2010

Labor Productivity and the Law of Decreasing Labor Content

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DEPARTMENT OF ECONOMICS

Working Paper

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Working Paper 2010-11

UNIVERSITY OF MASSACHUSETTS
AMHERST

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August 3, 2010

Abstract

This paper analyzes labor productivity and the *law of decreasing labor content* (LDLC) originally formulated by Farjoun and Machover (1983). First, it is shown that the standard measures of labor productivity may be rather misleading, owing to their emphasis on monetary aggregates. Instead, the conventional classical-Marxian labor values provide the theoretically and empirically sound measures of labor productivity. The notion of labor content and the LDLC are therefore central in order to understand the dynamics of capitalist economies. Second, some rigorous theoretical relations between different forms of profit-driven technical change and productivity are derived in a general input-output framework with fixed capital, which provide deterministic foundations to the LDLC. Third, the main theoretical propositions are analyzed empirically based on a new dataset of the German economy.

Keywords: labor productivity, law of falling labor content, technical change, labor values, Input-Output analysis.

JEL CLASSIFICATION SYSTEM: B51 (Socialist; Marxian; Sraffian), D57 (Input-Output Analysis), O33 (Technological Change: Choices and Consequences), C67 (Input-Output Models).

*We are grateful to Clopper Almon, Theodore Mariolis and Simon Mohun for comments and suggestions. Roberto Veneziani worked on this project while visiting the University of Massachusetts, Amherst. Their hospitality is gratefully acknowledged. The usual disclaimer applies.

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1 Introduction

In their influential book on *Laws of Chaos*, Farjoun and Machover (1983) formulate the celebrated *law of decreasing labor content* (henceforth, LDLC). According to the LDLC, if C is a commodity produced in a capitalist economy over a certain period of time, then "there is virtual certainty (probability very near 1) that the labor content of one unit of C will be lower at the end of the period than it was at the beginning" (ibid., p.97). According to Farjoun and Machover, the LDLC is a defining feature of capitalist economies: it is "the most basic dynamic law of capitalism, archetype of all capitalist development" (ibid., p.139).

Granting that the LDLC characterizes capitalist economies, two questions immediately arise. First, why is the LDLC relevant from a theoretical viewpoint? Farjoun and Machover consider the LDLC as self-evidently relevant because they see it as equivalent to the *law of increasing labor productivity* (see, for example, ibid., pp.11, 139 and passim). And labor productivity plays a key role in economic theories of growth and employment, including issues of innovation, structural change, income distribution, and so on. Yet the relevance of the LDLC for understanding trends in labor productivity is far from obvious: virtually all of the received productivity measures - as developed for instance in the UN's System of National Accounts (henceforth, SNA; UN, 1993. See also OECD, 2001; BLS, 2008) - focus on real GDP per unit of labor, or on some notion of 'real value added' per unit of labor, in order to measure the performance of (different sectors of) the economy. If the conventional SNA measures properly capture labor productivity, then one may argue that the notion of labor content is either misleading or at best redundant: the LDLC may indeed be "a prime example of a tendency that operates 'behind the backs' of the social protagonist, as though it were a law of nature" (Farjoun and Machover, 1983, p.84), but in order to explain some fundamental features of capitalist economies it would be unnecessary to uncover "some systematic connection between the visible and the invisible - between price and labor-content" (ibid.). It would be sufficient to analyze the dynamics of the price-based SNA measures. In other words, in principle, the relevance of the law of increasing labor productivity says very little about the relevance of the LDLC.

The second question is, how can the LDLC be derived, or deduced, from the functioning of capitalist market economies? What is the mechanism which explains "why individual actions motivated by considerations of price should in the long term result in a systematic effect on labor-content" (ibid., p.84)? Farjoun and Machover prove that the LDLC obtains in a probabilistic framework as the cumulative result of a sequence of technical changes such that the cost of physical inputs decreases while labor inputs remain constant. This type of innovations, however, represent a rather special case of the range of technical changes adopted in capitalist economies. Besides, the behavioral foundations of their analysis are not entirely spelled out, and fixed capital plays no essential role in their model, even though it is arguably central in capitalists' innovating decisions.

This paper analyzes both questions in a general input-output (henceforth, IO) framework. Indeed, it is shown, *contra* Farjoun and Machover, that a generalized IO approach provides a natural framework to formulate and derive the LDLC, and also to understand its theoretical relevance.

Section 2 addresses the first question and it shows the salience of the notion of labor content for the understanding of labor productivity. A thorough critical analysis of the standard SNA measures of sectoral as well as aggregate labor productivity is provided, from an IO perspective. The analysis of the structural features of the economy allowed by the IO framework forcefully shows that the SNA measures are inappropriate to capture production conditions, and shifts in efficiency and technology, owing to the central role of relative prices and final demand in their construction. Measures of sectoral and total labor productivity should be based on technological data as much as possible (subject to an unavoidable degree of aggregation), and they should not *definitionally* depend on price variables. Instead, the IO employment multipliers - that is, the labor values of classical-Marxian economic theory[1] - provide (in reciprocal form) theoretically sound measures of sectoral and economy-wide labor productivity, with purely technological foundations - insofar as IO coefficients can be interpreted as pure quantity magnitudes.

Thus, section 2 proves that the law of increasing labor productivity cannot be properly understood unless the LDLC is formulated. Yet the analysis also has broader implications for productivity analysis, because it shows that the shortcomings of the standard indices are more serious than it is acknowledged in the mainstream literature (e.g., Durand, 1994; Cassing, 1996; Schreyer, 2001) and that a proper understanding of labor productivity requires a focus on labor content. IO tables should always be an integral part of the SNA and the point of reference for all productivity measures at the macro- and meso-level of economic activity.[2]

Critiques of standard SNA productivity measures from an IO perspective and the use of employment multipliers to measure productivity are not novel (see, among the others, Gupta and Steedman, 1971; Steedman, 1983; Wolff, 1985, 1994; de Juan and Febrero, 2000; Almon, 2009).[3] This paper presents a new set of arguments and a unified theoretical framework for the analysis of productivity measures, which is based on a novel axiomatic method. Rather than comparing different measures in terms of their implications in various scenarios, this paper starts from first principles and formalizes some theoretically desirable properties that any measure of labor productivity should satisfy.[4] To be precise, the main axiom focuses only on *changes*

[1]Total labor costs and employment multipliers are identical in Leontief models, but can differ in more general economies. The analysis below can be extended to the general case by using the framework outlined in Flaschel (1983), albeit at the cost of a significant increase in technicalities.

[2]The importance of IO tables in productivity analysis is acknowledged in the mainstream literature (see, for example, Schreyer, 2001, p.50).

[3]In Richard Stone's original formulation of the UN's (1968) SNA, it is possible to find definitions of labor productivity that are conceptually analogous to the IO, or classical-Marxian, measures (e.g., UN, 1968, p.69). This paper suggests that it is quite unfortunate that this approach has been abandoned in the following revisions.

[4]The adoption of an axiomatic approach to analyze classical-Marxian themes is quite novel. For

in productivity and states that labor productivity at t in the production of good i has increased relative to the base period, if a unit increase of the net product of good i demands less labor than in the base period. This is a weak restriction and it incorporates the key intuitions behind the main productivity measures in the literature. Yet it characterizes the IO measures, whereas the conventional SNA indices do not satisfy it in general owing to their inherent dependence on relative prices and final demand.

The second major contribution of this paper, in section 3, is a rigorous analysis of the conditions under which profitable innovations lower labor values, thereby raising productivity and increasing consumption and investment opportunities. According to Farjoun and Machover (1983, p.141), "In [IO] theories of prices and profit the notion of labor-content can be defined, and the law can certainly be formulated. But it cannot be deduced or explained, because in these theories there is no general systematic connection between labour-content and price". This paper shows that this conclusion is not entirely correct and some systematic, deterministic connections between prices and labor content can be derived in general linear economies with fixed and circulating capital within the classical-Marxian tradition.

