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Abstract

We present a game-theoretic study of the selfish behavior of TCP users when they are allowed to use multiple concurrent TCP connections so as to maximize their goodputs or other utility functions. We refer to this as the TCP connection game. A central question we ask is whether there is a Nash Equilibrium in such a game, and if it exists, whether the network operates efficiently at such a Nash Equilibrium. Combined with the well known PFTK TCP Model [12], we study this question for three utility functions that differ in how they capture user behavior. The bad news is that the loss of efficiency or price of anarchy can be arbitrarily large if users have no resource limitations and are not socially responsible. The good news is that, if either of these two factors is considered, efficiency loss is bounded. This may partly explain why there will be no congestion collapse if many users use multiple connections.

1. Introduction

The conventional wisdom is that the stability of the Internet is due to TCP's congestion control mechanisms. For example, [1] uses a game-theoretic study to show that TCP-Reno and DropTail buffer can make the network operate efficiently when end points are allowed to adjust the increase and decrease parameters of TCP. On the other hand, [10] argues that both network and user behavior should be considered regarding the stability issue of the Internet. This paper is an attempt to understand the impact of user's behavior on the efficiency of the Internet.

Using a game-theoretic framework, we evaluate the impact of greedy behavior of users when allowed to open multiple TCP connections. One of our motivations comes from the observation of the trend that more and more users use some software agents (e.g.,FlashGet [3]) for concurrent downloading in order to accelerate file transfers [7]. Specifically, we are interested in a scenario in which a number of users compete for the capacity of a single bottleneck link. Users have infinite amounts of data to send, and they are allowed to open a number of concurrent connections. This scenario can be modelled as a non-cooperative game in which players are individual users. The strategy of each player is the number of TCP connections. Each player tries to maximize its own utility. We call this game the *TCP Connection Game*.

For this type of general game, we study three specific games that differ from each other in their utility functions. In *game 1*, a user's utility function corresponds to the long-term average goodput (packets per second transferred by the bottleneck link without being discarded). In *game 2*, users take into account the potential cost incurred on the system and on themselves. This cost is assumed proportional to the aggregate sending rate of all connections opened by all users. The cost incurred by each user is then the aggregate sending rate of all connections opened by this user. This cost not only can be interpreted as the packet sending cost for a user, but also the system-wide network resource consumed by the offered traffic. Thus in this sense, a user concerned with this cost can also be thought of as being socially responsible. In *game 3*, in addition to the packet sending cost, we introduce another term specific only to users and that accounts for the cost of maintaining open connections. We also allow users to have different computation powers. A more powerful user has more computation resource to support more TCP connections. For all three games, the Nash Equilibrium (NE) can be thought of as a combination of the number of connections of all users, at which no user can benefit from increasing or decreasing its number of connections. In this paper, we only study pure strategy NE.

We are interested in the following questions. Do there exist Nash Equilibria (NE) for these TCP connection games? If so, what is the loss of efficiency and price of anarchy of the network operating at a NE? How can the behaviors of users potentially affect the efficiency of the NE? For example, how can the socially responsible behavior of users affect the efficiency of the NE? Are users treated fairly at NE? Are NEs stable in the sense that any deviation from NE will converge back to NE?

We find that, in game 1, when users do not have any resource constraints and are not socially responsible, the loss of efficiency or the price of anarchy can be arbitrarily large. This is in contrast to the conclusion in [1] that the network operates efficiently with TCP-Reno loss recovery mechanism and DropTail queue even when users are capable of freely choosing additive increase and multiplicative decrease parameters of TCP. However, we find that in games 2 and 3, the efficiency loss is bounded if users are resource constrained and socially responsible. We also show that there exists a unique NE for game 2 and that it is locally stable, namely, any small deviation from NE will eventually converge back to NE. And, we have observed that it is very likely that the NE is globally stable as well. We also observe that integer NEs very likely exist for the real case that users are only allowed to open an integer number of connections. Last, in game 3, we show that a user with greater computation power is able to obtain greater goodput at the NE than a user with smaller computation power.

Related work. [1] studies a class of TCP games in which each user controls a single TCP connection, and the strategy of each user is a pair of values corresponding to the additive increase parameter α and multiplicative decrease parameter β. Our work differs from their work in that we let users use standard TCP but allow them to choose the number of concurrent connections. Congestion collapse is studied in [10] through a model of the interaction between network and user behavior. In [7], the authors study the impact of concurrent downloading on the fairness and system's transient behavior. In our game-theoretic study, we investigate the effect of the number of connections on TCP performance. Regarding this subject, there are also several related works, such as [13] and [9]. There are many TCP modelling works. Among them, we choose the known and the most accurate PFTK TCP model [12] as the basis of our analysis.

The rest of the paper is organized as follows. System optimization problem is addressed in Section 2. In Section 3, we study game 1 in which users are only interested in maximizing their goodput. In Section 4, we introduce the packet sending cost and social responsibility into user's utility function. Section 5 studies game 3 in which both cost and computation power are considered. We present the simulation study with NS in Section 6, and conclude the paper in Section 7.

2. System Optimization

In this section, we first define the TCP connection game, with specific utility function definitions left for the following sections. Then we address the system optimization problem, which is applicable to all utility functions studied in this paper.

Formally, in the TCP connection game, there are m ($m \geq 2$) TCP users with different Round Trip Time (RTT) R_i competing for the capacity C of a bottleneck link. Individual users are treated as players. A strategy n_i available to a user i is a feasible number of connections he/she can open concurrently. In practice, n_i takes positive integer values. We will first consider the case where n_i is a real-valued number, then we discuss the case where n_i is a positive integer number. Let S_i denote the feasible strategy set of player i, then $n_i \in S_i$. And the feasible strategy space of this game is $S = S_1 \times S_2 \cdots \times S_m$. Then, a feasible strategy tuple is a m-dimension vector $\mathbf{n} = (n_1, n_2, ..., n_m) \in S$. The objective of each player is to maximize its utility U_i by adjusting n_i. Various utility functions U_i are defined in the following sections to capture different user behaviors. Nash Equilibrium $\mathbf{n}_{ne}$ or $\mathbf{n}^* = (n_1^*, ..., n_m^*)$ [1] is defined as

$$n_i^* = \text{argmax}_{n_i \in S_i} U_i(n_1^*, n_2^*, ..., n_i, ..., n_m^*), \forall i$$

In the TCP connection game, from a system point of view, the aggregate goodput of all connections of all players always equals the bottleneck link capacity. Thus, there is nothing to be optimized regarding the aggregate goodput. However, we note that there is a cost associated with this aggregate goodput. For each user i, each connection will have a sending rate or offered rate B_i. Then, $\sum_{i=1}^m n_i B_i$, the aggregate of all these offered rates, drives the bottleneck link to full utilization. The larger $\sum_{i=1}^m n_i B_i$ is, the more network resource is utilized. Thus, we can think of B_i as a "cost" from the system's point of view and $\sum_{i=1}^m n_i B_i$ as an aggregate cost Φ. Thus, to get the highest efficiency, the system's optimization objective is to minimize this total cost while maintaining the bottleneck link fully utilized.

