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Abstract

Fiscal policy is needed to avoid dynamic inefficiency and maintain full employment in a modified Diamond OLG model with imperfect competition. A distributionally neutral tax scheme can maintain full employment in the face of variations in 'household confidence'. No variations in taxes will be needed if households correctly anticipate future taxes: the tax policy functions as an insurance scheme.

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1 Introduction

Overlapping generations models (OLG models) may give rise to dynamic inefficiency. The inefficiency case is typically dismissed as empirically irrelevant, but the dismissal presumes that the observed rate of return on capital is a good measure of the marginal product of capital. Fiscal policy is required to avoid inefficiency and maintain full employment in a modified Diamond model with imperfect competition.

Section 2 sets out the model. Taxation and public debt are added in section 3. We first consider the fiscal requirements in steady growth and then examine the implications of variations in 'household confidence' and saving. Section 4 contains a few concluding comments.

2 A modified Diamond model

Following Diamond (1965), all agents live for two periods. They work in the first period and live off their savings in the second. The utility function for a young agent in period t takes the standard CRRA form

$$U = \frac{c_{1,t}^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho}\frac{c_{2,t+1}^{1-\theta} - 1}{1-\theta}; \qquad \theta \geq 0 \tag{1}$$

where $c_{1,t}$ and $c_{2,t+1}$ are the level of consumption per capita when the agent is young and old, respectively; θ is the inverse of the intertemporal elasticity of substitution, and ρ is the discount rate. There is full employment and we take the labor supply to be inelastic. Normalizing the supply of an individual worker to one, the budget constraint is given by

$$c_{1,t} + \frac{1}{1+r_{t+1}}c_{2,t+1} = w_t \tag{2}$$

where r_{t+1} is the rate of return on savings and w_t is the real wage.

The maximization problem implies that

$$c_{1,t} = (1-s_t)w_t \tag{3}$$

where the saving rate s can be written

$$s_t = s(r_{t+1}) = \frac{(1+r_{t+1})^{(1-\theta)/\theta}}{(1+\rho)^{1/\theta} + (1+r_{t+1})^{(1-\theta)/\theta}} \tag{4}$$

Thus, total saving (= the capital stock in the following period) is given by

$$K_{t+1} = S_t = s(r_{t+1})L_t w_t \tag{5}$$

where L_t is the number of (young) workers at time t. We assume that L_t grows at the rate n.

With a Cobb-Douglas production function and logarithmic utility ($\theta = 1$), the model produces a unique and stable steady-growth path. The general case with $\theta \neq 1$ may have multiple steady-growth paths, but the qualitative conclusion is the same: the steady growth path(s) may be dynamically inefficient. The case of inefficiency is commonly dismissed since evidence suggests that capital's rate of return exceeds the growth rate (e.g. Romer 2006, chapter 2). The equality between the rate of return and the marginal product – which lies behind the dismissal – does not hold, however, if one abandons the assumption of perfect competition.

Consider a case with imperfect competition and a fixed-coefficient production function,

$$Y_t = \min\{L_t, \sigma K_t\} = L_t \leq \sigma K_t \tag{6}$$

$$\pi = \bar{\pi} \tag{7}$$

In equation (6), σ is the output-capital ratio at full capacity utilization. Disregarding labor hoarding, we assume that the labor constraint is binding, and units are normalized to give a labor productivity of one. The constant profit share in equation (7) is consistent with a profit-maximizing markup on marginal cost. The markup is constant if firms' perceived demand function is isoelastic, and marginal cost is equal to unit labor cost if firms have excess capital capacity. Excess capacity, in turn, may be desired for a variety of reasons, including entry deterrence (Spence 1977).

Using (5)-(7), and dividing through by K_t, the growth rate of the capital stock is given by

$$1 + \hat{K}_t = \frac{K_{t+1}}{K_t} = s(r_{t+1})(1 - \bar{\pi})\frac{Y_t}{K_t} = s(r_{t+1})(1 - \bar{\pi})u_t\sigma \tag{8}$$

where $u_t \leq 1$ is the utilization rate of capital and a hat over a variable is used to denote growth rates ($\hat{x} = (dx/dt)/x$).

The utilization and accumulation rates are constant in steady growth, full employment requires that $\hat{K} = \hat{L} = n$, and by definition we have $r = \pi u \sigma$. Thus, for given values of $\bar{\pi}, \sigma$ and n, equation (8) determines the steady-growth solution for utilization, u^*.[1] There is an upper bound on utilization, $u^* \leq 1$, and full-employment growth becomes impossible if there are no solutions satisfying this restriction: with a low saving rate, the capital stock cannot expand in line with the potential labor force, even at $u = 1$.

