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Fiscal and Monetary Policy Rules In an Unstable Economy

Soon Ryoo
Adelphi University, sryoo@adelphi.edu

Peter Skott
University of Massachusetts - Amherst, pskott@econs.umass.edu

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by

Ryoo, Soon
Skott, Peter

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Soon Ryoo[†] and Peter Skott[‡]

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Abstract

This paper examines the implications of different monetary and fiscal policy rules in an economy characterized by Harrodian instability. We show that (i) a monetary rule along Taylor lines can be stabilizing for low debt ratios but becomes de-stabilizing if the debt ratio exceeds a certain threshold, (ii) a 'Keynesian' fiscal policy rule can stabilize the economy at full employment, (iii) a fiscal 'austerity' rule that links fiscal parameters to deviations from a target debt ratio fails to adjust the 'warranted' to the 'natural' growth rate and destabilizes the warranted path, (iv) instability may arise from a combination of fiscal and monetary policy rules which separately would stabilize the system, and (v) austerity rules can in some circumstances enhance the stabilizing effects of monetary policy.

JEL classification: E12, E52, E62, E63

Key words: functional finance, fiscal policy rule, austerity, public debt, Harrodian instability

1 Introduction

Most of the current literature on fiscal and monetary policy rules consider a New Keynesian DSGE framework. The implications of a policy rule, however, depend on the properties of the economy in which it operates, and the conclusions derived from these models may not be robust.

One notable feature of DSGE models is the absence of endogenous tendencies to instability; fluctuations arise solely as the result of exogenous stochastic shocks. There are many potential sources of instability but multiplier-accelerator mechanisms in various forms have – rightly, in our view – received great attention in the Keynesian literature.[1] In this paper we take a benchmark Harrodian model as our point of departure and leave out other possible sources of instability (Minskian instability, for instance).

Extending the Harrodian analysis to a corporate economy with explicit financial assets, we examine the implications of different fiscal policy rules, both alone and in combination with a Taylor rule for monetary policy. The term 'policy rule' is used in a weak sense. The emphasis on fiscal and monetary *rules* is often motivated by the biases that allegedly arise from discretionary policy; monetary policy, it is argued, is subject to an inflation bias and fiscal policy suffers from deficit biases (Kopits 2001,

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[†]Department of Finance and Economics, Adelphi University, Garden City, NY 11530, USA; email: sryoo@.adelphi.edu

[‡](Corresponding author) Department of Economics, University of Massachusetts, Amherst, MA 01003,USA, and Aalborg University; email: pskott@econs.umass.edu.

[1]Harrod (1939), Samuelson (1939), Kaldor (1940) and Hicks (1950) are classic contributions. More recent work includes Skott (1989, 2015a), Chiarella et al. (2005), Fazzari et al. (2013), von Arnim and Barrales (2015).

Calmfors and Wren-Lewis (2011). We use the term 'policy rule' simply as a short-hand for systematic policy patterns, whether or not these patterns are the result of binding rules.

We show that (i) a monetary rule along Taylor lines can be stabilizing for low debt ratios but becomes de-stabilizing if the debt ratio exceeds a certain threshold, (ii) a Keynesian fiscal policy rule can bring the 'warranted growth rate' into line with the 'natural growth rate' and stabilize the economy at the warranted rate, (iii) a fiscal austerity rule (which links fiscal parameters to deviations from a target debt ratio) fails to adjust the warranted to the natural growth rate and destabilizes the warranted path, (iv) instability may arise from a combination of fiscal and monetary policy rules which separately would stabilize the system, and (v) austerity rules can in some circumstances enhance the stabilizing effects of monetary policy (compared to a completely passive fiscal policy).

The analysis is informed by a 'functional finance' approach (Lerner 1943).[2] We evaluate monetary and fiscal policies based on their ability to stabilize the economy at full employment, avoid inflation and ensure a desired level of investment. Public deficits and public debt emerge as the consequence of the policy but have no independent value.[3]

Our conclusions are very different from contemporary orthodoxy. A benchmark RBC model, first, has no aggregate demand problems. When New Keynesian imperfections are added, stochastic shocks and price rigidities may give rise to short-run aggregate demand problems, but there is no need for aggregate demand policy in the long run. A fully optimal policy may be quite complex, but Schmitt-Grohe and Uribe (2007) and Kirsinova et al. (2009) find minimal welfare losses compared to a fully optimal policy if monetary policy is used for short-run stabilization while taxes (or government spending) are set as an increasing (decreasing) function of the deviation of actual from target debt. This austerity rule for fiscal policy is destabilizing in our setting.

