

2010

# Likely values of the Higgs vacuum expectation value

JF Donoghue  
donoghue@physics.umass.edu

K Dutta

A Ross

M Tegmark

Follow this and additional works at: [https://scholarworks.umass.edu/physics\\_faculty\\_pubs](https://scholarworks.umass.edu/physics_faculty_pubs)



Part of the [Physics Commons](#)

---

## Recommended Citation

Donoghue, JF; Dutta, K; Ross, A; and Tegmark, M, "Likely values of the Higgs vacuum expectation value" (2010). *PHYSICAL REVIEW D*. 87.

Retrieved from [https://scholarworks.umass.edu/physics\\_faculty\\_pubs/87](https://scholarworks.umass.edu/physics_faculty_pubs/87)

This Article is brought to you for free and open access by the Physics at ScholarWorks@UMass Amherst. It has been accepted for inclusion in Physics Department Faculty Publication Series by an authorized administrator of ScholarWorks@UMass Amherst. For more information, please contact [scholarworks@library.umass.edu](mailto:scholarworks@library.umass.edu).

## Likely values of the Higgs vev

John F. Donoghue,<sup>1,\*</sup> Koushik Dutta,<sup>2,†</sup> Andreas Ross,<sup>3,‡</sup> and Max Tegmark<sup>4,§</sup>

<sup>1</sup>*Department of Physics, University of Massachusetts  
Amherst, MA 01003, USA*

<sup>2</sup>*Max-Planck-Institut für Physik (Werner-Heisenberg-Institut)  
Föhringer Ring 6, D-80805, München, Germany*

<sup>3</sup>*Department of Physics, Yale University  
New Haven, CT 06520, USA*

<sup>4</sup>*Department of Physics, Massachusetts Institute of Technology  
Cambridge, MA 02139, USA*

We make an estimate of the likelihood function for the Higgs vacuum expectation value by imposing anthropic constraints on the existence of atoms while allowing the other parameters of the Standard Model to also be variable. We argue that the most important extra ingredients are the Yukawa couplings, and for the intrinsic distribution of Yukawa couplings we use the scale invariant distribution which is favored phenomenologically. The result is successful phenomenologically, favoring values close to the observed vev. We also discuss modifications that can change these conclusions. Our work supports the hypothesis that the anthropic constraints could be the origin of the small value of the Higgs vev.

### 1. INTRODUCTION

It is known that if the masses of the light quarks and the electron were modestly different then nuclei and atoms would not exist [1–3]. Because the masses of fermions are proportional to the Higgs vacuum expectation value (vev), these bounds can be interpreted as constraints on possible values of the Higgs vev if the other parameters of the Standard Model are held fixed [1, 3]. This observation is interesting because it could provide an answer to one of the most significant puzzles of the Standard Model - often called the fine tuning problem or the hierarchy problem. If the underlying theory allows the existence of different values of the Higgs vev, we would only find ourselves in a region of the universe that contains atoms, and hence the vev may be constrained to a small range of possible values. This provides a motivation for theories with multiple possible values of the physical parameters, such as the string theory landscape [4].

However, in theories in which the basic parameters can take on multiple values, other parameters besides the Higgs vev will most likely also be variable. This would be the case in the string landscape picture. Since the atomic constraints are really on the up quark, down quark and electron masses, they translate to constraints on the product of the fermion Yukawa couplings and the Higgs vev, not the vev uniquely. Even without the exploration of specific theories, we might hope that the rough

conclusion is unchanged, namely that the scale of the electroweak sector must be reasonably close to the scale of the strong interactions in order that the masses of the light quarks and electron - the product of the electroweak interactions - be comparable to nuclear binding energies - primarily due to strong interactions.

In this paper we consider the more general case of allowing other parameters of the Standard Model to vary, and attempt to provide a likelihood distribution for the Higgs vev. One might think that this would be possible only with the knowledge of the full underlying theory, but we will primarily use data for this purpose. As we will argue in Sec. 2 and 3, this is possible because it is the Yukawa couplings that appear to have the most significant influence on the range of the vev. Because there are many masses, we then have experimental indication of the intrinsic probability distribution for the Yukawa couplings. The observed quark and lepton masses provide quite strong statistical evidence that this distribution is close to scale invariant [5, 6]. We will review this idea in Section 4 and provide further evidence in its favor in Section 5.

Applying such a scale invariant probability distribution for the Yukawa couplings, we investigate the likelihood distribution for the Higgs vev. This result is obtained by finding the relative probability that the  $u$ ,  $d$  quarks and the electron (governed by the scale invariant weight) fall in the anthropically allowed range. In this case our result is developed and displayed in Sec. 6. We can also study the effect of producing modest changes in our underlying assumptions. These are studied in Sec 7. We present our conclusions in Sec 8.

We are aware of many limitations of our work. Besides the assumptions that we state and explore, there are likely other effects (nucleosynthesis, cosmology, etc) that come into play, especially once we consider significant

---

\*Email: donoghue@physics.umass.edu

†Email: koushik@mppmu.mpg.de

‡Email: andreas.ross@yale.edu

§Email: tegmark@mit.edu

changes in the parameters of the Standard Model. However, one would expect that possible further anthropic constraints would only tighten the likelihood function in the neighborhood of the physical value. The goal of the present work is to obtain a sense of whether or not the atomic constraints could be the origin of the low value of the Higgs vev (within the context of landscape-like theories). Our work can be viewed as an attempt to quantify this by looking at what may be the dominant effects. Overall our conclusion is that it remains plausible that the atomic constraints are the origin of the low value of the Higgs vev.

## 2. GENERAL FRAMEWORK

The situation that we have in mind is similar to the string landscape picture in which there are very many possible values of each of the parameters. While in string theory the choices of parameters are discrete, the results appear to be so densely packed as to appear almost continuous<sup>1</sup>. We then describe the ensemble of such states by an intrinsic probability distribution or weight that specifies the probability of finding different values of the parameters. These probabilities would emerge from string theory and the weight encodes the shape of the string landscape. Let us call this weight  $\rho(v, \Gamma_i, g_i)$  where  $v$  is the Higgs vacuum expectation value,  $\Gamma_i$  are the Yukawa couplings, and  $g_i$  stands for the gauge couplings and all other parameters of the theory.

However, many combinations of the parameters do not lead to nuclei and atoms. There is an intrinsic selection effect that we would only find ourselves in friendly regions that include atoms. The shape of the intrinsic probability distribution in unfriendly regions of parameter space is then completely irrelevant for us and we are only concerned with the parameter subspace that leads to atoms. Let us denote by  $A(v, \Gamma_i, g_i)$  the function that is zero for all parameters that do not lead to atoms and unity for those that do. We will refer to this as the atomic function. In principle, the atomic function could also take into account not only the mere existence of atoms but also the probability of a physical environment of sufficient complexity developing with the atoms that are available with that parameter set. As one moves around the parameter space, especially near the allowed borders, greater or fewer numbers of atoms exist and/or would be produced in the early universe. With a reduced or enhanced set of atoms available, the resulting complexity might be greatly reduced or enhanced. However, such considerations are beyond our capabilities to calculate and we do

not consider them. Given the primary features uncovered below, it is unlikely that modifying the boundaries of the atomic function would have a large effect on our results. Moreover, it is certainly cleaner and more conservative to limit our discussion to the general physical characteristics of atoms and nuclei.

