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A Selective Review of Recent Quantitative Empirical Research in Marxist Political Economy

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Abstract

This paper surveys some of the quantitative empirical research in two areas of Marxist political economy: (a) Marxist national accounts, and (b) Marxist responses to the Sraffa-based critique of the 1970s. With respect to the first area, this paper explains the basic methodology underlying the construction of Marxist national accounts from traditional input-output data. With respect to the second area, it offers a short review of the theoretical literature surrounding the Sraffa-based critique of the 1970s, and subsequently discusses three Marxist responses in detail: the standard interpretation, the probabilistic interpretation and the new interpretation. It explains the basic theoretical positions of these three approaches and reviews the quantitative empirical work conducted within each.
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1. Introduction

Marxist political economy has a long and distinguished tradition of quantitative empirical analysis, going all the way back to Marx himself. Using tax returns data to infer patterns of top income distribution and nutrition data to understand conditions of the working class population in Volume I of *Capital*, Marx anticipated Thomas Piketty by more than 150 years and the discipline of development economics by about 100. Lenin's monumental work, *The Development of Capitalism in Russia*, had used extensive quantitative data on distribution of landholdings, small peasant industries and factory industries in Russia and Europe to argue against the Narodnik claim that capitalism could not develop in a backward country like Russia.

This robust tradition has been kept alive by the painstaking research of numerous Marxist scholars, who have used, over the years, improved statistical techniques and better measurement for quantitative empirical work. This paper will review some of this literature, focusing specifically on quantitative empirical research on two particular issues, Marxist national accounts and the value controversy. Therefore, many interesting and important lines of Marxist research will not be surveyed in this paper.

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Naturally then, this is a *selective* survey, and I make no claims about comprehensiveness. Some of the important areas that have been left out of this survey are: research that builds on the difference between labour and labour-power; analysis of financial crises; research on unemployment, the labour market and the reserve army of labour; research on the dynamics of accumulation by the Monthly Review school; research on institutional changes in the social structures of accumulation approach; research on consumption norms and behavior. But even this list is not exhaustive.

The rest of the paper is divided into two parts. The first part reviews, in section 2, a topic of empirical research that has widespread acceptance, both as to its necessity and broad methods, in Marxist political economy: construction of national accounts based on Marxist principles. The second part of the paper, running over sections 3, 4 and 5, reviews a literature that has seen more disagreement among Marxist scholars: the value controversy. In section 3, I will provide a quick review of theoretical debates in the labour theory of value that started with the so-called transformation problem in Volume III of Capital. I will argue that the Sraffa-based critique of the 1970s was a culmination of a long tradition that had criticized Marx, and, more importantly, that this episode was extremely productive for Marxist political economy. It challenged Marxist scholarship and elicited creative responses. While there are at least five different Marxist responses to the Sraffa-based critique, I will focus on the three that have generated impressive quantitative empirical work. In section 4, I will review the quantitative empirical research that has emerged in these three strands of Marxist political economy: the Standard Interpretation, the Probabilistic Interpretation, and the New Interpretation of Marxist value theory. Mathematical proofs and definitions are collected in an appendix.

2. Marxian National Accounts

The aim of Marxian national accounts is to generate estimates of the total value produced in an economy over a period of time, as also its component parts – constant capital, variable capital and surplus value. Since traditional national accounts are not based on a Marxian conceptual framework, estimates of Marxian categories cannot be directly read off from them. To use traditional national accounts for estimating Marxian categories two important distinctions have to be conceptualized and empirically operationalized: (a) the distinction between production and nonproduction activities; and (b) the distinction between productive and unproductive labour.

There is a long tradition that has attempted this task, including Mage (1963), Wolff (1977), Moseley (1982). In this paper, I will focus my comments on the ground-breaking work of Shaikh and Tonak (1994), which is, in a sense, a culmination of the previous literature on this issue and also the most comprehensive work till date.[1]

2.1. Theoretical Considerations

To understand the first distinction, we can divide the basic activities of social reproduction into two mutually exclusive and exhaustive groups: production and nonproduction. The difference between the two is crucial: while production results in the creation of new use values (wealth), nonproduction uses up wealth without creating new wealth. Nonproduction activities can, in turn, be divided into three mutually exclusive and exhaustive groups: distribution, social maintenance and personal consumption. Distribution involves activities that transfer use values, titles to use values or money from one set of

[1] See chapter 6 in Shaikh and Tonak (1994) for a critical review of the literature that precedes their work.

economic agents to another. Social maintenance refers to all activities that are geared towards the maintenance and reproduction of the social order. Personal consumption includes all activities involved in the maintenance and reproduction of individuals within the social order.

All schools of economic thought distinguish between production and consumption. Moreover they agree that production creates wealth and consumption uses up wealth. The difference between the neoclassical and classical-Marxian traditions arise from the characterization of the activities of distribution and social maintenance. For the neoclassical (and Keynesian) tradition, these activities are understood as production as long they are marketable and some entity is willing to pay for the activity or the product arising therefrom. The classical-Marxian tradition differs sharply by arguing that distribution and social maintenance should *not* be understood as production; rather, they should be understood as social consumption. This is because they *use up* rather than creating new use values.

These alternative classifications of the basic activities of social reproduction by the neoclassical/Keynesian and classical-Marxian traditions can be summarized in Figure 1. For the neoclassical/Keynesian tradition, personal consumption is total consumption and total production is the sum of distribution, social maintenance and production. In sharp contrast, for the classical-Marxian tradition, distribution and social maintenance together make up social consumption, and the sum of social and personal consumption give total consumption. Total production, on the other hand, is just production proper.

[Figure 1 about here]

To grasp the second distinction, i.e., the distinction between productive and unproductive labour, we can begin by noting that production, distribution and social maintenance can each involve three different types of labour – labour that creates use values for direct use, labour that creates use values for sale for income, and labour that creates use values for sale for profit. The classical-Marxian framework argues that the first type of labour creates use values, the second type creates use values and value, and the third creates use values, value and surplus value. With this understanding, the classical-Marxian tradition defines labour that produces surplus value as productive labour. All other labour is defined to be unproductive because it does not produce surplus value (and capital).[2]

2.2. Empirical Operationalization

The empirical operationalization of Marxian national accounts for the U.S. economy relies on three data sources: (a) input-output (IO) tables for benchmark years, (b) national income and product accounts (NIPA) for non-benchmark years, and (c) bureau of labour statistics (BLS) data for all years.

Traditional IO tables contain comprehensive information about two types of inter-industry flows in an economy over a period of time (usually a year). First, for each industry they provide information about the inputs used up and the value added created, the sum of the two being the gross output (revenue). Second, they provide information about the precise manner in which the gross product of any industry is used, i.e., either as an intermediate input in some industry or for a final use (personal consumption, investment, government purchases, or net export).

[2] Note that the difference between productive and unproductive labour has nothing to do with necessity, usefulness, or rationality. It is a narrow and precise distinction based on whether that labour creates surplus value.

Therefore, the first step in the construction of Marxian national accounts is to start with data from the published IO tables, re-classify industries to operationalize the distinction between production and nonproduction activities, and add the gross output for all industries in the former category to arrive at an estimate of total value produced. To do so, a researcher needs to answer the following question: which industries should be included in the production sector? To answer this question we need to keep a peculiarity of IO tables in mind. They record transactions in terms of producers' prices, instead of purchasers' prices. Hence, the value of a commodity gets realized in two steps. In the first step, the producer sells the commodity to a wholesaler or retailer, and in the second step, the wholesaler or retailer sells it to the final user. Thus, *the sum of the gross output of production and trade activities provide an estimate of the total value produced in the economy*.[3]

To be more precise, the production sector includes three types of industries: all industries that produce *goods* like agriculture, mining, manufacturing, etc.; all *productive services* industries like transportation, entertainment, lodging, cooking, healthcare, education, utilities, etc.; and all government enterprises. On the other hand, the trade sector includes the wholesale and retail trade industries, including government trading enterprises, and a part of the real estate sector – the part that deals with building, equipment and car rentals (because rental sale is akin to selling the commodity piecemeal over its lifetime).

Following Shaikh and Tonak (1994), we can call the production and trade sector together as the "primary flows" sector and note that the gross output of this sector is the total value produced in the economy. While industries in the primary flows sector derive their revenues from the production and sale of the commodity product, revenues of all other industries – which take the form of ground rent, finance charges and fees, interest charges, royalties of various types, insurance charges and taxes – are derivatives of primary flows in the following sense: they are either a recirculation of money flows generated by primary flows, or are a circulation of socially validated claims upon parts of primary flows, or both. Hence, we can club together all other industries into the "secondary flows" sector and note that their "output" cannot be counted as new value.

With an estimate of the total value (TV^*) as the sum of the gross output of the industries in the primary flows sector (production, GO_p, and trade, GO_t)

$$TV^* = GO_p + GO_t \qquad (1)$$

we move to the next step and estimate the constant capital (C^*) as the sum of material inputs (M_p') and depreciation (D_p) of the production sector

$$C^* = M_p' + D_p \, . \qquad (2)$$

It is worth pointing out that while we have included the gross output of the trade sector in the computation of total value, we have excluded the material inputs (M_t') and depreciation (D_t) of the trade sector from the calculation of constant capital. To understand the reason behind this let us compare two scenarios.

