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### **Constructing the invisible - Computer graphics and the end of Optical Media**

Ricardo Cedeño Montaña  
*Universidad de Antioquia Medellín, Colombia, ricardo.cedeno@udea.edu.co*

Christina Vagt  
*University of California Santa Barbara, vagt@gss.ucsb.edu*

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## **Constructing the invisible - Computer graphics and the end of Optical Media**

### **Cover Page Footnote**

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## Introduction

As computer graphics simulate optical hardware, they themselves cannot be considered optical media. Such is the argument Friedrich Kittler puts forward in a short article he first published in 2001.<sup>1</sup>

According to Kittler's media history, optical media faced their end with the emergence of computer graphic techniques such as raytracing and radiosity – an end that he introduces in *Optical Media* but that he does not fully elaborate in the few pages devoted to computers in that genealogy. In the following text on a media history of early computer graphics, we will on the one hand shed a ray of light on this end in the hope of reconstructing it, and on the other hand we will offer a description aimed at a media history of the imaginary. Our intention is not to argue for the end of optical media at the hands of algorithmic simulations of optical worlds. We are after something different. By turning our attention towards the algorithms and programs that drive these simulations, we want to foster a media historical reflection on the current situation in which images, commanded by techno-codes, delineate and structure the dominant code of communication with which we imagine.<sup>2</sup>

The path of our argumentation is grounded in two approaches to the creation of images and imagination, by the media philosopher Vilém Flusser and the media theorist Friedrich Kittler. In technical images the former sees a new kind of imagination based not on interpretation of the world but on calculating its effects, while for the latter imagination is merely an effect of the symbolic operations performed by media technology. Their views on images and media serve to highlight a fundamental contrast between the two: on the one hand we have the liberating power of technical images to break the chains imposed by the European colonial regime of interpretation via writing, and on the other hand the eschatological media history of the West that finds its end in the machinic chain of alphanumeric operations that generate images. In relation to the synthetic images, both positions seem to complement each other. In books such as *Kommunikologie* and *Into the*

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<sup>1</sup> Cf. Kittler, Friedrich: "Computergrafik. Eine halbtechnische Einführung". In *Paradigma Fotografie. Fotokritik am Ende des fotografischen Zeitalters*, edited by Herta Wolf, Frankfurt a. M.: Suhrkamp, 2002, 178–194. The English translation appeared one year before: Kittler, Friedrich, Ogger, Sara: "Computer Graphics: A Semi-Technical Introduction, *Grey Room* 02, Winter 2001, 30–45. The article is based on a talk he gave in 1998 (<https://archive.org/details/FriedrichKittlerVortragBasel1998Computergraphik>, last visited on 13. June 2018). Although the media historical analysis he offers of the techniques behind 3D computer graphics is contemporary with his long genealogy under the title *Optische Medien –Berlin Vorlesungen* 1999, published in 2002, he merely mentions them in passing in the preface of the book.

<sup>2</sup> Flusser, Vilém: *Medienkultur*, Frankfurt am Main: S. Fischer-Verlag, 1997, 134.

*Universe of Technical Images* Flusser culturally and theoretically widens the scope of the image regime in contrast to that of text, while Kittler maintains the primacy of the symbolic throughout all his writings.

Right in the first sentence of his *Semi-Technical Introduction* Kittler makes clear that for him optical media and computer graphics are not about the images they output, but rather about the algorithms that produce such images in the first place.

Computer images are the output of computer graphics. Computer graphics are software programs that, when run on the appropriate hardware, provide something to see and not just to read.<sup>3</sup>

Thus, computer graphics have a double existence, as a sensual array of shapes and colors on a screen and as a computer code on a memory and a processor. In a painstakingly “step-by-step process” an algorithm takes bit after bit “out of the memory” and assembles them on a screen. Computer graphics are indeed more than just images. They are rather computer programs. They are simultaneously something perceived by the human visual apparatus and something processed by the machine.

Even though the computer itself is not an optical medium, it has been clear since graphical processors became affordable and massively accessible during the 1990s, including at philosophy faculties and cultural history institutes, that such a machine was able to simulate all other machines and media, including imaging devices. It was also clear that the manipulation of other (optical) media through the computer knows no limits, except those imposed by the storage capacity of the machine and the time required to perform the algorithm. While in TV, film, and photography the optical hardware simply does what it has to do, the optic in the computer is virtual and thus only limited by the storage and processing resources. Before the sensual layer of perception, the computer graphics feign one optic (hardware), where only algorithms (software) reign.

Computer graphics are to these optical media what the optical media are to the eye. Just as the camera lens, literally as hardware, simulates the eye, which is literally wetware, so does software, as computer graphics,

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<sup>3</sup> Kittler, *Computergrafik*, 178.

simulate hardware.<sup>4</sup>

When software simulates hardware, then optics recede and the symbolic operations come to the forefront. In *Sealed Computers* (1992) the artist Maurizio Bolognini connects several computers to “jointly compute simple, generative graphic structures which, however, deliberately do not get displayed.”<sup>5</sup>

By preventing the continuously generated graphics to become visible, this artwork shows that in computer graphics even the visual output on the screen can be spared. But aside from this radical example, in general computer graphics only make optional the optics. Accordingly, the output of computer graphics, the digital image, is in no way a subject of reflections, but more a problem of how such reflections and refractions can be generated. The question posed by computer graphics and optical media reads, how does one decode historically an aesthetics that depends on optics? Kittler comes to this problem, during his time as Professor of Aesthetics and History of Media, because he observes that whether as hardware or software, it is optical operations, not images, that cause perception and *innerwordliness*. The Heideggerian thinking of a modernity commanded by images and presentations, the *Age of the World Picture*,<sup>6</sup> has left traces in Kittler’s and Flusser’s work. It is also very active in the problem of representation in the sciences and philosophy, without which the media theory of the turn of the century cannot be thought. With the emergence and spread of computer graphics during the last decades of the 20th century, this idea has received an upgrade. The within-the-world of the human *Daseins*, for the most part discreet and invisible, consists of more or less hard wired strings of scientific texts. After all, underneath all technical images and imaginations only symbolic operations run.