Assuming a solution with $u^* \leq 1$, adjustments in the utilization rate below this maximum play the same role as movements along the smooth production function in the standard specification. Fixed coefficients, however, bring the dynamic inefficiency problem into stark focus: for utilization rates below one, the marginal product of capital is zero, and the empirical observation of profit rates that exceed the rate of growth does not imply dynamic efficiency.

This case highlights another issue. So far investment has been determined passively by household saving, making dynamic inefficiency the only downside

[1] With fixed coefficients and a given profit share, the solution is unique, even if $\theta \neq 1$.

of high saving. The problem is transformed into one of aggregate demand if the level of investment is determined by profit-maximizing firms: firms will not maintain a constant rate of accumulation if they have persistent, unwanted excess capacity. Steady growth, in other words, requires utilization at the desired rate, $u = u^{**}$, and the unique steady-growth rate is given by $\hat{K}^* = s^*(1-\bar{\pi})u^{**}\sigma \gtreqless n$, where $s^* = s(\bar{\pi}u^{**}\sigma)$.[2] What appeared as a problem of dynamic inefficiency when investment adjusts passively to saving, now becomes a question of ensuring the existence of a steady-growth path with full employment.[3]

Imperfect competition is central to the argument. Under perfect competition, we would have $u = u^{**} = 1$,[4] the profit share becomes indeterminate, and equation (8) with $\hat{K} = n$ could be used to determine π (for $u = u^{**}$), rather than u with $\pi = \bar{\pi}$.

3 Public debt

Extending the model, we introduce a government that consumes (G_t), levies lumpsum taxes on the young and old generations (T_t^Y and T_t^O) and has debt (B_t).[5] Young households save in the form of fixed capital and government bonds. We assume that these assets are perfect substitutes and have the same rate of return.

The saving equation (5) now takes the form

$$K_{t+1} + B_{t+1} = S_t \tag{9}$$

while the public sector budget constraint is given by

$$G_t + (1 + r_t)B_t = B_{t+1} + T_t^Y + T_t^O \tag{10}$$

The young generation in period t maximizes (1) subject to a modified constraint,

$$c_{1,t} + \frac{1}{1 + r_{t+1}} c_{2,t+1} = w_t - \tau_t - \frac{1+n}{1 + r_{t+1}} \gamma_{t+1} \tag{11}$$

where $\tau_t \equiv T_t^Y / L_t$ and $\gamma_t = T_t^O / L_t$. This gives the following solution for saving

$$S_t = [s_t(w_t - \tau_t) + (1 - s_t)\frac{1+n}{1 + r_{t+1}} \gamma_{t+1}]L_t \tag{12}$$

where s_t is given by (4). Alternatively, saving can be written

$$S_t = \tilde{s}_t(w_t - \tau_t)L_t \tag{13}$$

[2] A desired utilization rate below unity ($u^{**} < 1$) may be inefficient. This inefficiency – which may derive from entry deterrence – cannot be addressed using fiscal policy. We use the term dynamic inefficiency to denote outcomes with $u < u^{**}$.

[3] Using a different terminology, this is the Harrodian problem of discrepancies between natural and 'warranted' growth rates.

[4] Price taking firms will produce at full capacity as long as $\pi > 0$.

[5] Since the labor supply is taken to be inelastic, it is irrelevant whether the taxes on the young are lumpsum or based on wage income.

where the young generation's saving rate out of the current disposable income ($\tilde{s}_t$) is given by

$$\tilde{s}_t = \frac{s_t(w_t - \tau_t) + (1 - s_t)\frac{1+n}{1+r_{t+1}}\gamma_{t+1}}{w_t - \tau_t} \tag{14}$$

Using (12)-(13) and dividing through by L_t, (9)-(10) can be rewritten,

$$\begin{aligned} (1+n)(k_{t+1} + b_{t+1}) &= s_t(w_t - \tau_t) + (1 - s_t)\frac{1+n}{1+r_{t+1}}\gamma_{t+1} && (15) \\ &= \tilde{s}_t(w_t - \tau_t) && (16) \\ g_t + (1 + r_t)b_t &= (1+n)b_{t+1} + \tau_t + \gamma_t && (17) \end{aligned}$$

where $g_t \equiv G_t/L_t, b_t \equiv B_t/L_t, k_t = K_t/L_t$.

