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The Effect of Price Postponement on the Coordination of a Two Stage Supply Chain Facing Consumer Returns

Thomas Lenk
University of Massachusetts Amherst

THE EFFECT OF PRICE POSTPONEMENT ON THE COORDINATION OF A TWO STAGE SUPPLY CHAIN FACING CONSUMER RETURNS

A Thesis Presented

by

THOMAS LENK

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Approved as to style and content by:

Ana Muriel, Chair

Erin Baker, Member

Thomas Brashear-Alejandro, Member

Mario A. Rotea, Department Head
Department of Mechanical and Industrial Engineering

ABSTRACT

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THOMAS LENK

M.S.I.E.O.R., UNIVERSITY OF MASSACHUSETTS AMHERST

Directed by: Professor Ana Muriel

In this thesis, we analyze the effect that price postponement has on the performance and coordination of a two-stage supply chain facing consumer returns. In an extended news-vendor setting with a single product, a single manufacturer and a single retailer who faces stochastic and price-dependend demand, we allow the retailer to postpone his decision on the retail price until after demand uncertainty is resolved. A certain percentage of sold products is returned, which results in reverse logistics costs and a full refund for both consumer and retailer.

In this setting we conduct an extensive computational study to investigate the value of considering returns in the optimization approach if the decision on the retail price is postponed. Moreover we analyze the value of the additional information gained by price postponement both if returns are considered or ignored. We present the impact on the equilibrium values of profits and decision variables for both the centralized and decentralized system and consider different incentive schemes to find out whether coordination is possible.

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CHAPTER 1

INTRODUCTION

“The price might not show if the item is a future release” is a statement to be found on the website of Great-Models.com. It is also an example for a price-postponement strategy. Great-Models.com is an online retail store providing buyers with scale models of helicopters, cars etc., accessories and decals (van Mieghem and Dada (1999) [52]). The statement suggests that the retailer will determine the price of the corresponding product after demand information from the pre-orders has been observed. Also other e-tailers, such as Amazon.com, apply postponement strategies and allow customers to place orders on products that have not been released yet (Granot and Yin (2005) [22]). Tackling uncertainty has become increasingly important during the last years due to global competition, increasing flexibility of markets and a broader range and individuality of products. In that context, “postponement of one form or another has become a marketing, manufacturing and logistics concept which is applied throughout the entire supply chain”(Granot and Yin (2005) [22]). Not only e-tailers make use of postponement strategies. Benetton (e.g., Signorelli and Heskett (1984) [43], Lee and Tang (1998) [31]) and Hewlett Packard have successfully implemented strategies of that kind and are often named as leading practitioners. Therefore they motivate the analysis of postponement strategies in supply chains.

Besides uncertainty, many enterprises and especially the internet retail companies mentioned above, face another challenge that has become more and more important during the last ten years: commercial returns. The question whether the influence of returned products should be analyzed can be answered easily by the statistics provided in Gentry (1999) [20]. In the U.S., return rates from consumers to manufacturers and retailers are often in the range of 6% to 15%. For some companies, in particular e-tailers or mail order companies, they can even go up to 35%. Alltogether, the value of returned products in the U.S. has been estimated to exceed $100 billion per year (Stock et al. (2002) [45]. The major cause for the increase during the last decade can be seen in the return policies that have been relaxed with the intention to attract a higher number of costumers. Due to the high impact on both the financial and operational performance of companies the field of Reverse Logistics (Rogers and Tibben-Lembke (1999) [37]) has emerged to analyze all the aspects that are associated with the return of a product - from the appropriate design of the supply chain over the technical and organizational aspects of handling returned products to the impact of returns on the supply chain partners' operational decisions and profits.

In this thesis we study the performance and coordination of a two-stage supply chain facing consumer returns and applying the concept of price postponement. With our work we intend to extend the research conducted by Ruiz-Benitez and Muriel (2007)[39], who analyze the effect of considering returns on the profits, order quantities and prices in a two stage supply chain under different incentive schemes. In an extended news vendor model with stochastic and price dependent demand we conduct extensive computational studies to investigate the impact of considering returns versus ignoring returns on the equilibrium values of profits and decision variables if the retailer postpones his decision on the retail price. Furthermore, we analyze the the value of the additional information gained by postponement both if returns are considered and ignored, i.e. we model a supply chain with and without a price postponement strategy and compare the respective equilibrium parameters. Also, as supply chains require the cooperation of independent players, we study both aspects under different incentive schemes, in particular a wholesale price-only contract and a buy-back contract, to find out whether coordination is possible. Finally, we present analytical results that support the findings of our computational work.

The thesis is structured as follows: Chapter 2 delivers an extensive literature review on the topics postponement of operational decisions, commercial returns and supply chain coordination and contracts. In chapter 3 we present the model, assumptions and important derivations of equilibrium parameters that our work will be based on. Chapter 4 contains our computational results comparing the strategies considering returns and ignoring returns under price postponement under a wholesale price-only and a buyback contract. Under the same incentive schemes, in chapter 5 we investigate the value of the additional information gained by postponing the decision on the retail price in the decentralized centralized system. Finally, chapter 6 sums up our main findings and proposes possible extensions of this work.

CHAPTER 2

LITERATURE REVIEW

In this chapter we review the literature to the three important areas and the related concepts that our work is based on. We start out by presenting research that has been done on postponement strategies. After that we review important work focused on consumer returns and reverse logistics. A major role in this part play the dissertation by Ruiz-Benitez (2007) [38] and the paper by Ruiz-Benitez and Muriel (2007) [39]. Finally we give a review on supply chain coordination and contracts.

2.1 Postponement Strategies

In the operations literature price postponement is understood as delaying the decision on the retail price to be placed until certain uncertain attributes of demand have been observed. In our case of a two stage supply chain, which we model as an extension of the classic newsvendor problem, this means that the retailer will first observe the wholesale price offered by the manufacturer and then place his order only based on an expectation of the retail price. The latter will be determined after demand uncertainty is resolved and the demand function is known. In general, Granot and Yin (2005) [23] consider the postponing of operational decisions such as pricing, production or ordering until some or all of the uncertain attributes of demand are resolved as a strategic mechanism to manage some of the risks associated with uncertain demand. As stated in the introduction, they claim that “postponement of one form or another has become a marketing, manufacturing and logistics concept which is applied throughout the entire supply chain”. Companies such as Benetton (e.g., Signorelli and Heskett (1984) [43], Lee and Tang (1998) [31]) and Hewlett Packard show the successful implementation of strategies of that kind and are often cited as leading practitioners.

In the operations literature the effect of postponement strategies has been extensively studied. Important work that our research is based on comes from Granot and Yin (2005) [22]. They study price and order postponement in a decentralized supply chain in the classical newsboy setting with multiplicative and price-dependent demand. For different types of contracts, competition, demand distributions, expected demand functions and timings of the retailer's operational decisions they examine profits and equilibrium values of the contract parameters. They find that, in most cases, a postponement strategy is beneficial for the supply chain partners. For a multiplicative demand model and an expected demand function $D(r)$ of the form $D(r) = 1 - r$ they find that - under price postponement - if a buyback option is offered, the equilibrium wholesale price, the profit allocation ratio and the retail price coincide with those in the corresponding deterministic model, in which the demand function coincides with the expected demand function in the price-postponement model. Further work on postponement in a decentralized setting has been done by Taylor (2002) [47]. In their model they investigate the postponement of upstream decisions. In particular, the point of time when the manufacturer decides to sell to the retailer is analyzed. Our focus, however, will be on the retailer's pricing decision just like in Granot and Yin (2005) [22], i.e. we investigate the downstream decisions.

For the centralized case the effect of various postponement strategies has been studied by different authors, e.g. Lee (1996) [30] and Lee and Tang (1997) [31]. A general analysis of the benefits of postponement can be found in Aviv and Federgruen (2001) [2]. For a two-stage supply chain van Mieghem and Dada (1999) [52] show, amongst other results, that postponement of either retail price or production always benefits (weakly) a centralized firm that has to decide on capacity, production (inventory) quantity and retail price. In other words, the expected value for perfect information (EVPI) of demand is non-negative in this particular setting. We will, in our work, come back to these results and investigate whether, in our case, the EVPI is also positive if consumer returns occur and if they are considered in the decision making process.

2.2 Consumer Returns

As stated in the introduction, the motivation to consider returns can be based on the statistics provided by Gentry (1999) [20]. Increasingly relaxed consumer return policies in the U.S. during the last decades with full refunds within 30 to 90 days (no questions asked) have contributed to very high levels of consumer returns. In percentages of total sales they range from 6% to 15% normally and even reach up to 35% for mail order companies and e-tailers. Guide et. al. (2006) [24] state that a similar development can be observed in the European Union (EU) caused by new EU internet sales policies and powerful resellers from the U.S. that have entered the market. The reasons for a product return can be different. According to Ferguson et al. (2005) [16] a large percentage is considered to be false failure returns (i.e. returns that have neither a functional nor a cosmetic defect), whereas only a minor part is due to real product failure. Table 2.1 shows the percentages of different reasons for the return of HP printers. True failure forms only 20% of their total returns. The customer's expectation or simply a change of mind play an equally important role. With each return causing costs of around 25% of the products price, a decrease in the product return rate is highly valueable.

Based on this motivation Ferguson et al. (2005) [16] present a target rebate contract that helps to reduce the percentage of false failure returns by increasing the retailer's sales effort. Specifically, the retailer receives a certain dollar amount for each unit of false failure returns below a target. In that way an incentive scheme

Reason for Return	***Percentage***	***Solution***
True failure	20	Design
Install/Basic Use	27	$Design, AfterSalesSupport$
Performance		Design, Retail Pre-Sales Qualification
Print Speed	25	
Print Quality		
Packaging	2.25	Design, Retail Pre-Sales Qualification
Sales Technique	12.75	Retail Pre-Sales Qualification
Consumer Behavior	12.5	Retail Policy

Table 2.1. Reasons for Returns of HP printers and suggested solutions (Ferguson et al. (2005) [16])

is established that leads to the reduction of false failure returns. Further research concerning an improved design of supply chains, in particular for end-of-life product returns, has been conducted by Guide et al. (2001) [24] and Fleischmann (2001) [17]. They point out the necessity of considering the trade-off between efficiency (cost efficiency) and responsiveness (time effectiveness) of the supply chain. They claim that increasing cost efficiency has to be weight up with increasing processing times during which obsolescence costs occur. Being more responsive a network will reduce the latter, but, however, incur higher costs.

In the classical setting of the newsvendor problem Vlachos and Decker (2003) [54] derive optimal order quantities under different return handling options (secondary markets, partial recovery or full recovery). Mostard and Teunter (2006) [34] allow for the returned products to be resold any number of times and find a closed form solution for the optimal order quantity in this setting.

The effect of considering returns on the equilibrium parameters in a two stage newsvendor setting is exhaustively studied by Ruiz-Benitez (2007) [38] and Ruiz-Benitez and Muriel (2007) [39]. In particular they compare profits, wholesale price, retail price and chosen order quantities with those associated with the classical newsboy problem ignoring returns. They find that not taking into account returns results in higher profits and better coordination than if they are considered. Furthermore they show the benefit of increasing the retailer's share of the return logistics costs for the profits of both players and the whole supply chain. Also, they find that buy-back contracts can have a negative effect on supply chain coordination if consumer returns are ignored. Finally, short studies of return allowance credit contracts (see also the working paper by Ruiz-Benitez and Muriel (2007) [40]) and price postponement in this setting are presented. Return allowance credits are another way of dealing with returns in a supply chain. Instead of taking back returned products the manufacturer offers the retailer a certain amount of money and guidelines about how to correctly dispose of the product (Corbett and Savaskan (1999) [10]). Large retail companies such as Wal-Mart specifically design distribution centers for that purpose. Allowance credits lie at 1.5% for Wal-Mart and between 3% to 5% for some other companies. Finally, Sullivan (1997) [46], Lariviere and Padmanabhan (1997) [28], and Bloom et al. (2000) [7] present research conducted on slotting allowances. The latter is considered a marketing practice for new products, in which the manufacturer creates an incentive for the retailer to carry new products by transferring a fixed sum of money. Ruiz-Benitez and Muriel (2007) [39] model a return allowance credit as a lump sum transfer of a percentage of the cost of either the total sales or the ordered quantity. The transfer is aimed to compensate the retailer for the costs associated with returns and thus to maintain retailer's good will.

As we will conduct an analysis concerning supply chain coordination under different incentive schemes, the third part of this review covers the respective literature in that field.

2.3 Supply Chain Contracts and Coordination

We start out by presenting literature for price-only contracts. Next we consider buy-back contracts, first under stochastic demand, then under stochastic and price dependend demand. At last, we review other types of contracts.

Operations literature speaks of a price-only contract if the only parameter to be determined is the wholesale price, set by the manufacturer. For a two-stage supply chain with a single retailer and a single manufacturer in a fixed price newsvendor setting with stochastic demand Lariviere and Porteus (2001) [29] discover the coefficient of variation to be an important factor for the performance of the supply chain and the split of profits. For increasing values the retailer becomes more price sensitive, the wholesale price decreases and the decentralized system looses efficiency. Both the supply chain's and the manufacturer's profits decrease, whereas the retailer's profits increase due to the lower wholesale price.

The phenomenon of "double marginalization" ((Spengler (1950) [44]) is a typical example for the necessity of incentive schemes. In a decentralized supply chain, as the retailer bases his decisions on the wholesale price instead of the lower production costs, his order quantity is smaller than for the centralized system. To better coordinate the system's performance, supply chain contracts have emerged to provide incentives that lead the decentralized system to a similar performance as in the centralized case. Lariviere (1999) [27], Tsay et al. (1999) [50] and Cachon (2003) [8] present extensive reviews on supply chain contracts and coordination literature.

A particular type of contract in this context is the buy-back contract. The effect of double marginalization is reduced, i.e. the retailer's order quantity is increased by the manufacturer's offer to buy back unsold quantities at the end of the selling season. The resulting higher payout for lower demand periods encourages the retailer to increase his inventory. It is shown by Pasternack (1985) [36] that a policy that does not allow for returns is not optimal. However, a policy that allows for unlimited returns under full credit is not optimal either. It is demonstrated that for an appropriate choice of wholesale and buyback price allowing the retailer unlimited returns at a partial credit achieves coordination of the supply chain. It has to be noted that buy back contracts are independent of the demand distribution, which can be considered another advantage of this type of coordination. Further research on buy-back-contracts has been conducted by Donohue (2000) [13] and Gong (2002) [21]. The former author shows that in a two stage production environment with two distinct production modes a contract that requires to determine the buy-back price and the two different wholesale prices corresponding to the production mode can coordinate the supply chain. The later author focuses on the study of buy back contracts under asymmetric information, which is of minor relevance for our work.

For the price-dependendent newsvendor model several authors have explored the effect of buy back contracts. Kandel (1996) [25]) claims that only if the manufacturer is able to establish resale price maintainance coordination is possible through buy-back contracts in this setting. Emmons and Gilbert (1998) [14] find that offering a buyback price for unsold products most of the time has a positive effect on the total supply chain profits, if demand is random and price-dependent and the wholesale price is given. Granot and Yin (2005a) [23] deliver another piece of work on buy-back contracts in a system with random and price-dependent demand. For a linear expected demand, they find closed form expressions for the optimal wholesale and buyback prices. If demand also is dependent on the retail price, Bernstein and Federgruen (2005) [6] find that coordination of the supply chain is not possible by offering a buyback option.

A comparison of the no returns policy (i.e. the manufacturer does not buy back unsold item) and the full returns policy (i.e. the retailer receives a full refund for all unsold products) under price-dependent demand can be found in Padmanabhan and Png (1997) [35]. Under no competition between retailers and uncertain demand, the retailer reacts with over-stocking to the offer of a returns policy. As a consequence the manufacturer is worse off than when he offers no returns. However, the manufacturer benefits from a returns policy if the retailers are competitive and demand is certain as retail competition is intensified. Also, the authors show that, under sufficiently low marginal production cost and uncertainty accepting returns from the retailer is beneficial for the manufacturer.

Research on alternative contracts has been conducted by various authors. For the fixed price newsvendor model, quantity flexibility (QF) contracts (Tsay (1999) [48], Tsay and Lovejoy (1999) [49], Anupindi and Bassok (1998) [1], sales-rebate contracts (Taylor (2002) [47]) and revenue-sharing contracts (Cachon and Lariviere (2005) [9]) have been studied. Analysis concerning price-discount sharing contracts can be found in Bernstein and Federgruen (2005) [6], the profit-sharing contract in Yao et al. (2002) [55]. In a two-period model with correlated demand Barnes-Schuster et al. (2002) [4] analyze the impact of supply chain contracts with options on channel coordination. In this setting, prior to the first period, the retailer not only determines the order quantities to be delivered at the beginning of each period, but additionally buys options that entitle him to order further units of the product at the beginning of the second period, after he has observed information about the demand in the first period. In a finite horizon model Bassok and Anupindi (1997) [5] derive the optimal order policy if the buyer faces a minimum quantity commitment and a discount price. Corbett and DeCroix (2001) [12] and Corbett et al. (2003) [11] present an analysis of shared-saving contracts applied to indirect materials. The latter are materials that have no relationship to the quantity produced of the final product(e.g. office supplies). They show that, although the supplier wants to increase his sales volume, the supply chain becomes more efficient if both can exert effort to reduce consumption, i.e. under a shared-savings contract both partners are better off if consumption decreases. At last, Corbett and Savaskan (1999) [10] claim that, in practice, contracts are rarely capable of achieving perfect coordination. They conclude that more complex contracts must be developed such as two-part tariffs instead of linear prices or even “menus” of contracts. Furthermore, to implement more sophisticated contracts of that kind, the presence of multiple flows of goods due to product returns may be a helpful mechanism.

In chapter 5 we have a closer look at the impact of the additional information gained by postponement. In particular economics literature provides several articles that discuss the influence of learning and different amounts of informativeness on decision variables and objective values. For cooperative games Epstein (1980) [15] presents a method that allows to analyze the differences in first-period decisions for varying levels of informativeness. The value of perfect information in a non-cooperative game setting is determined by Sakai (1985) [41] and Ulph and Ulph (1996) [51]. Gal-Or (1985) [18] and Vives (1989) [53] study the influence of information in oligopolies. A more general and less complicated approach is presented by Baker (2006) [3]. She presents a sequential decision problem that is very similar to our setting. A decision maker decides in period one, then observes an informative signal before he makes his second decision in period two. In our model the retailer first decides on the order quantity, then observes further demand information and then sets the retail price of the product. In this setting Baker (2006) [3] analyzes the comparative statics of risk and of learning. She finds that both are the same in the absence of risk aversion for first and second period decisions as well as for the value of information. Furthermore she concludes that risk attitude can be ignored when analyzing the qualitative effects of learning which allows studying the more simple case of risk neutrality without loss of generality. In our model we assume that both the retailer and the manufacturer have the same information on demand and both act according to the strategy they have agreed on earlier. As a consequence the literature that considers the value of information in a game setting is less relevant for our work. However, results that are valid for a single decision maker as well, like the ones presented in Baker (2006) [3], provide an interesting theoretical background for our computational results. Also work conducted on the value of sharing information between the supply chain partners as e.g. by Lee et al. (2000) [32] and Gavimeni et al. (1999) [19] is stated for matter of completeness only as we assume that both partners have the same information.

With the model and the analysis that we present in the following we intend to extend the work done on consumer returns and price postponement by Ruiz-Benitez(2007) [38]. To the best of our knowledge, no other work considers performance and coordination issues in the presence of commercial returns and price postponement.

CHAPTER 3

MODEL FORMULATION

In this chapter we present the model, corresponding assumptions and the derivations that our work is based on. In general, we study the same extended news vendor setting that is described by Ruiz-Benitez (2007) [38], but under a different timing of decisions. For simplicity and comparability reasons, we use the same notations and parameters.

3.1 The Model

The model consists of a single manufacturer, a single retailer and a single product. The retailer faces random and price dependent demand, i.e. the retail price r is not exogenously given but set by the retailer (after observing demand if he decides to postpone his decision). A selling season is composed of a single period during which the selling price remains constant. We do not allow for back ordering and we do not consider any costs associated with customer dissatisfaction. The product is produced by the manufacturer at a cost c with unlimited production capacity and sold to the retailer at a retail price of w. For unsold products we will include a buy-back price of s when considering buy-back contracts. In the case of wholesale-contracts the salvage value of v for unsold products will remain with the retailer. A percentage of α of total sales are returned to the retailer, who gives the respective customers a full refund of the retail price r. The manufacturer finally buys back the returns for the full wholesale price w and retrieves a salvage value of v_r for each item. As we further analyze the influence of different shares of the costs incurred by this process (shipping, inspection, possibly remanufacturing and final disposition) we include average logistics cost of l_1 and l_2 per unit for the manufacturer and the retailer, respectively. We define the parameter β as the retailer's share of the return logistic costs, i.e. $\beta = \frac{l1}{l1+l2} \times 100$. In reality, the value of β may be different for different industries depending on which partner takes responsibility in handling the returned products.

As in existing research on contracts literature, a few assumptions are made to have a meaningful problem. We require $r > w > c$ and $w > s > v \geq v_r$. Also, we assume $(1-\alpha)r > (1-\alpha)w + \alpha l_2$ and $(1-\alpha)w > c + \alpha(l_1 - v_r)$ to ensure that both the retailer's and manufacturer's profits are positive when considering returns. Further restrictions to the parameters are mentioned when needed.

Finally, the demand faced by the retailer, the order quantity and the product returns are captioned by the variables y, Q and R, respectively. We define $R = \alpha S$ the volume of product returns as a fixed percentage of the total sales ($S = \min(Q, y)$).

Figure 3.1 depicts the financial flows in this simple two-stage supply chain.

