of policy measures to improve the risk-taking behavior of the nonwealthy where private ‘social insurance’ markets fail.

On the other hand, many attempts at preserving the small independent entrepreneur through extending credit availability and crop insurance have failed (Carter and Coles 1997), though these failures may be due to forms of insurance that are not incentive compatible (Newbery 1989). For instance, insuring individual crops reintroduces the same agency problems as sharecropping and wage labor. By contrast, as we show below, allowing entrepreneurs to purchase insurance covering some general condition that is correlated with individual crop risk but that does not affect individual production incentives, can be effective in eliciting risk taking on the part of the nonwealthy without incurring efficiency losses. A crop insurance program in India, for example, based payments to individual farmers not on the output of their own plots but rather on average crop yields in larger agro-climatic regions to which they belong (Dandekar 1985). Disaster insurance for crops in the United States is similarly designed (Williams, Carriker, Barnaby and Harper 1993). Or insurance payments may be based on the exogenous source of the risk

itself, if this is measurable. An example of this is rainfall insurance, whereby the entrepreneur pays a fixed premium, and receives a schedule of returns depending upon the average rainfall in the region over the growing season.<sup>6</sup>

In this spirit, we show below that under plausible conditions reducing the exposure of the nonwealthy to stochastic fluctuations independent of their productive activities can induce increased entrepreneurial risk-taking, and hence can help sustain otherwise unsustainable asset redistributions. General social insurance can also allow access to credit markets for wealth-poor agents who would be otherwise excluded. Platteau, Murickan, Palatty and Delbar (1980), Sanderatne (1986), Ardington and Lund (1995) and Deaton and Case (1998) provide some evidence for this phenomenon.

In our analysis we shall use a ‘productivity enhancing’ criterion in place of the more familiar ‘Pareto improving’ criterion because we are studying policies aimed at egalitarian redistribution, to which the Pareto improving criterion need not apply. Nor need the usual ‘compensation criteria’ apply. According to the compensation criteria, gainers from the policy must be able to compensate losers, but potential losers must not be able to compensate potential gainers to forego the policy. However if redistribution is the goal, there is no reason to require that compensations be feasible.

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<sup>6</sup>Similarly, the taxation of agricultural income can be based on general growing conditions rather than measured farm output, thus combining insurance and revenue-producing goals. The idea is not new. The *Zabt* system of taxation, developed by the Mughal rulers of North India during the Sixteenth Century, based assessments on estimates of the productive capacities of the land rather than on actual harvests (Richards 1993):85ff.

This accounts for our definition of a policy as *productivity enhancing* if the gainers could compensate the losers and still remain better off, except that the implied compensation need not be implementable under the informational conditions and incentive constraints of the economy. A productivity enhancing egalitarian asset redistribution refers to a mandated reassignment to wealth-poor suppliers of labor services of residual claimancy and control over assets, which would not take place through competitive exchange but which is sustainable as a competitive equilibrium following the redistribution. The productivity gains associated with this class of redistributions arise from improvements in technical efficiency made possible by the improved incentives supported by reallocation of residual claimancy rights. As we will see, the efficient assignment of residual claimancy rights does not arise through private exchange because, while *ex post* assignment of rights to the poor is constrained Pareto efficient and hence sustainable in a competitive equilibrium, the *ex ante* distribution of rights is also constrained Pareto efficient, so given the *ex ante* distribution, private exchanges will not implement the *ex post* distribution of rights.

The model developed below shows that, exposed to the risk associated with residual claimancy, asset-poor entrepreneurs

- (a) may avoid buying projects that they could operate productively, even when they are financially capable of doing so, may sell rather than operate such projects that are transferred to them, and will choose suboptimal levels of risk for any project that they do retain and operate;
- (b) there exists a class of productivity enhancing egalitarian asset redistributions

that are sustainable as competitive equilibria but will not occur through private contracting even when loans are available to all entrepreneurs at the risk-free interest rate;

- (c) this class may be expanded by a offering fair insurance to nonwealthy asset holders that protects the entrepreneur against risk unassociated with the production process (e.g., health insurance, consumer goods price stabilization) or that protects independent entrepreneurs against ‘industry risk’ that is unrelated to the quality of their own decisions;
- (d) while competitive profit maximizing insurers may supply some forms of insurance of this type, they will generally do so in a suboptimal manner.

Our approach relates to the literature on wealth, risk-taking, and insurance as follows. Kihlstrom and Laffont (1979) and Banerjee and Newman (1991) develop models in which more risk averse agents become employees and less risk averse agents become entrepreneurs. They also showed that this situation involves allocational inefficiencies occasioned by incomplete markets for risk-sharing. Our model adds that with declining risk-aversion the nonwealthy will be employees and the wealthy entrepreneurs. In a related paper, Kanbur (1979) showed that when general equilibrium effects are included, redistributive taxation need not reduce the economic inequality occasioned by heterogeneous levels of risk aversion. In our model, egalitarian policies do not affect prices or the wage rate, and we take the profit rate as exogenous, as in the case of a small country operating without capital controls in an international economic system. Therefore Kanbur’s results do not obtain in our model. Two papers prior to the present contribution (Banerjee and

Newman 1993, Aghion and Bolton 1997) have modeled the dependence of the occupational distribution on the wealth distribution. Both assume risk-neutral agents and imperfect credit markets and, like us, find that redistribution can both improve productive efficiency and reduce wealth inequality. Our contribution in this regard is thus to extend their results to the case of risk-averse agents.

## 2 The Model

Consider a risk neutral employer who owns an asset and employs a worker. The worker receives a wage  $w$ , and the project uses non-depreciable capital goods with value  $\mathbf{k}$ . We assume the employer must supervise the worker to guarantee performance, with supervision costs  $m > 0$ . We also assume the project consists of a continuum of possible technologies of varying risk and expected return, with higher risk yielding higher expected return over some range. We summarize the choice of technology in an expected net revenue schedule  $g(\sigma)$ , which is a concave function of the standard deviation of revenue  $\sigma > 0$ , with a maximum at some  $\sigma^* > 0$ .<sup>7</sup>

We then write the employer's profits, net of the opportunity costs of capital,  $p(\sigma)$  as

$$p(\sigma) = \sigma z + g(\sigma) - \rho \mathbf{k} - m - w \quad (1)$$

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<sup>7</sup>This shape follows from two plausible assumptions. First, production techniques that offer positive expected return involve a strictly positive level of risk. Hence expected return is an increasing function of risk for low levels of risk. Second, firms have access to production techniques that have very high returns when successful, but with a low probability of success (e.g. a firm may lower costs by not diversifying its product line, or by assuming the availability of particular production inputs). Hence above a certain point expected return declines with increasing risk.

where  $z$  is a random variable with mean zero and standard deviation unity and  $\rho$  is the risk-free interest rate.

