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An Assessment of The Nonparametric Approach for Evaluating The Fit of Item Response Models

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AN ASSESSMENT OF THE NONPARAMETRIC APPROACH
FOR EVALUATING THE FIT OF ITEM RESPONSE MODELS

A Dissertation Presented

by

TIE LIANG

Submitted to the Graduate School of the
University of Massachusetts Amherst in partial fulfillment
of the requirements for the degree of

DOCTOR OF EDUCATION

February 2010

School of Education

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ABSTRACT

AN ASSESSMENT OF THE NONPARAMETRIC APPROACH FOR EVALUATING THE FIT OF ITEM RESPONSE MODELS

FEBRUARY, 2010

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As item response theory (IRT) has developed and is widely applied, investigating the fit of a parametric model becomes an important part of the measurement process when implementing IRT. The usefulness and successes of IRT applications rely heavily on the extent to which the model reflects the data, so it is necessary to evaluate model-data fit by gathering sufficient evidence before any model application. There is a lack of promising solutions on the detection of model misfit in IRT. In addition, commonly used fit statistics are not satisfactory in that they often do not possess desirable statistical properties and lack a means of examining the magnitude of misfit (e.g., via graphical inspections).

In this dissertation, a newly-proposed nonparametric approach, *RISE* was thoroughly and comprehensively studied. Specifically, the purposes of this study are to (a) examine the promising fit procedure, *RISE*, (b) compare the statistical properties of *RISE* with that of the commonly used goodness-of-fit procedures, and (c) investigate how *RISE* may be used to examine the consequences of model misfit.

To reach the above-mentioned goals, both a simulation study and empirical study were conducted. In the simulation study, four factors including ability distribution, sample size, test length and model were varied as the factors which may influence the

performance of a fit statistic. The results demonstrated that *RISE* outperformed G^2 and $S-X^2$ in that it controlled Type I error rates and provided adequate power under all conditions. In the empirical study, the three fit statistics were applied to one empirical data and the misfitting items were flagged. *RISE* and $S-X^2$ detected reasonable numbers of misfitting items while G^2 detected almost all items when sample size is large. To further demonstrate an advantage of *RISE*, the residual plot on each misfitting item was shown. Compared to G^2 and $S-X^2$, *RISE* gave a much clearer picture of the location and magnitude of misfit for each misfitting item.

Other than statistical properties and graphical displays, the score distribution and test characteristic curve (TCC) were investigated as model misfit consequence. The results indicated that for the given data, there was no practical consequence on classification before and after replacement of misfitting items detected by three fit statistics.

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CHAPTER 1

INTRODUCTION

1.1 Background

Item response theory (IRT) is a powerful scaling technique that provides appealing features such as the invariance of item and ability parameter values. Because of the attractive features IRT provides, parametric IRT models are used for a variety of measurement applications such as constructing tests, equating test scores, estimating proficiency levels, identifying differential item functioning (DIF), and selecting items for computerized adaptive testing (Embretson & Reise, 2000; Hambleton, Swaminathan, & Rogers, 1991). However, the success of specific IRT applications and the advantages of item response models are obtained when the fit between the model and the test data is satisfactory. Thus, it is crucial to understand the possible reasons for misfit and evaluate model-data fit as part of the process of validating the use of IRT models.

There are two general sources of misfit in IRT models: (1) violation of model assumptions pertaining to dimensionality/local independence and (2) failure to capture model-data relationships. For the most commonly-used IRT models to be applicable, the item pool must produce item responses that are predominately unidimensional. Fitting a unidimensional model to multidimensional data may produce model misfit that may have a consequential impact on the test results. Failure to capture model-data relationships is a specific type of assumption violation in which the model does not accurately reflect the true probabilities of a correct response (or does not accurately reflect the true probabilities of responding in a particular category for polytomous item responses). There are a few reasons why the model may not accurately reflect the true item characteristic curve (ICC). For example, for a multiple-choice item, the item may

contain multiple correct answers. Also, the wrong model may be selected (e.g., using a model that does not consider the influence of guessing on multiple choice test items).

1.2 Statement of the Problem

While many of the technical challenges that arise when applying IRT models to educational and psychological data have been resolved to some extent (e.g., model parameter estimation), the assessment of model fit remains a major hurdle to overcome for effective implementation (Hambleton & Han, 2004). Various methods and approaches have been suggested for detecting model misfit. Traditional methods for assessing goodness-of-fit in IRT are based on Pearson-type χ^2 statistics which are most widely used to assess model fit (e.g., Yen's (1981) Q_1 statistic). However, there are several drawbacks to using Pearson χ^2 statistics in assessing model fit. The two most common critiques about the χ^2 -based statistics are first, they are sensitive to sample size (Hambleton & Swaminathan, 1985). The distribution theory of χ^2 statistics, as well as most other formal tests of fit, is a large-sample theory. When the sample size is large, the statistical test rejects just about every model since with large sample sizes, statistical power is available to detect even very small model-test data discrepancies, and no statistical model for educational testing data is ever going to be strictly correct (i.e., the model will not reflect the true ICC *exactly* for all ability levels). Therefore, the first criticism is not necessarily because the χ^2 -based statistics are sensitive to sample size but that the hypothesis being tested is meaningless since it is always false (Cohen, 1994). Second, several popular fit statistics (e.g., G^2 and Yen's Q_1) are not distributed as a χ^2 . One reason is that the examinees are grouped based on IRT θ estimates, which contain error. Particularly speaking, parameter values are never known so the estimators of the parameters are treated as true values, which violates the

assumption underlying the χ^2 distribution (Stone & Zhang, 2003). Another reason is that the degrees of freedom are in question (Orlando & Thissen, 2000). As a result, the Type I error rate is inflated, in other words, many valid items are falsely identified as misfitting when in fact the model fits the data (Orlando & Thissen, 2000; Reise, 1990; Glas & Falcon, 2003; Stone & Hansen, 2000; DeMars, 2005).

Due to the limitations of χ^2 -based statistical methods for assessing model fit, it is prudent that another fit statistic be developed that avoids or limits the previous criticisms. This dissertation will focus on studying an approach for assessing model-fit proposed by Douglas & Cohen (2001) that compares a nonparametrically-derived ICC to the parametric ICC. This nonparametric approach, hereafter referred to as *RISE*, has exhibited controlled Type I error rates and adequate power in the dichotomous and polytomous case and provides a graphical representation of the misfit that may be used to judge the severity of the misfit (Wells & Bolt, 2008; Li & Wells, 2006; Liang & Wells, 2009, Liang & Wells, 2008). Beyond its attractive statistical properties, an additional advantage of the nonparametric approach is that it provides a convenient graphical representation of model misfit. This feature can provide the researcher or practitioner visual guidance about the location and type of misfit.

Concluding that a model is misfitting according to the hypothesis test allows a test developer to conclude that the model does not fit exactly in the population. However, it is possible that a misfitting model is still useful in that it approximates the true ICC very closely and that the consequences of the misfit are minimal. Therefore, it is important to consider the consequences of the misfit prior to rejecting the use of a model or item. In other words, it is worthwhile to investigate whether the misfit of the model has substantial practical consequences for the particular situation. The issue of evaluating practical consequences of model misfit has been given little attention in the

model checking literature in IRT (Hambleton & Han, 2004). However, it deserves more attention because it is possible that a statistical test indicates misfit while discrepancies between the test data and predictions from a model are of no practical consequence (van der Linden & Hambleton, 1997). Thus, this dissertation will also evaluate the importance of the practical consequences of model misfit together with statistical analyses and diagnostic plots for a fuller picture of fit assessment.

1.3 Objectives of the Study

The purposes of this study are to (a) review the currently used goodness-of-fit procedures, (b) further examine the promising fit procedure, *RISE*, (c) compare the statistical properties of *RISE* with that of the commonly used goodness-of-fit procedures, and (d) investigate how *RISE* may be used to examine the consequences of model misfit.

1.4 Outline of the Study

This study consists of five chapters. In this first chapter, the importance of model-data fit, an introduction of model-data fit problems, the purposes of the study and the outline of the study have been described. Chapter 2 presents a literature review of the IRT models in which the fit statistics will be applied, including *RISE*, and the comparative research on currently used goodness-of-fit test statistics. Chapter 3 describes the method for the study and also the design of the simulation studies that are conducted. Results will be summarized in Chapter 4. Finally, Chapter 5 will discuss the importance of the results and future research.

CHAPTER 2

REVIEW OF LITERATURE

This chapter begins with an introduction to assumptions of commonly used IRT models as well as a description of the models that will be used in the current dissertation. The fit statistics will then be briefly described followed by findings from empirical research on the fit statistics. Finally, based on the literature, the reasons for further exploring the fit statistic, *RISE*, are discussed.

2.1 IRT Assumptions and Models

- *IRT Models for Dichotomously Scored Data*

The three most popular unidimensional IRT models are the one-, two-, and three-parameter logistic models (1PLM, 2PLM and 3PLM, respectively). They have similar conceptual and mathematical formulations. The 3PLM may be expressed as follows:

$$P_i(\theta) = c_i + (1 - c_i) \frac{\exp[Da_i(\theta - b_i)]}{1 + \exp[Da_i(\theta - b_i)]}, \quad (1)$$

where $P_i(\theta)$ is the probability that an examinee with ability θ answers item i correctly, a_i is the slope (item discrimination) parameter, b_i is the location (item difficulty) parameter, c_i is the lower asymptote (pseudo-chance-level) parameter, and D is a scaling parameter. When D is 1.702, the probabilities will be within 0.01 of the normal ogive model.

The 2PLM and 1PLM are based on constraints or restrictions of the 3PLM. For example, the 2PLM sets the c -parameter to zero for all items. The 1PLM sets the c -parameter to zero and constrains the a -parameters to be equal for all items on an assessment.

- *IRT Models for Polytomously Scored Data*

1. Graded Response Model. The graded response model, developed by Samejima (1969), is appropriate to use when item responses are ordered categorical responses. For example, when Likert-type items are applied in surveys or questionnaires, GRM can be used to analyze those items. Two steps are needed to obtain the probability that an examinee responds to a particular category. The first step in estimating response probabilities involves computation of operating characteristic curves (OCCs), or in other words, measuring the probability that an examinee's response falls in or above a particular category given θ . The OCCs are an extension of the 2PLM and may be expressed as follows:

$$P_{ij}^*(\theta) = \frac{\exp[a_i(\theta - b_{ij})]}{1 + \exp[a_i(\theta - b_{ij})]}, \quad (2)$$

where $j=1, \dots, m_i$, m_i is equal to number of categories minus 1; $P_{ij}^*(\theta)$ is the probability of a randomly chosen examinee with proficiency of θ scoring x or above on item i ; a_i is the slope (item discrimination) parameter for item i . b_{ij} is the threshold (item difficulty) parameter, which is commonly interpreted as the trait level to respond in or above the particular threshold j of item i with 0.5 probability. GRM is an extension of 2PLM and belongs to “difference models.” Once OCCs are estimated, the actual category response curves (CRCs), which represent the probability of scoring in a particular category, are calculated by subtraction between two adjacent OCCs:

$$P_{ij}(\theta) = P_{ij}^*(\theta) - P_{i(j+1)}^*(\theta). \quad (3)$$

2. Generalized Partial Credit Model. Muraki (1992) developed the generalized partial credit model (GPCM) to be used for polytomous data that are based on completing steps (e.g., partial-credit items). The GPCM, classified as a “divide-by-total” model, may be expressed as follows:

$$P_{ix}(\theta) = \frac{\exp \left[\sum_{j=0}^x a_i(\theta - b_{ij}) \right]}{\sum_{r=0}^M \exp \left[\sum_{j=0}^r a_i(\theta - b_{ij}) \right]}, \quad (4)$$

where $\sum_{j=0}^0 a_i(\theta - b_{ij}) \equiv 0$. In the GPCM, b_{ij} is interpreted as the intersection (difficulty)

parameters where one category response is more likely to happen than previous responses given that the examinee has reached the previous step. Thus, b_{ij} is not necessarily ordered as occurs in the GRM. For example, if a GPCM item has 5 score categories, there are 4 difficulty parameters with each value at $b_1 = 0.2$, $b_2 = -1.3$, $b_3 = 1.7$, $b_4 = -0.5$, which indicate that the difficulty levels from easiest to hardest are step 2, step 4, step 1 and step 3. In addition, when the a parameter is constrained to be equal across items, the GPCM simplifies to the partial credit model (PCM; Masters, 1982).