3.1 Steady growth

By definition $w = 1-\pi$ and $r = \pi u \sigma$, and steady growth with dynamic efficiency requires that $b_t = b, \pi = \bar{\pi}, u_t = u^{**}$. Substituting these conditions into (15), using (17), and rearranging, we get

$$b = \frac{s^*(1-\bar{\pi}) - (1+n)k^*}{1 + n + s^*(r-n)} + \frac{1}{1+r}\gamma - \frac{s^*}{1+n+s^*(r-n)}g \tag{18}$$

where $r^* = \bar{\pi}\sigma u^{**}$ and $k^* = 1/(\sigma u^{**})$, and where $s^* = s(r^*)$ is determined by (4).

The required debt (b) depends *inversely* on public consumption (g) and *directly* on the level of taxes on the old generation (γ).[6] For any given γ, an increase in g implies that consumption has to contract in order to maintain equilibrium in the product market. This is achieved by increasing taxes on the young, and as a result the desired saving decreases. This, in turn, reduces the need for government debt as an outlet for saving. Analogously, with a given value of g, an increase in γ must be accompanied by a reduction in τ in order to maintain the level of consumption and equilibrium in the goods market. The intertemporal budget constraint is unchanged, but the disposable income of the young has gone up, and the amount of public debt must increase to meet the rise in saving.

3.2 Fluctuations in 'confidence'

The analysis can be extended to cover fluctuations in saving rates across generations. The fluctuations could be the result of variations in 'confidence'. Thus, assume (in line with most evidence) that $\theta > 1$ and note that the saving rate in period t depends on the expected rate of return and the expected future taxes (equations (13)-(14)). The introduction of confidence means that there is no

[6] An inverse relation between debt and government consumption is obtained in a non-OLG setting by Schlicht (2006).

longer perfect foresight, and actual saving can deviate from what would have been chosen under perfect foresight. Our primary concern in this paper, however, is not the sources of fluctuations in saving. To simplify the analysis, we therefore consider the effects of exogenous fluctuations in the young generation's saving rate out of current disposable income ($\tilde{s}_t$).

3.2.1 Distributionally neutral intervention

The fluctuations in the saving rate can be offset by a distributionally neutral policy intervention: institute a transfer to those young generations that are unduly pessimistic (tend to consume too little) and finance the transfer by taxing the same generation when it gets old.[7] Analogously, an overly optimistic generation can be taxed in the first period and compensated by a transfer in the second.

The aim is to achieve $u_t = u^{**}$ and $k_t = k^* = 1/(\sigma u^{**})$ so as to maintain full employment and avoid inefficiency. Using these targets and substituting (16) into (17), we get

$$\tau_t = \frac{(1+n)k^* + g + (1+r^*)b_t - \gamma_t - \tilde{s}_t(1-\bar{\pi})}{1-\tilde{s}_t} \tag{19}$$

Distributional neutrality means that a generation should not be favored (or punished) because of its degree of confidence. If b^* and γ^* denote the steady-growth values of b and γ along the optimal path when there are no variations in confidence, this requirement can be stated formally as

$$(1+r^*)(k^* + b_{t+1}) - \gamma_{t+1} = (1+r^*)(k^* + b^*) - \gamma^* \tag{20}$$

The expression on the left hand side of equation (20) gives the income available to an old generation in period $t+1$. Neutrality requires that this income be equal to the level that characterizes the steady growth path.[8]

Using (19) and (20), the equation for the tax on the young at time t can now be written

$$\tau_t = \frac{(1+n)k^* + g + (1+r^*)b^* - \gamma^* - \tilde{s}_t(1-\bar{\pi})}{1-\tilde{s}_t} \tag{21}$$

Hence,

$$\frac{\partial \tau_t}{\partial \tilde{s}_t} = -\frac{1-\bar{\pi}-\tau_t}{1-\tilde{s}_t} \tag{22}$$

As long as the disposable income of the young is positive ($1-\pi-\tau_t > 0$), we have $\frac{\partial \tau_t}{\partial \tilde{s}_t} < 0$.

[7] The transfer increases the saving of the young generation as well as its consumption. But the additional saving will be more than fully absorbed by the issue of government bonds, thus leading to a decline in the amount of saving that goes into fixed capital formation.

[8] The stabilization of output and the consumption of the old generation at their steady-growth values implies that the consumption of the young will also be at its steady-growth value.

3.2.2 Tax expectations

The above analysis uses systematic variations in τ_t and γ_{t+1} to get distributional neutrality across generations. We took the saving rate $\tilde{s}$ as exogenous, however. This combination of assumptions may seem unreasonable since in general the saving rate depends on the tax structure. The private sector's anticipation of future taxes does not, however, negate the possibility of distributionally neutral stabilization.

