

2009

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Chavez, C and Stranlund, JK, "A Note on Emissions Taxes and Incomplete Information" (2009). *Environmental and Resource Economics*. 178.

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A Note on Emissions Taxes and Incomplete Information

Carlos Chávez · John K. Stranlund

Received: 22 March 2008 / Accepted: 17 December 2008 / Published online: 4 January 2009
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Abstract In contrast with what we perceive is the conventional wisdom about setting a second-best emissions tax to control a uniformly mixed pollutant under uncertainty, we demonstrate that setting a uniform tax equal to expected marginal damage is not generally efficient under incomplete information about firms' abatement costs and damages from pollution. We show that efficient taxes will deviate from expected marginal damage if marginal damage is increasing and there is uncertainty about the slopes of the marginal abatement costs of regulated firms. Moreover, tax rates will vary across firms if a regulator can use observable firm-level characteristics to gain some information about how the firms' marginal abatement costs vary.

Keywords Emissions taxes · Asymmetric information · Incomplete information · Uncertainty

JEL Classification L51 · Q28

1 Introduction

In a first-best world an optimal tax to control emissions of a uniformly mixed pollutant involves a uniform per unit tax set equal to marginal damage from emissions at the efficient level of aggregate emissions. It is clear that many environmental economists' intuition about emissions taxes under incomplete information, particularly about firms' abatement

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costs, follows from the first-best result. That is, when a regulator is uncertain about firms' marginal abatement costs and perhaps marginal damage, the optimal tax is a uniform tax that is equal to expected marginal damage. The value of a uniform tax in this setting is probably the main reason for implementing price-based controls. A uniform tax leads to the distribution of emissions control that equates marginal abatement costs across sources of pollution: hence, despite the uncertainty about the level of aggregate control induced by a tax, the aggregate abatement costs of achieving the resulting level of control will be minimized.

This intuition is clearly evident in analyses of the relative efficiency of emissions taxes and competitive markets for transferable emissions quotas that began with [Weitzman \(1974\)](#) seminal work. The canonical analysis of price-based versus quantity-based emissions control features a tax set equal to expected marginal damage versus a competitive emissions trading program that produces an expected permit price that is equal to expected marginal damage. Even those that build on the difference between taxes and transferable permits under uncertainty by suggesting policies that combine price and quantity controls maintain a uniform pollution price. For example, [Roberts and Spence \(1976\)](#) note that one of the important consequences of their policy recommendation to combine price-based and quantity-based emissions control is that individual marginal abatement costs are equal and aggregate abatement costs are minimized. [Kwerel \(1977\)](#) does the same. Clearly, this result holds only if emissions are controlled by a single price.

It is well known that a revelation mechanism can be designed that motivates firms to truthfully reveal their cost functions so that a regulator can impose emissions taxes that deliver the first-best outcome ([Dasgupta et al. 1980](#), Sect. 2; [Baliga and Maskin 2003](#)). These taxes tend to be non-linear taxes that vary across firms. However, the revelation approach has not had a great influence on environmental policy debates; in fact, we know of no attempt to employ the revelation approach in real pollution control situations.

Like the majority of authors who work in this area, we take a second-best approach to derive efficient emissions taxes in this note; that is, we derive optimal per unit emissions taxes given a regulator's lack of information about firms' abatement costs and the damage function. We demonstrate that setting a uniform tax equal to expected marginal damage is not generally second-best optimal when a regulator has incomplete and asymmetric information about firms' abatement costs. In particular, asymmetric information about the slopes of firms' marginal abatement costs causes taxes to deviate from expected marginal damage. Moreover, second-best taxes will vary across firms if a regulator can use observable firm-level characteristics to gain some information about how the firms' marginal abatement costs vary. With this information, even though it is incomplete, using a tax to equalize the marginal abatement costs of all sources of pollution is not efficient.

2 The Basic Results

To demonstrate these results we consider a fixed number of n heterogeneous firms. These firms all emit the same uniformly mixed pollutant. Firm i is described by an abatement cost function $C(q_i, x_i, \varepsilon_i)$, where q_i is the firm's emissions, x_i is a vector of characteristics of firm i that a regulator can observe, and ε_i is a random parameter from the regulator's perspective but it is known to the firm. A firm's abatement cost function is strictly decreasing and strictly convex in the firm's emissions. While we assume that the functional form of abatement costs does not vary across firms, this is not necessary for our results. Although the form of C does

not vary, individual firm abatement cost functions vary with differences in their observable characteristics and the realizations of the random parameter.