To be precise, in this paper the n-commodity general equilibrium models analyzed by Roemer (1977, 1980) are generalized into two main directions. First, following the approach developed by Flaschel (2010), the circulating capital model is extended to the treatment of fixed capital proposed by Bródy (1970) in a seminal contribution. This is important because fixed capital - or, more precisely, capital tied up in production - is a key feature of capitalist economies and it is at the center of innovation processes but, as various authors have argued, the standard von Neumann framework has serious theoretical and empirical limitations.[5] Second, the analysis does not depend on *any* assumptions on prices: no condition on uniform profit rates is imposed and the conclusions hold for any vector of prices measured in terms of the wage unit. This extension is both empirically *and* theoretically relevant, because general equilibrium-type constructions (including uniform profit rate models) may be unsatisfactory as representations of allocation in market economies as argued, among the others, by Farjoun and Machover (1983).

In this general framework, different forms of technical change can be considered, and a deterministic theoretical foundation for the LDLC can be derived. In fact, it can be proved that profitable fixed-capital–using labor–saving innovations lead to productivity increases. Given that capital–using labor–saving technical change has characterized most of the phases in the evolution of capitalism (Marquetti, 2003), this result provides theoretical foundations for the conclusion that labor values tend to fall, and labor productivity tends to rise, over time in capitalist economies.[6] The formal analysis has also broader implications concerning the social effects of capital-

a seminal contribution, see Yoshihara (2010).

[5]See Bródy (1970) and more recently, Flaschel et al. (2010). For an extension of Roemer's (1977) model to von Neumann economies see Roemer (1979) and Dietzenbacher (1989).

[6]These results are consistent with the Marxian analysis of technical change and the historical tendencies of capitalism. See Foley (1986) and Duménil and Lévy (1995, 2003).

ists' individual decisions. For it can be proved that there is no clear-cut relationship between profitable technical change and social welfare in capitalist economies: capitalists' maximizing behavior is neither necessary nor sufficient for the implementation of productivity-enhancing and welfare-improving innovations.

The analysis in section 3 is related to the classical literature on technical change, distribution, and the evolution of capitalism (for recent contributions, see Duménil and Lévy, 2003; Foley, 2003; Petith, 2008). Yet unlike in the latter contributions, an explicit microeconomic perspective is adopted, which emphasizes capitalists' profit-maximizing behavior in highly disaggregated economies. Moreover, although the paper may shed some light on the influence of distributive conflict on technical change, the focus is not on the general relation between technical change and distribution, or on the much-debated effect of technical progress on profitability (see also Michl, 1994). Instead the effect of individually optimal capitalist decisions on productivity and social welfare is thoroughly explored. Finally, although the process generating innovations is not explicitly formalized, the analysis presented here can be supplemented with the classical-Marxian evolutionary model of technical change developed by Duménil and Lévy (1995, 2003).

The analysis is not purely theoretical, though. In section 4, an empirical appraisal of the main theoretical conclusions is provided, based on the new IO dataset of the German economy constructed by Kalmbach et al. (2005). The empirical evidence confirms the main conclusions: first, SNA measures of labor productivity can be rather misleading and quite different from the theoretically sound IO indices. Second, the LDLC holds for the German economy, a fact that is not easily visible by just looking at the IO tables. It should be noted, however, that the main aim of this paper is not to provide a fully rigorous econometric analysis of productivity measures, or of long-term trends of technical change in capitalist economies. The focus is primarily theoretical and methodological: the paper provides a general analysis of the relationships between prices, technical change, and labor productivity. From this viewpoint, the discussion of the German economy (1991-2000) does not aim to be exhaustive: it only illustrates the main theoretical points, and the empirical results should be taken as a first step towards a more detailed analysis.

2 Labor content and labor productivity

The point of departure of the analysis is the standard IO table 2, which shows economic activity in a particular year in the n sectors of the economy. The notation is standard: $p(t) = (p_1(t), ..., p_n(t))$ is the $1 \times n$ vector of prices of the n commodities at time t; $x_{ij}(t)$ is the amount of good i used as intermediate input in the production of good j; $x_i(t)$ is the gross output of good i; $f_i(t)$ is the final demand of good i.

At the most general level, labor productivity can be defined as a ratio between an index of output and an index of labor input. One possibility is to use *gross output* as a measure of real product and to define labor productivity as gross output per unit

Delivery from ↓ to →	Sector 1 … Sector n	Final Demand	row sum
Sector 1	$x_{11}(t)p_1(t) \ldots x_{1n}(t)p_1(t)$	$f_1(t)p_1(t)$	$x_1(t)p_1(t)$
.	. .	.	.
.	. .	.	.
.	. .	.	.
Sector n	$x_{n1}(t)p_n(t) \ldots x_{nn}(t)p_n(t)$	$f_n(t)p_n(t)$	$x_n(t)p_n(t)$
Value Added	$Y_1(t) \quad \ldots \quad Y_n(t)$	–	$Y(t)$
column sum	$x_1(t)p_1(t) \quad \ldots \quad x_n(t)p_n(t)$	$F(t)$	

Table 1: The standard form of an input-output table

of direct labor. As is well known, however, this measure is appropriate only in the rather special case of technical progress affecting all factors proportionally. Further, gross output based indices of productivity are sensitive to the degree of vertical integration: ceteris paribus, gross output based productivity rises as a consequence of outsourcing, even if there are no changes in technology and production conditions.

Therefore most of the literature focuses on value added.[7] Two methods are used to obtain *real* output measures starting from value added data. The *single deflation* method requires deflating all entries (both outputs and inputs) in the nominal table 2 by a common price deflator, say P. Single-deflated value added in sector i is then $Y_i^s(t) = Y_i(t)/P$, and at the aggregate level $Y^s(t) = \sum_{i=1}^n Y_i^s(t)$. Instead, the method of *double deflation* attempts to measure everything in constant prices, that is, with regard to table 2 it attempts to replace current prices $p(t)$ with the prices $p(0)$ of a base year $t = 0$. This method, however, cannot be directly applied to the row of values added in table 2, which are pure value magnitudes, and the double deflated sectoral values added $Y_i^d(t)$ are obtained indirectly by applying the accounting consistency requirement of the nominal table 2 to its analogue in constant prices. This means that $Y_i^d(t)$ is the value added that would have resulted in sector i, if the prices in table 2 had remained constant after the base year.

Thus, value added in base year prices remains a value magnitude and not a quantity independent of relative prices, and therefore both single- and double-deflated value added are problematic notions in productivity analysis. “Value added is ... not an immediately plausible measure of output: contrary to gross output, there is no physical quantity that corresponds to a volume measure of value-added” (Schreyer, 2001, p. 41). Rather than measures of sectoral real output, *single deflated* values, $Y_i^s(t)$, should be interpreted as indices of sectoral real incomes, with only a distant relation with technological conditions. Therefore any such measure as $Y_i^s(t)/L_i(t)$ - where $L_i(t)$ denotes the work hours employed in sector i - represents at best real purchasing power per unit of labor, rather than sectoral labor productivity. Instead, the economic meaning of sectoral *double deflated* value added is rather unclear: since $Y_i^d(t)$ in general differs from $Y_i^s(t)$, for any i, then $Y_i^d(t)$ does not measure output

[7] For an approach focusing on gross output, see Hart (1996) and Stiroh (2002).

\	$1 \dots n$		
1 . . . n	X	f	x
	$y^{d'}$	-	Y^d
	x'	F^d	-

Table 2: Elementary input-output table in matrix notation

correctly, and in addition it has nothing to do with real purchasing power. It is a purely fictitious quantity representing the income per worker that would have emerged *if* prices had remained constant at the level of the base year.