The share of investment in output (and the capital intensity of production), second, will be optimal if saving and investment are determined by intertemporally optimizing households with an infinite horizon. This optimality property is quite fragile and does not extend to models in which agents have a finite horizon. Skott and Ryoo (2014) analyze combinations of fiscal and monetary policy that are consistent with continuous full employment and a desired capital intensity in a standard OLG setting. The analysis shows how the implied trajectory for debt depends on, inter alia, the growth rate of the economy and the share of government consumption in total income: both increases in the growth rate and, somewhat paradoxically, increases in government consumption reduce the long-run debt ratio. The OLG structure and the assumptions about household behavior are quite orthodox in this and a companion paper which includes Keynesian problems of aggregate demand and long-run variations on confidence (Skott and Ryoo 2015). Similar long-run results linking the required debt ratio to government spending, the growth rate and the structure of taxation can be obtained in Keynesian models of a corporate economy (Schlicht 2006, Godley and Lavoie 2007, Ryoo and Skott 2013). The long-run analysis in these papers has serious limitations, however, if the steady growth path is unstable.

The model in this paper has steady-growth properties that are similar to those in Ryoo and Skott (2013). The structure of taxation is simpler in the present paper, but the main difference is the introduction of short-run dynamics and a more complicated policy problem. Ryoo and Skott (2013) analyzed how full employment could be maintained, starting from a position with the desired capital intensity and full employment, and assuming that the capital stock grows at the 'natural rate'. In this paper, the initial position is arbitrary and investment is determined endogenously along Harrodian

[2] Lerner's principle of functional finance

> "prescribes, first, the adjustment of total spending (by everybody in the economy, including the government) in order to eliminate both unemployment and inflation...; second, the adjustment of public holdings of money and of government bonds, by government borrowing or debt repayment, in order to achieve the rate of interest which results in the most desirable level of investment; and, third, the printing, hoarding or destruction of money as needed for carrying out the first two parts of the program." (Lerner 1943, p. 41)

[3] Skott (2015b) and Skott and Ryoo (2015) discuss functional finance in relation to 'secular stagnation.'

lines.

Stabilization issues have been addressed by a substantial (post-) Keynesian literature, including Lima and Setterfield (2008), Asada et al (2010), Franke (2015), Costa Lima et al. (2014) and Mason and Jayadev (2014). The papers by Franke and Mason and Jayadev are probably the closest to our analysis. Franke considers the use of monetary policy to stabilize a Harrodian economy. He does not include fiscal policy, however; there are no financial assets, and his specification of household behavior leaves out wealth effects. Like us, Mason and Jayadev analyze the implications of different fiscal rules and discuss interactions between monetary and fiscal policy, but the setting is different.[4] Using a short-run IS equation and a Phillips curve, Mason and Jayadev focus on how different policy assignments affect the stability of a short-run target equilibrium characterized by full employment and a constant debt ratio.

The analysis of fiscal policy and debt dynamics in an unstable economy inevitably becomes quite complex: it is hard to keep the number of state variables low. Asada et al. (2010) consider systems with up to nine state variables; Costa Lima et al. (2014) keep the system at five state variables but leave out Keynesian demand problems and do not include Harrodian elements (which typically add an extra state variable). Large systems may allow more 'realistic' assumptions but make it hard to identify robust results and disentangle the various mechanisms behind the results. Our approach in this paper is to (i) focus on a particular source of instability and (ii) specify policy rules in a way that keeps the dimension of the system as low as possible without sacrificing essential characteristics of the policies. Clearly this approach also has drawbacks. We return to these issues in the conclusion.

The rest of the paper falls in five sections. Section 2 lays out our extended Harrodian setting. Policy rules are introduced in Section 3 and section 4 analyzes the implications of the different rules for the stability properties of the full employment path. Section 5 presents some illustrative simulations. Section 6 concludes.