● *Assumptions of IRT Models*

Three important assumptions must be met before the previously IRT models are employed. The first assumption, unidimensionality, states that one ability is measured in a test. Although unidimensionality will not be strictly satisfied since there are several factors that influence test performance, IRT models are still useful as long as there is one predominate factor underlying the test data. The second assumption is local independence which states that the items are uncorrelated after controlling for the ability on the test. A third assumption is placed on the ICC shape. To apply a parametric model, it is assumed that the shape of the true ICC is the same as that posited by the parametric model.

2.2 Model-Data Fit Procedures

A general strategy for evaluating model fit at the item level involves comparing

observed data with model-predicted expectations (Hambleton & Han, 2005; Hambleton, Swaminathan, & Rogers, 1991; Rogers & Hattie, 1987). Both statistical significance tests and graphical analyses based on the residuals can be used for this purpose.

Broadly defined, a goodness-of-fit study is the evaluation of the similarity between observed and expected (predicted) outcomes. Within the context of IRT, this typically involves (1) estimating the parameters of an IRT model, (2) using those parameter estimates to predict, by way of the IRT model, examinee response patterns, and (3) comparing the predicted response patterns to actual observed examinee response patterns.

- *Traditional Method: χ^2 -Based Fit Statistics*

Many of the traditional χ^2 -based methods for assessing model fit are based on the following steps:

1. Estimate item and ability parameters.
2. Place examinees into groups along the ability scale using proficiency estimates.
3. Construct an observed score response distribution for each ability group by cross-classifying examinees using their ability estimates and score responses.
4. Form an expected score response distribution across score categories using item parameter estimates and ability estimates representing each ability group.
5. Compare observed and expected score response distributions using a χ^2 fit statistic or examination of residuals.

1. Yen's Q_1 . Yen's (1981) Q_1 index, proposed for use with dichotomous items, is calculated as follows:

$$Q_1 = \sum_{j=1}^{10} \frac{N_j (O_{ij} - E_{ij})^2}{E_{ij} (1 - E_{ij})}, \quad (5)$$

where N_j is the number of examinees in ability interval j ; O_{ij} is the observed proportion of examinees in interval j who answer item i correctly; and E_{ij} is the probability based on the model in interval j answering item i correctly. The number of intervals is fixed to be 10 (although, if the expected frequency in an interval is less than five, then the groups may be combined so that the expected frequency is greater than 5). Under the null hypothesis, Yen's Q_1 is distributed as a χ^2 with degrees of freedom equal 10 minus the number of parameters being estimated.

2. Likelihood Ratio Test. The likelihood ratio test fit statistic (McKinley & Mills, 1985), denoted G^2 , is reported by two commonly used commercial IRT software packages, BILOG-MG (Zimowski, Muraki, Mislevy & Bock, 2003) and PARSCALE (Muraki & Bock, 2003). G^2 is calculated for item i as follows:

$$G^2 = 2 \sum_{h=1}^{n_g} \left[r_{hi} \log \frac{r_{hi}}{N_h P_i(\bar{\theta}_h)} + (N_h - r_{hi}) \log \frac{N_h - r_{hi}}{N [1 - P_i(\bar{\theta}_h)]} \right], \quad (6)$$

where n_g indicates the number of intervals; r_{hi} represents the observed frequency of correct responses for item i in interval h ; N_h is the number of individuals in interval h ; and $P_i(\bar{\theta}_h)$ refers to the model-predicted proportion correct for item i at $\bar{\theta}_h$. $\bar{\theta}_h$ is the average of theta estimates in interval h . G^2 is distributed under the null hypothesis approximately as a χ^2 with degrees of freedom equal to the number of intervals. G^2 is basically a log-likelihood based statistic in which examinees are grouped into ability intervals based on their ability estimates, and then the empirical proportion correct against the model-based proportion correct within that interval is evaluated, accumulating across intervals.

3. Orlando and Thissen's $S-X^2$. Orlando and Thissen (2000) addressed the weakness of Yen's Q_1 statistic by grouping examinees based on their number correct

(NC) scores instead of their model-based ability estimates. The fit statistic, $S-X^2$, based on Yen's (1981) Q_1 statistic, is approximately distributed as a χ^2 with $G-m$ degrees of freedom, where m indicates the number of item parameters estimated and G is the number of score groups ($G=n-1$ when no groups have been collapsed, where n =number of score points). $S-X^2$ has been extended to test the fit of polytomous models (Bjorner, Smith, Stone & Sun, 2007; Kang & Chen, 2008). The general formula for testing the fit of a polytomous or dichotomous items is as follows:

$$S - X^2 = \sum_{t=t_{\min}}^{t_{\max}} \sum_{c_i=1}^{m_i} \frac{(O_{tci} - E_{tci})^2}{E_{tci}}, \quad (7)$$

where t is sum score, m is number of categories, O is observed frequency and E is expected frequency, i is item. Before calculating $S-X^2$, cells are collapsed to achieve expected cell frequencies above a certain number in order to avoid sparse expected frequencies.

4. Lagrange Multiplier (LM) Test. Glas (1998) proposed a Lagrange Multiplier (LM) procedure to evaluate item and ability parameter estimates for 2PLM and 3PLM. The rationale of LM tests is to test a restricted model against a more general alternative, where the restricted model is derived from the general model by imposing constraints on one or more parameters. The LM test is based on the evaluation of the first-order partial derivatives of the log-likelihood function of the general model, evaluated using the maximum likelihood estimates of the restricted model (Glas, 1998). As suggested by Glas, one advantage of using LM method for evaluation of item fit is that the asymptotic distribution of the statistics involved follows directly from asymptotic theory (Glas & Falc3n, 2003).

- *Graphical Displays of Model Data Fit*

In contrast to statistical analyses, the discrepancy between the model-based expectations and the observed data (i.e., residuals) may be examined visually via graphical representations to examine model fit. Such graphical displays, while less objective in that they do not provide error rate control as opposed to a strict statistical test, have proven to be useful in evaluating the complex relationship between model and data (Hambleton, Swaminathan, & Rogers, 1991). Residual plots provide information on the location and magnitude of possible misfit. Graphical displays are often best used in conjunction with statistical tests in that the test provides information regarding whether the model fits and the practical significance of the misfit.

- *Methods Involving Comparison of Observed Data with Simulated Data*

Hambleton and Han (2005) suggested assessing the goodness-of-fit of commonly used IRT models by comparing the observed score distribution to the distribution from simulated data according to the selected model, and using Komogrov-Smirnov test as a descriptive indicator of the difference in the distributions. The comparison of the observed and simulated score distribution can be based on all examinees to evaluate fit at the test level, or conditional on the observed performance of a particular item to evaluate fit at the item level. To compute the conditional test score given ability, the Lord-Wingersky recursive formula (Lord & Wingersky, 1984), and an extension of the Lord-Wingersky formula proposed by Wang, Kolen, and Harris (2000) are applied. Readers are referred to Wainer and Mislevy (1990), Kingston and Dorans (1985), Ankenmann (1994), Sinharay (2005) for illustrative examples of model data fit.

- *Bayesian Approach*

Under Bayesian framework, the posterior predictive model checking (PPMC) technique was proposed by Guttman (1967). It makes use of the posterior predictive

distribution of replicated data as a reference distribution for the observed data. In practice, data sets are simulated by generating parameter values from the predictive distribution. If the simulations and the data differ systematically, it is an indication of a potential model misfit.

Donoghue and Hombo (1999, 2001) derived a distribution for the fit statistic DHQ for dichotomous items. Their method uses posterior probabilities in computing item fit. The fit statistics proposed by Donoghue and Hombo (1999) are based on the assumption that the item parameters are fixed and known. Under this restrictive assumption, they demonstrated that the fit measure is asymptotically distributed as a quadratic form of a normal random variable. Donoghue and Hombo (2001) extended this method for use with polytomous data, and showed that the statistic is a quadratic form of normals as well in the extended application.

- *Nonparametric Approach (RISE)*

Douglas and Cohen (2001) proposed an approach in which a nonparametrically-based ICC is compared to the best-fitting parametric ICC. If the two ICCs differ substantially, then the parametric model is considered to be misfitting. The concept underlying this approach is based on the fact that a nonparametrically-derived ICC (e.g., kernel-smoothed estimates) provides a reasonable estimate to the true ICC under a weak set of assumptions (Douglas, 1997). Kernel smoothing is a popular nonparametric method developed by Ramsay (1991) and is based on the concept of *local averaging*. Local averaging is useful in estimating points on an ICC because $P(u = 1|\theta) = E(Y|\Theta = \theta)$. Therefore, a reasonable estimate of $P(u = 1|\theta)$ can be obtained by taking a weighted average of the response variable Y in which responses for examinees whose θ -value is close to a focal ability level, referred to as an evaluation point, contribute most heavily in estimating $P(u = 1|\theta)$. Since the

nonparametric approach imposes fewer restrictions on the shape of the ICC, it is concluded that the parametrically-based model is incorrect if it differs from the nonparametric ICC substantially. The following describes the steps used in the procedure of *RISE*.

- Estimate the category curve(s) using kernel smoothing, which is computed as follows:

$$P(\theta_j) = \frac{K\left[\frac{\hat{\theta}_i - \theta_j}{h}\right] Y_i}{\sum_{i=1}^n K\left[\frac{\hat{\theta}_i - \theta_j}{h}\right]}, \quad (8)$$

where K is Gaussian kernel, θ_j is evaluation point, θ_i is theta estimate, Y_i is examinee's response.

- Compute the optimal item parameter values for the parametric model of interest.
- Summarize the difference between the nonparametric and parametrically-based step ICCs for each item using the following statistic:

$$RISE_i = \sqrt{\frac{\sum_{k=1}^K \left[\frac{\sum_{q=1}^Q (\hat{P}_{qk} - \hat{P}_{qk}^{non})^2}{Q} \right]}{K-1}}, \quad (9)$$

where $\hat{P}_{qk}$ and $\hat{P}_{qk}^{non}$ represent points on the ICCs for the model-based and nonparametric methods for each category; Q is the number of evaluation points (e.g., $Q=100$); and K is the total number of categories. In the dichotomous case, $K=2$.

- To determine the significance level of the observed statistic, the following parametric bootstrapping procedure is performed to construct an empirical distribution for $RISE_i$ under H_0 (i.e., model fit).