In the following, we first introduce the known TCP sending rate model and its related per-connection goodput model in [12]. Throughout this paper, we will base our analysis on these two models.

The best known TCP sending rate model, full PFTK TCP model relating TCP sending rate B_i to loss rate p and RTT R_i, is given in [12], but it is too complicated for analysis. Therefore, we use a simplified version (recommended in the TFRC standard proposal [4]) given as

$$B_i = 1/(\mu R_i \sqrt{p} + T_{0,i}\nu(p^{3/2} + 32p^{7/2})) \qquad (1)$$

where $\mu = \sqrt{2b/3}$, $\nu = 3/2\sqrt{3b/2}$, $b = 1$ or 2, and $T_{0,i} = 4R_i$.

For per-connection goodput, we assume that the expected window size W of each connection is the same for all flows going through a congested bottleneck link [12], because we expect all connections to incur the same loss rate p at the bottleneck link queue. Let $\bar{W}$ denote this common window size. Then, the per-connection goodput of player i will be $G_i = \bar{W}/R_i$. Suppose that player i has n_i connections. Then, based on the bottleneck principle, the sum of goodputs of all connections of all players equals the link capacity C, i.e., $\sum_{i=1}^m n_i G_i = C$. Then, we have $\bar{W} = C/(\sum_{i=1}^m n_i/R_i)$ and $G_i = (C/R_i)/(\sum_{i=1}^m n_i/R_i)$.

We have the following formulation of the system optimization problem:

$$\min_{\mathbf{n}} \Phi = \sum_{i=1}^m n_i B_i \qquad (2)$$

subject to

$$B_i = 1/(\mu R_i \sqrt{p} + T_{0,i}\nu(p^{3/2} + 32p^{7/2})) \qquad (3)$$

$$B_i(1-p) = (C/R_i)/(\textstyle\sum_{j=1}^m n_j/R_j), \forall i \qquad (4)$$

$$n_i \in [1, ..., n_i^{max}]; m \geq 2; \mathbf{n} = \{n_1, ..., n_m\} \qquad (5)$$

Note, (4) indicates that per-connection goodput of a user is the product of its per-connection sending rate and the probability of successful transfer. Regarding the optimal operating point of the whole system, we have the following result.

Theorem 1 *In the TCP connection game, the system optimal cost is uniquely achieved at* $\mathbf{n}_{opt} = (1, 1, ..., 1)$.

Proof: First, we transform the objective function into a simpler form: $\Phi = C/(1-p)$. It is easy to see that we need to find the minimal feasible p to minimize Φ.

Note that p is a function of $\mathbf{n}$ (see (4)). If we take $T_{0,i} = 4R_i$, as recommended in [4], and let $\bar{\phi}(p) = \mu\sqrt{p} + 4\nu(p^{3/2} + 32p^{7/2})$, then (4) can be rewritten as $F(p, \mathbf{n}) =$

[1]For notational convenience, we use both $\mathbf{n}^*$ and $\mathbf{n}_{ne}$ interchangeably to denote the number of connections at NE. Similarly we use both p^* and p_{ne} to denote the loss rate at NE.

$(1-p)(\sum_{j=1}^{m} n_j/R_j) - C\bar{\phi} = 0$. Solving for p given a specific $\mathbf{n}$ is actually equivalent to solving the above equation. First note that,

$$\lim_{p\to 0} F = \sum_{j=1}^{m} n_j/R_j > 0; \lim_{p\to 1} F = -C\cdot\bar{\phi}(1) < 0$$

Furthermore, $F(p,\mathbf{n})$ is a strictly monotonic decreasing function of p since $\partial F/\partial p = -\sum_{j=1}^{m} n_j/R_j - C\bar{\phi} < 0$.

Thus, there must be a unique solution p in $F(p,\mathbf{n}) = 0$ for any given feasible $\mathbf{n}$. That is, p as a function of $\mathbf{n}$ is implicitly defined in $F(p,\mathbf{n}) = 0$. Note that p is a increasing function of $\mathbf{n}$, thus, minimal p_{opt} (satisfying $F(p,\mathbf{n}) = 0$) is uniquely achieved at $\mathbf{n}_{opt} = (1,1,...,1)$. Then, we see that the system cost expressed in (2) uniquely achieves minimum value at $\mathbf{n}_{opt}$ with $\Phi_{opt} = C/(1-p_{opt})$. ■

3. Game 1: Aggressive Users

In this section, we study the TCP connection game with goodput as the utility function. Users with this utility function are *aggressive* in the sense that they only care about goodput and have no resource limitations and are not socially responsible. We first identify the Nash Equilibria of this game, and then study how badly the system's performance is influenced by this selfish user behavior.

3.1. Nash Equilibrium

We first recall the basic definition of the TCP connection game in Section 2. The strategy set of player i is $S_i = \{1,2,3,4,..n_i^{max}\}$, where n_i^{max} is the maximum allowable number of connections for user i. We allow each player to maximize its aggregate goodput by adjusting its feasible number of connections n_i. Specifically, the utility of player i is represented as:

$$U_i = n_i G_i = (C n_i/R_i)/(\sum_{j=1}^{m} n_j/R_j) \qquad (6)$$

We call this *utility function 1* and the game with this utility function *Game 1*. The following lemma shows that Game 1 has a unique Nash Equilibrium (NE) at a boundary point in the strategy space.

Lemma 1 *There exists a unique Nash Equilibrium (NE) of the TCP connection game with utility function 1. At this NE, all players use their maximum number of allowable connections, that is, NE is* $(n_1^{max}, n_2^{max}, ..., n_m^{max})$.

Proof: Note that the strategy set of each player is a discrete set. To make the analysis easier, we first relax the strategy set of any player to be a real interval $[1, n_i^{max}]$. This relaxed version is a continuous kernel game [2].

For player i, consider the partial derivative

$$\partial G_i/\partial n_i = ((C/R_i)\sum_{j\neq i} n_j/R_j)/(\sum_{j=1}^{m} n_i/R_j)^2 \qquad (7)$$

Obviously, $\partial G_i/\partial n_i > 0$, thus, player i always has an incentive to increase its number of connections regardless of the number of connections used by other players. Since this is true for all players, the only NE is $\mathbf{n}^* = (n_1^{max}, n_2^{max}, ..., n_m^{max})$. Since the strategy set of the original discrete game is a subset of the strategy set of this continuous kernel game, and this NE is a feasible strategy in the original discrete game, we conclude that the original discrete game has a unique NE at $\mathbf{n}^*$. ■

Remarks. There is no fairness at the NE. Since the utility function is an increasing function of the number of connections opened by a user, user i with larger n_i^{max}/R_i will have a larger goodput than user j with smaller n_j^{max}/R_j.

3.2. Price of Anarchy and Loss of Efficiency

Price of Anarchy [5], is defined as the ratio of system performance at the worst NE and the system performance at the system optimal point. This value quantifies the loss of efficiency of the worst NE. Since there is a unique NE in Game 1, the price of anarchy is just the efficiency loss of this unique NE.