These well-known conceptual problems, though, are usually considered as minor, and in virtually all of the literature on labor productivity, value-added measures of real output, and in particular the double-deflated values, $Y_i^d(t)$ and $Y^d(t)$, are used. Sectoral and macroeconomic labor productivity are defined, respectively, as $\pi_i^c(t) = Y_i^d(t)/L_i(t)$, and $\pi^c(t) = Y^d(t)/L(t)$, where $L(t) = \sum_{i=1}^n L_i(t)$, and $\pi^c(t)$ can be decomposed as follows:

$$\pi^c(t) = \sum_i \left(\frac{L_i(t)}{L(t)}\right) \cdot \left(\frac{Y_i^d(t)}{L_i(t)}\right) = \sum_i \left(\frac{L_i(t)}{L(t)}\right) \cdot \pi_i^c(t). \tag{1}$$

Value added based indices are considered theoretically and empirically meaningful. Indices based on single-deflated value added are deemed appropriate to analyze issues relating to economic welfare, whereas "for the purposes of measuring efficiency and productivity [double deflated measures are] to be preferred" (Stoneman and Francis, 1994, p.425; see also Cassing, 1996). Several doubts can be raised on both claims, and in general on the standard approach to productivity analysis.

For any vector $z \in \mathcal{R}^n$, let z' denote its transpose[8] and let $\widehat{z}$ denote the diagonal matrix with z as its diagonal. In IO analysis, it is common to choose the units of the n commodities so that, in the base period, $p(0) = e' \equiv (1, ..., 1)$. The double or row-wise 'price deflated' table 2 can then be expressed in matrix notation as in table 2. Following common practice in IO analysis, the matrix of intermediate inputs X can be transformed into the matrix of input coefficients $A = X\hat{x}^{-1}$, and the $1 \times n$ vector of direct labor inputs $\ell = (\ell_1, ..., \ell_n)$ can be similarly transformed into a vector of labor coefficients $l = \ell\hat{x}^{-1}$.[9] Then, the macro-identity $Y^d = p(0)f = F^d$ behind table 2 can be expressed in matrix notation as follows

$$Y^d = p(0)y^d = p(0)(I - A)x = p(0)f = F^d.$$

[8] In what follows, vectors are always column vectors, unless otherwise stated.

[9] For the sake of notational simplicity, in the rest of the paper, the timing of vectors will be omitted, whenever this is clear from the context.

Instead, the labor time spent, directly and indirectly, in the production of the n goods is given by $v = (v_1, ..., v_n) = l(I - A)^{-1}$ and the IO, or classical-Marxian measures of sectoral labor productivity are defined as $\pi_i^m = 1/v_i$.[10] In the rest of this section, a general framework is provided to compare productivity measures. In order to avoid problems of interpretation, the structural coefficients (A, l) are considered as the parameters of a linear technology, as in standard IO practice.

One of the key problems of the SNA measures is that they are sensitive to changes in relative prices which do not reflect any shift in production conditions. Consider, for example, a simple economy with one capital good and one pure consumption good, such that at a given period t the technical coefficients, a_{ij}, are $0 < a_{11} < 1$, $a_{12} > 0$, and $a_{21} = a_{22} = 0$. If a single price deflator P is used, which includes prices of all sectors, as in standard index price theory, then quite puzzlingly real value added in sector 1 may be affected by changes occurring in sector 2 even if good 2 does not enter the production of good 1, either directly or indirectly.[11]

Productivity indices based on double deflated value added fare no better. Consider the IO matrix $\tilde{A}$ in constant prices where the standard normalization $p(0) = e'$ is not adopted, so that $\tilde{a}_{ij} = p_i(0)a_{ij}/p_j(0)$, for all i, j. Similarly, $\tilde{l}_j = l_j/p_j(0)$ and thus the same relationship holds for labor values: $\tilde{v}_j = v_j/p_j(0)$. Because the investment good sector is homogeneous with respect to inputs and outputs:

$$\pi_1^c = \frac{1 - p_1(0)a_{11}/p_1(0)}{l_1/p_1(0)} = \frac{1 - a_{11}}{l_1/p_1(0)} = \frac{p_1(0)}{v_1},$$

so that relative prices do not distort π_1^c, which coincides with the IO measure. For the consumption good sector, however, a different conclusion holds:

$$\begin{aligned} \pi_2^c &= \frac{1 - p_1(0)a_{12}/p_2(0)}{l_2/p_2(0)} = \frac{p_2(0) - p_1(0)a_{12}}{l_2} \neq \\ \frac{1}{v_2/p_2(0)} &= \frac{1}{(v_1/p_1(0))p_1(0)a_{12}/p_2(0) + l_2/p_2(0)} = \frac{1}{(v_1 a_{12} + l_2)/p_2(0)}. \end{aligned}$$

The numerator of π_2^c depends on relative prices, and thus on their structure and on the base period used. As a result, if $p(0)$ changes, π_2^c can change erratically without any changes in production conditions. To be sure, labor values are also measured relative to output value, but this only means that each time series of labor values is divided by the constant price of the corresponding good, which does not distort the internal structure of the time series itself: for any given j, $1/v_j$ is only rescaled and its growth rate is independent of prices. In general, whereas the indices π_j^c depend on the conceptually dubious double deflated values added, the vector v is derived from the meaningful, volume-oriented double deflated entries of the IO table $\tilde{A}$.

[10] v can be derived even if the assumptions of this paper are relaxed: see Gupta and Steedman (1971) for the treatment of fixed capital and imports, and Flaschel (1983) on joint production.

[11] In general, when output prices change relative to input prices, the single deflation method will detect variations in productivity even if production conditions are unchanged. For related analyses of the sensitivity of the SNA measures to changes in relative prices see Durand (1994), Hart (1996); and Almon (2009).

The previous conclusions can be generalized and made more rigorous, by analyzing alternative approaches in a unified framework, in which some desirable properties of productivity measures are defined ex ante. Let $e_i = (0, ..., 1, ..., 0)'$ be the $i-$th unity base vector. Definition 1 formalizes the notion of *increases* in labor productivity.[12]

Definition 1 (D1)

1. Labor productivity at t has increased *with regard to commodity* i, relative to the base period, if and only if an increase of the net product f by one unit of commodity i demands less labor than in the base period. Formally, let $x_i(t) = (I-A(t))^{-1}e_i$ and let $\ell_i(t) = l(t)x_i(t)$: labor productivity has increased if and only if $\ell_i(t) < \ell_i(0)$.

2. If $\ell(t) \leq \ell(0)$ then labor productivity at t has increased *in the whole economy*, with respect to the base period.

D1 does not aim to capture all aspects of labor productivity, and it only constrains *changes* in productivity. From an epistemological viewpoint, it can be seen as an axiom: whatever else a measure of productivity may do, it should satisfy D1, which sets some *minimal* restrictions on productivity measures. From this perspective, D1 has a number of attractive features. First, it has a firm technological foundation which captures only shifts in productive conditions and efficiency: purely monetary magnitudes are irrelevant and final demand plays only an auxiliary role.[13] This is certainly a desirable property of labor productivity measures, as many authors have argued (e.g. OECD, 2001). Second, by focusing on goods, rather than sectors, D1(1) incorporates the interdependencies between sectors and it allows one to capture the relation between technical change and social welfare. This may seem more controversial, but a similar concern for the role of intermediate inputs and vertical integration actually motivates the use of value-added based - as opposed to gross output based - indices in the mainstream literature (e.g. Schreyer, 2001, p.41ff): they are preferred because they capture interindustry transactions and "provide an indication of the importance of the productivity measurement for the economy as a whole. They indicate how much extra delivery to final demand per unit of primary inputs an industry generates" (Schreyer, 2001, p.42). Third, D1(2) may be deemed rather stringent, especially if n is large, as it requires (weakly) monotonic increases for all goods. From an axiomatic perspective, however, it sets a very weak and intuitive restriction on any productivity measure. This is even more evident if a (neoclassical) notion of productivity as measuring economic welfare is adopted, for in this case D1(2) is analogous to a paretian condition capturing vector-wise improvements in consumption and investment opportunities.

The next result states that D1 characterizes the classical-Marxian measures of labor productivity.

[12]The following notation holds for vector inequalities: for all x, $y \in \mathcal{R}^n$, $x \geqq y$ if and only if $x_i \geqq y_i$, all i; $x \geq y$ if and only if $x_i \geqq y_i$ and $x \neq y$; and $x > y$ if and only if $x_i > y_i$, all i.

