1993

THE EQUIVALENCE THEOREM AND GLOBAL ANOMALIES

JF Donoghue
donoghue@physics.umass.edu

J TANDEAN

The Equivalence Theorem and Global Anomalies

John F. Donoghue and Jusak Tandean

Department of Physics and Astronomy
University of Massachusetts
Amherst, MA 01003

February 1, 2008

Abstract

In the presence of some forms of global anomalies, the equivalence theorem, which relates the interactions of longitudinal gauge bosons to those of the Goldstone bosons, is not always valid. This can occur when the Goldstone sector contains an anomaly which is canceled in the gauge currents by the effects of a different sector of the theory. The example of the Standard Model without Higgs particles is used to illustrate this phenomena.

UMHEP-381

In the symmetry breaking of the Standard Model, three of the components of the complex Higgs doublet become the longitudinal degrees of freedom of the $W^{\pm}, Z^0$. In alternative mechanisms for symmetry breaking, such as Technicolor [1, 2], there also exist spin zero particles which turn into the longitudinal components of the gauge bosons. The equivalence theorem [3] says that the scattering amplitudes of longitudinal gauge bosons at high energy become equal to those of these spin zero Goldstone bosons in the original theory, up to corrections suppressed by powers of the energy

$$M(W_L^{\pm}, Z_L, \ldots) = M(w^{\pm}, z, \ldots) + O(M_W/E) \tag{1}$$

Here $w^{\pm}, z$ are the Goldstone fields. The equivalence theorem is useful because amplitudes involving the Goldstone bosons are generally easier to analyze than the full gauge boson amplitudes.

In this paper we discuss fermionic theories of symmetry breaking in which there are global but not gauge anomalies. We will see that this can lead to some counter examples to the equivalence theorem. Roughly stated, some global anomalies can modify the couplings of the Goldstone bosons, while not changing the gauge boson couplings. In most theories of Technicolor the quantum numbers have been arranged in a way such that this does not occur, but it remains a possibility in the larger framework of fermion-driven symmetry breaking.

It is generally accepted that the quantum numbers of the fermion of the theory must be chosen such that there are no anomalies in any of the currents coupled to gauge bosons. If the vector and axial vector gauge currents are described by matrices $T_V^{(a)}, T_A^{(a)}$

$$J_\mu^{(a)} = \bar{\psi}\gamma_\mu(T_V^{(a)} + T_A^{(a)}\gamma_5)\psi \tag{2}$$

the anomaly-free requirement is that

$$D^{abc} = Tr(T_A^{(a)}\{T_V^b, T_V^c\}) = 0 \tag{3}$$

For example, with the $SU(2)_L$ part of the neutral weak current and the electromagnetic current for one family

$$\begin{aligned} J_\mu^3 &= \frac{1}{2}[\bar{\gamma_\mu}(1+\gamma_5)u - \bar{d}\gamma_\mu(1+\gamma_5)d + \bar{\nu}\gamma_\mu(1+\gamma_5)\nu - \bar{e}\gamma_\mu(1+\gamma_5)e] \\ J_\mu^\gamma &= \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \bar{e}\gamma_\mu e \end{aligned} \tag{4}$$

the condition $D^{388} = 0$ occurs through the cancellation of the quark and electron contributions. In a one-doublet model of Technicolor, where the $SU(2)_L$ current is

$$J_\mu^{(3)} = \frac{1}{2}[\bar{U}\gamma_\mu(1+\gamma_5)U - \bar{D}\gamma_\mu(1+\gamma_5)D] \tag{5}$$

a cancellation with leptons is not used, so that the U(1) quantum numbers must be rearranged, i.e.,

$$J_\mu^\gamma = \frac{1}{2}[\bar{U}\gamma_\mu U - \bar{D}\gamma_\mu D] \tag{6}$$

in order to generate the anomaly-free condition within a single doublet.