Anticipating the possibility that emissions tax rates might vary across firms, let t_i be the tax that i faces. Even though the regulator does not know exactly how the firm will respond to this tax because of asymmetric information about the firm's abatement costs, it does know that it will choose its emissions to equate its marginal abatement costs to the tax. That is,

$$C_q(q_i, x_i, \varepsilon_i) + t_i = 0, \quad (1)$$

which implicitly defines the firm's emissions as

$$q_i = q(t_i, x_i, \varepsilon_i). \quad (2)$$

Moreover, the firm's marginal response to the tax is

$$q_t(t_i, x_i, \varepsilon_i) = -1/C_{qq}(q_i, x_i, \varepsilon_i) < 0. \quad (3)$$

Pollution damage is an increasing, convex, and uncertain function of aggregate emissions, $D(\sum q_i, \delta)$, where δ is a random variable. [Unless indicated otherwise, summations are over all regulated firms]. The regulator knows the joint distribution of $(x_1, \dots, x_n, \varepsilon_1, \dots, \varepsilon_n, \delta)$ so it can form an expectation of the social costs of pollution and its control, conditional on its observations of (x_1, \dots, x_n) . This is

$$E \left\{ \sum C(q_i, x_i, \varepsilon_i) + D \left(\sum q_i, \delta \right) \right\}. \quad (4)$$

The regulator chooses individual tax rates, (t_1, \dots, t_n) , to minimize (4) subject to its knowledge of how the firms will respond to their taxes, $q_i = q(t_i, x_i, \varepsilon_i)$, $i = 1, \dots, n$. Substitute these constraints into (4) to obtain the regulator's conditional expectation of the social cost function in terms of individual emissions taxes (t_1, \dots, t_n) :

$$E \left\{ \sum C(q(t_i, x_i, \varepsilon_i), x_i, \varepsilon_i) + D \left(\sum q(t_i, x_i, \varepsilon_i), \delta \right) \right\}. \quad (5)$$

Assuming that (5) is strictly convex in (t_1, \dots, t_n) and that optimality calls for a positive tax for each firm, the following first-order conditions uniquely identify the optimal tax rates:

$$\begin{aligned} & E \left(C_q(q_k(t_k, x_k, \varepsilon_k), x_k, \varepsilon_k) q_t(t_k, x_k, \varepsilon_k) \right) \\ & + E \left(D' \left(\sum q(t_i, x_i, \varepsilon_i), \delta \right) q_t(t_k, x_k, \varepsilon_k) \right) = 0, \quad k = 1, \dots, n. \end{aligned} \quad (6)$$

From (1) and (3), substitute $C_q(q_k, x_k, \varepsilon_k) = -t_k$ and $q_t(t_k, x_k, \varepsilon_k) = -1/C_{qq}(q_k, x_k, \varepsilon_k)$, $k = 1, \dots, n$, into (6) and rearrange the results to obtain

$$t_k = \frac{E \left(D' \left(\sum q(t_i, x_i, \varepsilon_i), \delta \right) \left(-1/C_{qq}(q_k, x_k, \varepsilon_k) \right) \right)}{E \left(-1/C_{qq}(q_k, x_k, \varepsilon_k) \right)}, \quad k = 1, \dots, n. \quad (7)$$

Finally, use the definition of the covariance between random variables to write (7) as

$$t_k = E \left(D' \left(\sum q(t_i, x_i, \varepsilon_i), \delta \right) \right) + \frac{\text{Cov} \left(D' \left(\sum q(t_i, x_i, \varepsilon_i), \delta \right), -1/C_{qq}(q_k, x_k, \varepsilon_k) \right)}{E \left(-1/C_{qq}(q_k, x_k, \varepsilon_k) \right)}, \quad k = 1, \dots, n, \quad (8)$$

where Cov denotes the covariance operator.