2 General setting

2.1 Harrodian benchmark

Consider a one-sector Harrodian economy without a public sector. Denoting aggregate output and the capital stock by Y and K, and assuming a Leontief technology, let $u = Y/K$ be the indicator of the rate of capacity utilization. A simple Harrodian specification relates the change in accumulation to the difference between actual and desired utilization:

$$\dot{g} = \lambda(u - u^d) \tag{1}$$

where $g = \hat{K} = I/K - \delta$ is the rate of accumulation and u^d represents the desired utilization rate. I and δ are real (gross) investment and the capital depreciation rate, respectively. Combining equation (1) with a traditional consumption function, the warranted growth rate is given by (see Appendix A)

$$g_w = (1 - c)u^d - \delta \tag{2}$$

where $1 - c$ is the saving rate.

For arbitrary exogenous values of u^d, c and δ, the warranted rate will only by a fluke be equal to an exogenously given natural rate, n.[5] A reconciliation between warranted and natural growth rates could be accomplished along Malthusian lines by endogenizing the natural rate (Leon-Ledesma and

[4] Using very different models, the interactions between monetary and fiscal policy have also been analyzed by, among others, Sargent and Wallace (1981) and Leeper (1991).

[5] The natural growth rate may include technical change; we use n to denote the sum of the growth rate of the labor force and the rate of Harrod-neutral technical change.

Thirlwall 2002, Dutt 2006). We consider it implausible, however, to suppose that the natural rate adjusts fully to whatever the warranted rate may be. Hence, part of the adjustment must fall on the warranted rate, and to simplify the exposition we take the natural rate as exogenous. The extent of the required adjustment in the warranted rate will be smaller if there is some flexibility in the natural rate, but the qualitative adjustment problem is the same, as long as the natural rate does not accommodate fully.

Adjustments in the warranted rate can come about through changes in u^d or $1 - c$. The choice of technique and the output-capital ratio may adjust in response to changes in factor prices (the Solow route); alternatively, the saving rate may adjust via Ramsey optimization, via differential saving rates for profit and wage income, or via fiscal policy. In this paper we focus on policy: monetary policy may affect the output-capital ratio and fiscal policy the rate of saving.

Harrod's second problem concerns the instability of the warranted growth path. The benchmark model implies that an accumulation rate above (below) the warranted rate induces a further increase (decrease) in the accumulation rate (see Appendix A). Thus, policy needs to stabilize the actual growth rate as well as ensure the adjustment of the warranted rate to the natural rate.

Subsections 2.2-2.3 consider the equalization of the warranted and natural rates; the stability question is the subject of section 3.

2.2 An extended Harrodian model

2.2.1 Interest rates and the choice of technique

The Leontief assumption in the benchmark model may seem highly restrictive. It can be seen, however, as a long-run implication of functional finance.

The desired level of investment is not constant in steady growth, and Lerner's prescription to set the interest rate so as to achieve the "most desirable level of investment" (Lerner 1943, p. 41) has long-run implications for the choice of technique. Like output and the capital stock, the level of investment will grow at the steady growth rate, keeping constant the capital-output ratio. The steady-growth version of 'the desired level of investment' therefore becomes 'the desired ratio of investment to output' or, equivalently, the 'desired output-capital ratio'. If σ is the desired output-capital ratio, functional finance prescribes that the interest rate be chosen so as to induce firms to choose a technique described by the Leontief production function,[6]

$$Y = \min\{\sigma K, L\} \tag{3}$$

L is employment in efficiency units and, without loss of generality, we have chosen units to get a labor productivity of one (if there is technical change, an efficiency unit of labor corresponds to a steadily decreasing amount labor time). The σ parameter defines the maximum output-capital ratio (given the chosen technique).

The steady-growth solution must have $r = \bar{r}$ where $\bar{r}$ is the real interest rate associated with the optimal capital intensity σ, but short-run variations in r may be used to stabilize the system at the steady growth path.[7] The inherited stock of fixed capital severely limits the scope for changes in the capital intensity in the short run and, as highlighted by the capital controversy, the possibility of reswitching and capital reversing makes the effects of changes in r on the choice of technique uncertain.

