

2013

Liberal Egalitarianism and the Harm Principle

Michele Lombardi

University of Glasgow, Michele.Lombardi@glasgow.ac.uk

Kaname Miyagishima

Waseda University

Roberto Veneziani

University of Massachusetts - Amherst

Follow this and additional works at: https://scholarworks.umass.edu/econ_workingpaper



Part of the [Economics Commons](#)

Recommended Citation

Lombardi, Michele; Miyagishima, Kaname; and Veneziani, Roberto, "Liberal Egalitarianism and the Harm Principle" (2013).
Economics Department Working Paper Series. 158.

Retrieved from https://scholarworks.umass.edu/econ_workingpaper/158

This Article is brought to you for free and open access by the Economics at ScholarWorks@UMass Amherst. It has been accepted for inclusion in Economics Department Working Paper Series by an authorized administrator of ScholarWorks@UMass Amherst. For more information, please contact scholarworks@library.umass.edu.

DEPARTMENT OF ECONOMICS

Working Paper

Liberal Egalitarianism and the Harm Principle

By

Michele Lombardi
Kaname Miyagishima
Roberto Veneziani

Working Paper 2013-07



**UNIVERSITY OF MASSACHUSETTS
AMHERST**

Liberal Egalitarianism and the Harm Principle

Michele Lombardi,¹ Kaname Miyagishima,² Roberto Veneziani³

May 22, 2013

¹Adam Smith Business School, University of Glasgow, Glasgow, G12 8QQ, United Kingdom.
E-mail: michele.lombardi@glasgow.ac.uk.

²JSPS Research Fellow, School of Political Science and Economics, Waseda University, 1-104
Totsukamachi, Shinjuku-ku, Tokyo, 169-8050, Japan. E-mail: kanamem@aoni.waseda.jp

³(Corresponding author) School of Economics and Finance, Queen Mary University of London,
London E1 4NS, United Kingdom, *and* Department of Economics, University of Massachusetts,
Amherst, USA. E-mail: r.veneziani@qmul.ac.uk.

Abstract

This paper analyses the implications of classical liberal and libertarian approaches for distributive justice in the context of social welfare orderings. An axiom capturing a liberal non-interfering view of society, named the Weak Harm Principle, is studied, whose roots can be traced back to John Stuart Mill's essay *On Liberty*. It is shown that liberal views of individual autonomy and freedom can provide consistent foundations for social welfare judgements, in both the finite and the infinite context. In particular, a liberal non-interfering approach can help to adjudicate some fundamental distributive issues relative to intergenerational justice. However, a surprisingly strong and general relation is established between liberal views of individual autonomy and non-interference, and egalitarian principles in the Rawlsian tradition.

JEL classification: D63; D70; Q01.

Keywords: Liberal principles, maximin, intergenerational equity, infinite utility streams.

Acknowledgements: Special thanks go to Geir Asheim, François Maniquet, Marco Mariotti and Peter Hammond, whose comments and suggestions have led to substantial improvements in the paper. We are grateful to J.C. Rodriguez Alcantud, N. Baigent, K. Basu, A. Carvajal, B. Dutta, M. Fleurbaey, K. Tadenuma, N. Yoshihara, and audiences at the University of Warwick (CRETA), the London School of Economics, the University of Maastricht, K.U. Leuven, Hitotsubashi University (Kunitachi), Waseda University (Tokyo), the University of Massachusetts (Amherst), the Midwest Political Science Association conference (Chicago), the New Directions in Welfare Conference (Oxford), the Royal Economic Society Conference (Guildford), the Logic, Game Theory and Social Choice conference (Tsukuba) and the Social Choice and Welfare Conference (Moscow) for useful comments and suggestions. The usual disclaimer applies.

1 Introduction

What are the implications of classical liberal and libertarian approaches for distributive justice? Can liberal views of individual autonomy and freedom provide consistent foundations for social welfare judgements? In particular, can a liberal non-interfering approach help to adjudicate some fundamental distributive issues relative to intergenerational justice? What is the relation between liberal political philosophy and the egalitarian tradition stemming from John Rawls's seminal book *A Theory of Justice* ([47])?

This paper addresses these questions, and in so doing it contributes to three different strands of the literature.

In some recent contributions, Mariotti and Veneziani ([45], [41]) have explored a new notion of respect for individual autonomy in social judgements, suited for Social Welfare Orderings (henceforth, SWOs), whose philosophical roots can be traced back to John Stuart Mill's essay *On Liberty*. The *Principle of Non-Interference* embodies the idea that "an individual has the right to prevent society from acting against him in all circumstances of change in his welfare, *provided* that the welfare of no other individual is affected" ([45], p.1).

Formally, the Principle Non-Interference (or Non-Interference, in short) can be illustrated as follows: in a society with two individuals, consider two allocations $u = (u_1, u_2)$ and $v = (v_1, v_2)$, describing the welfare levels of the two agents in two alternative scenarios. Suppose that, *for whatever reason*, u is strictly socially preferred to v . Suppose then that agent 1 either suffers a welfare loss, or enjoys a welfare increase in both allocations, while agent 2's welfare is unchanged, giving rise to two new allocations $u' = (u_1 + \varepsilon_u, u_2)$ and $v' = (v_1 + \varepsilon_v, v_2)$, with $\varepsilon_u \varepsilon_v > 0$. Non-Interference says that, if agent 1 strictly prefers u' to v' , then society should *not reverse* the strict preference between u and v to a strict preference for v' over u' . An agent "can veto society from a strict preference switch after a positive or negative change that affects only [her] and nobody else" ([45], p.2).

The veto power accorded to individuals is weak because a switch to indifference is admitted, *and* because Non-Interference is silent in a number of welfare configurations (e.g., if agent 1's welfare changes in opposite directions, $\varepsilon_u \varepsilon_v \not> 0$, or if she does not strictly prefer u' to v'). There are numerous non-dictatorial, and even anonymous SWOs that satisfy Non-Interference. Yet, surprisingly, Mariotti and Veneziani ([45]) prove that, in societies with a finite number of agents, dictatorial SWOs are the *only* ones compatible with Non-Interference among those satisfying Weak Pareto.¹ Lombardi and Veneziani ([40]) and Alcantud ([2]) have extended this result to societies with a countably infinite number of agents.

This impossibility proves the limitations of liberal approaches to Paretian social judge-

¹The Anonymity and Weak Pareto axioms are formally defined in section 3 below.

ments: there cannot be any ‘protected sphere’ for individuals even if nobody else is affected. As Mariotti and Veneziani ([45], p.2) put it, "Of the appeals of the individuals to be left alone because ‘nobody but me has been affected’, at least some will necessarily have to be overruled." The first contribution of this paper to the literature on liberal approaches is to analyse a specific, ethically relevant weakening of Non-Interference and provide a series of positive results, both in the finite and in the infinite context.

To be precise, we limit the bite of Non-Interference by giving individuals a veto power only in situations in which they suffer a *decrease* in welfare. Arguably, this captures the most intuitive aspect of a liberal ethics of Non-Interference, as it protects individuals in situations where they suffer a damage, while nobody else is affected: a switch in society’s strict preferences against an individual after she has incurred a welfare loss would represent a double punishment for her.

Formally, in the two-agent example above, we restrict Non-Interference to hold in situations where $\varepsilon_u, \varepsilon_v < 0$. We call this axiom the *Weak Harm Principle* - for it represents a strict weakening of the *Harm Principle* first introduced by Mariotti and Veneziani ([42]) - and show that a limited liberal ethics of Non-Interference can lead to consistent social judgements.²

The implications of liberal principles of non-interference (in conjunction with standard axioms in social choice), however, turn out to be fairly surprising. For there exists a strong formal and conceptual relation between liberal views, as incorporated in the Weak Harm Principle, and egalitarian social welfare relations (henceforth, SWRS). The analysis of this relation is the second main contribution of the paper.

Formally, we provide a number of fresh characterisations of widely used Rawlsian SWRS. Standard characterisations of the difference principle, or of its lexicographic extension, are based either on informational invariance and separability properties (see, e.g., d’Aspremont [19]; d’Aspremont and Gevers [20]) or on axioms with a marked egalitarian content such as the classic *Hammond Equity* axiom (Hammond [29], [30]).³

We prove that both the Rawlsian difference principle and its lexicographic extension can be characterised based on the Weak Harm Principle, together with standard efficiency, fairness and - where appropriate - continuity properties. The adoption of SWRS with a strong egalitarian bias can thus be justified based on a liberal principle of non-interference which is logically distinct from informational invariance and separability axioms, has no egalitarian

²Mariotti and Veneziani ([43]) analyse different restrictions of Non-Interference and characterise Nash-type orderings. For a related analysis of utilitarianism, see Mariotti and Veneziani ([44]).

³See also Tungodden ([57], [58]) and Bosmans and Ooghe ([15]). Similar axioms are used also in the infinite context; see, e.g., Lauwers ([35]), Asheim and Tungodden ([7]), Asheim et al. ([9]), Bossert et al. ([16]), Alcantud ([1]), Asheim and Zuber ([8]).

content and indeed has a marked individualistic flavour (in the sense of Hammond [31]).

This surprising relation between liberal approaches and egalitarian SWRS was originally established by Mariotti and Veneziani ([42]), who characterised the leximin SWO in finite societies based on the Harm principle. We extend and generalise their result in various directions.

First of all, as noted above, we focus on a strict weakening of the Harm principle. This is important both formally and conceptually. Formally, it has been argued that the characterisation in Mariotti and Veneziani ([42]) is less surprising than it seems, for under Anonymity the Harm Principle implies Hammond Equity (see Alcantud [2], Proposition 4). This conclusion does not hold with the Weak Harm Principle: even under Anonymity, the Weak Harm Principle and Hammond Equity are logically independent and the original insight of Mariotti and Veneziani ([42]) is therefore strengthened. Conceptually, by ruling out *only* a strict preference switch in social judgements, the Weak Harm Principle captures liberal and libertarian views more clearly than the Harm Principle, for it emphasises the negative prescription at the core of Mill’s analysis of non-interference and assigns a significantly weaker veto power to individuals.

Further, based on the Weak Harm Principle, we also provide new characterisations of Rawls’s difference principle. Compared to the leximin, the *maximin* SWR may be deemed undesirable because it defines rather large indifference classes. Yet, in a number of settings, its relatively simpler structure is a significant advantage, which allows one to capture the core egalitarian intuitions in a technically parsimonious way. Moreover, unlike the leximin, the maximin satisfies continuity and therefore egalitarian judgements based on the difference principle are more robust to small measurement mistakes, e.g. in empirical analysis. This probably explains the wide use of the maximin in modern theories of equality of opportunity (Roemer [48], [49]; Gotoh and Yoshihara [28]), in experimental approaches to distributive justice (Konow [34]; Bolton and Ockenfels [14]), in the analysis of the ethics of exhaustible resources and global warming (Solow [56]; Cairns and Long [21]; Roemer [51]; Llavador et al. [37]), and in the context of intergenerational justice (Silvestre [55]; Llavador et al. [36]).⁴ In the analysis of intergenerational justice and environmental economics, the maximin principle is often taken to embody the very notion of *sustainability* (Llavador et al. [38]).

Indeed, and this is the third main contribution of the paper, we analyse liberal and libertarian approaches to intergenerational justice. On the one hand, the intergenerational context provides a natural framework for the application of liberal principles of non-interference.

⁴Maximin preferences are prominent also outside of normative economics - for example, in decision theory and experimental economics. See, inter alia, the classic papers by Maskin ([46]); Barberà and Jackson ([11]); Gilboa and Schmeidler ([27]); and, more recently, de Castro et al. ([22]); Sarin and Vahid ([53]).

For there certainly are many economic decisions whose effects do not extend over time and leave the welfare of other generations unchanged. Moreover, liberal principles of non-interference seem to capture some widespread ethical intuitions in intergenerational justice (Wolf [60]). In the seminal Brundtland report, for example, sustainable development is defined precisely as “development that meets the needs of the present without compromising the ability of future generations to meet their needs” (Brundtland [17], p.43).