In general there are other currents which do have anomalies. For example, the left handed isospin current in the Standard Model.

$$J_\mu^{3q} = \frac{1}{2}[\bar{u}\gamma_\mu(1+\gamma_5)u - \bar{d}\gamma_\mu(1+\gamma_5)d], \tag{7}$$

which is classically a global symmetry current in the limit that the u, d quarks are massless, does have an anomaly [4]

$$\partial^\mu J_\mu = \frac{\alpha N_c}{12\pi}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} + \text{quark mass terms}. \tag{8}$$

By standard methods, this leads to the matrix element for $\pi^0 \to \gamma\gamma$ decay

$$M(\pi^0 \to \gamma\gamma) = \frac{\alpha N_c}{3\pi F\pi}\epsilon_{1\mu}p_{1\nu}\epsilon_{2\alpha}p_{2\beta} \tag{9}$$

which is in good agreement with the experimental value.

Fermionic theories of symmetry breaking make use of the fact that dynamical symmetry breaking of a global invariance can also at the same time break the underlying gauge symmetry. A common pedagogical example is given by QCD with massless u, d quarks[1], which has a global $SU(2)_L \times SU(2)_R$ chiral symmetry. This is dynamically broken to $SU(2)_V$, with an order parameter

$$< 0 \mid \bar{u}_L u_R \mid 0 >=< 0 \mid \bar{d}_L d_R \mid 0 >\neq 0 \tag{10}$$

and with the pions, $\pi^\pm\pi^0$, being the Goldstone bosons. However, the vacuum condensates of Eq. 10 also break the $SU(2)_L \times U(1)$, gauge invariance of the electroweak sector. If the Standard Model were to contain no Higgs bosons, these QCD interactions would provide the dynamical breaking of $SU(2)_L \times U(1)$, with the pions being eaten to form the longitudinal components of $W^{\pm}Z^0$, and masses given by

$$M_W^2 = \frac{1}{4} g_2^2 F_\pi^2 = (30MeV)^2$$
$$M_Z^2 = M_W^2 / cos^2\theta_W \tag{11}$$

Technicolor theories are often said to be like QCD because they are modeled on this pattern of symmetry breaking, but with a larger mass scale $[F_\pi \rightarrow v = 246GeV]$ and generally with different particle and quantum number assignments.

QCD can also provide a pedagogical example for the clash between the equivalence theorem and global anomalies. As mentioned above, there exist a coupling of π^0 to two photons. In the Higgsless Standard Model, is the longitudinal Z coupling $Z^0 \rightarrow \gamma\gamma$ coupling equal to that of $\pi^0 \rightarrow \gamma\gamma$, as stated by the equivalence theorem? Actually the transition of $Z_L^0 \rightarrow \gamma\gamma$ is forbidden for on-shell photons by Yang's theorem[5], However even for off- shell photons the relevant diagram vanishes in the Higgless Standard Model. The vertex would be generated by the triangle diagram of Fig. 1 with electrons and u, d quarks in the loop. The electrons and quarks cancel because their quantum numbers have been arranged to yield an anomaly-free current. In contrast the $\pi^0 \rightarrow \gamma\gamma$ is related to the triangle diagram with only u, d quarks in the loop. The difference between these implies that $M(Z_L^0 \rightarrow \gamma\gamma) \neq M(\pi^0 \rightarrow \gamma\gamma)$. However this is not as yet a true violation to the equivalence theorem because it is not a process which occurs at high energies $E >> M_W$.

In order to be convinced that the global anomalies can lead to a violation of the equivalence theorem, we have carried out the calculation for $e^+e^- \rightarrow Z_L^0\gamma$ and compared it to $e^+e^- \rightarrow \pi^0\gamma$ in the above model. Both of these proceed through an off-shell γ (and Z) coupling. We display only the intermediate photon result, although the Z^0 contribution is very similar. The $\pi^0\gamma$ final state determined by the matrix element of Eq. 9, and yields

$$\sigma(e^+e^- \rightarrow \pi^0\gamma) = \frac{\alpha^3}{24\pi^2 F_\pi^2}. \tag{12}$$