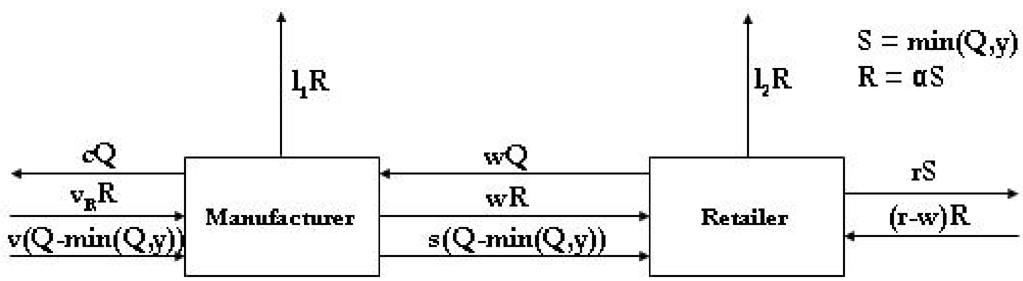

Figure 3.1. Financial Flows in the Two-Stage Supply Chain (Ruiz-Benitez (2007) [38])

Stochastic and Price-dependent Demand

As stated above we assume that the retailer faces a stochastic and price dependent demand. Literature offers two groups of price-dependent models: the additive demand model, $D(r, x) = y(r) + x$, (Mills (1959) [33]) and the multiplicative demand model,

$D(r, x) = y(r)x$, (Karlin and Carr (1962) [26]), where $y(r)$ is decreasing in the retail price r and x is the random component. Concerning the form of $y(r)$, broad studies can be found in literature , in particular for the linear case $y(r) = a - br(a > 0, b > 0)$ and the iso-elastic demand curve $y(r) = ar^{-b}(a > 0, b > 1)$. For reasons of consistency and comparability our model and assumptions are similar to the ones presented in Emmons & Gilbert (1998) [14], with the necessary extensions that allows us to include consumer returns. The expected demanded quantity, $D(r)$, defined on a closed interval $[c, \bar{r}]$ (i.e. $D(r) = 0$ for all $r \geq \bar{r}$) is assumed to be decreasing in the retail price r, continuous, nonnegative and twice differentiable. x is modeled as a positive random variable with mean equal to 1. Therefore the actual demand, y, will be presented as a product of the expected demand $D(r)$ and the random component x. The resulting density distribution function for the demand consequently can be expressed as follows:

$$g(y; r) = \frac{1}{D(r)} f(\frac{y}{D(r)}) \text{ for } y \geq 0, \tag{3.1}$$

where $f(\cdot)$ is the probability distribution function of x and $F(\cdot)$ is the corresponding cumulative distribution function. The distribution $F(\cdot)$ is assumed to be invertible and $f(\cdot)$ shall have a continuous derivative $f'(\cdot)$.

Information and Decision Strategies

When making their decisions both players are assumed to have full information about the expected demand function $D(r)$ and the distribution of x. Also, we assume, that both either include returns in their optimization problem or not, i.e. we do not consider asymmetric behavior. We define the following four possible strategies for the retailer:

- Policy CRP: The retailer both considers returns and postpones his decision on the retail price until demand uncertainty is resolved when optimizing his profits.

- Policy IRP: The retailer follows the postponement strategy but ignores commercial returns in his optimization model.

- Policy CR: The retailer decides on both the retail price and his order quantity before demand uncertainty is resolved.

- Policy IR: The retailer ignores commercial returns and does not follow the postponement strategy.

In the following section we summarize the most important derivations for the players' profits and the results of the analytical optimizations given they exist.

3.2 Profit Functions and Derivations when Considering Returns under Price Postponement

3.2.1 The Retailer's Problem

If the retailer postpones his pricing decision the sequence of events can be distinguished into three stages:

1. At first, before products are ordered or delivered, the manufacturer determines the wholesale price w, and the repurchase price s if he offers a buy back contract.

2. After that, the retailer determines his optimal order quantity and places the order to the retailer.

3. Finally, after observing demand uncertainty x, the retailer determines the selling price r.

Taking into account this sequence of events, we can present the following results obtained by Ruiz-Benitez (2007) [38] for $y(r)$ representing a linear demand curve, i.e. $y(r) = b(r-k)(k > 0, b < 0)$ (different forms of $y(r)$ lead to different results). The retailer's profit at the end of the selling season can be calculated as:

$$\Pi_R^{CRP}(Q, r, \bar{x}) = ((1-\alpha)r - \alpha(l_2 - w) - s)\min(Q, \bar{D}(r)) - (w-s)Q$$

$\bar{D}(r)$ represents the demand function after the stochastic variable $\bar{x}$ is fixed, i.e. demand becomes deterministic when uncertainty is resolved. To find the optimal retail price r, we distinguish between two cases:

1. If $Q \leq \bar{x}y(r)$, i.e., $r \leq y^{-1}(Q/\bar{x})$, then, we can write the retailer's profit as

$$\Pi_R^{CR}(Q, r, \bar{x}) = ((1-\alpha)r - \alpha(l_2 - w) - w)Q$$

This function increases in r and therefore the optimal retail price is $r^* = D^{-1}(Q/\bar{x}) = \frac{Q}{\bar{x}b} + k$, i.e. the retailer will always increase the selling price r until demand is no larger than available supply.

2. If $Q \geq \bar{x}y(r)$, i.e., $r \geq y^{-1}(Q/\bar{x})$, we can simplify the retailer's profit function to

$$\Pi_R^{CR}(Q, r, \bar{x}) = ((1-\alpha)r - \alpha(l_2 - w) - s)\bar{D}(r) - (w-s)Q$$

Taking the first derivative and setting it equal to zero, we get

$$\frac{\partial \Pi_R^{CR}(Q, r, \bar{x})}{\partial r} = (1-\alpha)\bar{x}b(r-k) + ((1-\alpha)r - \alpha(l_2 - w) - s)\bar{x}b$$

and thus, the optimal retail price is:

$$r^* = \frac{(1-\alpha)k + \alpha(l_2 - w) + s}{2(1-\alpha)}$$

Note that the realized demand does not have any influence on the optimal retail price. Only the parameter k of the expected demanded quantity $y(r)$ can be found in the formula.

The second derivative with respect to r is non-positive:

$$\frac{\partial^2 \Pi_R^{CRP}(Q, r, \bar{x})}{\partial r^2} = 2(1-\alpha)\bar{x}b < 0$$

Thus, the retailer's profit function is strictly concave and we get

$$r^* = \max\Big(\frac{(1-\alpha)k + \alpha(l_2 - w) + s}{2(1-\alpha)}, \frac{Q}{\bar{x}b} + k\Big)$$

The threshold for the optimal decision on the retail price in terms of the realized uncertainty, $\bar{x}$, therefore is:

$$\bar{x} \leq \frac{2(1-\alpha)Q}{b(-(1-\alpha)k + \alpha(l_2 - w) + s)}$$

Summarizing, given (w, s, Q), the retailer will choose the selling price r according to the following rule:

$$r^*(\bar{x}) = \begin{cases} \frac{Q}{\bar{x}b} + k & if \bar{x} > \frac{2(1-\alpha)Q}{b(-(1-\alpha)k + \alpha(l_2 - w) + s)} \\ \frac{(1-\alpha)k + \alpha(l_2 - w) + s}{2(1-\alpha)} & if \bar{x} \leq \frac{2(1-\alpha)Q}{b(-(1-\alpha)k + \alpha(l_2 - w) + s)} \end{cases} \quad (3.2)$$

The expected value of the retail price consequently is:

$$E[r^*] = k + \frac{-(1-\alpha)k + \alpha(l_2 - w) + s}{2(1-\alpha)} F(A) + \frac{Q}{b} \int_A^{\infty} \frac{1}{x} f(x) dx$$

where $A = \dfrac{2(1-\alpha)Q}{b(-(1-\alpha)k + \alpha(l_2 - w) + s)}$. At the point of his decision on the order quantity Q, however, the retailer lacks the knowledge of x. Therefore, his decision at this point of time is based on the knowledge about his own future behavior.

Using the knowledge about the dependency of the retail price on the chosen order quantity and the realized uncertainty his aim is to maximize his expected profits:

$$\Pi_R^{CRP}(Q,r^*) = E_x[((1-\alpha)r^* - \alpha(l_2 - w) - s)D(r^*)x] - (w-s)Q$$

If we replace the optimal selling price r^* by (3.2) we get the following term for the retailer's expected profits as a function of Q and r^*:

$$\begin{aligned}
\Pi_R^{CRP}(Q,r^*) &= ((1-\alpha)B - \alpha(l_2 - w) - s)b(B-k)\int_0^A xf(x)dx \\
&\quad +((1-\alpha)k - \alpha(l_2 - w) - s)Q(1-F(A)) \\
&\quad +\frac{(1-\alpha)Q^2}{b}\int_A^\infty \frac{1}{x}f(x)dx - (w-s)Q
\end{aligned}$$

where $A = \dfrac{2(1-\alpha)Q}{b(-(1-\alpha)k + \alpha(l_2 - w) + s)}$ and $B = \dfrac{(1-\alpha)k + \alpha(l_2 - w) + s}{2(1-\alpha)}$.

Taking first and second derivatives with respect to Q it can be shown that the retailer's expected profit function is strictly concave:

$$\begin{aligned}
\frac{\partial \Pi_R^{CR*}(Q,r)}{\partial Q} &= ((1-\alpha)k - \alpha(l_2 - w) - s)(1-F(A)) \\
&\quad +\frac{(1-\alpha)2Q}{b}\int_A^\infty \frac{1}{x}f(x)dx - (w-s) \qquad (3.3)
\end{aligned}$$

$$\frac{\partial^2 \Pi_R^{CRP}(Q,r)}{\partial Q^2} = \frac{(1-\alpha)2}{b}\int_A^\infty \frac{1}{x}f(x)dx < 0 \qquad (3.4)$$

since $b < 0$. Consequently, setting the first derivate with respect to Q equal to zero delivers a necessary and sufficient condition for the optimal order quantity, Q^*:

$$((1-\alpha)k - \alpha(l_2 - w) - s)(1-F(A)) + \frac{(1-\alpha)2Q}{b}\int_A^\infty \frac{1}{x}f(x)dx - (w-s) = 0 \quad (3.5)$$

Unfortunately as A depends on Q it is not possible to obtain a closed form expression for Q^*, which reasonably complicates any further analytical approaches. However, using equation 3.5 we obtain the following solution for the expected retail price in equilibrium:

Proposition 1 *In the two-stage news vendor-model considering consumer returns under price postponement the expected retail price under linear, stochastic and price-dependent demand is*

$$E[r^*] = \frac{(1-\alpha)k + \alpha(l_2 - w) + w}{2(1-\alpha)}$$

Proof.

$$\begin{aligned} E[r^*] &= k + \frac{-(1-\alpha)k + \alpha(l_2 - w) + s}{2(1-\alpha)} F(A) + \frac{Q}{b}\int_A^\infty \frac{1}{x} f(x)dx \\ &= k + \frac{w - (1-\alpha)k + \alpha(l_2 - w)}{2(1-\alpha)} \\ &= \frac{(1-\alpha)k + \alpha(l_2 - w) + w}{2(1-\alpha)} \end{aligned} \tag{3.6}$$

∎

3.2.2 The Manufacturer's Problem

The manufacturer decides on the wholesale price w (and the repurchase price s if he offers a buy-back contract) before the retailer makes any decision. As he is fully aware of the retailer's behavior he anticipates both the optimal order quantity and the expected retail price for each of his choices of w. He therefore maximizes the following function:

$$\begin{aligned} \Pi_M^{CRP}(Q^*, r^*, w, s) &= (w - c + v - s)Q^* - E_x[(\alpha(l_1 + w - v_r) + v - s)y(r^*)x] \\ &= (w - c + v - s)Q^* - (\alpha(l_1 + w - v_r) + v - s)E_x[y(r^*)x] \\ &= (w - c + v - s)Q^* - (\alpha(l_1 + w - v_r) + v - s)b(E_x[r^*x] \\ &\quad -kE_x[x]) \end{aligned}$$

Using the assumption on the expected value of the demand uncertainty $E_x[x] = 1$ and the law of conditional expectation with expression 3.2 for the optimal retail price we can simplify $E_x[r^*x]$:

$$\begin{aligned}
E_x[r^*x] &= E_x\left[\frac{(1-\alpha)k+\alpha(l_2-w)+s}{2(1-\alpha)}x\right]F(A)+E_x\left[\left(\frac{Q}{xb}+k\right)x\right](1-F(A)) \\
&= BE_x[x]F(A)+\left(E_x\left[\frac{Q}{b}\right]+E_x[kx]\right)(1-F(A)) \\
&= \left(B-\frac{Q}{b}-k\right)F(A)+\frac{Q}{b}+k
\end{aligned}$$

Substituting $E_x[r^*x]$ by this expression in the manufacturer's profit function and employing the relationship $b(B-k)=\frac{Q}{A}$, we get:

$$\begin{aligned}
\Pi_M^{CRP}(Q^*,r^*,w,s) &= (w-c+v-s)Q^*-(\alpha(l_1+w-v_r)+v-s) \\
&\quad \left[\left(\frac{Q^*}{A}-Q^*\right)F(A)+Q^*\right] \\
&= (w-c+v-s)Q^*-(\alpha(l_1+w-v_r)+v-s) \\
&\quad \left[1+\frac{1-A}{A}F(A)\right]Q^* \\
&= [w-c-\alpha(l_1+w-v_r) \\
&\quad -(\alpha(l_1+w-v_r)+v-s)\frac{1-A}{A}F(A)]Q^*
\end{aligned}$$

where $A=\dfrac{2(1-\alpha)Q}{b(-(1-\alpha)k+\alpha(l_2-w)+s)}$ and $B=\dfrac{(1-\alpha)k+\alpha(l_2-w)+s}{2(1-\alpha)}$. Unfortunately as we do not have a closed form expression for Q^*, the optimal wholesale price w^* cannot be obtained analytically. In our computational study we will apply the golden section method to find w^*. As this approach requires the function to be concave we plotted the manufacturer's profit function for the various input parameters and were able to ensure that it is concave.

Note that for the case of ignoring returns we can set the parameter α equal to zero in both the retailer's and manufacturer's problem. Doing so, we face similar difficulties in not being able to resolve the respective equations. Granot and Yin (2005) [22], however, show that there exists a one-to-one relationship between the equilibrium values of Q^* and w^* given by the first order condition for the optimal order quantity. We will specify the exact derivations and relationships in the next section.

3.2.3 The Centralized System

As for the centralized supply chain retailer and manufacturer are no longer separate players, the interaction between them and in particular the determination of the optimal wholesale price, i.e. stage 1, does not exist. The decisions to be made therefor only consist of placing the order or - in this case - production quantity and then, after uncertainty is resolved, determining the retail price. For the derivation we again start out by calculating the optimal selling price, r^*, and then, the optimal order quantity, Q^* based on the expectation for the former. The results are similar to the retailer's problem and can be obtained by simply replacing $(l_2 - w)$ with $(l - v_r)$, w with c and $s = 0$. For the optimal retail price we therefore can write

$$r^*(\bar{x}) = \begin{cases} \dfrac{Q}{\bar{x}b} + k & if x > \dfrac{2(1-\alpha)Q}{b(-(1-\alpha)k + \alpha(l - v_r))} \\ \dfrac{(1-\alpha)k + \alpha(l - v_r)}{2(1-\alpha)} & if \bar{x} \leq \dfrac{2(1-\alpha)Q}{b(-(1-\alpha)k + \alpha(l - v_r))} \end{cases} \tag{3.7}$$

The expected value of the retail price is calculated as follows:

$$E[r^*] = \frac{(1-\alpha)k + \alpha(l - v_r) + c}{2(1-\alpha)}$$

Similarly to the retailer's profit function concavity can also be shown for the centralized profits and therefore the sufficient and necessary condition for the optimal order quantity, Q^*, is given by:

$$((1-\alpha)k - \alpha(l - v_r))(1 - F(A')) + \frac{(1-\alpha)2Q}{b}\int_{A'}^{\infty}\frac{1}{x}f(x)dx - c = 0 \qquad (3.8)$$

where $A' = \dfrac{2(1-\alpha)Q}{b(-(1-\alpha)k + \alpha(l - v_r))}$. Again, we cannot resolve for Q since A' depends on Q.

3.3 Derivations for Policy IRP

Our aim in chapter 4 is to compare the equilibrium values for profits and decision variables under the policies CRP and IRP. Since the policy CRP was studied in the previous section, we start out by stating the most important results of the derivations for the policy IRP in order to perform the intended analysis. The derivations for both policy CR and IR, to which we compare our results in chapter 5, can be found in Ruiz-Benitez (2007) [38].

3.3.1 Profit Functions

The policy IRP resembles the price-dependent news vendor model under postponement except that we subtract the costs $(l_2 + r - w)$ and $(l_1 + w - v_R)$ associated with the returns R after the decisions have been made from the profit function of the retailer and the manufacturer, respectively. Granot and Yin (2005) [22] present the derivations for the associated model without returns for the case of the expected demand function equal to $D(r) = 1 - r$ and without a salvage value for unsold products. For our more general case of $D(r) = b(r - k)$ and the salvage values v and v_R for unsold and returned products we obtain similar results by setting the parameter α representing the percentage of total sales returned equal to zero in the derivations of the previous section and following the same backward induction procedure. The retail price for given values of Q, w and $\bar{x}$ consequently can be written as

$$r^*(\bar{x}) = \begin{cases} \frac{Q}{\bar{x}b} + k & if \bar{x} > \frac{2Q}{b(s-k)} \\ \frac{k+s}{2} & if \bar{x} \leq \frac{2Q}{b(s-k)} \end{cases} \tag{3.9}$$

This results in the following expected value that the retailer faces when making his decision on the order quantity Q:

$$E[r^*] = k - \frac{k-s}{2} F\left(\frac{2Q}{b(s-k)}\right) + \frac{Q}{b} \int_{\frac{2Q}{b(s-k)}}^{\infty} \frac{1}{x} f(x)dx \tag{3.10}$$

Accordingly, the first order condition for the optimal order quantity Q^* is:

$$(k-s)(1 - F\left(\frac{2Q}{b(s-k)}\right)) + \frac{2Q}{b} \int_{\frac{2Q}{b(s-k)}}^{\infty} \frac{1}{x} f(x)dx - (w-s) = 0 \tag{3.11}$$

The value of Q given by equation 3.11 is a global maximum as the second derivative of the retailer's profit with respect to Q is strictly negative, as shown before. Again it is not possible to obtain a closed form solution for the optimal order quantity. However, equation 3.11 can be resolved for w and thus, there exists a one-to-one relationship between w and Q:

$$w = k - (k-s)F\left(\frac{2Q}{b(s-k)}\right) + \frac{2Q}{b} \int_{\frac{2Q}{b(s-k)}}^{\infty} \frac{1}{x} f(x)dx \tag{3.12}$$

In their paper, Granot and Yin (2005) [22] make use of this possibility to express the manufacturer's profit function in terms of the optimal order quantity by replacing w by the term obtained from the retailer's first order condition. For the case of the expected demand function $D(r)$ equal to $D(r) = 1 - r$, they present an interval containing the optimal order quantity Q^* and closed form solutions for the wholesale price w and the repurchase price s if a buyback contract is offered. For the wholesale case further restrictions are necessary. If the density function of the uncertainty x, $f(x)$, is continuous and differentiable, and $xf(x)$ is increasing in x they again find an interval containing the optimal order quantity Q^* and implicit expressions for the wholesale and expected retail price. In the following we re-derive these results for our more general expected demand function $D(r) = b(r-k)$ and including salvage values for unsold and returned products. We note that Granot and Yin (2005) [22] show how the more general results without salvage values can be obtained from their specific results for $D(r) = b(r-k)$. We start out with the manufacturer's profit function if he ignores returns and the retailer postpones his decision on the retail price. It can be stated as follows:

$$\Pi_M^{IRP}(Q^*, r^*, w, s) = (w - c + v - s)Q^* - E_x[(v-s)D(r^*)x] \tag{3.13}$$

The expected value in the second part of the function can be simplified by considering expression 3.9 for the optimal retail price under policy IRP:

$$\begin{aligned}
E_x[(v-s)y(r^*)x] &= (v-s)E_x[b(r^*-k)x] \\
&= (v-s)b(E_x[r^*x] - kE_x[x]) \\
&= (v-s)b(E_x[r^*x] - k) \\
&= (v-s)b[\int_0^{\frac{2Q}{b(s-k)}} \frac{k+s}{2} xf(x)dx \\
&\quad + \int_{\frac{2Q}{b(s-k)}}^{\infty} (\frac{Q}{xb} + k)xf(x)dx - k] \\
&= (v-s)b\left(\frac{k+s}{2}\int_0^{\frac{2Q}{b(s-k)}} xf(x)dx + \frac{Q}{b}\int_{\frac{2Q}{b(s-k)}}^{\infty} f(x)dx\right) \\
&\quad +(v-s)b\left(k\int_{\frac{2Q}{b(s-k)}}^{\infty} xf(x)dx - k\right) \\
&= (v-s)b\left(\frac{s-k}{2}\int_0^{\frac{2Q}{b(s-k)}} xf(x)dx + \frac{Q}{b}\int_{\frac{2Q}{b(s-k)}}^{\infty} f(x)dx\right)
\end{aligned}$$

Using this simplification and equation 3.12 we can restate the manufacturer's profit:

$$
\begin{aligned}
\Pi_M^{IRP}(Q^*, s) &= (w - c + v - s)Q^* - E_x[(v - s)b(r^* - k)x] \\
&= \left(k - (k - s)F\left(\frac{2Q^*}{b(s-k)}\right) + \frac{2Q^*}{b}\int_{\frac{2Q^*}{b(s-k)}}^{\infty} \frac{1}{x}f(x)dx\right)Q^* \\
&\quad -(c + s - v)Q^* \\
&\quad -(v - s)b\left(\frac{s-k}{2}\int_0^{\frac{2Q^*}{b(s-k)}} xf(x)dx + \frac{Q^*}{b}\int_{\frac{2Q^*}{b(s-k)}}^{\infty} f(x)dx\right)
\end{aligned}
$$

3.3.2 Optimality Criteria under a Buyback Contract

We have now replaced the variable for the wholesale price w and thus continue working with a function in the variables Q^* and s. Taking first and second derivatives with respect to the optimal order quantity Q^* and the buyback offer s allows us to formulate two propositions concerning the optimal decision variables under a buyback and a wholesale contract. As the results for the latter can be easily derived from the buyback solutions, we begin with the former.