On the other hand, the application of liberal principles to intergenerational justice raises complex theoretical and technical issues. Lombardi and Veneziani ([40]) and Alcantud ([2]) have shown that there exists no fair and Paretian SWR that satisfies a fully non-interfering view in societies with a countably infinite number of agents. More generally, the analysis of distributive justice among an infinite number of generations is problematic for *all* of the main approaches, and impossibility results easily obtain (Lauwers [35]; Basu and Mitra [12]; Fleurbaey and Michel [24]; Zame [61]; Hara et al. [32]; Crespo et al. [18]). Several recent contributions have provided characterisation results for SWRs by dropping either completeness (Basu and Mitra [13]; Asheim and Tungodden [7]; Bossert et al. [16]; Asheim et al. [9]) or transitivity (Sakai [52]).⁵ But the definition of suitable anonymous and Paretian SWRs is still an open question in the infinite context (for a thorough discussion, see Asheim [3]).

Our main contribution to this literature is a novel analysis of liberal egalitarianism in economies with a countably infinite number of agents.

To be specific, we provide a new characterisation of one of the main extensions of the leximin SWR in infinitely-lived societies, namely the leximin overtaking proposed by Asheim and Tungodden ([7]). As in the finite-horizon case, we show that the Weak Harm Principle can be used to provide a simple and intuitive characterisation, without appealing to any informational invariance or separability property, or to axioms with an egalitarian content. Indeed, although we focus on a specific extension of the leximin that is prominent in the literature on evaluating infinite utility streams, our arguments can be modified to obtain new characterisations for *all* of the main approaches.

We also extend the analysis of Rawls’s difference principle to the intergenerational context. As already noted, if the leximin is adopted, social judgements are sensitive to tiny changes in welfare profiles and measurement errors. In the intergenerational context, an additional issue concerns the significant incompleteness of leximin SWRs which may significantly hamper social evaluation in a number of ethically relevant scenarios (see the discussion in Asheim et al. [5]). Therefore we provide a novel characterisation of the maximin *ordering*

⁵Asheim and Zuber ([8]) have recently proposed a complete and transitive extension of the leximin SWR which overcomes the impossibility by requiring only sensitivity to the interests of generations whose consumption has finite rank.

(more precisely, the *infimum rule*, Lauwers [35]) in societies with a countably infinite number of agents: based on the Weak Harm Principle, we identify a complete egalitarian criterion that allows for robust social evaluation of intergenerational distributive conflicts.

Our result differs from other characterisations in the literature in two key respects. Conceptually, the characterisation is again obtained by focusing on standard efficiency, fairness, and continuity properties together with a liberal principle of non-interference: neither egalitarian axioms, nor informational invariance or separability properties are necessary. Formally, unlike in Lauwers' ([35]) seminal paper, the proof of the characterisation results in the infinite context echoes very closely the proof in finite societies: perhaps surprisingly, both the axiomatic framework and the method of proof - and thus the underlying ethical intuitions - are essentially invariant.

In the light of our results, we can therefore provide some tentative answers to the questions posed in the opening paragraph. Liberal and libertarian approaches emphasising individual autonomy and freedom are logically consistent and provide useful guidance in social judgements (including in the analysis of intergenerational justice), provided the notion of non-interference is suitably restricted. Perhaps counterintuitively, however, a liberal non-interfering approach emphasising individual protection in circumstances of welfare losses leads straight to welfare egalitarianism. Based on the Weak Harm Principle, it is possible to provide a unified axiomatic framework to analyse a set of SWRs originating from Rawls's difference principle in a welfaristic framework. Thus, our analysis sheds new light on the normative foundations of standard egalitarian principles and provides a rigorous justification for the label 'liberal egalitarianism' usually associated with Rawls's approach.

The rest of the paper is structured as follows. Section 2 lays out the basic framework. Section 3 introduces our main liberal axiom and characterises the leximin SWO in economies with a finite number of agents. Section 4 analyses the implications of liberal views for robust (continuous) SWOs and derives a characterisation of the difference principle. Sections 5 and 6 extend the analysis to the intergenerational context. Section 7 concludes.

2 The framework

Let $X \equiv \mathbb{R}^{\mathbb{N}}$ be the set of countably infinite utility streams, where \mathbb{R} is the set of real numbers and \mathbb{N} is the set of natural numbers. An element of X is ${}_1u = (u_1, u_2, \dots)$ and u_t is the welfare level of agent t , or - in the intergenerational context - of a representative member of generation $t \in \mathbb{N}$. For $T \in \mathbb{N}$, ${}_1u_T = (u_1, \dots, u_T)$ denotes the T -head of ${}_1u$ and ${}_{T+1}u = (u_{T+1}, u_{T+2}, \dots)$ denotes its T -tail, so that ${}_1u = ({}_1u_T, {}_{T+1}u)$. $\min({}_1u) = \min\{u_1, u_2, \dots\}$ denotes the welfare level of the worst-off generation of ${}_1u$ whenever it exists. For $x \in \mathbb{R}$, ${}_{con}x = (x, x, x, \dots)$

denotes the stream of constant level of well-being equal to x .

A permutation π is a bijective mapping of \mathbb{N} onto itself. A permutation π of \mathbb{N} is finite if there is $T \in \mathbb{N}$ such that $\pi(t) = t, \forall t > T$, and Π is the set of all finite permutations of \mathbb{N} . For any ${}_1u \in X$ and any permutation π , let $\pi({}_1u) = (u_{\pi(t)})_{t \in \mathbb{N}}$ be a permutation of ${}_1u$. For any $T \in \mathbb{N}$ and ${}_1u \in X$, ${}_1\bar{u}_T$ is a permutation of ${}_1u_T$ such that the components are ranked in ascending order.

Let \succcurlyeq be a (binary) relation over X . For any ${}_1u, {}_1v \in X$, ${}_1u \succcurlyeq {}_1v$ stands for $({}_1u, {}_1v) \in \succcurlyeq$ and ${}_1u \not\succeq {}_1v$ for $({}_1u, {}_1v) \notin \succcurlyeq$; \succcurlyeq stands for “at least as good as”. The asymmetric factor \succ of \succcurlyeq is defined by ${}_1u \succ {}_1v$ if and only if ${}_1u \succcurlyeq {}_1v$ and ${}_1v \not\succeq {}_1u$, and the symmetric part \sim of \succcurlyeq is defined by ${}_1u \sim {}_1v$ if and only if ${}_1u \succcurlyeq {}_1v$ and ${}_1v \succcurlyeq {}_1u$. They stand, respectively, for “strictly better than” and “indifferent to”. A relation \succcurlyeq on X is said to be: *reflexive* if, for any ${}_1u \in X$, ${}_1u \succcurlyeq {}_1u$; and *transitive* if, for any ${}_1u, {}_1v, {}_1w \in X$, ${}_1u \succcurlyeq {}_1v \succcurlyeq {}_1w$ implies ${}_1u \succcurlyeq {}_1w$. \succcurlyeq is a quasi-ordering if it is reflexive and transitive. Let \succcurlyeq and \succcurlyeq' be relations on X , we say that \succcurlyeq' is an extension of \succcurlyeq if $\succcurlyeq \subseteq \succcurlyeq'$ and $\succ \subseteq \succ'$.

In this paper, we study some desirable properties of quasi-orderings, which incorporate notions of efficiency, fairness and liberal views of non-interference. In this section, we present some basic axioms that are used in the rest of the paper.

A property of SWRs that is a priori desirable is that they be able to rank all possible alternatives. Formally:

COMPLETENESS, **C**: for all ${}_1u, {}_1v \in X : {}_1u \neq {}_1v \Rightarrow {}_1u \succcurlyeq {}_1v$ or ${}_1v \succcurlyeq {}_1u$.

\succcurlyeq is an ordering if it is a complete quasi-ordering.

The standard way of capturing efficiency properties is by means of the Pareto axioms.⁶

STRONG PARETO, **SP**: $\forall {}_1u, {}_1v \in X : {}_1u > {}_1v \Rightarrow {}_1u \succ {}_1v$.

WEAK PARETO, **WP**: $\forall {}_1u, {}_1v \in X, \forall \epsilon > 0 : {}_1u \geq {}_1v + \text{con}\epsilon \Rightarrow {}_1u \succ {}_1v$.

A basic requirement of fairness is embodied in the following axiom, which requires social judgements to be neutral with respect to agents' identities.

FINITE ANONYMITY, **FA**: $\forall {}_1u \in X, \forall \pi \in \Pi, \pi({}_1u) \sim {}_1u$.

Finally, in the analysis of intergenerational justice, we follow the literature and consider two mainly technical requirements to deal with infinite-dimensional vectors (see, e.g., Asheim and Tungodden [7]; Basu and Mitra [13]; Asheim [3]; Asheim and Banerjee [4]).

⁶The notation for vector inequalities is as follows: for any ${}_1u, {}_1v \in X$, let ${}_1u \geq {}_1v$ if and only if $u_t \geq v_t, \forall t \in \mathbb{N}$; ${}_1u > {}_1v$ if and only if ${}_1u \geq {}_1v$ and ${}_1u \neq {}_1v$; and ${}_1u \gg {}_1v$ if and only if $u_t > v_t, \forall t \in \mathbb{N}$.

PREFERENCE CONTINUITY, **PC**: $\forall_1 u, {}_1 v \in X : \exists \tilde{T} \geq 1$ such that $({}_1 u_T, T+1 v) \succcurlyeq_1 v \forall T \geq \tilde{T} \Rightarrow {}_1 u \succcurlyeq_1 v$.

WEAK PREFERENCE CONTINUITY, **WPC**: $\forall_1 u, {}_1 v \in X : \exists \tilde{T} \geq 1$ such that $({}_1 u_T, T+1 v) \succcurlyeq_1 v \forall T \geq \tilde{T} \Rightarrow {}_1 u \succcurlyeq_1 v$.

These axioms establish “a link to the standard finite setting of distributive justice, by transforming the comparison of any two infinite utility paths to an infinite number of comparisons of utility paths each containing a finite number of generations” (Asheim and Tungodden [7]; p.223).

If there are only a finite set $\{1, \dots, T\} = N \subset \mathbb{N}$ of agents, or generations, X_T is the set of utility streams of X truncated at $T = |N|$, where $|N|$ is the cardinality of N . In order to simplify the notation, in economies with a finite number of agents the symbol u is used instead of ${}_1 u_T$. With obvious adaptations, the notation and the axioms spelled out above (except for **PC** and **WPC**) are carried over utility streams in X_T . Indeed, both **WP** and **FA** are logically equivalent to the standard weak Pareto and Anonymity axioms in finite economies, and so we shall use the same acronyms in both the finite and the infinite context.

3 The Weak Harm Principle

We study the implications of liberal views of non-interference in fair and Paretian social welfare judgements. In this section, we define and discuss the main liberal principle and then present a novel characterisation of the leximin ordering.

The key features of liberal, non-interfering views in social choice are captured by the Weak Harm Principle, according to which agents have a right to prevent society from punishing them in all situations in which they suffer a welfare loss, *provided no other agent is affected*. Formally:

WEAK HARM PRINCIPLE, **WHP**: $\forall u, v, u', v' \in X_T : u \succ v$ and u', v' are such that, $\exists i \in N$,

$$\begin{aligned} u'_i &< u_i \\ v'_i &< v_i \\ u'_j &= u_j \quad \forall j \neq i \\ v'_j &= v_j \quad \forall j \neq i \end{aligned}$$

implies $v' \not\succeq u'$ whenever $u'_i > v'_i$.

WHP captures a liberal view of non-interference whenever individual choices have no effect on others. The decrease in agent i 's welfare may be due to negligence or bad luck, but in any case **WHP** states that society should not strictly prefer v' over u' : having already suffered a welfare loss in both allocations, an adverse switch in society's strict preferences against agent i would represent an unjustified punishment for her.

WHP assigns a veto power to individuals in situations in which they suffer a harm and no other agent is affected. This veto power is weak in that it only applies to certain welfare configurations (individual preferences after the welfare loss must coincide with society's initial preferences) and, crucially, the individual cannot force society's preferences to coincide with her own.