The first term on the right hand side of (8) is expected marginal damage. Thus, the optimal emissions tax will be the same for every firm and will be equal to expected marginal damage if and only if the second term on the right hand side of (8) is zero. Moreover, the second term is zero if and only if the covariance term is zero. An important special case of this is when the slopes of the firms' marginal abatement cost functions are known. This case is important because it is common to model uncertainty about abatement costs as a random shift of only the intercept of marginal abatement costs, not their slopes. For example, in the "prices vs. quantities" literature, [Weitzman \(1974\)](#) focuses on this case, although not exclusively. The influential textbook treatment of this problem by [Baumol and Oates \(1988\)](#) takes this approach, as do many recent papers in this literature (e.g. [Hoel and Karp \(2002\)](#); [Newell and Pizer \(2003\)](#); [Moledina et al. \(2003\)](#), and [Quirion \(2004\)](#)). Perhaps the intuition that an optimal tax under asymmetric information about firms' abatement costs is set equal to expected marginal damage is due, at least in part, to the common simplifying assumption that the slopes of firms' marginal abatement costs are known.¹

Given asymmetric information about the slopes of firms' marginal abatement costs, (8) also suggests that optimal emissions taxes will vary across firms if regulators have at least some information about how observable firm characteristics affect their marginal abatement costs. This information could come from empirical studies of how observables like production and pollution control technologies and input and output levels determine firms' abatement costs. In some settings this information may be fairly coarse so that the number of distinct tax rates is small. For example, suppose in a particular control setting that several industries contribute to a pollution problem and that regulators know something about how abatement costs vary across these industries but not about how they vary within industries. Then, the number of distinct tax rates may simply be equal to the number of industries involved.² For another example, suppose that regulators have information about how abatement costs vary with abatement technologies but nothing else. Then the number of tax rates may be equal to the number of technologies employed.

3 The Role of Incomplete Information in the Determination of Emissions Taxes

In this section we use a relatively simple example to further explore the impact of incomplete information on optimal individual emissions taxes. Using a specification of marginal costs from [Weitzman \(1974, 1978\)](#) and [Laffont \(1977\)](#), assume that information about (x_1, \dots, x_n) allows the regulator to estimate the firms' marginal abatement cost functions as

$$-C_q(q_i, x_i, \varepsilon_i) = a_i + \alpha(\varepsilon_i) - \frac{b_i}{\beta(\varepsilon_i)}q_i, \quad i = 1, \dots, n. \quad (9)$$

The regulator is able to estimate the positive constants a_i and b_i , but with errors $\alpha(\varepsilon_i)$ and $\beta(\varepsilon_i)$. Assume $E(\alpha(\varepsilon_i)) = 0$, $E(\beta(\varepsilon_i)) = 1$, and each $\alpha(\varepsilon_i)$, $i = 1, \dots, n$, is independent of each $\beta(\varepsilon_i)$, $i = 1, \dots, n$. The latter assumption is made solely to simplify the analysis and

¹ Others in the "prices vs. quantities" literature have examined the role of uncertainty about the slopes of marginal cost functions, including [Weitzman \(1974\)](#) in his footnote on p. 486; [Malcomson \(1978\)](#) as well as [Weitzman \(1978\)](#) reply, and more recently, [Hoel and Karp \(2001\)](#). These papers do not address the issues that are important to us, namely that asymmetric information about the slopes of marginal abatement costs can make optimal taxes deviate from expected marginal damage and can make firms' individual tax rates vary under specific information conditions.

² There is precedent for this. Several authors (e.g., [Baranzini et al. \(2000\)](#) and [Bye and Nyborg \(2003\)](#)) note that carbon taxes in a number of European countries tend to be differentiated by industry. The possible reasons for this include differences in political leverage and revenue-raising potential across industries.

does not affect our main results. Note that in this example, we can let $x_i = (a_i, b_i)$. Finally, suppose that marginal damage is the linear function

$$D' \left(\sum q(t_i, x_i, \varepsilon_i), \delta \right) = c + \delta + d \sum q(t_i, x_i, \varepsilon_i), \quad (10)$$

where c and d are positive constants and δ is a random parameter with $E(\delta) = 0$.³