[6] A formalization of the argument can be found in Skott (1989, chapter 5) and Skott and Ryoo (2015). The capital controversy in the 1960s and 1970s highlighted the difficulties of constructing an aggregate production function and demonstrated, in particular, how theories that rely on movements along a smooth production function face intrinsic problems and contradiction. But the insights from the capital controversy do not imply that only one technique is available; nor do they invalidate the influence of the cost of finance on the choice of technique. For simplicity we take the cost of finance to be equal to the real rate of interest.

[7] For simplicity we assume that that $\bar{r}$ is unique. Depending on the set of potential techniques, multiple values of the interest rate may be consistent with the same choice of technique.

It therefore seems reasonable to treat the capital intensity – the Leontief production function (3) – as independent of short-term deviations of r from $\bar{r}$. This assumption does not exclude interest rate effects on investment. Firms typically want to hold excess capacity, and standard motivations for firms to maintain excess capacity imply that the desired utilization rate depends on the cost of finance (Skott 1989). Excess capacity, for instance, enables a firm to take advantage of sudden increases in demand, but this potential benefit has to be balanced against the costs of maintaining excess capacity. The costs are increasing in the cost of finance, and the desired utilization rate will therefore depend positively on the real interest rate. This relation between interest rate and desired utilization is not affected by the capital controversy. Using a linear specification we assume that

$$u^d = u^* + \theta(r - \bar{r}), \quad \theta > 0 \tag{4}$$

where u^* denotes the desired utilization rate at $r = \bar{r}$.

Equation (4) and the investment function (1) imply that the interest rate influences the rates of change of investment and aggregate demand but leave the rate of accumulation g as predetermined at any moment. Most short-run specifications of investment, by contrast, allow for contemporaneous effects of the interest rate on the level investment. Our extended Harrodian formulation in equations (4)-(6) therefore includes both level and rate-of-change effects:

$$g = a + a_1(u - u^d) - a_2(r - \bar{r}); \quad a_1 > 0, a_2 > 0 \tag{5}$$
$$\dot{a} = \lambda(u - u^d) \tag{6}$$

where u^d is given by (4).

2.2.2 Government consumption, taxes and public debt

The benchmark model has no public sector which is clearly a non-starter for an analysis of fiscal policy. Adding a public sector, we consider two potential fiscal instruments: government consumption and a tax rate.

For present purposes it seems reasonable to treat the long-run ratio of government consumption to the capital stock as structurally determined; the need for schools, bridges, police officers, etc. will typically depend on the size of the economy. Thus, we take the steady-growth value of the ratio of real government consumption (G) to the capital stock as exogenously given,

$$G/K = \bar{\gamma} \tag{7}$$

This exogeneity assumption glosses over the fact that the value of γ – the appropriate size of government – is strongly contested. Arguably, however, the different positions on this issue are largely tangential to the Harrodian problems of influencing the warranted rate.

Having fixed the long-run value of γ, the tax rate is determined endogenously by a fiscal rule. We assume a uniform tax rate τ on all household income, that is, on the sum of wages, dividends and (real) interest on both government and corporate debt. Formally, tax revenues are given by

$$T = \tau[pY - R + iB - \pi(B + D)] \tag{8}$$

where i and $r = i - \pi$ are the nominal and real interest rates; T is total nominal taxes (net of transfers); p is the goods price and $\pi = \hat{p}$ the rate of inflation; B is public debt; D and R are corporate debt and retained earnings;[8] 'hats' above a variable are used to denote a growth rate ($\hat{x} = (dx/dt)/x$).[9]

[8] We have

$$pY = W + \Pi = W + Div + iD + R$$

where W, Π, Div, D, R and i denote wages, profits, dividends, corporate debt, retained earnings and the nominal interest rate.

[9] Equation (8) assumes that households are taxed only on the real return on their financial assets. As an alternative,

Government debt is predetermined at any moment but evolves over time,

$$\begin{aligned}\dot{B} &= iB + pG - T \\ &= iB + \gamma pK - \tau(pY - R + rB - \pi D)\end{aligned} \tag{9}$$

Thus, the dynamics of the debt-capital ratio are given by

$$\dot{b} = (r-g)b + \gamma - \tau\left(u - \frac{R}{pK} - \pi\frac{D}{pK} + rb\right) \tag{10}$$

where $b = B/(pK)$.