Once the optics became simulated, media aesthetics shifted from physics to data processing and numerical procedures. This shift was already present in the information aesthetics proposed by German cyberneticist Max Bense<sup>7</sup> and more practically was implemented in the mid 1960’s in the artworks of early computer

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<sup>4</sup> Ibid., 182.

<sup>5</sup> Broeckmann, Andreas: Image, Process, Performance, Machine: Aspects of an Aesthetics of the Machinic. In: Grau, Oliver (ed.): *Media Art Histories*. Cambridge/Mass.: MIT Press, 2007, 205.

<sup>6</sup> Heidegger, Martin: “The Age of the World Picture”, in: *The Question Concerning Technology and Other Essays*, transl. by Lovitt, William, New York-London: Garland Publish., 1977, 115-154. On the problem of representation and image critique in the writings of Heidegger cf. Vagt, Christina: “Kosmographien. Heideggers Weltbildkritik und der Diagrammatische Grund”, in: Bothe, Thorsten and Suter, Robert (Ed.): *Prekäre Bilder*, München, DE: Fink, 2010, 159–174.

<sup>7</sup> Cf. Büscher, Barbara, Hoffmann, Christoph, Von Herrmann, Hans-Christian (Ed.): *Ästhetik als Programm. Max Bense / Daten und Streuungen*, Berlin, DE: Diaphanes, 2004.

artists such as the mathematician Frieder Nake. This problematic change brought by computer-generated images came first to the foreground with *Sketchpad* designed by Ivan Sutherland in 1963. Initially aimed at artists and draughtspeople, *Sketchpad*'s key innovation was to replace the abstract entities of a programming language with graphical images. A user was able to manipulate the vector graphics displayed on a screen by directly pointing at and interacting with the lines and points of a drawing via a light-pen. According to Nake, with *Sketchpad* emerged a graphical interface that split these new kinds of images between a visible surface and an invisible but manipulable *subface*; that is, a split between its sensual and computable existences, an interface that does not destroy the illusion of users, as those who draw on the screen.

The image as a digital image has become first and foremost *algorithmic*: now it has also an inwardness under its surface, in other words it is surface and subface simultaneously. Both, and this is decisive, are objectively available. The surface of the digital image is *visible*, while the subface is *manipulable*. The surface is there for the user, the subface for the processor (with a program). To subface belongs that and just that, that as data structure and algorithm is available.<sup>8</sup>

The processes on the subface include a chain of endless automatic operations of data processing such as searches, sorts, tabulations, interpolations, smooth, and tracing. In contrast to all other kinds of technical images, synthetic images consist specifically of a subface that is thoroughly manipulable and computable. As a matter of fact, it is this invisible subface that generates the visible surface, thereby concealing itself underneath. Acting as a media archaeologist Kittler investigates exclusively the subface or in other words the programs. “Conversely, computer graphics, because it is software, consists of algorithms and only of algorithms;”<sup>9</sup> and therefore the surface offers nothing of interest to him.

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<sup>8</sup> „Das Bild als digitales Bild ist zuvorderst *algorithmisch* geworden: Es besitzt nun auch eine unterflächliche Innerlichkeit bzw. ist Oberfläche und Unterfläche zugleich. Beide – das ist entscheidend – sind objektiv vorhanden. Die Oberfläche des digitalen Bildes ist *sichtbar*, während die Unterfläche *bearbeitbar* ist. Die Oberfläche besteht für den Benutzer, die Unterfläche für den Prozessor (mit Programm). Zur Unterfläche gehört das und nur das, was als Datenstruktur und Algorithmus vorhanden ist.“ (Nake, Frieder: “Das doppelte Bild”, in: *Digitale Form*, edited by Pratschke, Margarete, 3,2: 40–50, Bildwelten des Wissens, Berlin, DE: Akademie-Verlag, 2005, 47. Translated by the authors.)

<sup>9</sup> Kittler, *Computergrafik*, 183.

What applies to the early history of computer graphics is not enough to describe the entire universe of synthetic images. First, because unlike traditional images, synthetic images are not physical manifestations of the outside world, as Flusser points out the traditional image analysis consequentially cannot be applied to them. And second, because each new genre (type) of synthetic images made concrete by a computable surface produces “unexpected situations from among a given field of possibilities.”<sup>10</sup> In what follows, we also address the subface. However, whereas Kittler sketched his work along two programming paradigms, Raytracing and Radiosity, today, both paradigms are implemented in every 3D graphics program and they operate under the logical operator OR – OR, meaning you use one or the other. We complement the media archaeology of computer graphics with a layered model of three statistical procedures used to solve key problems on 3D graphics during the 1970’s. Furthermore, while Kittler fashions his history of computer graphics around the limitation on two paradigms whose tragic decision is between subject and no-subject, we are more interested in the fact that with computer graphics the history of optical media can be seen retrospectively as an escalation of the production of invisibility.

In the following section on the subface, we deal with three central problems of the early 3D computer graphics as functions in the construction of the invisible: the hidden vertices of a 3D object, the smoothing of edges and the hiding of the screen raster structure. In the last section, we address the surface and its users, and how both subface and surface are part of Kittler’s and Flusser’s techno-imagination.

## Subface

Technical images are products of techno scientific knowledge and texts. Computer graphics are different from other technical images in that they are created out of pure calculation. In contrast to film or analogue video, they are not symptoms of the chaotic world that needs to be organized. These types of images are made from statistics of points, where on the one hand the point is the geometrical unit, the pixel written on the screen, and on the other is the mathematical data unit calculated, whether it is sorted, interpolated or traced.