For this example, Eq. 8 can be written as

$$t_k = c + d \sum E(q(t_i, x_i, \varepsilon_i)) + dE(q(t_k, x_k, \varepsilon_k)) \text{Var}(\beta(\varepsilon_k)) \\ + d \sum_{i \neq k} E(q(t_i, x_i, \varepsilon_i)) \text{Cov}(\beta(\varepsilon_i), \beta(\varepsilon_k)) + \text{Cov}(\delta, \beta(\varepsilon_k)). \quad k = 1, \dots, n, \quad (11)$$

where $\text{Var}(\beta(\varepsilon_k))$ is the variance of $\beta(\varepsilon_k)$. (The derivation of Eq. 11 is presented in the Appendix). Note that the first two terms on the right side of (11) are the regulator's expectation of marginal damage, given individual taxes (t_1, \dots, t_n) and its estimates of the abatement cost parameters (a_i, b_i) , $i = 1, \dots, n$. As we noted in Sect. 2, an optimal policy is to set a single tax equal to expected marginal damage if the slopes of all the marginal abatement cost functions are known, because in this case all the variance and covariance terms in (11) are equal to zero.

However, the third term on the right hand side of (11) is clearly positive when marginal damage is increasing and the regulator is uncertain about the slope of firm k 's marginal abatement cost function. Thus the impact of this term on the firm's tax rate is to push it above expected marginal damage. Furthermore, the optimal tax rate increases as the regulator's uncertainty about $\beta(\varepsilon_k)$ increases (i.e., it has a higher variance). Recall from Eq. 3 that the reciprocal of the slope of a firm's marginal abatement cost function measures the emissions response of the firm to a marginal increase in its tax. Thus, the greater the regulator's uncertainty about a firm's marginal response to an emissions tax, the higher is its optimal tax. This effect also depends on the convexity of the damage function. For example, it is zero if marginal damage is a constant (i.e., $d = 0$). However, a more steeply sloped marginal damage function implies that each firms' tax rate exceeds expected marginal damage by a greater amount.

The reason that asymmetric information about the slope of a firm's marginal abatement cost function will tend to call for a tax that is higher than expected marginal damage is the following. A firm's reduction in emissions from a higher tax is greater when the slope of its marginal abatement cost function turns out to be greater than expected (i.e., the function is flatter than expected). Thus, setting a firm's tax somewhat higher than expected marginal damage reduces the welfare loss that occurs if the firm's marginal abatement cost function turns out to be flatter than expected by more than it increases the welfare loss if the firm's marginal abatement cost function is steeper than expected. It is optimal to set the tax higher than expected marginal damage to exploit this asymmetry; that is, to exploit the possibility that a higher tax will induce a greater emissions reduction from the firm if the slope of its marginal abatement cost is flatter than expected.

For a particular firm k , the fourth term on the right hand side of (11) involves the marginal damage associated with all the other firms' emissions times the covariances of the random factor of the slopes of their marginal abatement costs curves with the random factor of the slope of k 's marginal abatement cost curve. Of course, if the $\beta(\varepsilon_i)$'s are independently distributed this term is equal to zero; however, if they are not independently distributed it

³ Our primary reason for choosing this specification of the problem is that it produces a linear decomposition of the influences of uncertainty on efficient emissions taxes.

seems reasonable to assume that they would often vary together. In that case, the fourth term on the right hand side of (11) is positive, and the impact of this term on k 's tax rate is to push it further above expected marginal damage. We just noted that asymmetric information about the slope of a firm's marginal abatement cost function will tend to push its optimal tax above expected marginal damage to exploit the possibility that its marginal abatement cost function will be flatter than expected. This effect is reinforced if the slopes of all the firms' marginal abatement cost functions are positively correlated, because the slope of a firm's marginal abatement cost function will tend to be flatter than expected when other firms' marginal abatement cost slopes are flatter than expected.

The fifth term on the right hand side of (11) suggests that optimal tax rates may depend on correlated uncertainty between the damage function and the slope of k 's marginal abatement cost function. We do not have an a priori expectation of how δ and $\beta(\varepsilon_k)$ might vary together if at all. Let us note, however, that if δ and $\beta(\varepsilon_k)$ are independent of each other, or if there is no uncertainty in the damage function, $Cov(\delta, \beta(\varepsilon_k)) = 0$. However, if marginal damage is positively (negatively) correlated with the slopes of marginal abatement costs, then $Cov(\delta, \beta(\varepsilon_k)) > (<)0$, which implies an increase (reduction) in k 's tax rate.⁴ Perhaps the most important conclusions concerning uncertainty about the damage function is that the main results of our paper—that asymmetric information about the slopes of firms' marginal abatement costs will tend to make optimal taxes deviate from expected marginal damage and vary among firms—hold even if there is no uncertainty about the damage function. Moreover, damage uncertainty affects optimal tax rates only if it is correlated with the uncertainty about the slopes of polluting firms' marginal abatement costs.