1. Simulate item response data using the optimal item parameter estimates for each model obtained previously and N θ s sampled from a standard normal distribution.
2. Estimate the category curves and ICCs for the simulated data using kernel smoothing.
3. Compute the optimal values for item parameter estimates for each model, as described previously.
4. Compute $RISE_i$ for each item using equation (9).
5. Repeat steps 1 to 4 M times (e.g., $M=500$).

If the observed $RISE_i$ is greater than the 95th percentile of the constructed $RISE_i$, the conclusion is the model does not fit the data. In addition, for the nonparametric family, Wells and Bolt (2008), Li and Wells (2006), and Wells (2009), and Liang and Wells (2008) described the graphical representation of model misfit between nonparametric and parametric ICC to provide visual guidance about the location, type of misfit and magnitude of misfit.

2.3 Empirical Research on Fit Indices

In this section, a number of studies comparing the fit procedures are reviewed. The Type I error rate and power are two commonly used criteria that provide useful information regarding the statistical properties of a test statistic. As for graphical analysis, there are usually no clear criteria because it is more subjective and used as a supplement to significance tests. Model misfit consequence is one important issue as part of the fit procedure; therefore, a number of studies on this topic are also reviewed here. A summary of results from these studies is provided afterwards.

For dichotomous items, McKinley and Mills (1985) compared the performances of traditional goodness of fit statistics in detecting misfit. The fit statistics included were

Bock's χ^2 , Yen's Q_1 , Wright and Mead χ^2 (Wright & Mead, 1977; Yen, 1981), and the likelihood ratio χ^2 (G^2). Data were simulated to fit four models: 1PLM, 2PLM, 3PLM, and a two-factor linear model. Misfit was examined using an IRT model that was different from the model used to simulate item responses. By comparing Type I error rates, they found that the likelihood ratio statistics yielded the fewest erroneous rejections of fit when the data were simulated and calibrated using the same model. However, given controlled Type I error rates, Bock's (1972) index appeared to yield slightly better performances in terms of correct rejections of fit when the model used to simulate the items differed from the model used to calibrate the items. As a result, the authors could not conclude which measure was the best. Instead, they suggested that the choice of the fit procedures depend on the type of error that is considered to be the more serious error in a particular setting. With regard to the sample size issue, they concluded that sample sizes of 500 to 1000 were likely to yield the best results while a sample of 2000 seemed to make the fit procedures too sensitive.

Orlando and Thissen (2000) conducted a simulation study investigating the Type I error rates and empirical power for their new proposals of item fit statistics for dichotomously scored items. They compared $S-X^2$ and $S-G^2$ statistics with Yen's Q_1 and McKinley and Mills' log likelihood ratio statistic. They conducted three conditions in which the simulating and fitting models were identical to compute Type I error rates and three conditions involving model misspecifications to compute power. Results indicated that both Yen's (1981) Q_1 and McKinley and Mills's (1985) index had very high Type I error rates for dichotomous items, particularly for short tests. With sample sizes of 1,000, for a nominal α level of 0.05, empirical Type I error rate was around 0.95 for 10-item tests, between 0.10 and 0.29 for 40-item tests, and somewhat lower but still inflated for 80-item tests. The results also showed that $S-X^2$

performed better than the other statistics, in that its Type I error rates were close to the specified significance level while the others tended to reject the null hypothesis too often. In another study, Orlando & Thissen (2003) further investigated the utility of $S-X^2$ as a model fit index for the 3PLM. Three types of item misfit were simulated: An item that followed a nonmonotonic shaped; an item in which the probability of answering correctly does not reach 1 (i.e., the upper-asymptote is not 1); an item that exhibits a plateau over middle values of θ but follows a logistic curve before and after the plateau. The simulation compared the performance of $S-X^2$ to that of $Q_1 - X^2$, a Pearson χ^2 form of Yen's Q_1 index. Results indicated that the performance of $S-X^2$ was improved with increasing test length and sample size, and was superior to that of $Q_1 - X^2$ under most conditions.

Stone and Zhang (2003) presented results comparing the Type I error rates and empirical power for three alternatives for dichotomously scored items, including Orlando and Thissen's (2000) method, Stone's (2000a) resampling or pseudocounts-based method, and Donoghue and Hombo's (1999) *DHQ* index. As a frame of reference with traditional approaches to assessing model-data fit, Bock's (1972) χ^2 statistic was also examined. They reported excessively unacceptable Type I error rates for Bock's (1972) statistic, which is consistent with previous research. By contrast, the three new approaches appeared to offer promising alternatives to traditional methods. Among the three methods, they found that Stone's method tended to display more power to detect misfit in smaller samples, while Orlando and Thissen's method showed adequate power to detect misfit with a sample size of 2000 under the 2PLM or 3PLM simulated and 1PLM calibrated conditions, and the Donoghue and Hombo's method lacked adequate power at all sample sizes. However, none of the methods showed adequate power in detecting misfit under the 3PLM simulated and

2PLM calibrated conditions, which corroborates previous research (McKinley & Mills, 1985; Yen, 1981).

Glas and Falcón (2003) proposed another index based on a Lagrange multiplier (LM) test and compared this index with Orlando and Thissen's (2000) $S - G^2$ and the log-likelihood ratio test. The Type I error rate and power of the test of fit of the 3PLM was investigated. The test lengths in the simulation studies were 10, 20 and 40, and the examinee sample sizes were 500, 1000, and 4000. The results showed that the overall characteristics of the $S - G^2$ test, in terms of the ratio of hits and false alarms, were better than the overall characteristics of the LM test. They suggested that this difference could be due to differences in item parameters. As in Orlando and Thissen's procedure, for this index, examinees are grouped by number-correct scores rather than IRT estimated trait scores which were the grouping basis of the usual log-likelihood ratio index, and the LM index also takes into account the standard errors in the item parameter estimates.

For polytomous items, Stone and Hansen (2000) examined a Pearson's χ^2 type index similar to Yen's Q_1 (1981) or Bock's (1972) χ^2 and a log-likelihood ratio index similar to the statistic reported in PARSCALE's. Using true item parameters and true abilities, Type I error rates were somewhat more inflated for the log likelihood ratio index than for Pearson's χ^2 . However, using estimated abilities and true item parameters, Type I error rates were extremely inflated, especially for short tests. For example, for an eight-item test with a nominal α of 0.05, the Type I error rate ranged from 0.75 to 1.00 for the selected items displayed. The inflation was smaller with longer tests and for smaller sample sizes. The log likelihood index studied by Stone and Hansen differed somewhat from PARSCALE's index; item parameters were known rather than estimated, and .000001 was added to cells with an expected frequency of 0,

compared with PARSCALE's procedure of combining groups so that all cells have an expected frequency of at least 5.

DeMars (2005) examined the Type I error rates for PARSCALE's fit statistic G^2 for polytomous items. Using either the GRM or the PCM, data were simulated with 10 and 20 items. Type I error rates were found to be inflated for the shorter test length and for the GRM, and also for the longer test length when the ability distribution was uniform. Given the findings, the conclusion was that the fit index cannot be recommended for short tests.

Schrader, Ansley, and Kim (2004) examined the behavior of Q_1 and two new indices proposed by Orlando and Thissen (2000), $S-X^2$ and $S-G^2$, on polytomous data (e.g., constructed-response data) fit by the GPCM. In the study, those indices originally developed for dichotomous data were extended to more general forms for use with polytomous data. The misfitting items were from simulating a two-dimensional version of GPCM. They found that the Type I error rates from $S-X^2$ were very close to the significance level for various test lengths, while the Q_1 indices had inflated Type I error rates for shorter test lengths and more reasonable rates for the longest test. However, none of the selected indices had sufficient power to detect misfit under most of the two dimensional conditions. Based on the results, the authors suggested that the Q_1 indices were not useful in testing model fit, either due to inflated Type I error rates or insufficient power. Most of their results for polytomous data were found to be consistent with results found in Orlando and Thissen's studies for dichotomous data.

For a mix of dichotomous and polytomous items, Chon, Lee, and Ansley (2007) compared three different approaches for assessing IRT model-data fit using real test data containing both multiple choice and constructed response items. In order to evaluate fit for mixed data, they compared PARSCALE's G^2 as a traditional approach

with the generalized forms of Orlando and Thissen's $S-X^2$ and $S-G^2$. Some inconsistencies were found between traditional and new indices for assessing the fit of IRT models based on a mixed-format test. It was reported that the new indices indicated considerable better model fit than the traditional indices. Thus, the results from Chon, Lee, and Ansley (2007) provide a preliminary investigation for the model-data fit procedures.

Due to the unappealing features of statistical tests of misfit (e.g., a significant result does not imply a practically meaningful misfit), there is an urgent need for graphical procedures and displays for assessing model fit for both dichotomous and polytomous data. Some commercial IRT estimation software includes raw residual plots in the standard output—e.g., BILOG-MG (Zimowski, Muraki, Mislevy, & Bock, 2003). Among the widely used commercial software packages, BILOG-MG is one of the few that plots high quality graphs for identifying the fit in IRT models. The residual plots in BILOG-MG are proving to be popular and helpful in explaining model fit. Unfortunately, such fit plots are not available in PARSCALE (Mislevy & Bock, 2003), one of the most popular IRT software packages, which can handle polytomously-scored items.

Liang, Han and Hambleton (2009) developed software called *ResidPlots-2* which visually represents the discrepancy between the model-based expectations and the observed data. It provides a powerful tool for graphical residual analyses and is a valuable advance in convenience and flexibility for researchers and practitioners wishing to do graphical residual analysis. The advantages of this software include several features. First, it supports the most widely used IRT models including three dichotomous models (1PLM, 2PLM, 3PLM) and three polytomous models (GRM, GPCM, NRM). Second, it provides considerable flexibility with respect to the number

and size of the intervals for which the residuals are computed. Users are able to decide the number of intervals, choose the interval size to provide equal width or equal frequency, select the location of the data plot in each interval, eliminate intervals at the lower or higher end of the proficiency scale if desired, etc. Third, it allows users to decide what type of error bars they wish to have displayed. Users can specify the number of standard errors represented by the error bars (e.g., 2 SE). Fourth, it provides three sets of plots: (1) the item level, raw residual plots and standardized residual plots with error bars; (2) the test level, *ResidPlots-2* can show standardized residual distributions (PDF and CDF) with corresponding tables, item fit plots and score fit plots from both empirical and simulated data; and (3), observed test score distributions and predictive score distributions (PDF and CDF) are produced as well. The predictive test score distribution is based on simulation data generated from item and ability parameter estimates from the observed data. Fifth, some tables including basic item fit and person fit statistics and summaries of residuals are provided.

While the nonparametric approach, *RISE*, has exhibited controlled Type I error rates and adequate power under the studied conditions, a further promising advantage of this nonparametric approach demonstrated that it provides a graphical inspection of possible misfit. Wells and Bolt (2008) and Li and Wells (2006) examined *RISE*'s performance on 2PLM and GRM, respectively. Recently, Liang and Wells (2009) applied this nonparametric approach, *RISE*, to test the fit of the GPCM. In their study, the empirical Type I error rate and power of *RISE* were assessed for various test lengths (10-, 20-, and 40 items), sample sizes (500, 1000 and 2000), and type of assessment. What's more, Liang and Wells (2008) further explored the statistical properties of the proposed fit statistic, *RISE* on a mixed format test that included the 3PLM, 2PLM, and GPCM. The empirical Type I error rate and power of *RISE* were assessed for various

sample sizes (2000, 3000 and 5000). The results were compared to G^2 in PARSCALE (Muraki & Bock, 2003) and $S-X^2$ (Orlando & Thissen, 2000). The conclusion from the two papers was that compared to the two popular parametric methods, PARSCALE'S G^2 (Muraki & Bock, 2003) and $S-X^2$ (Orlando & Thissen, 2000), *RISE* better controlled Type I error rate and exhibited more power than $S-X^2$ and G^2 .