Let p_{ne} denote the loss rate when the system is at NE. The system cost at NE is:

$$\Phi_{ne} = \sum_{i=1}^{m} n_i^{max} B_{i,ne} = C/(1-p_{ne})$$

Then, the price of anarchy is given by:

$$L_{eff} = \Phi_{ne}/\Phi_{opt} = (1-p_{opt})/(1-p_{ne}) \qquad (8)$$

If the number of users m is fixed, then Φ_{opt} is a constant regardless of the values of n_i^{max}. But Φ_{ne} is an increasing function of n_i^{max}. The reason is as follows. Based on the proof of Theorem 1, we know that p is an increasing function of the number of connections and p asymptotically approaches one as n_i goes to ∞. Thus, when $n_i^{max} \to \infty$, $p_{ne} \to 1$. Then (8) indicates that the price of anarchy becomes unbounded and arbitrarily large.

It is interesting to note that the price of anarchy asymptotically approaches a constant when the population of users increases. We assume that all users have the same RTT $\bar{R}$, and they have the same maximal allowable number of connections $\bar{n}$, then p_{ne} is the solution of $(1-p_{ne})B_{ne} = C/(m\bar{n})$, and p_{opt} is the solution of $(1-p_{opt})B_{opt} = C/m$. The loss of efficiency is $L_{eff} = (1-p_{opt})/(1-p_{ne}) = \bar{n}B_{ne}/B_{opt}$.

Since $\lim_{m\to\infty} p_{opt} = 1$ and $\lim_{m\to\infty} p_{ne} = 1$, then

$$\lim_{m\to\infty} B_{opt} = 1/(\mu\bar{R} + 33T_0\nu) = \lim_{m\to\infty} B_{ne}$$

Thus, $\lim_{m\to\infty} L_{eff} = \bar{n}$. However we need to be cautious when interpreting this result. In this case, m is so large that the network cannot even support the case where each user opens only one connection ($p_{opt} \to 1$). Thus, the network cannot operate efficiently even at the system optimal point.

4. Game 2: Resource Constrained and Socially Responsible Users

The previous section shows that the price of anarchy can be arbitrarily large if users are only interested in maximizing

their goodputs. In this section, we will show that if users have some resource constraints and take some social responsibility by considering the cost to the system in their utility functions, then the price of anarchy is bounded.

Recall that we treat the aggregate sending rate from all connections opened by a player as the effort or cost incurred by that user. Let n_iB_i denote this cost. Note, this cost not only represents a cost to the system but also can be interpreted as the cost to the user for sending data. Then a user i may want to examine the tradeoff between the cost n_iB_i and the achieved goodputs when making a decision on how many connections to open, thus we can derive a utility function as follows

$$U_i = C(n_i/R_i)/(n_i/R_i + \sum_{k=1,k\neq i}^{m} n_k/R_k) - \beta n_i B_i \quad (9)$$

We call this *utility function 2* and the game with this utility function *Game 2*. Here, coefficient $\beta \in (0,1)$ represents a user's weight on the effort or cost. A smaller β means a user is less resource constrained and less socially responsible. If $\beta = 0$, this utility function becomes just the goodput, utility function 1.

In this section, first, we study a continuous kernel *symmetric* multiple player TCP connection game in which all users have the same Round Trip Time (RTT). We then consider two extensions. One is a discrete version of the *symmetric* multiple player TCP connection game. The other one is a continuous kernel *asymmetric* multiple player TCP connection game in which users have different RTTs.

4.1. Continuous Kernel Symmetric TCP Connection Game

In this game, since all users have the same RTT, the per-connection sending rate from all users are all the same. Thus, an arbitrary player i has the utility function given by (9) with B replacing B_i and all R terms being canceled out. Note that B is given by (3) and (4). B is a function of p which is in turn a function of $n_i, \forall i$. The strategy set for player i is a real interval $S_i = [1, \infty)$. Since all players take a real-valued number as a feasible strategy and the identity of a player is not important, we call this game a continuous kernel symmetric [2] TCP connection game with utility function 2.

Theorem 2 *There is a unique Nash Equilibrium (NE)* $\mathbf{n}^*$ *in the continuous kernel symmetric TCP connection game with utility function 2. At this NE, all players have the same number of connections. This NE is an interior point of the strategy space for* $m < m_0$ *and* $\mathbf{n}^* = (1, 1, ..., 1)$ *for* $m \geq m_0$. *Threshold* m_0 *is the largest* m *satisfying* $m(1-p^*)B^* \leq C$ *where* p^* *and* B^* *are respectively loss rate and per-connection sending rate at the NE.*

Proof: This proof consists of two parts. In the first part, we prove that the unique Nash Equilibrium is achieved at an interior point in the strategy space. In the second part, we present the results when the number of players is very large.

Part 1:

Player i tries to solve for its optimal strategy n_i^*, as a response to the strategies of all other players. Thus, if there is an interior point NE $\mathbf{n}^* = (n_1^*, ..., n_m^*)$, then it must be true that $\forall i, \partial U_i/\partial n_i^* = 0$, and $n_i^* = \mathrm{argmax}_{n_i \in S_i} U_i(n_1^*, ..., n_i, ..., n_m^*)$.

In the following, we first introduce a fact indicating that the stationary point satisfying $\partial U_i/\partial n_i = 0$ is actually the maximum point if it is in $[1, \infty)$. Then we show that there is a unique $\mathbf{n}^*$ satisfying $\partial U_i/\partial n_i^* = 0, \forall i$.

We need to seek all vectors $\mathbf{n}^*$ satisfying a set of m equations

$$\partial U_i/\partial n_i = 0, \forall i \in [1, 2, 3, ..., m] \quad (10)$$

In the following, we first prove that if $\mathbf{n}^*$ exists, $n_i^* = n_j^*, \forall i, j$. Then, we show that such $\mathbf{n}^*$ is actually unique by proving that there is only one p^* for which $n_i^* = n_j^*, \forall i, j$.

For an arbitrary player i, we have

$$\frac{\partial U_i}{\partial n_i} = \frac{Cn_{-i}}{(n_i + n_{-i})^2} - \frac{\beta}{\phi} - \frac{\beta n_i C \varphi}{(n_i + n_{-i})^2[(p-1)\varphi - \phi]} \quad (11)$$

where

$$\phi = \mu R\sqrt{p} + T_0\nu(p^{3/2} + 32p^{7/2}) = 1/B \quad (12)$$

$$\varphi = \frac{\mu R}{2\sqrt{p}} + T_0\nu(\frac{3}{2}\sqrt{p} + 112p^{5/2}) \quad (13)$$

and $n_{-i} = \sum_{k=1,k\neq i}^{m} n_k$ and $\varphi = d\phi/dp$.

