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# Growth Cycles With or Without Price Flexibility

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# DEPARTMENT OF ECONOMICS

## Working Paper

### **Growth Cycles with or without price flexibility**

by

Peter Skott

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**UNIVERSITY OF MASSACHUSETTS  
AMHERST**

# Growth cycles with or without price flexibility\*

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## Abstract

This note – written in response to von Arnim and Barrales (2015) – shows that (i) the Kaldor-Goodwin models in Skott (1989a, 1989b) and Skott and Zipperer (2012) provide good approximations to models with fast but finite adjustment of prices, (ii) the models can generate cyclical patterns that match the stylized facts, and (iii) an alternative model with instantaneous output adjustment and fixed prices produces a dynamic system that is virtually identical to the Kaldor-Goodwin; this model may describe parts of the service sector.

JEL codes: E12, E32

Key words: Endogenous cycles, Harroddian instability, price flexibility, rationing, labor hoarding, behavioral foundations.

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# 1 Introduction

The main thrust of my argument in Skott (1989a, 1989b) was that (i) Keynesian short-run equilibria (in which short-term expectations are taken to be met) should not be used as the basic building blocks for a dynamic theory; expectations must be disappointed in some short periods unless the economy follows a warranted growth path, (ii) there are good reasons to take seriously the possibility of Harrodian instability, (iii) local instability is perfectly compatible with bounded fluctuations; it can lead to an integrated story of growth and cycles, (iv) a stabilizing influence can come from a Marxian reserve-army mechanism, and (v) the reserve-army mechanism is empirically relevant in most advanced countries today; the state of the labor market (the employment rate) influences firms' production and investment decisions.

von Arnim and Barrales (2015) [AB] take a similar approach. The possibility of Harrodian instability is emphasized; their models include an 'output expansion function' that is almost identical to the one in Skott (1989a, 1989b); the employment rate is seen as a key element in the determination of output growth and accumulation. This is all common ground. In fact, AB's analysis is much closer to mine than to the benchmark post-Kaleckian growth model with its perfectly elastic labor supply, a focus on steady growth paths, and an assumption that these growth paths are stable.

In this note I focus on three issues, leaving aside minor points of disagreement. I shall consider, first, the question of price flexibility. AB's main criticism of my approach is, I believe, unfounded. The cyclical patterns produced by a Kaldor-Goodwin approach with perfect price flexibility approximate the patterns generated by an extended model with high but finite adjustment speeds. Since the perfect-flexibility version can reproduce the stylized patterns (as shown by Skott and Zipperer), the same will apply to versions with fast price adjustment. Ironically, matching the observed patterns in a way that is behaviorally plausible is likely to be much more thorny for AB's preferred approach.

Second, some of the main differences between AB and myself may be methodological. AB appear to downplay the role of 'behavioral analysis' and instead emphasize the importance of stylized facts. Their main criticism of my formulation is the alleged inability to reproduce observed cyclical patterns, and other comments throughout the paper suggest doubts about the usefulness of analyzing the microeconomic motivations behind macroeconomic relations. These doubts are widespread among heterodox economists; in my view, however, they are mistaken.

AB's criticism of the flex-price assumption and its implications, third, may be unwarranted. But the Marshallian structure of the Kaldor-Goodwin model can be questioned from another angle: in a service economy output could arguably be more flexible than prices. AB do not raise this issue and their model, like the Kaldor-Goodwin model, treats output as a state variable; it was brought up by Steve Fazzari when I presented the Skott-Zipperer paper at a conference in Berlin a few years ago. Surprisingly, perhaps, an economy with perfect output flexibility and fixed prices have dynamic properties that are quite similar to

those of the Kaldor-Goodwin model.

## 2 Price flexibility

AB's main critique of my formulation is that prices are taken to be perfectly flexible and that a model in which prices and profitability adjust to excess demand in the goods market cannot reproduce the observed cyclical patterns. As shown in Skott and Zipperer (2012) the latter criticism is invalid in the case of perfect flexibility; AB, however, claim that without fast adjustment the model must generate a "wrong  $u, \pi$  cycle" (p.10). The perfect flexibility assumption is clearly an approximation. If even the slightest deviation from perfect flexibility generates qualitatively different outcomes, a perfect-flexibility model therefore will command little interest.