WHP is weaker than the Principle of Non-Interference formulated by Mariotti and Veneziani ([45]) since it only focuses on welfare losses incurred by agents. However, it also represents a strict weakening of the Harm Principle (**HP**) proposed by Mariotti and Veneziani ([42]) because, unlike **HP**, it does not require that society's preferences over u' and v' be identical with agent i 's, but only that society should *not reverse* the strict preference between u and v to a strict preference for v' over u' (possibly except when i prefers otherwise). This weakening is important for both conceptual and formal reasons.

Conceptually, **WHP** aims to capture - in a welfaristic framework - a *negative freedom* that is central in classical liberal and libertarian approaches, namely, freedom from interference from society, when no other individual is affected. The name of the axiom itself is meant to echo J.S. Mill's famous formulation in his essay *On Liberty* (see Mariotti and Veneziani [41]). In this sense, by only requiring that agent i should not be punished in the SWR by changing social preferences against her, the liberal content of the axiom is much clearer and **WHP** strongly emphasises the negative prescription of the Harm Principle.

Formally, our weakening of **HP** has relevant implications for the analysis of liberal egalitarianism. Mariotti and Veneziani ([42]; Theorem 1, p.126) prove that, jointly with **SP**, **FA**, and **C**, **HP** characterises the leximin SWO, according to which that society is best which lexicographically maximises the welfare of its worst-off members.

The *leximin ordering* $\succsim^{LM} = \succ^{LM} \cup \sim^{LM}$ on X_T is formally defined as follows. The asymmetric factor \succ^{LM} of \succsim^{LM} is defined by:

$$u \succ^{LM} v \Leftrightarrow \bar{u}_1 > \bar{v}_1 \text{ or } [\exists i \in N \setminus \{1\} : \bar{u}_j = \bar{v}_j (\forall j \in N : j < i) \text{ and } \bar{u}_i > \bar{v}_i].$$

The symmetric factor \sim^{LM} of \succsim^{LM} is defined by:

$$u \sim^{LM} v \Leftrightarrow \bar{u}_i = \bar{v}_i, \forall i \in N.$$

The leximin SWO is usually considered to have a strong egalitarian bias, and so a characterisation based on a liberal principle with no explicit egalitarian content is surprising.

To clarify this point, note that the classic characterisation by Hammond ([29]) states that a SWR is the leximin ordering if and only if it satisfies **SP**, **FA**, **C**, and the following axiom.

HAMMOND EQUITY, **HE**: $\forall u, v \in X_T : u_i < v_i < v_j < u_j \exists i, j \in N, u_k = v_k \forall k \in N \setminus \{i, j\} \Rightarrow v \succ u$.

Unlike the **HP**, **HE** expresses a clear concern for equality, for it asserts that among any two welfare allocations which are not Pareto-ranked and differ only in two components, society should prefer the more egalitarian one.

Although **HE** and **HP** are conceptually distinct and logically independent, it has been argued that the characterisation of the leximin SWO in Mariotti and Veneziani ([42]) is formally unsurprising, because under **FA** and **C**, **HP** implies **HE** but the converse is not true (see Alcantud [2], Proposition 4).⁷ This objection does not hold if one considers **WHP**, instead. To see this, consider the following example.

Example 1 (Sufficientarianism) *Suppose that welfare units can be normalised so that a zero welfare level represents a decent living standard. Then one can define a SWR \succ^s on X_T according to which that society is best in which the highest number of people reach a decent living standard. Formally, $\forall u \in X_T$ let $P(u) = \{i \in N : u_i \geq 0\}$ and let $|P(u)|$ denote the cardinality of $P(u)$. Then $\forall u, v \in X_T$:*

$$u \succ^s v \Leftrightarrow |P(u)| \geq |P(v)|.$$

*It is immediate to see that \succ^s on X_T is an ordering and it satisfies **FA** and **WHP**, but violates both **HE** and **HP**.⁸*

Observe that the absence of any conceptual and formal relations between **WHP** and **HE**, even under **FA**, established in Example 1 is not a mere technical artefact. The Suppes-Sen grading principle, for instance, satisfies **FA** and **WHP** and violates **HE**, but one may object that this is due to its incompleteness. In contrast, the SWR in Example 1 is complete and it embodies a prominent approach to distributive justice in political philosophy and social choice (see, for example, Frankfurt [26] and Roemer [50]). Thus, even under **FA** and **C**, liberal principles of non-interference embody substantially different normative intuitions than standard equity axioms. Example 1 also highlights the theoretical relevance of our weakening of the Harm Principle, for the **WHP** is consistent with a wider class of SWOs,

⁷The argument is originally due to François Maniquet in unpublished correspondence.

⁸Consider, for example, two welfare profiles $u, v \in X_T$ such that $u = (2, -5, 1, 1, 1, \dots, 1)$ and $v = (-1, -1, 1, 1, 1, \dots, 1)$. By definition $u \succ^s v$, which violates **HE**.

including some - such as the sufficientarian - which embody some widely shared views on distributive justice.

Given this, it is perhaps surprising that the characterisation result provided in Mariotti and Veneziani ([42]) can be strengthened.⁹

Proposition 2 : *A SWR \succsim on X_T is the leximin ordering if and only if it satisfies **FA**, **SP**, **C**, and **WHP**.*

In the light of our discussion of **WHP** and Example 1, it is worth stressing some key theoretical implications of Proposition 2. First, it is possible to eschew impossibility results by weakening the Principle of Non-Interference proposed by Mariotti and Veneziani ([41]) while capturing some core liberal intuitions. For by Proposition 2 there exist anonymous and strongly Paretian SWOs consistent with liberal non-interfering views, as expressed in **WHP**.

Second, by Proposition 2 **HE** and **WHP** are equivalent in the presence of **FA**, **C**, and **SP**, even though they are logically independent. Actually, it can be proved that if $N = \{1, 2\}$, then under **SP** and **C**, **HE** implies **WHP**, but the converse is never true (see Mariotti and Veneziani [41]). Together with Example 1, this implies that Proposition 2 is far from trivial. For even under **C** and either **FA** or **SP**, **WHP** is not stronger than **HE**, and it is actually strictly weaker, at least in some cases.

Third, Proposition 2 puts the normative foundations of leximin under a rather different light. For, unlike in standard results, the egalitarian SWO is characterised without appealing to any axioms with a clear egalitarian content.¹⁰ Actually, **SP**, **C**, and **WHP** are compatible with some of the least egalitarian SWOs, namely the lexicographic dictatorships, which proves that **WHP** imposes no significant egalitarian restriction. As a result, Proposition 2 highlights the normative strength of Anonymity in determining the egalitarian outcome, an important insight which is not obvious in standard characterisations based on **HE**.

The next sections extend this intuition significantly and show that the counterintuitive egalitarian implications of liberal non-interfering principles are quite general and robust.

4 Liberal egalitarianism reconsidered

One common objection to the leximin SWO is its sensitivity to small changes in welfare profiles, and so to measurement errors and small variations in policies. Albeit possibly

⁹The properties in Proposition 2 are clearly independent. The proof of Proposition 2 is a generalisation of the proof of Theorem 1 in Mariotti and Veneziani ([42]) and is in Appendix 1.

¹⁰Nor to any invariance or separability axioms.

secondary in theoretical analyses, these issues are relevant in empirical applications and policy debates. In this section, we study the implications of liberal non-interfering approaches for social evaluations that are robust to small changes in welfare.

A standard way of capturing this property is by an interprofile condition requiring the SWO to vary continuously with changes in utility streams.

CONTINUITY, **CON**: $\forall u \in X_T$, the sets $\{v \in X_T | v \succ u\}$ and $\{v \in X_T | u \succ v\}$ are closed.

By Proposition 2, if **CON** is imposed in addition to **WHP**, **C**, **SP** and **FA** an impossibility result immediately obtains. Therefore we weaken our Paretian requirement to focus on **WP**. Strikingly, the combination of the five axioms characterises Rawls's difference principle.

The *maximin ordering* \succ^M on X_T is defined as follows: $\forall u, v \in X_T$,

$$u \succ^M v \Leftrightarrow \bar{u}_1 \geq \bar{v}_1.$$

Theorem 3 states that the standard requirements of fairness (**FA**), efficiency (**WP**), completeness (**C**), and continuity (**CON**), together with our liberal axiom characterise the maximin SWO.¹¹

Theorem 3 : A SWR \succ on X_T is the maximin ordering if and only if it satisfies **FA**, **WP**, **C**, **CON**, and **WHP**.

Proof. (\Rightarrow) Let \succ on X_T be the maximin ordering, i.e., $\succ = \succ^M$. It can be easily verified that \succ^M on X_T satisfies **FA**, **WP**, **C**, **CON**, and **WHP**.

(\Leftarrow) Let \succ on X_T be a SWR satisfying **FA**, **WP**, **C**, **CON**, and **WHP**. We show that \succ is the maximin SWO. We prove that, $\forall u, v \in X_T$,

$$u \succ^M v \Leftrightarrow u \succ v \tag{1}$$

and

$$u \sim^M v \Leftrightarrow u \sim v. \tag{2}$$

Note that as \succ on X_T satisfies **FA**, in what follows we can focus either on u and v , or on the ranked vectors \bar{u} and \bar{v} , without loss of generality.

First, we show that the implication (\Rightarrow) of (1) is satisfied. Take any $u, v \in X_T$. Suppose that $u \succ^M v \Leftrightarrow \bar{u}_1 > \bar{v}_1$. We proceed by contradiction, first proving that $v \succ u$ is impossible and then ruling out $v \sim u$.

¹¹The properties in Theorem 3 are clearly independent.

Suppose that $v \succ u$, or equivalently, $\bar{v} \succ \bar{u}$. As **WP** holds, $\bar{v}_j \geq \bar{u}_j$ for some $j \in N$, otherwise a contradiction immediately obtains. We proceed according to the following steps.

Step 1. Let

$$k = \min \{l \in N \mid \bar{v}_l \geq \bar{u}_l\}.$$

By **FA**, let $v_i = \bar{v}_k$ and let $u_i = \bar{u}_1$. Then, consider two real numbers $d_1, d_2 > 0$, and two vectors u', v^* - together with the corresponding ranked vectors $\bar{u}', \bar{v}^* \in X_T$ - formed from \bar{u}, \bar{v} as follows: \bar{u}_1 is lowered to $\bar{u}_1 - d_1 > \bar{v}_1$; \bar{v}_k is lowered to $\bar{u}_k > \bar{v}_k - d_2 > \bar{u}_1 - d_1$; and all other entries of \bar{u} and \bar{v} are unchanged. By construction $\bar{u}'_j > \bar{v}^*_j$ for all $j \leq k$, whereas by **WHP**, **C**, and **FA**, we have $\bar{v}^* \succ \bar{u}'$.

Step 2. Let

$$0 < \epsilon < \min\{\bar{u}'_j - \bar{v}^*_j \mid j \leq k\}$$

and define $\bar{v}' = \bar{v}^* + \text{con}\epsilon$. By construction, $\bar{v}^* \ll \bar{v}'$. **WP** implies $\bar{v}' \succ \bar{v}^*$. As $\bar{v}^* \succ \bar{u}'$, by *step 1*, the transitivity of \succ implies $\bar{v}' \succ \bar{u}'$.

If $\bar{u}'_j > \bar{v}'_j$ for all $j \in N$, **WP** implies $\bar{u}' \succ \bar{v}'$, a contradiction. Otherwise, let $\bar{v}'_l \geq \bar{u}'_l$ for some $l > k$. Then, let

$$k' = \min \{l \in N \mid \bar{v}'_l \geq \bar{u}'_l\}.$$

The above steps 1-2 can be applied to \bar{u}', \bar{v}' to derive vectors \bar{u}'', \bar{v}'' such that $\bar{u}''_j > \bar{v}''_j$ for all $j \leq k'$, whereas $\bar{v}'' \succ \bar{u}''$. By **WP**, a contradiction is obtained whenever $\bar{u}''_j > \bar{v}''_j$ for all $j \in N$. Otherwise, let $\bar{v}''_l \geq \bar{u}''_l$ for some $l > k'$. And so on. After a finite number s of iterations, two vectors \bar{u}^s, \bar{v}^s can be derived such that $\bar{v}^s \succ \bar{u}^s$, by steps 1-2, but $\bar{u}^s \succ \bar{v}^s$, by **WP**, a contradiction.