[13]The original net product f is irrelevant in D1, thanks to the linearity of the technology.

Proposition 1

> For a given commodity i, $\ell_i(t) < \ell_i(0)$ if and only if $\pi_i^m(t) > \pi_i^m(0)$. Furthermore, if the whole economy is considered $\ell(t) \leq \ell(0)$ if and only if $\pi_i^m(t) \geq \pi_i^m(0)$, for all $i = 1, \ldots, n$, with strict inequality for some i.

Proof:
By the definition of v, $L = \ell e = lx = l(I-A)^{-1}f = vf$. The latter expression implies $l_i(t) = v(t)e_i = v_i(t)$ and the desired result follows. □

In other words, labor productivity with regard to good i increases if and only if the amount of labor directly and indirectly embodied in good i decreases. Further, any index of aggregate labor productivity satisfies D1(2) if and only if it is monotonic in the vector of labor values. Proposition 1 provides theoretical foundations to the classical-Marxian indices as the appropriate indicators of labor productivity. To be sure, one may argue that the indices π_j^m have the disadvantage that they cannot be deduced only from data that characterize sector j, and it is this property that drives Proposition 1. Yet the standard value-added based measures cannot be defined based only on data from sector j, either, even though the dependence on the other sectors is less evident than in π_j^m. It is in fact *impossible* to formulate and interpret nominal value added Y_j – as well as 'real' value added Y_j^s, or Y_j^d – without reference to a price system (even if prices may not appear explicitly, owing to the normalization $p(0) = e'$). SNA measures do depend on the data of the other sectors via the price vector, but – unlike for π_j^m – the sectoral influences are unexplained and depend on the contingent institutional and market conditions of the base year. The rigorous technological foundation which characterizes the classical-Marxian indices is lost. Therefore, it should not be surprising that the standard SNA measures cannot correctly capture labor productivity either at the sectoral or at the aggregate level. This is proved in the following propositions.

Proposition 2 states that the SNA and the classical-Marxian indices of sectoral labor productivity coincide only in a very special case.

Proposition 2

> The equality $\pi_j^c = \pi_j^m = 1/v_j$, for all $j = 1, ..., n$ holds if and only if $\pi_j^c = \pi^c$, for all $j = 1, ..., n$.

Proof:
The result follows immediately noting that $\pi_j^c = \pi^c$, all $j = 1, ..., n$, holds if and only if $e' - e'A = \pi^c l$, or equivalently $(1/\pi^c)e' = l(I-A)^{-1} = v$.□

By Proposition 2, any differences in the two sectoral indices must be examined in relation to sectoral productivity differences. The next result instead shows that the SNA measure of aggregate productivity satisfies D1(2), *if final demand is constant.*

structure \ period	$t=0$		$t=1$	
matrix of intermediate inputs A	0.1	0.3	0.44	0.3
	0.4	0.3	0.1	0.3
labor inputs l	0.4	0.05	0.32	0.05

Table 3: A two-sector economy with profitable capital-using and labor-saving technical change (at constant prices $p(0) = e', w = 1$)

Proposition 3

> Suppose that $f(t) = f(0) = f > 0$. If $v(t) \leq v(0)$ then $\pi^c(t) > \pi^c(0)$. Furthermore, $\pi^c(t) > \pi^c(0)$ if and only if $v(t)f < v(0)f$.

Proof:
The result follows noting that $\pi^c(t) = p(0)f/L(t)$ and that the equality $L(t) = v(t)f$ holds, as shown in Proposition 1.$\square$

In other words, technical change yielding increases in productivity according to D1(2) implies a corresponding change in the SNA macroeconomic measure of labor productivity. Further, the change in technology decreases the expenditure of human labor for the production of a given vector of final demand f. Thus, Proposition 3 suggests that movements in the SNA aggregate measure map changes in the IO indicators, *if final demand is constant*. Yet Proposition 3 does not necessarily hold if final demand varies, nor does it hold at the sectoral level.

Consider the two-sector economy described in table 3, where process 1 is subject to technical change between $t = 0$ and $t = 1$. Let $p(0) = e'$ and assume $w = 1$. First, technical change in sector 1 is *capital-using and labor-saving*, in the sense that it increases the value of intermediate inputs, but it lowers labor costs, at current prices. Second, technical change is *profitable*, because unit costs in sector 1 decrease from 0.9 to 0.86. Third, the SNA sectoral productivity measure *increases* in sector 1 and remains *constant* in sector 2:

$$\pi_1^c(1) \approx 1.44 > \pi_1^c(0) = 1.25, \qquad \pi_2^c(1) = \pi_2^c(0) = 8.$$

Instead, fourth, the classical-Marxian measures, π_1^m, π_2^m, *decrease*:

$$\begin{aligned} \pi_1^m(1) &\approx 1.58 < \pi_1^m(0) \approx 1.70, \qquad \text{and} \\ \pi_1^m(1) &\approx 3.04 < \pi_1^m(0) \approx 3.09. \end{aligned}$$

The technical change described in table 3 leads to a sharp divergence in the standard indices, π_i^c, and the IO indices, π_i^m, which can move in opposite directions. Therefore, by Proposition 1, the example in table 3 proves that the SNA sectoral measures,

π_i^c, do not satisfy D1. Noting that these conclusions can be generalized to n-good economies, they can be summarized in the next Proposition.[14]

Proposition 4

> Suppose that $f(t) = f(0) = f > 0$. For any good i, if $\ell_i(t) < \ell_i(0)$ then $\pi_i^c(t)$ may increase, decrease, or remain constant relative to $\pi_i^c(0)$. Furthermore, it is possible to have $\ell(t) \leq \ell(0)$, but $\pi_i^c(t) \leqq \pi_i^c(0)$, for all i, with strict inequality for at least some i.

In other words, the standard sectoral productivity indices do not satisfy the minimal requirements set out in D1, even under the restrictive assumption of a constant final demand. The shortcomings of the SNA measures π_i^c derive primarily from the fact that they crucially rely on price information and do not properly reflect changes in technology. As a result, they can show increases in productivity in every sector even if the net production possibilities of the economy are deteriorating. Actually, by Proposition 3, the SNA aggregate index π^c does correctly reflect changes in the whole economy whenever final demand is constant, but table 3 shows that π^c and the sectoral measures π_j^c can actually move in opposite directions (in the example, π^c increases), if the sectoral allocation of labor changes appropriately (see equation 1). Hence, the SNA sectoral measures do not provide useful information concerning the sectors leading to movements in aggregate labor productivity.

It is worth stressing that the proof of Proposition 4 is completely general. In table 3, only *profitable* technical change is considered, but this is unnecessary to establish the proposition. It is however theoretically relevant because it shows that the result is not driven by some peculiar, or economically meaningless, combination of parameters. Further, none of the conclusions depends on the assumption of capital-using, labor-saving technical change, and it is easy to construct similar examples with other types of innovations.

Although the previous analysis has focused on sectoral productivity measures, the standard approach to aggregate productivity is also unsatisfactory, and the SNA measure π^c does not satisfy D1(2) in general. To see this, consider again a two-good economy with technical change between $t = 0$ and $t = 1$. At any t, let $L(t) = l(t)x(t)$, so that, by the definition of labor values, $L(t) = v(t)f(t) = v_1(t)f_1(t) + v_2(t)f_2(t)$. Then, dropping time subscripts for the sake of notational simplicity, for a given technology (A, l), the *net product transformation line* is given by:

$$f_2 = (L - v_1 f_1)/v_2 = L - \pi_2^m f_1/\pi_1^m, \quad \text{with} \quad \pi_1^m = 1/v_1, \pi_2^m = 1/v_2.$$

Figure 1 shows that if $\pi_1^m/\pi_2^m \neq p_2(0)/p_1(0)$, there can be a change in final demand from f^0 to f^1, and a simultaneous change in technology (A, l), such that $v(t) \leq v(0)$ and the net product transformation line shifts out, but $\pi^c(0) = p(0)f^0 > \pi^c(1) =$

[14] In table 3 the reciprocal of the direct labor time per unit of output, $1/l_i(t)$, also increases in sector 1 and remains constant in sector 2. Therefore Proposition 4 can be extended to the indices $\pi_i^l(t) = 1/l_i(t)$ which are also sometimes used in the mainstream literature to measure productivity.