In the first scenario, producers directly sell commodities to final users; in the second scenario, producers sell commodities to wholesaler/retailers at the "producers' price", and the latter then sell the

[3] All magnitudes in this section are measured in monetary units.

commodities to final users at the "purchasers' price". Note that there is no difference between the two scenarios as far as the process of production is concerned. Since a classical-Marxian perspective assumes that value can only be produced in the sphere of production, the total value and its components – constant capital, variable capital, and surplus value – are going to be the same in the two scenarios. The only difference between the two scenarios arises with regard to the distribution of surplus value. In the first scenario, the total surplus value is realized as the profit of the producer. In the second scenario, the total surplus value is realized as the profit of the producer *and* the total trade margin, i.e., the total revenue of the trade sector. Total trade margin, in turn, is the sum of intermediate inputs, wages and profits of the trade sector. Hence, when we calculate total value, we need to add the gross output of production and trade, but when we compute constant (and variable) capital, we need to exclude the trade sector (because the value of intermediate inputs in the trade sector is part of surplus value and not constant capital).

With estimates of total value and constant capital, we can compute Marxian value added (MVA^*) as

$$MVA^* = TV^* - C^* \qquad (3)$$

So far, we have not yet had to use the distinction between productive and unproductive labour, but in the next step, when we compute variable capital, it will become salient. Variable capital (V^*) is defined as the total wage bill of the productive workers in the production sector. Thus, the first task in computing variable capital is to separate out the productive and unproductive workers in the production sector.

The primary source of information on employment is the NIPA, which provides data on persons engaged in production (PEP) for all industries. PEP is the sum of full-time equivalent employees (FEE) and self-employed persons (SEP). The NIPA data does not have any information to distinguish productive and unproductive labour. Hence, we turn to employment data provided by the BLS, which distinguishes between supervisory workers and production/nonsupervisory workers. Using the ratio of the latter to total employment is a good estimate of the share of productive workers in any industry. Hence, if j denotes an industry in the production sector, then an estimate of productive workers in industry j is given by

$$L_j^p = \rho_j \times PEP_j \qquad (4)$$

where ρ_j is the ratio of productive workers to total employment (from BLS) in industry j, and PEP_j is the total employment (from NIPA) in industry j. Then, the sum total of all productive workers in production is given by

$$L^p = \sum_j L_j^p \,, \qquad (5)$$

where the subscript j runs over all industries in the production sector.

With an estimate of total productive workers in production in place, the next task is to compute their wage bill (or employee compensation). To do so we need to draw on data from both the NIPA and BLS. Unit wage of productive (production & nonsupervisory) workers is available from the BLS. But BLS wage data does not include wage supplements like employer contributions to social security and other pension funds. Since the cost of labour power should include total wage and wage equivalents, we turn

to data on employee compensation from the NIPA that includes supplements to wages. We can use both sets of information to define employee compensation for productive workers in industry j as

$$ec_j = \left(w_p'\right) \times \left(\frac{EC}{WS}\right)_j \qquad (6)$$

where w_p' is the unit wage of production & nonsupervisory workers (from BLS), EC is the employee compensation (from NIPA), and WS is wage & salaries (from BLS). The product of employee compensation and productive labour summed over all industries in the production sector provides an estimate of variable capital

$$V^* = \sum_j V_j = \sum_j \left(ec_j \times L_j^p\right) \qquad (7)$$

where the subscript j runs over all industries in the production sector, L_j^p is the productive labour in in industry j.[4]

The final step is to calculate surplus value as

$$S^* = MVA^* - V^* \qquad (8)$$

which allows us to compute the rate of surplus value as

$$sv^* = \frac{S^*}{V^*} \qquad (9)$$

and the rate of profit as

$$r^* = \frac{S^*}{K} \qquad (10)$$

where K is the total stock of fixed nonresidential gross private capital valued at replacement cost.

Using the methodology outlined above, Shaikh and Tonak (1994) report several interesting findings about the US economy over the period from 1948 to 1989. First, they find that there is a dramatic increase in the share of nonproduction workers in the total pool of workers. This reflects the corresponding increase in the share of nonproduction activities in the aggregate economy. Shaikh and Tonak (1994) offer the increase of nonproduction activities as a possible explanation of the productivity slowdown in the US economy from the early 1970s.[5] Second, they find that the rate profit has a pronounced tendency to fall over the period of analysis. This, according to them, supports Marx's hypothesis about the tendency for the rate of profit to fall over time. The third striking finding from Marxian national accounts is that the "social wage" of production workers have been negative for most of the post-war period. This means that production workers, as a group, have paid more taxes to the

[4] Mohun (2005) provides better estimates of productive labour and the wage bill of productive labour by doing away with some approximations that Shaikh and Tonak (1994) were forced to make due to unavailability of data.

[5] While there is broad agreement in the Marxist tradition on the need to distinguish productive and unproductive labour, there are alternative interpretations of the effect of unproductive labour on the macroeconomy. For alternative interpretations, see Paitaridis and Tsoulfidis (2012) and Mohun (2014). From a class perspective, Mohun's (2014) distinction between what he calls the "unproductive working class" and supervisors is important. His empirical work has emphasized the constancy of the share of the unproductive working class, and the rise in the share of supervisors, since the early 1960s. This observation has important insights to offer on the growing income and wealth inequality under neoliberal capitalism.

State than they have received back in benefits. The final interesting finding is that values are "close to" prices, a finding that I will examine in greater detail in the next two sections.

3. The Value Controversy

The relationship between the "value" and "price" of commodities has been an important issue of investigation in the classical-Marxian tradition for more than two centuries now. In the context of such discussions, "value" is defined as the total labour time directly and indirectly needed to produce a commodity with current technology and "price" refers to the set of prices that would arise in the long run when rates of profit are equalized across all industries (these long-run equilibrium prices are referred to in the literature as prices of production, or production prices, and are different from market prices). Both Ricardo and Marx were aware of the fact that relative prices (of production) and relative values were related to each other in complicated ways and need not necessarily coincide. But they had different ways of understanding this correspondence and divergence.

3.1. Ricardo, Marx and Bortkiewicz

Ricardo thought that the two ratios – relative prices and relative values – were approximately equal for individual commodities. After examining ample textual evidence Stigler (1958) summarized this contention with the famous quip that Ricardo had a 93% labour theory of value, with the 93% figure capturing the approximation involved. On the other hand, Marx understood that profit rate equalization and different compositions of capital across industries would necessarily lead to a deviation of relative prices (of production) from relative values. Hence, he conceptualized the equivalence between values and prices at the aggregate level (Marx 1991). For him, the process of equalization of the rate of profit was driven by the movement of capital across industries from lower (than average) to higher (than average) rates of profit. This process tended to push the economy, in the long run, towards a state where all industries earned a uniform rate of profit and commodities exchanged at prices of production.

For Marx, emergence of a uniform rate of profit (and prices of production) was primarily a way of conceptualizing the redistribution of surplus value across industries enforced by competitive pressures. He thought that *both* surplus value and total value would be preserved in this process of "transformation" of values into prices (of production). That is why he proposed the aggregate value-price equivalence as the cornerstone of his labour theory of value in Volume III of Capital: total value = total price, and total surplus value = total profits.

Immediately upon the publication of Volume III of Capital in 1894, critics (like the Austrian economist Eugen von Böhm-Bawerk) discovered two problems in Marx's transformation procedure: prices of inputs had not been transformed (even as prices of outputs had been), and the rate of profit had been calculated in value terms (whereas a consistent procedure would need to calculate it in price terms). While the two errors in Marx's procedure could be easily corrected, as shown by the German statistician-economist Ladislaus von Bortkiewicz in 1907, it was no longer possible, using the correct procedure, to derive both aggregate value-price equalities that Marx had asserted. In a sense, this was the first step away from Marx's analysis: *only one* of the aggregate value-price equalities could hold (Bortkiewicz 1949).

3.2. The Standard Interpretation and the Sraffa-Based Critique

Revival of interest in Marxian economics in the 1960s and 1970s saw further development and elaboration of the Bortkiewicz argument in the works of, among others, Francis Seton, Nobuo Okishio and Michio Morishima. The Seton-Okishio-Morishima contribution recast the question in the Leontief-Sraffa input-output framework, thereby making explicit the use-value basis and sectoral interdependence of capitalist production. At the same time, it generalized the analysis of Marx (5 industries) and Bortkiewicz (3 departments) to an *n*-commodity world. The Seton-Okishio-Morishima contribution proved that in a general circulating capital model, *relative* prices and a uniform rate of profit could be derived rigorously from data on technology and the real wage (commodity) bundle. But, an additional "normalization condition" was needed to derive absolute prices, and either of Marx's aggregate value-price equalities could be used for the purpose (Seton 1957, Okishio 1963, Morishima and Catephores 1978). This was the second step away from Marx's analysis: *none of* the aggregate value-price equalities could be derived; instead, one of them had to be *assumed as a normalization condition*.