With the atomic function we can obtain the total probability to find atoms in the landscape

$$P(A) = \int dv d\Gamma_i dg_i A(v, \Gamma_i, g_i) \rho(v, \Gamma_i, g_i) \quad (1)$$

where  $A$  denotes the existence of atoms. However, this total probability is not a quantity of interest. A more interesting one is constructed by omitting the integration over  $v$ ,

$$L(v) = \int d\Gamma_i dg_i A(v, \Gamma_i, g_i) \rho(v, \Gamma_i, g_i) \quad (2)$$

which we call the likelihood function. It is in fact the probability density  $\frac{dP}{dv}$  for atoms to exist.

Another useful assumption is the independence of parameters. This means that we assume that the intrinsic probability function factorizes into the product of separate weights. In formulas this implies

$$\rho(v, \Gamma_i, g_i) = \rho(v)\rho(\Gamma_e)\rho(\Gamma_u)\rho(\Gamma_d)\dots\dots \quad (3)$$

This is at least partially motivated by the vastness of the string landscape. If we hold all but one parameter fixed, there are likely other allowed vacua with this last parameter scanning over its allowed range. There is also some phenomenological evidence in favor of this from the distribution of quark and lepton masses, which all seem consistent with the same distribution. However, we note the approximate nature of this feature in our discussion of Sec. 7.

There also could be an a-priori distribution for the Higgs vev,  $\rho(v)$ , which is a property of the fundamental theory. We clearly do not know this function. However, it is expected that the vev can take on values in a very large range, at least up to a unification scale. Moreover, there are many additive contributions to  $v$  that come from quantum corrections. These add linearly in the respective couplings and this suggests that the overall distribution could be Taylor expanded in  $v$  about the observed value. If this is the case, then our considerations cover only a very small portion of the allowed range, and we treat the a-priori distribution as a constant in this narrow range. If the a-priori distribution in  $v$  were to be highly peaked in some direction, our results would be modified, and so this must count as an uncertainty in our method. Therefore, our assumption will be  $\rho(v) \sim \text{constant}$ , and unless we know more about the underlying theory, we feel that this is the most reasonable assumption under which to proceed.

The quantity we will explore in detail below is the probability to find atoms for a given value of  $v$ . It is obtained by taking a sample  $v$  and drawing the other

---

<sup>1</sup> For example, it has been estimated that there are  $10^{100}$  string vacua reproducing the Standard Model parameters within the present experimental error bars [4], and the density of states would be equally high in the neighborhood of these parameters.

relevant parameters randomly from the probability distributions we consider. Then we decide if the resulting configuration can yield atoms or not. With a large sample for a fixed value of  $v$  we can obtain the probability of having atoms by dividing the number of times we obtained atoms by the total number of simulations. Using the assumption of independence of the parameters introduced above, the quantity we obtain from our simulations then is

$$P(A|\text{given } v) = \int d\Gamma_i \int dg_j \left( \prod_{i,j} \rho(\Gamma_i) \rho(g_j) \right) A(\Gamma_i, g_j, v). \quad (4)$$

Now the likelihood function  $L(v)$  of Eq. (2) is obtained by the product of  $P(A|\text{given } v)$  and the intrinsic probability distribution  $\rho(v)$ . Under our assumption of a flat  $\rho(v) \sim \text{const.}$  in the range of interest, the likelihood function is then simply proportional to  $P(A|\text{given } v)$  of Eq. (4).

From the shape of the likelihood function  $L(v)$  we can infer which values of  $v$  are typical and which ones are highly improbable. Since  $L(v)$  is a probability density its shape itself is not a direct indicator of the most likely values of  $v$ . A peak in  $L(v)$  for example does not indicate the most likely values of  $v$ ; more meaningful quantities to give would be the median or other percentiles. A simpler way to explore the order of magnitude of the most likely values of  $v$  can be obtained by plotting our results for  $L(v)$  on a log-log scale. Since any probability distribution which has a finite value of its percentiles must fall off faster than  $1/v$  at large values of  $v$ , the log-log plots show us if and when the likelihood function falls off faster than  $1/v$ . If present, this point is then a reasonable estimate of the most likely values of  $v$ . If  $L(v)$  does not fall off faster than  $1/v$ , no constraints on the Higgs vev arise from the existence of atoms.

### 3. BRIEF SUMMARY OF ATOMIC CONSTRAINTS

To the extent that we understand how the Standard Model leads to the world that we observe, we should be able to describe the world that would result if we instead used parameters different from, but in the neighborhood of, those seen in Nature. Surprisingly, the structure of the elements changes dramatically for quite modest changes in the quark masses. In a recent paper [1], Damour and Donoghue have tightened and summarized the anthropic constraints on quark masses<sup>2</sup>. Here we briefly summarize these results.

The first constraint which results from the binding of nuclei gives an upper bound on the sum  $m_u + m_d$ . The

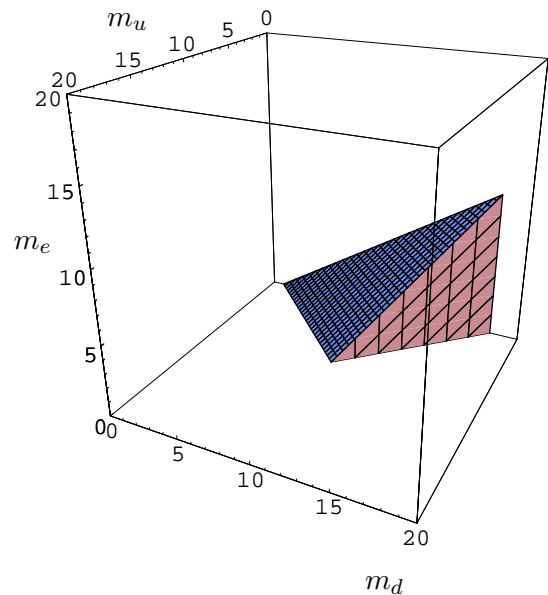


FIG. 1: The anthropic constraints on  $m_u$ ,  $m_d$ ,  $m_e$  in MeV units.

key feature here is that the pion mass-squared is proportional to this sum of masses, and as the pion mass gets larger nuclear binding quickly becomes weaker. The binding energy is small on the scale of QCD and is known to have opposing effects from an intermediate range attraction and a shorter range repulsion. The attractive component, heavily due to two pion exchange, is the most sensitive to the pion mass and weakening it leads to a lack of binding of nuclei. From [1] this constraint is

$$m_u + m_d \leq 18 \text{ MeV}. \quad (5)$$

The second constraint comes from the stability of protons. If protons could annihilate with electrons,  $p + e^- \rightarrow n + \nu_e$ , hydrogen would not exist. The proton and neutron mass difference gets contributions from the quark masses and from electromagnetic interactions. Using the best present estimates of these, the constraint becomes [1]

$$m_d - m_u - 1.67m_e \geq 0.83 \text{ MeV}. \quad (6)$$

The right hand side of the equation is linear in the electromagnetic fine structure constant. Modest variations in this number would not influence our results significantly. In providing this constraint, it has been assumed that the neutrino masses remain negligibly small. This feature is also anthropically required [7].

These constraints are summarized in Fig. 1. Note that the up quark and electron masses are able to vary down to zero mass, while the down quark mass is constrained to be non-zero. The dimensional scale is set by the QCD scale  $\Lambda_{QCD}$ , so that these constraints could be rephrased

<sup>2</sup> See also [2].

in terms of dimensionless ratios  $m_i/\Lambda_{QCD}^3$ .