Fact 1: Best response of a player is unique and it is the stationary point if the stationary point is in $[1, \infty)$**.** First we need to show that for any given n_{-i}, there is only one unique maximal point for U_i. Player i needs to solve the following equations to get a candidate for a maximal point n_i^m:

$$0 = \beta n_i - n_{-i}(1 - p - \beta)[\varphi(1-p)/\phi + 1] \quad (14)$$

$$0 = C\phi - (n_i + n_{-i})(1-p) \quad (15)$$

where (14) is a simplification of $\partial U_i/\partial n_i = 0$. We can think of n_i^m and p as implicit functions of n_{-i}. Note that for any given n_{-i}, there is a unique pair of (n_i^m, p) as the solution to (14) and (15). We can check that the unique stationary point n_i^m obtained from this implicit function is indeed a maximal point. We can enlarge the domain of U_i to be $(0, \infty)$, and notice that n_i^m is also a unique stationary point for this enlarged domain. Since $U_i(0, n_{-i}) = 0$ and $\lim_{n_i \to \infty} U_i = -\infty$, they are not larger than $U_i(n_i^m, n_{-i})$ given that n_i^m is indeed an interior point. Then we can conclude n_i^m is indeed a maximal point in domain $(0, \infty)$. If it is still a stationary interior point in $[1, \infty)$, then it also must be a maximal point. Otherwise if it is smaller than one, then the maximal point is one (the boundary point), which is discussed in Part 2 of this proof. We can show that $n_i^m = f_i(n_{-i})$ and $p = f_p(n_{-i})$ are continuous functions[2] on domain $n_{-i} \in (0, \infty)$. In addition, from implicit function theorem, we know that they are continuously differentiable.

Now, we go on to prove the existence and uniqueness of NE. Consider two arbitrary players i and j, and let $\delta_i n_i =$

[2] $f_i(n_{-i})$ is referred to as the best response or reaction function.

$\sum_{k=1,k\neq i}^{m} n_k$; $\delta_j n_j = \sum_{k=1,k\neq j}^{m} n_k$. When $\partial U_i/\partial n_i = \partial U_j/\partial n_j = 0$, we get

$$(1-p)[\delta_i + \beta\varphi/((1-p)\varphi+\phi)]/(1+\delta_i) - \beta = 0 \quad (16)$$
$$(1-p)[\delta_j + \beta\varphi/((1-p)\varphi+\phi)]/(1+\delta_j) - \beta = 0 \quad (17)$$

Let $\Delta = \beta\varphi/((1-p)\varphi+\phi)$, then combining (16) and (17) leads to

$$(\delta_i/(1+\delta_i) - \delta_j/(1+\delta_j)) + \triangle(1/(1+\delta_i) - 1/(1+\delta_j)) = 0 \quad (18)$$

For (18) to be true, we need either $\triangle = 1$ or $\delta_i = \delta_j$. We can show that $\Delta = 1$ cannot be true. We prove this by contradiction. Assume that it is true, then we can substitute it into (16), and get $\beta = 1-p$. Substituting $\beta = 1-p$ into $\Delta = 1$, we get $\phi = 0$. We know that $\phi = 0$ is impossible given that $p \in (0,1)$, thus $\Delta \neq 1$. Thus, the only possible solution is $\delta_i = \delta_j, \forall i, j$. This implies that $n_i^* = n_j^*$ at NE $\mathbf{n}^*$ if it exists.

In the following, we will prove that, when $n_i^* = n_j^*$, there exists a unique solution p^* for (10). Then we can conclude that there is a unique $\mathbf{n}^*$.

Since at NE all players have the same number of connections, from (11), we obtain

$$(m-1)/\beta - m/(1-p) + \varphi/((1-p)\varphi+\phi) = 0 \quad (19)$$

Let $F(p)$ denote the LHS of (19). Ideally, solving equation (19) with p as unknown, we can get loss rate at NE p^*. Then, substituting p^* back into (4), we can get $\mathbf{n}^*$ as the number of connections of all users at NE. However, (19) contains several powers of p such as $7/2$ and $5/2$, so it is impossible to get an algebraic solution of p. Thus, in the following, we examine several properties of $F(p)$, and based on these properties we make inferences about the behavior of NE. To get exact values of p^* and $\mathbf{n}^*$ for a given network setting, we can rely on Matlab for numerical solutions.

First, we will prove that (19) has only one solution for p in $(0,1)$. We note that $F(p)$ is a continuous function, and the domain of $F(p)$ is $p \in (0,1)$, and $\lim_{p\to 0} F(p) > 0$ and $\lim_{p\to 1} F(p) < 0$. We claim that $F(p)$ is a strictly monotonic decreasing function. If this claim is true, then there must be a single solution p^* for $F(p) = 0$. In the following, we prove this claim.

Consider the derivative

$$\begin{aligned}\frac{dF}{dp} &= \frac{-m}{(1-p)^2} + \frac{\varphi'\phi}{[(1-p)\varphi+\phi]^2}\\ &< \frac{-1}{(1-p)^2} + \frac{\varphi'\phi}{[(1-p)\varphi+\phi]^2}\\ &= \frac{-\phi^2 - 2(1-p)\varphi\phi - (1-p)^2(\varphi^2 - \varphi'\phi)}{(1-p)^2[(1-p)\varphi+\phi]^2} \quad (20)\end{aligned}$$

Thus, to prove that $\frac{dF}{dp} < 0$, we only need to prove that $\varphi^2 > \varphi'\phi$. This can be easily proved. See [14] for details.

If we substitute p^* into (4), together with the result that all users have the same number of connections at $\mathbf{n}^*$, we conclude that there is only one NE for this game and it is symmetric. That is, $n_i^* = \text{argmax}_{n_i \in S_i} U_i(n_1^*, ..., n_i, ..., n_m^*)$ and $n_i^* = n^*, \forall i$.

Part 2:

Now, we will show that if $m \geq m_0$ where m_0 is the largest m such that $m(1-p^*)B^* \leq C$, the NE is no longer an interior point of the strategy space. Instead, it is $\mathbf{n} = (1, 1, ..., 1)$.

Recall (19), and let

$$F(p,m) = m(1/\beta - 1/(1-p)) - 1/\beta + \varphi/((1-p)\varphi+\phi) \quad (21)$$

Given a value of m, we can plot a curve for $F(p,m)$ with p as x-axis and $F(p,m)$ as y-axis. Note that all these curves (with different m values) all meet at a single common point $(p_0, F(p_0, m))$ with $p_0 = 1-\beta$. Take any m_i and m_j and check that $F(p, m_i) = F(p, m_j)$ implies $p = 1-\beta$.

Recall that $F(p,m)$ is a monotonic decreasing function of p, and $F(p^*, m) = 0$. Since $F(p_0, m) < 0$, so p^* must be smaller than p_0. When $p < p_0$, we get

$$dF/dm = 1/\beta - 1/(1-p) = 1/(1-p_0) - 1/(1-p) > 0$$

Thus, as m increases, $F(p,m)$ is strictly monotonic increasing, and since $F(p,m)$ is a monotonic decreasing function of p, thus, we see that as m increases, for any given $F(p)$, p will be strictly increasing towards p_0. Then, it must be true that for $F(p^*) = 0$, as m increases, p approaches p_0.

Recall that at NE, we must have $(1-p^*)/\phi^* = C/(mn^*)$. Since all users must have at least one connection, i.e., $n^* \geq 1$, we have to make sure that $m(1-p^*)/\phi^* \leq C$. We know that as m increases, $p^* \to p_0 = 1-\beta$. Then ϕ^* as a function of p^* also increases to ϕ_0 (function of p_0). Thus, $(1-p^*)/\phi^*$ is bounded below by $(1-p_0)/\phi_0$. So, as m becomes larger and larger, eventually, $m(1-p^*)/\phi^*$ will be larger than C. Then the NE is no longer an interior point. Let m_0 denote this threshold, and it is the largest m satisfying

$$m(1-p^*)/\phi^* \leq C \quad (22)$$

Since it is difficult to obtain an explicit expression of p as a function of m, we rely on numerical method to identify m_0.