To substantiate their claim AB set up a simple model to analyze the case with finite adjustment speeds. The model, however, leaves out the employment rate and obscures the relation between output growth and the state of the product market. It may therefore be useful to examine the connection between perfect and finite adjustment speeds in more detail.

The level of output is predetermined in both the Kaldor-Goodwin model and AB's neo-Kaleckian model. Past production decisions were governed by demand expectations, and something has to give if these demand expectations fail to be met. Assuming that output cannot adjust instantaneously, there are three possibilities: the ex-post equality between investment and saving can be brought about by (i) direct rationing, (ii) unplanned changes in inventories, or (iii) changes in prices that affect excess demand.

I have made a case for the price mechanism. The argument is partly by elimination: rationing occurs but does not seem empirically important; inventories change over the cycle, but the changes are largely pro-cyclical which suggests that this is not the main adjustment.<sup>1</sup> Direct evidence also suggests that prices are much more flexible than commonly believed. At an anecdotal level, the prices I pay for butter, gas or airline tickets seem to change every week. More systematic evidence points in the same direction,<sup>2</sup> and I would still defend the assumption as a good approximation (with the caveat discussed below in section 4). The argument, however, is largely empirical, and my position may turn out to be wrong.<sup>3</sup> It is important therefore to stress that price flexibility is not central to the main story. It affects the movements in income distribution, but not the local instability and the cyclical patterns in employment and utilization.

The basic claim is that output growth depends on the degree of disequilibrium in the goods market: the level of output is predetermined, and a positive demand shock raises the growth rate of output. *If* there is perfect price flexibility the profit share can serve as an indicator of excess demand, as in Keynes

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<sup>1</sup>See Chiarella et al. (2005) for a model that includes inventory dynamics.

<sup>2</sup>E.g., Abe and Tonogi (2010).

<sup>3</sup>The price channel also seems interesting for theoretical reasons, as mentioned by AB: with perfect price flexibility, unemployment and instability cannot be blamed on rigid prices.

(1930). The growth of output therefore becomes a function of the profit share (an indicator of disequilibrium in the goods market) and the employment rate (an indicator of the state of the labor market).<sup>4</sup> This specification, however, is contingent on prices being perfectly flexible. If the assumption is changed – if, for instance, disequilibrium takes the form of direct rationing – the profit share cannot be used as an indicator of disequilibrium in the goods market, and the ‘output expansion function’ must be modified.

Skott (1988, 1989a, 1989b) analyze extreme cases with a fixed markup and perfect price flexibility. Now consider an intermediate case with fast but finite adjustment speeds for prices. By assumption output growth is determined by the disequilibrium in the goods market and the state of the labor market:

$$\hat{Y} = h(\Delta, e), \quad h_1 > 0, h_2 < 0 \quad (1)$$

where  $\Delta$  is a measure of the degree of excess demand,  $e$  the employment rate, and a hat over a variable denotes a growth rate ( $\hat{Y} = (dY/dt)/Y$ ). For simplicity, ignore inventories and assume that disequilibrium is reflected in a combination of (i) a profit share that deviates from what firms would have chosen, had they had perfect foresight about the state of demand and (ii) direct rationing of household consumption, i.e. consumption is determined as the difference between output and desired investment. With these assumptions, the degree of excess demand ( $\Delta$ ) can be represented by

$$\Delta = \phi\left(\frac{I-S}{Y}, \pi\right), \quad \phi_1 > 0, \phi_2 > 0 \quad (2)$$

where the desired saving rate  $S/Y$  depends on the profit share. The degree of direct rationing is given by  $(I-S)/Y$  and  $\pi$  is the profit share.<sup>5</sup>

The investment function in Skott (1989a, 1989b) includes both the utilization rate and the profit share. The profit share appeared because it captured the state of the goods market. With slow price adjustment, the counterpart would be to include  $\Delta$  instead of  $\pi$ . For present purposes, however, this complication is irrelevant and we may simply leave out the second argument. Thus, assume that

$$\frac{I}{Y} = f(u) \quad (3)$$

$$\frac{S}{Y} = g(\pi) \quad (4)$$

where  $u = Y/K$  is an indicator of utilization.

<sup>4</sup>See Skott (1989b) for a detailed behavioral analysis.