There are no known atomic constraints on the masses of the heavier quarks and leptons as long as they are significantly heavier than the up, down and electron. Heavy quarks decouple from low energy physics and have little influence on nuclei and atoms. Therefore, in our case the atomic function  $A(v, \Gamma_i, g_i)$  reduces to  $A(v, \Gamma_u, \Gamma_d, \Gamma_e)$ . However, the Higgs vev does influence the mass of the  $W$  gauge boson, and this in turn influences the rates of weak processes. Since weak interactions play a significant role in the  $pp$  cycle in stars, in particular the crucial reaction  $p + p \rightarrow d + e^+ + \nu$ , raising the  $W$  mass will slow the production of elements. So while atoms may exist, they may not be produced in favorable numbers. This constraint is complicated also by the dependence on the cosmological parameters governing stars. In [8], a special scenario was constructed in which the  $W$  mass can be taken to infinity if modifications are made to the usual picture of nucleosynthesis<sup>4</sup>. So while there may be additional constraints from the  $W$  mass which would further tighten the likelihood function, they have not been convincingly shown to be tighter than those of the atomic function, so we will ignore them in our analysis. Because further constraints lead to a narrowing of the likelihood function, omitting them is therefore a more conservative approach.

#### 4. THE DISTRIBUTION OF QUARK AND LEPTON MASSES

In a landscape picture, the Yukawa couplings would not be uniquely determined but would follow from some intrinsic distribution. Because there are enough masses, the observed masses can give us insight into what this distribution is without having to know the full underlying theory of the landscape [5, 6]<sup>5</sup>.

The intrinsic probability distribution for a quark or lepton Yukawa coupling is defined such that the fraction of values that appear at coupling  $\Gamma$  within a range  $d\Gamma$  is  $\rho(\Gamma) d\Gamma$ , with the normalization

$$1 = \int d\Gamma \rho(\Gamma). \quad (7)$$

In particular, we have explored a set of power law weights [5, 6]

$$\rho(\Gamma) = \begin{cases} \frac{N}{\Gamma^\delta} & \text{if } \Gamma_{\min} < \Gamma < \Gamma_{\max}, \\ 0 & \text{otherwise,} \end{cases} \quad (8)$$

<sup>3</sup> In fact,  $\Lambda_{QCD}$  serves as the comparison scale for all dimensional quantities in this work, so that the reference value for the Higgs vev  $v_0$  is also in units of  $\Lambda_{QCD}$ .

<sup>4</sup> See, however, [9] which raises problems with this scenario.

<sup>5</sup> The Yukawa interactions also influence quark mixing and the observed weight is consistent with the hierarchy of weak mixing elements [6, 10]. There are also possible implications for neutrino properties [6, 11].

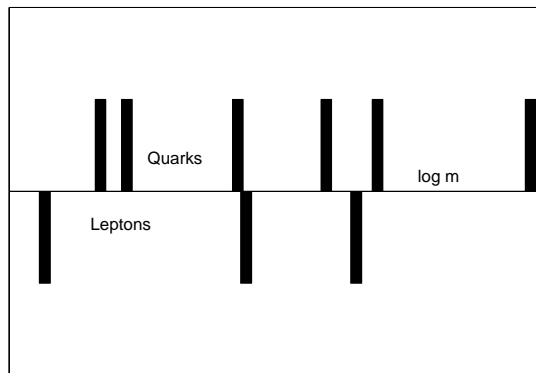


FIG. 2: Quark and lepton masses, defined at the energy  $\mu = M_W$ , on a logarithmic scale. A scale invariant weight corresponds to a uniform distribution on this scale. We use this feature to describe the distribution of Yukawa couplings

where the normalization constant is  $N = (1 - \delta)/[\Gamma_{\max}^{1-\delta} - \Gamma_{\min}^{1-\delta}]$  if  $\delta \neq 1$  and  $N = 1/\ln[\Gamma_{\max}/\Gamma_{\min}]$  if  $\delta = 1$ .

Such simple power law weights require at least one endpoint (or two for  $\delta = 1$ ) in order to be normalizable as in Eq. (7). When we determine the endpoints for the Yukawa distributions at large and low values, it is natural to use the renormalization group quasi-fixed point [12]  $\Gamma_{\max} \sim 1.26$  as the upper limit. In [5, 6] a lower endpoint  $\Gamma_{\min} \sim 1.18 \times 10^{-6}$  which corresponds to  $0.4m_e$  was used. This is explored more in the following section. With these ingredients, a likelihood analysis found that  $\delta = 1.02 \pm 0.08$ .

Of particular interest is the experimentally favored scale invariant distribution corresponds to a weight with  $\delta = 1$ :

$$\rho(\Gamma) = \frac{N}{\Gamma}. \quad (9)$$

While the evidence for the scale invariant weight is based on quantitative studies, the result can be seen qualitatively in Fig. 2. A scale invariant distribution is one which is a random uniform population on a logarithmic scale. Fig. 2 shows the Yukawa couplings for the quarks and leptons, at the scale  $\mu = M_W$ , plotted on a logarithmic scale. The result appears visually to be consistent with this idea, and in practice a scale invariant weight is highly favored<sup>6</sup>. We will use this as our primary weight for our analysis.

In exploring the uncertainties in the Higgs likelihood function, we will also consider weights which have no lower bound on the Yukawa couplings. This is only possible for  $\delta < 1$ , if the distribution is to be normalizable.

<sup>6</sup> Because the result is consistent with scale invariant, the choice of scale is not important and a similar result would be obtained using either the un-rescaled masses or with the Yukawa couplings defined at the Grand Unification scale.

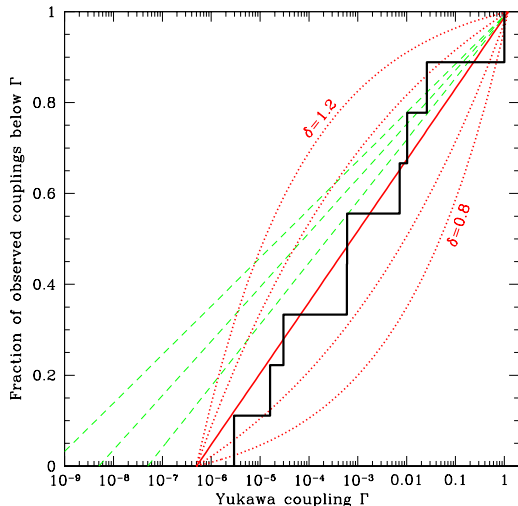


FIG. 3: The staircase-like curve shows the empirical distribution function, defined as the fraction of the 9 observed Yukawa couplings that lie below a given  $\Gamma$ -value. The other curves show the cumulative distribution functions predicted by our power-law probability distribution Ansatz. All assume  $\Gamma_{\max} = 1.2$ , and the straight lines correspond to the log-uniform  $\delta = 1$  case with different values of  $\Gamma_{\min}$ , the solid line having the best-fit value  $\Gamma_{\min} = 5 \times 10^{-7}$ . From top to bottom, the bent curves with the same endpoints take the  $\delta$ -values 1.2, 1.1, 1.0, 0.9 and 0.8, respectively. The  $\delta = 1$  line is seen to exhibit good consistency with the observed data.

While the possibility  $\Gamma_{\min} = 0$  is statistically disfavored (see next section), we found in [6] that the best fit value in that case is  $\delta = 0.86^{+0.04}_{-0.05}$ . We will use this in our tests of uncertainties in Sec 7.