■

Remarks: Note that the utility function of this TCP connection game is not *concave* in general. But we can still get an alternative but non-constructive proof of the existence of NE of this TCP game by modifying the proof of a general result (Theorem 4.3 in [2]). We can replace the strict convexity of cost function in that proof with the uniqueness of best response in TCP game, then the existence of NE is immediately obtained.

An illustrative example: NE as an interior point. We present an example to illustrate an interior-point NE in a continuous kernel TCP connection game with utility function 2. There are two players competing for a bottleneck link with capacity $C = 10$Mbps or 1250pkts/sec. They have the same RTT (240ms), and choose $\beta = 0.7$. To identify NE, we can plot the best response curves of these two players. For example, suppose we want to know the best response curve of

player 2. Given a specific number of connections n_1 of player 1, player 2 uses the simplified PFTK TCP model to maximize its utility defined in (9). We use optimization toolbox in Matlab to solve this optimization problem to get $f_2(n_1)$ as the best response to n_1 and plot the best response curve $f_2(n_1)$. Similarly we can plot the best response curve $f_1(n_2)$ for player 1. The intersecting point of these two curves is the NE. Figure 1 shows the simulation result, and we see that there is indeed one unique NE. And, it can be easily verified that this NE is the same as that predicted in Theorem 2.

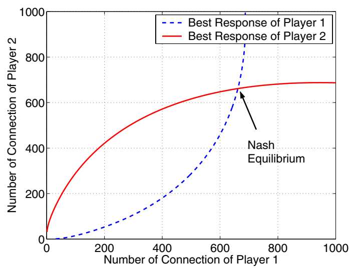

Figure 1. Best response curves intersect at NE.

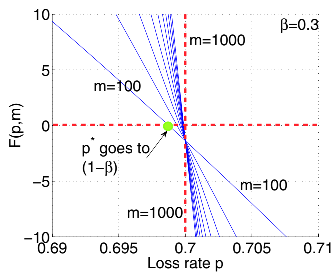

Figure 2. p^* **as a function of** m **when** $\beta = 0.3$.

An illustrative example: effects of population size. We use the same network settings as before. We take $\beta = 0.3$, and vary the number of players m from 100 to 1000. We plot all curves of $F(p, m)$ (defined in (21)) as a function of p in Figure 2. All these curves intersect at $p = 0.7$ as predicted by Theorem 2. And they intersect with $F(p, m) = 0$ at p^*s. This plot shows that n^* approaches $p_0 = 1 - \beta$ as m increases. In Figure 3, for several different β values, we plot the loss rate p^* at NE when the number of users m increases. As shown in this figure, for any given β, p^* approaches to $1 - \beta$ when m is not very large. However, when m is very large, Figure 4 shows that p grows more quickly, but still less than $\log m$. In summary, this example has verified the NE's behavior predicted in part 2 of the proof of Theorem 2.

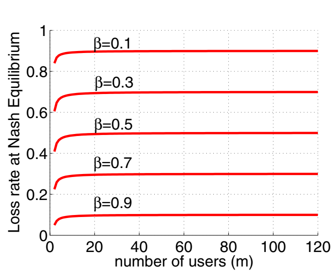

Figure 3. p^* **as a function of** m **for different** β **values.** m **varies from** 2 **to** 120.

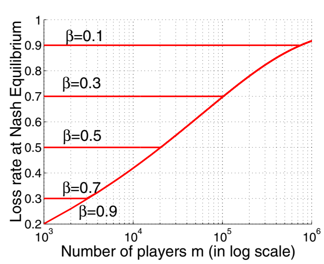

Figure 4. p^* **as a function of** m **for different** β **values.** m **varies from** 1000 **to** $1000,000$.

4.2. Loss of Efficiency

As in Section 2, we define the system optimization problem as minimizing the cost to maintain a busy bottleneck link. The loss of efficiency is defined as the ratio between the cost of the system at Nash Equilibrium and the system optimal cost. As before, the optimal system cost is $\Phi_{opt} = C/(1-p_{opt})$. Then, we can get the loss of efficiency of NE for this game.

Corollary 1 *In the continuous kernel symmetric TCP connection game, the loss of efficiency is* $L_{eff} = (1-p_{opt})/(1-p_{ne})$, *and it is always larger than or equal to one, but it is bounded.*

Proof: Consider that the system cost when the system is at NE:

$$\Phi_{ne} = B_{ne}(\sum_{i=1}^{m} n_i^*) = C/(1 - p*)$$

Then the loss of efficiency is given as

$$L_{eff} = \Phi_{ne}/\Phi_{opt} = (1 - p_{opt})/(1 - p_{ne}) \qquad (23)$$

We note that the loss of efficiency is always larger than or equal to one. Recall that p_{opt} must satisfy $(1 - p)m/\phi = C$, and p^* or p_{ne} must satisfy $(1 - p^*)mn^*/\phi* = C$. Then we have

$$(1 - p)/\phi = n^*(1 - p^*)/\phi^*$$

Since $n^* \geq 1$, then $p^* \geq p_{opt}$. As m increases, n^* decreases. Before n^* reaches 1, p^* is strictly larger than p_{opt}, and after that, $p^* = p_{opt}$. Then, it must be true that the maximal efficiency loss occurs when m is small.

Recall that $1 - p^* > \beta$ and p_{opt} is an increasing function of m, thus, we have

$$L_{eff} = (1 - p_{opt})/(1 - p^*) < (1 - p_{opt,m=2})/\beta \qquad (24)$$

This upper bound is a simple function of network parameters and user's aggressiveness coefficient β.

■

An illustrative example. We take the network settings in the previous examples, and choose $\beta = 0.7$. In Figure 5, we plot the loss rate of NE and system optimal point. As predicted, the loss rate of the NE is always greater than and equal to the loss rate of system optimal point. When m is sufficiently large, all users just use one connection, then the trajectory of loss rate increase will be the same as that of system optimal point.

In Figure 6, for several different values of β, we plot the loss of efficiency of NE as a function of m. The solid lines are the actual loss of efficiency, and the dashed lines are the upper bound computed from (24). As expected, the loss of efficiency is always upper bounded.

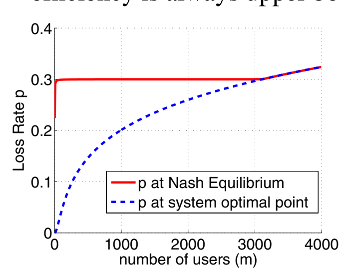

Figure 5. Loss rate as a function of number of players m **when** $\beta = 0.7$.

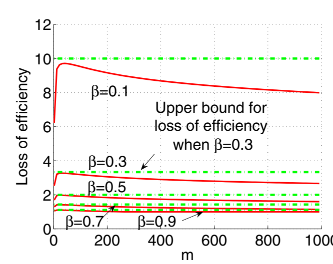

Figure 6. Loss of efficiency of NE as a function of m **when** $\beta = 0.7$.