2.4 Summary

The review of literature in this chapter highlights the limitations of statistical tests. Generally speaking, such tests tend to be narrowly focused on a particular aspect of the relationship between the model and the data, often summarizing the evaluation using a descriptive statistic or test result. Specifically speaking, there are three general problems with several of the commonly-used model fit statistics. First, it is well known that several of the statistics (e.g., G^2 and Yen's Q_1) are not distributed as a χ^2 because the examinees are grouped based on IRT θ estimates, which contain error, and because the degrees of freedom are in question (Orlando & Thissen, 2000); as a result, the Type I error rate is inflated under many conditions (when the Type I error rate is inflated, many valid items are falsely identified as misfitting when in fact the model fits the data). Second, even when the fit statistic is able to control the Type I error rate, at least for most test lengths (e.g., Orlando and Thissen's $S-X^2$), there is no accompanying method of assessing how poorly a model fits given a statistic that indicates misfit. This is problematic when assessing fit using very large samples because trivial misfit often leads to the rejection of the hypothesis that the model fits the data exactly in the population. Third, there are a limited number of statistical tests that can assess the fit of multiple IRT models used in a mixed format test. This is particularly problematic since many large-scale assessments utilize both dichotomous and polytomously-scored data. Graphical residual analyses, on the other hand, have

proven to be useful in evaluating the complex relationship between model and data (Hambleton, Swaminathan, & Rogers, 1991). The problem of relying on graphical evidence only is that it is less objective than a statistical test; as a result, it can be difficult to judge whether an item is providing reasonable fit or if the item misfit is practically meaningful. Therefore, it is helpful to use both statistical tests and graphical analysis to determine how well a model fits.

As discussed previously, the nonparametric approach for assessing fit appears to be promising due to the controlled Type I error rate, sufficient power to detect misfit, and the convenient graphical display of possible misfit that is available. The promising results of *RISE* support further study of the method in different testing contexts (e.g., non-normal ability distributions) and in exploring its use in examining the potential consequences of the misfit. Under these circumstances, the current study extends the use of *RISE* to more general and comprehensive applications by manipulating variety of factors (e.g., test length, sample size, IRT models, ability distribution). Results (e.g., type I error rate and power) will be compared to G^2 in PARSCALE (Muraki & Bock, 2003) and $S-X^2$ (Orlando & Thissen, 2000). In order to demonstrate its advantage of graphical representation of model misfit, some residual plots will be displayed from empirical data. Lastly, the consequences of potential misfit will be explored via the nonparametric approach using empirical data.

CHAPTER 3

METHOD

The nonparametric approach for assessing model fit was examined via a Monte Carlo simulation study and an analysis of empirical data. The Monte Carlo simulation study was performed to examine the empirical Type I error rate and power of the proposed statistic to detect misfitting items in a mixed format test under thirty-six conditions. In addition, the statistical properties of *RISE* were compared to a commonly reported fit statistic, G^2 , provided by the computer package, PARSCALE (Muraki & Bock, 2003), and a presently developed fit statistic, $S-X^2$, provided by the computer software, IRTFIT (Bjorner, Smith, Stone & Sun, 2007). An empirical data set was analyzed to explore the use of the nonparametric approach for assessing the practical meaningfulness of the misfit via the hypothesis test provided by *RISE*, inspection of residuals. Comparison of score distributions and test characteristic curves (TCCs) were also addressed as two aspects of model misfit consequence.

3.1 The Procedure of *RISE*

The *RISE* statistic was implemented for assessing the fit of multiple IRT models in a mixed-format as follows:

- Kernel smoothing was used to estimate the ICC for dichotomous data and OCC or CRC for polytomous data. In order to increase computational speed, the Fast Fourier Transformation (FFT) will be applied (Ramsay, 1991).
- Compute the optimal item parameter values for the parametric model of interest (i.e., 2PLM or 3PLM for dichotomous data, and GRM or GPCM for polytomous item responses). For the 2PLM and GRM, the optimal item parameter values were based on the nonparametric ICC using maximum likelihood estimation, the EM algorithm (Wells & Bolt, 2008, Li & Wells, 2006).

To maximize the log-likelihood, the Newton-Raphson procedure was conducted. However, two modifications were needed. One was to replace the observed proportion of correct response at each theta level by the kernel-smoothed probability at each evaluation point. The other was to compute the weight for each evaluation point instead of each theta value. For the 3PLM, a nonlinear model was fitted with three unknown parameters (a, b, c). The nonlinear model used a χ^2 merit function (Press, Teukolsky, Vetterling & Flannery, 1992) which was defined to determine best-fit parameters. The minimization procedure proceeded iteratively until χ^2 statistic stopped decreasing. And for the GPCM, least squares were used on the logit deviates (Liang & Wells, 2009). In other words, although the relationship between θ and each step ICC was nonlinear, the relationship between the logistic deviate, $\left(\log \left[\frac{P_{k+1}}{P_k} \right] \right)$, for each step ICC and θ was linear allowing the use of least squares to estimate the slope and difficulty for each step ICC.

- Summarize the difference between the nonparametric and parametrically-based step ICCs for each item using the following statistic:

$$RISE_i = \sqrt{\frac{\sum_{k=1}^{K-1} \left[\frac{\sum_{q=1}^Q (\hat{P}_{qk} - \hat{P}_{qk}^{non})^2}{Q} \right]}{K-1}} \quad (10)$$

where $\hat{P}_{qk}$ and $\hat{P}_{qk}^{non}$ represent points on the ICCs for the model-based and nonparametric methods for each category; Q is the number of evaluation points (e.g., $Q=100$); and K is the total number of categories. In the dichotomous case, $K=2$.

- To determine the significance level of the observed statistic, the following parametric bootstrapping procedure was performed to construct an empirical distribution for *RISE* under H_0 (i.e., model fit).
 1. Simulate item response data using the optimal item parameter estimates for each model obtained previously and N θ s sampled from a standard normal distribution.
 2. Estimate the category curves and ICCs for the simulated data using kernel smoothing.
 3. Compute the optimal values for item parameter estimates for each model, as described previously.
 4. Compute *RISE* for each item using equation (10).
 5. Repeat steps 1 to 4 M times (e.g., $M=500$).

If the observed $RISE_i$ was greater than the 95th percentile of the constructed $RISE_i$, then we concluded that the model did not fit the data.

3.2 Four Factors

A Monte Carlo simulation study was employed to examine the statistical properties of *RISE* for a mixed-format test. Four factors were varied: ability distribution, sample size, test length, and models.

Three ability distributions were considered to represent three shapes which are commonly found in practice. In particular, they were normal with mean of 0 and standard deviation of 1, positively skewed ($\gamma_1 = 0.95$) and negatively skewed distributions ($\gamma_1 = -0.95$).

The sample sizes were 500, 2,000 and 10,000 to represent small, medium and large sample sizes. Sample size 2,000 is typically used in many testing programs for anchor items; however, the small sample size 500 and large sample size 10,000 were

also examined for not only the purpose of generality but also the practical use since most traditional fit statistics do not perform well under these sample sizes which are actually not uncommon in some testing scenario.

Two test lengths that represent a small and relatively large assessment were investigated: 30 and 60 items. Those were typical test lengths in some testing programs.

Oftentimes, large-scale standardized tests, such as those used in statewide assessments, comprise both dichotomous (e.g., multiple-choice and short-answer) and polytomous (e.g., partial credit) item responses used to define the latent theta metric. To represent this context and cover variety of models efficiently, mixed format tests were simulated, so a mixture of binary and polytomous IRT models (3PLM, 2PLM, and GRM//GPCM) were assessed in terms of goodness-of-fit.

The percentage of misfitting items was not considered in the study because this factor does not influence either Type I error or power (Wells & Bolt, 2008). In summary, there were 36 conditions (3 ability distributions X 3 sample sizes X 2 test lengths X 2 formats).

In addition, the consequence of the misfit was explored by comparing the test distribution based on the raw data (or the nonparametric ICCs) versus the test distribution based on the parametric models. The procedure was illustrated below:

- Pick one simulation data (10,000 by 60) and identify misfitting items based on fit statistics
- Compute observed score distribution (CDF) and expected score distribution (CDF)
- Select score cuts and compute the CDF differences at those cuts
- Replace those detected misfitting items by fitting items based on item and ability parameter estimates to form a new dataset

- Compute observed score distribution (CDF) and expected score distribution (CDF) from the new dataset
- Compute the new CDF differences at the cuts again
- If the new difference was not significantly different from the old difference, then there was no model misfit consequence on those cuts of score distribution.

The true TCCs were compared with the estimated TCCs to illustrate model misfit consequences. The procedure for investigating misfit consequence on TCC is shown below:

- Pick one simulation data (10,000 by 60) and identify misfitting items based on fit statistics
- Compute true TCC and estimated TCC
- Select theta cuts and compute the TCC differences at those cuts
- Replace those detected misfitting items by fitting items based on item and ability parameter estimates to form a new dataset
- Compute true TCC and estimated TCC from the new dataset
- Compute the new TCC differences at the cuts again
- If the new difference was not significantly different from the old difference, then there was no model misfit consequence on those cuts of TCC.

3.3 Data Generation

Item response data were generated for each of the thirty-six conditions. The 30-item test consisted of twenty 3PLM items (representing multiple-choice items), five 2PLM items (representing short-answer items), and five GPCM (or GRM) items (representing five-category, polytomous responses) for three sample sizes: 500-, 2,000-, and 10,000-examinees. The item responses were simulated from three ability

distributions, respectively. Similarly, the 60-item test consisted of forty 3PLM items (representing multiple-choice items), ten 2PLM items (representing short-answer items), and ten GPCM (GRM) items (representing five-category, polytomous responses).

To assess the Type I error rate, 80% of the 3PLM items (16 items in the 30-item test, 32 items in the 60-item test), 80% of the 2PLM items (4 items in the 30-item test, 8 items in the 60-item test) and 80% of the GPCM (GRM) items (4 items in the 30-item test, 8 items in the 60-item test) were simulated from the respective parametric model. The generating item parameter values were obtained from a state-wide test that used a similar test format. Table 3.1 reports the generating item parameter values for the 60-item test. The values used in the 30-item conditions were a subset of those used in the 60-item condition.

To assess power, the remaining 20% of the total items were simulated from empirically-derived ICCs that did not follow their respective parametric models. The ICCs were obtained from a statewide assessment. Figure 1 reports the underlying ICCs for generating misfit based on kernel smoothed estimates for the 60-item test. Note that the percentage of misfitting items does not appear to influence either Type I error rate or power (Wells & Bolt, 2008), so other percentages were not examined in the study.

One-hundred replications were performed for each of the three sample size conditions. The underlying θ values were sampled from the three ability distributions.

3.4 Data Analyses

FORTTRAN code was written and implemented to perform the nonparametric approach to assess misfit and perform the parametric bootstrapping procedure. The computer software package, PARSCALE (Muraki & Bock, 2003), was used to obtain the fit statistic, G^2 , the computer software, IRTFIT (Bjorner et al., 2007) was used to obtain the fit statistic, $S-X^2$. All residual plots and score distribution plots based on

parametric approach were produced by the computer software ResidPlots-2 (Liang, Han & Hambleton, 2009).