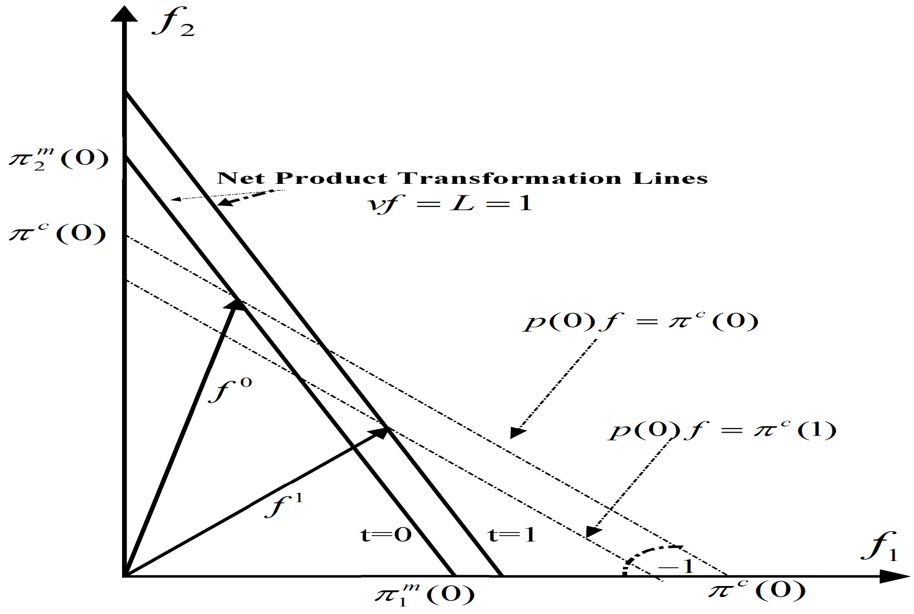

Figure 1: An increase in net production possibilities and a decrease in the conventional measure of aggregate labor productivity ($p(0) = e'$).

$p(0)f^1$. Noting that this argument can be easily generalized to n-good economies, it can be summarized in the next Proposition.

Proposition 5

> Suppose that $f(t) \neq f(0)$. If $\ell(t) \leq \ell(0)$, then $\pi^c(t)$ may increase, decrease, or remain constant relative to $\pi^c(0)$.

Proposition 5 concludes the theoretical analysis of labor productivity measures. The previous results prove that the SNA sectoral measures do not meet the requirement set out in D1(1). By Proposition 5, the SNA aggregate measure π^c does not satisfy the very weak condition in D1(2), either: it can detect a decline in productivity in the economy even if the net production possibilities unambiguously increase. Neither the sectoral nor the aggregate SNA productivity measures are adequate to capture shifts in technology and efficiency. Besides, Propositions 4 and 5 imply that, contrary to the received view, value added based measures are also inadequate to capture economic welfare, for an expansion of the net production possibilities increases social welfare.[15] Again, the problem with standard measures is that they are affected by changes in relative prices and final demand, independently from technical conditions. This suggests that the notion of labor content is essential to capture labor productivity, and the law of increasing labor productivity cannot be properly understood unless the LDLC is formulated.

[15]It is worth noting that Proposition 5 also applies to measures based on single deflated aggregate value added.

3 Technical change and the law of decreasing labor content

Section 2 proves that the classical-Marxian indices $\pi_j^m = 1/v_j$ represent the only theoretically sound measures of labor productivity, which capture both its technological and its welfare aspects, and thus the LDLC is crucial in order to understand the dynamics of a capitalist economy. In this section, some propositions are derived on the relationship between prices and productivity, by analyzing the conditions under which profitable innovations lower labor values.

Technologies are now more generally described by a 3-tuple (K, A, l), where K is a stock matrix whose generic entry K_{ij} denotes the amount of commodity i that is tied up (as inventory) in the production of commodity j.[16] Everything is expressed again per unit of commodity output. For the sake of simplicity, it is assumed that the output matrix is equal to the identity matrix, I, but all the results can be extended to technologies with multiple activities as well as joint production, provided the framework outlined in Flaschel (1983) to define labor content is adopted.

In order to avoid a number of uninteresting technicalities, and with no loss of generality, the following standard assumption is made on technology.

Assumption 1 (A1)

For any technology (K, A, l), A is productive and indecomposable, and $l > 0$.

Assumption 1 has two main implications. First, in this paper technical changes in the various sectors of the economy are considered separately and are assumed to occur in individual sectors.[17] Yet (A1) implies that the effects of sectoral innovations extend throughout the economy. Second, let $p_{wj} = p_j/w$ be the price of good j in terms of the wage unit, so that $p_w = p/w$ is the vector of wage prices. In what follows, it is not assumed that p_w represents long-run production prices: it may well be a vector of (normalized) market prices. By (A1), the Leontief inverse exists and is strictly positive, and so the next Lemma immediately follows, which extends a well-known property of prices of production with uniform profit rates to *any* vector of wage prices which allows for positive profits.

Lemma 1

Assume (A1). For any p_w such that $p_w > p_w A + l$, it follows that $p_w > v = l(I - A)^{-1} > 0$.

Thus, labor commanded prices are a useful upper estimate for embodied labor costs even if no restrictive assumption on uniform profit rates is made. It is worth noting that in the economy with fixed capital, the inequality $p_w > p_w A + l$ is a weak

[16] For a detailed explanation of the treatment of fixed capital see Bródy (1970) and Flaschel et al. (2010). In this section, it is still assumed that the matrix of depreciation of fixed capital is equal to zero, i.e. $A^\delta = 0$, but all the results can be extended to the matrix $\overline{A} = A + A^\delta$, and the corresponding labor values.

[17] The reader is referred to Bródy (1970) for the details of the prerequisites for an analysis of technical change in a Leontief IO system.

condition and it is only necessary for positive profits to occur in all sectors.

Let r_j be the profit rate on capital advanced in sector j. Definition 2 distinguishes various forms of technical change, depending on their effect on unit costs and on labor values, and on whether they tend to substitute labor for capital, or viceversa.

Definition 2

1. Technical change $(K_j, A_j, l_j) \mapsto (K_j^*, A_j^*, l_j^*)$ is *profitable* if and only if, at initially given prices p_w such that $p_{wj} = r_j p_w K_j + p_w A_j + l_j$ and $r_j > 0$:

$$r_j p_w K_j + p_w A_j + l_j > r_j p_w K_j^* + p_w A_j^* + l_j^*.$$

2. Technical change $(K_j, A_j, l_j) \mapsto (K_j^*, A_j^*, l_j^*)$ is *progressive* if and only if

$$v = vA + l > v^* A^* + l^* = v^*.$$

   Similarly, technical change is *regressive* if and only if $v < v^*$.

3. Technical change $(K_j, A_j, l_j) \mapsto (K_j^*, A_j^*, l_j^*)$ is: (i) *fixed-capital using (KU)* if and only if $K_j \leq K_j^*$ and *fixed-capital saving (KS)* if and only if $K_j \geq K_j^*$; (ii) *circulating capital using (CU)* if and only if $A_j \leq A_j^*$ and *circulating capital saving (CS)* if and only if $A_j \geq A_j^*$; and (iii) *labor using (LU)* if and only if $l_j \leq l_j^*$ and *labor saving (LS)* if and only if $l_j \geq l_j^*$.