2.2.3 Private consumption

Consumption decisions typically depend on non-human wealth and expected future wage income, as well as on current disposable income. The proportional consumption assumption in the benchmark model therefore needs to be modified.

In a corporate economy households' ownership of productive capital takes the form of equity. Disregarding housing and other, less important real assets, we assume that household wealth consists of equity and safe, short-term assets with a contractual rate of return.[10] The safe assets could include both bonds and bank deposits. For present purposes, however, the introduction of bank deposits would add very little, and we include only bonds. We treat corporate and government bonds as perfect substitutes.

Firms can issue new equity or buy back shares; they issue bonds, and they retain a proportion of their earnings. Our main focus in this paper is on public debt, and we simplify firms' financial behavior by assuming that (i) there is no net new issues of shares ($\dot{N} = 0$) by the corporate sector and that (ii) adjustments in the retention rate keep constant the debt-capital ratio for the corporate sector as a whole.[11] Formally,

$$D = \bar{d}pK \tag{11}$$

and

$$\dot{D} = \bar{d}pK(\pi + \hat{K}) \tag{12}$$

These assumptions imply that firms' finance constraint can be written

$$pI = R + \dot{D} + v\dot{N} = R + \dot{D} = R + \bar{d}pK(\pi + \hat{K}) \tag{13}$$

v is the price of shares and N the number of shares. The second equality follows from the assumption of no net equity issue and the third from differentiation of (11). Using (13) the ratio of retained earnings to capital can be written

$$\frac{R}{pK} = (1-\bar{d})g - \bar{d}\pi + \delta \tag{14}$$

where δ is the rate of depreciation of capital.

they could be taxed on the nominal return. Algebraically,

$$T = \tau(pY - R + iB)$$

This specification introduces inflation distortions: post-tax returns on bonds will be different if two otherwise identical economies have different inflation rates. These inflation effects would modify some of the quantitative results.

[10] Government bonds are taken to be short-term (always selling at par value). Treasury bills and notes make up about 80 percent of US government debt.

[11] The net issue of new equity in the US was positive but small (relative to total investment) in the early post war period but turned negative from the 1980s as buybacks increased.

Households' disposable income, Y^D, is given by

$$pY^D = [pY - R + iB] - \tau[pY + iB - R - \pi(D + B)] \tag{15}$$

and the consolidated budget constraint for the household sector can be written

$$pY^D = pC + v\dot{N} + \dot{B} + \dot{D} \tag{16}$$

By assumption, $\dot{N} = 0$ and equation (16) can be rewritten

$$pC = pY^D - (\dot{B} + \dot{D}) \tag{17}$$

Our description of consumption now follows the approach in Skott (1981, 1989) and Skott and Ryoo (2008). We assume that

$$(B + D)^* = \beta pC \tag{18}$$

$$(vN)^* = \alpha pC \tag{19}$$

where an asterisk is used to denote a target value, and β and α represent target stock-flow ratios. Assuming that households want to adjust their real bond holdings gradually towards the target level, we have

$$(\widehat{B + D})^d - \pi = \kappa \frac{(B + D)^* - (B + D)}{B + D} \tag{20}$$

or

$$(\dot{B} + \dot{D})^d = \kappa((B + D)^* - (B + D)) + \pi(B + D) \tag{21}$$

where $(\dot{B} + \dot{D})^d$ is the household flow demand for bonds. In equilibrium, this household flow demand must match the new bond issues, as given by equations (9) and (12)

Combining (17), (21), (18) and (12), and dividing through by pK, we now get the consumption function

$$\frac{C}{K} = c(1 - \tau)[u - \delta - (1 - \bar{d})g + rb] + c_\nu(b + \bar{d}) \tag{22}$$

where $c = 1/(1 + \kappa\beta)$ and $c_\nu = \kappa/(1 + \kappa\beta)$.[12] Bond holdings represent a fraction of wealth and the parameter c_ν therefore does not express the consumption propensity out of total wealth. If, say, $\beta/(\alpha + \beta) = 0.25$, a ballpark empirical estimate of 0.05 for the wealth effects on consumption corresponds to $c_\nu = 0.20$.