Simulation Analyses. The empirical Type I error rate and power were assessed for *RISE* as well as G^2 and $S-X^2$. The empirical Type I error rate was based on the proportion of items simulated using the three model combinations that were detected as misfitting at $\alpha = .01$ and $\alpha = .05$. Power was assessed based on the proportion of items simulated using the empirically-derived category curves shown at $\alpha = .01$ and $\alpha = .05$. However, power was interpreted for the specific statistic only for conditions in which the Type I error rate was controlled. Score distributions and TCCs before and after replacement of misfitting items were plotted to illustrate model misfit consequence.

Empirical Analyses. An additional advantage of the nonparametric approach, *RISE*, was the convenient graphical representation of model misfit. This feature can provide visual guidance about the location and type of misfit. The graphical features of the nonparametric approach were explored via the analysis of empirical data. The fit of the three model combinations was tested using *RISE* as well as G^2 and $S-X^2$. The data came from a large-scale assessment with thirty-two multiple-choice items, four short-answer items, and six partial-credit, five-category, polytomous items at sample sizes of 500, 2,000, and 10,000. Items flagged as misfitting according to the three statistical test statistics were compared. The misfit was further explored via a graphical representation. For each detected misfitting polytomous item, the empirical data and item characteristic curves were aggregated to form one raw residual plot. Model misfit consequence on score distribution was also investigated using the same procedure as the simulation study.

Table 3.1 *Generating Item Parameter Values*

Item	α	β	γ	β_1	β_2	β_3	β_4
1	1.027	-0.750	0.096				
2	0.985	-0.660	0.084				
3	1.098	-0.404	0.095				
4	1.379	0.247	0.135				
5	1.429	-0.850	0.094				
6	0.957	0.734	0.343				
7	1.099	-0.078	0.298				
8	1.268	0.122	0.221				
9	0.968	-0.334	0.134				
10	1.414	-0.041	0.156				
11	1.464	-0.382	0.306				
12	1.415	0.539	0.169				
13	0.880	0.228	0.159				
14	0.882	-0.674	0.158				
15	1.054	-0.460	0.239				
16	1.096	0.363	0.288				
17	0.996	-0.161	0.193				
18	1.499	-0.286	0.273				
19	1.541	-0.810	0.259				
20	1.724	-0.634	0.258				
21	1.460	0.747	0.070				
22	0.680	-0.418	0.191				
23	0.959	-0.300	0.284				
24	1.017	0.144	0.165				
25	1.011	0.134	0.162				
26	0.948	-0.298	0.281				
27	0.675	-0.400	0.189				
28	1.450	0.737	0.050				
29	0.721	-0.629	0.255				
30	0.839	-0.798	0.257				
31	1.496	-0.282	0.269				
32	0.995	-0.151	0.189				
33	0.952	0.267					
34	0.985	0.060					
35	1.151	0.284					
36	0.986	-1.311					
37	0.876	0.145					
38	0.946	-0.977					
39	1.013	0.173					
40	1.202	-0.004					
41	1.137			-1.054	-1.025	0.344	0.681
42	0.796			-0.090	-1.277	0.053	0.215
43	1.213			-0.030	-0.461	1.013	0.421
44	1.004			-0.839	-1.224	-1.083	-0.068
45	1.026			-1.035	-1.060	0.267	0.534

46	0.724	-0.010	-1.255	0.023	0.210
47	0.945	-0.777	-1.201	-1.139	-0.016
48	0.893	-1.001	-1.924	0.004	-0.237

Figure 3.1. Generating Empirical ICCs.

Item 9

Item 10

Item 11

Item 12

CHAPTER 4

RESULTS

4.1 Study 1-Simulation Study

4.1.1 Statistical Properties

Study 1 was a simulation study. In this study, the statistical properties of the three fit statistics were compared. Tables 4.1 through 4.12 summarize the Type I error rate and power for $\alpha = 0.05$ and $\alpha = 0.01$ under the thirty-six conditions from three fit statistics, *RISE*, G^2 and $S-X^2$. Each table covered results for one ability distribution, one model combination, three sample sizes, and two test lengths.

Table 4.1 reports the empirical Type I error rates for *RISE*, G^2 and $S-X^2$ based on normal ability distribution and 3PLM/2PLM/GPCM model combination. Table 4.2 reports results for 3PLM/2PLM/GRM model combination. Based on the two tables, the Type I error rate was controlled for *RISE* and highly deflated for $S-X^2$ especially under 500- and 2,000-examinee conditions for each model. On the other hand, G^2 exhibited inflated Type I error rates. For the 3PLM, the Type I error rate was slightly inflated for the 500- and 2,000-examinee conditions, moderately inflated for the 10,000-examinee condition. For the 2PLM, G^2 was slightly inflated for the 500- and 2,000-examinee conditions; it was highly inflated for the 10,000-examinee conditions. For the GPCM and GRM, G^2 was slightly inflated for the 500-examinee condition, moderately inflated for the 2,000-examinee condition and highly inflated for the 10,000-examinee condition.

In summary, *RISE* performed satisfactorily in that the Type I error rates were not inflated or affected by sample size and test length. $S-X^2$ exhibited highly deflated Type I error rates; however, as sample size increased and test length decreased, the Type I error rates increased toward the nominal alpha level. G^2 did not control Type I error

rates for most conditions and Type I error rates increased while sample size increased; however, test length did not have any impact on it.

Tables 4.3 and 4.4 report the empirical power rates for *RISE*, G^2 and $S-X^2$ across the conditions for the two model combinations in the mixed format test respectively. Because power is conditional on controlling the Type I error rate at the nominal alpha level ($\alpha=0.05$ and $\alpha=0.01$), the power rates corresponding to the Type I error rates greater than 0.07 were X-ed out for $\alpha=0.05$ and those greater than 0.03 were X-ed out for $\alpha=0.01$. It was not appropriate to compare power rates between the three fit statistics for those X-ed out cases.

In summary, as sample size increased, all three fit statistics exhibited increased power for each model. *RISE* exhibited more power than the valid power values of G^2 . On the other hand, compared to $S-X^2$, *RISE* exhibited more power to detect misfit under 500- and 2,000-examinee conditions for each model.

Table 4.5 reports the empirical Type I error rates for *RISE*, G^2 and $S-X^2$ based on negatively-skewed ability distribution and 3PLM/2PLM/GPCM model combination. Table 4.6 reports results for 3PLM/2PLM/GRM model combination. The two tables indicated that the Type I error rate across all conditions was controlled for *RISE* only. G^2 exhibited inflated Type I error rates. For the 3PLM, the Type I error rate was slightly inflated for the 500- and 2,000-examinee conditions and it was moderately inflated for the 10,000-examinee condition. For the 2PLM, G^2 was slightly inflated for the 500- and 2,000-examinee conditions; it was highly inflated for the 10,000-examinee conditions. For the GPCM and GRM, G^2 was slightly inflated for the 500-examinee condition, moderately inflated for the 2,000-examinee condition and highly inflated for the 10,000-examinee condition. $S-X^2$ only controlled Type I error rates for 500-examinee and 60-item condition for each model. It exhibited inflated Type I error rates for all

other conditions. For the 3PLM, the Type I error rate was slightly inflated for the 500-examinee with 30-item and 2,000-examinee with 60-item conditions; it was highly inflated for other conditions. For the 2PLM, $S-X^2$ was moderately inflated for the 500-examinee with 30-item and 2,000-examinee with 60-item conditions, highly inflated for other conditions. For the GPCM and GRM, $S-X^2$ was highly inflated for all conditions.

In summary, under the negatively-skewed ability distribution, *RISE* also outperformed the other two fit statistics in that the Type I error rates were not inflated and not affected by sample size and test length. G^2 did not control Type I error rates and showed increasingly inflated Type I error rates as sample size increased. $S-X^2$ did not control Type I error rates and exhibited a clear pattern that Type I error rates increased while sample size increased and test length decreased.

Tables 4.7 and 4.8 report the empirical power rates for *RISE*, G^2 and $S-X^2$ across the conditions for the two model combinations in the mixed format test respectively. Because power is conditional on controlling the Type I error rate at the nominal alpha level ($\alpha=0.05$ and $\alpha=0.01$), the power rates corresponding to the Type I error rates greater than 0.07 were X-ed out for $\alpha=0.05$ and those greater than 0.03 were X-ed out for $\alpha=0.01$. It was not appropriate to compare power rates between G^2 , $S-X^2$ and *RISE* for those X-ed out cases.

In summary, as sample size increased, all three fit statistics exhibited increased power for each model. *RISE* exhibited much more power than the valid power values of G^2 and $S-X^2$.

Table 4.9 reports the empirical Type I error rates for *RISE*, G^2 and $S-X^2$ based on positively-skewed ability distribution and 3PLM/2PLM/GPCM model combination.

Table 4.10 reports results for 3PLM/2PLM/GRM model combination. The two tables

demonstrated that the Type I error rate was controlled for *RISE* across all conditions. G^2 exhibited inflated Type I error rates. For the 3PLM and 2PLM, the Type I error rate was slightly inflated for the 500- and 2,000-examinee conditions, highly inflated for the 10,000-examinee condition. For the GPCM and GRM, G^2 was moderately inflated for the 500-examinee and the 2,000-examinee conditions and highly inflated for the 10,000-examinee condition. $S-X^2$ only controlled Type I error rates for 500-examinee and 60-item condition for 3PLM and 2PLM. It exhibited inflated Type I error rates for all other conditions. For the 3PLM, the Type I error rate was moderately inflated for the 500-examinee with 30-item and 2,000-examinee with 60-item conditions, highly inflated for other conditions. For the 2PLM, $S-X^2$ was moderately inflated for the 500-examinee with 30-item and 2,000-examinee with 60-item conditions, highly inflated for other conditions. For the GPCM and GRM, $S-X^2$ was highly inflated for all conditions.

In summary, under the positively-skewed ability distribution, *RISE* also outperformed the other two fit statistics in that the Type I error rates were not inflated and not affected by sample size and test length. G^2 did not control Type I error rates and showed increasingly inflated Type I error rates as sample size increased. $S-X^2$ did not control Type I error rates and had clear pattern that Type I error rates increased while sample size increased and test length decreased.

Tables 4.11 and 4.12 report the empirical power rates for *RISE*, G^2 and $S-X^2$ across the conditions for the two model combinations in the mixed format test respectively. Because power is conditional on controlling the Type I error rate at the nominal alpha level ($\alpha = 0.05$ and $\alpha = 0.01$), the power rates corresponding to the Type I error rates greater than 0.07 were X-ed out for $\alpha = 0.05$ and those greater than 0.03 were X-ed out

for $\alpha = 0.01$. It was not appropriate to compare power rates between G^2 , $S-X^2$ and $RISE$ for those X-ed out cases.

In summary, as sample size increased, all three fit statistics exhibited increased power for each model. $RISE$ exhibited much more power than the valid power values of G^2 and $S-X^2$.

4.1.2 Model Misfit Consequence

4.1.2.1 Score Distribution

One of the simulated data sets (normal ability distribution, 10,000-examinee, 60-item and 3PLM/2PLM/GPCM) was used to address the model misfit consequence on score distribution. Figures 4.1 through 4.3 report the observed score distribution (blue solid line) and the expected score distribution (red dotted line) before and after replacement of misfitting items detected by three fit statistics. However, to illustrate the misfit consequence on practical use, three cuts (40, 60, 80) were selected to quantify the difference in distributions attributed to the replacement of misfitting items. The difference in the percentage of examinees at each cut was 1%, 2% and 2% respectively. 0.005 was chosen to be a criterion, so the impact on score distribution before and after replacement was significant if the value in each cell was greater than 0.005. The reason to choose 0.005 as the criterion was because it was average of differences at all score points. Table 4.13 reports the results. Based on both the table and the plots, there was no significant model misfit consequence on score distribution for the given simulated data. It is important to note that this result may have occurred because the simulated misfit did not have a consequential effect on test score distributions.