To explore our result that optimal tax rates may vary across firms, let us simplify the problem by assuming that the $\beta(\varepsilon_i)$'s are identically, but not necessarily independently, distributed. In this case, $Var(\beta(\varepsilon_i))$ and $Cov(\delta, \beta(\varepsilon_i))$ are the same for each i , and $Cov(\beta(\varepsilon_i), \beta(\varepsilon_k))$ is the same for every pair of firms i and k . Then, using (11) we can calculate the difference between the tax rates for firms k and i as:

$$t_k - t_i = d \left[\sigma_\beta^2 - Cov_\beta \right] [E(q(t_k, x_k, \varepsilon_k)) - E(q(t_i, x_i, \varepsilon_i))], \quad (12)$$

where σ_β^2 denotes the variance of each $\beta(\varepsilon_i)$ and Cov_β denotes the covariance between any $\beta(\varepsilon_k)$ and $\beta(\varepsilon_i)$. Note that individual tax rates will not vary in our example if marginal damage is constant (i.e., $d = 0$), which we've already mentioned; if the variance and covariance terms are equal, which is highly unlikely, or if under the optimal policy the regulator's conditional expectations of the firms' emissions are the same. On this last point, in this example $E(q(t_i, x_i, \varepsilon_i)) = (a_i - t_i/b_i)$, $i = 1, \dots, n$. Then, $t_k - t_i = d \left[\sigma_\beta^2 - Cov_\beta \right] [(a_k - t_k)/b_k - (a_i - t_i)/b_i]$. Therefore, given $d \neq 0$ and $\sigma_\beta^2 \neq Cov_\beta$, the optimal policy is a uniform tax t if and only if $(a_k - t/b_k) = (a_i - t/b_i)$ for each pair of firms i and k . This is exceedingly unlikely, except when a regulator has such poor information about individual firms that it cannot distinguish its estimates of the parameters of their marginal abatement costs from one another.

⁴ Stavins (1996) examines how correlated uncertainty in the *intercepts* of marginal damage and a marginal abatement cost curve can impact the optimal choice of emissions taxes versus tradable emission permits, a point that was originally made by Weitzman (1974, footnote 1 on p. 485). In contrast the fifth term in Eq. 11 allows for the possibility that the *intercept* of the marginal damage function and the *slope* of a firm's marginal abatement cost function may be correlated and indicates how this might impact a firm's emissions tax.

4 Conclusion

We have demonstrated that asymmetric information about the slopes of firms' marginal abatement costs implies that optimal emissions taxes to control a uniformly mixed pollutant will generally differ from expected marginal damage. Moreover, this uncertainty leads to a policy of differentiated taxes, except when regulators do not have any knowledge of the variation of marginal abatement costs in the population of regulated firms. The extent of the deviations of optimal tax rates from expected marginal damage and their variation across firms are empirical matters that should be addressed in each pollution control setting.

Acknowledgements The authors are grateful to Matti Liski and one anonymous referee for their valuable comments and suggestions. We acknowledge support for this research provided by the Cooperative State Research Extension, Education Service, U. S. Department of Agriculture, Massachusetts Agricultural Experiment Station, and the Department of Resource Economics under Project No. MAS00871. Partial support was also provided by Conicyt-Chile, under Project Fondecyt No. 1060679, and Fondecyt International Cooperation under Project No 7060098.

Appendix: The Derivation of Equation (11)