The employer, who is risk neutral, maximizes  $\mathbf{E}p(\sigma)$ , the expected value of profits, giving first order condition

$$(\mathbf{E}p)_\sigma = g'(\sigma) = 0, \quad (2)$$

determining the expected profit-maximizing risk level  $\sigma^*$ .<sup>8</sup> We further assume that the project is part of a competitive system with free entry, so profits must be zero in equilibrium. Since the employer is risk-neutral, this means the equilibrium wage rate  $w^*$  is given by

$$w^* = g(\sigma^*) - \rho\mathbf{k} - m. \quad (3)$$

Suppose the wage-earner considers becoming an independent entrepreneur by renting capital and undertaking production. To abstract from problems of credit availability, we assume that the productive equipment constituting the asset may be rented at a per-period cost  $\rho\mathbf{k}$  where  $\rho$  is the risk-free interest rate. This is equivalent to assuming that the entrepreneur can borrow funds to purchase the asset at the risk-free rate. The independent entrepreneur's net payoff is then given by

$$y(\sigma) = \sigma z + g(\sigma) - \rho\mathbf{k}, \quad (4)$$

since being self-employed, the entrepreneur pays neither the wage nor the moni-

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<sup>8</sup>Here and throughout the paper, we use a variable subscript to a function to denote the partial derivative with respect to this variable.

toring cost (we assume that the effort level of the entrepreneur remains the same). Indeed, the fact that the entrepreneur does not incur the monitoring cost captures our assumption that productive efficiency improves when the entrepreneur ceases being a wage-earner and becomes the residual claimant.

Suppose the supplier of labor services has utility function  $u(\mathbf{w})$ , which is twice differentiable, increasing, and concave in wealth  $\mathbf{w}$ , and define

$$v(\sigma, \mu) = \mathbf{E}u(\mathbf{w}) = \int_{-\infty}^{\infty} u(\mu + \sigma z) dF(z), \quad (5)$$

where  $F(z)$  is the cumulative distribution of  $z$ . Thus  $v(\sigma, \mu)$  is the expected utility of the payoff  $\mu + \sigma z$ . We write the slope of the level curves  $v(\sigma, \mu) = \bar{v}$  where  $\bar{v} \in \mathbf{R}$ .

$$s(\sigma, \mu) = -\frac{v_{\sigma}}{v_{\mu}}, \quad (6)$$

and we write the Arrow-Pratt risk coefficient for the agent as

$$\lambda(\mathbf{w}) = -\frac{u''(\mathbf{w})}{u'(\mathbf{w})}.$$

We then have the following, due to Meyer (1987) and Sinn (1990):

*Proposition 1. Suppose  $v(\sigma, \mu)$  is defined by 5 and  $s(\sigma, \mu)$  is defined by (6). Then*

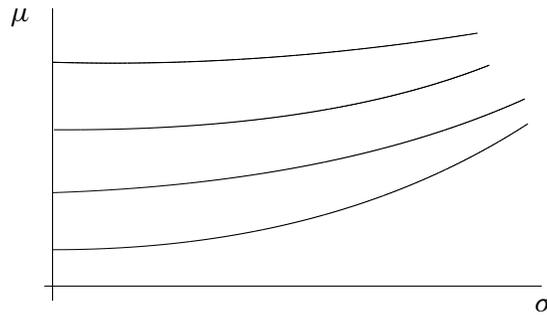
- (i) For  $\sigma > 0$ ,  $v_{\mu}(\sigma, \mu) > 0$  and  $v_{\sigma}(\sigma, \mu) < 0$ .*
- (ii)  $s(0, \mu) = 0$ .*
- (iii)  $s(\sigma, \mu) > 0$  when  $\sigma > 0$ ;*

(iv)  $v(\sigma, \mu)$  is concave;

(v)  $s_{\mu}(\sigma, \mu) < 0$  when  $\lambda'(\mathbf{w}) < 0$ ;

(vi)  $s_{\sigma}(\sigma, \mu) > 0$ .

This Proposition shows that  $v(\sigma, \mu)$  behaves like a utility function where  $\mu$  is a ‘good’ and  $\sigma$  is a ‘bad.’ The level curves  $v(\sigma, \mu) = \bar{v}$  are then indifference curves which, in the case of decreasing absolute risk aversion, are increasing, convex, flat at  $\sigma = 0$ , become flatter for increasing  $\mu$  when  $\sigma > 0$ , and become steeper for increasing  $\sigma$ . Movements to the north and to the west thus indicate both improved welfare and flatter indifference curves. These properties are illustrated in Figure 1. We henceforth assume the supplier of labor services exhibits decreasing absolute risk aversion, which means  $\lambda'(\mathbf{w}) < 0$ ; i.e., the agent becomes less risk averse as wealth increases.<sup>9</sup>



**Figure 1:** Indifference Curves of the Decreasingly Absolutely Risk Averse Agent with Utility Function  $v(\sigma, \mu)$

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<sup>9</sup>Virtually all empirical studies support decreasing absolute risk aversion. For a recent review of the literature, see Saha et al. (1994).

The entrepreneur then chooses  $\sigma$  to maximize

$$\pi(\sigma) \equiv v(\sigma, \mu(\sigma))$$

where

$$\mu(\sigma) \equiv \mathbf{E}y(\sigma) = g(\sigma) - \rho \mathbf{k}, \quad (7)$$

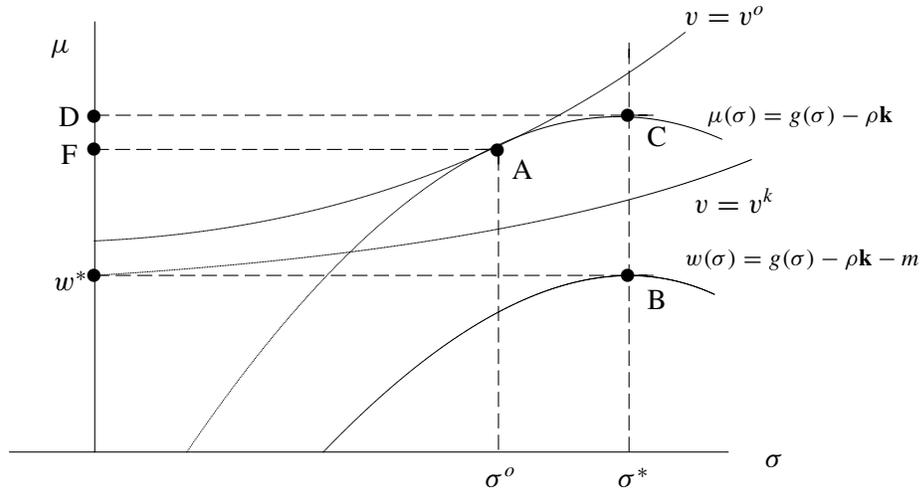
giving the first order condition

$$\pi_\sigma = v_\mu[g'(\sigma) - s(\sigma, \mu(\sigma))] = 0. \quad (8)$$

This indicates that the marginal rate of transformation of risk into expected payoffs,  $g'(\sigma)$ , must equal the marginal rate of substitution between risk and expected payoff,  $s(\sigma, \mu)$ . The entrepreneur's optimizing problem as residual claimant is depicted in Figure 2 as choosing the highest indifference curve of  $v(\sigma, \mu)$  that satisfies the constraint (7), which is just the tangency point at A, giving  $\sigma^o$ , which satisfies the first order condition (8). The entrepreneur's risk aversion implies  $s(\sigma, \mu) > 0$ , which by (8) requires that  $\sigma^o < \sigma^*$ , so the independent entrepreneur chooses a lower level of risk than the risk neutral employer.