4.1.2.2 TCC

The same simulation data were used to address the model misfit consequence on TCC. Figure 4.4 reports the nonparametric TCC (blue solid line) and the parametric

TCC (red dotted line) before and after replacement of misfitting items detected by *RISE*. However, to illustrate the misfit consequence on practical use, three cuts, -1.41, 0.33 and 1.11 were transformed from raw score scale to quantify the change of TCC difference. The cut of 0.5 was chosen because if the difference is larger than 0.5, it would be rounded to 1.0. Table 4.14 reports the results. Based on the table and the plots, there was no significant model misfit consequence on TCC for the given simulated data at the cuts.

4.2 Study 2-Empirical Study

4.2.1 Residual Plots on Misfitting Items

Three fit statistics were applied to one empirical data which contained 32 multiple choice items fitted by 3PLM, 4 short answer items fitted by 2PLM and 6 constructed response items fitted by GPCM. Three sample sizes (500, 2,000 and 10,000) were randomly drawn without replacement from a larger sample size. Table 4.15 summarized the number and percentage of misfitting items detected by each fit statistic at each sample size. The misfitting items were listed in the last column. At sample size 10,000, *RISE* and $S-X^2$ flagged very similar misfitting items while G^2 flagged almost all items as misfitting. Figure 4.5 through 4.12 show the residual plots on misfitting items detected by three fit statistics. Each misfitting polytomous item had one aggregated raw residual plot weighted by score categories.

4.2.2 Model Misfit Consequence on Score Distribution

Figure 4.13 through 4.15 report the observed score distribution (blue solid line) and the expected score distribution (red dotted line) before and after replacement of misfitting items detected by three fit statistics at three sample sizes. To further illustrate the misfit consequence quantitatively and practically, three cuts (25, 40, 55) were selected to assess the change of distribution difference. The percentage of examinees at

each cut was 1%, 2% and 4% respectively. Table 4.16 reports the results. Based on both the table and the plots, there was no meaningful model misfit consequence on score distribution for the given data at each sample size. In addition, the three fit statistics did not lead to any meaningful distribution difference before and after replacement. The finding was consistent with the examination of statistical properties in simulation study.

Table 4.1 *Empirical Type I Error Rate for RISE, PARSCALE's G^2 and IRTFIT's $S-X^2$ Statistic, Normal Ability Distribution, 3PLM/2PLM/GPCM Model Combination*

Sample Size	Test Length	Model	<i>RISE</i> (0.05)	G^2 (0.05)	$S-X^2$ (0.05)	<i>RISE</i> (0.01)	G^2 (0.01)	$S-X^2$ (0.01)
500	30	3PLM	0.038	0.069	0.027	0.009	0.026	0.007
		2PLM	0.041	0.085	0.005	0.011	0.045	0.000
		GPCM	0.050	0.060	0.008	0.012	0.008	0.000
	60	3PLM	0.036	0.070	0.008	0.007	0.031	0.003
		2PLM	0.045	0.070	0.003	0.009	0.023	0.001
		GPCM	0.052	0.079	0.001	0.010	0.021	0.000
2,000	30	3PLM	0.029	0.073	0.036	0.000	0.016	0.008
		2PLM	0.045	0.085	0.018	0.006	0.013	0.003
		GPCM	0.042	0.225	0.005	0.001	0.070	0.000
	60	3PLM	0.034	0.061	0.018	0.008	0.019	0.006
		2PLM	0.049	0.074	0.010	0.013	0.011	0.001
		GPCM	0.032	0.119	0.001	0.011	0.029	0.000
10,000	30	3PLM	0.040	0.251	0.040	0.004	0.095	0.006
		2PLM	0.039	0.538	0.055	0.002	0.283	0.008
		GPCM	0.053	0.995	0.063	0.012	0.995	0.013
	60	3PLM	0.036	0.321	0.032	0.000	0.139	0.005
		2PLM	0.042	0.466	0.031	0.004	0.230	0.004
		GPCM	0.048	0.931	0.026	0.005	0.843	0.008

Table 4.2 *Empirical Type I Error Rate for RISE, PARSCALE's G^2 and IRTFIT's $S-X^2$ Statistic, Normal Ability Distribution, 3PLM/2PLM/GRM Model Combination*

Sample Size	Test Length	Model	<i>RISE</i> (0.05)	G^2 (0.05)	$S-X^2$ (0.05)	<i>RISE</i> (0.01)	G^2 (0.01)	$S-X^2$ (0.01)
500	30	3PLM	0.031	0.063	0.022	0.011	0.016	0.008
		2PLM	0.041	0.065	0.010	0.006	0.040	0.004
		GRM	0.032	0.078	0.009	0.010	0.018	0.001
	60	3PLM	0.043	0.083	0.013	0.005	0.038	0.002
		2PLM	0.045	0.085	0.003	0.008	0.038	0.000
		GRM	0.036	0.064	0.001	0.007	0.013	0.000
2,000	30	3PLM	0.041	0.076	0.037	0.000	0.021	0.011
		2PLM	0.047	0.063	0.025	0.012	0.010	0.009
		GRM	0.040	0.420	0.016	0.009	0.185	0.005
	60	3PLM	0.039	0.064	0.018	0.007	0.016	0.006
		2PLM	0.050	0.073	0.011	0.010	0.009	0.002
		GRM	0.032	0.398	0.007	0.000	0.176	0.000
10,000	30	3PLM	0.048	0.484	0.043	0.004	0.241	0.013
		2PLM	0.046	0.843	0.051	0.002	0.618	0.010
		GRM	0.053	1.000	0.064	0.005	1.000	0.008
	60	3PLM	0.042	0.293	0.039	0.006	0.114	0.007
		2PLM	0.052	0.489	0.040	0.001	0.223	0.004
		GRM	0.044	1.000	0.050	0.005	1.000	0.001

Table 4.3 *Empirical Detection Rate for RISE, PARSCALE's G^2 and IRTFIT's $S-X^2$ Statistic, Normal Ability Distribution, 3PLM/2PLM/GPCM Model Combination*

Sample Size	Test Length	Model	<i>RISE</i> (0.05)	G^2 (0.05)	$S-X^2$ (0.05)	<i>RISE</i> (0.01)	G^2 (0.01)	$S-X^2$ (0.01)
500	30	3PLM	0.298	0.153	0.035	0.145	0.028	0.010
		2PLM	0.270	0.140	0.070	0.129	0.010	0.010
		GPCM	0.341	0.130	0.150	0.172	0.010	0.080
	60	3PLM	0.369	0.489	0.021	0.200	0.049	0.006
		2PLM	0.310	0.095	0.010	0.178	0.015	0.005
		GPCM	0.289	0.160	0.050	0.103	0.045	0.020
2,000	30	3PLM	0.596	0.288	0.253	0.371	0.138	0.103
		2PLM	0.892	0.290	0.400	0.406	0.070	0.180
		GPCM	0.910	0.780	0.830	0.790	0.560	0.680
	60	3PLM	0.612	0.636	0.053	0.322	0.419	0.093
		2PLM	0.785	0.330	0.185	0.555	0.135	0.085
		GPCM	0.873	0.685	0.445	0.626	0.440	0.275
10,000	30	3PLM	0.999	0.995	0.985	0.999	0.965	0.941
		2PLM	1.000	1.000	1.000	1.000	0.980	1.000
		GPCM	1.000	1.000	1.000	1.000	1.000	1.000
	60	3PLM	0.998	0.990	0.969	0.960	0.976	0.938
		2PLM	1.000	1.000	1.000	1.000	0.995	0.980
		GPCM	1.000	1.000	1.000	1.000	1.000	1.000

X-ed out cells represent conditions in which the test statistic exhibited inflated Type I error rates.

Table 4.4 *Empirical Detection Rate for RISE, PARSCALE's G^2 and IRTFIT's $S-X^2$ Statistic, Normal Ability Distribution, 3PLM/2PLM/GRM Model Combination*

Sample Size	Test Length	Model	<i>RISE</i> (0.05)	G^2 (0.05)	$S-X^2$ (0.05)	<i>RISE</i> (0.01)	G^2 (0.01)	$S-X^2$ (0.01)
500	30	3PLM	0.303	0.065	0.029	0.188	0.028	0.020
		2PLM	0.277	0.120	0.066	0.220	0.020	0.030
		GRM	0.346	0.110	0.143	0.272	0.010	0.100
	60	3PLM	0.374	0.019	0.022	0.240	0.074	0.003
		2PLM	0.337	0.080	0.011	0.208	0.025	0.007
		GRM	0.299	0.075	0.080	0.203	0.010	0.025
2,000	30	3PLM	0.636	0.178	0.257	0.471	0.063	0.106
		2PLM	0.899	0.240	0.523	0.506	0.080	0.189
		GRM	0.901	0.730	0.801	0.799	0.520	0.643
	60	3PLM	0.598	0.460	0.153	0.330	0.236	0.093
		2PLM	0.858	0.245	0.295	0.505	0.060	0.185
		GRM	0.871	0.575	0.445	0.626	0.285	0.294
10,000	30	3PLM	0.996	0.965	0.980	0.990	0.890	0.932
		2PLM	1.000	1.000	1.000	1.000	0.990	1.000
		GRM	1.000	1.000	1.000	1.000	1.000	1.000
	60	3PLM	0.998	0.991	0.969	0.960	0.974	0.938
		2PLM	1.000	1.000	1.000	1.000	1.000	1.000
		GRM	1.000	1.000	0.986	0.934	1.000	0.934

X-ed out cells represent conditions in which the test statistic exhibited inflated Type I error rates.