<sup>5</sup>The value of the short-run equilibrium profit share may depend on employment. In this case a modified indicator could be used:

$$\Delta = \phi\left(\frac{I-S}{Y}, \pi, \pi^*\right), \quad \phi_1 > 0, \phi_2 > 0, \phi_3 < 0$$

where  $\pi^* = \xi(e)$  represents the profit share in short-run equilibrium. This extension is irrelevant for present purposes.

Substituting (2)-(4) into the output expansion function (1), we have<sup>6</sup>

$$\hat{Y} = h(\phi(f(u) - g(\pi), \pi), e) \quad (5)$$

The degree of disequilibrium – the deviation of actual demand curves from the demand curves that would have justified firms' current levels of output – does not depend on whether firms choose to respond to the disequilibrium by changing prices or by rationing. Putting it differently, for a given level of output, an increase in the price of goods may change the degree of quantity rationing but not the underlying disequilibrium. Using (2)-(4), this independence property implies that the excess demand indicator satisfies the condition<sup>7</sup>

$$\frac{d\phi}{d\pi} = -\phi_1 g' + \phi_2 = 0 \quad \text{for all } u, \pi, e \quad (6)$$

Assuming that the labor force in efficiency units grows at the rate  $n$  and ignoring depreciation, the dynamics for utilization and employment are given by

$$\hat{u} = \hat{Y} - \hat{K} = h(\phi(f(u) - g(\pi), \pi), e) - uf(u) \quad (7)$$

$$\hat{e} = h(\phi(f(u) - g(\pi), \pi), e) - n \quad (8)$$

With finite adjustment speeds for prices, a dynamic equation for the profit share can now be added,

$$\dot{\pi} = \mu \frac{I - S}{Y} = \mu[f(u) - g(\pi)] \quad (9)$$

Equations (7)-(9) describe a 3D system of differential equations. The Jacobian of the system can be written

$$J(u, e, \pi) = \begin{pmatrix} h_1 \phi_1 f' - uf' - f & h_e & h_1 \phi_2 - h_1 \phi_1 g' \\ h_1 \phi_1 f' & h_e & h_1 \phi_2 - h_1 \phi_1 g' \\ \mu f' & 0 & -\mu g' \end{pmatrix} \quad (10)$$

Using (6), the Jacobian matrix simplifies to

$$J(u, e, \pi) = \begin{pmatrix} h_1 \phi_1 f' - uf' - f & h_e & 0 \\ h_1 \phi_1 f' & h_e & 0 \\ \mu f' & 0 & -\mu g' \end{pmatrix} \quad (11)$$

Thus, the condition (6) – and the fact that it holds for all values of  $u, e, \pi$  – implies that the 3D system becomes separable.

The self-contained 2D dynamics for  $(u, e)$  are given by

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<sup>6</sup>The terminology is not attractive and in later work I have sometimes referred to the function which describes output growth as simply 'the growth function'.

<sup>7</sup>The independence of the degree of disequilibrium of the price response implies that the disequilibrium can be written as a function of  $u$ . Thus, equation (6) holds for all values of the state variables, not just at stationary points.

$$J(u, e) = \begin{pmatrix} h_1\phi_1f' - uf' - f & h_e \\ h_1\phi_1f' & h_e \end{pmatrix} \quad (12)$$

The derivative  $h_1$  describes the sensitivity of output growth to a change in excess demand and  $\phi_1f'$  the sensitivity of excess demand to a rise in utilization. Neither of these derivatives depends on whether the disequilibrium adjustment is via prices or rationing.

Now consider the limiting case where  $\mu \rightarrow \infty$ .<sup>8</sup> The dynamics for  $\pi$  is stable, and if the adjustment speed goes to infinity, we get

$$\pi \rightarrow \theta(u) = g^{-1}(f(u)) \quad (13)$$

$$\Delta \rightarrow \phi(0, \theta(u)) \quad (14)$$

with

$$\theta' = \frac{f'}{g'} \quad (15)$$

Using (15) and (6), we have

$$h_1\phi_1f' = h_1\phi_2\frac{f'}{g'} = h_1\phi_2\theta' \quad (16)$$

and the Jacobian can now be written

$$J(u, e) = \begin{pmatrix} h_1\phi_2\theta' - uf' - f & h_e \\ h_1\phi_2\theta' & h_e \end{pmatrix} \quad (17)$$

In short, a move from perfect to finite price flexibility does not change the stability properties of the  $(u, e)$ -subsystem: the dynamics of the  $(u, e)$  system in (12) is independent of whether the adjustment is via prices or rationing.<sup>9</sup> Moreover, with fast price adjustment, the Jacobian converges to (17) which (with notational changes) is identical to the one in Skott (1989a, 1989b); fast but finite adjustment can be approximated by the perfect-flexibility system.