## 5. FURTHER EVIDENCE ON THE FERMION MASS DISTRIBUTION

Since the Bayesian result of Ref. [6]  $\delta = -1.02 \pm 0.08$  from a likelihood analysis does not address the question of whether the best-fit distribution is actually consistent with the data, we have studied a set of frequentist Kolmogorov-Smirnov (KS) tests. The observational input into a KS-test is the so-called empirical distribution function  $F_{\text{obs}}(\Gamma)$ , defined as the fraction of the 9 observed Yukawa couplings that lie below a given  $\Gamma$ -value. This function is plotted in Fig. 3, and the horizontal location of the staircase-shaped steps correspond to the 9 rank-ordered Yukawa couplings for  $(e, \mu, \tau, u, d, s, c, b, t)$ , taken from the Particle Data Group compilation [13]. For any particular choice of our model parameters  $(\delta, \Gamma_{\min}, \Gamma_{\max})$ , we can compute a predicted cumulative probability distribution function

$$F(\Gamma) \equiv \int_{\Gamma_{\min}}^{\Gamma} \rho(\Gamma') d\Gamma' \quad (10)$$

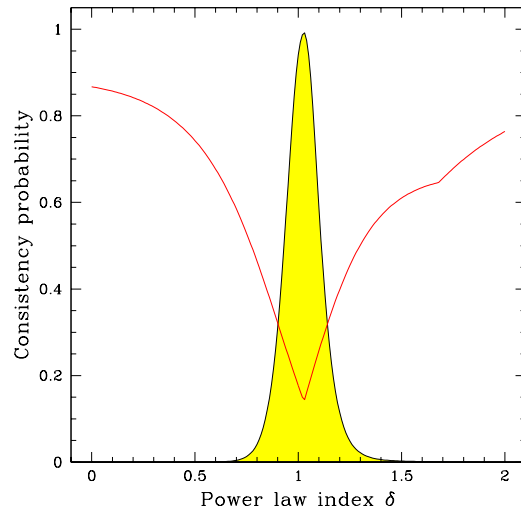


FIG. 4: The probability that our model is consistent with 9 observed Yukawa couplings is shown as a function of the power law index  $\delta$ , assuming the best-fit value  $\Gamma_{\min} = 5 \times 10^{-7}$ . The V-shaped curve shows the maximal difference between the predicted and observed distribution functions from Fig. 3.

and compare how well it agrees with  $F_{\text{obs}}(\Gamma)$ . Fig. 3 illustrates this for variations in both  $\delta$  and  $\Gamma_{\min}$ . The KS-test uses as a goodness-of-fit statistic the maximum vertical discrepancy  $|F_{\text{obs}}(\Gamma) - F(\Gamma)|$  between theory and observation. The corresponding consistency probability is shown in Figs. 4 and 5 for variations in  $\delta$  and  $\Gamma_{\min}$ , respectively, keeping a fixed upper cutoff  $\Gamma_{\max} = 1.2$ .

The constraints on  $\delta$  from Fig. 4 nicely reproduce the previous likelihood result using frequentist methods. Fig. 4 shows that we can reject the null hypothesis that our model is correct at high significance if  $\delta$  deviates substantially from unity, and also that the data are perfectly consistent with  $\delta = 1$ . This approach also enables us to place a bound on the lower endpoint  $\Gamma_{\min}$ , as seen in Fig. 5. Normalizability alone requires a lower endpoint when  $\delta \geq 1$ , but for any  $\delta$ -value, it is clear from Fig. 3 that the fit becomes poor if the left endpoint is dragged sufficiently far to the left of the leftmost data point. Fig. 5 shows that the best fit value is  $\Gamma_{\min} \approx 5 \times 10^{-7}$  for the scale-invariant case, although the constraints are rather weak, with  $1 - \sigma$  and  $2 - \sigma$  bounds (32% and 0.045% consistency probability) corresponding to  $\Gamma_{\min} \approx 7 \times 10^{-9}$  and  $\approx 5 \times 10^{-11}$ , respectively.

In Ref. [6] a Bayesian approach was employed where the dependence of the likelihood function on the exponent  $\delta$  was studied for a fixed  $\Gamma_{\min}$ . Here we extend this analysis by investigating also the dependence of the likelihood on  $\Gamma_{\min}$ . As in [6] we fix the upper endpoint to be  $\Gamma_{\max} = 1.26$  which is motivated by the quasi-fixed point of the Standard Model. In Fig. 6 we show the contour plot of the log-likelihood function as a function of the lower Yukawa endpoint  $\Gamma_{\min}$  and the exponent  $\delta$ . The

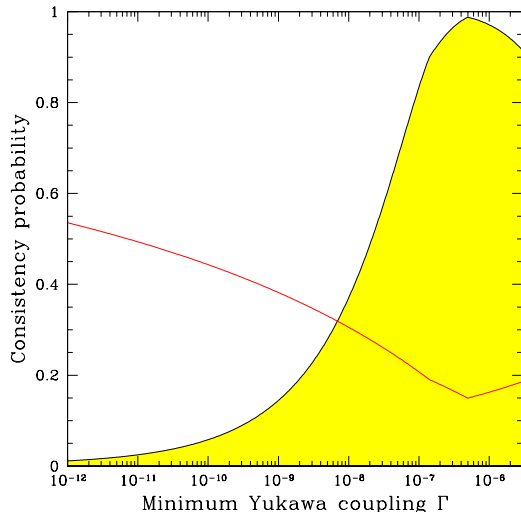


FIG. 5: The probability that our model is consistent with 9 observed Yukawa couplings is shown as a function of the lower cutoff  $\Gamma_{min}$ , assuming the scale-invariant power law index  $\delta = 1$ . The V-shaped curve shows the maximal difference between the predicted and observed distribution functions from Fig. 3.

darkest area is the  $1 - \sigma$  range, the next one is the  $2 - \sigma$  range etc. Here the  $n - \sigma$  range is taken as the parameter space where the log-likelihood is at most  $n^2/2$  below its maximum.

We see that for all values of  $\delta$  the data favors a lower endpoint for the distribution, and for any fixed value of  $\delta$  the likelihood function increases monotonically up to the highest possible value  $\Gamma_{min} = \Gamma_e$ . The best fit, i.e. the highest likelihood, is found for  $\delta = 1.06$  and  $\Gamma_{min} = \Gamma_e$ . Any probability distribution with  $\Gamma_{min} = \Gamma_e$  seems of course very unnatural since one out of the nine measured Yukawas would lie exactly on the endpoint of the probability distribution, but the likelihood analysis does not take that into account. For  $\delta < 1$  where the power law weights do not require a lower endpoint  $\Gamma_{min}$ , the data says that such a scenario with  $\Gamma_{min} = 0$  is disfavored by over  $2 - \sigma$ .