On the surface, computer graphics are images that simulate the hardware of optical media, mostly the centuries-old central perspective and the optics of cameras. These types of images don’t have an optical input. As a consequence, on the subface a computer graphics program uses a large chain of data processing

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<sup>10</sup> Flusser, Vilém. *Writings*. University of Minnesota Press, 2002. p. 116

algorithms and statistics in order to sort, interpolate, and trace countless collections of points across the space. All these mathematical and statistical operations are at the root of what Paul Virilio called the statistical image.<sup>11</sup> They also constitute the material layer of a type of imagery only possible with the universally discreet data processing we call computer.<sup>12</sup>

Synthetic images are hyper-granular entities in which each grain, each point, is the site for countless calculations. It is thus only through the complete datafication of the point that the computer is able to calculate and give shape to the formless. In the following part we sketch a short media history of computer graphics focusing on three different historical problems faced during the 1970's: first, the sorting of all vertices in a 3D model according to their relation with a hypothetical observer; second, the interpolation of the shading and orientation data of all vertices to color an object; and third, the calculation for each pixel on the screen of their values of illumination to produce shadows, reflections, and refractions.

### Hiding lines

The history of the hidden-surface problem faced by computer scientists during the 1960's and 1970's illustrates a genealogy of the computational image centered not on the superficial visibility of the image but on the underlying techniques that make the image possible.<sup>13</sup> The solutions to this problem marked a historical turning point in the bi-dimensional representation of three dimensional scenes because these solutions were driven by statistical modes of structuring the objects themselves and not by geometrical operations to organize the sight as it is the case of the central perspective. While the central perspective is a diagrammatic visual method for drawing based on measurements and geometrical relations, computer graphics are based on algorithms and data processing and only emerge on the surface of the monitor, through the application of systems of mathematical equations.<sup>14</sup> A computer graphics program breaks a scene into discrete data, mostly the spatial coordinates and color information of each object. It then arranges these data in lists and sorts them according to a particular criterion, e.g. depth.

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<sup>11</sup> Cf. Virilio, Paul: *The Vision Machine*, Indianapolis, IN: Indiana University Press, 1994, 75.

<sup>12</sup> “[...] der universalen diskreten Datenverarbeitung namens Computer“ (Kittler, Friedrich: *Optische Medien*, Berlin, DE: Merve Verlag, 2002, 16. Translated by the authors.)

<sup>13</sup> Cf. Gaboury, Jacob: “Hidden Surface Problems: On the Digital Image as Material Object.” *Journal of Visual Culture* 14, no. 1 (April 1, 2015): 40–60.

<sup>14</sup> “[...] durch Anwendung mathematischer Gleichungssysteme“ (Kittler, *Optische Medien*, 319. Translated by the authors.)



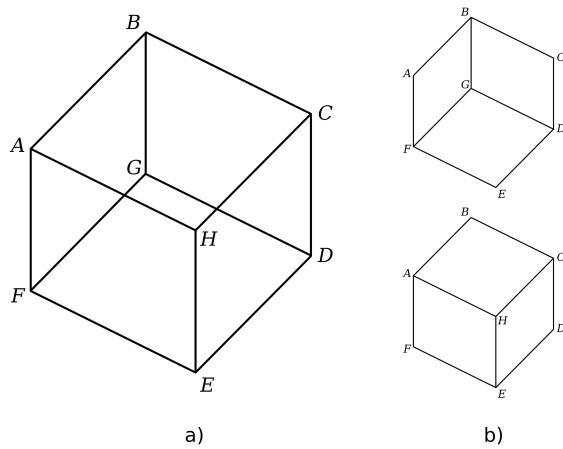


Figure 1: What vertex should be hidden?

In figure 1 we illustrate a very simple case of the hidden-surface problem. On the left side (a) there is a cube with all its edges and vertices visible. This is an *ambiguous* image because it is difficult to discern which vertex, G or H, is on the back of the cube and therefore not visible. We are confronted with a situation in which, if the position of a hypothetical observer is unknown, we would have two options, as shown on the right side of figure 1 (b). The problem is then to decide which vertices and lines in the scene would be hidden in the rendered image. In order to generate an *unambiguous* image of the cube, a computer program must first list and arrange all the vertices from the nearest to the farthest according to their positions in the space and their spatial relation to the observer. Before a computer can image any visual output, it must perform an exhaustive search and ordering of all points in a scene. Removing the hidden lines from simple objects such as our cube was trivial, the challenge however was to find a computationally efficient way to do that in real-time for situations with several complex objects.

3D objects regardless of their complexity are nothing but a collection of flat faces and each face is a thorough list of the coordinates of all its vertices and the

connections between them.<sup>15</sup> In other words, they are composed exclusively of addresses and values. Any vertex will be listed in a coordinate system as  $q = [x, y, z, w]$ . Such a system was introduced by August Ferdinand Möbius in 1827. It uses four addresses instead of the usual three of the Cartesian coordinates, producing a new kind of analytical geometry.<sup>16</sup> In contrast to the Cartesian coordinates, the inclusion of the additional operator ( $w$ ) allows this system to intersect parallel lines in the space, thus breaking one of the fundamental axioms of the Euclidean geometry. This scientific concept is able to mathematically describe the geometry of cameras by projecting points to infinity using a finite coordinate system. The Möbius' coordinate system takes the equations of a 3D space and depicts them in a 2D surface. Thus, all points and lines converge on a single point on the bi-dimensional surface and in that way the functioning of the pinhole camera is treated analytically. From the very beginning, computer graphics do not operate in the Euclidean space but in a projective space that collapses at one point.

In 1974, the pioneer of computer graphics, Ivan Sutherland, together with two other researchers, analyzed ten different hidden-surface algorithms and organized them into a taxonomy that highlights their various approaches and techniques. Their taxonomy provided the means to identify the underlying principles shared by all Hidden Surface algorithms. They concluded that in order to structure the objects for rendering synthetic images without any ambiguity concerning what is visible all these algorithms relied on two basic principles: sorting and coherence.<sup>17</sup> Thus, the creation of 3D scenes depends on the one hand on the statistical ordering of all the vertices of the objects and on the other on describing any complex surface as a set of flat polygons and straight edges.