Proposition 2 *When ignoring returns under price postponement in a news-vendor setting with linear and price-dependent expected demand the manufacturer globally maximizes his expected profit by offering a wholesale price* $w^* = \frac{k+c}{2}$ *and a buyback price* $s^* = \frac{k+v}{2}$ *if he commits to a buyback contract. In equilibrium the retailer's optimal order quantity* Q^* *satisfies*

$$k - (k - v)F\left(\frac{4Q^*}{b(v-k)}\right) + \frac{4Q^*}{b}\int_{\frac{4Q^*}{b(v-k)}}^{\infty} \frac{1}{x}f(x)dx - c = 0, \tag{3.14}$$

and lies in the interval $\left[0, \frac{b(v-k)}{4}B\right]$ *if the probability distribution for the uncertainty term* x *has positive values not beyond an upper bound* B*. In addition we have the following expressions:*

$$
\begin{aligned}
E[r^*] &= \frac{3k+c}{4} \\
\Pi_M^{IRP*} &= \frac{Q^*}{2}\left[k-c+(v-k)F\left(\frac{4Q^*}{b(v-k)}\right)\right] \\
&\quad -\frac{b}{8}\left((v-k)^2\int_0^{\frac{4Q^*}{b(v-k)}} xf(x)dx-\alpha(c-k)(2l_1+k+c-2v_R)\right) \\
Pi_R^{IRP*} &= \frac{Q^*}{4}\left[k-c+(v-k)F\left(\frac{4Q^*}{b(v-k)}\right)\right] \\
&\quad -\frac{b}{16}\left((v-k)^2\int_0^{\frac{4Q^*}{b(v-k)}} xf(x)dx-\alpha(c-k)(4l_2+k-c)\right)
\end{aligned}
$$

Note that in the case of no returns occurring, i.e. $\alpha = 0$, we get:

$$
\begin{aligned}
\Pi_M^{IRP}* &= 2E[\Pi_R^{IRP}]^* \\
&= \frac{Q^*}{2}\left[k-c+(v-k)F\left(\frac{4Q^*}{b(v-k)}\right)\right]-\frac{b}{8}(v-k)^2\int_0^{\frac{4Q^*}{b(v-k)}} xf(x)dx
\end{aligned}
$$

Proof. In order to show the results described before, we take first and second derivatives with respect to the variables Q^* and s:

$$
\begin{aligned}
\frac{\partial\Pi_M^{IRP}(Q^*,s)}{\partial s} &= [F\left(\frac{2Q^*}{b(s-k)}\right)+(k-s)f(\frac{2Q^*}{b(s-k)})\frac{2Q^*}{b(s-k)^2} \\
&\quad +\frac{2Q^*}{b}\frac{b(s-k)}{2Q^*}f(\frac{2Q^*}{b(s-k)})\frac{2Q^*}{b(s-k)^2}]Q^*-Q^* \\
&\quad +b[\frac{s-k}{2}\int_0^{\frac{2Q^*}{b(s-k)}} xf(x)dx+\frac{Q^*}{b}\int_{\frac{2Q^*}{b(s-k)}}^{\infty} f(x)dx] \\
&\quad -(v-s)b[\frac{1}{2}\int_0^{\frac{2Q^*}{b(s-k)}} xf(x)dx-\frac{s-k}{2}\frac{2Q^*}{b(s-k)} \\
&\quad f\left(\frac{2Q^*}{b(s-k)}\right)\frac{2Q^*}{b(s-k)^2}+\frac{Q^*}{b}f\left(\frac{2Q^*}{b(s-k)}\right)\frac{2Q^*}{b(s-k)^2}] \\
&= F\left(\frac{2Q^*}{b(s-k)}\right)Q^*-Q^*+\frac{b(s-k)}{2}\int_0^{\frac{2Q^*}{b(s-k)}} xf(x)dx \\
&\quad +Q^*\int_{\frac{2Q^*}{b(s-k)}}^{\infty} f(x)dx-\frac{(v-s)b}{2}\int_0^{\frac{2Q^*}{b(s-k)}} xf(x)dx \\
&= \frac{b(2s-k-v)}{2}\int_0^{\frac{2Q^*}{b(s-k)}} xf(x)dx
\end{aligned}
$$

In case of existence of stationary point(s), we find a unique value of s which is $s = \frac{k+v}{2}$. Taking the first derivative with respect to the optimal order quantity Q^* results in:

$$\begin{aligned}
\frac{\partial \Pi_M^{IRP}(Q^*, s)}{\partial Q^*} &= k - (k-s)F\left(\frac{2Q^*}{b(s-k)}\right) + \frac{2Q^*}{b}\int_{\frac{2Q^*}{b(s-k)}}^{\infty} \frac{1}{x} f(x)dx \\
&+ [-(k-s)f\left(\frac{2Q^*}{b(s-k)}\right)\frac{2}{b(s-k)} + \frac{2}{b}\int_{\frac{2Q^*}{b(s-k)}}^{\infty} \frac{1}{x} f(x)dx \\
&- \frac{2Q^*}{b}\frac{b(s-k)}{2Q^*} f\left(\frac{2Q^*}{b(s-k)}\right)\frac{2}{b(s-k)}]Q^* - (c+s-v) \\
&- (v-s)b[\frac{s-k}{2}\frac{2Q^*}{b(s-k)} f\left(\frac{2Q^*}{b(s-k)}\right)\frac{2}{b(s-k)} \\
&+ \frac{1}{b}\int_{\frac{2Q^*}{b(s-k)}}^{\infty} f(x)dx - \frac{Q^*}{b} f(\frac{2Q^*}{b(s-k)})\frac{2}{b(s-k)}] \\
&= k - (k-s)F\left(\frac{2Q^*}{b(s-k)}\right) + \frac{2Q^*}{b}\int_{\frac{2Q^*}{b(s-k)}}^{\infty} \frac{1}{x} f(x)dx \\
&+ \frac{2Q^*}{b}\int_{\frac{2Q^*}{b(s-k)}}^{\infty} \frac{1}{x} f(x)dx - (c+s-v) \\
&- (v-s)\left[1 - F\left(\frac{2Q^*}{b(s-k)}\right)\right] \\
&= k - (k-v)F\left(\frac{2Q^*}{b(s-k)}\right) + \frac{4Q^*}{b}\int_{\frac{2Q^*}{b(s-k)}}^{\infty} \frac{1}{x} f(x)dx - c \qquad (3.15)
\end{aligned}$$

Evaluating the later at $s = \frac{k+v}{2}$ and setting it equal to zero leads to equation 3.14:

$$\begin{aligned}
\frac{\partial \Pi_M^{IRP}(Q^*, s^* = \frac{k+v}{2})}{\partial Q^*} &= k - (k-v)F\left(\frac{2Q^*}{b(\frac{k+v}{2} - k)}\right) + \frac{4Q^*}{b}\int_{\frac{2Q^*}{b(\frac{k+v}{2}-k)}}^{\infty} \frac{1}{x} f(x)dx - c \\
&= k - (k-v)F\left(\frac{4Q^*}{b(v-k)}\right) + \frac{4Q^*}{b}\int_{\frac{4Q^*}{b(v-k)}}^{\infty} \frac{1}{x} f(x)dx - c = 0
\end{aligned}$$

Analyzing the first derivative with respect to Q^* at $Q^* = 0$ and for $Q \to \infty$ allows us to draw conclusions about the existence of stationary points:

$$\frac{\partial \Pi_M^{IRP}(Q^* = 0, s^* = \frac{k+v}{2})}{\partial Q^*} = k - c > 0 \tag{3.16}$$

and

$$lim_{Q\to\infty} \frac{\partial \Pi_M^{IRP}(Q^*, s^* = \frac{k+v}{2})}{\partial Q^*} = v - c < 0, \tag{3.17}$$

because $k > c > v$ to insure a meaningful problem. Thus we conclude that there exists one unique stationary point $(s^* = \frac{k+v}{2}, Q^*)$. Forming the second derivatives with respect to each of the two variables and evaluating them at $(s^* = \frac{k+v}{2}, Q^*)$ results in the following Hessian Matrix:

$$H\left(Q^*, s^* = \frac{k+v}{2}\right) = \begin{pmatrix} \frac{4}{b}\int_{\frac{4Q^*}{b(v-k}}^{\infty} \frac{1}{x} f(x)dx & 0 \\ 0 & b\int_0^{\frac{4Q^*}{b(v-k}} xf(x)dx \end{pmatrix},$$

which is negative definite as $b < 0$ and both integral terms are positive. Consequently, the point $(s^* = \frac{k+v}{2}, Q^*)$ globally maximizes the manufacturer's profit.

The closed form expression for w^* is obtained by inserting $s^* = \frac{k+v}{2}$ into equation 3.12 and using expression 3.14 to simplify the term:

$$\begin{aligned} w^* &= k - (k-s)F\left(\frac{2Q}{b(s-k)}\right) + \frac{2Q}{b}\int_{\frac{2Q}{b(s-k)}}^{\infty} \frac{1}{x} f(x)dx \\ &= k - \frac{k-v}{2} F\left(\frac{4Q}{b(v-k)}\right) + \frac{2Q}{b}\int_{\frac{4Q}{b(v-k)}}^{\infty} \frac{1}{x} f(x)dx \\ &= k + \frac{1}{2}\left[-(k-v)F\left(\frac{4Q}{b(v-k)}\right) + \frac{4Q}{b}\int_{\frac{4Q}{b(v-k)}}^{\infty} \frac{1}{x} f(x)dx\right] \\ &= k + \frac{1}{2}(c-k) \\ &= \frac{k+c}{2} \end{aligned}$$

Similarly, we obtain the following term for the expected retail price $E[r^*]$ in equilibrium:

$$
\begin{aligned}
E[r^*] &= k - \frac{k - \frac{k+v}{2}}{2} F\left(\frac{2Q}{b\left(\frac{k+v}{2} - k\right)}\right) + \frac{Q}{b}\int_{\frac{2Q}{b\left(\frac{k+v}{2}-k\right)}}^{\infty} \frac{1}{x} f(x)dx \\
&= k + \frac{1}{4}\left[-(k-v)F\left(\frac{4Q}{b(v-k)}\right) + \frac{4Q}{b}\int_{\frac{4Q}{b(v-k)}}^{\infty} \frac{1}{x} f(x)dx\right] \\
&= k + \frac{1}{4}(c-k) \\
&= \frac{3k+c}{4}
\end{aligned}
$$

The optimal expected profits are obtained in a similar manner. Note that we need to subtract the costs associated with the returns, which were not considered during the optimization process. For the case of $f(x)$ only being defined on an interval [0,B] all upper integral bounds equal to infinity are replaced by j. As a consequence we analyze the first derivative of the manufacturer's profit function at the order quantity Q^* for which the last integral term turns to zero, i.e. for $\frac{4Q^*}{b(v-k)} = B$ or $Q^* = \frac{b(v-k)}{4}B$.

∎

3.3.3 Optimality Criteria under a Wholesale Contract

Based on the derivations above we can formulate the following results for the wholesale price contract setting.

Proposition 3 *When no re-purchase price is offered under price postponement in a news-vendor setting with linear, price-dependent expected and ignoring returns under price postponement, the retailer's equilibrium order quantity Q^* is the unique solution that satisfies*

$$
k - (k-v)F\left(\frac{2Q^*}{b(s-k)}\right) + \frac{4Q^*}{b}\int_{\frac{2Q^*}{b(s-k)}}^{\infty} \frac{1}{x} f(x)dx - c = 0
$$

and, in equilibrium:

$$
\begin{aligned}
w^* &= \frac{1}{2}\left[k + c - (k+v)F\left(\frac{2Q^*}{b(-k)}\right)\right] \\
E[r^*] &= \frac{1}{4}\left[3k + c - (k+v)F\left(\frac{2Q^*}{b(-k)}\right)\right]
\end{aligned}
$$

if the density function of x, $f(x)$, is continuous and differentiable, and $xf(x)$ is increasing in x.

Proof.

If no repurchase price is offered the manufacturer's profit is only a function of the retailer's optimal order quantity Q^*. From 3.15 we get the first derivative with respect to Q^* by setting the variables for the wholesale price s and the salvage value of unsold products v (v remains with the retailer in the wholesale case) equal to zero:

$$
\frac{d\Pi_M^{IRP}(Q^*)}{dQ^*} = k - kF\left(\frac{2Q^*}{b(-k)}\right) + \frac{4Q^*}{b}\int_{\frac{2Q^*}{b(-k)}}^{\infty} \frac{1}{x} f(x)dx - c
$$

After some simplifications the second derivative with respect to Q^* can be expressed as:

$$
\begin{aligned}
\frac{d^2\Pi_M^{IRP}(Q^*)}{dQ^{*2}} &= \frac{2}{b} f\left(\frac{2Q^*}{b(-k)}\right) + \frac{4}{b}\int_{\frac{2Q^*}{b(-k)}}^{\infty} \frac{1}{x} f(x)dx - \frac{4}{b} f\left(\frac{2Q^*}{b(-k)}\right) \\
&= \frac{4}{b}\int_{\frac{2Q^*}{b(-k)}}^{\infty} \frac{1}{x} f(x)dx - \frac{2}{b} f\left(\frac{2Q^*}{b(-k)}\right) \qquad (3.18)
\end{aligned}
$$

As we are unable to evaluate the sign of expression 3.18 we continue and form the third derivative:

$$
\begin{aligned}
\frac{d^3\Pi_M^{IRP}(Q^*)}{dQ^{*3}} &= -\frac{4}{b}\frac{1}{Q^*} f\left(\frac{2Q^*}{b(-k)}\right) - \frac{4}{b^2(-k)} f'\left(\frac{2Q^*}{b(-k)}\right) \\
&= -\frac{4}{b}\left[\frac{1}{Q^*} f\left(\frac{2Q^*}{b(-k)}\right) + \frac{1}{b(-k)} f'\left(\frac{2Q^*}{b(-k)}\right)\right] \qquad (3.19)
\end{aligned}
$$

If we demand $xf(x)$ to be increasing in x, we get:

$$
f\left(\frac{2Q^*}{b(-k)}\right) + \frac{2Q^*}{b(-k)} f'\left((\frac{2Q^*}{b(-k)}\right) \geq 0
$$

And after rearranging the variables in this expression we find:

$$\frac{1}{Q^*}f\left(\frac{2Q^*}{b(-k)}\right)+\frac{1}{b(-k)}f'\left(\frac{2Q^*}{b(-k)}\right)>\frac{1}{Q^*}f\left(\frac{2Q^*}{b(-k)}\right)+\frac{1}{b(-k)}f'\left(\frac{2Q^*}{b(-k)}\right)\geq 0$$

Therefore, as $b < 0$ by definition, the third derivative 3.19 of the manufacturer's profit is positive and the consequently the second derivative in 3.18 is strictly increasing. Analyzing the first derivative with respect to Q^* at $Q^* = 0$ and for $Q \to \infty$ and the second derivative for $Q \to \infty$ allows us to draw conclusions about the existence of a unique solution:

$$\frac{d\Pi_M^{IRP}(Q^*=0)}{dQ^*}=k-c>0, \tag{3.20}$$

$$lim_{Q\to\infty}\frac{d\Pi_M^{IRP}(Q^*)}{dQ^*}=-c<0 \tag{3.21}$$

and

$$lim_{Q\to\infty}\frac{d^2\Pi_M^{IRP}(Q^*)}{dQ^{*2}}=0 \tag{3.22}$$

Based on that we conclude that the second derivative is negative for all values of $Q < \infty$ and thus there exists a unique solution Q^* that satisfies the first order condition

$$\frac{d\Pi_M^{IRP}(Q^*)}{dQ^*}=k-kF\left(\frac{2Q^*}{b(-k)}\right)+\frac{4Q^*}{b}\int_{\frac{2Q^*}{b(-k)}}^{\infty}\frac{1}{x}f(x)dx-c=0$$

The expression for w^* is obtained by inserting the implicit expression for Q^* and $s = 0$ into equation 3.12. Similarly, using the obtained terms for w^*, Q^* and $s = 0$ we obtain the stated expressions for the expected retail price and each of the partners profits. ∎

CHAPTER 4

CONSIDERING VERSUS IGNORING RETURNS UNDER PRICE POSTPONEMENT

In this chapter we compare the policies CRP and IRP, i.e. we evaluate the behavior of the supply chain if the retailer postpones his decision on the retail price and either does take into account returns or not. We perform a computational study for the system under both a wholesale and a buyback contract. In order to analyze the value of considering returns, we compare our results under price postponement to the outcomes in Ruiz-Benitez and Muriel (2007)[39], where the decision on the retail price is not delayed. Chapter 5 then will focus on the value of the additional information gained by postponing the pricing decision, i.e. we compare the systems performance for considering returns with and without postponement.

To perform computational work we need to establish specific parameter values and probability distribution functions. For the following analysis we use a uniform distribution on the interval (0,2) to represent the probability distribution function of the uncertainty term x, i.e. $x \sim U(0,2)$. For the expected demand of the form $D(r) = b(r-k)$ we assume the parameter values $(b,k) = (-3,5)$. Both Emmons & Gilbert (1998) [14] and Ruiz-Benitez (2007) [38] use the same parameter specifications in their work. In our base case we will consider a return volume of $\alpha = 20\%$, a retailer's share of the retrun logistics costs of $\beta = 5\%$, salvage values $v = v_R = 0$ and also production costs $c = 1$ if not stated otherwise. We conduct a sensitivity analysis in the latter parameters in order to give our observations more weight (see table 4.1). Also we include further computational results for different values of b and k in the appendix and mention important differences. Note that using the assumption

b	k	c	β	α	l	v	v_R
-3	5	$[1, 3]$	$[5\%, 95\%]$	$[6\%, 35\%]$	$[1, 3]$	$[0, 1]$	$[0.5v, v]$
-3	2.5	$[0.2, 1.6]$	$[5\%, 95\%]$	$[6\%, 20\%]$	$[1, 3]$	$[0, c]$	$[0.5v, v]$
-1.5	10	$[1, 3]$	$[5\%, 95\%]$	$[6\%, 35\%]$	$[1, 3]$	$[0, 1]$	$[0.5v, v]$

Table 4.1. Scope of the sensitivity analysis

of a uniform distribution for the distribution of x allows to simplify the derivations we presented so far. However, the problem of being unable to resolve the first order condition for Q remains. Also, note that we will use the same parameter values as stated here as our base case throughout our further computational work. To investigate the effect of considering versus ignoring returns under price postponement we analyze the equilibrium values for the decision variables as well as the supply chain partners' and the overall profits for the decentralized case. To evaluate the coordination of the system we establish the profits in the centralized setting as a benchmark.

4.1 CRP vs. IRP under a Wholesale Price-Only Contract

We start out by presenting our observations for the defined base case without a buyback offer by the manufacturer, i.e. unsold products remain with the retailer at a salvage value of v.

Figure 4.1 shows the optimal order quantity Q, the expected retail price $E[r]$ and the wholesale price w for different production cost values c. We find that the optimal order quantity is higher when both players ignore returns. The retail and wholesale price, however, are lower if returns are not taken into account. These relationships seem intuitive. When considering returns, prices are higher to cope with the costs caused by the expected number of returned products and lower sales come with a lower absolute number of returns.

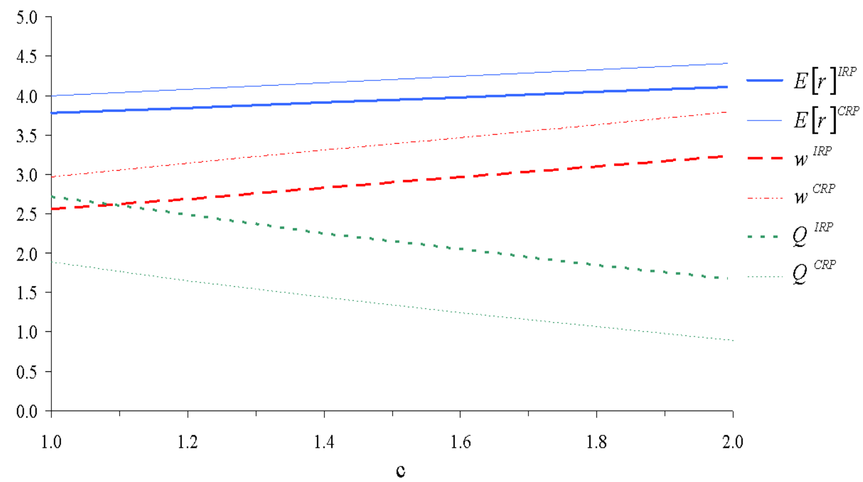

Figure 4.1. Optimal decision variables for different production cost values and for policies IRP and CRP when no repurchase price is offered

The left graph in figure 4.2 contains the expected supply chain profits for the centralized and decentralized system as a function of the production costs c. We find that *under price postponement and no buyback offer considering returns leads to higher profits for the whole supply chain than ignoring returns*, as indicated by the blue lines. This observation stands in contrast to the results delivered by Ruiz-Benitez and Muriel (2007)[39], who conduct similar analysis for the case without price postponement. They find that both the supply chain profits and the profits of the partners are higher if returns are ignored as indicated by the right graph in figure 4.2. Under price postponement the preference for considering returns even increases for higher production costs c, i.e. while profits decline the difference between IRP and CRP becomes larger. As the absolute gap between order quantities remains fairly constant for higher values of c (see Figure 4.1) the impact of the returned goods becomes more accentuated by the increasing value of the product.

From Figure 4.2 we also discover that the absolute profit gap to the centralized system considering returns decreases for higher production costs c. The percentage values in table 4.2 show the relative differences between the two systems, calculated

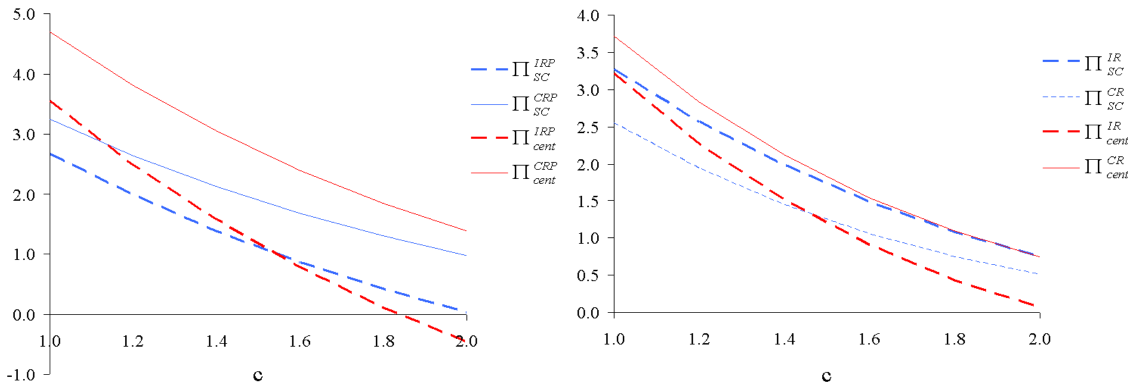

Figure 4.2. Total supply chain profits for the centralized and decentralized systems under policies CRP and IRP and CR and IR for $\alpha = 20\%$, $\beta = 5\%$ and varying production costs c under a wholesale contract

as $\frac{\Pi_{cent} - \Pi_{decent}}{\Pi_{cent}}$. As the wholesale price increases at a lower rate than c (see Figure 4.1), the relative difference between the two parameters becomes smaller. As a consequence the effect of double marginalization is reduced and the optimal order quantity for the decentralized system lies closer to the optimal Q in the centralized case. Thus, *coordination is better for higher product values.* Also in the context of double marginalization, we additionally observe the following: *for the case of ignoring returns the centralized system is outperformed by the decentralized system for high values of c* (see again Figure 4.2). In the decentralized system the retailer's order quantity is lower than in the centralized case as he purchases the products for the wholesale price w and not the production costs c. Thus he sees a lower profit margin. However, ignoring returns leads the decentralized system to a higher order quantity than optimal under returns. Consequently, the effect is compensated for higher values of c when the difference between w and c becomes smaller. The decentralized system's order quantity ends up being closer to the optimal centralized one when considering returns.