## 6. THE LIKELIHOOD FUNCTION FOR THE HIGGS VEV

We now combine our two key ingredients, the anthropic constraints and the probability distribution for the Yukawa couplings which is phenomenologically successful in describing the observed Yukawa couplings, in order to estimate the likelihood distribution for the Higgs vev. Our approximation of the full problem, under the assumptions described in Section 2, consists of

$$L(v) = \int d\Gamma_i A(v, \Gamma_u, \Gamma_d, \Gamma_e) \rho(\Gamma_i) \quad (11)$$

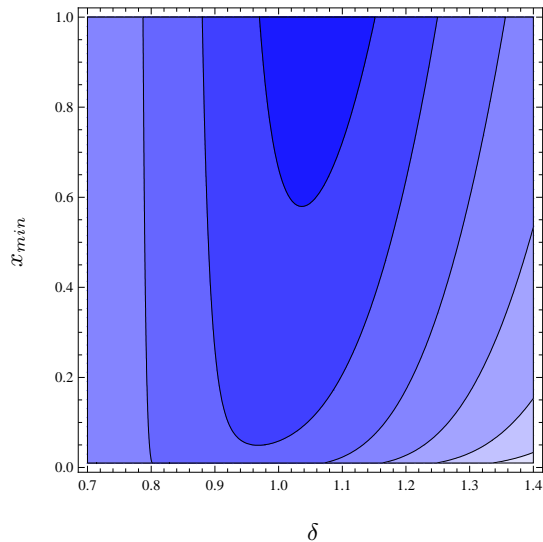


FIG. 6: Contour plot of the log-likelihood as a function of the lower endpoint for the Yukawa couplings  $\Gamma_{min}$  and the exponent  $\delta$ . We normalize  $\Gamma_{min}$  relative to the lowest observed Yukawa coupling,  $\Gamma_e$ , by defining  $x_{min} = \Gamma_{min}/\Gamma_e$ . The darkest area is the  $1\sigma$  range, the second darkest area marks the  $2\sigma$  range etc.

where a product over all charged fermions  $i$  is understood. In comparison with Eq. (2), the gauge couplings have been dropped due to our focus on the primary constraints due to the Yukawa couplings. The potential dependence on  $v$  in  $\rho(v, \Gamma_i \dots)$  is no longer present due to the assumption that the intrinsic probability distribution in  $v$  is roughly flat over our the allowed atomic window<sup>7</sup>. The atomic function  $A(v, \Gamma_u, \Gamma_d, \Gamma_e)$  is summarized in Fig. 1 and  $\rho(\Gamma_i)$  is the probability distribution for the Yukawa couplings where we use  $\delta = 1$ ,  $\Gamma_{min} = 0.4\Gamma_e$  and  $\Gamma_{max} = 1.26$ . The normalization of  $L(v)$  is irrelevant, we are only interested in estimating its shape. We calculate  $L(v)$  numerically by randomly populating the Yukawa couplings using the appropriate weight at different values of the vev. In particular we generate a set of three Yukawa couplings for the up-type quarks, for the down-type quarks and for the leptons. The smallest Yukawa coupling of each set is defined to be  $\Gamma_u$ ,  $\Gamma_d$  and  $\Gamma_e$  respectively. The relative probability of satisfying the atomic constraints then yields the likelihood function.

Let us briefly reiterate here the main assumptions which go into our analysis and explain the logic of how the application of a scale invariant probability distribution for the Yukawa couplings can yield a constraint on the scale of the Higgs vev. First of all, we note that we extract the observed Higgs vev from measurements other

<sup>7</sup> we will return to these issues in our discussion of the uncertainties

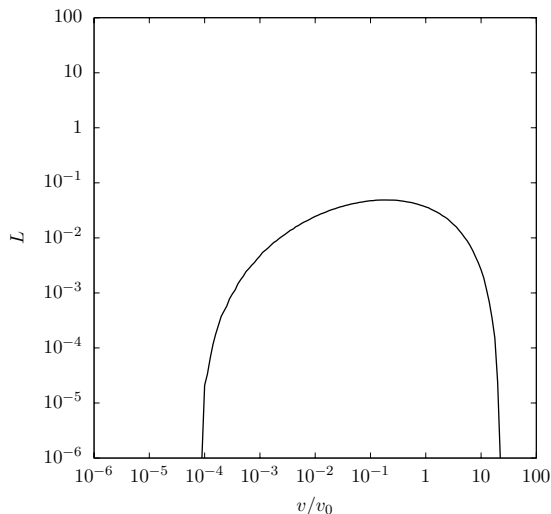


FIG. 7: The likelihood function  $L$  for the Higgs vev as a function of  $v/v_0$  ( $v_0$  is the observed Higgs vev of 246 GeV), constructed with our favored scale invariant weight. The normalization gives the fraction of simulations which satisfied the anthropic constraints.

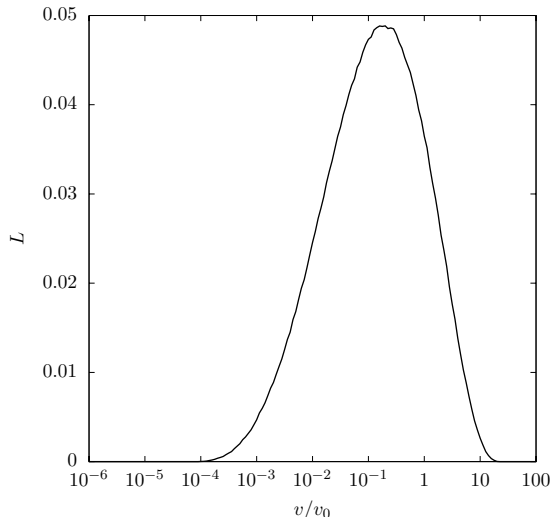


FIG. 8: This shows the same likelihood function  $L$  as in Fig. 7, but with the vertical axis on a linear scale.

than the fermion masses, such as  $M_W$ . Now, our crucial assumption that the Higgs vev and the Yukawa couplings are statistically independent is used to infer the probability distribution for the Yukawa couplings independently of the Higgs vev. In practice, it essentially means that we infer the scale invariant weight for the Yukawa couplings directly from the observed scale invariant weight for the quark and lepton masses for a fixed value of the Higgs vev. The probability distribution for the Yukawa couplings is taken to be a power law with an exponential close to the scale invariant case of  $\delta = 1$ . For  $\delta = 1$  it

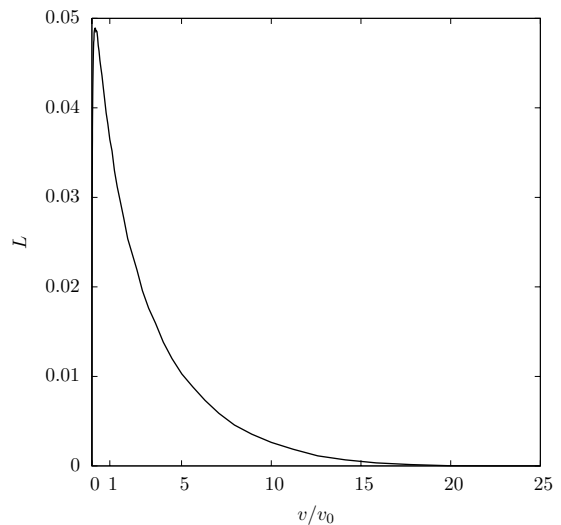


FIG. 9: The likelihood function  $L$  shown in Fig.7 and Fig. 8 is shown here with both the horizontal and vertical axes on a linear scale.

requires lower and upper endpoints. Whereas the lower endpoint of the distribution is inferred from the measured Yukawa couplings, for the upper endpoint the renormalization group quasi-fixed point has been used. Again, the assumption of statistical independence is crucial when we extract these endpoints and use them universally for any Higgs vev. Thus, we know that the Yukawa couplings have to not only be uniformly distributed on a log scale in a stretch extending over roughly six orders of magnitude, but we know where on the log scale this stretch is located. Finally, a scale enters through the atomic constraints on the light fermion masses, and together with the probability distribution of Yukawa couplings, it yields our constraints on the Higgs vev. Consider for example a large Higgs vev of  $10^{16}$  GeV. Since our Yukawa couplings from a scale invariant weight are required to lie roughly between  $10^{-6}$  and 1, it would be impossible to get light fermion masses in the MeV range for such a large value of the Higgs vev.