AB claim that the Kaldor-Goodwin model cannot generate the observed clockwise pattern in utilization-profit space. The claim is correct for the 2D model: assuming Harroddian instability, the baseline model of perfect price flexibility implies that the profit share is an increasing function of utilization; there are no cycles in  $(u, \pi)$ .<sup>10</sup> Empirically, however, there are lags in accumulation

<sup>8</sup>In the other extreme case – when  $\mu = 0$  – the dynamic equation for  $\pi$  simply drops out, and we are left with the 2D system (12).

<sup>9</sup>This invariance result is quite surprising. In general, adding an extra state variable to a 2D system does not lead to separability (as exemplified by a comparison between the 2D and 3D versions of the Barbosa-Filho and Taylor model). The separability in this particular case depends on the specification of accumulation in (3) as well as on condition (6). The system ceases to be separable if accumulation depends on the profit share for reasons that are unrelated to the degree of disequilibrium in the goods market.

<sup>10</sup>AB correctly point out that Harroddian instability is necessary for the relation to be increasing and the model to produce instability and cyclical fluctuations. They seem to consider this a weakness; I am not sure why. The dynamic properties of a system depend on the specification of the system, and Harroddian instability was introduced precisely because it is implied by a plausible specification of investment behavior.



– it takes a while to plan and implement increases in the capital stock – and as shown in Skott and Zipperer (2012), an extended model which treats the accumulation rate as a state variable can produce the observed clockwise cycles in  $u, \pi$ . The dimension of the system will increase to four if both the profit share and the accumulation rate are treated as state variables. But there is no reason to expect a discontinuity in the cyclical patterns as a result of this increase: if the 3D system with perfect price adjustment approximates the 4D system with fast but finite adjustment, the qualitative cyclical patterns will be preserved for sufficiently large adjustment speeds.

This ability to reproduce the observed patterns is achieved with simple specifications that were not designed specifically to match the patterns. If matching the patterns is the main concern, an approach based on price adjustments in response to goods market disequilibrium offers many possible, empirically justified modifications. The saving behavior, for instance, has been kept exceedingly simple. Saving is not fully determined by contemporaneous profits; saving out of wage income (whether positive or negative), lags in consumption (because of habit formation, say), or autonomous government spending will affect the equilibrium condition for the goods market and influence the patterns. Not all possible modifications will work in the 'right direction'. But it seems rather peculiar to dismiss a Kaldor-Goodwin approach on the basis that fast but finite adjustment speeds of prices may fail to generate the observed  $u, \pi$  pattern in some simple specifications. The dismissal is particularly strange if the proposed alternative has greater difficulties accounting for the  $u, \pi$  pattern.

AB's preferred model has nominal prices and wages adjust in response to utilization and employment rates. In order to obtain the observed patterns they assume that the wage share rises in response to an increase in capital utilization. But why would high utilization favor workers? Why would wages increase more than prices at high rate of capital utilization, *holding constant the employment rate*? AB point to a paper by Diallo et al. (2011) which suggests that variations in the nominal wage are strongly influenced by the bargaining of insiders. Accepting the role of insiders, however, it remains unclear why an increase in a firm's capital utilization rate would lead to an increase in the firm's product real wage, holding constant the variables that describe the employment rate and the utilization rate of labor.<sup>11</sup> High utilization rates of capital may lead to an increase in a firm's prices and profitability; as a result the firm's workers may gain bargaining power and see an increase in their nominal wage and consumption real wage (their nominal wage relative to the general price level). But it seems implausible to assume that an increase in a firm's utilization will squeeze its profit share

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<sup>11</sup>Diallo et al. allow labor utilization to vary. In the theoretical model, however, they assume that both the ratio of the labor force to the capital stock and the employment rate are constant. These assumptions are analogous to Barbosa-Filho and Taylor's (2006) assumption that the employment rate and capital utilization move together. There is no theoretical justification for the assumptions, and empirically they do not hold. The wage share, moreover, is given by the ratio of the real wage to the productivity of labor. The latter is proportional to the utilization rate of labor; the wage share therefore can be decreasing in utilization even if the real wage is increasing.