Sorting algorithms are related to the history of processing large quantities of items of data. In data processing, sorting means the “rearrangement of items into ascending or descending order.”<sup>18</sup> This is a basic technique whose implementation in a machine is as old as the electro-mechanical tabulating machines designed by Hermann Hollerith during the 1880s for the Census Bureau of the United States. At the end of the 19th century, the ever-increasing population of the United States had become an acute problem for tabulating the voluminous census data. In Hollerith's design, any recorded individual data became a “statistical item” and sorting these

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<sup>15</sup> Sutherland, Ivan E., Sproull, Robert F., Schumacker, Robert A.: “A Characterization of Ten Hidden-Surface Algorithms”, *ACM Comput. Surv.* 6, No. 1 (1974): 1–55. p. 3.

<sup>16</sup> Möbius, August Ferdinand: *Gesammelte Werke*. Bd. 1. Leipzig, DE: Hirzel, 1885, p.VIII.

<sup>17</sup> Sutherland et al: “A Characterization of Ten Hidden-Surface Algorithms,” 3.

<sup>18</sup> Knuth, Donald E.: *The Art of Computer Programming*, Volume 3: (2Nd Ed.) Sorting and Searching. Redwood City, CA, USA: Addison Wesley Longman Publishing Co., Inc., 1998, 1.

data according to a particular criterion a matter of “counting or tallying such statistical items separately or in combination by means of mechanical counters.”<sup>19</sup> By the time digital computers arrived at the end of the 1940’s, machinic sorting was a well-established technique and a sorting algorithm was promptly implemented in one of the first computers: the EDVAC.<sup>20</sup>

A sorting routine was specially important for the designers of the EDVAC, because an efficient ordering code would not only offer solutions for differential equations, but would also show the “combinatorial ‘decision-making’ aspects of algorithms.”<sup>21</sup> During the early 1950s, the EDVAC was primarily used for ballistic trajectory calculations at the Aberdeen Proving Ground, Maryland. In that historical context, the problem of deciding what is visible in computer graphics is related to the long story of data processing that reaches back to counting large populations and deciding where to aim and fire missiles. In a projective space, a sorting algorithm orders polygons according to the closest vertex to the observer. Using such a criterion, it tabulates the vertices as visible or hidden. This algorithm places the statistical processing of data as the basis for any kind of computer imagery. During the years following Sutherland’s taxonomy another statistical approach emerged. Following the principle of impenetrability of matter, where two bodies cannot occupy the same space at the same time, this new approach compared and buffered the positions of all elements coinciding in the same pixel according to their depth. The granularity of depth buffering or z-buffering algorithms as described by Wolfgang Straßer and Edwin Catmull in 1974 has allowed hidden-surface algorithms to be implemented today as hardware operations in all kind of devices.

Once the geometric positions of all the vertices of each polygon or each pixel in a scene are sorted according to a geometric quantity, e.g. the z-axis, the next problem in the quest for simulating optical images became to decide how to shade each surface and for that again the solution was found on statistics.

### Shading Surfaces

To hide the non-visible surfaces of the objects in a 3D scene is not enough to achieve realistic synthetic images. During the early 1970s researchers in computer graphics such as Henri Gouraud and Bui-Tuong Phong strived “to generate an image that

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<sup>19</sup> Hollerith, Herman: *Art of Compiling Statistics*. US395782 (A), issued 8. Januar 1889, 4.

<sup>20</sup> Von Neumann, John: “First Draft of a Report on the EDVAC”. Moore School of Electrical Engineering. University of Pennsylvania, 30. Juni 1945, 3.

<sup>21</sup> Knuth, *The Art of Computer Programming, Volume 3*, p. 384.

approximates the real object closely enough to provide a certain degree of realism.”<sup>22</sup> In order to do so they created algorithms that computed the shading of an object as a continuous function of the data of all the vertices that make the object.

The shading problem is relatively simple to describe: how to produce the illusion of a smooth curved surface when the given surface is in fact described by a set of small flat faces?<sup>23</sup> Figure 2 illustrates one such set of flat faces, ABCD. In essence, a shading algorithm calculates the value of grey for point P, as the interpolation of the shading and the orientation of each vertex in ABCD. On a curved surface, it estimates a different shade of grey at each raster point of the screen. The faceted structure of the object, a principle Sutherland called coherence, simplified the computation of the shading because the less faces an object has, the faster the computation of the shading of the whole surface.

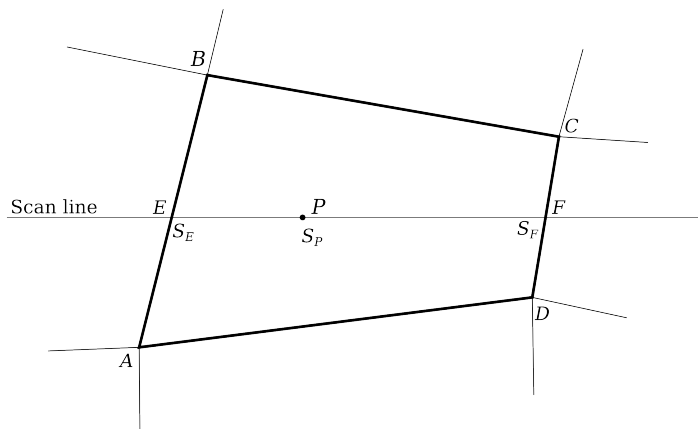


Figure 2: Projection of one polygon intersected by the scan line.

Several approaches to calculate the grey value of P emerged during the early 1970s, again mainly at the University of Utah. In 1971 Gouraud, a graduate student of Sutherland, devised an algorithm that first interpolates the shading values along the

<sup>22</sup> Phong, Bui Toung: “Illumination of Computer Generated Pictures“, Communication of the ACM 18, No. 6 (1975): 311–7. p. 311.