The tradeoff between the allocative gains and suboptimal risk losses that occur when the asset is assigned to the asset-poor entrepreneur is illustrated in Figure 2. This figure depicts both the pre-transfer allocation in which the employer chooses  $\sigma^*$  and pays  $w^*$ , and the post-asset-transfer situation indicated by point A. The allocative gain associated with the transfer is the increase in the expected return from  $w^*$  to the point  $D$ , or just  $m$ , the saving in monitoring input. The suboptimal

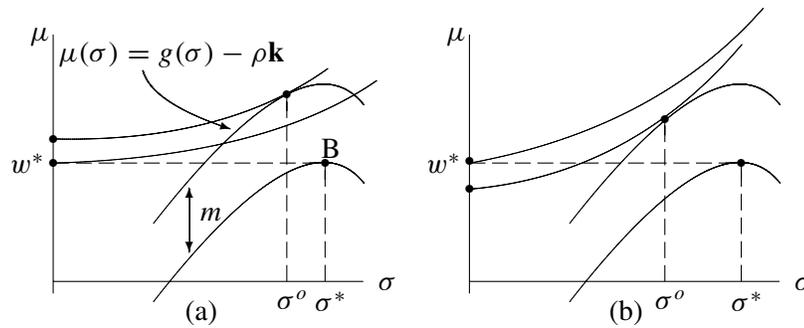
risk loss is  $D - F$ , reflecting the fact that the risk averse entrepreneur prefers point  $A$  to point  $C$  on the risk-return schedule. There is no reason, of course, to expect the gains to exceed the costs.



**Figure 2:** The Tradeoff between Gain in Expected Return and Lost in Suboptimal Risk Taking

To compare the welfare of the supplier of labor services as entrepreneur as opposed to wage-earner, note that when the employer chooses  $\sigma$ , by (3) the equilibrium payoff to the employee occurs at the maximum point  $\sigma^*$  of the schedule  $w = g(\sigma) - \rho\mathbf{k} - m$ , as shown at point  $B$  in Figure 3(a). Figure 3(a) shows the case where the agent is better off as entrepreneur rather than employee, since the indifference curve through  $(\sigma^o, \mu(\sigma^o))$  is higher than the indifference curve through  $(0, w^*)$ . By contrast, Figure 3(b) shows the case where the agent is better off working for the employer. Notice that in this case the agent has higher expected income as residual claimant than as wage-earner, but is exposed to an excessive

level of risk. The differences between the two cases is the greater degree of risk aversion assumed in the second case, as is indicated by the steeper indifference locus. Competitive equilibrium for the first case implies that the agent acquire the asset and in the second that the agent work for the employer, so in both cases the competitive assignment of residual claimancy and control rights would appear to implement an efficient solution.



**Figure 3:** Comparing Wage Earning and Independent Production

Note that in (a) the agent is better off as residual claimant, and in (b) the reverse is true.

### 3 Wealth Redistribution

We now consider whether the analysis would be altered by an outright transfer of  $\mathbf{k}$  to the entrepreneur, thus obviating the need to rent these assets. It might well be thought that the result would not change, as the agent's per period return from selling the asset  $\rho\mathbf{k}$  is exactly the rental cost, so the asset transfer simply converts a direct cost (the cost of renting the capital) into an opportunity cost (the forgone cost of renting the capital to another agent), seemingly leaving the analysis unaffected. But

this inference is unwarranted. Suppose the entrepreneur has wealth  $\mathbf{w}$  not associated with entrepreneurship, and earns a secure income  $\rho\mathbf{w}$  on this wealth. Then we have

*Theorem 1. If the entrepreneur satisfies decreasing absolute risk aversion, the level of risk the entrepreneur assumes is an increasing function of wealth  $\mathbf{w}$ .*

The intuition behind this result is straightforward. Increasing the entrepreneur's wealth flattens the indifference curves in  $\sigma$ - $\mu$  space, so the optimal production point moves closer to the maximum on the risk-return schedule. To prove the theorem, note that with wealth  $\mathbf{w}$ , (7) now becomes

$$\mu(\sigma) = \rho(\mathbf{w} - \mathbf{k}) + g(\sigma),$$

and the entrepreneur as before chooses  $\sigma$  to maximize  $\pi(\sigma) \equiv v(\sigma, \mu(\sigma))$ , giving the first order condition (8), which we totally differentiate with respect to  $\mathbf{w}$  to obtain

$$\pi_{\sigma\sigma} \frac{d\sigma}{d\mathbf{w}} + \pi_{\sigma\mathbf{w}} = 0.$$

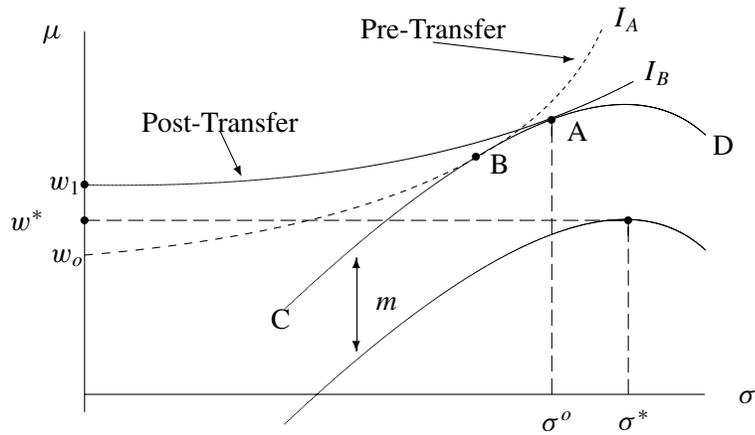
Now  $\pi_{\sigma\sigma} < 0$  by the second order condition, and

$$\pi_{\sigma\mathbf{w}} = -\rho v_{\mu} s_{\mu} > 0,$$

since  $s_{\mu}(\sigma_{\mathbf{w}}, \mu_{\mathbf{w}}) < 0$  by Proposition 1v. Thus  $d\sigma/d\mathbf{w} > 0$ . ■

It follows that there exist wealth transfers of the following form: before the transfer, the agent prefers to work for an owner whose capital stock is  $\mathbf{k}$ . When an amount  $\mathbf{k}$  of wealth is transferred to the agent, indifference curves become flatter,

and in the new situation holding the productive asset and becoming an independent entrepreneur is the preferred alternative. The transfer is productivity enhancing because the increase in technical efficiency (elimination of  $m$ ) is not offset by the output losses occasioned by the suboptimal risk level.