Table 4.5 *Empirical Type I Error Rate for RISE, PARSCALE's G^2 and IRTFIT's $S-X^2$ Statistic, Negatively-Skewed Ability Distribution, 3PLM/2PLM/GPCM Model Combination*

Sample Size	Test Length	Model	<i>RISE</i> (0.05)	G^2 (0.05)	$S-X^2$ (0.05)	<i>RISE</i> (0.01)	G^2 (0.01)	$S-X^2$ (0.01)
500	30	3PLM	0.029	0.070	0.194	0.000	0.017	0.057
		2PLM	0.031	0.073	0.273	0.011	0.005	0.100
		GPCM	0.038	0.100	0.980	0.010	0.040	0.870
	60	3PLM	0.033	0.082	0.031	0.008	0.020	0.005
		2PLM	0.045	0.114	0.015	0.009	0.020	0.003
		GPCM	0.030	0.168	0.035	0.007	0.068	0.014
2,000	30	3PLM	0.040	0.126	0.807	0.001	0.038	0.654
		2PLM	0.041	0.143	0.988	0.000	0.043	0.908
		GPCM	0.047	0.638	1.000	0.004	0.423	1.000
	60	3PLM	0.035	0.093	0.284	0.003	0.030	0.137
		2PLM	0.049	0.108	0.460	0.010	0.016	0.361
		GPCM	0.038	0.403	0.981	0.002	0.213	0.891
10,000	30	3PLM	0.046	0.584	0.995	0.005	0.393	0.986
		2PLM	0.042	0.855	1.000	0.011	0.638	1.000
		GPCM	0.050	1.000	1.000	0.012	1.000	1.000
	60	3PLM	0.041	0.309	0.943	0.006	0.157	0.866
		2PLM	0.053	0.619	1.000	0.010	0.365	1.000
		GPCM	0.044	0.990	1.000	0.005	0.974	1.000

Table 4.6 *Empirical Type I Error Rate for RISE, PARSCALE's G^2 and IRTFIT's $S-X^2$ Statistic, Negatively-Skewed Ability Distribution, 3PLM/2PLM/GRM Model Combination*

Sample Size	Test Length	Model	<i>RISE</i> (0.05)	G^2 (0.05)	$S-X^2$ (0.05)	<i>RISE</i> (0.01)	G^2 (0.01)	$S-X^2$ (0.01)
500	30	3PLM	0.043	0.071	0.201	0.008	0.011	0.101
		2PLM	0.039	0.075	0.236	0.012	0.018	0.115
		GRM	0.042	0.090	0.889	0.005	0.010	0.745
	60	3PLM	0.030	0.074	0.042	0.007	0.018	0.012
		2PLM	0.047	0.074	0.027	0.001	0.021	0.014
		GRM	0.026	0.103	0.045	0.004	0.025	0.010
2,000	30	3PLM	0.035	0.152	0.798	0.000	0.061	0.545
		2PLM	0.049	0.180	0.899	0.000	0.063	0.677
		GRM	0.047	0.420	0.936	0.010	0.180	0.821
	60	3PLM	0.046	0.096	0.275	0.006	0.033	0.121
		2PLM	0.051	0.106	0.303	0.005	0.031	0.190
		GRM	0.048	0.331	0.875	0.002	0.123	0.744
10,000	30	3PLM	0.029	0.789	0.991	0.004	0.633	0.936
		2PLM	0.042	0.990	0.990	0.013	0.960	0.990
		GRM	0.053	1.000	0.998	0.012	1.000	0.975
	60	3PLM	0.035	0.377	0.967	0.007	0.197	0.860
		2PLM	0.050	0.748	0.987	0.009	0.454	0.900
		GRM	0.047	1.000	0.997	0.006	1.000	0.997

Table 4.7 *Empirical Detection Rate for RISE, PARSCALE's G^2 and IRTFIT's $S-X^2$ Statistic, Negatively-Skewed Ability Distribution, 3PLM/2PLM/GPCM Model Combination*

Sample Size	Test Length	Model	<i>RISE</i> (0.05)	G^2 (0.05)	$S-X^2$ (0.05)	<i>RISE</i> (0.01)	G^2 (0.01)	$S-X^2$ (0.01)
500	30	3PLM	0.410	0.190	0.235	0.258	0.118	0.088
		2PLM	0.367	0.050	0.200	0.230	0.000	0.080
		GPCM	0.446	0.120	0.730	0.372	0.020	0.053
	60	3PLM	0.394	0.178	0.173	0.390	0.255	0.099
		2PLM	0.327	0.195	0.060	0.308	0.065	0.085
		GPCM	0.289	0.100	0.055	0.203	0.050	0.030
2,000	30	3PLM	0.696	0.445	0.638	0.571	0.210	0.458
		2PLM	0.899	0.100	0.970	0.508	0.020	0.870
		GPCM	0.903	0.700	1.000	0.801	0.530	1.000
	60	3PLM	0.798	0.695	0.704	0.730	0.516	0.563
		2PLM	0.888	0.225	0.330	0.624	0.080	0.225
		GPCM	0.801	0.695	1.000	0.613	0.430	0.980
10,000	30	3PLM	0.987	0.908	0.973	0.900	0.899	0.943
		2PLM	0.934	0.860	1.000	0.911	0.660	1.000
		GPCM	1.000	1.000	1.000	1.000	1.000	1.000
	60	3PLM	0.998	0.999	0.964	0.960	0.999	0.945
		2PLM	0.867	0.565	0.585	0.802	0.450	0.520
		GPCM	1.000	1.000	1.000	0.925	1.000	1.000

X-ed out cells represent conditions in which the test statistic exhibited inflated Type I error rates.

Table 4.8 *Empirical Detection Rate for RISE, PARSCALE's G^2 and IRTFIT's $S-X^2$ Statistic, Negatively-Skewed Ability Distribution, 3PLM/2PLM/GRM Model Combination*

Sample Size	Test Length	Model	<i>RISE</i> (0.05)	G^2 (0.05)	$S-X^2$ (0.05)	<i>RISE</i> (0.01)	G^2 (0.01)	$S-X^2$ (0.01)
500	30	3PLM	0.387	0.080	0.214	0.385	0.043	0.100
		2PLM	0.567	0.110	0.200	0.303	0.020	0.108
		GRM	0.546	0.110	0.719	0.327	0.030	0.374
	60	3PLM	0.574	0.254	0.144	0.409	0.200	0.109
		2PLM	0.627	0.155	0.105	0.391	0.065	0.085
		GRM	0.689	0.545	0.090	0.602	0.505	0.060
2,000	30	3PLM	0.769	0.255	0.583	0.517	0.113	0.428
		2PLM	0.899	0.150	0.896	0.680	0.030	0.863
		GRM	0.924	0.990	0.998	0.801	0.860	0.975
	60	3PLM	0.888	0.529	0.478	0.703	0.339	0.336
		2PLM	0.765	0.220	0.560	0.602	0.140	0.257
		GRM	0.910	0.930	0.877	0.747	0.825	0.819
10,000	30	3PLM	0.991	0.998	0.995	0.847	0.995	0.920
		2PLM	0.920	0.980	0.999	0.811	0.920	0.898
		GRM	0.998	1.000	0.990	0.998	1.000	0.887
	60	3PLM	0.998	0.999	0.987	0.940	0.995	0.865
		2PLM	0.864	0.505	0.880	0.720	0.380	0.534
		GRM	1.000	1.000	0.911	0.931	1.000	0.904

X-ed out cells represent conditions in which the test statistic exhibited inflated Type I error rates.

Table 4.9 *Empirical Type I Error Rate for RISE, PARSCALE's G^2 and IRTFIT's $S-X^2$ Statistic, Positively-Skewed Ability Distribution, 3PLM/2PLM/GPCM Model Combination*

Sample Size	Test Length	Model	<i>RISE</i> (0.05)	G^2 (0.05)	$S-X^2$ (0.05)	<i>RISE</i> (0.01)	G^2 (0.01)	$S-X^2$ (0.01)
500	30	3PLM	0.034	0.072	0.255	0.013	0.291	0.105
		2PLM	0.042	0.086	0.360	0.015	0.248	0.158
		GPCM	0.041	0.095	0.830	0.007	0.038	0.673
	60	3PLM	0.039	0.278	0.041	0.008	0.218	0.014
		2PLM	0.045	0.179	0.027	0.007	0.188	0.005
		GPCM	0.048	0.369	0.211	0.001	0.210	0.090
2,000	30	3PLM	0.029	0.089	0.955	0.000	0.016	0.868
		2PLM	0.032	0.080	0.963	0.009	0.015	0.903
		GPCM	0.028	0.480	0.990	0.011	0.298	0.990
	60	3PLM	0.033	0.093	0.393	0.003	0.025	0.184
		2PLM	0.040	0.125	0.530	0.004	0.043	0.281
		GPCM	0.037	0.454	0.884	0.002	0.330	0.790
10,000	30	3PLM	0.050	0.653	0.990	0.006	0.434	0.990
		2PLM	0.045	0.803	0.990	0.002	0.743	0.990
		GPCM	0.045	1.000	0.990	0.012	0.998	0.990
	60	3PLM	0.051	0.632	0.988	0.010	0.445	0.973
		2PLM	0.053	0.750	0.990	0.014	0.609	0.986
		GPCM	0.036	0.980	0.990	0.005	0.950	0.990

Table 4.10 *Empirical Type I Error Rate for RISE, PARSCALE's G^2 and IRTFIT's $S-X^2$ Statistic, Positively-Skewed Ability Distribution, 3PLM/2PLM/GRM Model Combination*

Sample Size	Test Length	Model	<i>RISE</i> (0.05)	G^2 (0.05)	$S-X^2$ (0.05)	<i>RISE</i> (0.01)	G^2 (0.01)	$S-X^2$ (0.01)
500	30	3PLM	0.037	0.079	0.254	0.011	0.021	0.111
		2PLM	0.042	0.081	0.348	0.009	0.040	0.160
		GRM	0.039	0.096	0.720	0.008	0.032	0.668
	60	3PLM	0.035	0.078	0.053	0.012	0.025	0.029
		2PLM	0.042	0.074	0.042	0.004	0.021	0.007
		GRM	0.031	0.108	0.198	0.009	0.029	0.099
2,000	30	3PLM	0.039	0.155	0.897	0.002	0.078	0.756
		2PLM	0.045	0.177	0.906	0.012	0.084	0.880
		GRM	0.047	0.431	0.989	0.008	0.191	0.937
	60	3PLM	0.048	0.106	0.476	0.011	0.037	0.281
		2PLM	0.049	0.098	0.501	0.010	0.032	0.384
		GRM	0.050	0.344	0.834	0.007	0.145	0.795
10,000	30	3PLM	0.030	0.798	0.997	0.008	0.679	0.875
		2PLM	0.041	0.992	0.997	0.009	0.961	0.834
		GRM	0.054	0.998	0.991	0.015	0.906	0.821
	60	3PLM	0.039	0.434	0.974	0.008	0.302	0.924
		2PLM	0.049	0.876	0.899	0.010	0.547	0.826
		GRM	0.048	1.000	0.990	0.007	1.000	0.900

Table 4.11 *Empirical Detection Rate for RISE, PARSCALE's G^2 and IRTFIT's $S-X^2$ Statistic, Positively-Skewed Ability Distribution, 3PLM/2PLM/GPCM Model Combination*

Sample Size	Test Length	Model	<i>RISE</i> (0.05)	G^2 (0.05)	$S-X^2$ (0.05)	<i>RISE</i> (0.01)	G^2 (0.01)	$S-X^2$ (0.01)
500	30	3PLM	0.576	0.250	0.145	0.385	0.735	0.083
		2PLM	0.620	0.250	0.440	0.260	0.940	0.150
		GPCM	0.602	0.120	0.140	0.345	0.010	0.040
	60	3PLM	0.488	0.255	0.191	0.390	0.603	0.096
		2PLM	0.593	0.305	0.060	0.347	0.220	0.050
		GPCM	0.607	0.185	0.140	0.500	0.065	0.045
2,000	30	3PLM	0.766	0.273	0.620	0.508	0.253	0.463
		2PLM	0.903	0.280	0.980	0.730	0.040	0.980
		GPCM	0.917	0.580	0.970	0.824	0.390	0.900
	60	3PLM	0.769	0.274	0.308	0.533	0.198	0.230
		2PLM	0.821	0.335	0.610	0.706	0.040	0.330
		GPCM	0.905	0.310	0.540	0.741	0.100	0.275
10,000	30	3PLM	0.978	0.405	0.985	0.902	0.263	0.965
		2PLM	0.946	0.800	0.990	0.918	0.650	0.990
		GPCM	0.999	1.000	0.990	0.899	1.000	0.990
	60	3PLM	0.998	0.518	0.849	0.970	0.383	0.766
		2PLM	0.905	0.895	0.990	0.862	0.795	0.990
		GPCM	1.000	1.000	0.990	0.956	0.985	0.990

X-ed out cells represent conditions in which the test statistic exhibited inflated Type I error rates.