For our main result, we consider the case seen in Nature where only the  $u$ ,  $d$  quarks and the electron fall within the anthropic window - all others quarks and leptons are heavier and should not be part of stable atoms. We discuss alternatives in the next section. We implement this constraint by requiring that the second lightest up-type quark, the  $c$  quark, does not lie within the anthropic window sketched in Fig. 1 when the  $m_u$  axis is replaced by  $m_c$ , and analogously for the second lightest down-type quark.

For our favored scale invariant weight, the result is shown as Fig. 7, 8 and 9 using log-log, log-linear and linear-linear coordinate axes. This is our primary result. We see that the distribution is peaked near the value  $v_0$  observed in Nature and it extends over several orders of magnitude. The median value in this distribution is



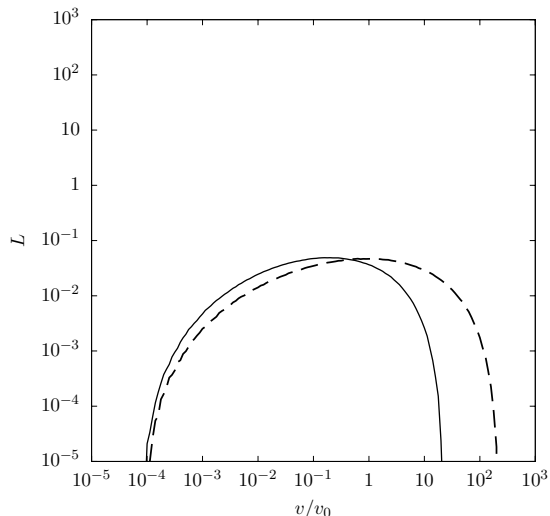


FIG. 10: The likelihood function for different values of the lower endpoint in the Yukawa distribution,  $\Gamma_{min} = 0.4\Gamma_e$  (solid) and  $\Gamma_{min} = 0.04\Gamma_e$  (dashed).

$v = 2.25 v_0$ . The  $2 - \sigma$  range extends from  $0.10 v_0$  to  $11.7 v_0$ . We observe that there is a steep upper cutoff in the allowed values of the vev which comes from the lower endpoint  $\Gamma_{min}$  present in the scale invariant distribution of the Yukawa couplings. The relevant point where the likelihood function falls off faster than  $1/v$  is located at a few times  $v_0$ . We conclude that  $v_0$  would be a very typical value for the Higgs vev whereas values  $\gg 10 \times v_0$  would be very unlikely.

## 7. UNCERTAINTIES

In this section, we consider the changes in the likelihood function if we modify some of the features of our analysis. The two greatest effects come from the variation or removal of the lower endpoint in the fermion mass distribution, and from the possibility of extra quarks or leptons within the anthropic window. The first of these has the potential to significantly modify our results.

First we can consider the changes if we use a different value of the lower cutoff. Changing the lower endpoint by one order of magnitude within the context of the scale invariant weight produces the modification shown in Fig. 10. The plot shows that for  $\Gamma_{min} = 0.04\Gamma_e$  the likelihood distribution favors larger values of the Higgs vev. In this case, smaller allowed values of the Yukawa couplings are compensated by larger values of the Higgs vev to satisfy the atomic constraint. We see that the qualitative features of the spectrum remain. However, the value of  $v$  where the likelihood starts to fall off faster than  $1/v$  is roughly 10 times higher if we divide  $\Gamma_{min}$  by a factor of 10. We clearly need to address this issue of how much the most likely Higgs vev depend on the cutoff at low Yukawa couplings in the weight.

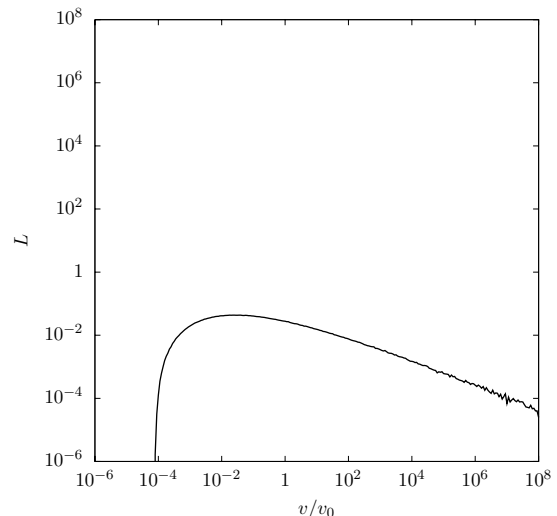


FIG. 11: The likelihood function resulting from a power law weight of exponent  $\delta = 0.86$  without any lower endpoint  $\Gamma_{min}$ .

One major concern about the result of the previous section is that the shape of the likelihood function is determined at large values of the vev by the fact that the scale invariant weight has a lower endpoint to the Yukawa distribution. We address this issue by considering a power weight with  $\delta$  less than unity, with no lower endpoint. Thus the power law behavior of the weight at low Yukawa couplings is not cut off at all and extends all the way down to zero. The Yukawa couplings can therefore become arbitrarily small, without being in violent disagreement with the overall fermion mass distribution. Such a distribution has the potential to allow arbitrarily large values for the vev. In these cases, there would be situations where a high value of the vev is counter-balanced by very small Yukawas in order to satisfy the atomic constraints.

In Fig. 11, we show the likelihood function that is obtained for  $\delta = 0.86$  and  $\Gamma_{min} = 0$ . While the maximum of the likelihood function remains close to the observed value we see that it never falls off faster than  $1/v$  in the region studied which extends up to  $10^8 \times v_0$ . That means the most likely values of the Higgs vev in this scenario would be very large. Because the power law weight is valid down to zero Yukawa couplings, we lose the constraints for the Higgs vev to be in the neighborhood of the observed  $v_0$ . While our analysis in Sec. 5 has shown that  $\Gamma_{min} = 0$  is disfavored by over  $2\sigma$  this issue remains a serious caveat. It motivates further top-down studies of properties of the string theory landscape in order to identify the existence of a lower cutoff in the quark mass weight.

Another uncertainty is the interesting possibility that more quarks beyond  $u, d, e$  fall in the atomic window. If we treat all the quarks independently as we have been doing we find that it is reasonably common that more quarks do fall in the allowed atomic window. For exam-

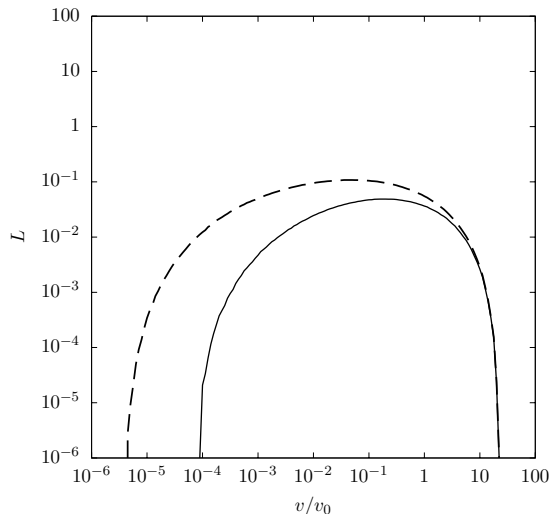


FIG. 12: The likelihood function allowing any number of light quarks in the anthropic window (dashed), along with that allowing  $u, d, e$  only (solid).

ple, with the scale invariant weight the likelihood function including extra quarks in this range is shown in Fig. 12. The allowance of extra small masses to fall in the atomic window makes the distribution peaked around smaller values in comparison to when we do not allow them to fall inside the window. The value of  $v$  where the likelihood starts falling off faster than  $1/v$  and therefore the most likely Higgs vev remains almost unchanged. We do not see the disadvantage of this situation for the existence of atoms.