**Figure 4:** Example of a Productivity Enhancing Asset Redistribution

This is illustrated in Figure 4. In this figure, the before-transfer indifference curves for the agent are the dashed curves. Clearly wage labor dominates independent production. After the transfer, indicated by the solid curves, the decrease in risk aversion of the agent renders independent production superior to wage labor.<sup>10</sup>

*Theorem 2. Starting from an competitive equilibrium with a given distribution of wealth, there is some  $m_{min}$  such that for  $m \geq m_{min}$ , there is a productivity enhancing*

<sup>10</sup>The utility levels corresponding to the dashed and the solid indifference curves are of course not the same. In particular, the dashed indifference curve through point  $(0, w^*)$  corresponds to a lower utility level than the solid indifference curve through  $(0, w^*)$ , since in the latter case the agent has higher wealth.

*redistribution of wealth that can be sustained in a competitive equilibrium.*

Proof: We know from Proposition 1v that  $s(\mu, \sigma, w + \mathbf{k}) < s(\mu, \sigma, w)$ , the transfer of wealth  $\mathbf{k}$  flattening the indifference curves. Suppose  $I_A$  and  $I_B$  are the before and after redistribution indifference curves tangent to the independent entrepreneur's production frontier DABC (Figure 4), respectively. Let  $w_o$  be the point where  $I_A$  intersects the  $\mu$ -axis, and let  $w_1$  be the point where  $I_B$  intersects the  $\mu$ -axis, so  $w_o < w_1$ . Then as long as  $m$  is such that  $w^* \in (w_o, w_1)$ , the entrepreneur would not have chosen to acquire the asset prior to the transfer yet prefers holding the asset to selling it and working for the employer. Thus the transfer is sustainable. This result also demonstrates that the gains to the entrepreneur are sufficient to compensate the previous owner of the asset as the entrepreneur's returns to holding the asset exceed the opportunity cost  $\rho\mathbf{k}$ , which is identical to the required compensation. ■

An egalitarian wealth transfer may thus be productivity enhancing, although the compensation that rendered the transaction a Pareto improvement is not generally implementable, since a lump sum wealth transfer  $\mathbf{k}$  to the former owner (or equivalently, an enforceable commitment of the entrepreneur to pay  $\rho\mathbf{k}$  per period) would simply induce the entrepreneur to sell rather than operate the asset.

Credit market constraints played no part in this demonstration, as the entrepreneur was assumed to be able to borrow at the competitive risk-free interest rate  $\rho$ . However if the asset poor do face credit constraints insofar as a transfer of wealth may alleviate these constraints a second class of productivity enhancing asset transfers may exist. To see this assume that the cost of borrowing to the entrepreneur is

$r(\mathbf{w})$  where  $\mathbf{w} \geq 0$  is the total collateralizable wealth of the entrepreneur, where

$$r'(\mathbf{w}) < 0 \quad \text{and} \quad \lim_{\mathbf{w} \rightarrow \infty} r(\mathbf{w}) = \rho. \quad (9)$$

We have

*Theorem 3. Suppose a credit constrained worker with wealth  $\mathbf{w}$  faces an interest rate  $r(\mathbf{w})$  satisfying (9), and a fraction  $\kappa$  of the value  $\mathbf{k}$  of the capital requirements of the project can serve as collateral on a loan. Then for sufficiently large  $\mathbf{k}$  the transfer of the capital good to the agent is productivity enhancing.*

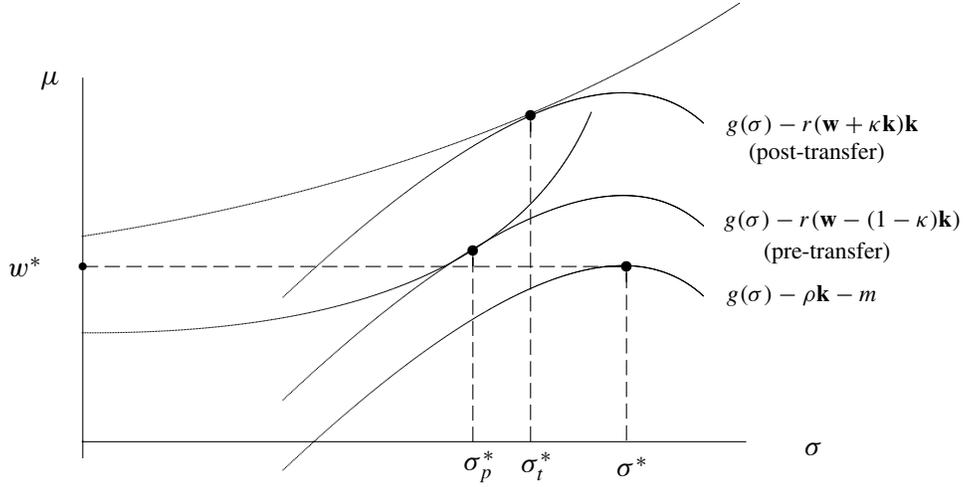
*Proof:* A entrepreneur with wealth  $\mathbf{w}$  can acquire the capital good at per period cost of  $r(\mathbf{w} - (1 - \kappa)\mathbf{k})\mathbf{k}$ . The expected income  $\mu_p$  for the entrepreneur who purchases the asset is

$$\mu_p(\sigma) = g(\sigma) - r(\mathbf{w} - (1 - \kappa)\mathbf{k})\mathbf{k},$$

while the expected income  $\mu_t$  for the entrepreneur who has acquired the asset by transfer is

$$\mu_t(\sigma) = g(\sigma) - r(\mathbf{w} + \kappa\mathbf{k})\mathbf{k}.$$

Choose  $\sigma_p^*$  and  $\sigma_t^*$  to maximize  $v(\sigma, \mu_p(\sigma))$ , and  $v(\sigma, \mu_t(\sigma))$ , respectively. The agent who is employed and receiving the wage  $w^*$  would not benefit from purchasing the asset if  $v(\sigma_p^*, \mu_p(\sigma_p^*)) < v^*(0, w^*)$ , which is clearly true for sufficiently large  $\mathbf{k}$ . The same agent having received the asset  $\mathbf{k}$  by transfer would prefer to hold the asset if  $v(\mu_t(\sigma_t^*), \sigma_t^*) > v^*(0, w^*)$ . A productivity enhancing asset transfer thus