Based on AB's chosen criterion – explaining the tendency of profit shares to fall at high levels of utilization – an approach which links adjustments in prices and profit shares to excess demand in the goods market would seem more promising than arguments which emphasizes the effects of utilization and employment on bargaining power.

### 3 Behavioral analysis and stylized facts

As argued in section 2, AB's dismissal of the Kaldor-Goodwin approach on empirical grounds is unjustified. The basis for their dismissal, however, may be indicative of a broader, methodological disagreement.

The tenor of AB's discussion is to look for specifications that can reproduce the reduced-form patterns for utilization, employment and the profit share. The punchline in their section 3 is a simulation which shows that with suitable parameter values the neo-Kaleckian 3D "system can indeed generate the three stylized perpetual fluctuations" (p.20). Other indications can be found in footnote 12 which suggests that Skott "considers output growth, accumulation and price setting the rational decisions of a (representative) firm, whereas here these processes are viewed as the structural outcomes of the complex interplay of a multitude of agents" and in footnote 5 where Skott's equations are described as "truly behavioral"; by contrast the term 'behavioral' is put in inverted commas in the description of the equations in the extended Barbosa-Filho&Taylor model (p.19).

The relation between microeconomic behavior and macroeconomic relations is complex, both because of aggregation problems and because the macroeconomic environment may influence behavior at the micro level. It is also true that a purely theoretical analysis of plausible behavior is insufficient; often it will not even enable one to establish unambiguously the sign of a particular effect. Nevertheless, an exclusive focus on stylized facts carries with it significant dangers.

The Lucas-inspired revolution in macroeconomics has produced flawed models and bad policy. But the misguided way in which mainstream macroeconomics has tried to provide rigorous 'microeconomic foundations' does not justify a neglect of microeconomic behavior. A behavioral analysis can have real power (and can also inform empirical work). The specification of investment provides an example. Investment may be insensitive to variations in utilization rates in the short run, but minimally rational firms with an eye on profitability will react strongly to large and sustained movements in their utilization rates. Why invest if you already have plenty of unwanted excess capacity? The extension of the standard Keynesian stability condition to the long run therefore is hard to defend (Skott 2012).<sup>12</sup> At a general methodological level, it is dangerous to play down the motivations and constraints at the microeconomic level (see Skott 2014 for further discussion of these issues).

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<sup>12</sup>AB seem to agree with this point. Their model therefore is not Kaleckian in the sense that I have used the term.

Returning to the analysis of cyclical patterns, AB's analysis seems to be guided by the question, 'what sign patterns in the Jacobian can give us the observed pattern of correlations?'. It may be more fruitful to turn the question around and ask, 'what are the plausible behavioral relations and what are their implications for the cyclical patterns?'. This is the approach in Skott and Zipperer (2012). Building on earlier work, we examined different specifications econometrically. Our findings were consistent with the behavioral relations in the Kaldor-Goodwin specifications, and the cyclical implications matched the stylized patterns in the US data.<sup>13</sup>

## 4 Output flexibility

AB question the sensitivity of prices and profit shares to changes in demand. Disequilibrium also leads to 'forced saving' in their model, but the adjustment in saving happens through direct rationing: consumption is given by the difference between the predetermined level of output and the desired level of investment. This rationing assumption does not seem to match actual behavior. Is there any evidence that consumers as a whole curtail their consumption in times of high demand because the products they want are sold out? There are examples of individual goods that get rationed - flights can be full or there may be a wait list for a particularly popular new car - but generalized rationing of this kind is not a feature of market economies. Thus, it is implausible to suggest that direct rationing forces an increase in aggregate saving in cases of excess demand. Even more implausible would seem the notion that consumption automatically increases - without price movements - in cases of excess supply.

We are left with a problem: if we accept that there is little evidence of direct rationing and if, following AB, we dismiss price adjustment and leave out inventories, there would seem to be no adjustment mechanisms. This problem could be resolved if output were flexible.

The treatment of output as a predetermined state variable makes perfect sense if the model describes manufacturing or agricultural goods that cannot be produced instantaneously. It is more questionable for large parts of the service sector. The output of a hairdresser cannot be stored and the act of production cannot be separated from the act of consumption. This does not mean that there are no predetermined variables in the production of services. Cooks, hairdressers and retail workers have to be hired and they need kitchens, hair salons and shops to work in. Neither capital nor labor can be adjusted instantaneously. Employment and the stock of capital are predetermined at any moment but the utilization of labor and capital depends on the level of demand.<sup>14</sup>

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<sup>13</sup>Abe (2014) examines the Japanese case.