However at this point it is useful to recognize a known flaw in our approximation that the Yukawa distributions of each flavor are treated as independent. Even if the original Yukawa couplings of the theory are distributed independently, the final output governing masses will not be independent. The original Yukawas exist in a  $3 \times 3$  complex matrix for each charge sector. The diagonalization of this matrix yields the final eigenvalues as well as rotation angles that go into the CKM and PMNS weak mixing matrices. It is well known that upon diagonalization, the eigenvalues of a matrix repel each other. In random matrix theory this leads to a repulsion of the final eigenvalues. For our case, this says that the  $u, d, e$  distributions will be independent, since they come from different charge sectors, hence different matrices. However, the likelihood of two quarks *of the same charge* falling in the allowed atomic range will be decreased by this repulsion. We have studied this effect by generating random Yukawa couplings in a  $3 \times 3$  matrix and diagonalizing. As discussed in [6], in this case in order to approximate a scale invariant distribution of the final eigenvalues we start with an initial weight with  $\delta = 1.16$ , and we use  $\Gamma_{min} = 0.4\Gamma_e$ . The resulting likelihood function is shown in Fig. 13, where the solid curve only allows  $u, d, e$  in the anthropic window and where the dashed curve re-

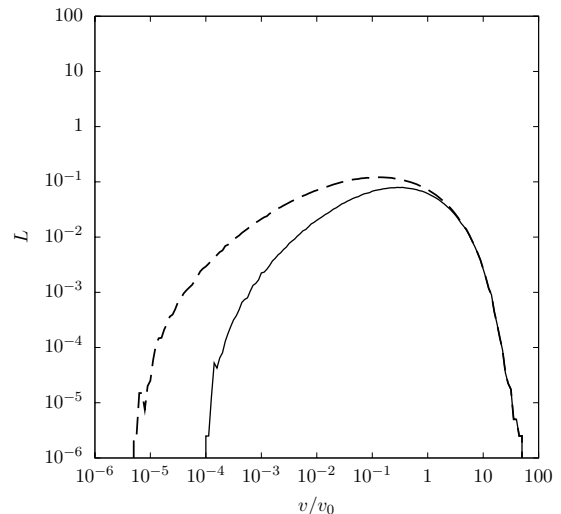


FIG. 13: The likelihood function resulting from a biunitary diagonalization of the Yukawa matrices allowing any number of light quarks in the anthropic window (dashed), along with that allowing  $u, d, e$  only (solid).

sults from allowing any number of light quarks in the anthropic window. While the falloff at higher  $v$  is now less steep than in the result from diagonal simulations without matrix diagonalizations of Fig. 12, it is clearly falling off much faster than  $1/v$  and the most likely value of  $v$  would be below  $10 \times v_0$ . Comparing the result from the diagonalization of the Yukawa matrices in Fig. 13 with the corresponding result from diagonal simulations without matrix diagonalizations shown in Fig. 12, we note that the two curves in Fig. 13 are closer together than the ones in Fig. 12 which is due to the repulsion of eigenvalues in the matrix diagonalization case.

Since our work is based on statistical independence of  $v$  and  $\Gamma_i$ , i.e.  $\rho(v, \Gamma_i) = \rho(v)\rho(\Gamma_i)$ , there could be even more dramatic problems if there were correlations between  $v$  and  $\Gamma_i$ . For example, if in our weight  $\rho(\Gamma_i)$  both endpoints  $\Gamma_{min}, \Gamma_{max}$  were proportional to  $1/v$ , there would be no anthropic constraints on  $v$  since varying  $v$  would keep the distribution of the masses the same. Because we measure the Yukawa couplings at a single value of  $v$ , we cannot address this.

We have mentioned previously the possibility that further anthropic constraints could come into play. These could change the detailed shape of the likelihood function because they could eliminate regions of parameter space which are somewhat different from our world. However, since our parameters are clearly consistent with the constraints, the elimination of other regions would likely narrow the resulting likelihood function for  $v$ . This would change the shape of the function, but would not modify our basic conclusions about the compatibility of our value of  $v$  with the allowed anthropic range.

One might also wonder if the likelihood function should also take into account the other great anthropic con-

straint, that on the clumping of matter in the universe to form stars and planets, which limits in particular the cosmological constant. The likelihood function for the cosmological constant has been studied by Vilenkin and Garriga [14]. The parameters of the Standard Model do of course also influence the cosmological constant. For example a shift in the value of the up quark mass (or the Higgs vev) by one part in  $10^{40}$  would shift the cosmological constant by 100%. However, for the cosmological constant to have an anthropic selection, the possible values of  $\Lambda$  must be densely packed and there should be other slightly different combinations of parameters that yield an anthropically allowed cosmological constant. For the rather narrow range of  $v$  that we probe, it seems very reasonable that the clumping of matter constraint has little influence on the likelihood of the Higgs vev.

## 8. CONCLUSIONS

The fundamental question that we are addressing is whether it is plausible that it is the atomic constraints that explain the low value of the Higgs vev in landscape theories. This is known to be the case at fixed values of the Yukawa couplings, but since these parameters also may vary in landscape theories, and since their magnitudes seem to be peaked at low values, the answer is less obvious in a more general context. We have used experimental information on the distribution of masses to address this issue. We find that even if the Yukawa couplings are allowed to scan in a way that is favored phenomenologically, the likely values of the Higgs vev are close to the one observed in Nature. This supports the hypothesis that these constraints favor Higgs values similar to ours.

This can be interpreted as a motivation for further exploration of landscape theories. The value of the Higgs vev and that of the cosmological constant are two great “fine-tuning” problems of the fundamental interactions, and the presence of a landscape would change the way that we approach the issue of fine-tuning [15]. This is because both of these problems appear to have plausible resolutions through anthropic constraints that are appropriate for landscape theories<sup>8</sup>.

In [8], it was argued that the anthropic constraints on the quark masses cannot be used to constrain the Higgs vev, by constructing a plausible scenario in which the weak interactions do not appear. In the context of <sup>8</sup> However, the “strong CP problem” is a fine-tuning puzzle that does not appear to have an anthropic resolution [16].

our exploration in this paper, this could be realized by taking  $v \rightarrow \infty$ ,  $\Gamma \rightarrow 0$ , with their product fixed. This situation may arise in our scenario for power law weights with exponent  $\delta < 1$  which extend down to zero Yukawa couplings. Such weights are disfavored by more than  $2 - \sigma$  in comparison to weights with a lower cutoff  $\Gamma_{min}$  of the order of  $\Gamma_e$  as we inferred from the measured fermion spectrum. Nevertheless they are a serious caveat to an anthropic constraint for the Higgs vev. For the preferred weights with a lower cutoff  $\Gamma_{min}$  of the order of  $\Gamma_e$  the most likely values are close to that seen in Nature.

One of the strengths of our approach is that it does not rely on the ultraviolet completion of the fundamental underlying theory. Aside from our assumption of statistical independence, this input comes from the data on the quark and lepton masses. Both the issues of statistical independence and of weights without a lower cutoff could possibly be addressed in top-down studies of the landscape.

The likelihood function that we have constructed is an estimator that tries to quantify the effect of the possible variation of the fundamental parameters on the range of allowed values for the Higgs vev. Further understanding of anthropic constraints may be able to narrow the likelihood function further. Even if the range is narrowed, this is more of a consistency check than a prediction of landscape theories, since we already know that the observed value of the vev is anthropically allowed. However, it does provide further motivation for landscape theories and suggests that within these theories the hierarchy and fine-tuning problems associated with the vev are not as serious obstacles as they are in other theories.