Definition 2 generalizes the definitions in Roemer (1977) to economies with capital tied up in production *and* to any vector of wage prices, p_w: profits are treated as a mere residual and no assumptions are made on the uniformity of profit rates or on the determination of p_w.[18] It is worth noting that in Definition 2(3), innovations are defined in physical terms and they are monotonic in all produced inputs. Although this may seem a stringent condition in an n-good space, it is in line with the definitions of capital-using (or capital-saving) technical changes used in policy debates and with intuitive notions of the mechanization process that has characterized much of capitalist development. However, as argued below, the main results of this paper can be extended to more general types of technical change.[19]

Next, define the following auxiliary intermediate input matrix:

$$A^{*+} = max\{A^*, A\} \geq A^*.$$

If j is the sector subject to technical change, the auxiliary matrix A^{*+} is CU with respect to A if $A_{ij}^* > A_{ij}$, for at least some i. Based on A^{*+}, a specific class of innovations is considered below and the following assumption is made:

[18] In Roemer (1977), cost-reducing innovations are called *viable*, but the notion of *profitability* more explicitly conveys the idea of monetary, rather than physical, magnitudes.

[19] As Roemer (1977, p.410) notes, it is also not restrictive to focus on technical changes where all labor values change in the same direction. If technical change occurs in one sector at a time, this will not produce value changes in opposite directions in different sectors.

Assumption 2 (A2)

For any profitable KU–LS technical change $(K_j, A_j, l_j) \mapsto (K_j^*, A_j^*, l_j^*)$, the following inequality holds: $p_w A_j + l_j > p_w A_j^{*+} + l_j^*$.

Assumption 2 states that the main part of the cost-reduction process occurs via changes in the capital that is tied up in production, which allows for significant reductions in labor costs. Instead, changes in intermediate inputs are unsystematic and secondary, and therefore profitable even if the auxiliary matrix A^{*+} is considered. (A2) rules out only secondary profitable technical changes, and yields no major loss of generality in the analysis of LS innovations. Then, the first key result on technical change in general economies with fixed capital can be derived.

Theorem 1

Assume (A1). Let $p_w > p_w A + l$. Under (A2), all KU–LS profitable technical changes are progressive. However, there are KU–LS progressive technical changes which are not profitable.

Theorem 1 is quite general and by no means obvious. For it proves that cost-reducing innovations that substitute fixed capital for labor are progressive, even if no stringent assumption is made concerning the effect of technical change on intermediate inputs. Therefore, in general, LS innovations will reduce the labor content of goods and increase net production possibilities. Yet profitable KU-LS innovations do not fully exploit the potential of technical progress to increase labor productivity. For there exist feasible technologies that will not be adopted by capitalists that would yield social welfare improvements by increasing net production possibilities.

The proof that profitable KU-LS innovations increase consumption and investment opportunities has relevant implications for the LDLC and the understanding of capitalist economies. For it derives a systematic relationship between certain forms of technical change, profit maximizing behavior, and labor values. Empirically, one may conjecture that distributive conflict and increasing wages have introduced a bias in the direction of technical change towards KU-LS changes that may partly explain the secular increase in labor productivity observed in capitalist economies. Theoretically, although class conflict is not analyzed in this paper, one may construct a plausible scenario in which wage increases induce KU-LS technical change, and so a decrease in labor content. This argument may provide microfoundations to the LDLC, which need not be based on - but, of course, can be supplemented by - probabilistic considerations. The price implications of technical changes may indeed be chaotic, as Farjoun and Machover argued, but the quantity implications investigated in this paper are independent of such chaotic behavior.

The result in Theorem 1, however, cannot be extended to other types of innovations. Theorem 2 proves that there may be profitable KS-LU innovations that reduce the economy's net production possibilities, and thus social welfare.

Theorem 2

Assume (A1)-(A2). Let $p_w > p_w A + l$. All KS-LU progressive technical changes are weakly profitable. However, there are KS–LU profitable technical changes which are not progressive. More precisely, a technical change is progressive if and only if $v_j > vA_j^* + l_j^*$.

Together with Theorem 1, Theorem 2 provides a full description of technical change in a capitalist economy with capital tied up in production. Theorem 2 characterizes the conditions under which KS-LU progressive technical change occurs: KS-LU innovations are progressive, and thus increase social welfare, if and only if they reduce the labor content of a commodity in terms of the *old* labor values. Thus, Theorem 2 implies that the problematic situation with respect to technological regress is, generally speaking, the labor-using case. To be specific, labor productivity falls if the following inequalities hold simultaneously

$$r_j p_w K_j + p_w A_j + l_j > r_j p_w K_j^* + p_w A_j^* + l_j^*, \quad l_j \leq l_j^*, \quad v_j < vA_j^* + l_j^*.$$

In Theorem 2, labor values move all in the same direction, i.e., if labor productivity falls in some sectors, then it falls in all of them. Therefore it is unambiguously clear whether the set of net production possibilities expands or contracts. In the KS-LU case with $v_j < vA_j^* + l_j^*$, it contracts, as the labor contents of all commodities rise. Hence capitalist choices leading to KS-LU technical change may have adverse effects on economic development, since they may undermine the LDLC and thus decrease consumption and investment opportunities, and periods characterized by KS-LU technical change may be plagued by productivity slowdowns.

Theorems 1 and 2 generalize Roemer's (1977) results in economies with circulating capital and they identify some systematic connections "between the visible and the invisible - between price and labour-content" (Farjoun and Machover, 1983, p.84). As noted above, given the KU-LS nature of technical progress in actual capitalist economies, Theorem 1 sheds some light on the LDLC, by identifying a link between profit-driven individual actions and the behavior of labor content. Instead, Theorem 2 can be interpreted as identifying another (potential) failure of the invisible hand. The case $v_j = vA_j^* + l_j^*$ is the dividing line that separates strictly falling from strictly rising labor contents. This dividing line is expressed in terms of labor values, and thus it is not visible to agents in the economy, who take their profit-maximizing decisions based on price magnitudes. As a result, individually rational decisions may lead to socially suboptimal outcomes.

As a final remark, it is worth stressing again the generality of Theorems 1 and 2. Although the innovations considered are defined in physical terms, consistently with Definition 2(3), it is possible to derive both results using weaker notions of technical change based on the cost of fixed capital $p_w K_j$.

4 Productivity measures and the LDLC: Empirical results

This section provides an empirical illustration of the main concepts and propositions discussed above. For this purpose, the IO dataset constructed by Kalmbach et al. (2005) in their study of the German economy (1991 – 2000) is considered. Kalmbach et al. group the 71 original sectors into seven macro-sectors. They divide the industrial sector into agriculture, manufacturing, and construction. Within manufacturing itself, they further distinguish more traditional industries from the so-called '*export core*' (a crucial subsector in an export-oriented country like Germany), which comprises the four single production sectors with the highest exports: chemical, pharmaceuticals, machinery, and motor vehicles. They also distinguish between three main types of services: business-related services, consumer services, and social services. For their aggregation, Kalmbach et al. adopt a broad definition of business-related services by including wholesale trade, communications, finance, leasing, computer and related services, research and development services, in addition to business-related services *in a narrow sense*. Consumer services instead include: retail trade, repair, transport, insurance, real estate services, and personal services. Table 4 summarizes the seven (macro) sectors thus obtained and the sectoral output shares (in percentages, for the year 2000).

1 :	Agriculture	1.33
2 :	Manufacturing, the export core	12.37
3 :	Other manufacturing	22.55
4 :	Construction	6.29
5 :	Business-related services	21.36
6 :	Consumer services	23.35
7 :	Social services	12.75

Table 4: The 7-sectoral structure of the economy.

The technological coefficients of the 7-sectoral aggregation are reported in table 4, which shows the intermediate IO matrix A of the German economy for the year 1995 per 10^6 Euro of output value. The double-deflated coefficients $\tilde{a}_{ij}$ are used to characterize the entries of A. There are also (not shown) a depreciation matrix, A^δ, a fixed capital matrix, K, and a vector of labor coefficients, l.