<sup>14</sup>For some segments of the workforce there is an important qualifier: an increasingly 'flexible' labor market in which workers are called in 'as needed' shifts the cost of underutilization from firms to workers.

Consider a simple 'flex-output economy'.<sup>15</sup> There is excess capacity of both labor and capital, and output adjusts instantaneously to the level of demand (within the limits of labor and capital capacity). Using a Leontief production function, the capacity constraints are given by

$$Y \leq \min\{\lambda L, \sigma K\}$$

Short-run demand expectations are being met – the economy is in short-run equilibrium – if the utilization rate of labor is at the desired rate.<sup>16</sup> Unanticipated demand shocks are absorbed by movements in output and utilization. Thus, the equalization of saving and investment can be achieved without direct rationing or adjustments in prices and profit shares. As a benchmark case, assume that the real wage is fixed,  $w/p = \omega = \bar{\omega}$ .

Output is no longer a state variable, but the same behavioral reasoning that led to the output expansion function in the Kaldor-Goodwin case now yields an 'employment expansion function': employment changes in response to demand signals from the output market (the actual utilization rate of labor) and the state of the labor market (the employment rate):

$$\hat{L} = h(y, e); \quad h_y > 0, h_e < 0 \quad (18)$$

where  $y = Y/L$  is a measure of labor utilization.

The investment function also needs slight modification compared to the Kaldor-Goodwin formulation. Capital adjusts more sluggishly than labor, and this can be captured by having the accumulation rate depend on the labor-capital ratio, rather than the output-capital ratio:

$$\hat{K} = f(l); \quad f' > 0 \quad (19)$$

where  $l = L/K$  is the employment-capital ratio.<sup>17</sup> Here again, the behavioral argument for the specification is analogous to the Kaldor-Goodwin case: capital accumulation responds mainly to changes in the slow-moving state variable

<sup>15</sup>This economy has similarities with the model in Diallo et al. (2011). The similarities include an endogenous determination of the utilization of labor and the influence of labor utilization on the growth of employment. Unlike the present model, however, Diallo et al. take output to be a state variable. They also specify accumulation in a very different way, assume that labor and capital utilization move together, introduce monetary policy in the form of a Taylor rule, and focus on a 2D subsystem of a higher-dimension system.

<sup>16</sup>Employment is a state variable but adjusts faster than capital. Short-run equilibrium therefore is defined with reference to the utilization of labor.

<sup>17</sup>To get an exact parallel to equation (3), the investment-labor ratio could be specified as an increasing function of the labor-capital ratio,

$$\frac{I}{L} = \tilde{f}(l); \quad \tilde{f}' > 0$$

The accumulation rate would then be given by

$$\hat{K} = l \tilde{f}(l) = f(l)$$

The multiplicative form of the term  $l\tilde{f}(l)$  has no significance. If, say,  $\tilde{f}(l) = (-a + bl)/l$  then  $f(l) = -a + bl$  becomes linear.

(the output-capital ratio in the flex-price economy and the labor-capital ratio in the flex-output economy).<sup>18</sup> Thus, equations (18)-(19) and (1) and (3) can be derived from the same basic behavioral assumptions of goal oriented behavior (profit maximization); the differences arise because of changes in the assumptions about technologies and the character of the output.

Retaining the saving function (4) and assuming no depreciation, the output labor ratio is determined by the condition

$$\frac{S}{K} = g(\pi) \frac{Y}{K} = g(\pi)yl = f(l) = \hat{K} \quad (20)$$

Shocks to demand affect labor productivity (labor utilization) and a constant real wage therefore does not imply constancy of the profit share; the profit share is increasing in labor utilization:

$$\pi = 1 - \frac{\bar{\omega}}{y} \quad (21)$$

Combining (20)-(21) and assuming Harrodian instability ( $f'(l) > g(\pi)y$  for all  $y \leq \lambda$ ), we get.