The Seton-Okishio-Morishima framework of the labour theory of value, sometimes known as the "standard interpretation", was subjected to a serious Sraffa-based critique in the 1970s, most prominently by Ian Steedman (1977). The main thrust of the critique was an argument of redundancy. Given data on technology and the real wage bundle, one could calculate the uniform rate of profit and prices of production. Hence, there was no need of value categories. Value was conceptually redundant. Even though his critique was not new – the same point had been made previously by Paul Samuelson (1971) and seven decades earlier by the Russian economist Dmitriev (1974/1902) – it was forceful and provocative. Paradoxically, the Sraffa-based critique was also extremely productive for Marxist political economy. In responding to the Sraffa-based critique, Marxist political economy renewed itself. By the early 1980s, one could discern several strands of Marxist political economy that has emerged as a response to the Sraffa-based critique. Many of these strands opened up new, or continued older, lines of scholarly work and research. Without claiming to be comprehensive, I would direct the attention of readers to five strands.

3.3. Marxist Responses to the Sraffa-Based Critique

The first response that developed in the 1970s through the work of scholars like Ben Fine, Laurence Harris, Simon Mohun and others emphasized the difference between the Ricardian understanding of value as "embodied labour" and the Marxian understanding as "abstract labour". While this strand opened up interesting theoretical questions, to the best of my knowledge, it did not materialize into a progressive research program with quantitative empirical work (for a recent exposition of this strand, see Fine, Lapavitsas and Saad-Filho 2004). So, I will not pursue it here.

The second response came from within the Seton-Okishio-Morishima framework in the form of the Fundamental Marxian Theorem (FMT). The FMT demonstrated that positive profits can arise if and only if there is positive surplus value. Since it is a "if and only if" claim, the FMT is a weak response to the Sraffa-based critique. It shows that surplus value is necessary for profits, but equally well that profits are necessary for surplus value. Moreover, the FMT did not lead to any empirical work. Hence, I will not pursue it here.

The third response emerged in the work of Anwar Shaikh (1977, 1984). His main claim was similar to Ricardo's: value and price magnitudes are *approximately* equal. In his 1984 paper, Shaikh developed a

theoretical argument demonstrating that the deviation of prices from values would be "small" and then used data from the Italian and US economies to show that his claim is empirically valid. Anwar Shaikh's work on this question has given rise to a large literature that I will review below.

The fourth response came from the work of two mathematicians, Emmanuel Farjoun and Moshé Machover, who brought a probabilistic approach to political economy. Farjoun and Machover (1983) argued that most economic variables – like price, rate of profit, wage – are non-degenerate random variables, each with their probability distribution functions. This means that equilibrium in a capitalist economy should be characterized by a *distribution* of the rate of profit, instead of a single, uniform rate of profit. Looked at from within a probabilistic perspective, the Sraffa-based critique is based on the erroneous postulate of *a* uniform rate of profit as characterizing long run equilibrium; hence, its conclusions are invalid. In addition to offering a rebuttal of the Sraffa-based critique, Farjoun and Machover (1983) also developed a positive theory of the distribution of the rate of profit and the value-price deviation. Using a heuristic guide from statistical mechanics, they argued that the rate of profit is likely to have a 2-parameter *gamma distribution*; and, relying on the central limit theorem, they argued that *specific price* (price per unit of labour value) is distributed as a Gaussian random variable. They presented some empirical evidence in support of their theory which has been recently followed. I will review this strand below.

The final response came through the "new interpretation" (NI) of Marxian economics, developed independently by Duncan Foley and Gérard Duménil in the late 1970s. The NI emphasizes Marx's insistence that the value-price equivalence be conceptualized at the aggregate level. But instead of trying to derive such an equivalence from more primitive principles, the NI *defines* the labour theory of value to be such an equivalence. The twin conceptual innovations of "value of money" and "value of labour power" anchored the aggregate equivalence, and created a theoretically informed and consistent accounting framework. The accounting framework, in turn, opened up a progressive research program that has given rise to a large empirical literature investigating issues of technology, distribution and crisis in capitalism. I will review this strand of the literature below.

4. New Directions in Empirical Research

Marxist economics that grew out of and in response to the Sraffa-based critique of value theory that I reviewed above has opened up at least three interesting strands of empirical research. The first strand falls within the "standard interpretation" of Marx's value theory and pursues the following question: how "close" are value magnitudes to price magnitudes? The second strand falls within what I would call the "probabilistic interpretation" of Marx's value theory and pursues a different set of questions: how are profit rates and unit prices distributed in long run equilibrium? The third strand locates itself within the "new interpretation" of Marx's value theory and has pursued yet another different set of questions: how do value magnitudes relate to technology, distribution and crisis tendencies in capitalist economies?

4.1. Standard Interpretation Empirics

The basic framework for empirical analysis of the value-price relationship in the standard interpretation literature has been a bivariate regression model where relative price is regressed on relative value. Such an empirical model is derived from a theoretical relationship between relative prices and relative values

mediated by ratios of integrated capital-labour ratios. I will give a very brief sketch of the argument here; for details see appendix A and Shaikh (1984).

Let $p_{ij} = \frac{p_i}{p_j}$ and $\lambda_{ij} = \frac{\lambda_i}{\lambda_j}$ be the relative price and value of commodity i and j respectively; then

$$p_{ij} = \lambda_{ij} \times \left\{\frac{1+\frac{r}{w}k_i^T}{1+\frac{r}{w}k_j^T}\right\} \qquad (11)$$

where r and w refer to the uniform rate of profit and wage rate, respectively, and

$$k_i^T \equiv \frac{K_i^T}{L_i^T} = \frac{K_i+K_i^{(1)}+K_i^{(2)}+K_i^{(3)}+\cdots}{L_i+L_i^{(1)}+L_i^{(1)}+L_i^{(2)}+L_i^{(3)}+\cdots} \qquad (12)$$

is the integrated capital-labour ratio, i.e., the ratio of integrated capital stock, K_i^T, and integrated labour, L_i^T. The integrated labour, L_i^T, is the sum of the direct labour required to produce commodity i, and the sum of the labour required to produce the means of production required in the production of commodity i, and the labour required to produce the means of production of the means of production required in the production of commodity i, and so on (integrated capital, K_i^T, is defined in an analogous manner).

The theoretical results contained in (11) and (12) have two important implications. First, the result was derived without imposing *any* aggregate normalization condition. In particular, it avoids imposing either of the aggregate equalities that Marx thought would hold: total prices = total value, and total surplus value = total profits. This increases the generality and applicability of the result, and also distinguishes from the NI, where an aggregate value-price equality *is* the labour theory of value.

Second, it offers a *partial rebuttal* of the claim that price and values must diverge from each other because sectoral organic compositions are very different from each other. It is *partial* because it accepts that prices and values will diverge, and it is a *rebuttal* because it shows that the divergence will be small. To see the latter claim, note that the integrated capital-labour ratio is a *weighted average* of actual capital-labour ratios for all stages of production going back indefinitely. Hence, its variation across sectors is bound to be much smaller than variation in actual capital labour ratios. To be more precise, this argument suggests that even when actual capital-labour ratios k_i and k_j are very different from each other, we can have $k_i^T \cong k_j^T$ as long as the economy is sufficiently interdependent. This means that the term multiplying relative values on the RHS of (11) is "close" to unity. Hence, this opens up the possibility that relative prices and relative values might be "close" to each other even when there is large variation in capital-labour ratios across sectors.

The model in (11) and (12) is also the basis of empirical analysis of the claim that values and prices are approximately equal. There are two methods of analysis that have been used in the literature, a regression-based method and a non-regression-based method.

4.1.1. Regression-based Methodology

Taking logarithms of (11) gives the bivariate regression model that has been used for empirical analysis

$$\ln(p_{ij}) = \alpha + \beta \times \ln(\lambda_{ij}) + u_{ij} \qquad (13)$$

where a comparison with (11) suggests that

$$\alpha + u_{ij} = \ln\left(\frac{1+\frac{r}{w}k_i^T}{1+\frac{r}{w}k_j^T}\right)$$

A high value of R^2 from the estimation of (13) – most researchers using this method have found values of the R^2 in excess of 0.9 – is interpreted as evidence in support of the standard interpretation (or Ricardian) hypothesis that relative values and relative prices are approximately equal (Shaikh 1984, Cockshott and Cottrell 1997, and Tsoulfidis and Maniatis 2002).