<sup>23</sup> Gouraud, Henri: “Continuous Shading of Curved Surfaces“. IEEE Transactions on Computers C-20, Nr. 6 (June 1971): 623–9. p. 629.

edges of the faces E and F in figure 2; and second, it outputs the shading for each pixel along the scan line by interpolating the shading on the intersection between the edges and the scan line, SE and SF in figure 2.<sup>24</sup> Using that chain of interpolations adjusted by a coefficient to express the relative position of E, F, and P, in 1971 Gouraud was able to render point by point the smooth curved surface of a 3D model based on his wife's face.<sup>25</sup> In this synthetic human face the value of any SP takes into consideration the shading of adjacent faces in which the surface has been divided in an attempt to make the edges between all the faces of the surface imperceptible.

However, this early digital face did not conceal its faceted structure enough. Coherence has a side effect. It produces abrupt changes of luminance gradient where the surface bends sharply, making visible the faceted structure and destroying the illusion of smoothness. In 1865, the physicist Ernst Mach observed that “every new turn of a curve, every projection or depression of a surface, involves a deviation of some space-sensation from the mean of the surrounding field on which attention is directed.”<sup>26</sup> This optical effect, called Mach Bands, made it impossible to hide the inner structure of the 3D model from the human visual system because the high spatial frequencies that carry information about highlights and fine details get amplified along the polygon boundaries, making them appear brighter than the surrounding low spatial frequencies carrying information about the global illumination of the surface. Thus, the surface of the 3D model was not smooth, revealing its faceted structure. Increasing the number of faces to eliminate such undesired highlights was impractical because of the exponential increase of time in order to compute all the interpolations. Therefore, coherence had to be overcome by considering the orientation of each vertex towards the observer and imitating physical shading situations such as diffuse and specular reflections, data that Gouraud's algorithm did not compute.<sup>27</sup>

A more effective algorithm at giving such illusions was developed shortly after by another PhD student at the University of Utah, Bui-Tuong Phong. Phong was concerned with mechanically simulating light and the optics of its transmission. In 1973, he wrote an algorithm that outputs the shading of each point of a 3D surface based on the parameters of the light source and of the surface plus the light reflected

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<sup>24</sup> Ibid, p. 626.

<sup>25</sup> Bellin, Isabelle. “Images de synthèse : palme de la longévité pour l'ombrage de Gouraud”. Document Interstices. Interstices, 15, September 2008 (<https://interstices.info/ombrage-gouraud>).

<sup>26</sup> Mach, Ernst: *Contributions to the Analysis of the Sensations*. Trans. C. M. Williams. Chicago, IL: The Open Court Publishing Company, 1897. p. 97

<sup>27</sup> Phong, “Illumination of Computer Generated Pictures,” p. 315.

off towards the observer. In contrast to Gouraud, Phong's shader interpolates at each raster point not the shading values of the vertices but the curvature of the surface and the diffuse and specular reflections caused by the illumination of the scene.<sup>28</sup> In order to calculate the shading of any point P on any surface, this shader assumes that the position and angle of both the light source and the observer are located at infinity and both become parametrised as variables (i and s) in the shading equation.<sup>29</sup> As these images appear from pure mathematical calculations, Phong's shader does not conform to a mode of representation by means of the senses but rather to the latter as an intellectual representation of these things as they actually are in themselves.

In order to compute the shading of a surface, Gouraud and Phong were using one of the ancient techniques to compute the path and position of celestial bodies. Studied by astronomers, mathematicians, and statisticians for centuries, interpolation is a mathematical concept that estimates the “parabolic curve which shall pass through any given points.”<sup>30</sup> Similarly, since at least the 19th century draftsmen have used the French curve, a drafting tool shaped by segments of curves with different radii, ellipses, hyperbolas, and parabolas to manually find and draw long but continuous curves that pass through a given set of points. During the WWII such sinuous but irregular curves of the draftsmen became formalized by Isaac Jacob Schoenberg in his theory of splines at the same place where the sorting algorithm of Hollerith was being implemented in the EDVAC, the Ballistic Research Laboratories at Aberdeen Proving Ground, in Maryland. In Schoenberg's spline theory “any spline curve ... may be represented in one and only one” segmented function that joins smoothly the coefficients computed from multiple points.<sup>31</sup> So during the 1960s when the digital computer became widespread, splines assumed an important role first “as the premier tool for data fitting and computer-aided geometric design.”<sup>32</sup> Interpolation was then ready to join sorting and to infiltrate the technical images via the theory of splines. By first hiding the non-visible vertices and

<sup>28</sup> Ibid.

<sup>29</sup> Phong's shading equation to calculate the shading at any point P on a surface is:  $S_p = C_p[\cos(i)(t-d)] + W_{(i)}[\cos(s)]^n$ , where for any shading point  $S_p$ , s is the angle between the reflected light and the line of sight. Ibid

<sup>30</sup> Newton, Isaac: *Newton's Principia: the mathematical principles of natural philosophy* (1687). Translated by A. Motte (1729) New York, NY: First American edition by N. W. Chittenden. New York: Daniel Adee, 1846, Book 3, Lemma V.

<sup>31</sup> Schoenberg, I. J.: „Contributions To The Problem Of Approximation Of Equidistant Data By Analytic Functions: Part A.—On The Problem Of Smoothing Or Graduation. A First Class Of Analytic Approximation Formulae“. *Quarterly of Applied Mathematics* 4, No. 1 (1946): 45–99. p. 72.

<sup>32</sup> Askey, R, and C de Boor: “I. J. Schoenberg (1903–1990).” *Journal of Approximation Theory* 63.1 (1990): II-2 *ScienceDirect*. Web. p.1.

then smoothing the edges of each face, statistics took command of the digital image.

### Tracing Rays

Within a computer screen, there is a raster where a picture element, the pixel, gets a Cartesian address and a color value. According to Kittler, this roots computer images in a medium of war: radar.<sup>33</sup> First, this green and circular warning system provided a model for the accurate localization of each pixel on the screen. Second, used for detecting and following enemy airplanes, this military medium also provided a model for tracing the path of light particles in a 3D scene in order to determine the color of each individual pixel on the observer's plane.