Therefore, by **C**, it must be $\bar{u} \succ \bar{v}$ whenever $\bar{u} \succ^M \bar{v}$. We have to rule out the possibility that $\bar{u} \sim \bar{v}$. We proceed by contradiction. Suppose that $\bar{u} \sim \bar{v}$. Since $\bar{v}_1 < \bar{u}_1$, there exists $\epsilon > 0$ such that $\bar{v}^\epsilon = \bar{v} + \text{con}\epsilon$ and $\bar{v}_1^\epsilon < \bar{u}_1$ so that $\bar{u} \succ^M \bar{v}^\epsilon$. However, by **WP** and transitivity of \succ it follows that $\bar{v}^\epsilon \succ \bar{u}$. Then the above reasoning can be applied to \bar{v}^ϵ and \bar{u} to obtain the desired contradiction.

Now, we show that the implication (\Rightarrow) of (2) is met as well. Suppose that $u \sim^M v \Leftrightarrow \bar{u}_1 = \bar{v}_1$. Assume, to the contrary, that $u \not\sim v$, or equivalently, $\bar{u} \not\sim \bar{v}$. By **C**, without loss of generality, let $\bar{u} \succ \bar{v}$. By **FA**, it must be $\bar{u} \neq \bar{v}$. As $\bar{u} \succ \bar{v}$, it follows from **CON** that there exists a neighbourhood $S(\bar{v})$ of \bar{v} such that $\bar{u} \succ v'$ for all $v' \in S(\bar{v})$. Then, there exists $v' \in S(\bar{v})$ such that $v' \gg \bar{v}$ and $\bar{u} \succ v'$, so that $\bar{u} \succ v'$ but $v' \succ^M \bar{u}$. By the implication (1) proved above, it follows that $v' \succ \bar{u}$, a contradiction. ■

Theorem 3 has two main implications in the context of our analysis. First, it provides another way out of the impossibility result by Mariotti and Veneziani ([45]): if their Principle of Non-Interference is replaced by **WHP**, then there exist anonymous and (weakly) Paretian liberal SWOs that are also continuous. This is particularly interesting given that the consistency between Weak Pareto, continuity properties, and liberal principles in the spirit of Sen’s celebrated *Minimal Liberalism* axiom has been recently called into question by Kaplow and Shavell ([33]).

Second, Theorem 3 provides a novel characterisation of the maximin that generalises the key insight of the previous section. Standard characterisations of the difference principle use either informational invariance and separability properties (d’Aspremont and Gevers [20]; Segal and Sobel [54]) or axioms incorporating a clear inequality aversion such as **HE** (Bosmans and Ooghe [15]) or the Pigou-Dalton principle (Fleurbaey and Tungodden [25]).

Unlike informational invariance axioms, **WHP** has a clear ethical foundation, but it has no egalitarian content. Thus, Theorem 3 characterises an egalitarian principle using an axiom, **WHP**, which only incorporates a liberal, non-interfering view of society.

5 The Weak Harm Principle and intergenerational justice

In the previous sections, we have studied the implications of liberal principles of non-interference in societies with a finite number of agents and have shown that consistent fair and Paretian liberal social judgements are possible. We now extend our analysis to societies with an infinite number of agents. A liberal non-interfering approach seems particularly appropriate in the analysis of intergenerational distributive issues: although the welfare of a generation is often affected by decisions taken by their predecessors, there certainly are many economic decisions whose effects do not extend over time and leave the welfare of other generations unchanged. In this section (and the next), we show that a consistent fair and Paretian liberal approach to intergenerational justice is indeed possible.

The extension of the main liberal principle to the analysis of intergenerational justice is rather straightforward and needs no further comment, except possibly noting that in this context, **WHP** is weakened to hold only for pairs of welfare allocations whose tails can be Pareto-ranked.

WEAK HARM PRINCIPLE, **WHP***: $\forall_1 u, {}_1 v, {}_1 u', {}_1 v' \in X : {}_1 u \succ {}_1 v$ and $\exists T \geq 1, \exists \epsilon \geq 0$ such

that ${}_1v \equiv ({}_1v_T, ({}_{T+1}u + \text{con}\epsilon))$, and ${}_1u', {}_1v'$ are such that, $\exists i \leq T$,

$$\begin{aligned} u'_i &< u_i, \\ v'_i &< v_i, \\ u'_j &= u_j, \forall j \neq i, \\ v'_j &= v_j, \forall j \neq i, \end{aligned}$$

implies ${}_1v' \not\succ {}_1u'$ whenever $u'_i > v'_i$.

As already noted, economies with an infinite number of agents raise several issues concerning the existence and the characterisation of SWOs, and different definitions of the main criteria (including utilitarianism, egalitarianism, the Nash ordering, and so on) can be provided in order to compare (countably) infinite utility streams. Here, we provide a novel characterisation of one of the main approaches in the literature, namely the leximin overtaking recently formalised by Asheim and Tungodden ([7]), in the tradition of Atsumi ([10]) and von Weizsäcker ([59]). However, as argued at the end of this section, our key results are robust and **WHP*** can be used to provide normative foundations to *all* of the main extensions of the leximin SWR in the literature. Perhaps surprisingly, liberal views of non-interference *in general* lead to egalitarian SWRs even in the intergenerational context.

The leximin overtaking criterion is defined as follows.

Definition. (Asheim and Tungodden [7]; Definition 2, p.224) For all ${}_1u, {}_1v \in X$, ${}_1u \sim^{LM*} {}_1v \Leftrightarrow \exists \tilde{T} \geq 1$ such that $\forall T \geq \tilde{T}$: ${}_1\bar{u}_T = {}_1\bar{v}_T$; and ${}_1u \succ^{LM*} {}_1v \Leftrightarrow \exists \tilde{T} \geq 1$ such that $\forall T \geq \tilde{T}$ $\exists t \in \{1, \dots, T\}$: $\bar{u}_s = \bar{v}_s \forall 1 \leq s < t$ and $\bar{u}_t > \bar{v}_t$.

In order to characterise the leximin overtaking, we also require that the SWR be at least able to compare profiles with the same tail. This seems an obviously desirable property which imposes a minimum requirement of completeness on the SWR.

MINIMAL COMPLETENESS, MC: $\forall {}_1u, {}_1v \in X$, $\exists T \geq 1$ $({}_1u_T, {}_{T+1}v) \neq {}_1v \Rightarrow ({}_1u_T, {}_{T+1}v) \succ {}_1v$ or ${}_1v \succ ({}_1u_T, {}_{T+1}v)$.

Theorem 4 proves that Finite Anonymity, Strong Pareto, Weak Harm Principle, Minimal Completeness and Weak Preference Continuity characterise the leximin overtaking.¹²

Theorem 4 : \succ is an extension of \succ^{LM*} if and only if \succ satisfies **FA**, **SP**, **MC**, **WHP***, and **WPC**.

¹²The proof that the properties in Theorem 4 are independent is in Appendix 2.

Proof (\Rightarrow) Let $\succcurlyeq^{LM^*} \subseteq \succcurlyeq$. It is easy to see that \succcurlyeq meets **FA** and **SP**. By observing that \succcurlyeq^{LM^*} is complete for comparisons between utility streams with the same tail it is also easy to see that \succcurlyeq satisfies **MC** and **WPC**.

We show that \succcurlyeq meets **WHP***. Take any ${}_1u, {}_1v, {}_1u', {}_1v' \in X$ such that ${}_1u \succcurlyeq {}_1v$, and $\exists T \geq 1, \exists \epsilon \geq 0$ such that ${}_1v \equiv ({}_1v_T, ({}_{T+1}u + \text{con}\epsilon))$, and ${}_1u', {}_1v'$ are such that $\exists i \leq T, u'_i < u_i, v'_i < v_i$, and $u'_j = u_j, v'_j = v_j \forall j \neq i$. We show that ${}_1u' \succcurlyeq {}_1v'$ whenever $u'_i > v'_i$.

Because \succcurlyeq^{LM^*} is complete for comparisons between utility streams whose tails differ by a nonnegative constant, ${}_1u \succcurlyeq^{LM^*} {}_1v$. Therefore by definition, $\exists \tilde{T} \geq 1 : \forall T' \geq \tilde{T} \exists t \in \{1, \dots, T'\} \bar{u}_s = \bar{v}_s \forall 1 \leq s < t$ and $\bar{u}_t > \bar{v}_t$. Take any $T' \geq \tilde{T}$. Theorem 1 in Mariotti and Veneziani ([42]; 126) implies that there exists $t^* \leq t \leq T'$ such that $\bar{u}'_s = \bar{v}'_s \forall 1 \leq s < t^*$ and $\bar{v}'_{t^*} < \bar{u}'_{t^*}$. As it holds true for any $T' \geq \tilde{T}$, it follows that ${}_1u' \succcurlyeq {}_1v'$ as $\succcurlyeq^{LM^*} \subseteq \succcurlyeq$.

(\Leftarrow) Suppose that \succcurlyeq satisfies **FA**, **SP**, **MC**, **WHP***, and **WPC**. We show that $\sim^{LM^*} \subseteq \sim$ and $\succcurlyeq^{LM^*} \subseteq \succcurlyeq$. Take any ${}_1u, {}_1v \in X$.

Suppose that ${}_1u \sim^{LM^*} {}_1v$. By definition, $\exists \tilde{T} \geq 1 : \forall T \geq \tilde{T} \bar{u}_T = \bar{v}_T$, and so ${}_{T+1}u = {}_{T+1}v$, for any $T \geq \tilde{T}$. It follows that ${}_1u \sim {}_1v$, by **FA**.

Suppose that ${}_1u \succcurlyeq^{LM^*} {}_1v$. By definition, $\exists \tilde{T} \geq 1 : \forall T \geq \tilde{T} \exists t \in \{1, \dots, T\}$ such that $\bar{u}_s = \bar{v}_s \forall 1 \leq s < t$ and $\bar{u}_t > \bar{v}_t$. Take any such T and consider ${}_1w \equiv ({}_1u_T, {}_{T+1}v)$. Note that ${}_1w \succcurlyeq^{LM^*} {}_1v$. We show that ${}_1w \succcurlyeq {}_1v$. By **FA** and transitivity, we can consider ${}_1\bar{w} \equiv ({}_1\bar{u}_T, {}_{T+1}v)$ and ${}_1\bar{v} \equiv ({}_1\bar{v}_T, {}_{T+1}v)$. By **MC**, suppose that ${}_1\bar{v} \succcurlyeq {}_1\bar{w}$. We distinguish two cases.