	IRP			CRP		
	decentr.	centr.	%Δ	decentr.	centr.	%Δ
production cost $c = 1$						
$E[r^*]$	3.776	3.000	-20.6%	3.992	3.375	-15.5%
Q^*	2.707	6.578	142.9%	1.886	4.787	153.8%
Π_{SC}	2.664	3.550	33.2%	3.246	4.695	44.6%
production cost $c = 2$						
$E[r^*]$	4.113	3.500	-14.9%	4.407	4.000	-9.2%
Q^*	1.663	3.787	127.7%	0.892	2.096	135.0%
Π_{SC}	0.031	-0.464	-1587.3%	0.976	1.384	41.8%
production cost $c = 3$						
$E[r^*]$	4.427	4.000	-9.6%	4.788	4.625	-3.4%
Q^*	0.903	1.985	119.9%	0.238	0.531	123.5%
Π_{SC}	-0.991	-1.943	-96.1%	0.101	0.141	39.8%

Table 4.2. Comparison of the centralized and decentralized system for $\alpha = 20\%$ and $\beta = 5\%$ under the wholesale contract

Looking at the profits of the single partners in figure 4.3 , we observe that - for the chosen value of $\beta = 0.05$, i.e. the manufacturer faces 95% of the reverse logistics costs l - the latter clearly is better off when considering returns. The retailer, however, carrying the smaller part of l, prefers to ignore returns. We will further investigate these assymetric preferences in the upcoming sensitivity analysis. We further note that the growing preference for CRP in the production costs c is mainly contributed by the manufacturer whereas the retailer's profit gap to policy IRP remains fairly constant.

Our major observations for the wholesale contract under price postponement and the parameter assumptions of our base case can be summed up as follows:

- Order quantities are lower, retail and wholesale price higher when returns are considered.
- Considering returns leads to higher expected total profits for the supply chain.

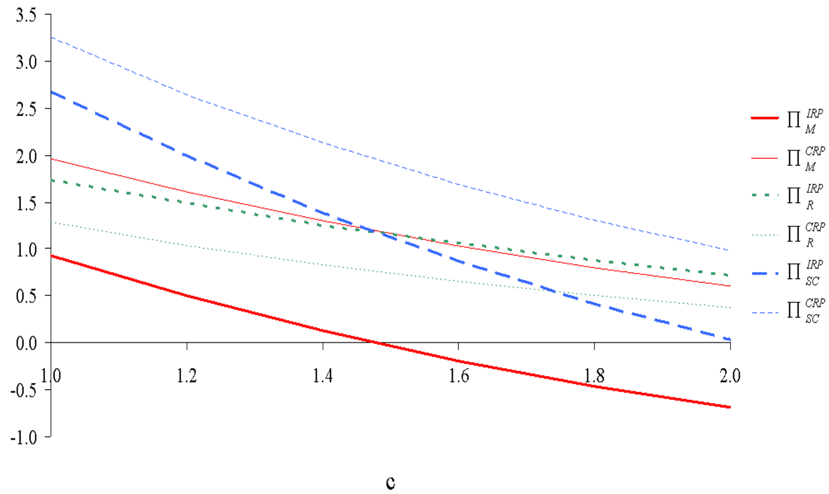

Figure 4.3. Profit functions in c of the decentralized system for the policies IRP and CRP for $\alpha = 20\%$ and $\beta = 5\%$ under a wholesale contract

- Higher production costs result in better coordination of the decentralized supply chain both when considering and ignoring returns, and in greater profit loss associated with ignoring returns.
- Although the supply chain is better off if returns are included, this strategy may be profitable for only one of the partners.

To investigate whether these conclusions stay valid for different choices of the parameters we conduct a sensitivity analysis in the production cost c, the return rate α, the retailer's share of the return logistics cost β, the salvage values for unsold and returned products, v and v_R, as well as for the parameters b and k, which determine the price elasticity and market size of the associated expected linear demand function $D(r) = b(r - k)$. In short, we find that our first three observations remain valid - more or less accentuated - during the whole scope of our computations. Concerning the single players' profits we determine the share of return logistics costs to be the decisive factor for their preference of the return strategy. Moreover we find that under postponement the latter also leads to better coordination of the supply chain if the retailer faces the higher share. Finally we point out the following difference in influence of parameters: Under higher production costs c coordination improves and considering over ignoring returns becomes more profitable for *both* partners. An increase in the driving factors for the impact of returns, α, l and v_R, makes considering returns more preferable, but coordination gets worse and the preferences of the single players concerning their strategy remain dependent on the share β of the return logistics costs. In what follows we describe the sensitivity analysis in detail.

Different share of return logistics costs

Our analysis so far was based on a value for β of $\beta = 0.05$, which means that the manufacturer carries 95% of the return logistics cost. Figure 4.4 displays the behavior of both the decision variables (left graph) and the profits if the parameter β is varied from 5% to 95%. We observe that the wholesale price w under policy CRP decreases in β whereas the expected retail price stays (fairly) constant and the optimal order quantity increases only slightly. The decrease in w obviously is based on the decreasing return costs for the manufacturer and, at the same time, compensates the retailer's decreasing profit margin. As a consequence of the lower wholesale price the retail price stays constant and the expected profits of both partners and of the total supply chain increase (slightly) as can be seen from Figure 4.4. In other words, *increasing the retailer's share of the return logistics costs improves the coordination of the supply chain.* We point out that Ruiz-Benitez and Muriel (2007)[39] observe the same in their comparison of the policies CR and IR, i.e. without price postponement.

Considering the profits of each of the supply chain partners in the decentralized system for considering versus ignoring returns, we find that we have to distinguish between different values for the retailer's share of the return logistics costs β: *When the retailer faces a low share of the return logistics costs, the manufacturer prefers to*

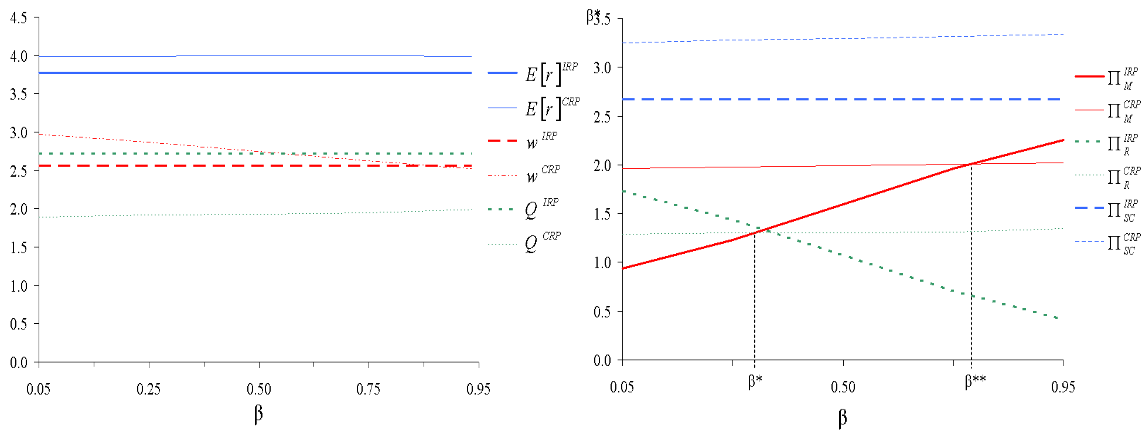

Figure 4.4. Optimal decision variables and profits for different values of β under policies IRP and CRP and $\alpha = 20\%$, no repurchase price offered

consider returns whereas the retailer is better off if the latter are ignored. The opposite is the case for high values of β. Figure 4.3 represents the partner's and the supply chain profits as a function of the production costs c for our base case with $\alpha = 20\%$ and $\beta = 5\%$. For this choice of the parameters the retailer's profits are higher if both partners ignore returns whereas the manufacturer is better off by far if returns are considered as he carries 95% of the return logistics costs. The supply chain partners' preferences concerning the return policy, however, change for increasing values of β. The right graph in Figure 4.4 shows that below a certain value β^{**} the manufacturer is better off if returns are considered in the system. If the retailer's share falls below β^* he prefers to ignore returns and thus again, considering returns is suboptimal for one of the partners. Figure 4.4, however, also shows that, for the chosen parameters, there exists a range (β^*, β^{**}) in which both partners prefer to consider returns.

We note, however, that for higher production costs as shown in figure 4.5 ($c = 3$), both players will prefer considering returns no matter who bears the higher logistics cost. We point out that the production costs are the only driving factor for this change. Neither an increased return percentage nor higher logistics cost result in the

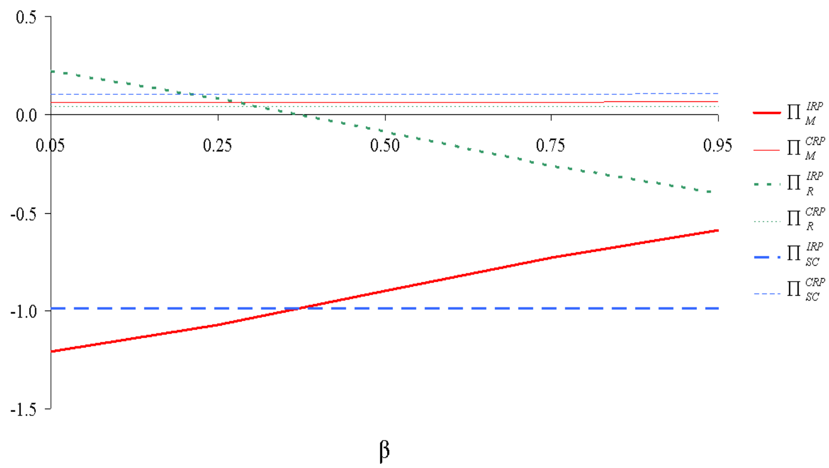

Figure 4.5. Profits in the decentralized system under policies CRP and IRP for $\alpha = 20\%$, $c = 3$ and varying share of return logistics costs β under a wholesale contract

CRP becoming the best choice for both players. Therefore, in a setting in which both players are allowed to choose their respective policy, these observations have to be considered as they might lead to asymmetric behavior. Possible consequences of the latter in a similar setting without price postponement are studied in Schmitt (2007) [42].

Different percentage of sales returned and different logistics costs

For an increased volume of returns α and for higher return logistics cost l we observe similar behavior in the decision variables as for increased values of c; this is basically the effect of increased costs associated with returns. Figure 4.6 shows that higher costs associated with returns lead to an increase in the expected optimal retail price $E[r^*]$ and the optimal wholesale offer w^* while the optimal order quantity Q^* decreases. Note again that the increase of the wholesale price becomes lower if the the manufacturer's share of the return logistics cost $1 - \beta$ decreases.

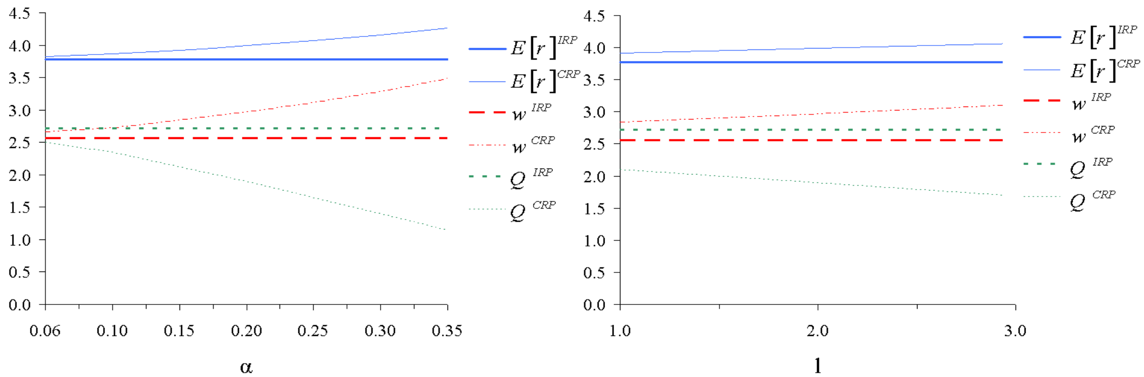

Figure 4.6. Optimal decision variables for different volumes of returns α and return logistics cost l under policies IRP and CRP and $\beta = 5\%$, $l = 2$ (left graph), $\alpha = 20\%$ (right graph); no repurchase price offered

From Figure 4.7 we find that both for higher percentages of sales returned and higher return costs, considering returns becomes more and more profitable for the supply chain, the reason simply being the higher mistake in the optimization process associated with ignoring returns (not a decreased double marginalization effect). For the displayed case of $\beta = 5\%$ the manufacturer prefers to consider returns whereas the retailer would be better off to ignore them. Again these preferences change for higher values of β.

As stated earlier, however, higher return costs do not have the same effect as the production costs c on each of the partners' preferences regarding considering or ignoring returns, i.e. they do not lead to the former being more profitable for both players no matter who shares the higher part of the return costs l. Table 4.3 shows that, for constant production costs $c = 1$, a significant increase in both return rate and logistics costs from $(\alpha, l) = (6\%, 1)$ to $(35\%, 3)$ does not change the single partner's preferences concerning the optimal strategy. In fact, higher costs caused by returns significantly increase both the absolute and relative difference to the unprefered strategy (see bold values and their counterparts). Also in contrast

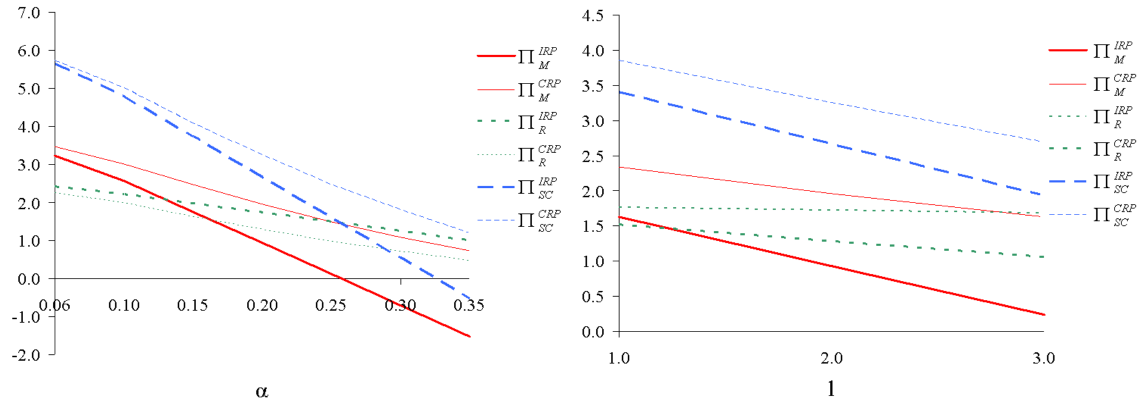

Figure 4.7. Expected profits for different volumes of returns α ($l = 2$) and return logistics cost l ($\alpha = 20\%$) under policies IRP and CRP and $\beta = 5\%$; no repurchase price offered

to the positive influence of higher production costs c on the coordination of the supply chain, we point out that both higher returns and higher values of l lead to an increase in the relative difference to the profits of the centralized system, i.e. coordination becomes worse as shown by the percentage values at the bottom of table 4.3: As an example, the profits of the centralized system are 38.8% higher than in the decentralized system for 6% returns and return logistics costs of $l = 1$. For $\alpha = 35\%$ and $l = 3$ the gap has increased to 43.3% of the profits in the decentralized supply chain. As a sidenote, table 4.3 again shows the positive influence on supply chain profits of an increase in the retailer's share β of the return logistics costs. For high costs associated with returns the coordination gap decreases from 60.2% to 43.3%. Also note the negative percentage numbers under policy IRP indicating that the decentralized system outperforms the centralized system for high costs associated with returns.

	$\alpha = 6\%, l = 1$		$\alpha = 35\%, l = 3$	
	IRP	CRP	IRP	CRP
$\beta = 5\%$				
Π_M	3.432	**3.614**	-2.739	**0.405**
Π_R	**2.422**	2.348	**0.937**	0.252
Π_{SC}	5.854	5.962	-1.802	0.656
$\%\Delta$	38.5%	39.1%	−69.3%	60.2%
$\beta = 95\%$				
Π_M	**3.630**	3.626	**0.731**	0.446
Π_R	2.223	**2.348**	-2.532	**0.288**
Π_{SC}	5.854	**5.974**	-1.802	**0.734**
$\%\Delta$	38.5%	38.8%	−69.3%	43.3%

Table 4.3. Influence of increased return volumes α and return logistics costs l on the partner's policy preferences (bold values) and on supply chain coordination (percentage difference to the centralized system) for $c = 1$ and different values for the retailer's share β of l

		IRP			**CRP**		
v	v_R	$E[r^*]$	w^*	Q^*	$E[r^*]$	w^*	Q^*
$\beta = 5\%$							
0.2	**0.1**	3.766	2.532	2.811	3.976	2.928	1.971
0.6	**0.3**	3.743	2.487	3.064	3.940	2.855	2.181
1.0	**0.5**	3.715	2.429	3.419	3.885	2.746	2.512
$\beta = 95\%$							
0.2	**0.1**	3.761	2.523	2.759	3.976	2.476	2.029
0.6	**0.3**	3.736	2.471	2.852	3.936	2.396	2.153
1.0	**0.5**	3.700	2.400	2.983	3.889	2.303	2.304
$\beta = 5\%$							
0.2	**0.2**	3.766	2.532	2.811	3.968	2.911	1.995
0.6	**0.6**	3.743	2.487	3.064	3.915	2.806	2.262
1.0	**1.0**	3.715	2.429	3.419	3.849	2.672	2.654
$\beta = 95\%$							
0.2	**0.2**	3.766	2.532	2.811	3.968	2.460	2.106
0.6	**0.6**	3.743	2.487	3.064	3.926	2.377	2.382
1.0	**1.0**	3.715	2.429	3.419	3.852	2.229	2.896

Table 4.4. Optimal values of decision variables for increasing salvage values under the wholesale contract; base case parameter specifications

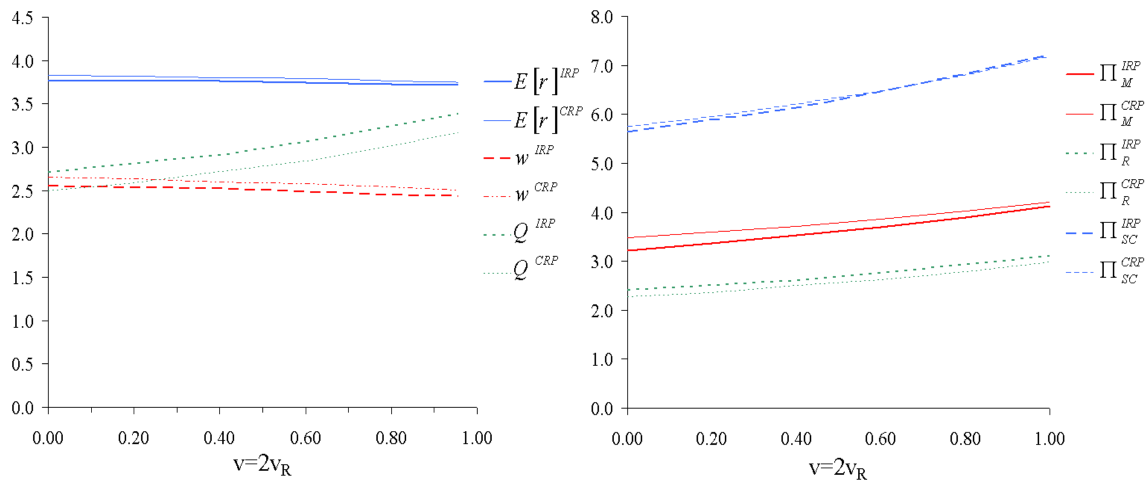

Figure 4.8. Optimal values of the decision variables and respective expected profits as a function of the salvage values v for unsold products and v_R for returned products, which are assumed to be equal; no repurchase price offered

Positive salvage values

So far we have only considered both the salvage value for unsold products v and for returned products v_R to be equal to zero. Note that under a wholesale price contract surplus inventory at the end of the selling season stays with the retailer and thus he realizes the salvage value v. Products returned by the consumer are bought back for the full wholesale price w by the manufacturer, who retrieves an average value of v_R per item. The following computational results show that positive salvage values have the opposite effect on the behavior of the optimal values of the decision variables and the expected profits than the two parameters α and l. This is intuitive, since it reduces the costs associated with returns. Figure 4.8 assumes a return rate of $\alpha = 20\%$, production costs of $c = 1$, return logistics cost of $l = 2$, of which 5% is carried by the retailer, and that the salvage value for returned products is half the salvage value for unsold products, i.e. $v_R = 0.5v$. We observe the expected: the optimal wholesale and retail prices are higher when considering returns, but the absolute difference decreases (slightly). For a low manufacturer's share of the return

		IRP				CRP			
v	v_R	Π_M	Π_R	Π_{SC}	$\%\Delta$	Π_M	Π_R	Π_{SC}	$\%\Delta$
$\beta = 5\%$									
0.2	**0.1**	1.100	1.823	2.923	33.4%	2.047	1.355	3.402	45.8%
0.6	**0.3**	1.475	2.045	3.520	34.8%	2.246	1.532	3.778	49.5%
1.0	**0.5**	1.934	2.352	4.286	40.0%	2.501	1.819	4.320	58.2%
$\beta = 95\%$									
0.2	**0.1**	2.378	0.451	2.829	37.8.4%	2.084	1.388	3.472	42.9%
0.6	**0.3**	2.636	0.527	3.163	50.0%	2.218	1.524	3.742	51.0%
1.0	**0.5**	2.912	0.640	3.552	68.9%	2.365	1.693	4.058	68.4%
$\beta = 5\%$									
0.2	**0.2**	1.174	1.823	2.997	34.2%	2.085	1.382	3.467	45.4%
0.6	**0.6**	1.701	2.045	3.746	37.3%	2.369	1.622	3.991	48.4%
1.0	**1.0**	2.320	2.352	4.672	44.6%	2.734	1.975	4.709	56.1%
$\beta = 95\%$									
0.2	**0.2**	2.506	0.490	2.997	34.2%	2.153	1.444	3.597	40.2%
0.6	**0.6**	3.058	0.688	3.746	37.1%	2.467	1.669	4.136	43.2%
1.0	**1.0**	3.708	0.964	4.672	44.5%	2.886	2.101	4.986	47.4%

Table 4.5. Optimal expected profits for increasing salvage values under the wholesale contract; base case parameter specifications

logistics costs, w^*_{CRP} drops below the optimal wholesale price when ignoring returns as can be seen in table 4.4. The retailer still orders less when considering returns with the gap to IRP slightly increasing for higher salvage values.