The $Z^0\gamma$ amplitude with the quarks in the triangle diagram requires the full loop amplitude which has been given by Adler. For massless quarks one finds

$$\sigma(e^+e^- Z\gamma)_{quark} = \frac{\alpha^2 g_2^2}{96\pi^2 M_Z^2 cos^2\theta_w}\left(1-\frac{M_Z^4}{q^4}\right)\left(1-\frac{M_Z^2}{q^2-M_Z^2}\right)^2 \tag{13}$$

Because the vector boson mass is related to F_π as given in Eq. 11, Eq. 13 would satisfy the equivalence theorem if only quarks were to be included. However the full calculation of the $Z\gamma$ final state requires both quarks and leptons in the triangle diagram, and the lepton couplings have been arranged to cancel the effects of quarks, such that if all the fermions are massless one finds

$$\sigma(e^+e^- \rightarrow Z_L^0\gamma)_{TOT} = 0 \tag{14}$$

We have not calculated any higher order diagrams leading to the $Z\gamma$ final state.

None of the existing discussions[3] of the equivalence theorem take into account the possibilities of global anomalies in the Goldstone boson sector. Indeed most proofs are firmly within the context of the Standard Model with Higgs particles, and the Higgs particles do not have any anomalous couplings. In order to incorporate global anomalies into the treatment of longitudinal gauge bosons, one may use effective Lagrangians. Consider a theory beyond the Standard Model which has two sectors. One is strongly interacting and contains the fields which generate the Goldstone bosons. The only role of the other sector is to cancel the anomalies in the gauge currents, and we will assume that it consists of weakly interacting particles. If the scale of the strong sector is well above M_W, its effect at low energy can be described by an effective Lagrangian involving only the Goldstone fields. The result is rather similar to the effective Lagrangian found when one integrates out a very heavy fermion[6]. In that case, the heavy fermion does not completely decouple, but leaves behind the effect of the anomaly. Likewise, in our example the low energy theory must contain an explicit Wess-Zumino-Witten[7] anomaly Lagrangian in order to represent the contribution to the anomaly of the heavy particles. We have

$$\begin{aligned}\mathcal{L}_{TOT} &= L_{Goldstone} + L_{lepton} \\ \mathcal{L}_{Goldstone} &= L_{reg} + L_{anomaly}\end{aligned} \tag{15}$$

Here L_{lepton} is the usual $SU(2)_L \times U(1)$ invariant Lagranging of the weakly interacting sector. The usual Goldstone boson effective Lagrangian is

$$\begin{aligned}
\mathcal{L}_{reg} &= \frac{v}{4} Tr(D_\mu U D^\mu U^\dagger) + \ldots \\
U &= exp\left(i\frac{\vec{\tau}\cdot\vec{\phi}}{v}\right) \\
D_\mu U &= \partial_\mu U - ig_2\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu U + ig_1 U\frac{\tau_3}{2}B_\mu
\end{aligned} \tag{16}$$

The Wess Zumino Witten anomaly Lagrangian depends on the quantum number assignments of the fundamental fields. For a weak doublet in an $SU(N)$ vector theory, the complete anomaly Lagrangian is given in Ref. 8. We display here the portion involving two gauge bosons

$$\begin{aligned}
&L_{anomaly} = -i\frac{N_{TC}}{48\pi^2}\epsilon^{\mu\nu\alpha\beta}Tr[\partial_\mu r_\nu U^\dagger \ell_\alpha U R_\beta + \partial_\mu \ell_\nu U r_\alpha U^\dagger L_\beta \\
&+(r_\mu\partial_\nu r_\alpha + \partial_\mu r_\nu r_\alpha)R_\beta + (\ell_\mu\partial_\nu\ell_\alpha + \partial_\nu\ell_\alpha + \partial_\mu\ell_\nu\ell_\alpha)L_\beta \\
&+\frac{1}{2}(r_\mu R_\nu r_\alpha R_\beta - \ell_\mu L_\nu \ell_\alpha L_\beta) - r_\mu U^\dagger \ell_\nu U R_\alpha R_\beta + \ell_\mu U r_\nu U^\dagger L_\alpha L_\beta]
\end{aligned} \tag{17}$$