After hiding the non-visible vertices of a 3D model and smoothing its faceted surface, the third problem in the quest for deceitfully realistic synthetic images was to calculate the global illumination information in a 3D scene that determines the color of each individual pixel of the image. Phong's shading model assumed light sources were points infinitely distant from the objects. If realistic pictures were to be generated they must include lights as points within the limits of the scene as well as their physical coefficients of transmission and reflection. At the end of the 1970's, research in computer graphics focused on the simulation of shadows, the reflection of light from object to object, and the transmission of light through transparent objects. This research concluded in algorithms that find out which rays of lights actually contribute to the image and what mixture of red, green, and blue they have. Since the 3D objects are in a parametrized projective space and the screen is a grid of small squares, the solution to this problem was provided by classical ray optics that since Descartes have been applied to the laws of reflection and refraction.<sup>34</sup> Raytracing, an algorithm of the 20th century, becomes thus the site for the machinic combination of the optics of early modernity with the infinite projective space of the 19th century.

A raytracing program locates the observer behind a raster, where each pixel is a "rectangular region whose corners are four sample points."<sup>35</sup> Through each of these sample points a recursive raytracing algorithm such as the one designed by Turner Whitted in 1979 sends a series of rays from the observer to explore a 3D simulated scene. The rays will then detect and interact with the objects in the scene and if possible reach the points of light. This sounds counter-intuitive because the

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<sup>33</sup> Kittler, *Optische Medien*, 178.

<sup>34</sup> *Ibid.*, 186.

<sup>35</sup> Whitted, Turner: „An Improved Illumination Model for Shaded Display“. *Communications of the ACM* 23, No. 6 (Juni 1980): 343–9. p. 346.

obvious approach would be to trace the “light rays emanating from a source ... through their paths until they strike the viewer.”<sup>36</sup> However, such a method would be a huge waste of time as a very depressing number of such rays will ever reach the viewer. After exploring the scene enough, the algorithm determines the contributions of each ray to the color of the pixel and determines the colors of the pixel in a weighted average in which some rays are given more importance than others.

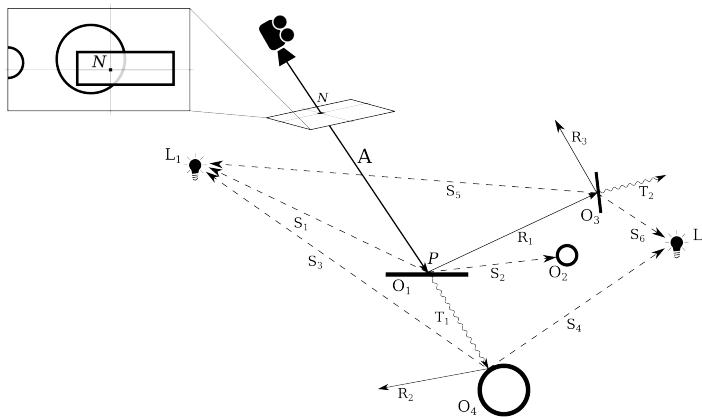


Figure 3: Path of the ray A.

Figure 3 illustrates the simple case of tracing the ray A. The scene has two light sources,  $L_1$  and  $L_2$ , and four objects,  $O_1$ ,  $O_2$ ,  $O_3$ , and  $O_4$ . Imitating radar, the algorithm emits a ray A that passes through the image plane through the pixel N and hits  $O_1$  at point P. From there two rays,  $S_1$  and  $S_2$ , are sent to the two lights.  $S_1$  hits  $L_1$  without interruption meaning that P receives direct illumination from that light. Before reaching  $L_2$ ,  $S_2$  hits an object thus generating a shadow. The semi transparent and reflective surface of  $O_1$  transmits a ray  $T_1$  to an object behind and reflects a ray  $R_1$  to the object on the right. At both objects the process of tracing back the transmissions and reflections of each ray begins again. Such a process is repeated recursively until there is enough information to have an educated approximation of the color of the pixel N. Obviously that recursion cannot run infinitely because

<sup>36</sup> Ibid., 345.



neither the machine nor the human has an infinite amount of memory or patience to wait for an image.

A mantra in computer graphics reads: the larger the memory of the computer, the larger the raster of the image plane; and the greater the number of rays and bounces that the computer is able to calculate. Whitted already pointed out that computer graphics simulate a continuous phenomenon with a discrete number of samples.<sup>37</sup> However large the memory and the raster, under close inspection, computer graphics would always make apparent to the observer the fragmented structure of the computer image, thus breaking the surface of the spell of continuity.

## Surface

The current synthetic images cannot hide their very construction entirely. Kittler's dream of genuine computer graphics designed not after the human eye and their perspective but after calculable light and their quantum physical conditions won't be fulfilled until further developments in physics.<sup>38</sup> First in such utopian state images would do, what hitherto they have tried to conceal – the impossibility of a true representation of the world because “no algorithm can produce a world picture at once fully detailed and fully integral.”<sup>39</sup>

However, as our short media history of 3D synthetic images during the 1970's has shown, the technical development of computer graphics is not concerned with a full representation of the world but with its statistical approximation. Computer graphics settled for an initial presentation of the world as a statistically organized cluster of points, that when assembled by sorting, interpolation, and tracing can yield all sorts of possible forms. This is still valid whether such operations to create graphics are carried out by software, directly on hardware, or by much more complex techniques of today. Consequently, the problem is to understand the handling of these statistically assembled clusters of points, that instead of presenting the real chaos of the world, they produce the universe of technical images.

This end of optical media in the algorithmic procedures of computer graphics does not only bring techniques back to the surface of media history. Kittler's media materialism pursues at the same time a deconstruction of the

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<sup>37</sup> Ibid, p. 346.