With respect to the profits we find that the supply chain remains better off if returns are considered. However the absolute gap between the two policies is getting smaller as can be seen from table 4.5 for a return volume of $\alpha = 20\%$. We note that for $\alpha = 6\%$ and the extreme case $\beta = 5\%$, $v = v_R = c$ ignoring returns is the slightly better choice for the decentralized supply chain. Concerning the overall difference to the performance of the centralized system we find from table 4.5 that coordination becomes worse under both policies. For $\beta = 95\%$ and $v = 2v_R = c$ the profits of the centralized system are 68.9% and 68.4% higher than for the decentralized supply chain under policy IRP and CRP, respectively. Looking at the profits of the single partners we observe that in particular the manufacturer's profit under IRP catches up

(b,k)		$\alpha = 6\%, l = 1$			$\alpha = 20\%, l = 3$		
		IRP	CRP	Δ	IRP	CRP	Δ
		$\beta = 5\%$					
$(b,k) = (-3, 2.5)$	$E[r^*]$	2.053	2.090	0.037	2.053	2.405	0.352
	w^*	1.607	1.677	0.071	1.607	2.230	0.623
	Q^*	0.840	0.729	-0.111	0.840	0.100	-0.740
		$\beta = 95\%$					
	$E[r^*]$	2.053	2.095	0.042	2.053	2.279	0.226
	w^*	1.607	1.630	0.023	1.607	1.583	-0.024
	Q^*	0.840	0.724	-0.117	0.840	0.322	-0.518
		$\beta = 5\%$					
$(b,k) = (-3, 5)$	$E[r^*]$	3.776	3.776	0.000	3.776	3.992	0.216
	w^*	2.552	2.552	0.000	2.552	2.959	0.406
	Q^*	2.707	2.707	0.000	2.707	1.886	-0.822
		$\beta = 95\%$					
	$E[r^*]$	3.776	3.811	0.034	3.776	3.990	0.214
	w^*	2.552	2.561	0.008	2.552	2.505	-0.048
	Q^*	2.707	2.559	-0.149	2.707	1.986	-0.722
		$\beta = 5\%$					
$(b,k) = (-1.5, 10)$	$E[r^*]$	7.180	7.210	0.030	7.180	7.373	0.193
	w^*	4.361	4.417	0.056	4.361	4.722	0.361
	Q^*	3.408	3.288	-0.120	3.408	2.842	-0.566
		$\beta = 95\%$					
	$E[r^*]$	7.180	7.232	0.052	7.180	7.383	0.203
	w^*	4.361	4.343	-0.018	4.361	4.291	-0.070
	Q^*	3.408	3.271	-0.137	3.408	2.910	-0.498

Table 4.6. Optimal values of the decision variables for different market sizes and price elasticities under a buyback contract

with respect to policy CRP, whereas the absolute gap between the retailer's profits under the different strategies stays fairly constant. At last we note that also the positive effect of high retailer's shares of l on the supply chain profits remains valid for positive salvage values.

Different market size and price elasticity

To complete our investigations on wholesale contracts we conducted the same sensitivity analysis for different values of b and k, where b represents the price elasticity and bk the market size. The two additional scenarios consist of a market with half the size than under the base case and a market with half of the base case price elasticity.

Table 4.6 shows the equilibrium values of the decision variables for the three scenarios and for different return volumes, return logistics costs and shares of costs. We observe that for the values under both IRP and CRP reducing market size results in lower values and reducing price elasticity in higher ones. The influence of the retailer's share and higher costs associated with returns remain valid under policy CRP.

Looking at the profits in table 4.7 we also make similar observations as before. For both a lower market size and lower price elasticity, the supply chain is clearly better off if returns are considered. The preferences of the single partners again depend on the value of β. Finally, we point out that the profit gap to the centralized system lies in a range from 37% to 45% for all parameter specifications whereas lower price elasticity has a negative effect on coordination if returns are considered. The effect of the remaining parameter on coordination, remains the same for all three scenarios, more or less accentuated.

4.2 CRP vs. IRP under a Buyback Contract

Before we present the results of our computations, we shortly recall the manufacturer's expected profit function under CRP and our derivations under IRP. In proposition 2 we have shown that, under ignoring returns, there exists a unique optimum for the manufacturer's profit function. When considering returns, however, we were unable to prove analytically neither the existence of a maximum nor the concavity of the function. Therefore, for our computational work we ensured the latter two properties by plotting the manufacturer's profit for the different parameter values. Figure 4.9 presents two different views for our base case parameter specifications.

		$\alpha = 6\%, l = 1$		$\alpha = 20\%, l = 3$	
		IRP	CRP	IRP	CRP
		$\beta = 5\%$			
$(b, k) = (-3, 2.5)$	Π_M	0.304	**0.384**	-0.430	**0.078**
	Π_R	**0.277**	0.239	**0.170**	0.048
	Π_{SC}	0.581	**0.624**	-0.260	**0.126**
	$\%\Delta$	29.1%	36.9%	−117.9%	44.2%
		$\beta = 95\%$			
	Π_M	0.377	**0.386**	0.052	**0.082**
	Π_R	0.204	**0.234**	-0.312	**0.050**
	Π_{SC}	0.581	**0.621**	-0.260	**0.132**
	$\%\Delta$	29.1%	37.6%	−117.9%	37.8%
$(b, k) = (-3, 5)$		$\beta = 5\%$			
	Π_M	3.432	**3.614**	0.934	**1.962**
	Π_R	**2.422**	2.348	**1.730**	1.284
	Π_{SC}	5.854	**5.962**	2.664	**3.246**
	$\%\Delta$	38.5%	39.1%	33.2%	44.6%
		$\beta = 95\%$			
	Π_M	**3.630**	3.626	**2.256**	2.020
	Π_R	2.223	**2.348**	0.409	**1.342**
	Π_{SC}	5.854	**5.974**	2.664	**3.362**
	$\%\Delta$	38.5%	38.8%	33.2%	39.7%
$(b, k) = (-1.5, 10)$		$\beta = 5\%$			
	Π_M	10.107	**10.291**	6.159	**7.132**
	Π_R	**6.945**	6.929	**5.204**	4.895
	Π_{SC}	17.052	**17.220**	11.363	**12.027**
	$\%\Delta$	41.2%	40.6%	46.0%	44.5%
	Π_M	**10.323**	10.157	**7.681**	7.255
	Π_R	6.476	**6.834**	3.681	**4.964**
	Π_{SC}	16.799	**16.991**	11.363	**12.219**
	$\%\Delta$	41.2%	40.4%	46.0%	42.2%

Table 4.7. Equilibrium profits for different market sizes and price elasticities under a wholesale contract

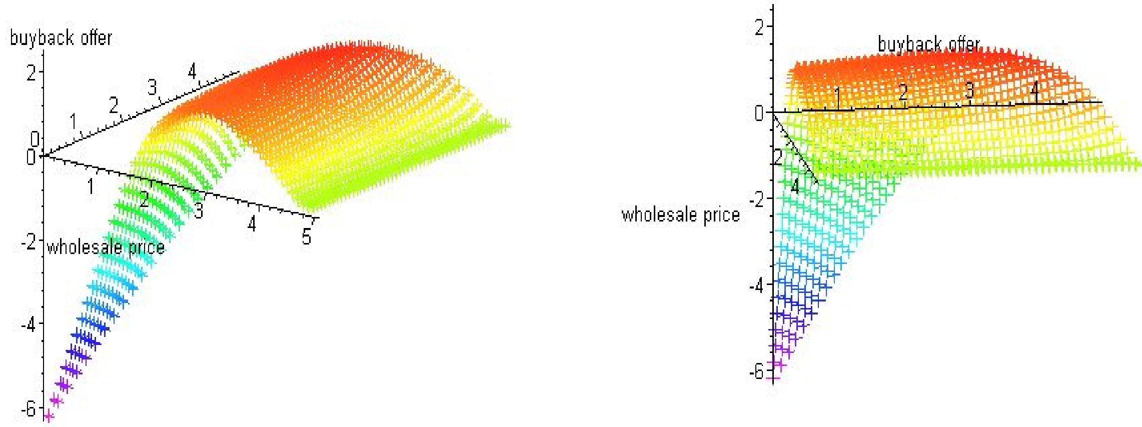

Figure 4.9. Expected profit function of the manufacturer for the base case specifications

In their comparison of considering versus ignoring returns without price postponement, Ruiz-Benitez and Muriel (2007)[39] show that buyback contracts do not coordinate the decentralized system. However, if returns are considered, there exist values for the wholesale price w and the buyback offer s under which both the retailer and the manufacturer are better off than under the wholesale contract. Yet, for the particular case of ignoring returns, buybacks may have a negative effect on the supply chain profits compared to a setting in which unsold inventory remains with the retailer. Even though ignoring returns under no postponement may lead to worse coordination under buybacks than under a wholesale contract, it remains the better policy for a supply chain facing returns - if the retail price is set before the order quantity is chosen. In the following we present our computational results when the latter decision is postponed. We start out by analyzing the base case defined earlier and vary the parameter c for the production costs. Then, we conduct a sensitivity analysis in order to formulate more general results on the comparison of CRP and IRP under buybacks and the difference to the wholesale contract we discussed in the previous section. We observe the following under the parameter specifictations of our base case:

1. Under both policy IRP and CRP supply chain coordination improves when a buyback contract is offered (Figure 4.10). When ignoring returns the decentralized system outperforms the centralized system (Table 4.8).

2. Under both policies IRP and CRP the manufacturer is better off under a buyback contract than under a whole sale contract (Figure 4.10).

3. The retailer (slightly) prefers the wholesale contract for small production costs c. For high values of c he is (slightly) better off under a buyback contract (Table 4.8).

4. The values of the decision variables are higher under buybacks.

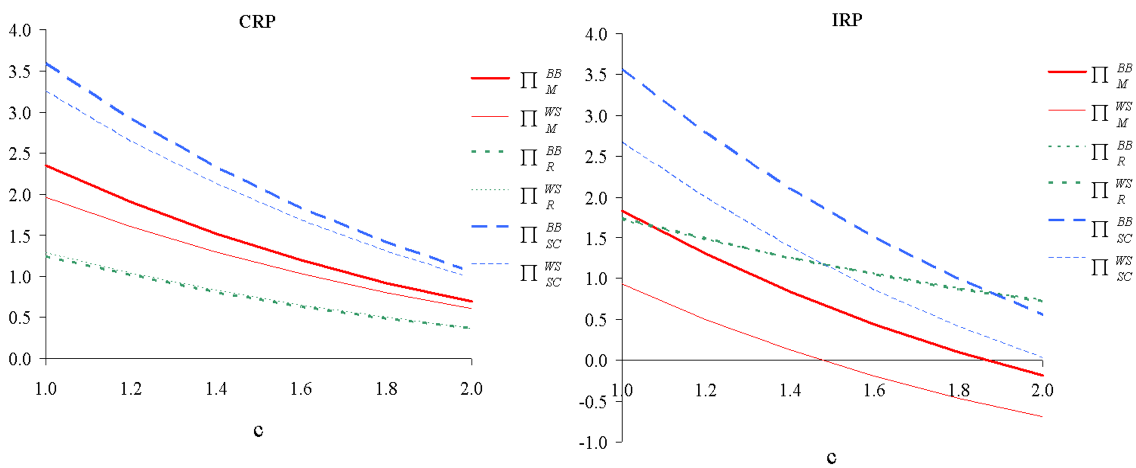

Figure 4.10. Expected profits in equilibrium under a buyback and a wholesale contract for strategies CRP(left) and IRP(right); $\alpha = 20\%$, $\beta = 5\%$

Figure 4.10 nicely compares the equilibrium profits under a buyback and a wholesale contract for both strategies, CRP and IRP, and our base case parameter specifications. Both graphs show the better coordination of the decentralized system under

	CRP		**IRP**	
	Buyback-Cont.	Wholesale Cont.	Buyback-Cont.	Wholesale Cont.
	production cost $c = 1$			
Π_M	**2.346**	1.962	**1.835**	0.934
Π_R	1.241	**1.284**	1.727	**1.730**
Π_{SC}	**3.587**	3.246	**3.562**	2.664
$\%\Delta$	30.9%	44.6%	-0.4%	33.2%
	production cost $c = 3$			
Π_M	**0.070**	0.064	**-0.942**	-1.209
Π_R	**0.038**	0.037	**0.234**	0.218
Π_{SC}	**0.109**	0.101	**-0.707**	-0.991
$\%\Delta$	29.4%	39.8%	-174.7%	-96.1%

Table 4.8. Percent difference to full coordination under a buyback and a wholesale contract for strategies IRP and CRP; base case parameter specifications

buyback contracts, which is valid for all values of $c = 1$ to $c = 3$ although the gap decreases. A numerical example for the base case is given in table 4.8. The coordination under considering returns improves from a 44.6% difference to 30.9%. For ignoring returns we even find that the relative profit gap of 33.2% turns into a negative difference of -0.4%, i.e. profits in the centralized system are 0.4% lower than in the decentralized system. As already found for the wholesale contract, higher production costs c lead to better coordination. For policy CRP this effect is stronger under a wholesale contract, whereas under strategy IRP higher product values result in a by far more signficant increase under the buyback contract (from -0.4% to -174.7% compared to 33.2% to -96.1%).

The major reason for the higher total profits under buybacks is a gain for the manufacturer, whereas the retailer seems indifferent between the two forms of contracts. Looking at figure 4.11 we see that, under both strategies, the buyback offer encourages the retailer to order higher quantities as the risk of having items left over at the end of the season is shared with the manufacturer. The higher order quantity and a higher wholesale price (w increases stronger by far than $E[r]$) seems to overcompensate the additional expected costs for taking back unsold inventory and thus the manufacturer is clearly better off under buybacks. Although the retailer should profit from the lower risk he faces, the higher wholesale price takes away the advantage.

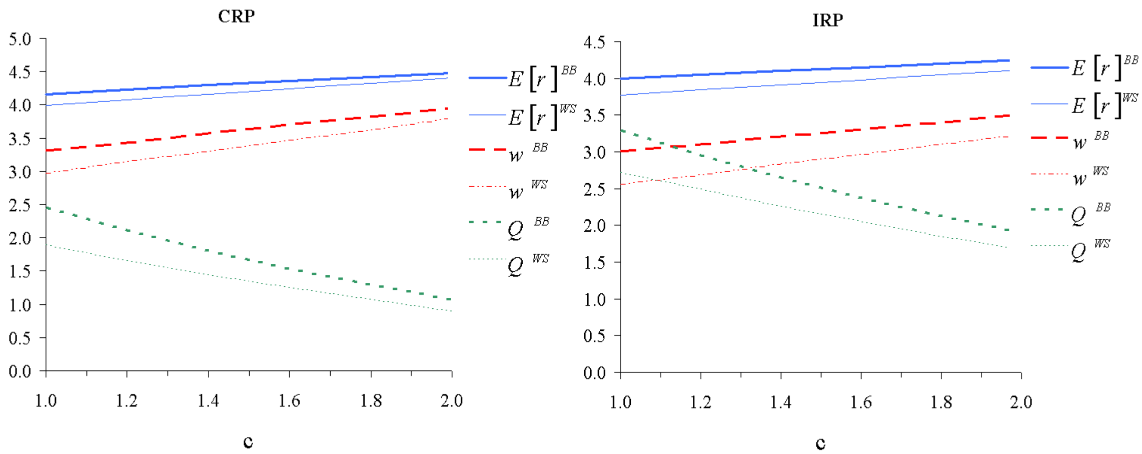

Figure 4.11. Optimal values of the decision variables under a buyback and a wholesale contract for strategies CRP(left) and IRP(right); $\alpha = 20\%$, $\beta = 5\%$

Before we verify and extend these four base case observations in a sensitivity analysis, we focus on the comparison between the policies IRP and CRP under a buyback contract as we originally did for the wholesale contract. We observe the same relationships as under the wholesale contract:

For a return volume of $\alpha = 20\%$ and a retailer share of the logistics cost of $\beta = 5\%$, table 4.9 shows the equilibrium values of the decision variables and the expected profits for different production cost c. As under the wholesale contract, we find that the retail price under CRP is higher while the order quantity is lower than under policy IRP. The fact that the manufacturer shares the major part of the return logistics costs, results in the wholesale price being higher if returns are considered. Also, the manufacturer offers a buyback price that lies above the value when ignoring returns and increases in c, which seems intuitive. We will see later that the buyback offer is similarly influenced by the manufacturer's share of the return logistics cost as

	$c=1$		$c=2$		$c=3$	
	CRP	IRP	CRP	IRP	CRP	IRP
$E[r^*]$	4.163	4.000	4.483	4.250	4.803	4.500
w^*	3.302	3.000	3.941	3.500	4.581	4.000
s^*	2.777	2.500	2.853	2.500	2.944	2.500
Q^*	2.453	3.289	1.067	1.894	0.276	0.993
Π_M	**2.346**	1.835	**0.691**	-0.187	**0.070**	-0.942
Π_R	1.241	**1.727**	0.366	**0.739**	0.038	**0.234**
Π_{SC}	3.587	3.562	1.057	0.552	0.109	-0.707

Table 4.9. Equilibrium values under a buyback contract for decision variables and profits for the strategies CRP and IRP; return volume of $\alpha = 20\%$, retailer's share of l of $\beta = 5\%$ contract

the wholesale price w. Looking at the profits, we again notice that the profitability of considering returns versus ignoring returns grows for a higher product value c. Also note that only the manufacturer is better off when considering returns as he faces a high share of the return logistics cost l. We will see that also under buybacks these preferences change for a different share of l.

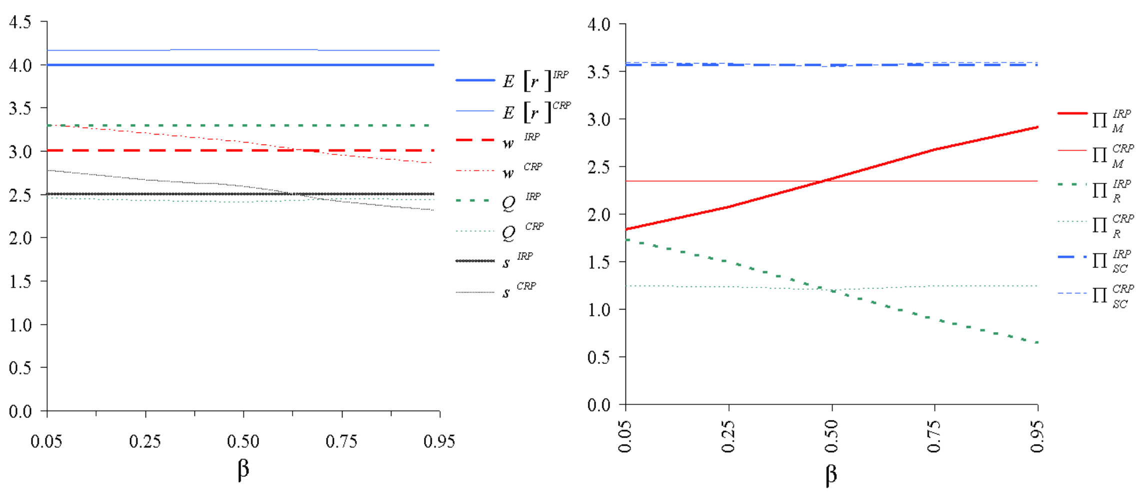

Figure 4.12. Optimal values of decision variables and expected profits under a buyback contract for different shares of return logistics costs β; base case parameter specifications

	$\beta = 5\%$		$\beta = 95\%$	
	CRP	IRP	CRP	IRP
$E[r^*]$	**4.803**	4.500	**4.802**	4.500
w^*	**4.581**	4.000	**4.129**	4.000
s^*	**2.944**	2.500	2.472	**2.500**
Q^*	0.276	**0.993**	0.278	**0.993**
Π_M	**0.070**	-0.942	**0.070**	-0.402
Π_R	0.038	**0.234**	**0.039**	-0.306
Π_{SC}	**0.109**	-0.707	**0.109**	-0.707

Table 4.10. Equilibrium values for decision variables and profits under a buyback contract for high production costs and different share of return logistics costs; return volume of $\alpha = 20\%$

Different Share of Return Logistics costs

As indicated in the previous section, we observe the typical behavior of both equilibrium values of the decision variables and profits for varying values of β also under buybacks. Figure 4.12 shows that, under buybacks, only the wholesale price and the buyback offer when considering returns are affected by changing values of β. With the retailer facing higher return costs the values for both variables decrease, thus compensating the retailer. The retail price and order quantity, as a consequence, remain (fairly) independent of the retailer's share of logistics costs. Concerning profits, we again observe that the preference for considering returns changes to the partner who carries a high part of the return costs (the values of β for which preferences change are not the same, as might be concluded from figure 4.12). *In contrast to the wholesale contract, we were unable to observe a significant increase of total profits if the retailer carries a high share of the return logistics costs.* On the contrary, we found examples in which total profits decrease (slightly).

Similar to the wholesale setting, we observe that the influence of β on the preferences of each of the players disappears for high production costs c. The reduced effect of double marginalization leads to a growing β-range, in which considering returns is the optimal strategy for both partners, as is indicated by the bold profit values in

	CRP		IRP	
	Buyback-Cont.	Wholesale Cont.	Buyback-Cont.	Wholesale Cont.
	production cost $c = 1$			
Π_M	**2.346**	2.020	**2.915**	2.256
Π_R	1.236	**1.342**	**0.648**	0.409
Π_{SC}	**3.582**	3.362	**3.562**	2.664
$\%\Delta$	31.1%	39.7%	−0.4%	33.2%
	production cost $c = 3$			
Π_M	**0.070**	0.065	**-0.402**	-0.590
Π_R	**0.039**	0.038	**-0.306**	-0.401
Π_{SC}	**0.109**	0.103	**-0.707**	-0.991
$\%\Delta$	29.0%	37.0%	−174.7%	−96.1%

Table 4.11. Comparison of equilibrium values for profits when the retailer shares 95% of the return logistics cost between buyback and wholesale contract for different production costs

the numerical example in table 4.10: the manufacturer's preference is already independent of β and the retailer's preference for IRP has decreased significantly if we compare his profits to table 4.9 for $c = 1$.

Finally, coming back to the four base case observations for the comparison of buyback and wholesale contracts we made at the beginning, we find the following from table 4.11: Observation 1 remains valid for all values of β, i.e. for both CRP and IRP buyback contracts lead to higher profits for the decentralized supply chain with all other parameters fixed to the base case specifications, also if β *and* c are increased. In contrast to policy CRP under a wholesale contract, we found that there is no significant increase of profits for the retailer facing a higher share of the return logistics costs under buybacks. As a consequence the difference between the profits under the two contracts decreases in β when considering returns, whereas it stays constant when ignoring them. Also observation 2, the manufacturer's preference for buybacks, does not change for higher values of β. In observation 3 we stated that, for both IRP and CRP the retailer is (slightly) better off under the wholesale contract for a low share of return costs and low production costs while he prefers buybacks for high product values. Comparing table 4.11 for $\beta = 95\%$ with table 4.8 shows that the statement remains valid also for high production costs except that, under IRP, the retailer's preference for buybacks becomes independent from c and is more significant than when considering returns. At last, concerning the difference between the decision variables, values under buybacks remain higher, as stated in observation 4, and the gap stays fairly constant for higher values of β.