In order to calculate the labor values of the seven sectors, the formula $v = l(I - A - A^\delta)^{-1}$ is used in each of the ten years under consideration. The classical-Marxian measures, π_j^m, are then derived as the reciprocal of the entries of v. Instead, dividing each of the 70 real value added items (per 10^6 Euro output value) by the corresponding labor coefficient (per 10^6 Euro output value) one obtains the conventional measures of labor productivity, π_j^c. The time series of the two productivity measures for six of the seven sectors are shown in figure 2.[20]

The empirical evidence confirms the main conclusions of the paper. Concerning the measurement of labor productivity, the data shows that the two series π_j^m, π_j^c are

[20]Social services are omitted because they are subject to processes that in general are not determined by profit-maximizing firms. Details of the computations of the time series of the two indices are available from the authors upon request.

	1	2	3	4	5	6	7
1 :	0.030	0.000	0.047	0.000	0.000	0.002	0.002
2 :	0.081	0.241	0.050	0.021	0.003	0.008	0.014
3 :	0.159	0.226	0.338	0.286	0.030	0.060	0.065
4 :	0.010	0.005	0.009	0.020	0.007	0.034	0.020
5 :	0.137	0.107	0.126	0.088	0.291	0.118	0.080
6 :	0.032	0.044	0.045	0.100	0.071	0.139	0.044
7 :	0.034	0.008	0.013	0.007	0.009	0.014	0.025

Table 5: Technological coefficients of the 7-sectoral aggregation (FRG,1995).

very different, as expected from the analysis in section 2. First of all, apart from the remarkable exception of sector 3, the *levels* of the two measures are sharply different in all sectors and in virtually every year of the sample, with no recognizable overall pattern (in some sectors the standard measures are higher than the IO indices, but the opposite happens in other sectors) and with differences even in the relative ranking of sectors in terms of their labor productivity. By Proposition 2 above, this is to be expected, given the wide sectoral differences in productivity. Secondly, even the *qualitative behavior* of the two indices over time is very different, as expected from Proposition 4. In sector 4, both the *trend* and the year-on-year behavior of the two variables are markedly different. The Marxian measure of productivity has risen over time, while the conventional SNA measure shows a sharp increase immediately after the German reunification but a significant decline thereafter. Even setting aside the construction sector (where measurement problems may play a role), in various instances the two indices provide opposite verdicts concerning the *direction of change* of labor productivity over time. Particularly striking examples are sector 2: 1995-96 (and to a lesser extent 1997-98); sector 3: 1994-95 (and to a lesser extent 1997-98); sector 5: 1993-1995; and last but not least sector 6: 1997-2000, which is characterized by a similar, if less pronounced, overall pattern as sector 4.[21]

In sum, the *theoretical* differences between the two measures do give rise to significant *empirical* discrepancies. The standard SNA indices π_j^c lack theoretical foundations, as argued in section 2 above, and they can also be very misleading in empirical analysis, as the evidence in figure 2 forcefully shows.

Concerning the relation between prices, profits, and labor values, all the tables in figure 2 show that the LDLC holds for the German economy (1991-2000). The classical-Marxian indices of labor productivity show a clear upward trend in all sectors. This result seems robust and it is consistent with the findings of previ-

[21]It is worth noting in passing that sector 7, social services (not shown in figure 2), has a similar pattern as sector 6.

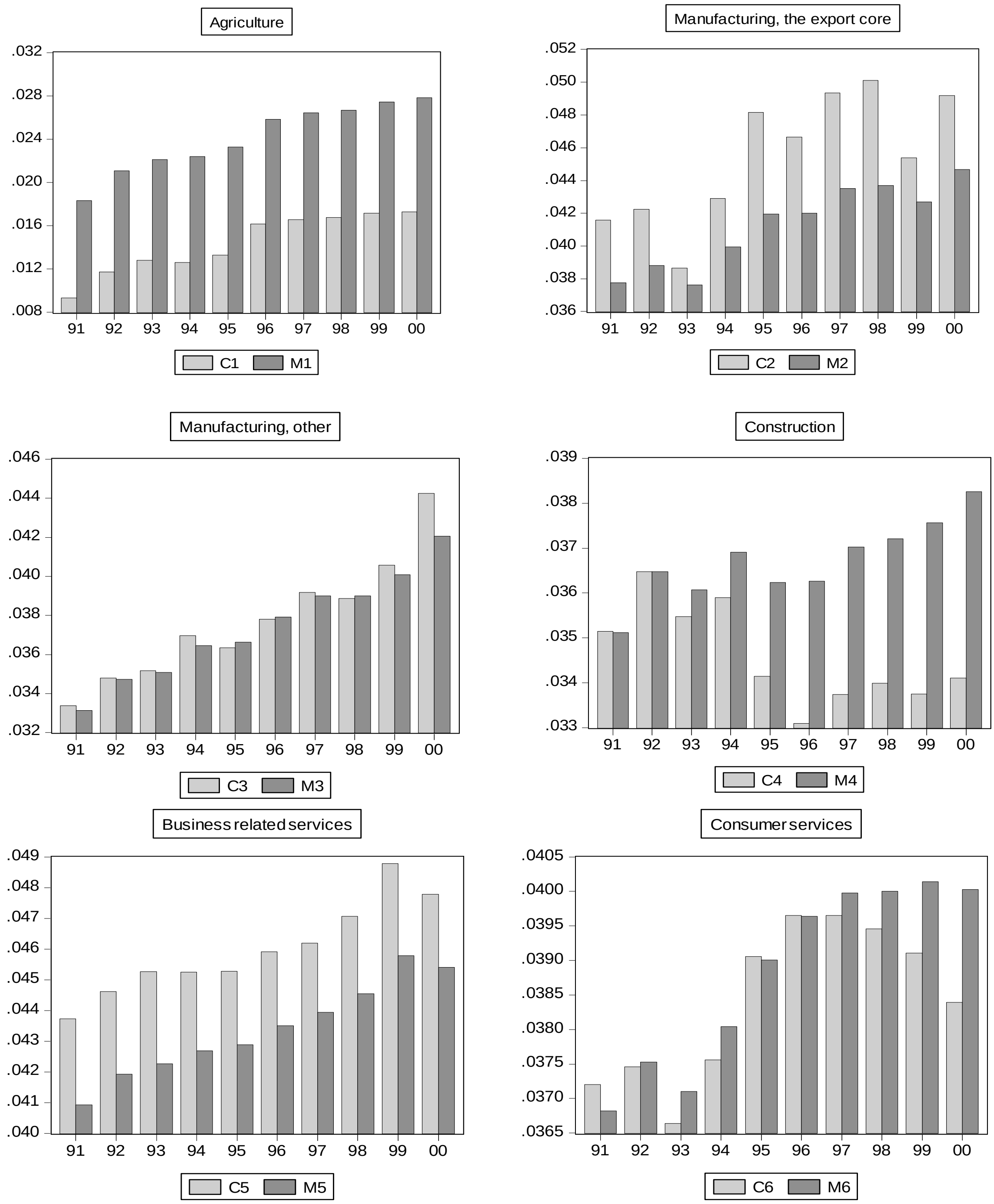

Figure 2: Comparing conventional and Marxian labor productivity indices: $\pi_j^c, 1/v_j$

ous studies (see, for example, Gupta and Steedman, 1971; Wolff, 1985; de Juan and Febrero; 2000), even though only few contributions explicitly focus on sectoral productivities.

5 Conclusions

This paper analyzes the *law of decreasing labor content* (LDLC) originally formulated by Farjoun and Machover (1983). First, the issue of the relevance of the LDLC is addressed. It is argued that the IO indices based on the classical-Marxian labor values are the only theoretically sound measures of labor productivity. Instead conventional indices based on real value added per worker are theoretically questionable and less reliable empirically. The notion of labor content is necessary to understand labor productivity and the LDLC is central in order to understand the dynamics of capitalist economies. Indeed, "Without the concept of labour-content, economic theory would be condemned to scratching the surface of phenomena, and would be unable to consider, let alone explain, certain basic tendencies of the capitalist mode of production" (Farjoun and Machover, 1983, p.97).