Case 1. ${}_1\bar{v} \succcurlyeq {}_1\bar{w}$

As **SP** holds it must be the case that $\bar{v}_l > \bar{w}_l$ for some $l > t$. Let

$$k = \min\{t < l \leq T | \bar{v}_l > \bar{w}_l\}.$$

By **FA**, let $v_i = \bar{v}_k$ and let $w_i = \bar{w}_{k-g}$, for some $1 \leq g < k$, where $\bar{w}_{k-g} > \bar{v}_{k-g}$. Then, let two real numbers $d_1, d_2 > 0$, and consider vectors ${}_1w', {}_1v'$ formed from ${}_1\bar{w}, {}_1\bar{v}$ as follows: \bar{w}_{k-g} is lowered to $\bar{w}_{k-g} - d_1$ such that $\bar{w}_{k-g} - d_1 > \bar{v}_{k-g}$; \bar{v}_k is lowered to $\bar{v}_k - d_2$ such that $\bar{w}_k > \bar{v}_k - d_2 > \bar{w}_{k-g} - d_1$; and all other entries of ${}_1\bar{w}$ and ${}_1\bar{v}$ are unchanged. By **FA**, consider ${}_1\bar{w}' = ({}_1\bar{w}'_T, {}_{T+1}v)$ and ${}_1\bar{v}' = ({}_1\bar{v}'_T, {}_{T+1}v)$. By construction $\bar{w}'_j \geq \bar{v}'_j$ for all $j \leq k$, with $\bar{w}'_{k-g} > \bar{v}'_{k-g}$, whereas **WHP***, combined with **MC** and **FA**, implies ${}_1\bar{v}' \succcurlyeq {}_1\bar{w}'$. Furthermore, by **SP**, it is possible to choose $d_1, d_2 > 0$, such that ${}_1\bar{v}' \succcurlyeq {}_1\bar{w}'$, without loss of generality. Consider two cases:

a) Suppose that $\bar{v}_k > \bar{w}_k$, but $\bar{w}_l \geq \bar{v}_l$ for all $l > k$. It follows that ${}_1\bar{w}' > {}_1\bar{v}'$, and so **SP** implies that ${}_1\bar{w}' \succcurlyeq {}_1\bar{v}'$, a contradiction.

b) Suppose that $\bar{v}_l > \bar{w}_l$ for some $l > k$. Note that by construction $\bar{v}'_l = \bar{v}_l$ and $\bar{w}'_l = \bar{w}_l$ for

all $l > k$. Then, let

$$k' = \min\{k < l \leq T | \bar{v}'_l > \bar{w}'_l\}.$$

The above argument can be applied to ${}_1\bar{w}'$, ${}_1\bar{v}'$ to derive vectors ${}_1\bar{w}''$, ${}_1\bar{v}''$ such that $\bar{w}''_j \geq \bar{v}''_j$ for all $j \leq k'$, whereas **WHP***, combined with **MC**, **FA**, and **SP**, implies ${}_1\bar{v}'' \succ {}_1\bar{w}''$. And so on. After a finite number of iterations s , two vectors ${}_1\bar{w}^s$, ${}_1\bar{v}^s$ can be derived such that, by **WHP***, combined with **MC**, **FA**, and **SP**, we have that ${}_1\bar{v}^s \succ {}_1\bar{w}^s$, but **SP** implies ${}_1\bar{w}^s \succ {}_1\bar{v}^s$, yielding a contradiction.

Case 2. ${}_1\bar{v} \sim {}_1\bar{w}$

Since, by our supposition, $\bar{v}_t < \bar{u}_t \equiv \bar{w}_t$, there exists $\epsilon > 0$ such that $\bar{v}_t < \bar{w}_t - \epsilon < \bar{w}_t$. Let ${}_1\bar{w}^\epsilon \in X$ be a vector such that $\bar{w}^\epsilon_t = \bar{w}_t - \epsilon$ and $\bar{w}^\epsilon_j = \bar{w}_j$ for all $j \neq t$. It follows that ${}_1\bar{w}^\epsilon \succ^{LM*} {}_1\bar{v}$ but ${}_1\bar{v} \succ {}_1\bar{w}^\epsilon$ by **SP** and the transitivity of \succ . Hence, the argument of *Case 1* above can be applied to ${}_1\bar{v}$ and ${}_1\bar{w}^\epsilon$, yielding the desired contradiction.

It follows from **MC** that ${}_1\bar{w} \succ {}_1\bar{v}$. **FA**, combined with the transitivity of \succ , implies that $({}_1u_T, {}_{T+1}v) \succ {}_1v$. Since $T \geq \tilde{T}$ is arbitrary, **WPC** implies ${}_1u \succ {}_1v$, as desired. ■

Theorem 4 shows that, if the principle of non-interference analysed by Lombardi and Veneziani ([40]) and Alcantud ([2]) in the intergenerational context is suitably restricted to hold only for welfare losses, then possibility results for liberal, fair and Paretian social judgements do emerge. Indeed, Theorem 4 provides a novel characterisation of one of the main extensions of the leximin criterion to economies with an infinite number of agents, based on **WHP***, thus confirming the striking link between a liberal and libertarian concern for individual autonomy, and egalitarian criteria.

These conclusions are robust and can be extended to alternative definitions of the leximin criterion.¹³ For example, if **WPC** is replaced with a stronger continuity requirement, a stronger version of the leximin overtaking (the *S-Leximin*, see Asheim and Tungodden, [7]; Definition 1, p.224) can easily be derived. Perhaps more interestingly, Bossert et al. ([16]) have dropped continuity properties and have characterised a larger class of extensions of the leximin criterion satisfying **SP**, **FA**, and an infinite version of **HE**.¹⁴ Lombardi and Veneziani ([39]) have shown that it is possible to provide a characterisation of the leximin relation defined by Bossert et al. ([16]) based on **SP**, **FA**, and the Weak Harm Principle. Further,

¹³It is worth noting in passing that Theorem 4 can be further strengthened by requiring **WHP*** to hold only for vectors with the same tail, namely $\epsilon = 0$.

¹⁴Formally, the relationship between the characterisation of the leximin by Bossert et al. ([16]) and that by Asheim and Tungodden ([7]) is analogous to the relationship between the characterisation of the utilitarian SWR by Basu and Mitra ([13]) and the characterisations of the more restrictive utilitarian SWR induced by the overtaking criterion (see the discussion in Bossert et al. [16]; p.580).

the Weak Harm Principle can be used - instead of various versions of the Hammond equity axiom - to characterise the leximin SWR proposed by Sakai ([52]), which drops transitivity but retains completeness; and the *time-invariant leximin overtaking* proposed by Asheim et al. ([5]).¹⁵

In summary, in the intergenerational context too, liberalism implies equality.¹⁶

6 Liberal egalitarianism extended

In section 4, we argued that a potential shortcoming of the leximin criterion is its sensitivity to infinitesimal changes in welfare profiles and explored the implications of liberal principles together with a continuity requirement that incorporates a concern for robustness in social judgements. In the context of intergenerational distributive justice, a further problem of the various extensions of the leximin criterion is their incompleteness, which makes them unable to produce social judgements in a large class of pairwise comparisons of welfare profiles.

In this section, we complete our analysis of liberal principles of non-interference by analysing the implications of **WHP*** in adjudicating intergenerational distributive conflicts when social welfare criteria are required to be continuous *and* to be able to adjudicate *all* distributive conflicts. This is by no means a trivial question, for it is well known that continuity is a problematic requirement for SWOs in economies with an infinite number of agents and impossibility results easily arise (Hara et al. [32]; Zame [61]).

As a first step, we shall slightly restrict our state space as follows:

$$\bar{X} \equiv \{u \in X \mid \exists M \in \mathbb{R} : |u_t| \leq M \ \forall t \in \mathbb{N}\}.$$

This is a mild restriction, which yields no significant loss of generality and follows a common practice in the literature on intergenerational justice (e.g., Lauwers [35]; Basu and Mitra [12], [13]; Zame [61]; Hara et al. [32]; Asheim [3]; Asheim and Banerjee [4]).¹⁷

The main axioms incorporating completeness, fairness, efficiency, and liberal non-interference are the same as in previous sections, given the domain restriction.

¹⁵As compared to the standard overtaking criterion, the time invariant version does not rely on a natural ordering of generations. Thus, it is possible to drop **WPC** and replace it with a similar consistency axiom that does not entail a preference for earlier generations.

¹⁶The proofs of the above claims are available from the authors upon request.

¹⁷The restriction can also be motivated theoretically. For example, Mariotti and Veneziani ([43], [44]) argue that opportunities should be conceptualised as chances in life, or *probabilities* of success, and study allocation criteria on the T -dimensional unit box. We note in passing that our main conclusions continue to hold even if one allows for welfare profiles that are unbounded above, albeit at the cost of some changes in the axiomatic system.

Because the set of infinite bounded vectors has no natural topology, we follow Lauwers ([35]) and define continuity based on the sup metric.

SUP CONTINUITY, \mathbf{CON}_{d_∞} : $\forall_1 u \in \bar{X}$: there is a sequence of vectors $\{1v^k\}_{k=1}^\infty$ such that $\lim_{k \rightarrow \infty} 1v^k = 1v \in \bar{X}$ with respect to the sup metric d_∞ , and $1v^k \succcurlyeq 1u$ (*resp.*, $1u \succcurlyeq 1v^k$) $\forall k \in \mathbb{N} \Rightarrow 1u \not\succeq 1v$ (*resp.*, $1v \not\succeq 1u$).

Observe that in general \mathbf{CON}_{d_∞} is weaker than the standard continuity axiom but it is equivalent to the latter if the SWR is complete.¹⁸

Our next result extends the key insights on liberal egalitarianism to the intergenerational context. Formally, the *maximin* SWO \succcurlyeq^{M^*} on \bar{X} can be defined as follows:

$$\forall_1 u, 1v \in \bar{X} : 1u \succcurlyeq^{M^*} 1v \Leftrightarrow \inf_{t \in \mathbb{N}} u_t \geq \inf_{t \in \mathbb{N}} v_t.$$

Theorem 5 proves that Finite Anonymity, Weak Pareto, Completeness, Sup Continuity, Weak Harm Principle, and Preference Continuity characterise \succcurlyeq^{M^*} on \bar{X} .¹⁹

Theorem 5 *A SWR \succcurlyeq on \bar{X} is the maximin SWO if and only if it satisfies **FA**, **WP**, **C**, \mathbf{CON}_{d_∞} , **WHP***, and **PC**.*

Proof. (\Rightarrow) Let \succcurlyeq on \bar{X} be the maximin SWO, i.e., $\succcurlyeq = \succcurlyeq^{M^*}$. It can be easily verified that \succcurlyeq^{M^*} on \bar{X} satisfies **FA**, **WP**, **C**, \mathbf{CON}_{d_∞} , **WHP***, and **PC**.

(\Leftarrow) Let \succcurlyeq on \bar{X} be a SWR satisfying **FA**, **WP**, **C**, \mathbf{CON}_{d_∞} , **WHP***, and **PC**. We show that \succcurlyeq is the maximin SWO. To this end, it suffices to show that $\forall_1 u, 1v \in \bar{X}$,

$$\inf_{t \in \mathbb{N}} u_t > \inf_{t \in \mathbb{N}} v_t \Rightarrow 1u \succ 1v \tag{3}$$

and

$$\inf_{t \in \mathbb{N}} u_t = \inf_{t \in \mathbb{N}} v_t \Rightarrow 1u \sim 1v. \tag{4}$$

Consider (3). Take any $1u, 1v \in \bar{X}$ such that $\inf_{t \in \mathbb{N}} u_t > \inf_{t \in \mathbb{N}} v_t$. In order to prove that $1u \succ 1v$, we first demonstrate that

$$\exists T \geq 1, \forall t \geq T, \forall \epsilon > 0 : (1\hat{x}_t, {}_{t+1}v + {}_{con}\epsilon) \succcurlyeq 1v, \tag{5}$$

¹⁸It is also weaker than the *Continuity* axiom recently proposed by Asheim et al. ([6], p.271), although the two properties are equivalent for complete SWRs.

¹⁹The proof that the properties in Theorem 5 are independent is in Appendix 2. It is worth noting in passing that the characterisation of the maximin SWO can also be obtained without the full force of completeness, by adopting an axiom similar to **MC** above. We thank Geir Asheim for suggesting this to us.

where

$$\hat{x} = \frac{\inf_{t \in \mathbb{N}} u_t + \inf_{t \in \mathbb{N}} v_t}{2}.$$

We proceed by contradiction. Assume that (5) fails. Since \succsim satisfies **C**, it follows that for any $T \geq 1$ there exist $t \geq T$ and $\epsilon > 0$ such that ${}_1v \succ ({}_1\hat{x}_t, {}_{t+1}v + {}_{con}\epsilon)$. Since $\hat{x} > \inf_{t \in \mathbb{N}} v_t$, it follows that there exists $T^* \geq 1$ such that $\hat{x} > v_{T^*} \geq \min\{v_1, \dots, v_{T^*}\}$. By the contradicting hypothesis, and since \succsim satisfies **C**, there exist $t^* \geq T^*$ and $\epsilon > 0$ such that ${}_1v \succ ({}_1\hat{x}_{t^*}, {}_{t^*+1}v + {}_{con}\epsilon)$. For the sake of notational simplicity, let $({}_1\hat{x}_{t^*}, {}_{t^*+1}v + {}_{con}\epsilon) \equiv {}_1x$. Observe that $\hat{x} > \min\{v_1, \dots, v_{T^*}\} \geq \min\{v_1, \dots, v_{t^*}\} \equiv {}_1\underline{v}_T$.