But the magnitude of the R^2 is probably not the best way to test the underlying theory. To see this note that the theory in (11) and (12) does not have *any* implications for the R^2 in the bivariate regression model (13) estimated by OLS. Instead, the theory suggests that $k_i^T \cong k_j^T$ so that the researcher needs to test whether

$$\frac{1+\frac{r}{w}k_i^T}{1+\frac{r}{w}k_j^T} = 1 \qquad (14)$$

which is equivalent to testing if

$$\ln\left(\frac{1+\frac{r}{w}k_i^T}{1+\frac{r}{w}k_j^T}\right) = 0 \qquad (15)$$

Hence, because the expected value of the error term in (13) is zero by construction, the theory in (11) would lead to the test of the following joint null hypothesis

$$H_0: \alpha = 0 \;\&\; \beta = 1 \qquad (16)$$

with respect to the parameters in (13) and can be conducted as a F-test. Moreover, the test of the null hypothesis in (16) has a straightforward graphical interpretation: in a bivariate regression of log relative prices on log relative values, as represented by (13), the regression line passes through the origin and has a slope of unity.

None of the papers that have used this methodology have tested the null hypothesis in (16), including Shaikh (1984), Petrovic (1987), Cockshott and Cottrell (1997), and Tsoulfidis and Maniatis (2002). This could be taken up in future research.

4.1.2. Non-Regression-based Methodologies

Non-regression-based methodologies have proceeded by constructing various measures of "distance" between the vector of prices and the vector of values. To operationalize this methodology let us define the following n-vector (where there are n sectors in the economy):

$$\boldsymbol{x} = \left[\frac{p_1}{\lambda_1}\ \frac{p_2}{\lambda_2}\ \frac{p_3}{\lambda_3}\ldots\ \frac{p_n}{\lambda_n}\right] \qquad (17)$$

Each element of the *n*-vector is the ratio of the price – price of production, or market price or some other price – and the value of the product of some sector in the economy (with the latter often multiplied by the nominal wage rate to make its units comparable to values). In this approach, the basic method of analysis is to define a measure of "distance" between $\boldsymbol{x}$ and the unit vector $\boldsymbol{i}$ (which is an *n*-vector of ones) and see whether it is relatively small.

Petrovic (1987) uses a measure of distance called root-mean-square-per-cent-error (RMS%E), which is the (positive) square root of the (arithmetic) mean squared (percent) deviation between $\boldsymbol{x}$ and $\boldsymbol{i}$:

$$RMS\%E = \sqrt{\frac{1}{n}\sum_j \left(\frac{p_j}{\lambda_j} - 1\right)^2}. \qquad (18)$$

Ochoa (1989) uses a similar measure of distance called mean absolute deviation (MAD), which is (arithmetic) mean of the absolute deviation between elements of $\boldsymbol{x}$ and $\boldsymbol{i}$

$$MAD = \frac{1}{n}\sum_i \left|\frac{p_j}{\lambda_j} - 1\right| \qquad (19)$$

and Shaikh (1998) uses a related measure called mean absolute weighted deviation (MAWD), which is an weighted average of the absolute deviation between elements of $\boldsymbol{x}$ and $\boldsymbol{i}$

$$MAWD = \sum_j \left|\frac{p_j}{\lambda_j} - 1\right| \mu_j$$

where $\mu_j = \lambda_j z_j / \sum \lambda_j z_j$ is the weight applied to the j-th term.

All these measures suffer from the problem that their value depends on the choice of the *numeraire*. To deal with this problem, Steedman and Tomkins (1998), propose an alternative measure of distance:

$$d = \sqrt{2(1 - \cos\alpha)} \qquad (20)$$

where α is the angle between the vectors $\boldsymbol{x}$ and $\boldsymbol{i}$**.** Motivated by the discussion in Steedman and Tomkins (1998), Fröhlich (2013) uses the tangent of the angle between the vectors $\boldsymbol{x}$ and $\boldsymbol{i}$ as his preferred measure of (angular) distance*:*

$$\tan\alpha = \frac{\sigma_x}{\mu_x}, \qquad (21)$$

where σ_x and μ_x are the standard deviation and mean, respectively, of the elements of the vector $\boldsymbol{x}$.

While the problem of numeraire dependence is solved through the method proposed by Steedman and Tomkins (1998), all these non-regression-based methods suffer from another, common, problem: there is no benchmark value to compare with and identify whether some observed measure of distance is "small". For instance, there is no know whether a calculated value of RMS%E of 4.45% - the smallest value in Table 1 in Petrovic (1987) - is "small", or that a calculated value of α of 3.159 degrees – the smallest value in Table A1 in Fröhlich (2013) – is "small". One might think that the benchmark of comparison is zero (0% for RMS%E, and 0 degrees for α). But that is incorrect. The benchmark should be the typical "distance" from zero, not the point of comparison (zero) itself. It is the lack of information about the benchmark distance from zero that makes the comparisons difficult.

To put this point using the language of statistics, one would say that the calculated value of the "distance" (no matter how it is defined) is a point estimate of the true distance between the two vectors. But to assign any significance to the point estimate, one would also need the standard error. The fact that this method does not compute any entity that is analogous to a standard error prevents a researcher from assigning any significance to the calculated "distance". This is a serious drawback of these non-regression-based methods.

One remedy would be to conduct a simple t-test of the hypothesis that the mean of the elements of the vector $\boldsymbol{x}$ is equal to unity. In conducting such a test, one would be taking the mean of the difference of the elements of the vector $\boldsymbol{x}$ from unity and scaling it by the variance of the sum of the elements of the vector $\boldsymbol{x}$**.** While this would be a valid method, the difficulty would now pertain to the computation of the variance of the sum of the elements of the vector $\boldsymbol{x}$ because the elements are admittedly not independent. Thus, to compute the variance of the sum, a researcher would have to compute covariance terms. The simplest way to avoid this complicated covariance computation is to use the regression-based method and conduct the hypothesis test outlined in (16).

4.2. Probabilistic Interpretation Empirics

The probabilistic approach to political economy starts from the recognition that all commonly used economic variables – price, value, rate of profit – are non-degenerate random variables and argues for replacing the deterministic with a stochastic conception of equilibrium (Farjoun and Machover 1983). Both Marx and his critics (e.g., proponents of the Sraffa-based critique) use a deterministic conception of equilibrium, which is understood as a long-run "ideal" state of the system characterized by a uniform rate of profit and prices of production. Farjoun and Machover (1983) argue that such a conception is fundamentally flawed because real economies never have a *single* rate of profit, but rather a probability distribution of profit rates. Hence, a stochastic conception of equilibrium of the system would be characterized by the *distribution* of the rate of profit in equilibrium, instead of a single uniform value.

With this basic idea animating their understanding, Farjoun and Machover (1983) develop arguments to derive the equilibrium distribution of two key variables of interest to Marxist political economy, the rate of profit and what they call the "specific price" (which is defined as the price per unit of embodied labour). The empirical work that follows from this framework is to test whether actually observed profit rates and specific prices have the distribution that is posited by the theoretical arguments.

What is the distribution of the rate of profit in capitalist economies? Based on an analogy from statistical mechanics, Farjoun and Machover (1983) hypothesize that the rate of profit has a *gamma distribution* (details of the two-parameter gamma distribution can be found in appendix B).

> In seeking theoretical expression for the distribution of R [rate of profit], a useful heuristic guide is provided by statistical mechanics. In a gas at equilibrium, the total kinetic energy of all the molecules is a given quantity. It can be shown that the 'most chaotic' partition of this total kinetic energy among the molecules results in a gamma distribution. Now, if we consider that in any given short period, there is a more-or-less fixed amount of social surplus … and that capitalist competition is a very disorderly mechanism for partitioning this surplus among capitalists in the form of profit, then the analogy of statistical mechanics suggest that R may also have a gamma distribution. (Farjoun and Machover 1983, pp. 68)

The next task is to derive the distribution of the specific price

$$\Psi(i) = \frac{\mathrm{p}(i)}{\Lambda(i)} \qquad (22)$$

where i indexes transactions taking place in some period T, $\mathrm{p}(i)$ is the price paid in transaction i, and $\Lambda(i)$ is the total amount of human labour embodied in the commodities that participated in transaction i. Thus, the specific price, $\Psi(i)$, is the price paid per unit of labour content in transaction i. An argument that relies on decomposing total price of any commodity into the sum of non-labour inputs, labour input

and profit, and iterating back through the various stages of production of the non-material inputs can be used to show that the numerator in (22) is the sum of two terms

$$\mathrm{p}(i) = w(i) + \pi(i) \qquad (23)$$

where $w(i)$ is the sum of total wages paid to all workers who participated directly and indirectly in the production of commodities involved in transaction i, and $\pi(i)$ is the sum of total profits earned by all firms directly and indirectly involved in the production of commodities involved in transaction i.[6] If there are m firms in the economy, then the sum in (23) can be broken into value-added contributions (because value added is the sum of wages and profits) coming from each of these firms (some of which may be zero). Moreover, since this logic holds for each transaction, we can dispense with the index i. Thus,

$$\Psi = \sum_{j=1}^{m} \frac{w_j + \pi_j}{\Lambda} \qquad (24)$$

where the sum in (24) runs over the total number of firms in the economy. Since any capitalist economy will have a large number of firms, some version of the central limit theorem can be invoked on (24) to suggest that the specific price is distributed as a normal random variable. Farjoun and Machover (1983) also derive the mean and variance of Ψ as $\mathbb{E}(\Psi) = 2$, and $var(\Psi) = 1/3$, and thus propose that

$$\Psi \sim \mathbb{N}(2, 1/3) \qquad (25)$$

What does the distribution of the observed rate of profit look like? Farjoun and Machover (1983; ch. 8) present data on profit rates in the British non-oil manufacturing Industry in 1972 and 1981. After plotting the empirical distribution (Figure 5 and 7), they impose a gamma distribution with shape and scale parameters 4.72 and 32, respectively, on top. The empirical distribution seems to visually match the imposed gamma distribution. This can be interpreted as suggestive evidence in support of the claim that the rate of profit is distributed as a gamma random variable.