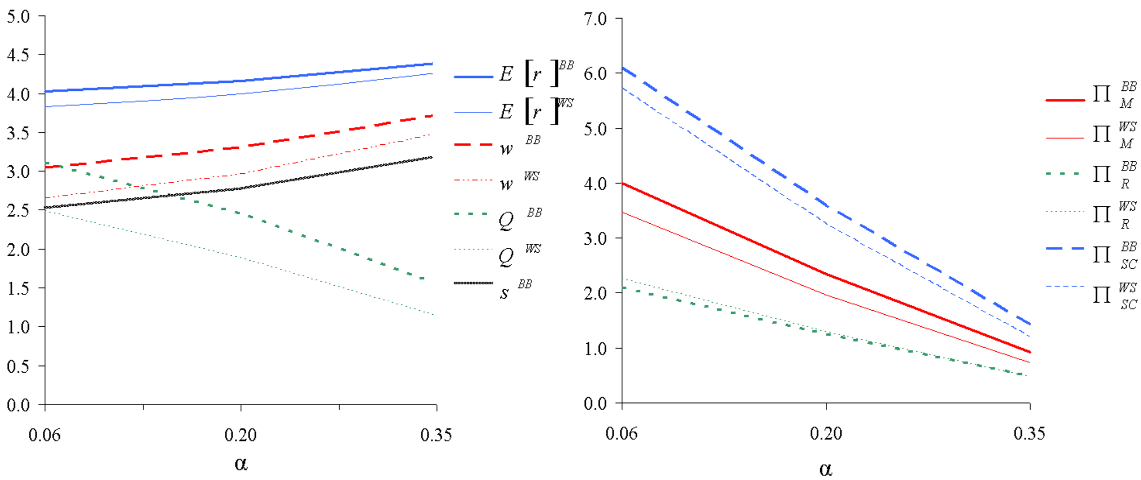

Figure 4.13. Comparison of optimal values of decision variables and expected profits when considering returns under a buyback and a wholesale contract for different return volumes α; base case parameter specifications

Different return volumes and return logistics costs

Varying the parameters α (return volume) and l (return logistics costs) in the intervals $[6\%, 35\%]$ $[1, 3]$ respectively allows us to further generalize observations 1-4. When considering returns we find that the buyback contract better coordinates the decentralized supply chain within these parameter ranges, even though the absolute gap to the total profits decreases. The same holds for the manufacturer, who remains better off under buybacks for all levels of costs associated with the returns. The retailer's preference concerening the contract form tends towards the wholesale contract for low return costs and towards the buyback contract for high costs associated with returns. The relationships of the decision variables between the two contract forms (observation 4) remain valid, too. Similarly to the profits, the gaps are getting smaller. Figure 4.13 shows these observations for policy CRP as a function of α and for $l = 2$. Increasing the return logistics cost l for a fixed α leads to similar graphs, whereas both parameters accentuate each other as they are multiplied in the profit functions.

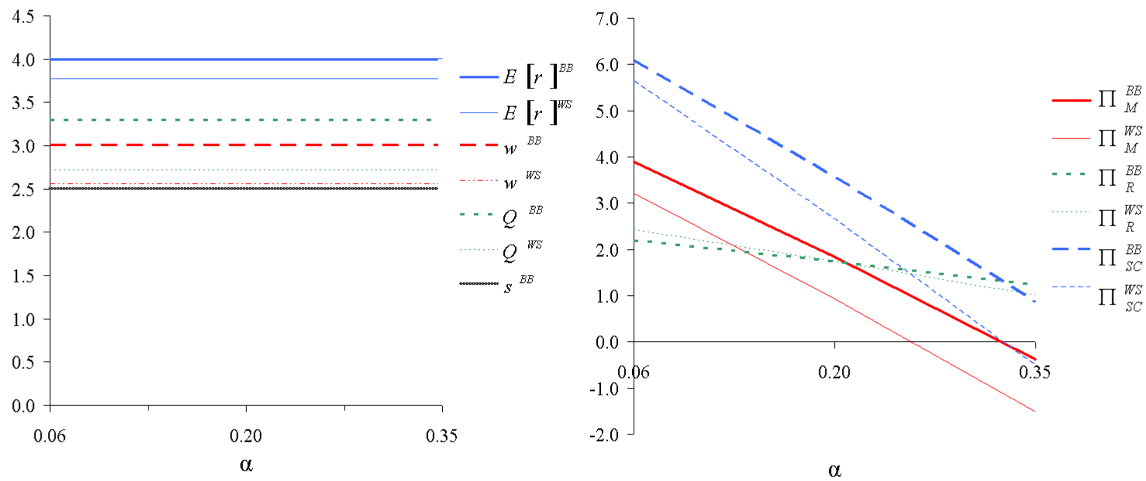

Figure 4.14. Comparison of optimal values of decision variables and expected profits when ignoring returns under a buyback and a wholesale contract for different return volumes α; base case parameter specifications

When returns are ignored we also can confirm our observations 1-4. We note that the buyback contract leads to increasingly better results than the wholesale contract, thus the total profit gap and the difference between the manufacturer's profits under the different contracts increase. The retailer also profits from a buyback offer for higher costs associated with returns. The decision variables are not influenced by the return costs as the latter are not considered in the optimization problem.

Apart from the difference between the wholesale and buyback contract, we shortly summarize the comparison of CRP and IRP under buybacks. Table 4.12 shows that

	$\alpha = 6\%, l = 1$			$\alpha = 35\%, l = 3$		
	IRP	CRP	Δ	IRP	CRP	Δ
	$\beta = 5\%$					
$E[r^*]$	4.000	**4.010**	0.010	4.000	**4.521**	0.521
w^*	3.000	**3.017**	0.017	3.000	**3.962**	0.962
s^*	**2.500**	2.488	-0.012	2.500	**3.432**	0.932
Q^*	**3.289**	3.154	-0.135	**3.289**	1.126	-2.163
	$\beta = 95\%$					
$E[r^*]$	4.000	**4.016**	0.016	4.000	**4.519**	0.519
w^*	**3.000**	2.972	-0.028	**3.000**	2.503	-0.497
s^*	**2.500**	2.442	-0.058	**2.500**	1.946	-0.554
Q^*	**3.289**	3.127	-0.161	**3.289**	1.117	-2.172

Table 4.12. Comparison of equilibrium values for decision variables under a buyback contract for strategies CRP and IRP; the retailer carries 5% of the return logistics costs

	$\alpha = 6\%, l = 1$		$\alpha = 35\%, l = 3$	
	IRP	CRP	IRP	CRP
	$\beta = 5\%$			
Π_M	4.064	**4.144**	-1.368	**0.525**
Π_R	**2.198**	2.156	**1.180**	0.282
Π_{SC}	6.262	**6.300**	-0.188	**0.807**
$\%\Delta$	29.5%	31.6%	-1525.2%	30.3%
	$\beta = 95\%$			
Π_M	**4.226**	4.145	**1.467**	0.525
Π_R	2.036	**2.125**	-1.655	**0.282**
Π_{SC}	6.262	**6.270**	-0.188	**0.807**
$\%\Delta$	29.5%	32.2%	-259.5%	30.3%

Table 4.13. Comparison of equilibrium values for profits under a buyback contract for strategies CRP and IRP; the retailer carries 5% of the return logistics costs

also under buybacks higher costs associated with returns lead to a higher expected retail price $E[r^*]$, a higher wholesale price w^* and a higher buyback offer s^* (as the values change with α and l the slope of the latter two is higher for a high share of the return costs for the manufacturer), whereas the optimal order quantity Q^* decreases. Table 4.13 indicates the increasing profitability of policy CRP for higher α and l, but also the negative effect on coordination, which stands in contrast to the positive effect of the production cost c on the difference to the centralized system. Note the manufacturer and retailer's preferences for the displayed case of $\beta = 5\%$. They switch for $\beta = 95\%$ independently of how high the return costs are, which is again in contrast to high production costs c.

Positive Salvage values

To continue our analysis we take a look at positive salvage values v and v_R for unsold and returned products. Just like under the wholesale contract we observe that a positive salvage value v_R has the opposite effect on profits and decision variables than α and l because it decreases the costs associated with returns. From table 4.14 we find that both retail and wholesale price decrease when considering returns and thus reduce the gap to policy IRP under which the both parameters remain unaffected. The buyback offer increases under both policies and ends up being (almost) equal to the wholesale price for $v = v_R = c$. The optimal order quantity Q^* increases stronger when ignoring returns.

The most important observations when looking at the respective profits in table 4.15 are the *improvement of coordination of the decentralized system under considering returns, which is contrary to the wholesale contract* and the conclusion that *already small salvage values lead to strategy IRP delivering higher total profits than strategy CRP*. For $v = 0.2$ and $v_R = 0.1$ total profits under IRP are 3.855 whereas considering returns leads to 3.805. The gaps to the respective centralized system are

		IRP				CRP			
v	v_R	$E[r^*]$	w^*	s^*	Q^*	$E[r^*]$	w^*	s^*	Q^*
$\beta = 5\%$									
0.2	**0.1**	4.000	3.000	2.600	3.466	4.150	3.276	2.844	2.632
0.6	**0.3**	4.000	3.000	2.800	3.994	4.150	3.274	3.062	3.113
1.0	**0.5**	4.000	3.000	3.000	6.000	4.110	3.195	3.190	4.923
$\beta = 95\%$									
0.2	**0.1**	4.000	3.000	2.600	3.466	4.167	2.858	2.447	2.601
0.6	**0.3**	4.000	3.000	2.800	3.994	4.154	2.833	2.619	3.088
1.0	**0.5**	4.000	3.000	3.000	6.000	4.101	2.728	2.717	4.783
$\beta = 5\%$									
0.2	**0.2**	4.000	3.000	2.600	3.466	4.150	3.276	2.844	2.633
0.6	**0.6**	4.000	3.000	2.800	3.994	4.111	3.197	2.964	3.217
1.0	**1.0**	4.000	3.000	3.000	6.000	4.073	3.120	3.115	5.144
$\beta = 95\%$									
0.2	**0.2**	4.000	3.000	2.600	3.466	4.156	2.836	2.416	2.629
0.6	**0.6**	4.000	3.000	2.800	3.994	4.122	2.770	2.553	3.222
1.0	**1.0**	4.000	3.000	3.000	6.000	4.070	2.666	2.660	5.109

Table 4.14. Optimal values of decision variables for increasing salvage values under the buyback contract; base case parameter specifications

3.8% and 28.4%, which shows that for the centralized system still considering returns is the better approach. Comparing each of the partners' profit gains, we find for both strategies that it is the manufacturer who profits most from a positive salvage value no matter who faces the higher share of the returns costs. This seems intuitive as the latter actually realizes the salvage value for the returned and unsold products. The retailer only profits from the lower wholesale and higher buyback price the manufacturer offers.

Coming back to the difference between the wholesale and buyback contract we find all of our four original observations confirmed. Supply chain profits even increase stronger under buybacks for higher salvage values no matter if returns are ignored or considered. Also the manufacturer's preference for buyback contracts becomes more accentuated. Under policy CRP even the retailer ends up being better off eventually for high salvage values, whereas under policy IRP, the wholesale contract remains his

		IRP				CRP			
v	v_R	Π_M	Π_R	Π_{SC}	$\%\Delta$	Π_M	Π_R	Π_{SC}	$\%\Delta$
$\beta = 5\%$									
0.2	**0.1**	2.050	1.805	3.855	1.1%	2.478	1.327	3.805	30.4%
0.6	**0.3**	2.569	2.005	4.574	3.7%	2.824	1.471	4.295	31.5%
1.0	**0.5**	3.360	2.340	5.700	5.3%	3.407	1.889	5.297	29.0%
$\beta = 95\%$									
0.2	**0.1**	3.130	0.725	3.855	1.1%	2.480	1.283	3.763	31.8%
0.6	**0.3**	3.649	0.925	4.574	3.7%	2.824	1.454	4.278	32.0%
1.0	**0.5**	4.440	1.260	5.700	5.3%	3.402	1.914	5.316	28.6%
$\beta = 5\%$									
0.2	**0.2**	2.110	1.805	3.915	2.7%	2.520	1.328	3.848	31.1%
0.6	**0.6**	2.749	2.005	4.754	8.1%	2.955	1.598	4.553	30.1%
1.0	**1.0**	3.660	2.340	6.000	12.5%	3.662	2.052	5.714	28.6%
$\beta = 95\%$									
0.2	**0.2**	3.190	0.725	3.915	2.7%	2.521	1.315	3.836	31.5%
0.6	**0.6**	3.829	0.925	4.754	8.1%	2.958	1.570	4.529	30.8%
1.0	**1.0**	4.740	1.260	6.000	12.5%	3.661	2.060	5.721	28.5%

Table 4.15. Optimal expected profits and percentage difference to the centralized system for increasing salvage values under the wholesale contract; base case parameter specifications

favorite setting. This confirms our observations so far indicating, that the retailer's gain from agreeing to a buyback contract is either not significant or negative.

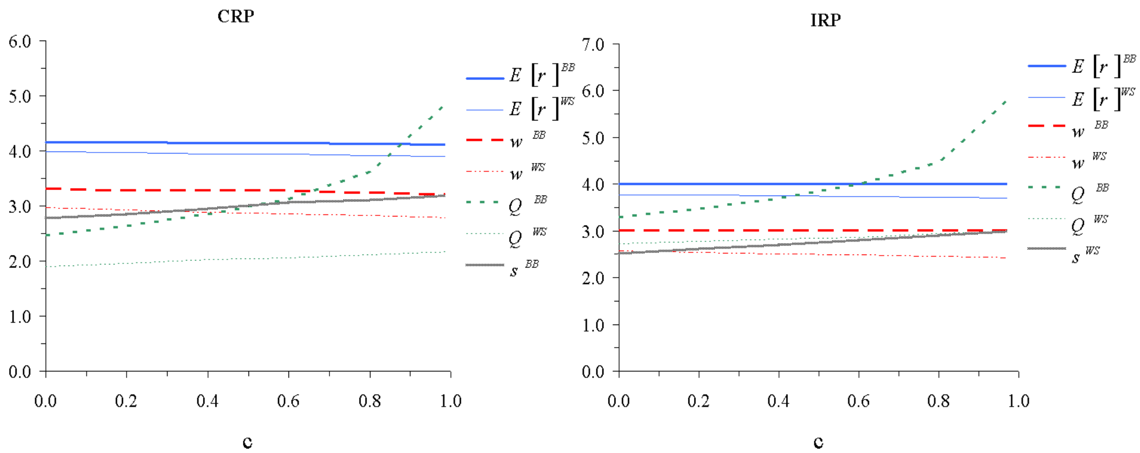

Figure 4.15. Comparison of optimal values of decision variables and expected profits under a buyback and a wholesale contract for different salvage values and shares of return logistics costs; base case parameter specifications

Observation 4 concerning the optimal values of the decision variables is confirmed as well. All are higher under the buyback contract, whereas the difference between optimal order quantities increases significantly for high salvage values as the only incentive to not order an infinite amount for $v = v_R = c$ are the return logistics costs.

Different market size and price elasticity

Finally, to complete our analysis of buyback contracts we again conducted a similar sensitivity analysis for different values of b and k, i.e. for different price elasticities and market sizes. We chose the same specifications as under the wholesale contract: a market with half the size than under the base case and a market with half of the base case's price elasticity.

Table 4.16 shows the equilibrium values of the decision variables for the three scenarios and for different return volumes, return logistics costs and shares of costs.

		$\alpha = 6\%, l = 1$			$\alpha = 20\%, l = 2$		
		IRP	CRP	Δ	IRP	CRP	Δ
		$\beta = 5\%$					
$(b, k) = (-3, 2.5)$	$E[r^*]$	2.125	**2.139**	0.014	2.125	**2.296**	0.171
	w^*	1.750	**1.775**	0.025	1.750	**2.068**	0.318
	s^*	**1.250**	1.221	-0.029	1.250	**1.487**	0.237
	Q^*	**0.947**	0.858	-0.088	**0.947**	0.384	-0.562
		$\beta = 95\%$					
	$E[r^*]$	2.125	**2.141**	0.016	2.125	**2.297**	0.172
	w^*	**1.750**	1.721	-0.029	**1.750**	1.620	-0.130
	s^*	**1.250**	1.177	-0.073	**1.250**	1.058	-0.192
	Q^*	**0.947**	0.858	-0.088	**0.947**	0.385	-0.562
$(b, k) = (-3, 5)$		$\beta = 5\%$					
	$E[r^*]$	4.000	**4.010**	0.010	4.000	**4.163**	0.163
	w^*	3.000	**3.017**	0.017	3.000	**3.302**	0.302
	s^*	**2.500**	2.488	-0.012	2.500	**2.777**	0.277
	Q^*	**3.289**	3.154	-0.135	**3.289**	2.453	-0.836
		$\beta = 95\%$					
	$E[r^*]$	4.000	**4.016**	0.016	4.000	**4.162**	0.162
	w^*	**3.000**	2.972	-0.028	**3.000**	2.850	-0.150
	s^*	**2.500**	2.442	-0.058	**2.500**	2.308	-0.192
	Q^*	**3.289**	3.127	-0.161	**3.289**	2.434	-0.854
$(b, k) = (-1.5, 10)$		$\beta = 5\%$					
	$E[r^*]$	7.750	**7.756**	0.006	7.750	**7.893**	0.143
	w^*	5.500	**5.510**	0.010	5.500	**5.760**	0.260
	s^*	5.000	**5.000**	0.000	5.000	**5.233**	0.233
	Q^*	**4.407**	4.325	-0.082	**4.407**	3.856	-0.551
		$\beta = 95\%$					
	$E[r^*]$	7.750	**7.751**	0.001	7.750	**7.889**	0.139
	w^*	**5.500**	5.441	-0.059	**5.500**	5.303	-0.197
	s^*	**5.000**	4.913	-0.087	**5.000**	4.758	-0.242
	Q^*	**4.407**	4.308	-0.099	**4.407**	3.838	-0.569

Table 4.16. Optimal values of the decision variables for different market sizes and price elasticities under a buyback contract

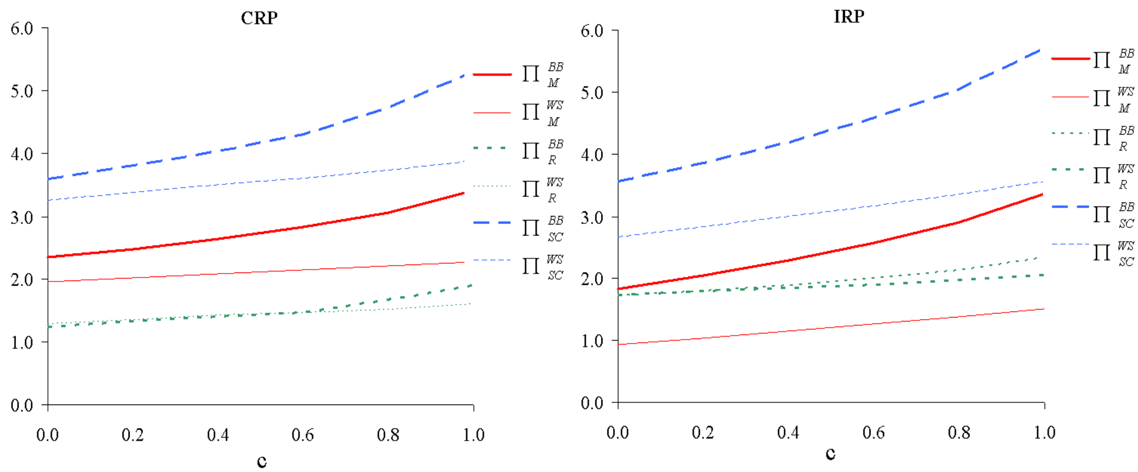

Figure 4.16. Comparison of optimal values of decision variables and expected profits under a buyback and a wholesale contract for different salvage values and shares of return logistics costs; base case parameter specifications

We observe, that all relationships found so far remain valid. The values of the decision variables are significantly lower for $k = 2.5$ and significantly higher for $b = -1.5$. The values under considering returns are similarly influenced as found earlier.

Looking at the profits in table 4.17 we find a slight difference to our observations so far. We note that for a lower market size the supply chain is clearly better off if returns are considered. The preferences of the single partners again depend on the value of β. The latter is also true for lower price elasticity. However, we find that for $b = -1.5$ if the supply chain prefers one strategy (for high values of α, l and v) it is better off when ignoring returns. This seems intuitive as a unit increase in price causes a lower change in demand and thus less returns than for higher price elasticities. Finally, we point out that the profit gap to the centralized system lies in a range from 27% to 34% for all parameter specifications.

		$\alpha = 6\%, l = 1$		$\alpha = 20\%, l = 2$	
		IRP	CRP	IRP	CRP
		$\beta = 5\%$			
$(b, k) = (-3, 2.5)$	Π_M	0.378	**0.426**	-0.261	**0.090**
	Π_R	**0.252**	0.233	**0.173**	0.053
	Π_{SC}	0.630	**0.659**	-0.087	**0.143**
	$\%\Delta$	19.0%	29.5%	-550.1%	27.0%
		$\beta = 95\%$			
	Π_M	**0.439**	0.426	**0.144**	0.090
	Π_R	0.191	**0.232**	-0.232	**0.053**
	Π_{SC}	0.630	**0.658**	-0.087	**0.143**
	$\%\Delta$	19.0%	29.7%	-550.1%	27.2%
		$\beta = 5\%$			
$(b, k) = (-3, 5)$	Π_M	4.064	**4.144**	1.835	**2.346**
	Π_R	**2.198**	2.156	**1.727**	1.241
	Π_{SC}	6.262	**6.300**	3.562	**3.587**
	$\%\Delta$	29.5%	31.6%	-0.4%	30.9%
		$\beta = 95\%$			
	Π_M	**4.226**	4.145	**2.915**	2.346
	Π_R	2.036	**2.125**	0.647	**1.236**
	Π_{SC}	6.262	**6.270**	3.562	**3.582**
	$\%\Delta$	29.5%	32.2%	-0.4%	31.1%
		$\beta = 5\%$			
$(b, k) = (-1.5, 10)$	Π_M	12.051	**12.108**	8.362	**8.683**
	Π_R	**6.213**	6.191	**5.092**	4.518
	Π_{SC}	18.263	**18.299**	**13.454**	13.202
	$\%\Delta$	31.9%	32.4%	23.3%	31.6%
	Π_M	**12.233**	12.108	**9.577**	8.682
	Π_R	6.030	**6.200**	3.877	**4.518**
	Π_{SC}	18.263	**18.308**	**13.454**	13.200
	$\%\Delta$	30.3%	32.3%	23.3%	31.6%

Table 4.17. Equilibrium profits for different market sizes and price elasticities under a buyback contract

4.3 Similarities to the Deterministic Model

In their paper on price and order postponement in a similar model without returns, Granot and Yin (2005) [22] observe that, under buybacks, both the optimal wholesale price w^* and the optimal expected retail price $E[r^*]$ are the same as in the deterministic model, in which the demand function corresponds to the expected demand function in the stochastic models. As a consequence, this also holds for the model with returns when the latter are ignored as the decision variables are unaffected under this policy. For strategy CRP we find the following:

Proposition 4 *If returns are considered in the optimization process, the optimal expected retail price $E[r^*](w)$ as a function of the wholesale price w is the same in the stochastic and deterministic model, in which the demand function corresponds to the expected demand $D(r) = b(r - k)$.*

Proof. In order to prove proposition 4 we set up the retailer's profit Π_{RD}^{CR} under deterministic demand:

$$\Pi_{RD}^{CR}(w, r) = [(1 - \alpha)r - w - \alpha(l_2 - w)]b(r - k)$$

Taking the first derivative with respect to the retail price r we get:

$$\frac{\partial \Pi_{RD}^{CR}(w, r)}{\partial r} = (1 - \alpha)b(r - k) + [(1 - \alpha)r - w - \alpha(l2 - w)]b \tag{4.1}$$

Setting term 4.1 equal to zero and simplifying results in the following stationary point for a given wholesale price w:

$$r^* = \frac{(1 - \alpha)k + w + \alpha(l_2 - w)}{2(1 - \alpha)} \tag{4.2}$$

Taking the second derivative with respect to r proves concavity as $b < 0$:

$$\frac{\partial^2 \Pi_{RD}^{CR}(w,r)}{\partial r^2} = (1-\alpha)b + (1-\alpha)b = 2(1-\alpha)b < 0$$

And thus, the stationary point in 4.2 is the optimal retail price for a given wholesale price w and corresponds to the result in proposition 2 for stochastic and price dependent demand. ∎

To show that also the equilibrium value for the retail price is the same in both models, we would need to show that the optimal wholesale prices are equal. But, as we are not able to derive an analytical result for w in the stochastic model, we can only rely on the results of our computational work.