**Figure 5:** A Productivity Enhancing Redistribution where the Entrepreneur Faces a Credit Constraint

requires that:

$$v(\sigma_p^*, \mu_p(\sigma_p^*)) < v^*(0, w^*) < v(\mu_t(\sigma_t^*), \sigma_t^*).$$

Suppose the first inequality is satisfied. Since

$$\mu_t(\sigma) - \mu_p(\sigma) = [r(\mathbf{w} - (1 - \kappa)\mathbf{k}) - r(\mathbf{w} + \kappa\mathbf{k})]\mathbf{k} > 0,$$

it is clear that, for sufficiently large  $\mathbf{k}$ , the second inequality will be satisfied as well at  $\sigma = \sigma_p^*$ , and hence *a fortiori* at  $\sigma = \sigma_t^*$ . ■

Figure 5 illustrates a productivity enhancing redistribution to the credit constrained wealth poor entrepreneur.

Thus where a wealth transfer will alleviate the credit market constraints faced by the wealth poor, productivity enhancing redistributions may exist even were

the entrepreneurs' risk aversion unaffected by the transfer. Hence wealth related credit constraints and wealth related risk aversion provides the basis for productivity enhancing asset redistributions. The two mechanisms are analogous in that in both cases the transfer of the asset reduces the costs associated with the assignment of residual claimancy and control rights to the wealth poor, attenuating suboptimal risk taking and the costs of risk exposure in the first, and reducing the opportunity cost of ownership in the second.

#### **4 Insurance**

It follows that measures that render the entrepreneur less risk averse, or lessen the risk involved in production, lessen the risk allocation losses associated with the reassignment of residual claimancy and control rights to low-wealth entrepreneurs. An entrepreneur who acquires the productive asset through an egalitarian redistribution policy, but who would otherwise prefer to sell this asset, could be induced by such measures to remain residual claimant on the use of the asset. In addition, such measures would reduce the losses from risk avoidance by entrepreneurs willingly engaged in independent production. We shall suggest two plausible measures of this type. The first involves insuring entrepreneurs against forms of risk exogenous to the production process, and the second involves insuring entrepreneurs against public risk—risk correlated with the risk of independent production, but which is publicly observable.

Suppose the entrepreneur's wealth independent from participating in production,  $w$ , has a stochastic element  $\gamma\zeta$  of mean zero distributed independently from  $z$ ,

where  $\gamma > 0$  is a constant. We call such a stochastic element *exogenous risk*, and we term a reduction in  $\gamma$  a *reduction in exogenous risk* (as opposed to the endogenous risk  $\sigma z$  that chosen by the entrepreneur).

It seems plausible that lowering exogenous risk would lead a entrepreneur to increase endogenous risk, since we know from Proposition 1vi that the marginal rate of substitution between risk and expected return,  $s(\sigma, \mu)$ , increases as the level of risk increases. Therefore we would expect a reduction in the exogenous risk faced by an independent entrepreneur to lower the ‘marginal cost’ of risk taking in production, and hence increase voluntary risk-taking. In fact, however, we need a condition stronger than decreasing absolute risk aversion to conclude that this is the case. We have

Theorem 4. *Let  $\lambda(\mathbf{w}) = -u''(\mathbf{w})/u'(\mathbf{w})$ , the Arrow-Pratt risk coefficient at wealth level  $\mathbf{w}$  for an agent with utility function  $u(\mathbf{w})$  exhibiting decreasing absolute risk aversion. Then if*

$$\lambda''(\mathbf{w}) > \lambda(\mathbf{w})\lambda'(\mathbf{w}), \quad (10)$$

*a reduction in exogenous risk leads the entrepreneur to increase the level of risk in production.*

Proof: We show in Lemma 1 in the Appendix that in the presence of exogenous risk, we can still describe the entrepreneur as optimizing in  $\sigma$ - $\mu$  space, and the indifference curves have the same properties when  $\gamma > 0$ , as when  $\gamma = 0$ . Then we show in Proposition 2 in the Appendix that under the conditions stated in Theorem 4 that lowering  $\gamma$  flattens the entrepreneur’s indifference curves at each point in  $\sigma$ - $\rho$  space, and hence induces the entrepreneur to assume more risk. ■

For what utility functions does Theorem 4's condition 10 hold? Note that for constant absolute risk aversion, which implies a utility function of the form  $u(\mathbf{w}) = \alpha - \beta e^{-\gamma \mathbf{w}}$ , exogenous risk does not affect the entrepreneur's choice of endogenous risk.<sup>11</sup> We have not succeeded in finding a decreasing absolute risk aversion utility function for which condition (10) is violated. Indeed, as the next series of corollaries demonstrate, all decreasing absolute risk averse utility functions that we have found in the research literature satisfy (10). We conclude that Theorem 4 has quite general application.

The most obvious candidates are of course the logarithmic and power law utility functions, which satisfy decreasing absolute and constant relative risk aversion (meaning  $\mathbf{w}\lambda(\mathbf{w})$  is constant). We have

*Corollary 4.1. If the entrepreneur exhibits constant relative risk aversion, a reduction in exogenous risk  $\gamma\zeta$  leads the entrepreneur to increase the level of risk  $\sigma z$  in production.*

Proof: Constant relative risk aversion means that  $w\lambda(w)$  is constant, which implies  $\lambda'(\mathbf{w}) < 0$  and  $\lambda''(\mathbf{w}) = -2\lambda'(\mathbf{w})/\mathbf{w} > 0 > \lambda(\mathbf{w})\lambda'(\mathbf{w})$ . ■

Another plausible candidate is a utility function whose Arrow-Pratt coefficient declines according to a power law. We then have

*Corollary 4.2. For any  $\alpha, \beta > 0$  there is an increasing, concave utility function*

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<sup>11</sup>To see this, note that with exponential utility, (20) in the Appendix shows that exogenous risk merely multiplies the utility function by a constant.

$u(\mathbf{w})$  with Arrow-Pratt risk coefficient

$$\lambda(\mathbf{w}) = \alpha \mathbf{w}^{-\beta}.$$

If the entrepreneur has such a utility function, a reduction in exogenous risk  $\gamma\zeta$  leads the entrepreneur to increase the level of risk  $\sigma z$  in production.