In a recent study of the German economy for 2000 and 2004, Fröhlich (2013) has strengthened the visual results in Farjoun and Machover (1983) with more rigorous statistical tests. He finds that the empirical distribution of the rate of profit in Germany is well approximated by a gamma distribution with shape and scale parameters of 2.78 and 20.29 in 2004, and 2.03 and 15.65 in 2000. Moreover, a two-sample Kolmogorov-Smirnov test is not able to reject the null hypothesis that rate of profit data in 2000 and 2004 were drawn the same continuous distribution (p-value = 0.74).[7]

Fröhlich (2013) also tests the proposition that the "specific price" (price per unit of labour content) is distributed as $\mathbb{N}(2, 1/3)$. He finds that the mean of specific price is close to 2 in both 2000 and 2004. But the distribution is better approximated as log-normal instead of normal. His results suggests that the logarithm of specific price is distributed as $\mathbb{N}(0.66, 0.21)$.

[6] This is the same argument that is used in part by Shaikh (1984). See appendix A for details, especially the derivation of (A4), (A5) and (A6).

[7] A two-sample Kolmogorov-Smirnov test is used to test the null hypothesis that two empirical distributions are drawn from the same theoretical distribution. Operationalizing the test involves calculating the "distance" between the two empirical distribution functions and testing whether it is "small". Standard statistical packages like R and STATA have built-in functions to conduct the test.

Future work in this tradition would need to address at least two issues. First, the argument for arriving at the distribution of the rate of profit is based on an analogy from statistical mechanics. The analogy needs to be replaced with a more rigorous derivation, as Farjoun and Machover (1983) suggest. Second, the distribution of specific price is a normal distribution. But this is odd because both price and value (as labour content) have only positive magnitudes. Hence, the distribution needs to rule out negative values. The evidence presented by Fröhlich (2013) suggests that the specific price might be distributed as a log-normal random variable (which is consistent with the fact that it is only positive). This discrepancy between evidence and theory should be used to go back and re-work the theory of the distribution of specific price.

4.3. New Interpretation Empirics

For the new interpretation, the labour theory of value *is* a consistent *ex post* accounting framework that retains the proportionality of profits and unpaid labour time at the aggregate level. Two conceptual innovations proposed independently by Foley (1982) and Duménil (1983/1984) are needed for constructing the accounting framework.[8]

The first conceptual innovation is to *define* the value of money (m) as the ratio of the total labour embodied in the net output and the money value added

$$m = \frac{\sum_i \lambda_i y_i}{\sum_i p_i y_i} \qquad (26)$$

where i indexes commodities, λ_i denotes value (sum of direct and indirect labour embodied), p_i refers to price, and y_i refers to net ouput. The equality in (26) is the basic accounting identity of the new interpretation. It ensures that at the aggregate level, money value added (measured in prices) and labour embodied (measured in terms of hours of labour) are equal. According to the new interpretation, (26) *is* the labour theory of value. It allows free conversion of monetary into social labour time equivalent magnitudes, and vice versa.

The second conceptual innovation is to *define* the value of labour power (vlp) as the product of the nominal wage rate and the value of money

$$vlp = w \times m \qquad (27)$$

This definition of the value of labour power departs from the standard way of defining it. This departure rests on some deep theoretical considerations. In a capitalist society, the capacity to work becomes a commodity and is called labour power. It is different from all other commodities in at least two significant ways. First, it is not produced in capitalist enterprises using wage labour in the same way that other commodities are produced. Second, being a capacity, it is present in the person of the worker. It is regenerated within the person of the worker through a complex process involving interactions within families and supported by consumption of material and nonmaterial 'things'. So, it is not meaningful to try to define the labour embodied in the commodity labour power.

This leaves open two possibilities for defining the value of labour power. The traditional method has been to define it as the labour embodied in the commodity bundle that an average worker consumes. But if commodities do not exchange in proportion to their values, which is the normal situation in

[8] I draw on Mohun (2004) for some of the discussion in this sub-section.

capitalism, the monetary cost of purchasing the commodity bundle would *not* be proportional to its value, i.e., the labour embodied in it. That is why the new interpretation abandons this definition and proposes a second definition: value of labour power is the social labour time equivalent of the nominal wage, i.e., the product of the nominal wage and the value of money as given by (27).[9]

The definitions in (26) and (27) show immediately that aggregate profits equals the product of the value of money and aggregate surplus value, and that the ratio of aggregate wages and profits is equal to the rate of surplus value (for details see appendix C). Hence, this framework retains a central insight of Marx's labour theory of value: at the aggregate level, the source of profits is unpaid labour.

4.3.1. Profitability Analysis

Quantitative empirical work within the new interpretation framework has focused on analyzing trends in capital accumulation, technological change and income distribution.[10] Since the rate of profit is a key determinant of capital accumulation, the typical approach in the analysis of accumulation and crisis, is to start by studying a carefully constructed time series of the rate of profit.

4.3.1.1. Medium Run Analysis

The time span of analysis in this tradition is the medium or long run. Hence, fluctuations of aggregate demand is typically abstracted from. To study the drivers of profitability, a decomposition of the rate of profit into the share of profit and the output-capital ratio is used, i.e.,

$$r \equiv \frac{\Pi}{K} = \left(\frac{\Pi}{Y}\right) \times \left(\frac{Y}{K}\right) \qquad (28)$$

where r is the rate of profit, Π is total profits, K is the stock of fixed capital, and Y is value added. In decomposition in (28), the profit hare, Π/Y represents income distribution between capitalists and workers, and technology is represented by the output-capital ratio Y/K. Changes in either or both factors is used to explain the observed movement of the rate of profit. This is followed by a detailed analysis of the evolution of both these components by further decomposing them.

The profit share is decomposed as follows:

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[9] Over the past few decades, a single system (SS) interpretation has also been developed. The SS interpretation is similar to the new interpretation in many respects but has a key difference: it does not agree that the commodity labour power needs a special (or asymmetric) treatment, and insists that the value of all commodities be computed in the same way in which the value of labour power has been computed in (27). This tradition has many variants and even though there are disagreements on other elements of the framework, they all share a common non-standard definition of value: $\boldsymbol{\lambda}_{SS} = m_{SS}\boldsymbol{pA} + \boldsymbol{l}$, where the subscript SS refers to "single system", $\boldsymbol{\lambda}_{SS}$ is the vector of values, m_{SS} is the value of money, and symbols without the subscript have their usual meaning. This non-standard definition of value goes back to Wolff, Roberts and Callari (1982), and continues to be used in the works of Fred Moseley (e.g., Moseley 2000) and the proponents of the temporal single system interpretation of Marx's value theory (e.g., Kliman and McGlone 1999). To the best of my knowledge, none of these authors have offered any convincing argument for abandoning the standard definition of value, $\boldsymbol{\lambda} = \boldsymbol{\lambda A} + \boldsymbol{l}$, and replacing with the non-standard one that they use. Hence, I will not pursue this literature here.

[10] While the framework developed through the new interpretation does not preclude investigating issues related to the divergence of value and prices of individual commodities such an investigation has not been taken up for quantitative empirical research till now. Mohun (2004) suggests that such issues are "at best of only doubtful interest" in the new interpretation framework. It is not clear why.

$$\frac{\Pi}{Y} = \frac{Y - W}{Y} = 1 - \frac{W}{Y} = 1 - \frac{w}{y}$$

where W is the total wage bill, $w = W/(L * P_y)$ is the real product wage, $y = Y/(L * P_y)$ is the real labour productivity, and P_y is some index of price of value added (e.g., GDP deflator).

The output-capital ratio is decomposed as

$$\frac{Y}{K} = \left(\frac{Y_r}{K_r}\right) \times \left(\frac{P_y}{P_k}\right)$$

where Y_r is real value added, K_r is real capital stock, P_y is an index of price of value added, and P_k is an index of price of capital stock.

Change in, or the growth rate of, each of the components in these decompositions is studied and then related to changes in broader political economic factors like the contours of class struggle, evolution of state policies, and other relevant factors. This approach has been used to study long run capitalist evolution as also episodes of crises under capitalism (Duménil *et al*. 1984, Duménil *et al*. 1985, Michl 1988, Duménil and Lévy 1993, Foley and Michl 1999, Marquetti, Filho and Lautert 2010, Duménil and Lévy 2004, Duménil and Lévy 2011, Basu and Vasudevan 2013, Mohun 2013).