where

$$\begin{aligned}
L_\mu &= \partial_\mu U U^\dagger, R_\mu = U^\dagger L_\mu U = U^\dagger \partial_\mu U \\
-i\ell_\mu &= g_1(Q - \frac{\tau_3}{2})B_\mu + g_2\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu \\
&= eA_\mu Q + \frac{g_2 Z_\mu}{cos\theta_w}[\frac{\tau_3}{2} - sin^2\theta_w Q] + \frac{g_2}{\sqrt{2}}(\tau_+ W_\mu^- + \tau_- W_\mu^+) \\
-ir_\mu &= g_1 Q B_\mu = eQA_\mu - \frac{g_2 Z_\mu}{cos\theta_w} sin^2\theta_w Q
\end{aligned} \tag{18}$$

Given this effective Lagrangian, one can make a gauge change which removes most of the manifestations of the Goldstone bosons. For this "unitary gauge" we transform

$$g_2\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu \rightarrow g_2\frac{\vec{\tau}}{2}\cdot\vec{W}'_\mu = U g_2\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu U^\dagger - i\partial_\mu U U^\dagger \tag{19}$$

The lowest order Lagrangian becomes

$$L_2 = \frac{g_2^2 F_\pi^2}{4}(W_\mu^+ W^{-\mu} + \frac{g_2^2 F_\pi^2}{8cos^2\theta_w} A_\mu A^\mu) \tag{20}$$

i.e., only the gauge boson mass term survives. However the Goldstone fields do not disappear from the anomaly. While the specific form is not instructive, we have verified that the anomaly Lagrangian still contains both U and $\partial_\mu U$ after the gauge transformation of Eq. 18. Thus one does not entirely remove the Goldstone degree of freedom by going to the unitary gauge. This result is an indication, within the framework of effective Lagrangians, of the inequivalence of some of the Goldstone and gauge boson couplings

Most models of Techicolor do not share this problem. The reason is that in constructing a new technicolor model one generally requires that anomaly cancellations occur completely within the new strongly interacting sector as in Eq. 6. To do otherwise would be uneconomical because one would require further fermions ("leptons") outside of the Technicolor sector in order to make the theory free of gauge anomalies. However there is no requirement that forbids such an arrangement of fermions, as can be seen from the fact that Nature has chosen these quantum number assignments for the quarks and leptons of the Standard Model. Of course, the QCD effects described above have negligible effects on W and Z physics at the TeV scale. There may, however, be TeV scale theories in the class of possible examples of fermionic symmetry breaking which do contain global anomalies for the Goldstone bosons.

Acknowledgements: We thank A. Manohar, H. Georgi and G. Valencia for useful discussions. This work was supported in part by the U.S. National Science Foundation.

References

[1] L. Susskind, Phys. Rev. **D20**, 2619 (1979).
S. Weinberg, Phys. Rev. **D19**, 1277 (1979).

[2] E. Farhi and L. Susskind, Phys. Rep. **C74**, 277 (1981).

[3] J.M. Cornwall, D.N. Leven and G. Tiktopoulos, Phys. Rev. **D10**, 1145 (1974).
B.W. Lee, C. Quigg and H.B. Thacker, Phys. Rev. **D16**, 1519 (1977).
M. Chanowitz and M. K. Gaillard, Nucl. Phys. **B261**, 379 (1985).

J. Bagger and C. Schmidt, Phys. Rev. **D41**, 269 (1990).
H. Veltman, Phys. Rev. **D41**, 2294 (1990).

[4] S.L. Adler, Phys. Rev. **177**, 2426 (1969).
J.S. Bell and R. Jackiw, Nuovo Cim. **60A**, 47 (1967).

[5] C.N. Yang, Phys. Rev. **77**, 242 (1950).

[6] E. D'Hoker and E. Farhi, Nucl. Phys. **B248**, 59 (1984); **B248**, 77 (1984).

[7] J. Wess and B. Zumino, Phys. Lett. **B37**, 95 (1971).
E. Witten, Nucl. Phys. **B223**, 422 (1983).

[8] N. Pak and P. Rossi, Nucl. Phys. **B250**, 279 (1985).