The optimal wholesale price for the deterministic model can be obtained, as before, by forming first and second derivatives with respect to w of the manufacturer's profit Π_{MD}^{CR}, which can be stated as follows:

$$\begin{aligned}\Pi_{MD}^{CR}(w) &= [(1-\alpha)w - c - \alpha(l_1 - v_R)]b(r(w) - k) \\ &= [(1-\alpha)w - c - \alpha(l_1 - v_R)]\frac{b}{2(1-\alpha)}[(1-\alpha)w - (1-\alpha)k + \alpha l_2]\end{aligned}$$

The resulting first order condition for the optimal wholesale price

$$2(1-\alpha)w - (1-\alpha)k + \alpha(l_1 - l_2 - v_R) - c = 0$$

is satisfied by:

$$w_{CD}^* = \frac{(1-\alpha)k + c + \alpha(l_1 - l_2 - v_R)}{2(1-\alpha)} \tag{4.3}$$

The second derivative delivers the sufficient condition for 4.3 to be the global maximum:

$$\frac{\partial^2 \Pi_{RM}^{CR}(w)}{\partial r^2} = (1-\alpha)b < 0$$

The resulting equilibrium retail price for the deterministic model finally is obtained by inserting the optimal wholesale price into term 4.2:

$$r^*_{CD} = \frac{3(1-\alpha)k + c + \alpha(l-w)}{4(1-\alpha)} \tag{4.4}$$

We note the similarity to the equilibrium values for both the stochastic and deterministic system ignoring returns:

$$w^*_{IRP} = w^*_{ID} = \frac{k+c}{2} \tag{4.5}$$

and

$$E[r^*_{IRP}] = r^*_{ID} = \frac{3k+c}{4} \tag{4.6}$$

Comparing our computational results for the optimal wholesale price w with the results under deterministic demand, we find only small differences in the second digit, which can easily be caused by numerical inaccuracies. For the buyback case we use a combined golden-section method for a two-dimensional function to find the optimal wholesale and buyback offer, which stops if the change in the manufacturer's profit between two (w, s)-combinations is smaller than 0.0001. As our computations indicate, the function is very flat in an (0.05,0.05)x(0.05,0.05) area around its optimum. If we use the closed form solution for w obtained for the deterministic model to calculate the optimal wholesale and buyback offer under stochastic demand, we find values (w, s) with a manufacturer's profit that is 0.001 units higher than our optimal results. The gap between the values of the optimal decision variables seems to be a constant low second digit value. Altogether, although we are unable to prove analytically that the equality of the optimal wholesale price in both models, our computational results suggest the correctness of this relationship.

4.4 Conclusions

In this chapter we have investigated and compared the decentralized supply chain's performance and the behavior of the decision variables under the policies IRP and CRP. We draw the following general conclusions for the different incentive schemes:

Results under a Wholesale Contract

- Like under no postponement, when considering returns order quantities are lower than when ignoring returns. The retail price, however, is higher whereas the optimal wholesale price may increase or decrease depending on the manufacturer's share of the return logistics costs.

- Considering returns leads to higher supply chain profits than ignoring returns. This result stands in constrast to the findings by Ruiz-Benitez and Muriel (2007)[39] without postponement.

- Although the supply chain is better off if returns are included, this strategy may be profitable for only one of the partners depending on who carries the lower part of the costs associated with returns.

- Also under price postponement, supply chain profits are higher when the retailer faces the higher share of the return logistics cost.

- CRP becomes more profitable for higher costs associated with returns. Coordination, however, gets worse.

- Coordination is better for higher product values.

- It is possible that the decentralized system achieves better results than the centralized supply chain if returns are ignored.

- Different market sizes and price elasticities do not seem to influence the nature of the found relationships

Differences under a Buyback Contract

- Coordination is improved by buyback contracts no matter if returns are ignored or considered.
- For very low salvage values considering returns leads to higher supply chain profits. This preference, however, changes quickly for increasing values of v, v_R and also for lower price elasticity b.
- The manufacturer profits the most from agreeing to a buyback contract. The retailer's gain is either not significant or negative.
- The values of all decision variables are higher under buybacks.
- The retail price and the profits of each of the players are (fairly) independent when considering returns.
- Coordination stays fairly constant under considering returns for different market sizes and price elasticities whereas the nature of all other relationships remains valid.

CHAPTER 5

THE VALUE OF PRICE POSTPONEMENT

In chapter 4 we have focused on the comparison of considering and ignoring returns under price postponement. Chapter 5 now is intended to clarify the value of postponing the decision on the retail price until after demand uncertainty is resolved. We therefore compare the policies CRP and CR as well as IRP and IR to see the effect of postponement on the decision variables and profits when returns are considered or not. In particular, we focus on the value of the additional information that is gained by the postponed decision on the retail price.

In the following we start out with an analysis of the centralized system under the respective policies. After that, we investigate the effect on the decentralized system and each of the partners. Our results are based on the same computational work as in chapter 4 and the derivations in chapter 3 for policies IRP and CRP. For the underlying profit functions and optimality criteria under the cases without price postponement, IR and CR, we refer to Ruiz-Benitez (2007) [38] for detailed derivations. As a starting point we again choose the parameter specifications $(\alpha, \beta, l) = (20\%, 5\%, 2)$ with salvage values v and v_r equal to zero as the base case. A similar sensitivity analysis as in chapter 4 is conducted to provide more generality.

5.1 Performance of the Centralized System

Intuition tells us for the case of one decision maker - like in the centralized supply chain - who correctly optimizes, that each bit of additional information leads to better results and thus, in our case, to higher profits. When not considering certain available aspects like the presence of returns, however, this may not necessarily be the case and thus lead to suboptimal decisions.

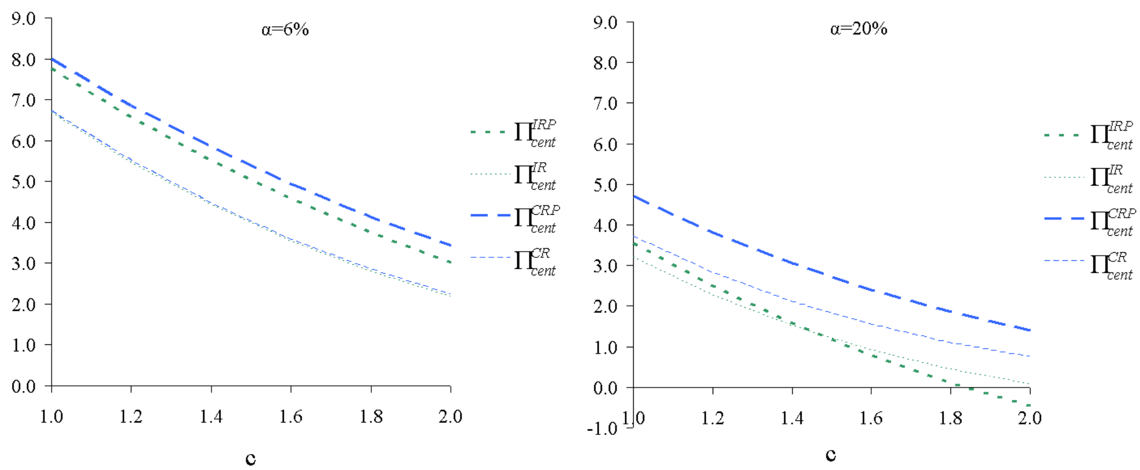

Figure 5.1. Profits in the centralized system under policies CRP, IRP, CR and IR as a function of the production costs c for different return volumes α

From figure 5.1 we find that the centralized system that considers returns achieves higher profits if the decision on the retail price is postponed until after demand uncertainty is resolved. With the profits decreasing in c but the gap between CRP and CR staying fairly constant, the relative value of the additional information becomes significantly higher for higher production costs. While for lower return volumes policy IRP remains relatively close to the profits under CRP and the system seems to be almost indifferent under no postponement, the difference between considering and ignoring returns becomes distinct for higher values α.

The fact that for increasing production costs profits under policy IRP drop below the ones under IR is reflected by the behavior of the decision variables. In figure 5.2 we observe that the order quantity and the expected retail price under policy IRP represent the highest and lowest value, respectively, among the four strategies and, consequently generate the largest quantity of returns. For high costs associated with returns the additional value of the information on demand gained by postponement

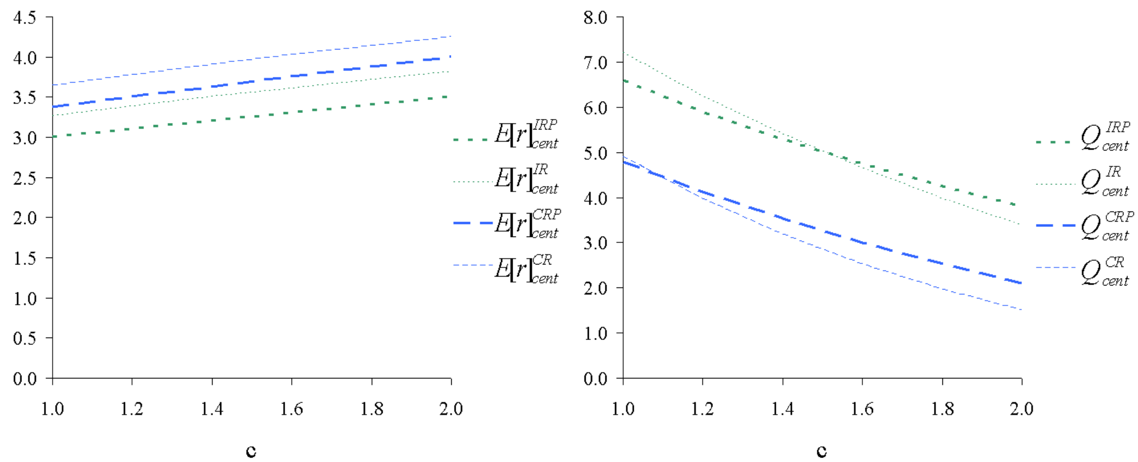

Figure 5.2. Equilibrium values of the decision variables in the centralized system under policies CRP, IRP, CR and IR as a function of the production costs c for a return volume of $\alpha = 20\%$

does no longer compensate the costs generated by the suboptimal behavior. Increasing the return volume α with all other parameters fixed to the base case specifications does exactly reflect these relationships. As can be seen from table 5.1 the relative gap between CRP and CR increases significantly indicating the importance of valuable demand information if return costs are high. For lower return costs, for example, caused by a positive salvage value shown in table 5.2, the value of the additional demand information decreases and ends up being zero for $v = 2v_R = c$.

In order to analyze the general value of the additional information gained by postponement in the presence of returns, we also compare the total supply chain profits to a model in which no returns occur. From table 5.3 we draw two conclusions: Postponement is more valuable in a system that faces returns and in which the salvage values are low, i.e. the return costs high. As the value of the additional information under no returns increases with higher salvage values, however, it eventually ends up being higher than under returns.

	Π_{cent}^{CR}	Π_{cent}^{CRP}	Δ	$\%\Delta$	Π_{cent}^{IR}	Π_{cent}^{IRP}	Δ	$\%\Delta$
α	$c = 1$							
0.06	6.728	7.999	1.271	18.9%	6.689	7.750	1.061	15.9%
0.2	3.713	4.695	0.982	26.5%	3.213	3.550	0.337	10.5%
0.35	1.282	1.855	0.573	44.7%	-0.512	-0.950	-0.439	-85.7%
α	$c = 2$							
0.06	2.226	3.416	1.190	53.5%	2.169	3.001	0.832	38.3%
0.2	0.750	1.384	0.634	84.5%	0.067	-0.464	-0.531	-793.7%
0.35	0.048	0.156	0.108	224.5%	-2.185	-4.177	-1.992	-91.2%

Table 5.1. Value of price postponement in the centralized system when ignoring and considering returns as a function of the return volume α for different production costs c

	Π_{cent}^{CR}	Π_{cent}^{CRP}	Δ	$\%\Delta$	Π_{cent}^{IR}	Π_{cent}^{IRP}	Δ	$\%\Delta$
$v = 2v_R$	$\alpha = 6\%$							
0.2	7.232	8.303	1.071	14.8%	7.196	8.071	0.875	12.2%
0.6	8.511	9.083	0.573	6.7%	8.483	8.884	0.401	4.7%
1.0	10.398	10.398	0.000	0.0%	10.380	10.200	-0.180	-1.7%
$v = 2v_R$	$\alpha = 20\%$							
0.2	4.109	4.962	0.853	20.8%	3.652	3.897	0.246	6.7%
0.6	5.157	5.649	0.492	9.5%	4.803	4.744	-0.059	-1.2%
1.0	6.834	6.834	0.000	0.0%	6.600	6.001	-0.599	-9.1%

Table 5.2. Value of price postponement in the centralized system when ignoring and considering returns as a function of the salvage values $v = 2 * v_R$ for different return volumes α

	Π_{cent}^{CR}	Π_{cent}^{CRP}	Δ	$\%\Delta$	Π_{cent}	Π_{cent}^{PP}	Δ	$\%\Delta$
c	$\alpha = 6\%$							
1.0	6.728	7.999	1.271	18.9%	8.179	9.550	1.371	16.8%
2.0	2.226	3.416	1.190	53.5%	3.070	4.486	1.416	46.1%
c	$\alpha = 20\%$							
1.0	3.713	4.695	0.982	26.5%	8.179	9.550	1.371	16.8%
2.0	0.750	1.384	0.634	84.5%	3.070	4.486	1.416	46.1%
v	$\alpha = 6\%$							
0.2	7.232	8.303	1.071	14.8%	8.625	9.860	1.234	14.3%
0.6	8.511	9.083	0.573	6.7%	9.120	10.658	1.538	16.9%
v	$\alpha = 20\%$							
0.2	4.109	4.962	0.853	20.8%	8.625	9.860	1.234	14.3%
0.6	5.157	5.649	0.492	9.5%	9.120	10.658	1.538	16.9%

Table 5.3. Comparison of the value of price postponement in the centralized system with and without returns as a function of the production costs c and the salvage values $v = 2 * v_R$ for different return volumes α

5.2 The Decentralized System

With the centralized system ignoring returns under price postponement we have found an example in which the value of additional demand information can become negative, the reason being the incorrect optimization approach. In this section we analyze the decentralized system's profits and values of decision variables with and without postponement. In particular, we focus on the factors return volume α and salvage values v and v_R that have shown to be the driving ones for the value of information.

5.2.1 Considering Returns with and without Price Postponement

Ruiz-Benitez (2007) [38] conduct a computational study for the comparison of the policies CRP and CR with a sensitivity analysis in the production costs c, the return volume α and the retailer's share β of the return logistics costs for both a wholesale and a buyback contract. When considering returns, they find for the chosen set of parameter specifications:

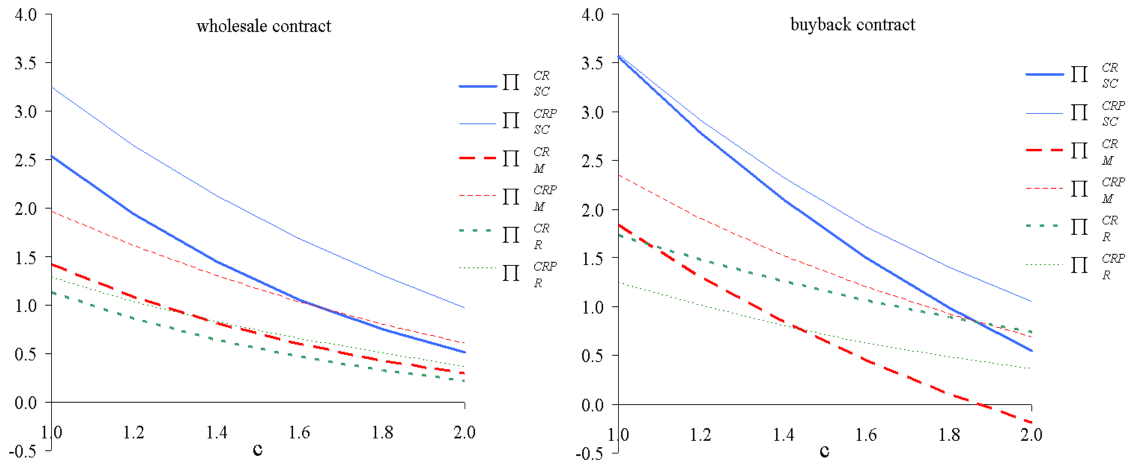

Figure 5.3. Profits in the decentralized system under policies CRP and CR for the two different contract forms as a function of the production costs c; base case parameter specifications

1. Price postponement improves coordination of the supply chain and enhances the profits of both the retailer and the manufacturer if returns are considered (confirmed by table 5.5).
2. Under a wholesale contract:
   - the manufacturer profits the most from the additional demand information (see figure 5.3 and table 5.5).
   - Order quantities and the retail price decrease while the wholesale price increases under postponement (see figure 5.4).
3. Under a buyback contract:
   - Benefits from the postponement strategy are distributed more equally among the two supply chain partner (see figure 5.3 and table 5.5).
   - Different retailer's shares of return logistics do neither impact the partners' nor total supply chain profits.

	$E[r]^{CRP}$	w^{CRP}	s^{CRP}	Q^{CRP}	Π_M^{CRP}	Π_R^{CRP}	Π_{SC}^{CRP}
calculated	4.163	3.302	2.777	2.453	2.346	1.241	3.587
conjecture	4.188	3.350	2.850	2.393	2.348	1.174	3.521
error	0.024	0.048	0.073	-0.059	0.002	-0.067	-0.065

Table 5.4. Numerical error of our computations based on the conjecture in section 4.3

- The buyback offer and the wholesale price do not change when the decision on the retail price is postponed.

In our study, in which we additionally consider positive salvage values v and v_R, we find the same results. Only concerning the last two points our computations do not exactly confirm the stated conclusion. From figure 5.4 we see that the buyback offer s and the wholesale price w are not exactly the same with and without postponement. Table 5.5 indicates that the retailer's profit decreases slightly for higher values of β if the pricing decision is postponed under buybacks. We note, however, that the absolute differences may lie within our numerical error. Table 5.4 shows an example for the numerical error if we assume that the closed form solution for the wholesale price in the deterministic model is the same in the stochastic model as stated in section 4.3.

As expected, table 5.5 nicely shows the positive value of additional information both under buybacks and the wholesale contract. Also we find the increasing value for higher return volumes. Another interesting aspect is the larger relative gain from postponement for the retailer under buybacks whereas under the wholesale contract he profits less.

For the centralized system we observed that increasing salvage values have a decreasing effect on the value of additional demand information. The same is true for the decentralized system. Yet, during our complete sensitivity analysis we find that the value of price postponement under buybacks remains positive. Under wholesale

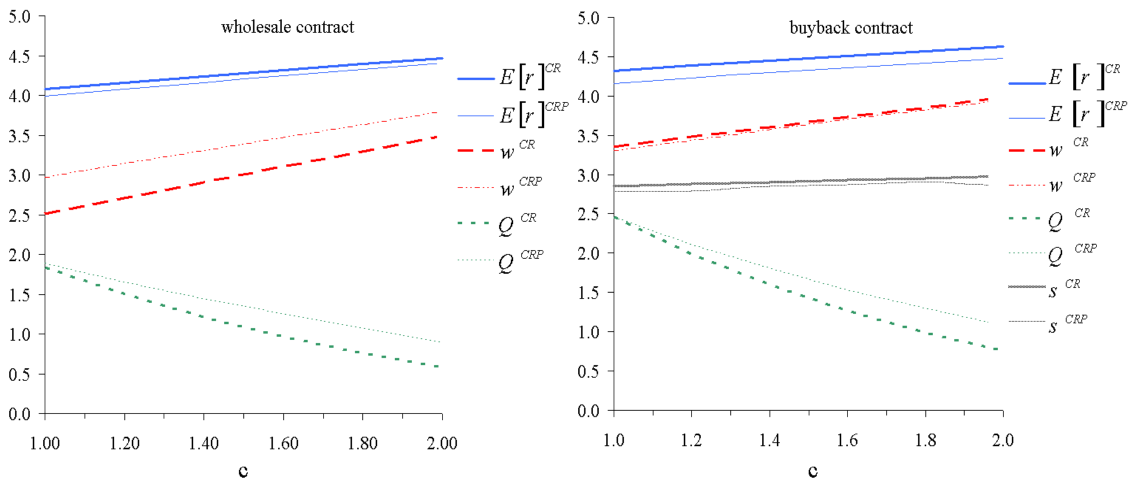

Figure 5.4. Decision variables in the decentralized system under policies CRP and CR for the two different contract forms as a function of the production costs c; base case parameter specifications

contracts, however, it is possible, that for high salvage values or low price elasticity the value becomes negative as can be seen from table 5.6. This goes along with our observation that for low return costs and low risk for unsold inventory the value of information decreases when returns are considered. The supply chain therefore is better coordinated when the decision on prices is not postponed in this case.