Taxes can be used as an insurance mechanism when future taxes are anticipated. Consider the extreme case where taxes are perfectly foreseen. Formally, let γ_{t+1} be determined by

$$(1+n)\gamma_{t+1} = (1+n)\gamma^* + \beta(1+r_{t+1})(w_t - \tau_t - c_{1,t}) \tag{23}$$

$$\beta = \frac{r_{t+1} - r^*}{1 + r_{t+1}} \tag{24}$$

The tax scheme in (23)-(24) combines lumpsum taxes (γ^*) with a proportional tax on capital income (β). The proportional tax rate is conditional on the realized rate of return, and the conditionality ensures that the private after-tax rate of return always will be r^*. Thus, using (23)-(24), the household budget constraint (11) can be rewritten

$$\begin{aligned} c_{2,t+1} &= (w_t - \tau_t - c_{1,t})(1+r_{t+1}) - (r_{t+1} - r^*)(w_t - \tau_t - c_{1,t}) - (1+n)\gamma^* \\ &= (1+r^*)(w_t - \tau_t - c_{1,t}) - (1+n)\gamma^* \end{aligned} \tag{25}$$

The budget constraint becomes independent of 'confidence'. Consequently, the tax rate on the young should be set at the steady-growth level, $\tau_t = \tau^*$, and no variations are required.

3.2.3 Non-neutral intervention

Sections 3.2.1-3.2.2 assume that the private sector is subject to swings in confidence but that the government correctly anticipates the rate of return on capital and has the ability to implement fairly sophisticated tax schemes. Intergenerational neutrality and perfect government foresight are not required, however, for the stabilization of k at k^*.

Consider the simple case in which capital income is taxed at a constant (time-invariant) rate (β),

$$\gamma_t = \beta(k_t + b_t)(1 + r_t) \tag{26}$$

Unlike in section 3.2.2, β need not satisfy (24) and – returning to the case without private sector anticipation of future taxes – we take the variations in the young's saving rate out of current disposable income ($\tilde{s}_t$) to be exogenous.[9]

[9] With the proportional tax on capital income the relation (14) between $\tilde{s}$ and s implies – using (16) – that

$$\tilde{s} = \frac{s}{1 - (1-s)\beta}$$

Hence, the argument could also be phrased in terms of exogenous movements in s.

By assumption the profit is constant ($\pi = \bar{\pi}$) and the policy is designed to keep $k_t = k^*$ (and thus $r_t = r^*$).

Combining these assumptions and (26) with equations (16) and (17), the debt dynamics can be written

$$b_{t+1} = A_t - B_t b_t \tag{27}$$

where

$$A_t = \frac{\tilde{s}_t[1 - \bar{\pi} - g + \beta(1 + r^*)k^*] - k^*(1 + n)}{(1 + n)(1 - \tilde{s}_t)} \tag{28}$$

$$B_t = (1 - \beta)\frac{1 + r^*}{1 + n}\frac{\tilde{s}_t}{1 - \tilde{s}_t} \tag{29}$$

For a constant value of $\tilde{s}$ and a sufficiently large tax rate β, the difference equation (27) has a unique, stable stationary point,

$$b^{**} = \frac{A}{1 + B} \tag{30}$$

Random fluctuations in $\tilde{s}$ generate fluctuations in A and B (and thereby in b, also in the long run), but if the ratio $A_t/(1 + B_t)$ is bounded then so are the fluctuations in b.

4 Conclusion

In a modified Diamond model, fiscal policy and public debt are needed to avoid dynamic inefficiency and maintain full employment. This intervention need not affect the inter-generational distribution: fluctuations in household 'confidence' can be addressed through distributionally neutral policies.

The results raise doubts about the current policy obsession with debt and its negative aspects. In particular, austerity measures that reduce entitlement programs like social security and medicare may easily aggravate the 'debt problem': these changes correspond to a rise in the tax on the older generation, and in the model this increases the debt that is required to maintain dynamic efficiency and full employment. A reduction in government consumption, likewise, requires an increase in the long-run debt.

The model is abstract and has obvious limitations. There may be intra-generational distributional issues if heterogeneity within generations is allowed, and incentive problems may need to be considered if there are non-lumpsum taxes, to name just a couple questions that the model ignores. Even the saving assumptions that are at the center of the analysis can be questioned: only the young save, leading to the counterfactual implication that there is no saving out of capital income. The fixed-coefficient production function, finally, may seem restrictive. But fixed coefficients do not prevent efficiency under perfect competition; imperfect competition is the more important assumption.

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