$$y = \phi(l); \quad \phi' > 0 \quad (22)$$

If the labor force grows at a constant rate,  $n$ , we now get a two-dimensional system in the state variables  $e$  and  $l$ :

$$\hat{l} = h(y, e) - f(l) \quad (23)$$

$$\hat{e} = h(y, e) - n \quad (24)$$

This dynamic system has the same mathematical structure as the Kaldor-Goodwin model, and we get clockwise cycles in  $(e, l)$  space. Both the output-capital ratio (the utilization rate of capital) and the profit share are increasing functions of the labor-capital ratio. Observationally, therefore, the flex-output economy looks like the Kaldor-Goodwin economy with respect to the predicted patterns for employment, capital utilization and the profit share. The similarity between the dynamic patterns of flex-price and flex-output systems suggests that actual economies – which may contain both flex-price and flex-output sectors – may exhibit these patterns too.

In terms of predictions, the main difference between the two models concerns the cyclical variation in labor productivity; labor productivity is assumed constant in the basic Kaldor-Goodwin but varies with utilization in the flex-output economy. It should be noted, however, that in the Kaldor-Goodwin flex-price model a constant labor productivity was introduced merely to simplify the analysis: labor hoarding and cyclical variations in productivity also

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<sup>18</sup>Accumulation may also depend on the fast variable (the utilization rate for labor, in this case). The dependence has to be weak, however, in order to ensure the stability (and meaningfulness) of the (ultra-) short run equilibrium. In the flex-output economy the stability condition is simply the standard Keynesian restriction that investment respond less strongly than saving to changes in output in the (ultra-) short run.

characterize non-service sectors, and indeed these variations help reconcile the relative magnitudes of the predicted fluctuations in employment, utilization and the profit share with empirical observations (Skott and Zipperer 2012).

Both flex-price and flex-output models can be extended. The accumulation rate can be treated as a state variable, for instance, as in Skott and Zipperer (2012); the saving function can be modified to allow for saving out of wages and habit formation, or to take into account financial stocks; inventories can be included; distinct sectors with different properties can be identified. Going beyond the abstract modeling of a pure capitalist system, there is clearly room for a host of possible extensions to improve the empirical fit; a public sector, fiscal and monetary policy, and foreign trade are among the obvious candidates.<sup>19</sup> But perhaps the most important task is detailed empirical work to examine the validity of the main assumptions underlying the different models, including the question of price and output flexibility.

For present purposes another extension of the flex-output model may be of particular interest: the assumption of fixed real wages can be relaxed. At its simplest, this relaxation could involve making the real wage a function of labor utilization and the employment rate:

$$\omega = \psi(y, e) \tag{25}$$

The signs of the partials in this relation are ambiguous, a priori. A positive partial with respect to  $y$  would be expected in businesses where tips (or other forms of output-dependent bonuses) make up a significant part of wages; a negative partial would be more likely in businesses like airlines that have fixed wage rates and raise prices when planes fill up. The extension in equation (25) would change the precise relation between the labor-capital ratio and the profit share, but the qualitative properties of the model would probably be unchanged for plausible magnitudes of the partials.

More significant changes involve adding a dynamic element to the determination of the real wage.<sup>20</sup> As an example, the real wage could be given by<sup>21</sup>

$$\omega = \psi(a, y, e); \quad \psi_a > 0 \tag{26}$$

$$\dot{a} = \theta(l, e, \omega) \tag{27}$$

Here again the partials of the equations are ambiguous, and in this case the properties of the extended system may depend critically on the precise specification. It should be noted, perhaps, that equations (25)-(27) describe the real wage without any reference to wage and price inflation. It would be important to consider the formation of money wages and prices separately if inflation affects

<sup>19</sup>Franke (2014) analyze the stabilizing effect of Taylor rules; Ryoo and Skott (2015) consider fiscal and monetary policy rules in an economy with Harroddian instability.

<sup>20</sup>This is the main focus of Diallo et al. (2011).

<sup>21</sup>The state variable  $a$  captures the gradual effects of labor utilization and employment on distribution. The system includes a simple Goodwin specification as a special case; if  $\omega = \psi(a, y, e) = a$  and  $\dot{a} = \theta(l, e, \omega) = -\gamma + \rho e$  the system reduces to Goodwin's real-wage Phillips curve.

policy and/or private-sector behavior directly; neither AB nor the flex-price and flex-output models in this note include inflation effects of this kind.

## 5 Conclusion

AB's analysis of growth and cycles has strong similarities with my own work. The properties of the investment function, for instance, have figured prominently in (post-) Keynesian debates and, in my view, the theoretical and empirical evidence strongly favors a Harroddian perspective; AB do not appear to disagree.