5.2.2 Ignoring Returns with and without Price Postponement

Our results for both the centralized system and the decentralized system so far have shown that the additional information gained from postponing the decision on the retail price is profitable in most cases if the supply chain optimizes correctly, i.e. considers returns. When ignoring returns, we have found for the centralized system that postponement can lead to worse results for high production costs and high costs associated with returns. Now considering the decentralized system we make similar observations:

	CR			CRP			%Δ		
(α, β)	Π_M	Π_R	Π_{SC}	Π_M	Π_R	Π_{SC}	Π_M	Π_R	Π_{SC}
	wholesale contract								
$(6\%, 5\%)$	**2.765**	2.082	4.846	**3.476**	2.257	5.733	25.7	8.4	18.3
$(6\%, 95\%)$	**2.797**	2.088	4.885	**3.500**	2.267	5.767	25.1	8.5	18.0
$(20\%, 5\%)$	**1.411**	1.128	2.539	**1.962**	1.284	3.246	39.0	13.8	27.8
$(20\%, 95\%)$	**1.490**	1.154	2.644	**2.020**	1.342	3.362	35.6	16.3	27.2
	buyback contract								
$(6\%, 5\%)$	**3.364**	1.683	5.047	**3.998**	2.088	6.086	18.8	24.0	20.6
$(6\%, 95\%)$	**3.364**	1.683	5.047	**3.998**	2.067	6.065	18.9	22.8	20.2
$(20\%, 5\%)$	**1.857**	0.929	2.786	**2.346**	1.241	3.587	26.4	33.6	28.8
$(20\%, 95\%)$	**1.857**	0.929	2.786	**2.346**	1.236	3.582	26.3	33.0	28.6

Table 5.5. Comparison of the value of price postponement in the decentralized system under a wholesale and a buyback contract for different return volumes α and shares β of logistics cost

	CR			CRP			%Δ		
$v = 2v_R$	Π_M	Π_R	Π_{SC}	Π_M	Π_R	Π_{SC}	Π_M	Π_R	Π_{SC}
	wholesale contract								
0.20	**1.631**	1.276	2.907	**2.084**	1.388	3.472	27.8	8.8	19.4
0.40	**1.792**	1.421	3.214	**2.149**	1.460	3.610	19.9	2.8	12.3
0.60	**1.981**	1.594	3.576	**2.218**	1.524	3.742	11.9	-4.4	4.6
0.80	**2.205**	1.805	4.010	**2.289**	1.614	3.903	3.8	-10.6	-2.7
	buyback contract								
0.20	**2.054**	1.028	3.083	**2.480**	1.283	3.763	20.7	24.7	22.1
0.40	**2.290**	1.147	3.437	**2.636**	1.371	4.007	15.1	19.6	16.6
0.60	**2.579**	1.291	3.870	**2.824**	1.454	4.278	9.5	12.6	10.6
0.80	**2.941**	1.472	4.413	**3.054**	1.700	4.755	3.9	15.5	7.7

Table 5.6. Comparison of the value of price postponement in the decentralized system considering returns under a wholesale and a buyback contract for different salvage values; base case parameter specifications

		IR			IRP			%Δ		
α	c	Π_M	Π_R	Π_{SC}	Π_M	Π_R	Π_{SC}	Π_M	Π_R	Π_{SC}
		wholesale contract								
6%	**1**	**2.880**	2.282	5.162	**3.223**	2.411	5.633	11.9	5.6	9.1
	2	**0.969**	0.834	1.803	**1.221**	1.087	2.308	26.1	30.3	28.0
	3	**0.190**	0.219	0.410	0.177	**0.380**	0.557	-7.3	73.3	35.9
20%	**1**	**1.576**	1.698	3.274	0.934	**1.730**	2.664	-40.7	1.9	-18.6
	2	0.151	**0.596**	0.747	-0.688	**0.719**	0.031	-555.8	20.8	-95.8
	3	-0.207	**0.149**	-0.058	-1.209	**0.218**	-0.991	-484.3	46.2	-1615.7
α	c	buyback contract								
6%	**1**	**3.396**	1.874	5.270	**3.893**	2.189	6.082	14.6	16.9	15.4
	2	**1.116**	0.691	1.807	**1.514**	1.007	2.520	35.6	45.6	39.4
	3	**0.215**	0.183	0.399	0.297	**0.360**	0.658	38.0	96.7	65.0
20%	**1**	**1.778**	1.466	3.244	**1.835**	1.728	3.562	3.2	17.8	9.8
	2	0.139	**0.509**	0.648	-0.187	**0.739**	0.552	-234.6	45.2	-14.9
	3	-0.247	**0.126**	-0.121	-0.942	**0.234**	-0.707	-281.2	86.3	-483.1

Table 5.7. Comparison of the value of price postponement in the decentralized system ignoring returns under a wholesale and a buyback contract; base case parameter specifications

From table 5.7 we find that for both contract forms postponement leads to increasing profits for higher production costs if the return volume is low. If the system faces a large percentage of sales returned, however, an increase in c has a negative effect on total profits. Note again, that under buybacks the value of the additional information is higher than under the wholesale contract. Also we point out that an increase in return volumes has a stronger negative effect on the manufacturer than on the retailer. This can also nicely be seen from figure 5.5, which shows the supply chain partners' and the total profits under the two contracts with and without postponement as a function of the return volume α. We observe that postponement remains more profitable until a higher values of α under buybacks than under the wholesale contract. Yet, it is also the manufacturer who carries the major losses as he faces the larger share of the return logistics costs.

The graphs of the decision variables in figure 5.6 allow us to explain the development of the profits: As returns are not considered in the optimization process, all

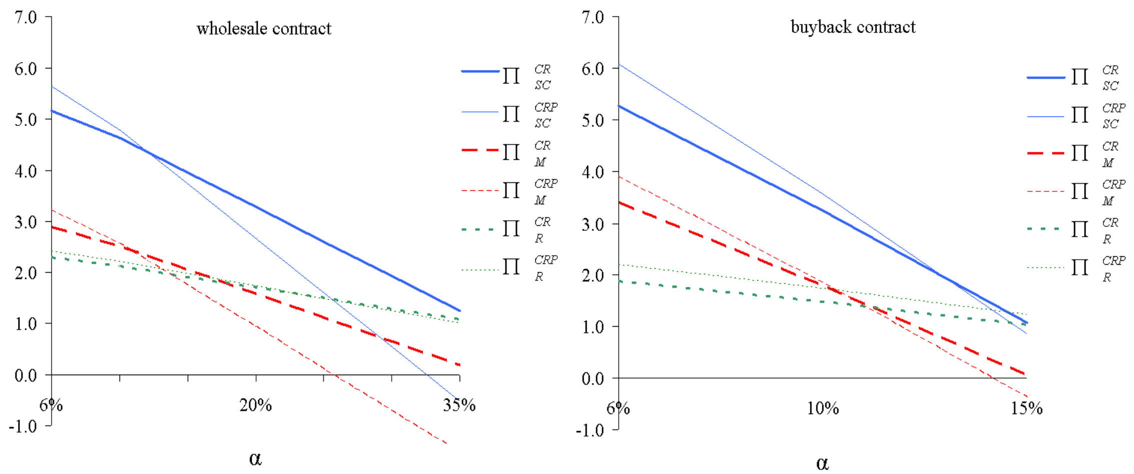

Figure 5.5. Profits in the decentralized system under policies IRP and IR for the two different contract forms as a function of the return volume α; base case parameter specifications

equilibrium parameters remain constant for different return volumes. As the retail price without postponement is higher, demand and eventually absolute returns will be lower than under policy IRP. Although the retailer places higher orders under IR and therefore faces a higher number of unsold items, the savings in return costs due to the lower absolute number of returns seem to be higher than the return costs under policy CRP. These explanations are confirmed when looking at higher return logistics costs l and a lower market size bk, both being driving factors for the absolute and relative costs associated with returns.

5.3 Conclusions

Altogether we have determined the costs associated with returns to be the driving factor for the value of the additional demand information gained through price postponement:

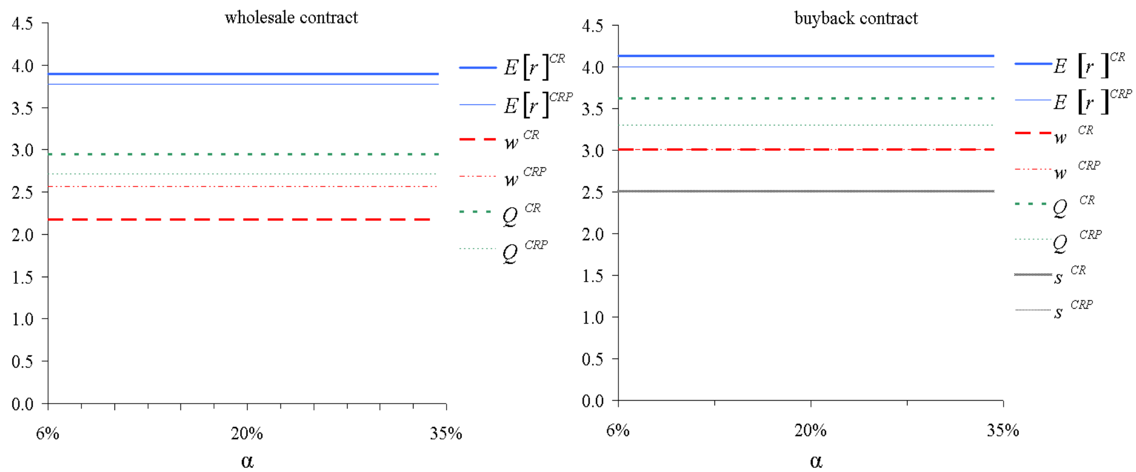

Figure 5.6. Decision variables in the decentralized system under policies IRP and IR for the two different contract forms as a function of the return volume α; base case parameter specifications

- For the centralized system and in most of the cases for the decentralized system that considers returns, we find that higher costs associated with returns make the additional information on demand more valuable.
- Interestingly, for the decentralized system that ignores returns, we find that higher return costs have a negative influence on the value of additional information.
- Lower market size and lower price elasticity have a similar influence on the profitability of price postponement as higher return volumes and higher salvage values, respectively, and thus are important factors in the decision making process.

CHAPTER 6

CONCLUSIONS AND POSSIBLE EXTENSIONS

In this thesis we have studied the application of a price postponement strategy in the decision making process of a two stage supply chain that faces consumer returns. As a model we have presented an extended news-vendor setting with stochastic and price dependent demand. We have found that, under price postponement, the qualitative relationships between the decision variables when considering and ignoring returns are similar to a setting without postponement. Concerning profits we have observed that considering returns is the better choice under a wholesale contract and also in a buyback setting, if salvage values are low. Ignoring returns, which seems the better choice under no postponement in most settings according to Ruiz-Benitez and Muriel (2007)[39], becomes interesting under buybacks if salvage values increase. The buyback contract form achieves better coordination no matter which policy is preferred.

Additionally we have observed that, naturally, the centralized system that considers returns gains from the additional demand information. We have found an example of a negative value of additional information in the integrated system that ignores returns. Surprisingly, there exist parameter combinations for which knowing less about demand leads to higher profits. Less surprisingly, we have observed both positive and negative values in the decentralized case.

Altogether, taking into account policies with and without postponement, we can extend the findings by Ruiz-Benitez and Muriel (2007)[39] and deliver a qualitative chart of effective decision policies for the supply chain. We recall that under no postponement ignoring returns under a buyback contract leads to improved supply chain performance. As we have shown, the influence of the return rate α, the return logistics costs l and the salvage value of returned products v_R can be grouped and summarized as 'costs associated with returns'. This allows us to present in Figure 6.1 the (two) best strategies for scenarios with different return costs and salvage values of unsold products. The policies under price postponement generally deliver the best results. As shown in chapter 4 considering returns under postponement (quickly) loses its profitability with increasing high salvage value of unsold products. It becomes more preferable for high costs associated with returns. If both return costs and salvage values are high, however, the positive effect of postponement is reduced and eventually, for a salvage value close to 100% of the production costs, ignoring returns without postponement leads to improved performance of the supply chain. Naturally, as analytical results seem impossible and we rely on computational work, further variations of the demand function can emphasize the generality of the results. For the model without returns, Granot and Yin (2005) [22] additionally conduct an analysis for exponential and negative power expected demand and show further similarities to the corresponding deterministic model. Their results motivate a further comparison to the value of postponement in the presence of returns. Another interesting idea to extend the findings in this thesis could be a game theoretic analyis. We have found that not always both partners do profit from the additional information gained by price postponement depending on the share of return logistics costs. A game theoretic analysis as the one being conducted in Schmid (2007) [42] could deliver further interesting insights on the value of sharing information in a setting, in which both players are free in the decision on their strategy.

Finally, contracts that coordinate the supply chain under postponed pricing and consumer returns should be developed. Ruiz Benitez (2007) [38] extends profit and

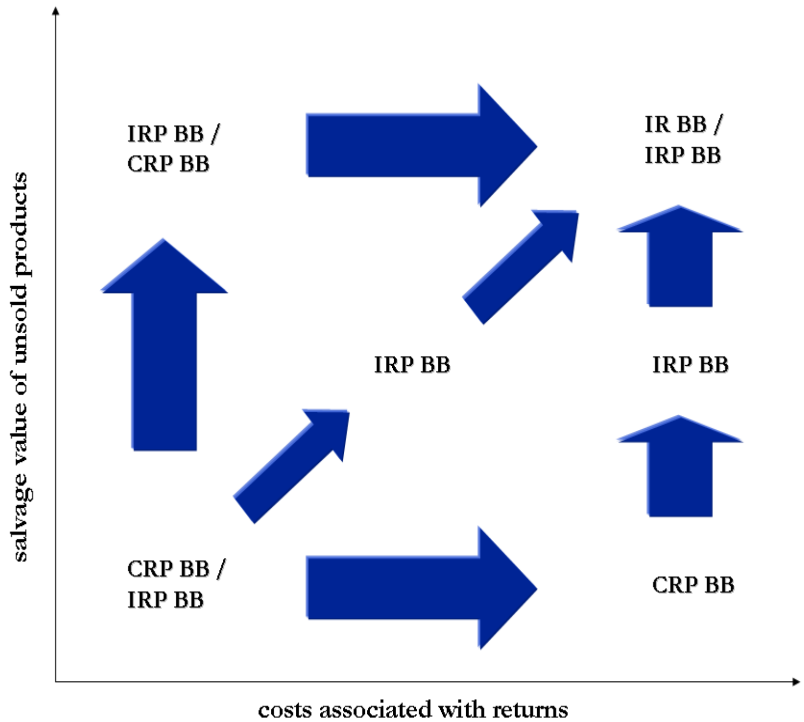

Figure 6.1. Optimal strategies for the whole supply chain for different salvage values and costs associated with returns

revenue sharing contracts to the case with consumer returns. We expect them to coordinate the supply chain in the price postponement case as well.

BIBLIOGRAPHY

[1] Anupindi, R., and Bassok, Y. Analysis of supply contracts with total minimun commitment. *IIE Transactions 30*, 8 (1998), 723–734.

[2] Aviv, Y., and Federgruen, A. Design for postponement: A comprehensive characterization of its benefits under unknown demand distributions. *Operations Research 49*, 4 (2001), 578–598.

[3] Baker, E. Increasing risk and increasing informativeness: Equivalence theorems. *Operations Research 54*, 2 (2006), 26–36.

[4] Barnes-Schuster, D., Y. Bassok, and Anupindi, R. Coordination and flexibility in supply contracts with options. *Manufacturing and Service Operations Management 4*, 3 (2002), 171–207.

[5] Bassok, Y., and Anupindi, R. Analysis of supply contracts with total minimun commitment. *IIE Transactions 29*, 5 (1997), 373–381.

[6] Bernstein, F., and Federgruen, A. Decentralized supply chains with competing retailer under demand uncertainty. *Management Science 51*, 3 (2005), 409–426.

[7] Bloom, P.N., G.T. Gundlach, and J.P.Cannon. Slotting allowances and fees: Schools of thought and the views of practicing managers. *Journal of Marketing 64*, 2 (2000), 92–108.

[8] Cachon, G. P. *Handbooks in Operations Research and Management Science: Supply Chain Management*. North-Holland, 2003, ch. Supply Chain Coordination with Contracts.

[9] Cachon, G. P., and Lariviere, M. A. Supply chain coordination with revenue-sharing contracts: Strenghts and limitations. *Management Science 51*, 1 (2005), 30–44.

[10] Corbett, C.J., and Savaskan, R.C. *Quantitative Models for Supply Chain Management*. Boston, Kluwer, 1999, ch. Contracting and coordination in closed-loop supply chains.

[11] Corbett, J.C., G. A. DeCroix, and Ha, A.Y. Optimal shared saving contracts in supply chains: Linear contracts and double moral hazard. Tech. rep.

[12] Corbett, J.C., and DeCroix, G. A. Shared-savings contracts for indirect materials in supply chains: Channel profits and environmental impacts. *Management Science 47*, 7 (2001), 881–893.

[13] Donohue, K.L. Efficient supply contracts for fashion goods with forecast updating and two production modes. *Management Science 46*, 11 (2000), 1397–1411.

[14] Emmons, H., and Gilbert, S.M. The role of returns policies in pricing and inventory decisions for catalogue goods. *Management Science 44*, 2 (1998), 276–283.

[15] Epstein, L. G. Decision making and the temporal resolution of uncertainty. *International Economic Review 21*, 2 (1980), 269–283.

[16] Ferguson, M., V.D.R. Guide Jr., and Souza, G.C. Supply chain coordination for false failure returns. 2005.

[17] Fleischmann, M. Quantitative models for reverse logistics. In *Lecture Notes in Economics and Mathematical Systems*, vol. 501) address = Berlin.

[18] Gal-Or, E. Information sharing in oligopoly. *Econometrica 53* (1985).

[19] Gavirneni, S., T. Kapuscinski, and Tayur, S. Value of information in capacitated supply chains. *Management Science 45* (1999).

[20] Gentry, C.R. Reducing the cost of returns. *Chain Store Age 75*, 10 (1999), 124–126.

[21] Gong, Q. Optimal buy-back contracts with asymmetric information. Working paper, Kellog School of Management, Northwestern University, 2000.

[22] Granot, D., and Yin, S. Price and order postponement in a decentralized newsvendor model with multiplicative and price-dependent demand. Tech. rep.

[23] Granot, D., and Yin, S. On the effectiveness of returns policies in the price-dependent newsvendor model. *Naval Research Logistics 52* (2005).

[24] Guide, V.D.R. Jr., G.C. Souza L.N. Van Wassenhove, and Blackburn, J.D. Time value of commercial product returns. *Management Science 52*, 8 (2006), 1200–1214.

[25] Kandel, E. The right to return. *J. Law Econom. 39* (1996).

[26] Karlin, S., and Carr, C.R. *Studies in Applied Probability and Management Science*. Stanford University Press, Stanford, CA, 1962, ch. Prices and optimal inventory policy.

[27] Lariviere, M. *Quantitative Models for Supply Chain Management*. Boston, Kluwer, 1999, ch. Supply chain contracting and coordination with stochastic demand.

[28] Lariviere, M.A., and Padmanabhan, V. Slotting allowances and new product introductions. *Marketing Science 16*, 2 (1997), 112–128.

[29] Lariviere, M.A., and Porteus, E.L. Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing & Service Operations Management 3*, 4 (2001), 293–305.

[30] Lee, H.L. Effective management of inventory and service through product process redesign. *Operations Research 44*, 1 (1996), 151–159.

[31] Lee, H.L., and Tang, C.S. Modelling the costs and benefits of delayed product differentiation. *Management Science 43*, 1 (1998), 40–53.

[32] Lee, H.L., K. C. So, and Tang, C.S. The value of information sharing in a two-level supply chain. *Management Science 46* (2000).

[33] Mills, E.S. Uncertainty and price theory. *Quarterly Journal of Economics 73* (1959).

[34] Mostard, J., and Teunter, R. The newsboy problem with resalable returns: A single period model and case study. *European Journal of Operational Research 169* (2006), 81–96.

[35] Padmanabhan, V., and Png, I.P.L. Manufacturer's returns poplicy and retail competition. *Marketing Science 16*, 1 (1997), 81–94.

[36] Pasternack, B. Optimal pricing and return policies for perishable commodities. *Marketing Science 4*, 2 (1985), 166–176.

[37] Rogers, D., and Tibben-Lembke, R.S. *Going Backwards: Reverse Logistics Trends and Practices*. Pittsburgh: RLEC Press, 1999.

[38] Ruiz-Benitez, R. *Supply Chain Coordination in the Presence of Consumer Returns*. Dissertation.

[39] Ruiz-Benitez, R., and Muriel, A. Supply chain coordination in the presence of consumer returns. 2007.

[40] Ruiz-Benitez, R., and Muriel, A. Supply chain coordination in the presence of consumer returns: The return allowance credit contract. 2007.

[41] Sakai, Y. The value of information in a simple duopoly model. *Journal of Economic Theory 36* (1985).

[42] Schmid, H. Impacts of asymmetric decision policies and consumer behavior on supply chain coordination under consumer returns.

[43] Signorelli, S., and Heskett, J.L. Benetton (a) and (b). *Harvard Business School case* (1984).

[44] Spengler, J.J. Vertical integration and antitrust policy. *Journal of Political Economy 58* (1950).

[45] Stock, J., T. Speh H., and Shear, H. Many happy (product) returns. *Harvard Business Review 80*, 7 (2002), 16–17.

[46] Sullivan, M.W. Slotting allowances and the market for new products. *Journal of Law and Economics 40*, 2 (1997), 461–493.

[47] Taylor, T. Supply chain coordination under channel rebates with sales effort effects. *Management Science 48*, 8 (2002), 992–1007.

[48] Tsay, A.A. Quantity-flexibility contract and supplier-customer incentives. *Management Science 45*, 10 (1999), 1339–1358.

[49] Tsay, A.A., and Lovejoy, W.S. Quantity-flexibility contracts and supply chain performance. *Manufacturing & Service Operations Management 1*, 2 (1999), 89–111.

[50] Tsay, A.A., S. Nahmias, and Agrawal, N. *Modeling supply chain contracts: A review*. Boston, Kluwer, 1999, ch. Supply chain contracting and coordination with stochastic demand.

[51] Ulph, A., and Ulph, D. *Who gains from learning about global warming?* 1996.

[52] Van Mieghem, J.A., and Dada, M. Price versus production postponement: Capacity and competition. *Management Science 45*, 12 (1999), 1631–1649.

[53] Vives, X. Technological competition, uncertainty, and oligopoly. *Journal of Economic Theory 48* (1989).

[54] Vlachos, D., and Dekker, R. Return handling options and order quantities for single period products. *European Journal of Operational Research 151* (2003), 38–52.

[55] Yao, L., Y. Chen, and Yan, H. Analysis of a supply contract for coordinating the newsvendor with price dependent demand. Tech. rep.