Table 4.12 *Empirical Detection Rate for RISE, PARSCALE's G^2 and IRTFIT's $S-X^2$ Statistic, Positively-Skewed Ability Distribution, 3PLM/2PLM/GRM Model Combination*

Sample Size	Test Length	Model	<i>RISE</i> (0.05)	G^2 (0.05)	$S-X^2$ (0.05)	<i>RISE</i> (0.01)	G^2 (0.01)	$S-X^2$ (0.01)
500	30	3PLM	0.478	0.087	0.219	0.365	0.057	0.108
		2PLM	0.676	0.102	0.340	0.404	0.078	0.127
		GRM	0.656	0.097	0.178	0.487	0.089	0.135
	60	3PLM	0.669	0.205	0.119	0.490	0.199	0.106
		2PLM	0.624	0.159	0.080	0.320	0.107	0.040
		GRM	0.599	0.578	0.145	0.371	0.505	0.055
2,000	30	3PLM	0.788	0.267	0.637	0.574	0.131	0.437
		2PLM	0.901	0.234	0.782	0.668	0.097	0.780
		GRM	0.925	0.879	0.896	0.812	0.765	0.700
	60	3PLM	0.812	0.433	0.408	0.730	0.324	0.303
		2PLM	0.795	0.395	0.511	0.649	0.230	0.430
		GRM	0.901	0.901	0.593	0.751	0.825	0.457
10,000	30	3PLM	0.997	0.999	0.944	0.889	0.995	0.906
		2PLM	0.984	0.990	0.877	0.810	0.943	0.789
		GRM	0.995	0.995	0.991	0.990	0.887	0.990
	60	3PLM	0.998	0.978	0.903	0.940	0.902	0.742
		2PLM	0.900	0.934	0.866	0.831	0.877	0.735
		GRM	0.999	1.000	0.875	0.939	0.990	0.802

X-ed out cells represent conditions in which the test statistic exhibited inflated Type I error rates.

Table 4.13 *The Difference of The Difference of Score Distributions Before and After Replacement in Simulation Study*

Cuts	$RISE$ (0.005)	G^2 (0.005)	$S-X^2$ (0.005)
40	0.002	0.001	0.001
60	0.001	0.000	0.001
80	0.001	0.001	0.000

Table 4.14 *The Difference of The Difference of TCCs Before and After Replacement in Simulation Study*

Cuts	<i>RISE</i> (0.5)
-1.41	0.421
-0.33	0.497
1.11	0.499

Table 4.15 *Misfitting Items Detected by Each Fit Statistic at Each Sample Size in Empirical Study*

Sample Size	Fit Statistics	Number	Percentage	Item ID
500	<i>RISE</i>	1	2%	18
	G^2	5	12%	12,16,17,23,30
	$S-X^2$	0	0%	No Items Flagged
2,000	<i>RISE</i>	3	7%	12,18,37
	G^2	8	19%	2,5,10,14,16,37,38,40
	$S-X^2$	1	2%	37
10,000	<i>RISE</i>	7	17%	12,16,18,23,30,37,40
	G^2	40	95%	All Items Except 1 and 15
	$S-X^2$	8	19%	12,16,18,23,28,30,37,40

Table 4.16 *The Difference of The Difference of Score Distributions Before and After Replacement in Empirical Study*

Sample Size	Cuts	<i>RISE</i> (0.005)	G^2 (0.005)	$S-X^2$ (0.005)
500	25	0.003	0.002	
	40	0.002	0.002	
	55	0.003	0.004	
2,000	25	0.003	0.001	0.001
	40	0.003	0.001	0.001
	55	0.002	0.000	0.000
10,000	25	0.003	0.004	0.003
	40	0.003	0.003	0.003
	55	0.004	0.005	0.003

X-ed out cells represent conditions in which no misfitting items were detected.

Figure 4.1. Score Distribution Before and After Replacement of Misfitting Items Detected by RISE.

Figure 4.2. Score Distribution Before and After Replacement of Misfitting Items Detected by G^2 .

Figure 4.3. Score Distribution Before and After Replacement of Misfitting Items Detected by $S-X^2$.

Figure 4.4. TCCs Before and After Replacement of Misfitting Items Detected by *RISE*.

Figure 4.5. Misfitting Items Detected by *RISE* at Sample Size 500.

Figure 4.6. Misfitting Items Detected by G^2 at Sample Size 500.

Figure 4.7. Misfitting Items Detected by *RISE* at Sample Size 2,000.

Figure 4.8. Misfitting Items Detected by G^2 at Sample Size 2,000.

Figure 4.9. Misfitting Items Detected by $S-X^2$ at Sample Size 2,000.

Figure 4.10. Misfitting Items Detected by *RISE* at Sample Size 10,000.

Figure 4.11. Misfitting Items Detected by G^2 at Sample Size 10,000.

Figure 4.12. Misfitting Items Detected by $S-X^2$ at Sample Size 10,000.

Figure 4.13. Score Distribution Before and After Replacement of Misfitting Items Detected by Three Fit Statistics at Sample Size 500.

RISE

G^2

$S-X^2$ (no replacement)

Figure 4.14. Score Distribution Before and After Replacement of Misfitting Items Detected by Three Fit Statistics at Sample Size 2,000.

RISE

G^2

$S-X^2$

Figure 4.15. Score Distribution Before and After Replacement of Misfitting Items Detected by Three Fit Statistics at Sample Size 10,000.
RISE

CHAPTER 5

CONCLUSION

Model fit is an important aspect in IRT in that misfit between an IRT model and data can undermine the invariance property which is key to all IRT applications such as IRT equating, differential item functioning and computerized adaptive testing. Given the importance of model fit in IRT, the lack of research investigating the influence of misfit as well as the methods to detect misfit, is surprising. Several of the currently available fit statistics (e.g., G^2 and Yen's Q_1) are unappealing due to inflated Type I error rates and their insensitivity to the location and magnitude of the misfit. In addition, there has been little research on evaluation of model misfit consequences although this topic should be considered seriously in deciding the use of a model in particular situations.

The purpose of this dissertation was to address the above-mentioned two issues in model data fit evaluation procedures. In the aspect of investigating goodness-of-fit statistics, the nonparametric approach was extended and compared to two other commonly used and well-accepted fit statistics through statistical properties and graphical displays. Non-inflated Type I error rates are important since it is vital to retain valid items since item development is often expensive. In addition, adequate power to identify misfitting items is also important since IRT applications rely on the model fitting the data. Graphical displays, although they suffer from some degree of subjective interpretation, can often provide meaningful insights into the nature of model-data misfit and assist in interpreting the meaningfulness of an effect from a statistical significance test.

The newly-proposed nonparametric approach, *RISE*, overcomes the main problems of traditional fit statistics such as arbitrariness of grouping and inaccuracy of

the null distribution assumption. Essentially, the idea of *RISE* is based on the weak assumption proposed by Douglas and Cohen (2001) that for moderate sample sizes, nonparametrically-derived ICC will be close to the parametric ICC. The significance level of *RISE* is determined by the established parametric bootstrapping procedure which forms an empirical distribution of the difference between the nonparametric ICC and parametric ICC. Beyond its statistical properties mentioned above, an additional advantage of the nonparametric approach is the convenient graphical representation of model misfit. This feature can provide the researcher or practitioner with visual guidance about the location and type of misfit. On the other hand, in the aspect of examining model misfit consequences, both the test score distribution and the TCC were used to address this issue. Basically, misfitting items detected by three fit statistics were replaced by fitting items to see the change of score distribution or TCC. If the change was not significant for classification, the conclusion was that there was no practical consequence for the given data and condition.

In the dissertation, Study 1 was a simulation study to examine the statistical properties of *RISE* thoroughly and comprehensively. Ability distribution, sample size, test length and model were four factors considered as they may influence performance of fit. Among the thirty-six conditions from the four factors, *RISE* outperformed G^2 and $S-X^2$ in that it controlled Type I error rates and provided adequate power. However, G^2 exhibited different degrees of inflated Type I error rates on all conditions while less power than *RISE*. $S-X^2$ did not control Type I error rates for the conditions involved in skewed ability distributions except the 500-examinee and 60-item condition, which may be due to the requirement of distribution assumption of $S-X^2$ and thus the limited use of the software to compute $S-X^2$. However, it did exhibit controlled Type I error rates for the conditions under normal ability distributions although the power was much

lower than *RISE* especially for small and medium sample sizes. In the simulation study, model misfit consequences on the score distribution and TCC were also examined. The score distribution and TCC did not change significantly before and after replacement of the misfitting items. This result is probably due to the small amount of misfit simulated at the test score level.

In Study 2, the three fit statistics were applied to empirical data for three sample sizes. The number of misfitting items detected increased by sample size among all three fit statistics. *RISE* and $S-X^2$ detected comparable number of misfitting items while G^2 detected a lot (e.g., at sample size 10,000, G^2 detected 95% items as misfitting items).

Graphical displays were implemented in this study to help judge the accuracy of detection. One advantage of the nonparametric approach is that the difference between the nonparametric and parametric ICCs provides evidence regarding the location and magnitude of misfit of each detected item. One phenomenon for the parametric approach, especially G^2 , was that the reasonably good items shown by the residual plots were still detected by the fit statistic as misfitting items. Model misfit consequence on score distributions with small, medium and large sample sizes and real data were also investigated. The conclusion was the same as the simulation study. However, the results demonstrated the capability of *RISE* in the accuracy of misfit item detection.

Although the results in the dissertation provided strong evidence that the proposed *RISE* is a convincing fit statistic for the studied conditions, further exploration is still necessary to understand the nonparametric approach in depth. For instance, other smoothing techniques to produce the nonparametric ICC can be implemented to compare with kernel smoothed ICCs. This may influence the fitting ICC and thus the whole *RISE* procedure. It may also be worthwhile to examine the statistical properties for other models. In the current study, only four models, 2PLM, 3PLM, GRM and

GPCM were applied. In future research, the 1PLM/PCM combination may be of interest since it is also commonly used in some testing programs (e.g., state achievement testing and the health field). Furthermore, it may be useful to explore the performance between a variety of parameter estimation methods in obtaining the optimal item parameter values as the robustness of item parameter estimation techniques is desired in the nonparametric approach. Other than parameter estimation techniques, the number of evaluation points may also influence the parameter estimation procedure, so future research will include this factor. In addition, model misfit consequences on score distributions can be further assessed by simulating large amount of misfit which is expected to make a difference. Also, other aspects of misfit consequence on the invariance property and parameter estimates can be conducted. Finally, it would be worthwhile to compare *RISE* with newly emerging fit statistics (e.g., Bayesian approach) as according to the research literature, Bayesian approach has also exhibited good performance (Sinharay, 2005, Stone, 2000a).

While it is unreasonable to expect IRT models to provide perfect fit to the test data, the evidence is substantial that they provide an excellent framework for solving measurement problems. Model-data fit is a systematic and comprehensive procedure (see Hambleton & Han, 2005). Although it is clear from this study that the nonparametric approach is promising to identify misfitting items and provide the graphical representation of misfit, more and more problems may still be ahead and have not been solved. For example, as IRT has been widely accepted and applied in variety of fields, more and more practical problems such as how to measure and understand model misfit consequence are challenging. All in all, the new method is expected to bring a new direction for improving the utility of IRT modeling for measurement problem-solving in educational and psychological testing.

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