<sup>38</sup> “Computer graphics would deserve the name only if they could render to vision what appears unseen—the optical partial values of quantum-physically distributed particle dynamics.” (Kittler, *Computer Graphics*, 44).

<sup>39</sup> Ibid, 43.

subject—as it was common in the continental philosophy of the post WWII era, but rarely in a direct connection with technical media. His historiography of optical media, including its supposed end in computer graphics, contains or even forces a philosophy without a subject.<sup>40</sup> As discussed above, from the perspective of computer graphics, at least since the Phong shader, the subject is nothing more than a variable  $s$  in an algorithm. In Kittler's argument in *Optical Media*, we uncover that already in the 1950's the electronic visual media had in any case already reduced the imaginary to a minimum:

In contrast to film, television was already no longer optics. It is possible to hold a film reel up to the sun and see what every frame shows. It is possible to intercept television signals, but not to look at them, because they only exist as electronic signals. The eyes can only access these signals at the beginning and end of the transmission chain, in the studio and on the screen. Digital image processing thus ultimately represents the liquidation of this last remainder of the imaginary.<sup>41</sup>

After this dreary end of the imaginary through electronic image processing Kittler calls, as he so frequently does, for the three registers of structuralist psychoanalysis: the symbolic, the imaginary, and the real. These registers also play a key role in his narration of the technical and warring history of the central perspective, photography, film, and television in order to tragically confine the subject. The psychoanalysis of Jacques Lacan, playfully demoted by Kittler to a “set of conceptual tools”, is not only turned into media theory, but also historicized: the subject first has to pass through the mirror stadium in form of cinema, before the hallucinatory effect of the real can be finally abandoned under the growing conditions of the electronic television.

*Le reel* refers only to that which has neither a figure, like the imaginary, nor a syntax, like the symbolic. In other words, combinational systems and processes of visual perception cannot access the real, but [...] this is

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<sup>40</sup> Vgl. Serres, Michel: “Gnomon: Die Anfänge Der Geometrie in Griechenland”. In *Elemente einer Geschichte der Wissenschaften*, edited by Serres, Michel, Stengers, Isabelle, Latour, Bruno et al., Frankfurt a.M, 1995, p. 108–175, here p. 125.

<sup>41</sup> Kittler, *Optical Media*, 226.

precisely why it can only be stored and processed by technical media.<sup>42</sup>

Only technical media, not bodies, can store and process the real (that has neither images nor symbols). Technical media do not address the Cartesian subject nor the Heideggerian *Dasein*, but the real, which can be associated with sound and noise, but not with text or image.<sup>43</sup> Therefore a philosophy of the subject is no longer needed and is replaced by media theory. This polemic against the subject centered philosophy resonates every time that Kittler addresses the liquidation of the imaginary or the epistemological weakness of images. In his media history, the Lacanian derivative signified ‘the real’ remains a curious reference. It always occurs within a topology of the so-called Borromean rings, an old mathematical structure that illustrates the Lacanian model of psychoanalysis and that unites its three registers in a continuous structure.<sup>44</sup> Should the imaginary in the time of computer graphics really be at stake, then so will be the symbolic and the real, and consequently the subject in psychoanalysis—the Borromean rings would dissolve completely. From a psychoanalytical perspective, the Kittlerian transfer of the Lacanian triad in film (imaginary), gramophone (real), and typewriter (symbolic) is not entirely well stitched.<sup>45</sup> Yet it is in the connection between computer graphics and the tragedy of the rational subject that the strength of his approach lies: The matrix of structuralist psychoanalysis prevents a relapse into a teleological narration of the progress of technical media.

In form of a techno-mathematical program, Kittler sketches a compressed history of the Cartesian subject (e.g. in the form of raytracing), whose optic in the physical and mathematical precipices of the 19th century (e.g. in form of the shading problems) turns insane and in the machinic statistics of the 20th century literally vanishes from the image plane (e.g. in form of calculations for light energy and shadows). Finally, everything shrivels to a single parameter in a virtual and endless mathematical space—a point among points. Such narcissistic injury of humanity in the form of technical media fits in the series, the Copernican turn, the Theory of Evolution, and the discovery of the unconscious. Computer graphics are, just like the whole media history of Kittler a form of a history of being (*Seinsgeschichte*) of technology, that in the very Heideggerian sense does not tolerate other subjects or theories of truth next to it.

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<sup>42</sup> Ibid, 40.

<sup>43</sup> Kittler, *Computer Graphics*, 44.

<sup>44</sup> Cf. Bitsch, Annette: “*always crashing in the same car*”. *Jacques Lacans Mathematik des Unbewussten*. Weimar: VDG Verlag und Datenbank für Geisteswissenschaften, 2001, 337.

<sup>45</sup> Cf. Kittler, Friedrich: *Film Grammophon Typewriter*, Berlin, DE: Brinkmann und Bose, 1986.

Of course, there are certain places that suggest a different way or opening. After the 19th century, when mathematics began calculating surfaces and thermodynamics, physics dared the step from mechanics to the field, and the subject played madly atop the infinite surfaces of a Möbius strip, only one possible answer to the questions of history and subjectivity posed by the 19th century emerged—and here Kittler cites no longer Lacan but Flusser: the digital computer.<sup>46</sup> In the global calculations of luminance and color values for each pixel—*Shading* and *Raytracing*—performed by a 3D graphics program, the surface in which all geometry is calculated as a coherent mesh of triangles and rectangles simply disappears. Moreover, in *Shading*, in accordance with the projective space, a dimensionless point depicts itself, thus opening the virtual infinitude of the space. In contrast, *Radiosity* closes again the virtual system: in order to follow the light rays to the eye, *Radiosity* only calculates the light energy of the scene. There is no need for an observer because “a visible world is no longer derived from rays and surface points, but rather from illuminating and illuminated surfaces.”<sup>47</sup> Here the subject depicts itself no more, instead it depicts the orthogonal surface of a computer chip.<sup>48</sup> At the end of the *Semi-Technical Introduction* the computer imagines itself.