Proof: We can assume  $\beta \neq 1$ , since the  $\beta = 1$  case follows from the previous Corollary. To find  $u(\mathbf{w})$ , we write the identity  $\lambda(\mathbf{w}) = -u''(\mathbf{w})/u'(\mathbf{w})$  in the form

$$\frac{d}{d\mathbf{w}} \log u'(\mathbf{w}) = -\alpha \mathbf{w}^{-\beta}$$

and integrate twice, getting

$$u(\mathbf{w}) = \int e^{-\frac{\alpha}{1-\beta} \mathbf{w}^{1-\beta}} d\mathbf{w}.$$

By the Fundamental Theorem of the Calculus,  $u(\mathbf{w})$  has the desired properties.<sup>12</sup>

We can then calculate directly that

$$\lambda''(\mathbf{w}) - \lambda(\mathbf{w})\lambda'(\mathbf{w}) = \alpha\beta\mathbf{w}^{-2(1+\beta)} (\alpha\mathbf{w} + (1 + \beta)\mathbf{w}^\beta) > 0.$$

This completes the proof. ■

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<sup>12</sup>This utility function has the closed form

$$u(\mathbf{w}) = -\Gamma \left[ \frac{1}{1-\beta}, \frac{\alpha}{1-\beta} \mathbf{w}^{1-\beta} \right],$$

for  $\beta \in (0, 1)$ , where  $\Gamma$  is the incomplete Gamma function (Wolfram 1996).

Finally, consider the following utility function:

$$u(\mathbf{w}) = -e^{-\alpha \mathbf{w}^\beta}, \quad (11)$$

with  $\beta < 1$ ,  $\alpha\beta > 0$ . This function satisfies decreasing relative risk aversion for  $\alpha < 0$ , constant relative risk aversion for  $\alpha = 0$ , and increasing relative risk aversion for  $\alpha > 0$  (Saha et al. 1994). We have

*Corollary 4.3. Suppose the entrepreneur has a utility function of the form (11) for any  $\beta < 1$  and any  $\alpha$  such that the utility function is increasing in wealth. Then a reduction in exogenous risk  $\gamma\zeta$  leads the entrepreneur to increase the level of risk  $\sigma z$  in production.*

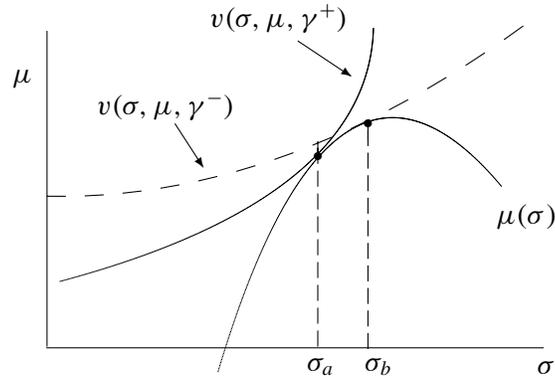
*Proof:* A direct computation shows that utility is increasing in wealth if and only if  $\alpha\beta > 0$ . Also,

$$\lambda''(\mathbf{w}) - \lambda(\mathbf{w})\lambda'(\mathbf{w}) = \frac{1 - \beta}{\mathbf{w}^3} [(3 - \beta) + w^{2\beta}\alpha^2\beta^2 + 2\alpha\beta w^\beta(2 - \beta)],$$

which is positive for all  $\beta < 1$ ,  $\alpha\beta > 0$ . ■

Theorem 4 is illustrated in Figure 6 for the case where  $z + \gamma\zeta$  form a linear class (e.g., both  $z$  and  $\zeta$  are normally or uniformly distributed). In the figure, the reduction in exogenous risk leads the entrepreneur to increase endogenous risk from point  $\sigma_a$  to  $\sigma_b$ .

We conclude that an economic policy measure that reduces the degree of uncertainty facing entrepreneurs unrelated to the productive asset itself, for instance health insurance, consumer goods price stabilization, or business cycle stabilization,



**Figure 6:** Reducing Exogenous Risk Leads to Increased Production Risk.

Note: The dashed curve is the entrepreneur's indifference curve under conditions of reduced exogenous risk. The risk level is displaced from  $\sigma_a$  to  $\sigma_b$ .

may induce nonwealthy entrepreneurs to assume a higher level of risk exposure in production and thus increase the scope of application of productivity enhancing egalitarian redistributions.

A second measure with similar properties is insurance against public risk. Suppose the random variable  $\eta$  is positively correlated with the stochastic element  $z$  in production, and is publicly observable at the end of the production period, hence is contractible. We call  $\eta$  a *production-related public risk*. Average rainfall in the region over the growing season, for instance, is a form of production-related public risk. Consider a market for a fair insurance policy on production-related public risk that pays entrepreneurs a premium  $l$  and obliges the entrepreneur to pay back an amount  $b\eta$  at the end of the production period. We call this a *public risk insurance policy*, and we call  $b$  the *payback rate*. We say the market in public risk insurance is

*competitive* if the buyer is free to choose the premium and the payback rate, subject to the insurance being fair. We have

Theorem 5. *Suppose  $\sigma$  is not contractible, but there is a production-related public risk variable  $\eta$ . Consider insurance in which the entrepreneur receives a lump sum  $l^*$  and pays the insurer  $b^*\eta$  when  $\eta$  is observed at the end of the period, and  $b^*$  is chosen so that the insurance is fair (i.e.,  $l^* = b^*\mathbf{E}\eta$ ). Then*

- (a)  *$(l, b)$  can be chosen so that a profit-maximizing entrepreneur will purchase the policy and will choose the optimal risk level  $\sigma^*$ .*
- (b) *if  $(l, b)$  is chosen to maximize the entrepreneur's payoff, subject to being fair, the resulting level of entrepreneurial risk is socially optimal if and only if  $\eta$  is perfectly correlated with  $z$ .*

The intuition underlying Theorem 5 is that the socially optimal insurance policy  $(l^*, b^*)$  induces risk neutral behavior by restricting the entrepreneur's choice to no insurance at all or more insurance than the entrepreneur would choose in a competitive environment. Profit maximizing entrepreneurs would demand a lower level of insurance. The reason for the difference is that only when the degree of risk and the public signal are perfectly correlated does the insurance policy that renders the standard deviation of income invariant to the choice of risk level by the entrepreneur (inducing risk neutral behavior by the entrepreneur) also minimize the standard deviation of income (corresponding to the entrepreneur's desired fair insurance policy).

Proof: Since the insurance is fair,  $l = b\mu_\eta$ , where  $\mu_\eta = \mathbf{E}\eta$ . The entrepreneur's

payoff net of the opportunity cost of capital is then

$$y(\sigma, b) = \rho(\mathbf{w} - \mathbf{k}) + \sigma z + g(\sigma) - b(\eta - \mu_\eta) \quad (12)$$

which is a random variable with mean

$$\mu(\sigma) = \rho(\mathbf{w} - \mathbf{k}) + g(\sigma)$$

and standard deviation  $\tau$  given by

$$\tau^2 = \sigma^2 - 2b\sigma r_{z\eta}\sigma_\eta + b^2\sigma_\eta^2, \quad (13)$$

where  $r_{z\eta}$  is the correlation between  $z$  and  $\eta$  and  $\sigma_\eta$  is the standard deviation of  $\eta$ .