## Acknowledgments

J.D. acknowledges support from the National Science Foundation grant PHY- 055304. K. D would like to acknowledge the financial support by the DFG cluster of excellence “Origin and Structure of the Universe”. A.R has been supported in part by the Department of Energy grant No. DE-FG-02-92ER40704. MT was supported by NSF grants AST-0134999 and AST-05-06556, a grant from the John Templeton foundation and a fellowships from the David and Lucille Packard Foundation. In addition both J.D. and M.T. thank the Foundational Questions Institute for support and for conference travel that aided this collaboration.

- 
- [1] T. Damour and J. F. Donoghue, “Constraints on the variability of quark masses from nuclear binding,” *Phys. Rev. D* **78**, 014014 (2008) [arXiv:0712.2968 [hep-ph]].  
 [2] J. Barrow and F. Tipler, *The Anthropic Cosmological Principle* (Clarendon Press, Oxford, 1986).

- C. J. Hogan, “Why the universe is just so,” *Rev. Mod. Phys.* **72**, 1149 (2000) [arXiv:astro-ph/9909295].  
 R. N. Cahn, “The eighteen arbitrary parameters of the standard model in your everyday life,” *Rev. Mod. Phys.* **68**, 951 (1996).

- L. B. Okun, *Usp. Phys. Nauk*, **161**, 177 (1991)(Soviet Phys. Usp. **34**, 818).
- R. L. Jaffe, A. Jenkins and I. Kimchi, “Quark Masses: An Environmental Impact Statement,” *Phys. Rev. D* **79**, 065014 (2009) [arXiv:0809.1647 [hep-ph]].
- [3] V. Agrawal, S.M. Barr, J.F. Donoghue and D. Seckel, “Anthropic considerations in multiple-domain theories and the scale of electroweak symmetry breaking,” *Phys. Rev. Lett.* **80**, 1822 (1998) hep-ph/9801253.
- V. Agrawal, S.M. Barr, J.F. Donoghue and D. Seckel, “The anthropic principle and the mass scale of the standard model,” *Phys. Rev.* **D57**, 5480 (1998) hep-ph/9707380.
- [4] M. R. Douglas, “The statistics of string / M theory vacua,” *JHEP* **0305**, 046 (2003) [arXiv:hep-th/0303194].
- S. Ashok and M. R. Douglas, “Counting flux vacua,” arXiv:hep-th/0307049.
- L. Susskind, “The anthropic landscape of string theory,” arXiv:hep-th/0302219.
- R. Kallosh and A. Linde, “M-theory, cosmological constant and anthropic principle,” *Phys. Rev. D* **67**, 023510 (2003) [arXiv:hep-th/0208157].
- R. Bousso and J. Polchinski, “Quantization of four-form fluxes and dynamical neutralization of the cosmological constant,” *JHEP* **0006**, 006 (2000) [arXiv:hep-th/0004134].
- [5] J. F. Donoghue, “The weight for random quark masses,” *Phys. Rev. D* **57**, 5499 (1998) [arXiv:hep-ph/9712333].
- [6] J. F. Donoghue, K. Dutta and A. Ross, “Quark and lepton masses and mixing in the landscape,” *Phys. Rev. D* **73**, 113002 (2006) [arXiv:hep-ph/0511219].
- [7] M. Tegmark, A. Vilenkin and L. Pogosian, “Anthropic predictions for neutrino masses,” *Phys. Rev. D* **71**, 103523 (2005) [arXiv:astro-ph/0304536].
- [8] R. Harnik, G. D. Kribs and G. Perez, “A universe without weak interactions,” *Phys. Rev. D* **74**, 035006 (2006) [arXiv:hep-ph/0604027].
- [9] L. Clavelli and R. E. . White, “Problems in a weakless universe,” arXiv:hep-ph/0609050.
- [10] G. W. Gibbons, S. Gielen, C. N. Pope and N. Turok, “Measures on Mixing Angles,” *Phys. Rev. D* **79**, 013009 (2009) [arXiv:0810.4813 [hep-ph]].
- G. W. Gibbons, S. Gielen, C. N. Pope and N. Turok, “Naturalness of CP Violation in the Standard Model,” *Phys. Rev. Lett.* **102**, 121802 (2009) [arXiv:0810.4368 [hep-ph]].
- [11] L. J. Hall, M. P. Salem and T. Watari, “Statistical Understanding of Quark and Lepton Masses in Gaussian Landscapes,” *Phys. Rev. D* **76**, 093001 (2007) [arXiv:0707.3446 [hep-ph]].
- L. J. Hall, M. P. Salem and T. Watari, “Quark and Lepton Masses from Gaussian Landscapes,” *Phys. Rev. Lett.* **100**, 141801 (2008) [arXiv:0707.3444 [hep-ph]].
- L. J. Hall, H. Murayama and N. Weiner, “Neutrino mass anarchy,” *Phys. Rev. Lett.* **84**, 2572 (2000) [arXiv:hep-ph/9911341].
- N. Haba and H. Murayama, “Anarchy and hierarchy,” *Phys. Rev. D* **63**, 053010 (2001) [arXiv:hep-ph/0009174].
- A. de Gouvea and H. Murayama, “Statistical test of anarchy,” *Phys. Lett. B* **573**, 94 (2003) [arXiv:hep-ph/0301050].
- J. R. Espinosa, “Anarchy in the neutrino sector?,” arXiv:hep-ph/0306019.
- L. J. Hall, M. P. Salem and T. Watari, “Neutrino mixing and mass hierarchy in Gaussian landscapes,” *Phys. Rev. D* **79**, 025010 (2009) [arXiv:0810.2561 [hep-th]].
- [12] B. Pendleton and G. G. Ross, “Mass And Mixing Angle Predictions From Infrared Fixed Points,” *Phys. Lett. B* **98**, 291 (1981).
- C. T. Hill, “Quark And Lepton Masses From Renormalization Group Fixed Points,” *Phys. Rev. D* **24**, 691 (1981).
- C. T. Hill, C. N. Leung and S. Rao, “Renormalization Group Fixed Points And The Higgs Boson Spectrum,” *Nucl. Phys. B* **262**, 517 (1985).
- M. Lanzagorta and G. G. Ross, “Infrared fixed points revisited,” *Phys. Lett. B* **349**, 319 (1995) [arXiv:hep-ph/9501394].
- [13] S. Eidelman *et al.* [Particle Data Group], “Review of particle physics,” *Phys. Lett. B* **592**, 1 (2004).
- [14] J. Garriga and A. Vilenkin, “Solutions to the cosmological constant problems,” *Phys. Rev. D* **64**, 023517 (2001) [arXiv:hep-th/0011262].
- [15] J. F. Donoghue, “The fine-tuning problems of particle physics and anthropic mechanisms,” arXiv:0710.4080 [hep-ph]. ‘Universe or Multiverse’ ed. by Bernard Carr pp 231-246 (Cambridge Univ. Press).
- [16] J. F. Donoghue, “Dynamics of M theory vacua”, *Phys. Rev. D* **69**, 106012 (2004) [Erratum-ibid. *D* **69**, 129901 (2004)] arXiv:hep-th/0310203].
- T. Banks, M. Dine and E. Gorbatov, “Is There A String Theory Landscape,” *JHEP* **0408**, 058 (2004). [arXiv:hep-th/0309170].
- L. Ubaldi, “Effects of theta on the deuteron binding energy and the triple-alpha process,” *Phys. Rev. D* **81**, 025011 (2010). arXiv:0811.1599 [hep-ph].