Let ${}_1\bar{v} = ({}_1\bar{v}_{t^*, t^*+1} v)$. By **FA** and transitivity, ${}_1\bar{v} \succ {}_1x$. Suppose that ${}_1x_{t^*} \gg {}_1\bar{v}_{t^*}$. Then, there exists $0 < a < \min\{\min\{x_t - \bar{v}_t | t \leq t^*\}, \frac{\epsilon}{2}\}$ such that $x_t \geq \bar{v}_t + a$ for all $t \in \mathbb{N}$. But then **WP** implies ${}_1x \succ {}_1\bar{v}$ yielding a contradiction.

Therefore, suppose that for some $1 < t \leq t^*$ we have that $\bar{v}_t \geq x_t = \hat{x}$. We proceed according to the following steps.

Step 1. Let

$$q = \min\{1 < t \leq t^* | \bar{v}_t \geq x_t = \hat{x}\}.$$

Then, consider two real numbers $d_1, d_2 > 0$, and two vectors ${}_1x^1, {}_1v'$ - together with the corresponding ranked vectors ${}_1\bar{x}^1 = ({}_1\bar{x}_{t^*, t^*+1}^1 x)$, ${}_1\bar{v}' = ({}_1\bar{v}'_{t^*, t^*+1} v) \in \bar{X}$ - formed from ${}_1x, {}_1\bar{v}$ as follows: x_q is lowered to $x_q^1 = x_q - d_1 = \hat{x} - d_1 > \bar{v}_1 = {}_1\underline{v}_T$; \bar{v}_q is lowered to $v'_q = \bar{v}_q - d_2$ where $\hat{x} > \bar{v}_q - d_2 > \hat{x} - d_1$; and all other entries of ${}_1x$ and ${}_1\bar{v}$ are unchanged. By construction $\bar{x}_t^1 > \bar{v}'_t$ for all $1 \leq t \leq q$, whereas by **WHP***, **C**, **FA**, we have

$${}_1\bar{v}' \succsim {}_1\bar{x}^1. \quad (6)$$

Step 2. Let

$$0 < k < \min\left\{\min\{\bar{x}_t^1 - \bar{v}'_t | t \leq q\}, \frac{\epsilon}{2t^*}\right\} < \epsilon \quad (7)$$

and define ${}_1\bar{v}^1 = {}_1\bar{v}' + {}_{con}k$. By construction, $\bar{v}_t^1 \geq \bar{v}'_t + k$ for all $t \in \mathbb{N}$, and so **WP** implies ${}_1\bar{v}^1 \succ {}_1\bar{v}'$. By (6) and transitivity, it follows that ${}_1\bar{v}^1 \succ {}_1\bar{x}^1$.

Suppose that ${}_1\bar{x}_{t^*}^1 \gg {}_1\bar{v}_{t^*}^1$. Then, since $\inf_{t \in \mathbb{N}} \bar{x}_t^1 > \inf_{t \in \mathbb{N}} \bar{v}_t^1$ and ${}_{t^*+1}\bar{x}^1 \equiv {}_{t^*+1}v + {}_{con}\epsilon \gg {}_{t^*+1}\bar{v}^1 \equiv {}_{t^*+1}v + {}_{con}k$, there exists $a \in (0, \min\{\min\{\bar{x}_t^1 - \bar{v}'_t | t \leq t^*\}, \frac{k}{2t^*}\})$ such that $\bar{x}_t^1 \geq \bar{v}_t^1 + a$ for all $t \in \mathbb{N}$. **WP** implies ${}_1\bar{x}^1 \succ {}_1\bar{v}^1$ yielding a contradiction. Otherwise, let $\bar{v}_t^1 \geq \bar{x}_t^1$ for some t , with $q < t \leq t^*$. Let

$$q' = \min\{q < t \leq t^* | \bar{v}_t^1 \geq \bar{x}_t^1\}.$$

Noting that by (7), $\epsilon - k = \epsilon' > 0$ so that ${}_{t^*+1}\bar{x}^1 - {}_{t^*+1}\bar{v}^1 = {}_{t^*+1}\epsilon' \gg {}_{con}0$, the above steps 1-2 can be applied to ${}_1\bar{x}^1, {}_1\bar{v}^1$ to derive vectors ${}_1\bar{x}^2, {}_1\bar{v}^2$ such that $\bar{x}_t^2 > \bar{v}_t^2$ for all

$1 \leq t \leq q'$, whereas ${}_1\bar{v}^2 \succ {}_1\bar{x}^2$. By **WP**, a contradiction can be obtained whenever ${}_1\bar{x}_{t^*}^2 \gg {}_1\bar{v}_{t^*}^2$. Otherwise, let $\bar{x}_t^2 \leq \bar{v}_t^2$ for some $q' < t \leq t^*$. And so on. After a finite number $s \leq t^*$ of iterations, two vectors ${}_1\bar{x}^s$ and ${}_1\bar{v}^s$ can be derived such that ${}_1\bar{v}^s \succ {}_1\bar{x}^s$, by steps 1-2, but ${}_1\bar{x}_{t^*}^s \gg {}_1\bar{v}_{t^*}^s$, and so ${}_1\bar{x}^s \succ {}_1\bar{v}^s$ can be obtained by applying **WP**, a contradiction. This completes the proof of (5).

Next, we prove that ${}_{con}\hat{x} \succcurlyeq {}_1v$ holds. To this end, define a sequence of vectors $\{{}_{con}h^{-1}\}_{h \in \mathbb{N}}$. Because (5) holds, it follows that there exists $T \geq 1$ such that $({}_1\hat{x}_t, {}_{t+1}v + {}_{con}h^{-1}) \succcurlyeq {}_1v$ for all $t \geq T$ and all $h \in \mathbb{N}$. Fix any $t \geq T$. Then, since $\lim_{h \rightarrow \infty} ({}_1\hat{x}_t, {}_{t+1}v + {}_{con}h^{-1}) = ({}_1\hat{x}_t, {}_{t+1}v)$ and $({}_1\hat{x}_t, {}_{t+1}v + {}_{con}h^{-1}) \succcurlyeq {}_1v$ for any $h > 0$, **CON** $_{d_\infty}$ and **C** imply that $({}_1\hat{x}_t, {}_{t+1}v) \succcurlyeq {}_1v$. Because $t \geq T$ is arbitrary, it follows that $({}_1\hat{x}_t, {}_{t+1}v) \succcurlyeq {}_1v$ for all $t \geq T$. **PC** implies that ${}_{con}\hat{x} \succcurlyeq {}_1v$, as sought. Finally, noting that by construction, ${}_1u \gg {}_{con}\hat{x}$ and $\inf_{t \in \mathbb{N}} u_t > \hat{x}$, **WP** implies that ${}_1u \succ {}_{con}\hat{x}$, and so by transitivity we conclude that ${}_1u \succ {}_1v$, as sought.

Next, we show that (4) holds as well. Suppose that $\inf_{t \in \mathbb{N}} u_t = \inf_{t \in \mathbb{N}} v_t$. Let $m \in \mathbb{N}$; then, ${}_1u + {}_{con}m^{-1} \in \bar{X}$ and ${}_1u - {}_{con}m^{-1} \in \bar{X}$. Then, for each $m \in \mathbb{N}$, $\inf_{t \in \mathbb{N}} u_t + m^{-1} > \inf_{t \in \mathbb{N}} v_t > \inf_{t \in \mathbb{N}} u_t - m^{-1}$. It follows from (3) that ${}_1u + {}_{con}m^{-1} \succcurlyeq {}_1v \succcurlyeq {}_1u - {}_{con}m^{-1}$ for each $m \in \mathbb{N}$. Since $\lim_{m \rightarrow \infty} {}_{con}m^{-1} = 0$, **CON** $_{d_\infty}$ and **C** imply that ${}_1u \succcurlyeq {}_1v$ and ${}_1v \succcurlyeq {}_1u$, as sought. ■

Theorem 5 establishes an interesting possibility result for liberal approaches in economies with an infinite number of agents. For it proves that there exist fair, Paretian and continuous social welfare *orderings* that respect a liberal principle of non-interference. Indeed, the maximin SWO satisfies even the stronger version of **WHP** (analogous to that presented in section 3) extended to hold for *any* countably infinite streams.

Further, Theorem 5 provides a novel, and interesting characterisation of the maximin SWO in the intergenerational context. Lauwers ([35]) characterises the maximin SWO in the infinite context by focusing on Weak Pareto, Anonymity, Continuity, Repetition Approximation and either a strong version of **HE**,²⁰ or Ordinal Level Comparability. Theorem 5 provides a completely different foundation to the maximin SWO, because **WHP** is logically and theoretically distinct both from axioms with an egalitarian content, such as **HE**, and from informational invariance conditions.

Theorem 5 thus confirms the main intuitions concerning the relation between liberal and egalitarian approaches: the application of **WHP**, together with standard fairness, efficiency, and continuity properties leads straight to intergenerational welfare egalitarianism.

²⁰Formally, for any two bounded infinite vectors ${}_1u, {}_1v$ such that $u_i \geq v_i \geq v_j \geq u_j$ for some $i, j \in \mathbb{N}$ and $u_k = v_k \forall k \in \mathbb{N} \setminus \{i, j\}$, ${}_1v \succcurlyeq {}_1u$ (Lauwers [35], p.46).

7 Conclusions

A number of recent contributions have raised serious doubts on the possibility of a fair and efficient liberal approach to distributive justice that incorporates a fully non-interfering view. This paper has shown that possibility results do emerge, in societies with both a finite and an infinite number of agents, provided the bite of non-interference is limited in an ethically relevant way. Anonymous and Paretian criteria exist which incorporate a notion of protection of individuals (or generations) from unjustified interference, in situations in which they suffer a welfare loss, provided no other agent (or generation) is affected.

A weaker version of a liberal axiom - the Harm Principle - recently proposed by Mariotti and Veneziani ([42]), together with standard properties, allows us to derive a set of new characterisations of the maximin and of its lexicographic refinement, including in the intergenerational context. This is surprising, because the Weak Harm Principle is meant to capture a liberal and libertarian requirement of non-interference and it incorporates no obvious egalitarian content. Thus, our results shed new light on the ethical foundations of the egalitarian approaches stemming from Rawls's difference principle, and provide new meaning to the label of *liberal egalitarianism* usually attached to Rawls's theory.

From the viewpoint of liberal approaches emphasising a notion of individual autonomy, or freedom, however, our results have a rather counterintuitive implication. For they prove that, in various contexts, liberal non-interfering principles lead straight to welfare egalitarianism.

8 Appendix 1: Proof of Proposition 2

Proof of Proposition 2

(\Rightarrow) Let \succsim on X_T be the leximin ordering, i.e., $\succsim = \succsim^{LM}$. It is clear that leximin ordering satisfies **C**, **SP** and **FA**. Moreover, since **WHP** is weaker than **HP**, the proof that \succsim^{LM} on X_T meets **WHP** follows from the proof of necessity of **HP** provided by Mariotti and Veneziani (2009, Theorem 1, p.126).

(\Leftarrow) Let \succsim on X_T be a SWO satisfying **SP**, **FA**, **C**, and **WHP**. We show that \succsim on X_T is the leximin SWO. Thus, we should prove that, $\forall u, v \in X_T$,

$$u \sim^{LM} v \Leftrightarrow u \sim v \tag{8}$$

and

$$u \succ^{LM} v \Leftrightarrow u \succ v \tag{9}$$

First, we prove the implication (\Rightarrow) of (8). If $u \sim^{LM} v$, then $\bar{u} = \bar{v}$, and so $u \sim v$, by **FA**.