4.3.1.2. Short Run Analysis

The profit rate decomposition analysis in the new interpretation framework is related to a larger and older literature that has also used profit rate decomposition to study causes of crisis. This literature goes back to at least Weisskopf (1979) but is different from the new interpretation framework in that its primary focus is the short run. That is why, in addition to technology and distribution, it uses a third term to capture demand problems – capacity utilization rate – in the decomposition of the profit rate. Letting Z refers to capacity output, the three-term decomposition can be written as

$$r \equiv \frac{\Pi}{K} = \left(\frac{\Pi}{Y}\right) \times \left(\frac{\mathrm{Y}}{Z}\right) \times \left(\frac{\mathrm{Z}}{K}\right)$$

where the first term on the right Π/Y is the profit share, the second term Y/Z is the capacity utilization rate, and the last term Z/K is the capacity-capital ratio. Much as in the medium run analysis, each of the term in the decomposition is further broken into its real and price components for further detailed study. Following the pioneering work of Weisskopf (1979), many scholars have used and extended this strand of the literature (e.g., Henley 1987, Bakir and Campbell 2009, Kotz 2009, Izquierdo 2013).

4.3.2. Biased Technical Change

The analysis of technical change under capitalism has also allowed the new interpretation to link with broader debates on growth and distribution in heterodox macroeconomics (Foley and Michl 1999, Duménil and Lévy 2003, Michl 2009). For a large group of capitalist countries and over long periods of time, it is observed that technical change is biased towards labour, i.e., labour productivity increases even as the output-capital ratio declines. In the language of macroeconomics, such a pattern of technical change would be characterized as labour saving capital using. It also matches the depiction of the process of technical change under capitalism developed by Marx. Hence, Foley and Michl (1999) call

this Marx-biased technical change (MBTC) and Duménil and Lévy (2003) describe it as trajectories à la Marx.

There are two reasons why such patterns of technical change might be interesting. First, since the share of profits in national income is relatively stable over long periods of time, especially before the 1980s, such periods of rising labour productivity and declining output-capital ratio puts downward pressure on the rate of profit and could precipitate a period of crisis in capitalism. Hence, MBTC can become an important component of explanations of crisis tendencies in capitalist economies.

Second, MBTC gives rise to an important proposition relating to the viability of technical change, i.e., whether a new technique of production will be adopted by profit maximizing capitalist firms? The viability condition, in turn, can be used to derive competing testable implications about income distribution corresponding to a neoclassical and a classical-Marxian viewpoint. The neoclassical viewpoint suggests that the observed profit share should be equal to the viability parameter (equivalently, that the observed real wage rate should equal the marginal product of labour),

$$\pi = \pi^* \qquad (29)$$

and the classical-Marxian viewpoint claims that the profit share should be less than the viability parameter (equivalently, that the real wage should exceed the marginal product of labour)

$$\pi < \pi^* \qquad (30)$$

where the viability parameter is defined as

$$\pi^* = 1 + \frac{(-\chi)(1+\gamma)}{(1+\eta)(\gamma-\chi)} \qquad (31)$$

with χ referring to the growth rate of the output-capital ratio, η denoting the expected growth rate of the real wage rate, and γ referring to the growth rate of labour productivity (for details, see appendix D).

Basu (2010) considers two variants of the model in (29), (30) and (31). The first variant has zero expected growth in the real wage rate, i.e., $\eta = 0$; and the second has the expected real wage rate growing in tandem with labour productivity, i.e., $\gamma = \eta$. The second variant is more realistic because it accords well with historical data suggesting roughly equal growth rate of productivity and wages before the 1980s. Hence, Basu (2010) uses the second variant to conduct empirical analysis in a cross country regression framework with the following model

$$\pi_i^* = \alpha + \beta\pi_i + \text{controls} + u_i \qquad (32)$$

To test the neoclassical versus the classical-Marxian models, he tests the joint null hypothesis

$$H_0: \alpha = 0 \;\&\; \beta = 1 \qquad (33)$$

with a F-test. Since the neoclassical model suggests that the viability parameter and the actual profit share are equal, rejection of the null hypothesis (33) can be interpreted as evidence against the neoclassical model.

Using data from the Extended Penn World Tables 2.1, Basu (2010) conducts the test for the period 1963-2000 for two sets of countries: (a) 25 OECD countries; and (b) 117 countries (which includes 25 OECD

countries). He reports high values of the F-statistic (which are significant at standard levels) for both sets of countries and concludes that there is strong evidence against the neoclassical viewpoint.[11] Using data from the period 1963 to 1998, Michl (2009) also presents strong evidence – from cross sectional, time series and pooled regressions – against the neoclassical model.

5. Conclusion

In this paper I have reviewed two strands of recent quantitative empirical research in Marxist political economy. I started out by reviewing a literature that is widely accepted – both as to its necessity and basic methodology – in the Marxist tradition, viz., construction of national income and product accounts that is consistent with Marxist political economy. Starting with traditional input-output (and national accounts) data, the operationalization of two key conceptual features can take us to Marxist national accounts: the distinction between production and nonproduction activities, and the distinction between productive and nonproductive labour.

There is a long and distinguished literature that has worked on this issue, including Mage (1963), Wolff (1977), and Moseley (1982), and has culminated in Shaikh and Tonak (1994). There is an urgent need to carry this work forward by applying Shaikh and Tonak's (1994) methodology, with certain modifications that may be necessary (e.g., Mohun 2005), to construct and disseminate Marxist national accounts for as many countries as data limitations make possible. This is an area that can fruitfully engage future Marxist scholars and it is encouraging to see that work on these issues has already started (e.g., Paitaridis and Tsoulfidis 2012, and Rotta 2014).

The second area I reviewed in this paper is related to the value controversy. Value is a foundational concept in Marxian political economy, and its interpretation and relationship to observable variables denominated in prices has been a source of lively debate and, often, bitter controversy for more than a century. Starting almost immediately with the publication of Volume III of Capital in 1894, critics have argued that Marx work in the three volumes of Capital does not provide an internally consistent labour theory of value. Repeating points made earlier by critics like Dmitriev and Samuleson, Ian Steedman (1977) elaborated this critique – what he called a Sraffa-based critique – in a forceful manner in the 1970s. The main claim in the critique was that value categories are redundant to the analysis of capitalism because prices of production and a uniform rate of profit can be computed directly from data on technology and distribution (real wage bundle).

One can discern at least five different responses to the Sraffa-based critique that I have briefly indicated in section 3.3. In this paper I have focused on the three Marxist strands that have not only responded to the redundancy critique but also opened up progressive research programs with nontrivial empirical components. Each of these responses accepts the fact that prices of production and a uniform rate of

[11] An alternative way to test for the existence of Marx-biased technical change is to use cointegration analysis. The basic idea is straightforward. Labour saving technical change increases under wage pressure; hence, there is a causal relationship running from real wages to labour productivity. Since both real wages and labour productivity have unit roots, an eminently plausible hypothesis is that they are cointegrated, implying that there is a long run relationship between the two variables. Moreover, it is this long run relationship that, driven by Marx-biased technical change, gives rise to non-trending factor shares. This approach has been explored by Marquetti (2004) for the US economy and by de Souza (2014) for a panel of countries.

profit can be computed from data on technology and distribution. But, each argue in their own way, that this does not make value categories redundant.

The first response was developed through the work of Anwar Shaikh in the late 1970s. The argument contained in Shaikh (1984) provides, to my mind, a partial rebuttal of the Sraffa-based critique by converting the question into an empirical one. According to this "standard interpretation" response the interesting question is not that values and prices diverge, which is uncontroversial, but by how much? In essence, this strand advances Ricardo's neglected claim that values and prices are approximately equal. In Shaikh (1984), a theoretical argument is constructed to show that even in the face of wide divergence of organic composition of capital across sectors, relative prices can be "close" to relative values because they depend on integrated capital labour ratios and not actual capital-labour ratios. Using data from several capitalist economies, this strand then demonstrates that relative values and relative prices are strongly correlated. While this is an important first step, I have pointed out in section 4.1.1 that the methods employed for such empirical analysis could be improved upon.

The second response emerged from the work of Farjoun and Machover (1983), who argued for a probabilistic approach to Marxist political economy. Farjoun and Machover's rebuttal of the Sraffa-based critique rested on asking a simple question: is there any justification for the assumption of a single, uniform rate of profit? The answer is: no. Empirical evidence shows that in any economy at any point in time, there is a distribution of the rate of profit, rather than a single value of the rate of profit. Thus, long run equilibrium of capitalist economies are characterized by an equilibrium distribution of the rate of profit, and not a single uniform rate of profit. Once the assumption of a uniform rate of profit is abandoned, the whole Sraffa-based critique collapses.