Effects of user's aggressiveness. β represents a user's preference of how much effort he/she is willing to expend to get the desired goodput share of the bottleneck capacity. Intuitively, as β gets larger, a user is likely to use less effort, then the number of connections at NE will be smaller, and the loss rate of NE will be smaller.

This can be verified by looking at the relationship between loss rate of NE and β. Recall that p^* is the solution to the following equation $F = (m-1)/\beta - m/(1-p) + \varphi/((1-p)\varphi + \phi) = 0$. Then

$$\partial p/\partial \beta = -F'_\beta/F'_p = \frac{(m-1)/\beta^2}{-\frac{m}{(1-p)^2} + \frac{\varphi'\phi}{((1-p)\varphi+\phi)^2}} < 0$$

This indicates that p is a decreasing function of β. Since n^* is a increasing function of p, we know that if β increases, at the NE, users open fewer and fewer connections.

We can expect the system cost (the aggregate costs of all users) to decrease as β increases. Recall that the system cost at the NE is $f = \sum_{i=1}^{m} n_i^* B^* = C/(1-p^*)$. As p^* is a decreasing function of β, we see that as β becomes larger and larger (users becomes less and less aggressive), the required system effort will be smaller and smaller. In addition, since p_{opt} is independent of β, from the above discussion on p^*, we see that L_{eff} is a decreasing function of β. This is understandable, as users become less and less aggressive (larger β), the NE will be more and more efficient.

As an example, we use the same network settings as before, and fix the number of users to be 100, but vary β from 0.05 to 0.99. We expect that the loss rate at the NE to decrease as β increases, and finally reach the loss rate of the system optimal point. This means that β is so large that NE is no longer an interior point of the stategy space and all users are so conservative that everyone just opens one connection, as shown in Figure 7. On the other hand, as users becomes more and more aggressive (β decreases), at the NE, users open more and more connections. This means that the whole system will need more and more effort to keep the same aggregate goodputs. Figure 8 shows the loss of efficiency decreases as users become less and less aggressive, which is expected.

It would also be interesting to understand the situation where different users have different βs. This will be a topic of our future research.

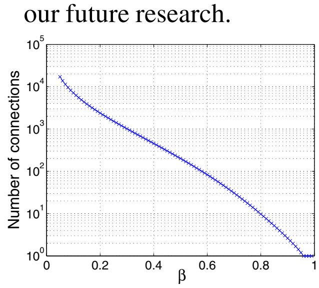

Figure 7. Number of connections at NE as a function of β when $m = 100$.

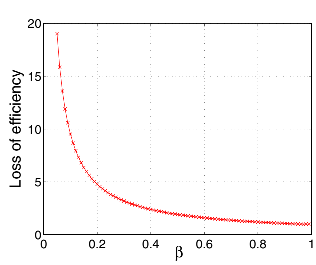

Figure 8. Loss of efficiency at NE as a function of β when $m = 100$.

4.3. Stability of Nash Equilibrium

A natural question is whether the unique NE of this game is stable. As defined in [2, 6], if some player deviates from NE by an arbitrary amount in the feasible strategy set, and other players observe this and they adjust their responses optimally based on some fixed ordering of moves, and if this adjustment process converges to the original NE, then we say that this NE is *globally stable* with respect to this adjustment scheme. Correspondingly we can define local stability by restricting the stability domain to be some ε-neighborhood of NE. As pointed out in [2], stability condition is the same regardless of the adjustment schemes when there are only two players. Here, we only study the stability of a two-player TCP connection game, and this adjustment scheme is also called *best reply dynamics*. The response or reaction function of each player is determined by solving its optimization problem. Recall (14) and (15), from which we obtain $n_1(t+1) = f_1(n_2(t))$ and $n_2(t+1) = f_2(n_1(t))$ where t indicates discrete time step.

Some sufficient conditions (needed for contraction mapping) for the global stability of NE is given in [6], but they are not satisfied in this game. Checking the best response curves in Figure 1 shows that contraction mapping is not true when the game is in state $(1, 1)$, namely both players using only one connection. Nevertheless, we are able to show that the unique NE is locally stable.

Theorem 3 *In the two-player symmetric continuous kernel TCP connection game with utility function 2, the unique NE is locally stable.*

The basic idea of the proof of this theorem is as follows. Since we can derive the exact form of the first derivative of the best response function and this derivative is continuous, we can check that the absolute value of this derivative is strictly smaller than one at the NE. Then we are able to show that locally at NE, there exists a contraction mapping driving the system to the NE if the deviation from the NE is sufficiently small, based on the Banach contraction mapping theorem [8] and the mean value theorem. The detailed proof is in [14].

As for the global stability, we simulated a large range of network parameters, and found that $f_1(n_2)$ and $f_2(n_1)$ were always concave functions. Since the concavity of the best response function and the uniqueness of NE imply global stability (proved in [14]), we conjecture that the NE is very likely to be globally stable.

4.4. Extension 1: Integer TCP Connection Game

In this section, we consider a more practical TCP connection game in which each player can only choose a positive integer number of connections. That is, each player's strategy set is $\mathbb{N}$. We call this the *Integer TCP Connection Game*.

To study this game, we use the results of the corresponding continuous kernel game. Note that if the pure strategy NE in the continuous game is an integer vector, then it must be a NE of the corresponding integer game. The more interesting case is where the pure strategy NE of the continuous game is a non-integer vector $\mathbf{n}^* = (n^*, n^*, ..., n^*)$. When this happens, we can approximate n^* by taking floor n_f^* and ceiling n_c^* of n^* to

get 2^m integer-valued vectors.

In the following, first, we will show that, at these integer-valued vectors, the utility deviation of each player from the non-integer NE is bounded. As the number of users increases, this bound approaches zero. For convenience, we call such a vector an *approximate Nash Equilibrium*[3]. Next, we demonstrate that this integer game must have pure strategy NE(s) at some of these integer vectors given that some pathological cases never occur.

We start with a simple example of a two-player game. Figure 9 shows that in the continuous version of the game, the intersecting point of the best response curves of two players is a fraction number $(661.5, 661.5)$. If we restrict the strategy space of each player to be $\mathbb{N}$, we can approximate the continuous game NE with the floors and ceilings of the NE vector to get four vectors: $(661, 661)$, $(661, 662)$, $(662, 661)$ and $(662, 662)$.

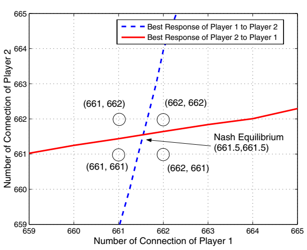

Figure 9. An observed case where integer NE must exist.

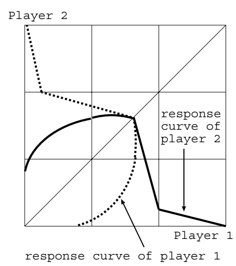

Figure 10. A pathological case.