Next, we prove the implication (\Rightarrow) of (9). Suppose that $u \succ^{LM} v$, and so, by definition $\bar{u}_1 > \bar{v}_1$ or $\exists t \in \{2, \dots, T\}$ such that $\bar{u}_s = \bar{v}_s \forall 1 \leq s < t$ and $\bar{u}_t > \bar{v}_t$. Suppose, by contradiction, that $v \succ u$. Note that since \succcurlyeq satisfies **FA**, in what follows we can focus, without loss of generality, either on u and v , or on the ranked vectors \bar{u} and \bar{v} . Therefore, suppose $\bar{v} \succ \bar{u}$. As **SP** holds it must be the case that $\bar{v}_l > \bar{u}_l$ for some $l > t$. Let

$$k = \min\{t < l \leq T \mid \bar{v}_l > \bar{u}_l\}.$$

By **FA**, let $v_i = \bar{v}_k$ and let $u_i = \bar{u}_{k-g}$, for some $1 \leq g < k$, where $\bar{u}_{k-g} > \bar{v}_{k-g}$. Then, let two real numbers $d_1, d_2 > 0$, and consider vectors u', v' and the corresponding ranked vectors \bar{u}', \bar{v}' formed from \bar{u}, \bar{v} as follows: first, \bar{u}_{k-g} is lowered to $\bar{u}_{k-g} - d_1$ such that $\bar{u}_{k-g} - d_1 > \bar{v}_{k-g}$; next, \bar{v}_k is lowered to $\bar{v}_k - d_2$ such that $\bar{u}_k > \bar{v}_k - d_2 > \bar{u}_{k-g} - d_1$; finally, all other entries of \bar{u} and \bar{v} are unchanged. By construction $\bar{u}'_j \geq \bar{v}'_j$ for all $j \leq k$, with $\bar{u}'_{k-g} > \bar{v}'_{k-g}$, whereas **WHP**, combined with **C**, and **FA**, implies $\bar{v}' \succcurlyeq \bar{u}'$. By **SP**, $d_1, d_2 > 0$ can be chosen so that $\bar{v}' \succ \bar{u}'$, without loss of generality. Consider two cases:

a) Suppose that $\bar{v}_k > \bar{u}_k$, but $\bar{u}_l \geq \bar{v}_l$ for all $l > k$. It follows that $\bar{u}' > \bar{v}'$, and so **SP** implies that $\bar{u}' \succ \bar{v}'$, a contradiction.

b) Suppose that $\bar{v}_l > \bar{u}_l$ for some $l > k$. Note that by construction $\bar{v}'_l = \bar{v}_l$ and $\bar{u}'_l = \bar{u}_l$ for all $l > k$. Then, let

$$k' = \min\{k < l \leq T \mid \bar{v}'_l > \bar{u}'_l\}.$$

The above argument can be applied to \bar{u}', \bar{v}' to derive vectors \bar{u}'', \bar{v}'' such that $\bar{u}''_j \geq \bar{v}''_j$ for all $j \leq k'$, whereas **WHP**, combined with **FA**, **C**, and **SP**, implies $\bar{v}'' \succ \bar{u}''$. And so on. After a finite number of iterations s , two vectors \bar{u}^s, \bar{v}^s can be derived such that, by **WHP**, combined with **FA**, **C**, and **SP**, we have that $\bar{v}^s \succ \bar{u}^s$, but $\bar{u}^s > \bar{v}^s$ so that **SP** implies $\bar{u}^s \succ \bar{v}^s$, yielding a contradiction.

We have proved that if $u \succ^{LM} v$ then $u \succcurlyeq v$. Suppose now, by contradiction, that $v \sim u$, or equivalently $\bar{v} \sim \bar{u}$. Since, by our supposition, $\bar{v}_t < \bar{u}_t$, there exists $\epsilon > 0$ such that $\bar{v}_t < \bar{u}_t - \epsilon < \bar{u}_t$. Let $\bar{u}^\epsilon \in X_T$ be a vector such that $\bar{u}^\epsilon_t = \bar{u}_t - \epsilon$ and $\bar{u}^\epsilon_j = \bar{u}_j$ for all $j \neq t$. It follows that $\bar{u}^\epsilon \succ^{LM} \bar{v}$ but $\bar{v} \succ \bar{u}^\epsilon$ by **SP** and the transitivity of \succcurlyeq . Hence, the above argument can be applied to \bar{v} and \bar{u}^ϵ , yielding the desired contradiction. ■

9 Appendix 2: Independence of Axioms

The proofs of the independence of the axioms used to characterise the finite maximin and leximin SWOs are obvious and therefore they are omitted. It is worth noting, however, that some of the examples below can be easily adapted to apply to the finite context.

Independence of axioms used in Theorem 4

In order to complete the proof of Theorem 4, we show that the axioms are tight.

For an example violating *only* **FA**, define \succsim on X as follows: $\forall {}_1u, {}_1v \in X$,

$${}_1u \sim {}_1v \Leftrightarrow {}_1u = {}_1v,$$

$${}_1u \succ {}_1v \Leftrightarrow \text{either } u_1 > v_1, \text{ or } \exists T \in \mathbb{N} \setminus \{1\} : u_t = v_t \ \forall t < T \text{ and } u_T > v_T.$$

The SWR \succsim on X is not an extension of the leximin SWR \succsim^{LM^*} . The SWR \succsim on X satisfies all axioms except **FA**.

For an example violating *only* **SP**, define \succsim on X as follows: $\forall {}_1u, {}_1v \in X$, ${}_1u \sim {}_1v$. The SWR \succsim on X is not an extension of the leximin SWR \succsim^{LM^*} . The SWR \succsim on X satisfies all axioms except **SP**.

For an example violating *only* **WHP***, define \succsim on X as follows: $\forall {}_1u, {}_1v \in X$,

$${}_1u \sim {}_1v \Leftrightarrow \exists \tilde{T} \geq 1 \text{ such that } \forall T \geq \tilde{T} : {}_1\bar{u}_T = {}_1\bar{v}_T,$$

$${}_1u \succ {}_1v \Leftrightarrow \exists \tilde{T} \geq 1 \text{ such that } \forall T \geq \tilde{T}, \exists t \in \{1, \dots, T\} \text{ with } \bar{u}_s = \bar{v}_s \ (\forall t < s \leq T) \text{ and } \bar{u}_t > \bar{v}_t.$$

The SWR \succsim on X is not an extension of the leximin SWR \succsim^{LM^*} . The SWR \succsim on X satisfies all axioms except **WHP***.

For an example violating *only* **MC**, let for any $T \in \mathbb{N}$ and ${}_1u \in X$, $\rho_T({}_1u_T)$ be a permutation of ${}_1u_T$. Then define \succsim on X as follows: $\forall {}_1u, {}_1v \in X$,

$${}_1u \sim {}_1v \Leftrightarrow \exists \tilde{T} \geq 1 \text{ such that } \forall T \geq \tilde{T} : {}_1u_T = \rho_T({}_1v_T) \text{ for some permutation } \rho_T;$$

$${}_1u \succ {}_1v \Leftrightarrow \exists \tilde{T} \geq 1 \text{ such that } \forall T \geq \tilde{T} : {}_1u_T > \rho_T({}_1v_T) \text{ for some permutation } \rho_T.$$

The SWR \succsim on X is not an extension of the leximin SWR \succsim^{LM^*} . The SWR \succsim on X satisfies all axioms except **MC**.

For an example violating *only* **WPC**, let \succsim on X be the leximin defined in Bossert et al. (2007; p. 586). The SWR \succsim on X is not an extension of the leximin SWR \succsim^{LM^*} . The SWR \succsim on X satisfies all axioms except **WPC**. [To see that **WPC** is violated, for all $x, y \in \mathbb{R}$, let $rep(x, y) \equiv (x, y, x, y, \dots)$ and consider the profiles ${}_1u = (2, rep(1, \frac{1}{2}))$ and ${}_1v = (3, rep(0, \frac{3}{5}))$. Then, $({}_1u_T, {}_{T+1}v) \succ {}_1v, \forall T \in \mathbb{N} \setminus \{1\}$ but ${}_1u \not\succeq {}_1v$.]

Independence of axioms used in Theorem 5

In order to complete the proof of Theorem 5, we show that the axioms are tight.

For an example violating *only* **FA**, define \succsim on \bar{X} as follows: $\forall_1 u, {}_1 v \in \bar{X}$,

$${}_1 u \succsim {}_1 v \Leftrightarrow u_1 \geq v_1.$$

\succsim is a SWO on \bar{X} and it satisfies all axioms except **FA**.

For an example violating *only* **WP**, define \succsim on \bar{X} as follows: $\forall_1 u, {}_1 v \in \bar{X}$, ${}_1 u \sim {}_1 v$. \succsim is a SWO on \bar{X} and it satisfies all axioms except **WP**.

For an example violating *only* **PC**, define \succsim on \bar{X} as follows: $\forall_1 u, {}_1 v \in \bar{X}$,

$${}_1 u \succsim {}_1 v \Leftrightarrow \liminf_{t \in \mathbb{N}} u_t \geq \liminf_{t \in \mathbb{N}} v_t.$$

\succsim is a SWO on \bar{X} and it satisfies all axioms except **PC**. [To see that **PC** is violated, consider the profiles ${}_1 u = {}_{con}0$ and ${}_1 v = {}_{con}1$. By construction, $({}_1 u_T, {}_{T+1} v) \sim {}_1 v \forall T \geq 2$, but ${}_1 v \succ {}_1 u$.]

Let the following notation hold for the next two examples. Define \bar{X}^* as follows:

$$\bar{X}^* = \{ {}_1 u \in \bar{X} \mid \min_{t \in \mathbb{N}} u_t \text{ exists} \}.$$

For all ${}_1 u \in \bar{X}^*$, let $t({}_1 u)$ be one of the generations such that $u_{t({}_1 u)} = \min_{t \in \mathbb{N}} u_t$.

For an example violating *only* **WHP***, define \succsim on \bar{X} as follows: $\forall_1 u, {}_1 v \in \bar{X}$,

$$(i) \text{ if } {}_1 u, {}_1 v \in \bar{X}^*, \text{ then } {}_1 u \succsim {}_1 v \Leftrightarrow \frac{\min_{t \in \mathbb{N}} u_t + \inf_{t \in \mathbb{N} \setminus \{t({}_1 u)\}} u_t}{2} \geq \frac{\min_{t \in \mathbb{N}} v_t + \inf_{t \in \mathbb{N} \setminus \{t({}_1 v)\}} v_t}{2};$$

$$(ii) \text{ if } {}_1 u \in \bar{X}^*, {}_1 v \in \bar{X} \setminus \bar{X}^*, \text{ then } {}_1 u \succsim {}_1 v \Leftrightarrow \frac{\min_{t \in \mathbb{N}} u_t + \inf_{t \in \mathbb{N} \setminus \{t({}_1 u)\}} u_t}{2} \geq \inf_{t \in \mathbb{N}} v_t;$$

$$(iii) \text{ otherwise, } {}_1 u \succsim {}_1 v \Leftrightarrow \inf_{t \in \mathbb{N}} u_t \geq \inf_{t \in \mathbb{N}} v_t.$$

\succsim is a SWO on \bar{X} and it satisfies all axioms except **WHP***. [To see that **WHP*** is violated, consider the profiles ${}_1 u = (1, {}_{con}6)$, ${}_1 v = {}_{con}3$, ${}_1 u' = (1, 3, {}_{con}6)$, and ${}_1 v' = (3, 2, {}_{con}3)$. By the definition of \succsim , ${}_1 u \succ {}_1 v$, but ${}_1 v' \succ {}_1 u'$, which contradicts **WHP***.]