Second, the dynamics of labor productivity in capitalist economies is analyzed in a general linear model with fixed capital. It is proved that capitalists' maximizing behavior is neither necessary nor sufficient for the implementation of productivity-enhancing and welfare improving innovations. Further, it is shown that the type of capital-using labor-saving profitable innovations that have characterized capitalist economies tend to lower labor values, which may provide a deterministic foundation for the LDLC. Some empirical evidence is also provided, which shows that the LDLC holds in the German economy after the reunification.

The analysis in this paper can be extended in various directions. From the empirical viewpoint, the discussion in section 4 is preliminary and only a first step towards a comprehensive investigation of alternative productivity measures. Further, a systematic econometric investigation of the theoretical relations between technical change and productivity explored in section 3 would be interesting.

From the viewpoint of economic theory, the main conclusions have some broad implications that may be worth exploring further. The analysis of productivity measures sheds some new light on old debates between classical and neoclassical approaches. As Glyn (2004, p.6) aptly noted, "there has always been a tension between the classical notion of productivity which is tied directly to the conditions of production ... and the neoclassical view where productivity is measured by the appropriation of income." This paper shows that the neoclassical value-added based measures are indeed inappropriate to capture efficiency and technological conditions, and the classical, IO perspective is the appropriate one. Interestingly, however, it is not clear that value-added measures are satisfactory as indicators of income appropriation and economic welfare, either.

Further, the analysis suggests that the strength of the labor theory of value may not lie primarily in the prediction of price movements (even though total labor costs are an important – if not the central – component in actual price changes when measured in terms of the wage-unit). Instead, the so-called "Dual System Approach" to the Marxian labor theory of value may be more relevant, whereby labor values and (actual or production) prices are considered side by side, and labor values are important as part of a system of national accounts. In reciprocal form, they provide measures of labor productivity that identify the implications of technical change, a central phenomenon in capitalist economies. This interpretation can be traced back to Marx himself and his discussion of the reciprocal relationship between labor values and the measurement of labor productivity in *Capital I* (Chapter 1, section 1). The exploration of these implications must be left here for future research, however.

6 Appendix: Proofs of Theorems 1 and 2

Proof of Theorem 1:

In order to prove the first part of the statement we need to consider three cases.
Case 1. Suppose that $A_j^* = A_j$, so that $A^* = A$. Then by (A2) it follows that $l \geq l^*$ and by (A1) $v^* < v$.

Case 2. Suppose that $A_j^* \leq A_j$. Then, by definition $\left(A_j^*, l_j^*\right)$ is CS-LS with respect to (A_j, l_j), according to Definition 2(3), and it is immediate to show that $v^* < v$.

Case 3. Suppose that $A_{ij}^* > A_{ij}$, for at least some i. Then, consider the auxiliary matrix A^{*+} and define the vector of auxiliary labor values $v^{*+} = v^{*+}A^{*+} + l^*$. Note that, according to Definition 2(3), $\left(A_j^{*+}, l_j^*\right)$ is CU-LS with respect to (A_j, l_j), and by (A2) $p_w A_j + l_j > p_w A_j^{*+} + l_j^*$, or equivalently, $p_w(A_j^{*+} - A) - (l - l^*) \leq 0$.

Next, by Lemma 1, we know that $0 < v < p_w$, so that the latter inequality implies

$$v(A^{*+} - A) - (l - l^*) \leq 0,$$

and thus

$$vA^{*+} + l^* \leq vA + l = v.$$

By recursive application of the latter inequality, we get:

$$v(t+1) = v(t)A^{*+} + l^* \leq v(t),$$

$t = 0, 1, 2, 3, \ldots$, with $v(0) = v$. This sequence is bounded below and monotonically decreasing and thus it converges to the vector

$$v(\infty)A^{*+} + l^* = v(\infty) = v^{*+}.$$

Therefore, by (A1) it follows that $v^{*+} < v$, so that $\left(A_j^{*+}, l_j^*\right)$ is progressive with respect to (A_j, l_j). Finally, note that by definition $\left(A_j^*, l_j^*\right)$ is CS-LS with respect to $\left(A_j^{*+}, l_j^*\right)$, according to Definition 2(3) and therefore it is immediate to prove that $v^* < v^{*+}$, which implies $v > v^{*+} > v^*$.

The second part of the statement follows noting that there may be CU-LS technical changes with $v^* < v$, such that $p_w A_j + l_j \leqq p_w A_j^* + l_j^*$ at the initial price vector $p_w > p_w A + l$, because the latter is not proportional to v in general, and noting that for KU-LS technical changes if $p_w A_j + l_j \leqq p_w A_j^* + l_j^*$ then $r_j p_w K_j + p_w A_j + l_j < r_j p_w K_j^* + p_w A_j^* + l_j^*$.$\square$

Remark: The recursive argument used in the proof of case 3 can be modified to provide an alternative demonstration of Proposition 8 in Roemer (1977).

Proof of Theorem 2:

1. Consider KS-LU progressive technical change $(K_j, A_j, l_j) \rightarrow \left(K_j^*, A_j^*, l_j^*\right)$. If $p_w A_j + l_j \geqq p_w A_j^* + l_j^*$, then the desired result immediately follows noting that technical change is KS, so that $K_j \geq K_j^*$ and therefore $r_j p_w K_j > r_j p_w K_j^*$ at initial prices p_w such that $p_w > p_w A + l$. Therefore suppose $p_w A_j + l_j < p_w A_j^* + l_j^*$. Since technical change is progressive, then by Lemma 1 $p_w > v > v^*$. The latter inequalities imply that $p_w > p_w A^* + l^*$. Suppose, by way of contradiction, that $p_{wj} = r_j p_w K_j + p_w A_j + l_j < r_j p_w K_j^* + p_w A_j^* + l_j^*$. The latter inequality implies that the KU-LS technical change $\left(K_j^*, A_j^*, l_j^*\right) \rightarrow (K_j, A_j, l_j)$ is profitable and therefore, since the premises of Theorem 1 are satisfied, it is progressive so that $v^* > v$, a contradiction. Therefore, we have $p_{wj} = r_j p_w K_j + p_w A_j + l_j \geqq r_j p_w K_j^* + p_w A_j^* + l_j^*$.

2. In order to prove the second part of the statement, note that if KS-LU technical change $(K_j, A_j, l_j) \rightarrow \left(K_j^*, A_j^*, l_j^*\right)$ is profitable, and thus $r_j p_w K_j + p_w A_j + l_j > r_j p_w K_j^* + p_w A_j^* + l_j^*$, this has no implication on the inequality $v_j \gtreqless v A_j^* + l_j^*$. Then, we prove that technical change is progressive if and only if $v_j > v A_j^* + l_j^*$.

First, note that $v_j > v A_j^* + l_j^*$ implies $vA^* + l^* \leq vA + l = v$, and therefore it is possible to construct an infinite sequence

$$v(t+1) = v(t)A^* + l^* \leq v(t),\ t = 0, 1, 2, 3, \ldots,$$

with $v(0) = v$, which is monotonically decreasing, and bounded below, and thus converges to $v(\infty)A^* + l^* = v(\infty) = v^*$, $v^* > 0$. By (A1) it follows that $v > v^*$.

Next, note that if $v_j = v A_j^* + l_j^*$, technical change is neither progressive nor regressive. Finally, suppose $v_j < v A_j^* + l_j^*$. Then $v \leq vA^* + l^*$ and we can consider the following monotonically increasing sequence

$$v(t) \leq v(t)A^* + l^* = v(t+1),\ t = 0, 1, 2, 3, \ldots,$$

with $v(0) = v$. By Lemma 1, $v < p_w$ and by profitability it follows that $p_w A^* + l^* \leq p_w$. Therefore:

$$v(t) \leq v(t)A^* + l^* = v(t+1) < p_w A^* + l^* \leq p_w,\ t = 0, 1, 2, 3, \ldots,$$

so that the sequence is bounded above by the vector p_w, and therefore it converges to:

$$v(\infty) = v(\infty)A^* + l^* = v^*, \quad v^* > 0.$$

By (A1) $v < v^*$ must hold.$\square$

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