If the insurer chooses  $b$ , the entrepreneur's first order condition (8) now becomes

$\tilde{u}_\sigma = v_\mu[\mu_\sigma - s(\mu, \tau)\tau_\sigma] = 0$ , or

$$\tilde{u}_\sigma = v_\mu \left[ g'(\sigma) - s(\mu, \tau) \frac{\sigma - br_{z\eta}\sigma_\eta}{\tau} \right] = 0. \quad (14)$$

Since the Pareto-efficient level  $\sigma^*$  satisfies  $g'(\sigma^*) = 0$ , the entrepreneur will be induced to choose this level when

$$b^* = \frac{\sigma^*}{r_{z\eta}\sigma_\eta}. \quad (15)$$

The payout rate  $b^*$  is that which renders the standard deviation of income invariant with respect to the choice of risk and thus induces the entrepreneur to choose  $\sigma$  to

maximize expected income. The corresponding premium is then  $l^* = b^* \mu_\eta$ .

Thus there is a fair public insurance policy that induces the socially optimal level of risk-taking. Will such a policy be offered on a competitive insurance market? Suppose the insurance market is competitive, so the entrepreneur can choose the payback rate to maximize expected payoff. The entrepreneur then chooses  $\sigma$  and  $b$  to maximize

$$\tilde{u}(\sigma, b) = \mathbf{E}[u(y(\sigma, b))].$$

Because varying  $b$  does not affect the expected net cost of the insurance (since  $l = b\mu_\eta$ ), reductions in the standard deviation of income are costless and the optimal choice  $b^o$  is that which minimizes  $\tau^2$ , giving the first order condition

$$\frac{\partial(\tau^2)}{\partial b} = 2b\sigma_\eta^2 - 2r_{z\eta}\sigma\sigma_\eta = 0, \quad (16)$$

From this we get

$$b^o = \frac{\sigma r_{z\eta}}{\sigma_\eta}. \quad (17)$$

Substituting  $b^o$  in (13) gives

$$\tau = \sigma \sqrt{1 - r_{z\eta}^2}.$$

The optimal risk level  $\sigma$  then follows from the first order condition (14), which becomes

$$g'(\sigma^o) = s(\mu(\sigma^o), \tau(\sigma^o)) \sqrt{1 - r_{z\eta}^2}. \quad (18)$$

This is satisfied by  $\gamma'(\sigma) = 0$  only when  $r_{z\eta} = 1$ . Further, comparing (15) and (17)

for  $r_{z\eta} < 1$ , the payback rate that implements the socially optimal risk level  $b^*$  will exceed that which would be offered on a competitive insurance market. ■

There are three reasons why the market in public risk insurance may fail. First, as we have seen in Theorem 5, the competitively determined insurance rate does not achieve the socially optimal outcome. Second, the market in public risk insurance is subject to adverse selection if  $r_{z\eta}$  differs among entrepreneurs and is not public knowledge. Third, a private industry selling public risk insurance may not be able to operate as approximately risk neutral, since the signal  $\eta$  is a macroeconomic variable that is perfectly correlated for all insurance purchasers, so insurance companies cannot use the law of large numbers to handle the volatility of their payouts. Moreover, if there is uncertainty concerning  $\mu_\eta$ , or if  $\mu_\eta$  shifts over time, the insurance companies' risk position becomes even more precarious. Thus government policy might be needed to implement this outcome.

Of course an analysis of the defects of the market solution to the independent entrepreneur's risk problem must be complemented by an analysis of the defects of the public sector as an insurance provider. In particular, in the absence of a mechanism guaranteeing their accountability, public decision-makers will choose the level and type of independent entrepreneur insurance to meet multiple objectives, of which fostering socially efficient production is only one.

## 5 Conclusion

The efficiency-equality trade off has conventionally been thought to arise from the distorting disincentive effects of taxes and transfers on the motivation to engage in

hard work, risk-taking and other productivity-enhancing behaviors. A more contemporary consideration of efficiency-equality relationships adds that when some agents are asset poor and hence cannot be assigned residual claimancy status with respect to their own actions, incentive distortions may arise not from governmental interventions alone, but additionally from the incomplete nature of contracts governing risk-taking and effort combined with limited wealth holding by entrepreneurs.

In this setting redistribution of assets to the wealth poor may allow residual claimancy to be assigned to those performing services that are not easily contractible, thus attenuating the associated incentive problems, and possibly inducing an efficiency-equality complementarity rather than tradeoff. Specifically, we have shown that even if the poor face no credit market constraints, when the level of risk aversion depends on the asset position of the individual, there may exist a multiplicity of equilibrium assignments of residual claimancy and control rights associated with wealth ownership. Analogous results hold for cases where risk aversion is independent of wealth but in which the cost of capital to entrepreneurs depends on their wealth level. Some of these multiple distributions of wealth may be both more equal and more efficient (in a well defined sense) than others.<sup>13</sup>

However, as we have seen, this way of representing equality-efficiency relationships also suggests possible allocational inefficiencies arising from the suboptimal risk taking likely to be implemented by the asset poor. We have shown that insurance against exogenous and public risk may broaden the scope for productivity enhanc-

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<sup>13</sup>As in the work of Galor and Zeira (1993), Durlauf (1996) and subsequent contributions, our model thus demonstrates the non-ergodic nature of the wealth distribution.

ing egalitarian asset redistribution but that socially optimal levels of insurance will not be offered by private providers. Thus a governmental intervention combining asset redistribution and insurance may be warranted under some conditions.

## 6 Appendix

Suppose an entrepreneur has wealth  $\mathbf{w}$  and income

$$y(\sigma) = \rho\mathbf{w} + \gamma\zeta + \sigma z + g(\sigma), \quad (19)$$

where  $\zeta$  is a random variable with mean zero, positive variance, and cumulative distribution  $G(\zeta)$ ,  $\gamma > 0$ , and the remaining terms are defined as above. Then if  $\gamma = 0$ , the random variables  $\{y(\sigma) | \sigma > 0\}$  form a linear class:

**Definition.** A family  $\mathcal{F} = \{y_\alpha\}$  of random variables with finite means  $\{\mu_\alpha\}$  and standard deviations  $\{\sigma_\alpha\}$  is said to belong to a *linear class* if there is a random variable  $z$  such that

$$\frac{y_\alpha - \mu_\alpha}{\sigma_\alpha} = z$$

for all  $\alpha$ . We call the random variable  $z$  the *generator*  $\mathcal{F}$ .