In addition to the rebuttal of the Sraffa-based critique, Farjoun and Machover (1983) also develop a positive theory. They argue that the rate of profit follows a gamma distribution, and develop an elaborate argument to show that the specific price (price per unit of labour embodied) follows a normal distribution. While the distribution of the rate of profit is derived on the basis of an analogy from statistical mechanics, the distribution of the specific price is derived by taking recourse to some central limit theorem. Evidence presented by Fröhlich (2013) for the German economy shows that the rate of profit does indeed follow a gamma distribution but that the specific price follows a log normal distribution (instead of a normal distribution). To strengthen the "probabilistic interpretation" framework, a better and more rigorous argument for deriving the distribution of the rate of profit is needed; moreover, since specific price is bounded away from zero, the argument for deriving its distribution as normal needs to be revisited.

The third response emerged as the "new interpretation" of Marx's value theory, developed independently by Foley (1982) and Duménil (1983/1984). The new interpretation's rebuttal of the Sraffa-based critique relies on developing a theoretically informed, internally consistent *ex post* accounting framework. Two definitions anchor this accounting framework. First, the value of money is defined as the ratio of the total labour embodied in the net output (measured in labour time units) and the money value added (measured in prices). Second, the value of labour power is defined as the product of the nominal money wage and the value of money. These two definitions imply that profits are proportion to unpaid labour time, and value added is proportional to total labour time (the latter by definition). This is the way in which the redundancy argument is addressed in the new interpretation:

since value and price are equal at the aggregate level, the question of redundancy itself becomes redundant.

Even though the deviation of individual values and prices could be investigated in the new interpretation framework, researchers have studied other questions. One approach that has been used frequently is a profit rate decomposition analysis to study medium and long run dynamics of capitalist economies. Researchers using this framework have also connected to the larger heterodox macroeconomics literature by analyzing issues of technical change and income distribution.

This brief review of recent quantitative empirical research in Marxist political economy has highlighted some of the interesting issues that are currently being investigated. While the details are important, it is also important to highlight the big picture. Marxist political economy has used a serious critique as a productive impulse. It has used the Sraffa-based critique to reinvent itself, making its arguments theoretically robust and its empirical basis ever more firm. Future researchers need to continue this project.

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Appendix

A. Value-Price Relationship in Shaikh (1984)

The price of any commodity is the sum of materials (i.e., non-labour) costs, wage costs and profits; hence

$$p = wL + \pi + M \qquad \text{(A1)}$$

where p is the price, w is the wage rate, L is the labour input, π is the profit, and M is the material cost (including depreciation), each of these expressed per unit of the commodity. The material cost M is the price received by some other capitalist firm. Hence, it can be decomposed into the sum of three analogous terms as

$$M = wL^{(1)} + \pi^{(1)} + M^{(1)} \qquad \text{(A2)}$$

so that substituting (A2) in (A1) gives

$$p = wL + \pi + wL^{(1)} + \pi^{(1)} + M^{(1)} \qquad \text{(A3)}$$

If this process of substitution is carried to the limit so that the residual material cost approaches zero, the original price will become the sum of two terms

$$p = W^T + \pi^T \qquad \text{(A4)}$$

where W^T is total direct and indirect labour cost incurred in producing one unit of the commodity

$$W^T = wL^T \equiv w\left(L + L^{(1)} + L^{(2)} + L^{(3)} + \cdots\right) \qquad \text{(A5)}$$

and π^T is the total direct and indirect profits earned in producing one unit of the commodity

$$\pi^T \equiv \left(\pi + \pi^{(1)} + \pi^{(2)} + \pi^{(3)} + \cdots\right) \qquad \text{(A6)}$$

The sum of the direct and indirect labour that is required for producing one unit of the commodity is its value, λ, i.e.,

$$\lambda = \left(L + L^{(1)} + L^{(2)} + L^{(3)} + \cdots\right) \qquad \text{(A7)}$$

Substituting (A7) in (A5) gives the link between price and value

$$p = wL^T + \pi^T = wL^T\left(1 + \frac{\pi^T}{wL^T}\right) = w\lambda(1 + y) \qquad \text{(A8)}$$

where y is the integrated profit-wage ratio.

Conducting this analysis for any two commodities indexed i and j and taking the ratio of their prices we get

$$p_{ij} = \lambda_{ij} y_{ij} \qquad \text{(A9)}$$

where

$$p_{ij} \equiv \frac{p_i}{p_j}; \quad \lambda_{ij} \equiv \frac{\lambda_i}{\lambda_j}; \quad \text{and } y_{ij} \equiv \frac{(1+y_i)}{(1+y_j)} \qquad \text{(A10)}$$

The relationship in (A9) and (A10) applies to any arbitrary set of prices. Hence, they apply to prices of production. But with prices of production comes a uniform rate of profit r so that

$$\pi = rK \text{ and } \pi^T = r\,K^T \qquad \text{(A11)}$$

where K and K^T are the capital stock and the integrated capital stock respectively. Thus, the integrated profit-wage ratio now becomes

$$z = \frac{\pi^T}{wL^T} = \left(\frac{r}{w}\right)\left(\frac{K^T}{L^T}\right) \qquad \text{(A12)}$$

Denoting prices of production with the superscript n (to stand for neo-Ricardian), and substituting the expression for the integrated capital stock (A12) in (A8) shows that

$$p_{ij}^n = \lambda_{ij} z_{ij} \qquad \text{(A13)}$$

where

$$p_{ij}^n \equiv \frac{p_i^n}{p_j^n}; \quad \lambda_{ij} \equiv \frac{\lambda_i}{\lambda_j}; \quad \text{and } z_{ij} \equiv \frac{\left(1+\frac{r}{w}k_i^T\right)}{\left(1+\frac{r}{w}k_j^T\right)} \qquad \text{(A14)}$$

This is the expression in (11) and (12) in the text.

B. Gamma Distribution

If X has a gamma distribution with shape parameter k and scale parameter b, then X has the probability density function given by

$$f(x) = \frac{1}{\Gamma(\mathrm{k})\,\mathrm{b}^{\mathrm{k}}} x^{k-1} e^{-x/b}, \; 0 < x < \infty \qquad \text{(B1)}$$

where $\Gamma(k)$ is the gamma function defined by

$$\Gamma(k) = \int_0^\infty s^{k-1} e^{-s} ds, \quad k \in (0, \infty). \qquad \text{(B2)}$$

The following properties of X are useful.

1. If X has a gamma distribution with shape parameter k and scale parameter b, then
   a. $\mathbb{E}(X) = kb$
   b. $Var(X) = kb^2$
2. If X has a gamma distribution with shape parameter k and scale parameter b, then
   a. $\mathbb{E}(X^n) = b^n \times \frac{\Gamma(n+k)}{\Gamma(k)}$, for $n \geq 0$
   b. $\mathbb{E}(X^n) = b^n n^{(k)} = b^n n\,(n-1)(n-2)\cdots(n-k+1)$ if $n \in \mathbb{N}$.
3. If X has a gamma distribution with shape parameter k and scale parameter b, then its moment generating function is given by

$$\mathbb{E}(e^{tX}) = \frac{1}{(1-bt)^k}, \; t < \frac{1}{b} \qquad \text{(B3)}$$

C. New Interpretation

In the context of a n-commodity economy, let $\boldsymbol{x}$ and $\boldsymbol{y}$ denote $(n \times 1)$ vectors of net and gross output, respectively; let $\boldsymbol{A}$ denote the $(n \times n)$ input-output matrix. Let $\boldsymbol{\lambda}$ and $\boldsymbol{l}$ denote $(1 \times n)$ vectors of values (i.e., labour embodied) and direct labour inputs, respectively. Then, we have the following relationships:

$$\boldsymbol{\lambda} = \boldsymbol{\lambda A} + \boldsymbol{l} \qquad \text{(C1)}$$

$$\boldsymbol{x} = \boldsymbol{A x} + \boldsymbol{y} \qquad \text{(C2)}$$

Post-multiplying (C1) by $\boldsymbol{x}$ and pre-multiplying (C2) by $\boldsymbol{\lambda}$, and equating the two gives

$$\boldsymbol{lx} = \boldsymbol{\lambda y} \qquad \text{(C3)}$$

which shows that the total labour involved in producing the gross output is embodied in the net output. Since the money value of the net output is the sum of wages and profits, we have

$$\boldsymbol{py} = w\boldsymbol{lx} + \Pi = W + \Pi \qquad \text{(C4)}$$

where $\boldsymbol{p}$ is the $(1 \times n)$ vectors of prices, w is the nominal wage rate, $W(= w\boldsymbol{lx})$ is the total wage bill, and Π is aggregate profits. By definition, surplus value (S) is the difference between the total labour embodied in the net output and variable capital. Thus,

$$S \equiv \boldsymbol{lx} - w\boldsymbol{lx}m = \left(\frac{\boldsymbol{lx}}{m} - W\right)m = (\boldsymbol{py} - W)m = \Pi m$$

where m is the value of money defined in (26). Since the rate of exploitation, e, is the ratio of surplus value and variable capital, we get

$$e = \frac{\Pi m}{W m} = \frac{\Pi}{W}$$

D. Generalized Viability Condition

A technique of production is the pair (x, ρ), where x is labour productivity and ρ is the output-capital ratio, and the set of all currently existing techniques of production represent technology. Technological progress is addition of new techniques to technology. In a capitalist society, the distribution of income between the two fundamental classes can be represented by

$$w = x - kv \qquad \text{(D1)}$$

where w is the wage rate and $k = x/\rho$ is the capital intensity of a technique of production. Represented in $v - w$ space, the relationship in (D1) is a downward sloping straight line with an intercept x and slope $-k$, as represented in Figure 2. We can call it the distribution schedule because it represents the class struggle over the distribution of income in capitalism.