For an example violating *only* **CON_{d∞}**, define \succsim on \bar{X} as follows: $\forall_1 u, {}_1 v \in \bar{X}$,

$$(i) \text{ if } \inf_{t \in \mathbb{N}} u_t > \inf_{t \in \mathbb{N}} v_t, \text{ then } {}_1 u \succ {}_1 v;$$

$$(ii) \text{ if } {}_1 u, {}_1 v \in \bar{X}^* \text{ and } u_{t({}_1 u)} = v_{t({}_1 v)}, \text{ then } {}_1 u \succsim {}_1 v \Leftrightarrow \inf_{t \in \mathbb{N} \setminus \{t({}_1 u)\}} u_t \geq \inf_{t \in \mathbb{N} \setminus \{t({}_1 v)\}} v_t;$$

$$(iii) \text{ if } {}_1 u \in \bar{X} \setminus \bar{X}^*, {}_1 v \in \bar{X}^*, \text{ and } \inf_{t \in \mathbb{N}} u_t = \min_{t \in \mathbb{N}} v_t, \text{ then } {}_1 u \succ {}_1 v;$$

$$(iv) \text{ if } {}_1 u, {}_1 v \in \bar{X} \setminus \bar{X}^*, \text{ and } \inf_{t \in \mathbb{N}} u_t = \inf_{t \in \mathbb{N}} v_t, \text{ then } {}_1 u \sim {}_1 v.$$

\succsim is a SWO on \bar{X} and it satisfies all axioms except \mathbf{CON}_{d_∞} . [To see that \mathbf{CON}_{d_∞} is violated, consider the profiles ${}_1u^k = (\frac{1}{k}, {}_{con}1)$, $k \in \mathbb{N}$, and ${}_1v = (0, {}_{con}2)$. Observe that ${}_1v \in \bar{X}^*$, ${}_1u^k \in \bar{X}^* \forall k \in \mathbb{N}$ and $\lim_{k \rightarrow \infty} {}_1u^k = (0, {}_{con}1) \in \bar{X}^*$. By the definition of \succsim , ${}_1u^k \succsim {}_1v \forall k \in \mathbb{N}$, but ${}_1v \succ (0, {}_{con}1)$, which contradicts \mathbf{CON}_{d_∞} .]

For an example violating *only* \mathbf{C} , define \succsim on \bar{X} as follows: $\forall {}_1u, {}_1v \in \bar{X}$,

$$\begin{aligned} {}_1u \sim {}_1v &\Leftrightarrow {}_1u = \pi({}_1v) \text{ for some } \pi \in \Pi; \\ {}_1u \succ {}_1v &\Leftrightarrow \exists \epsilon > 0 : {}_1u \geq \pi({}_1v) + {}_{con}\epsilon, \text{ for some } \pi \in \Pi. \end{aligned}$$

\succsim is a SWR on \bar{X} and it satisfies all axioms except \mathbf{C} .

References

- [1] Alcantud, J.C. (2012): The impossibility of social evaluations of infinite streams with strict inequality aversion. *SAET Bulletin*. Forthcoming. doi: 10.1007/s00355-012-0687-x.
- [2] Alcantud, J.C. (2013): Liberal approaches to ranking infinite utility streams: When can we avoid interference? *Social Choice and Welfare*. Forthcoming. doi: 10.1007/s00355-012-0687-x.
- [3] Asheim, G. B. (2010): Intergenerational equity. *Annual Review of Economics* 2: 197–222.
- [4] Asheim, G. B., Banerjee, K. (2010): Fixed-step anonymous overtaking and catching-up. *International Journal of Economic Theory* 6: 149–165.
- [5] Asheim, G. B., d’Aspremont, C., Banerjee, K. (2010): Generalized time-invariant overtaking. *Journal of Mathematical Economics* 46: 519–533.
- [6] Asheim, G. B., Mitra, T., Tungodden, B. (2012): Sustainable recursive social welfare functions. *Economic Theory* 49: 267–292.
- [7] Asheim, G. B., Tungodden, B. (2004): Resolving distributional conflicts between generations. *Economic Theory* 24: 221–230.

- [8] Asheim, G. B., Zuber, S. (2012): A complete and strongly anonymous leximin relation on infinite streams. *Social Choice and Welfare*. Forthcoming. doi: 10.1007/s00355-012-0705-z.
- [9] Asheim, G.B., Mitra, T., Tungodden, B. (2007): A new equity condition for infinite utility streams and the possibility of being Paretian. In: Roemer, J.E., Suzumura, K. (eds.) *Intergenerational Equity and Sustainability*, pp. 55–68, MacMillan, London.
- [10] Atsumi, H. (1965): Neoclassical growth and the efficient program of capital accumulation. *Review of Economic Studies* 32: 127–136.
- [11] Barberà, S., Jackson, M.O. (1988): Maximin, leximin, and the protective criterion: characterizations and comparisons. *Journal of Economic Theory* 46: 34–44.
- [12] Basu, K., Mitra, T. (2003): Aggregating infinite utility streams with intergenerational equity: the impossibility of being Paretian. *Econometrica* 71: 1557–1563.
- [13] Basu, K., Mitra, T. (2007): Utilitarianism for infinite utility streams: a new welfare criterion and its axiomatic characterization. *Journal of Economic Theory* 133: 350–373.
- [14] Bolton, G.E., Ockenfels, A. (2006): Inequality Aversion, Efficiency, and Maximin Preferences in Simple Distribution Experiments: Comment. *American Economic Review* 96: 1906–1911.
- [15] Bosmans, K., Ooghe, E. (2006): A Characterization of maximin. Mimeo, Katholieke Universiteit Leuven.
- [16] Bossert, W., Sprumont, Y., Suzumura, K. (2007): Ordering infinite utility streams. *Journal of Economic Theory* 135: 179–189.
- [17] Brundtland, G.H. (ed.) (1987): *Our Common Future*, Oxford University Press, Oxford.
- [18] Crespo, J.A., Nuñez, C., Rincón–Zapatero, J.P. (2009): On the impossibility of representing infinite utility streams. *Economic Theory* 40: 47–56.
- [19] d’Aspremont, C. (1985): Axioms for social welfare orderings. In: Hurwicz, L., Schmeidler, D., Sonnenschein, H. (eds.) *Social Goals and Social Organization: Essays in Memory of Elisha Pazner*, pp.19–76, Cambridge University Press, Cambridge.
- [20] d’Aspremont, C., Gevers, L. (2002): Social Welfare Functionals and Interpersonal Comparability. In: Arrow, K.J., Sen, A.K., Suzumura, K. (eds.) *Handbook of Social Choice and Welfare vol. 1*, pp.459–541, North Holland, Amsterdam.

- [21] Cairns, R., Long, N.V. (2006): Maximin: A direct approach to sustainability. *Environment and Development Economics* 11: 275–300.
- [22] de Castro, L.I., Pesce, M., Yannelis, N.C. (2011): Core and equilibria under ambiguity. *Economic Theory* 48: 519–548.
- [23] Engelmann, D., Strobel, M. (2004): Inequality Aversion, Efficiency, and Maximin Preferences in Simple Distribution Experiments. *American Economic Review* 94: 857–69.
- [24] Fleurbaey, M., Michel, P. (2003): Intertemporal equity and the extension of the Ramsey Criterion. *Journal of Mathematical Economics* 39: 777–802.
- [25] Fleurbaey, M., Tungodden, B. (2010): The tyranny of non-aggregation versus the tyranny of aggregation in social choices: a real dilemma. *Economic Theory* 44: 399–414.
- [26] Frankfurt, H. (1987): Equality as a Moral Ideal. *Ethics* 98: 21–43.
- [27] Gilboa, I., Schmeidler, D. (1989): Maximin expected utility with non-unique prior. *Journal of Mathematical Economics* 18: 141–153.
- [28] Gotoh, R., Yoshihara, N. (2003): A Class of Fair Distribution Rules à la Rawls and Sen. *Economic Theory* 22: 63–88.
- [29] Hammond, P.J. (1976): Equity, Arrow’s conditions and Rawls’ difference principle. *Econometrica* 44: 793–804.
- [30] Hammond, P.J. (1979): Equity in two–person situations: Some consequences. *Econometrica* 47: 1127–1135.
- [31] Hammond, P.J. (1996): Consequentialist decision theory and utilitarian ethics. In: Farina, F., Hahn, F., Vannucci, S. (eds.) *Ethics, Rationality, and Economic Behaviour*, pp.92–118, Clarendon Press, Oxford.
- [32] Hara, C., Shinotsuka, T., Suzumura, K., Xu, Y. (2008): Continuity and egalitarianism in the evaluation of infinite utility streams. *Social Choice and Welfare*, 31: 179–191.
- [33] Kaplow, L., Shavell, S. (2001): Any Non-welfarist Method of Policy Assessment Violates the Pareto Principle. *Journal of Political Economy* 109: 281–286.
- [34] Konow, K. (2002): Which is the fairest one of all? A positive analysis of justice theories. *Journal of Economic Literature* 41: 1188–1239.
- [35] Lauwers, L. (1997): Rawlsian equity and generalised utilitarianism with an infinite population. *Economic Theory* 9: 143–150.

- [36] Llavador, H., Roemer, J.E., Silvestre, J. (2010): Intergenerational justice when future worlds are uncertain. *Journal of Mathematical Economics* 46: 728–761.
- [37] Llavador, H., Roemer, J.E., Silvestre, J. (2013): A Dynamic Analysis of Human Welfare in a Warming Planet. *Journal of Public Economics*, forthcoming.
- [38] Llavador, H., Roemer, J.E., Silvestre, J. (2013): *Sustainability for a warming planet*, ms. in preparation.
- [39] Lombardi, M., Veneziani, R. (2009): Liberal Egalitarianism and the Harm Principle. Working Paper 649, Queen Mary University of London.
- [40] Lombardi, M., Veneziani R. (2012): Treading a fine line: characterizations and impossibilities for liberal principles in infinitely-lived societies. *The B.E. Journal of Theoretical Economics*, 9. doi: 10.1515/1935-1704.1899.
- [41] Mariotti, M., Veneziani, R. (2009): The paradoxes of the liberal ethics of non-interference. WP 653, Queen Mary University of London.
- [42] Mariotti, M., Veneziani, R. (2009): Non-interference implies equality. *Social Choice and Welfare* 32: 123–128.
- [43] Mariotti, M., Veneziani, R. (2012): Opportunities as chances: maximising the probability that everybody succeeds, DP 9, University of Massachusetts, Amherst.
- [44] Mariotti, M., Veneziani, R. (2012): Allocating chances of success in finite and infinite societies: The Utilitarian criterion. *Journal of Mathematical Economics* 48, 226–236.
- [45] Mariotti, M., Veneziani, R. (2013): On the impossibility of complete non-interference in Paretian social judgements. *Journal of Economic Theory*, forthcoming.
- [46] Maskin, E. (1979): Decision-making under ignorance with implications for social choice. *Theory and Decision* 11: 319–337.
- [47] Rawls, J. (1971): *A Theory of Justice*, Harvard University Press, Cambridge.
- [48] Roemer, J.E. (1998): *Equality of opportunity*. Cambridge, MA: Harvard University Press.
- [49] Roemer, J.E. (2002): Equality of opportunity: A progress report. *Social Choice and Welfare* 19: 455–471.
- [50] Roemer, J.E. (2004): Eclectic Distributional Ethics. *Politics, Philosophy and Economics* 3: 267–281.

- [51] Roemer, J.E. (2011): The ethics of distribution in a warming planet. *Environmental and resource economics* 48, 363–390.
- [52] Sakai, T. (2010): Intergenerational equity and an explicit construction of welfare criteria. *Social Choice and Welfare* 35: 393–414.
- [53] Sarin, R., Vahid, F. (2001): Predicting how people play games: a simple dynamic model of choice. *Games and Economic Behavior* 34: 104–122.
- [54] Segal, U., Sobel, J. (2002): Min, Max, and Sum. *Journal of Economic Theory* 106: 126–150.
- [55] Silvestre, J. (2002): Progress and conservation under Rawls’s maximin principle. *Social Choice and Welfare* 19: 1–27.
- [56] Solow, R. M. (1974): Intergenerational equity and exhaustible resources. *Review of Economic Studies* 41: 29–45.
- [57] Tungodden, B. (1999): Hammond equity: A generalization. Discussion paper 20/99, Norwegian School of Economics and Business Administration.
- [58] Tungodden, B. (2000): Egalitarianism: Is leximin the only option? *Economics and Philosophy* 16: 229–245.
- [59] von Weizsäcker, C.C. (1965): Existence of optimal program of accumulation for an infinite time horizon. *Review of Economic Studies* 32: 85–104.
- [60] Wolf, C. (1995): Contemporary Property Rights, Lockean Provisos, and the Interests of Future Generations. *Ethics* 105: 791–818.
- [61] Zame, W.R. (2007): Can intergenerational equity be operationalized? *Theoretical Economics* 2: 187–202.