Lemma 1. *Suppose  $u(\cdot)$  is twice differentiable, increasing, and concave, and let*

$$\hat{u}(\mathbf{w}, \gamma) = \int_{-\infty}^{\infty} u(\mathbf{w} + \gamma\zeta) dG(\zeta), \quad (20)$$

*where  $\zeta$  is a random variable with mean zero, finite variance, and cumulative distribution  $G(\zeta)$ . Then Proposition 1 holds for  $\hat{u}(\mathbf{w}, \gamma)$ , where  $\gamma$  is a parameter.*

Proof: We show that  $\hat{u}(\mathbf{w}, \gamma)$  satisfies the conditions of Proposition 1, where  $\mathbf{w}$  is the argument to the utility function and  $\gamma$  is a parameter. Let us write, for any function  $f$ ,

$$\mathbf{E}_\zeta f = \int_{-\infty}^{\infty} f(\mathbf{w} + \gamma\zeta) dG(\zeta)$$

so  $\hat{u}(\mathbf{w}, \gamma) = \mathbf{E}_\zeta u$ . Then using the same notation, we have  $\hat{u}_{\mathbf{w}}(\mathbf{w}, \gamma) = \mathbf{E}_\zeta u_{\mathbf{w}} > 0$  and  $\hat{u}_{\mathbf{w}\mathbf{w}}(\mathbf{w}, \gamma) = \mathbf{E}_\zeta u_{\mathbf{w}\mathbf{w}} < 0$ , so  $\hat{u}(\cdot, \gamma)$  is twice differentiable, increasing, and concave. ■

**Remark.** Lemma 1 shows that even when there is a stochastic element  $\gamma\zeta$  in the non-production-related income of the agent, Proposition 1 continues to hold. In this case we write the utility function as  $u(\mathbf{w}, \gamma)$  and the mean-variance utility function as  $v(\sigma, \mu, \gamma)$ , with indifference curve slopes  $s(\sigma, \mu, \gamma)$ . It remains to determine the behavior of  $s_\gamma(\sigma, \mu, \gamma)$ . For this we will use

Lemma 2. *Suppose  $g(x)$  and  $r(x)$  are defined on a (possibly infinite) interval  $(a, b)$ . Suppose  $g(x)$  changes sign, from negative to positive, exactly once,  $\int_a^b g(x) dG(x) = 0$ , and  $r'(x) > 0$ . Then*

$$\int_a^b r(x)g(x) dG(x) > 0.$$

Proof: Suppose  $g(x_0) = 0$  and let  $r_0 = r(x_0)$ . Then

$$\int_a^b r(x)g(x) dG(x) = \int_a^b (r(x) - r_0)g(x) dG(x) > 0,$$

since the integrand is always positive for  $x \neq x_0$ .

Proposition 2. *Suppose  $u(\cdot)$  is twice differentiable, increasing, concave, and exhibits decreasing absolute risk aversion. Define  $\hat{u}(\mathbf{w}, \gamma)$  by (20), and let  $\hat{\lambda}(\mathbf{w}, \gamma)$*

be the Arrow-Pratt risk coefficient for  $\hat{u}(\mathbf{w}, \gamma)$ . Then  $\hat{\lambda}_{\mathbf{w}\mathbf{w}} \geq \hat{\lambda}\hat{\lambda}_{\mathbf{w}}$  implies  $\hat{\lambda}_\gamma > 0$ ; i.e., increasing the agent's exposure to  $\zeta$  increases the agent's risk aversion with respect to  $\mathbf{w}$ .

Proof: Since  $\hat{\lambda} = -\mathbf{E}_\zeta u_{\mathbf{w}\mathbf{w}}/\mathbf{E}_\zeta u_{\mathbf{w}}$  (using the notation of Lemma 1), we have

$$\hat{\lambda}_\gamma(\mathbf{w}, \gamma) = -\frac{\mathbf{E}_\zeta \left[ \zeta (\hat{\lambda}(\mathbf{w}, \gamma) u_{\mathbf{w}\mathbf{w}} + u_{\mathbf{w}\mathbf{w}\mathbf{w}}) \right]}{\mathbf{E}_\zeta u_{\mathbf{w}}}. \quad (21)$$

Now  $\lambda(\mathbf{w}) = -u_{\mathbf{w}\mathbf{w}}(\mathbf{w})/u_{\mathbf{w}}(\mathbf{w})$  implies  $u_{\mathbf{w}}\lambda_{\mathbf{w}} = -(\lambda u_{\mathbf{w}\mathbf{w}} + u_{\mathbf{w}\mathbf{w}\mathbf{w}})$ . Suppose  $\lambda_{\mathbf{w}\mathbf{w}} \geq \lambda\lambda_{\mathbf{w}}$ . Then

$$\frac{d(u_{\mathbf{w}}\lambda_{\mathbf{w}})}{d\mathbf{w}} = u_{\mathbf{w}\mathbf{w}}\lambda_{\mathbf{w}} + u_{\mathbf{w}}\lambda_{\mathbf{w}\mathbf{w}} = u_{\mathbf{w}}[-\lambda\lambda_{\mathbf{w}} + \lambda_{\mathbf{w}\mathbf{w}}] > 0.$$

Notice that  $\zeta$  is increasing and since  $\mathbf{E}_\zeta \zeta = 0$ , it changes sign exactly once, going from negative to positive. It follows from Lemma 2 that

$$\mathbf{E}_\zeta [\zeta (\lambda u_{\mathbf{w}\mathbf{w}} + u_{\mathbf{w}\mathbf{w}\mathbf{w}})] = \mathbf{E}_\zeta [\zeta u_{\mathbf{w}}\lambda_{\mathbf{w}}] > 0. \quad (22)$$

When  $\gamma = 0$  we have  $\hat{\lambda}(\mathbf{w}, \gamma) = \lambda(\mathbf{w})$ , so (22) and (21) imply  $\hat{\lambda}_\gamma(\mathbf{w}, 0) > 0$ , so  $\hat{\lambda}(\mathbf{w}, \gamma) > \lambda(\mathbf{w})$  for some nonempty interval  $0 \leq \gamma < \epsilon$ . Moreover, the same reasoning implies that for any  $\gamma$  such that  $\hat{\lambda}(\mathbf{w}, \gamma) \geq \lambda(\mathbf{w})$ , we have  $\hat{\lambda}_\gamma(\mathbf{w}, \gamma) > 0$ . It follows that  $\hat{\lambda}(\mathbf{w}, \gamma) > \lambda(\mathbf{w})$  for all  $\gamma > 0$  and  $\hat{\lambda}_\gamma(\mathbf{w}, \gamma) > 0$ . ■

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