Firms' pricing and output decisions have received less attention, and this question is more open, both theoretically and empirically. My reading of the evidence suggests that traditional Keynesian presumptions of fixed prices (or relatively sticky markups) have limited support and that disequilibrium in the goods market rapidly feeds into price movements. This view was embodied in the Kaldor-Goodwin model with perfect price flexibility. The model can produce the stylized patterns considered by AB, and there is no reason to expect that an extended model with fast but finite price adjustment will not be able to do the same. Adding a new state variable or changing parameter values affect the properties of a dynamic system, but there is nothing particularly restrictive about assuming perfect price flexibility; a model with this assumption can provide a good approximation if price adjustment is fast.

The argument for perfect price flexibility comes with a caveat: parts of the economy – particularly in the service sector – may be characterized by (near-) perfect output flexibility. In these sectors the adjustment to demand shocks can come via output rather than price movements (in many cases, like airlines, both prices and output adjust). Surprisingly, perhaps, a flex-output economy of this kind can be described by a dynamic system that is almost identical to the system for the flex-price economy.

Implicitly, AB's paper raises broader, methodological issues. AB appear to judge models on their ability to match stylized patterns. It is relatively easy, however, to match a small number of stylized facts if assumptions can be chosen freely. For a model to be interesting, the assumptions have to be plausible, and the plausibility of behavioral macroeconomic equations cannot be judged without reference to microeconomic behavior. It is not a matter of unidirectional microeconomic foundations – institutions and the overall macroeconomic environment influence microeconomic behavior. But "truly behavioral" analysis (to use AB's terms) should be an essential part of macroeconomic analysis.

## References

- [1] Abe, T. (2014). "Cyclical patterns in Japanese manufacturing firms: analyzing three post-Keynesian models". *Evolutionary and Institutional Economic Review*, 11 (1), pp. 1–18.

- [2] Abe, N. and Tonogi, A. (2010) "Micro and macro price dynamics in daily data". *Journal of Monetary Economics*, 57, pp. 716-728.
- [3] Barbosa-Filho, N.H. and Taylor, L. (2006) "Distributive and demand cycles in the US economy – a structuralist Goodwin model". *Metroeconomica*, 57 (3), pp. 389-411.
- [4] Chiarella, C., Flaschel, P. and Franke, R. (2005) *Foundations for a disequilibrium theory of the business cycle*. Cambridge: Cambridge University Press.
- [5] Diallo, M.B, Flaschel, P., Krolzig, H.-M. and Proano, C. (2011) "Reconsidering the dynamic interaction between real wages and macroeconomic activity". *Research in World Economy*, 2 (1), pp. 77-93.
- [6] Franke, R. (2014) "Can the New Macroeconomic Consensus Tame Harro-dian Instability?". Mimeo.
- [7] Keynes, J.M. (1930) *A treatise on money*. London and Basingstoke: Macmillan.
- [8] Ryoo, S. and Skott, P. (2015) "Functional finance in an unstable economy". Mimeo.
- [9] Skott, P. (1988). "Dynamics of a Keynesian economy under different mon-etary regimes". In P. Flaschel and M. Kruger (eds), *Recent Approaches to Economic Dynamics*, Verlag Peter Lang, Frankfurt am Main.
- [10] Skott, P. (1989a). Effective demand, class struggle and cyclical growth. *International Economic Review*, 30 (1), pp. 231–247.
- [11] Skott, P. (1989b). *Conflict and effective demand in economic growth*. Cam-bridge: Cambridge University Press.
- [12] Skott, P. (2012). "Theoretical and empirical shortcomings of the Kaleckian investment function". *Metroeconomica*, 63 (1), pp. 109–138.
- [13] Skott, P. (2014) "Pluralism, the Lucas critique, and the integration of macroeconomics and microeconomics". *Review of Political Economy*, 26 (4), pp. 503-515.
- [14] Skott, P. and Zipperer, B. (2012). "An empirical evaluation of three post-Keynesian models". *Intervention – European Journal of Economics and Economic Policies*, 9 (2), pp. 277–308.
- [15] von Arnim, R. and Barrales, J. (2015) "Demand-driven Goodwin cycles with Kaldorian and Kaleckian features". *Review of Keynesian Economics*, this issue..