How much performance loss will be incurred due to this integer constraint on strategy space? Recall that for an arbitrary user i, its utility at the NE is given as $U_i^* = Cn^*/(mn^*) - \beta n^* B^* = C/m - \beta n^*/\phi^*$. Among all such integer approximate NEs, the worst case *goodput loss* could happen when user i opens $(n^* - 1)$ connections while all others open $(n^* + 1)$ connections. Let G_l denote this goodput lower bound, then $G_l = C(n^* - 1)/(mn^* + m - 2)$. On the other hand, the worst case *cost increase* could happen when user i opens $(n^* + 1)$ connections while all others open $(n^* - 1)$ connections. Then, the cost increase upper bound is $J_u = \beta(n^* + 1)/\phi_u$, where ϕ_u is the solution of

$$(1 - p_u)/\phi_u = C/(mn^* - m + 2)$$

Then, the upper bound of utility loss is given as

$$\begin{aligned}\Delta U =& G^* - G_l - \beta(J^* - J_u)\\ =& C\left(\frac{1}{m} - \frac{1 - 2/(n^*+1)}{m - 2/(n^*+1)}\right) + \beta\left[n^*\left(\frac{1}{\phi_u} - \frac{1}{\phi^*}\right) + \frac{1}{\phi_u}\right]\end{aligned}$$

Thus the utility loss of any user at any approximate integer NE is bounded by ΔU, and this bound approaches zero as m increases. The system performance loss at any approximate NE from that of the NE in continuous game also approaches zero.

[3]Note, these approximate integer Nash Equilibria are different from ε-Nash Equilibrium defined in [2].

Now the next question is whether these approximate NEs are possibly NEs in the integer game? For this question, we have the following result.

Theorem 4 *In the integer symmetric TCP connection game with utility function 2, if the Nash Equilibrium of the corresponding continuous game is a non-integer vector* $\boldsymbol{n}^* = (n^*, ..., n^*)$ *with* n_f^* *and* n_c^* *denoting the floor and ceiling of* n^**, then there must exist pure-strategy integer Nash Equilibrium provided that the following condition is satisfied: the best response for player* $i, \forall i$ *is always chosen from* n_f^* *and* n_c^* *given that all other players choose either* n_f^* *and* n_c^*.

A detailed proof of this theorem is in [14]. To illustrate this theorem, we recall Figure 9 and observe that when player 2 chooses 662 as its strategy, the best *integer* response of player 1 must be either 661 or 662, since for each strategy of player 2, player 1 has a unique best response and all other strategies monotonically decrease in utility as they get away from the best response (recall that there is only one unique interior-point maximum for utility function 2). Similarly we have the following arguments: when player 2 chooses 661, the best *integer* response of player 1 must be either 661 or 662; when player 1 chooses 661 or 662, the best *integer* response of player 2 must be chosen from 661 and 662. This observation is exactly the condition required for Theorem 4.

There is some pathological case in which there could be no pure strategy NE. Figure 10 shows such an example. If we assume that the integer closest to the response curve is the integer with the highest uility, then, we can see that the game shown in this figure has no pure strategy integer NE even though there is a fractional NE in the continuous game. Actually, we never saw this case in our simulations.

Since the condition required for Theorem 4 is satisfied in all of our simulations, we conjecture that pure strategy integer NE always exists for the integer TCP connection game. This will be a topic of our future research.

4.5. Extension 2: Asymmetric TCP Connection Game

The *asymmetric* game differs from the previous *symmetric* game in that users have different Round Trip Time (RTT). For player i, we have the following utility function

$$U_i = (Cn_i/R_i)(n_i/R_i + \sum_{k=1, k\neq i}^{m} n_k/R_k) - \beta n_i B_i \quad (25)$$

where B_i is given in (3) and (4).

Theorem 5 *There is a unique Nash Equilibrium (NE) in the continuous kernel asymmetric TCP connection game. This NE is an interior point of the strategy space given that the number of users is not larger than* m_0 *given in (26). At this interior-point NE, for any two players* i *and* j*, we have* $n_i^*/n_j^* = R_i/R_j$.

Since the proof is very similar to that of Theorem 2, we only sketch the basic idea as follows. First we need to set $\bar{\phi} = \mu\sqrt{p} + 4\nu(p^{3/2} + 32p^{7/2})$, $\phi_i = R_i\bar{\phi}$ and $\varphi_i = R_i\bar{\varphi}$. Then following a similar procedure in the proof of Theorem 2, we can derive $\partial U_i/\partial n_i$ and $\partial U_j/\partial n_j$ for any two players, and

let $\delta_i n_i/R_i = \sum_{k\neq i} n_k/R_k$; $\delta_j n_j/R_j = \sum_{k\neq j} n_k/R_k$. Then we can show that $\delta_i = \delta_j$, thus, $n_i^*/n_j^* = R_i/R_j$.

If we sort the number of connections in an ascending order as $n_1^*, n_2^*, ..., n_m^*$, then as we increase the number of users, all n_i^*s will simultaneously decrease but maintain their relative proportional relationship. As m reaches a large enough number where n_1^* must be less than 1, then player 1 will just maintain one connection. From then on, as m continues increasing, NE will no longer be an interior point. To maintain an interior-point NE, m must be smaller than m_0, where m_0 is the largest m satisfying

$$m(1-p^*)/(R_1\bar{\phi}^*) \leq C \tag{26}$$

It is easy to see that at this interior-point NE, users have the same utility, and the efficiency loss of the NE is bounded.

5. Game 3

Recall that cost $\beta n_i B_i$ considered in Section 4 contains the cost to the whole system and the cost to a user at packet level. In this section, we introduce another term specific only to users and that accounts for the cost of maintaining open connections. Specifically, we use αn_i to represent the computation cost, and call α the computation power coefficient. Intuitively, the more connections a user opens, the more computation power he/she needs. αn_i can be thought of as the resource requirement on CPU power, memory, etc. Thus, we can consider a more comprehensive utility function including both packet sending cost and computation resource limitation. We refer to this as *utility function 3*, given as:

$$U_i = (Cn_i)/(n_i + \sum_{k=1,k\neq i}^{m} n_k)(1-\beta/(1-p)) - \alpha n_i \tag{27}$$

We refer to the game with this utility function as *Game 3*.

Theorem 6 *There is a unique Nash Equilibrium (NE) n_α^* in the continuous kernel symmetric TCP connection game with utility function 3. At this NE, all players have the same number of connections. This NE is an interior point of the strategy space for $m < m_{0,\alpha}$ and $\mathbf{n}_\alpha^* = (1,1,...,1)$ for $m \geq m_{0,\alpha}$, where $m_{0,\alpha}$ is the largest m such that $m(1-p_\alpha^*)/\phi_\alpha^* \leq C$, and p_α^* is the loss rate at the NE.*

Following a similar procedure in the proof of Theorem 2, we can prove that there is a unique NE $\mathbf{n}_\alpha^* = (n_\alpha^*, n_\alpha^*, ..., n_\alpha^*)$. See [14] for details. Since at the NE, we must have $(1-p_\alpha^*)/\phi_\alpha^* = C/(mn_\alpha^*)$, and since all users must have at least one connection, i.e., $n_\alpha^* \geq 1$, we have to make sure that $m \leq m_{0,\alpha}$ where $m_{0,\alpha}$ is the largest m such that $m(1-p_\alpha^*)/\phi_\alpha^* \leq C$. We need to rely on numerical method to identify $m_{0,\alpha}$. Similar to Theorem 2, we have $p_\alpha^* < p_{0,\alpha}$ where $p_{0,\alpha}$ is the solution of $1-p_{0,\alpha} = \alpha\phi_{0,\alpha} + \beta$. Thus, $1-p_\alpha^* > \beta$. And we know that as m increases, $p^* \to p_{0,\alpha}$. Then ϕ_α^* as a function of p_α^* also increases to $\phi_{0,\alpha}$ (function of $p_{0,\alpha}$). Thus, $(1-p_\alpha^*)/\phi_\alpha^*$ is bounded. So, when m becomes larger and larger, eventually, $m(1-p_\alpha^*)/\phi_\alpha^*$ will be larger than C, which means that all users only use one connection at the NE.