Flusser calls this closing of the virtual system ‘techno imagination’. The basis for these new images is not a concrete scene, but an abstract concept, in this case the code of the graphic processor. The history of technical images, that according to Flusser starts with photography, marks a radical caesura in human communication, as radical at least as the invention of writing. The techno-image not only promises the end of the classical imagination, but also the end of national languages or Western ‘occidental’ history, because the mediation of text through communication with images can be left aside, as Flusser argues. These images are however a new type of images. They are rendered by texts. They do not refer to any concrete scene but to abstract and scientific concepts. And in contrast to all other images they belong to a very different ontological category of images for they are “a revolutionary code.”<sup>49</sup> Techno-images dissolve traditional structures of communication, and the more technical they get, the more they are used for political purposes. Flusser, who himself had to flee from NS-occupied Prague to Brasil in 1939, draws a direct line from the use of film and photography for political propaganda in the early 20<sup>th</sup> Century, to the Rumanian revolution of 1989 that was “made by television”. Flusser is fully aware of the anti-political impact techno-images can have, especially when he tries to

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<sup>46</sup> Kittler, *Computer Graphics*, p. 40.

<sup>47</sup> *Ibid.* 39.

<sup>48</sup> *Ibid.* 41.

<sup>49</sup> “[...] ein revolutionärer Code [...]” (Flusser, *Kommunikologie*, 140. Translated by the authors.)

imagine what would happen if such a techno-image-revolution would take place in the US - would that be the end of political decision-making as we know it?<sup>50</sup>

Flusser insists on two statements that are interesting here: First, techno-images are completely different from traditional images yet they communicate; and second, their specificity lies not in their production's method, not in the materials they are made of, and not in their structure, but in their significance.<sup>51</sup>

Above all, he distinguishes techno-images by their "untruthfulness" - they are difficult to decode. Like every "urtext", their untruthfulness consists in that they are received by a receiver who doesn't know the code. The structure of the not yet understood code contains the danger that by structuring the communications structures in such a way, those who send information are rendered an incompetent layer of writers and those who receive, the receivers, are an alphabetic mass and for this reason only obey the sense of what is given to them by the sender.<sup>52</sup> Flusser's criticism of techno-images is a critic of the mass-media and their incompetent producers. Such a critique is at its core political. Towards the end of the 1980s, Flusser sketches a resistance through a new art of techno-imagination and the outline of a new consciousness that is no longer fashioned after the linearity of Western history. Here, according to Flusser's prediction, a new set of categories such as inter-subjectivity, point of inflexion, and proximity will substitute for all dual categories such as earlier-later, when-then, true-false, real-unreal. The historical consciousness doesn't know how to deal with this new set of categories. Evoked by Flusser, in this new techno-imagination the distinction between recognition, desire, and experience as well as their institutional manifestations, science, politics, and art, emerge as completely meaningless.<sup>53</sup>

Speaking in a similar enthusiastic fashion, but from the subface of techno-images, Kittler refers to Flusser and his interpretation of the computer as the "abolition of all dimensionality".<sup>54</sup> The successful abolition of all spatial dimensions to the zero dimensionality of information or points implies that nothing can be concealed because nothing is signified, nothing it represented, not any longer. Instead, the surface is completely free for presentation. Consistently, Kittler started to practice graphics programming in 32-bit-systems as a recreational hobby as well as teaching it to undergraduate students of cultural history in the late 1990's.

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<sup>50</sup> Cf. Flusser, Vilém: *Lob der Oberflächlichkeit*, Düsseldorf, DE: Bollmann, 1993, 236-243.

<sup>51</sup> Cf. Flusser, *Kommunikologie*, 139.

<sup>52</sup> Cf. *ibid.* p.149.

<sup>53</sup> Cf. *ibid.* p. 222.

<sup>54</sup> Kittler, *Optical Media*, 226.

Kittler and Flusser left us plenty of techno-imagination: the former is drawn to the subface of images, and the latter praises their surface. We agree with both of them, that the imaginary, the movement from concrete scenes to abstract concepts or codes, has been relocated through computer graphics. Underneath the surface of any technical image operate a series of scientific concepts and mathematical procedures. Yet, images remain only as intermediary forms, many of them never even reaching any human eyes because they are only exchanged among devices and filter as spam by machines.<sup>55</sup> The millions of images circulating daily in social media and mobile phones, the universe of technical trash images, contributes to a great extent to the growing culture of self-surveillance. Will the quick as lightning circulation and distribution of image-surface, enabled by the algorithmic subface, cause in turn the emergence again of strategies for self-withdrawal from representation—as visual artist Hito Steyerl claims?<sup>56</sup> The work of contemporary artists such as Steyerl critically addresses current trends towards self-quantification and hypervisibility of computer vision and strategies to resist this domination of images.<sup>57</sup> Techno- imagination relies on consumption as well as production of techno-images.

The history of optical media has not (yet) ended, as Kittler once claimed. By the hand of computer graphics it has rather dissociated itself from historical categories such as representation and traditional imagination reduced to visibility. Thus, the historiography of optical media should tackle the procedures for the construction of the invisible as well as the visible, engaging both with surfaces and subfaces and think of images beyond representation and visibility.

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<sup>55</sup> Cf. Steyerl, Hito: “The Spam of the Earth. Withdrawal from Representation.” *E-Flux* 32 (2012). <http://www.e-flux.com/journal/32/68260/the-spam-of-the-earth-withdrawal-from-representation/> (last visited on 2. October 2016).

<sup>56</sup> Will more and more people try to withdraw from the omnipresent surveillance by photo and video technologies, now that elite techno clubs in Berlin have instituted a strict camera ban? (Cf. Ibid.)

<sup>57</sup> Cf. Hito Steyerl, *How not to be seen: A Fucking Didactic Educational.MOV File*, 2013. (<https://www.artforum.com/video/hito-steyerl-how-not-to-be-seen-a-fucking-didactic-educational-mov-file-2013-51651>). Last visited 14. June 2018.

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