A firm i 's choice of emissions given a tax t_i is the solution to $-C_q(q_i, x_i, \varepsilon_i) = t_i$. With (9), solving for i 's emissions yields

$$q(t_i, x_i, \varepsilon_i) = \frac{(a_i + \alpha(\varepsilon_i) - t_i)\beta(\varepsilon_i)}{b_i}, \quad (\text{A1})$$

with expectation

$$E(q(t_i, x_i, \varepsilon_i)) = \frac{(a_i - t_i)}{b_i}. \quad (\text{A2})$$

Moreover, the firm's marginal response to a change in the tax is

$$q_t(t_i, x_i, \varepsilon_i) = -\beta(\varepsilon_i)/b_i, \quad (\text{A3})$$

with expectation

$$E(q_t(t_i, x_i, \varepsilon_i)) = -1/b_i. \quad (\text{A4})$$

Substitute (A3) into Eq. 7 to write firm k 's optimal tax as

$$\begin{aligned} t_k &= \frac{E(D'(\sum q(t_i, x_i, \varepsilon_i), \delta) (-\beta(\varepsilon_k)/b_k))}{-1/b_k} \\ &= E\left(D'(\sum q(t_i, x_i, \varepsilon_i), \delta) \cdot \beta(\varepsilon_k)\right) \end{aligned} \quad (\text{A5})$$

Substitute $D'(\sum q(t_i, x_i, \varepsilon_i), \delta) = c + \delta + d \sum q(t_i, x_i, \varepsilon_i)$ (Eq. 10 in the text) and (A1) into (A5) to obtain

$$t_k = E(c\beta(\varepsilon_k) + \delta\beta(\varepsilon_k)) + d \sum \frac{(a_i - t_i)E(\beta(\varepsilon_i)\beta(\varepsilon_k)) + E(\alpha(\varepsilon_i)\beta(\varepsilon_i)\beta(\varepsilon_k))}{b_i} \quad (\text{A6})$$

Since each $\alpha(\varepsilon_i)$, $i = 1, \dots, n$, is independent of each $\beta(\varepsilon_i)$, $i = 1, \dots, n$, and $E(\alpha(\varepsilon_i)) = 0$, $E(\alpha(\varepsilon_i)\beta(\varepsilon_i)\beta(\varepsilon_k)) = 0$. Moreover, since $E(\beta(\varepsilon_k)) = 1$, and c is a constant, $E(c\beta(\varepsilon_k) +$

$\delta\beta(\varepsilon_k) = c + E(\delta\beta(\varepsilon_k))$. Finally, from (A2), $E(q(t_i, x_i, \varepsilon_i)) = (a_i - t_i)/b_i$. Therefore, (A6) can be written as

$$t_k = c + d \sum E(q(t_i, x_i, \varepsilon_i)) E(\beta(\varepsilon_i)\beta(\varepsilon_k)) + E(\delta\beta(\varepsilon_k)). \quad (\text{A7})$$

Using the definition of covariance:

$$\begin{aligned} E(\beta(\varepsilon_i)\beta(\varepsilon_k)) &= \text{Cov}(\beta(\varepsilon_i), \beta(\varepsilon_k)) + E(\beta(\varepsilon_i)) E(\beta(\varepsilon_k)) \\ &= \text{Cov}(\beta(\varepsilon_i), \beta(\varepsilon_k)) + 1, \end{aligned} \quad (\text{A8})$$

and

$$E(\delta\beta(\varepsilon_k)) = \text{Cov}(\delta, \beta(\varepsilon_k)) + E(\delta) E(\beta(\varepsilon_k)) = \text{Cov}(\delta, \beta(\varepsilon_k)). \quad (\text{A9})$$

Substitute (A8) and (A9) into (A7) to obtain

$$t_k = c + d \sum E(q(t_i, x_i, \varepsilon_i)) (\text{Cov}(\beta(\varepsilon_i), \beta(\varepsilon_k)) + 1) + \text{Cov}(\delta, \beta(\varepsilon_k)). \quad (\text{A10})$$

Again from the definition of covariance, $\text{Cov}(\beta(\varepsilon_k), \beta(\varepsilon_k)) = \text{Var}(\beta(\varepsilon_k))$, where $\text{Var}(\beta(\varepsilon_k))$ is the variance of $\beta(\varepsilon_k)$. Using this, (A10) can be written as

$$\begin{aligned} t_k &= c + d \sum E(q(t_i, x_i, \varepsilon_i)) + dE(q(t_k, x_k, \varepsilon_k)) \text{Var}(\beta(\varepsilon_k)) \\ &\quad + d \sum_{i \neq k} E(q(t_i, x_i, \varepsilon_i)) \text{Cov}(\beta(\varepsilon_i), \beta(\varepsilon_k)) + \text{Cov}(\delta, \beta(\varepsilon_k)), \end{aligned}$$

which is Eq. 11.

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