Comparison between Game 2 and Game 3.

Since α represents a user's computation power limitation, introducing α will make users more conservative. Thus, we might expect that at the NE of Game 3, users will open fewer number of connections than at the NE of Game 2. And, as the number of users increases, users will be more quickly to tend to open just one connection in Game 3. This intuition is formalized in the following lemma.

Lemma 2 *The interior-point Nash Equilibrium (NE) of Game 3 will give a lower loss rate and smaller number of connections than the NE of Game 2. And, as the number of users increases, the interior-point NE of Game 3 will more quickly become the boundary NE $(1,1,..,1)$.*

See [14] for a detailed proof.

Loss of Efficiency.

As before, the loss of efficiency of Game 3 is $L_{eff} = (1-p_{opt})/(1-p_{ne,\alpha})$. Similar to Game 2, the loss of efficiency is always larger than or equal to 1, but it is upper-bounded. Recall that $1-p_\alpha^* > \beta$. And since p_{opt} is an increasing function of m, we have

$$L_{eff} = (1-p_{opt})/(1-p_\alpha^*) < (1-p_{opt,m=2})/\beta$$

Even though this upper bound is the same as that of Game 2, we see that the actual efficiency loss of this game is smaller than that of Game 2 since $p_\alpha^* < p^*$.

The findings in this section again indicate that we might not expect large efficiency loss or congestion collapse in reality.

Users with different computation power.

We might as well be interested in the case where users have different computation power. Then, we can represent a user's utility function as: $U_i = n_i C/(n_i + \sum_{k=1,k\neq i}^{m} n_k) - \alpha_i n_i$ where $\alpha_i \neq \alpha_j, \forall i \neq j$. For this game, we have the following result. A detailed proof is given in [14].

Theorem 7 *In the continuous kernel multiple player TCP connection game with users having different computation power, when $m < m_0$, there exists an interior-point NE, where m_0 is the largest m such that $\frac{C\phi^*(\sum \alpha_k - (m-1)\alpha_1)}{(1-p^*)\sum \alpha_k} \geq 1$. At this NE, a more powerful user will have more connections and higher goodput and utility.*

6. NS Simulations

We use NS simulations in this section to verify the analytical results derived in previous sections. We consider a single bottleneck link with capacity 10Mbps or 1250pkt/sec, competed by users who are allowed to open multiple concurrent connections. Due to space limitation, we only present here an example simulation result on utility function 2.

First, we show to what extent the simplified PFTK model captures the TCP behavior observed in NS simulations. Figure 11 and Figure 12 show respectively the comparison of loss rate and goodput among those measured in NS simulation, estimated by simplified PFTK model [12], and estimated by Square-Root-P model [11]. In Figure 11, to compute the estimated loss rate p of the simplified PFTK TCP model, we numerically solve for p by using the measured TCP sending

rate B in NS. Similarly, we use B to solve for p for Square-Root-P model. Figure 11 shows that Square-Root-P model is completely useless when the number of connections gets large. The simplified PFTK TCP model gives a good estimate of loss rate. In addition, Figure 12 shows that the simplified PFTK TCP model gives a very good estimate of measured per-connection goodput.

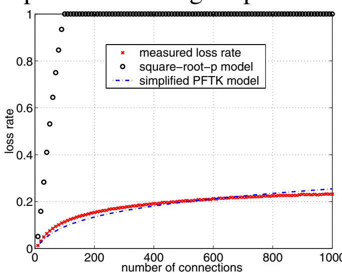

Figure 11. Loss rate.

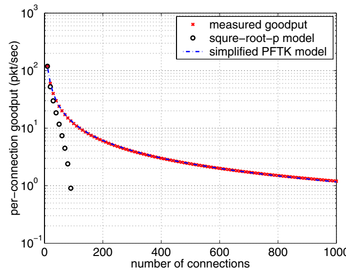

Figure 12. Goodput.

Figure 13 illustrates the existence of a unique Nash Equilibrium observed in NS simulation of a two-player symmetric TCP connection game. Both users have the same two-way propagation delay 40ms. The bottleneck link queue is a RED queue with a target queuing delay 10ms. Each user uses utility function 2 with aggressive coefficient $\beta = 0.8$. Figure 14 shows that the predicted NE by our analysis is very close to the one observed in NS simulations in Figure 13.

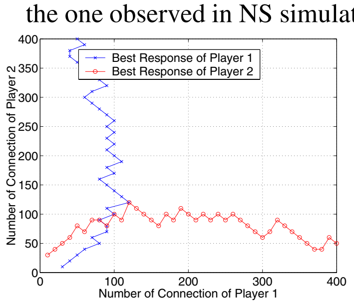

Figure 13. Nash Equilibrium observed in NS simulation.

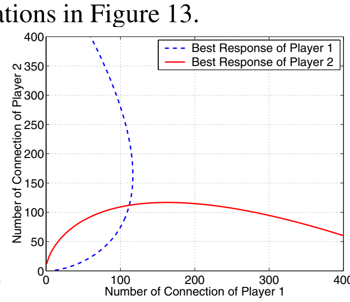

Figure 14. Nash Equilibrium computed by PFTK model.

7. Conclusions

In this paper, we studied a particular selfish behavior of TCP users in which users are allowed to open multiple concurrent connections to maximize their individual goodputs or other utilities. Since such a strategic usage of TCP is easy to realize through some software agents (e.g.,FlashGet [3]) and its potential impact could be harmful [7], it is important to understand its implication on the stability of the Internet. To this end, we modelled users as players in a non-cooperative non-zero-sum game competing for the capacity of a single bottleneck link, referred to as the *TCP Connection Game*. We used different utility functions to model different user behaviors, and used the well known PFTK TCP model [12] as the basis of our analysis.

We demonstrated analytically that there was always a unique Nash Equilibrium (NE) in all variants of TCP connection games we studied. Our results indicate that, at the NE, the loss of efficiency or price of anarchy can be arbitrarily large if users have no resource limitations and are not socially responsible. However, if either of these two factors is considered, the efficiency loss is bounded. And in game 2, the game capturing the user's cost and social responsibility, we have also shown that the unique NE is always locally stable and is globally stable if the game satisfying certain conditions which are actually observed in all our simulations. And the integer NEs always exist when users are restricted to use only an integer number of connections and if some pathological case never occurs. In summary, the general message is that this selfish usage of TCP might not lead to the congestion collapse.

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