Suppose the current best-practice technique of production is given by (x, ρ) and a new technique (x', ρ') becomes available with $x' = x(1 + \gamma)$, and $\rho' = \rho(1 = \chi)$. Note that each technique of production corresponds to a different distribution schedule in Figure 2. Now, suppose the current (real)

wage rate of w and capitalist firms expected it to grow at some positive rate $\eta > 0$, so that $w' = w(1 + \eta)$, and let π denote the current profit share, i.e., $\pi = (1 - w/x)$. Is the new technique of production "viable", i.e., would a profit-maximizing capitalist firm choose the new technique of production?

[Figure 2 about here]

Let v_n^e denote the expected rate of profit that would arise if the new technique of production were to be adopted. Then,

$$v_n^e = \left(1 - \frac{w'}{x'}\right)\rho' \qquad \text{(D2)}$$

If, on the other hand, the current technique of production continues to be used but the wage rate increases to w', the profit rate would be

$$v_o^e = \left(1 - \frac{w'}{x}\right)\rho \qquad \text{(D3)}$$

To understand the viability condition, let us start by looking at the wage rate. If the expected wage rate w' is bigger than the "switch point" wage rate (the wage rate given by the intersection of the two distribution schedules)

$$w' > w^* \qquad \text{(D4)}$$

then the corresponding relationship among the profit rates is given by

$$v_n^e > v_o^e \qquad \text{(D5)}$$

i.e., the expected profit rate when using the new technique of production is higher than the expected profit rate with the old technique. Hence profit maximizing capitalist firms will adopt the new technique of production, i.e., the new technique of production will be viable. A little algebra shows that (D4) will be satisfied if

$$\pi < 1 + \frac{\chi(1+\gamma)}{(1+\eta)(\gamma-\chi)} = \pi^* \qquad \text{(D6)}$$

which is the model in (29), (30) and (31) in the main text.

In the above analysis, the switch point wage rate (w^*) plays an important role. If the expected wage rate if higher than w^* then the firm will switch from the old to the new technique of production; if the expected wage rate is lower or equal to w^*, then the firm will continue using the old technique. It is in this sense that the intersection of the two distribution schedules is a *switch* point.

But how does this relate to the neoclassical model? Recall that in a neoclassical framework, technology is represented by a smooth production function, e.g., a Cobb-Douglas production function. In such a set-up, even a small change in the wage rate leads to a new technique of production being chosen. This is because in a neoclassical set-up the curve that is analogous to ABC in Figure 2 (the north-east frontier of the intersection of the two distribution schedules) would be a smooth convex (to the origin) curve. Hence, every point on such a curve would be a switch point. Thus, in terms of the classical-Marxian model, the neoclassical position is represented by the condition that $w' = w^*$ or $\pi = \pi^*$.

To summarize: the neoclassical model implies that $\pi = \pi^*$and the classical-Marxian model implies that $\pi < \pi^*$. These two alternatives are tested in the empirical model in (32) and (33) in the main text. For more details see Foley and Michl (1999, ch. 7), Michl (2009), and Basu (2010).

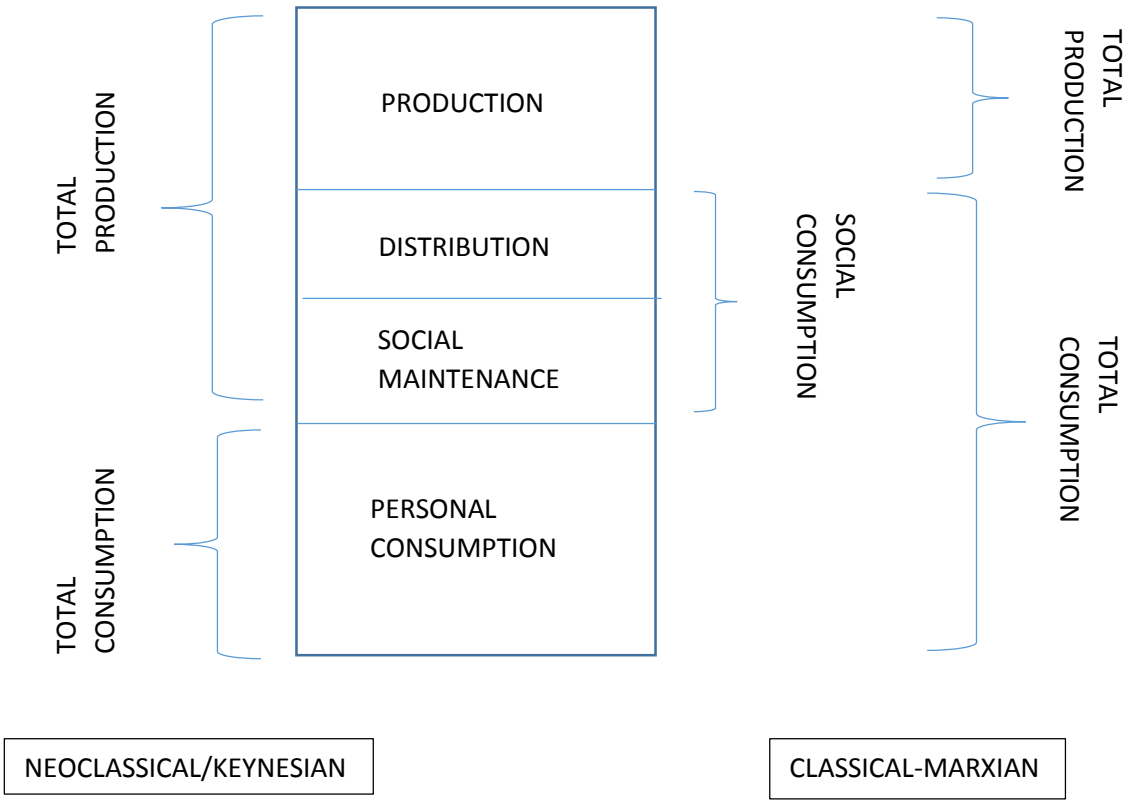

Figure 1: Alternative classifications of the four basic activities of social reproduction in the Neoclassical/Keynesian and Classical-Marxian traditions.

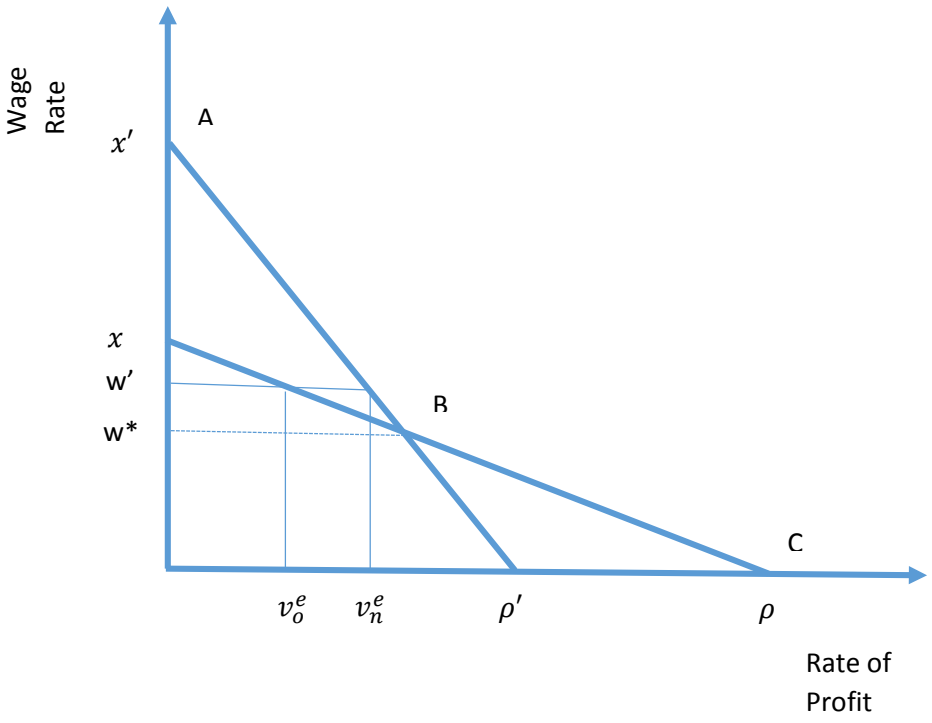

Figure 2: Marx-biased Technical Change and the Choice of Technique