It should be noted that the target value α for the ratio of equity to consumption plays no role for aggregate consumption. Since there is no net issue of new equity, households as a group are unable to spend current income on the purchase of new shares; any attempt to do so simply generates capital gains as the price of shares is bid up until the target stock-flow ratio has been reached.[13] The target value β for the ratio of bonds to consumption affects the consumption rates c and c_ν, and the value of β could depend on expected rates of return as well as on expected future incomes. An increase in the interest rate r, for instance, could shift the desired portfolio towards bonds and raise $\beta/(\alpha + \beta)$; the income effect from higher returns, on the other hand, reduces the need to save for retirement which may lead to a decline in $\alpha + \beta$. The net effect on the target stock-flow ratio β and thereby the parameters c and c_ν is ambiguous. We leave out these complications and treat β as a constant.

[12] The alternative tax specification in which the tax rate applies to nominal disposable income would modify this equation, introducing a negative effect of inflation on consumption: if households are being taxed on the nominal return ($i(B + D)$), not the real return ($r(B + D)$), an increase in inflation would raise taxation and reduce consumption.

[13] These induced capital gains are the key to Kaldor's 'neo-Pasinetti theorem'; Kaldor (1966), Skott (1981).

2.2.4 Inflation

The benchmark model is silent about inflation. Our extended model assumes an expectations-augmented Phillips curve:

$$\pi = \pi^e + \eta_1(u - u^*) + \eta_2(k - k^*) \tag{23}$$

where π^e is expected inflation and k is the ratio of capital K to the labor force, L_f. The value of k is related to the employment rate e via[14]

$$e = uk.$$

k^* in (23) refers to the level of k that is required for full employment steady growth, i.e., $k^* = e^*/u^*$ where e^* represents full employment. It is assumed throughout this paper that full employment is well defined and determined independently of aggregate demand.[15]

The specification (23) generalizes a standard equation in which unanticipated inflation is determined by the employment rate e: it allows the utilization rate to influence inflation directly, in addition to its indirect effects via the employment rate.[16]

We assume an adaptive inflation expectations,

$$\dot{\pi}^e = \mu(\pi - \pi^e) \tag{24}$$

and, substituting (23) in (24), we have:

$$\dot{\pi}^e = \mu\eta_1(u - u^*) + \mu\eta_2(k - k^*) \tag{25}$$

2.3 Steady growth

Using (7) and (22), the equilibrium condition for the goods market – $Y = C + I + G$ – becomes

$$u = c(1-\tau)[u - \delta - (1-\bar{d})g + rb] + c_\nu(b + \bar{d}) + g + \delta + \bar{\gamma} \tag{26}$$

or

$$\begin{aligned} u \quad = \quad & \frac{1 - c(1-\tau)(1-\bar{d})}{1 - c(1-\tau)} g + \frac{c(1-\tau)r + c_\nu}{1 - c(1-\tau)} b \\ & + \frac{1}{1 - c(1-\tau)} \bar{\gamma} + \frac{c_\nu}{1 - c(1-\tau)} \bar{d} + \delta \end{aligned} \tag{27}$$

In steady growth with full employment we have $\dot{b} = \dot{a} = 0, r = \bar{r}, u^d = u^*$ and $g = n$. Hence – using equations (6), (27), (14) and (10) – we can solve for the steady-growth value of b and the associated

[14] The employment rate can be written

$$e = \frac{L}{L_f} = \frac{L}{Y}\frac{Y}{K}\frac{K}{L_f} = uk$$

where (by choice of units) $L/Y = 1$

[15] Hysteresis and money illusion make this simplifying assumption questionable. For present purposes, however, the assumption is relatively harmless.

[16] If

$$\pi = \pi^e + f(e) + g(u)$$

then, using a first-order approximation, we have

$$\begin{aligned} \pi &= \pi^e + f(e) + g(u) \\ &= \pi^e + f'(e^*)[k^*(u - u^*) + u^*(k - k^*)] + g'(u)(u - u^*) \end{aligned}$$

tax rate τ :

$$b^* = \frac{(1-c)(u^* - \bar{\gamma} - n - \delta)}{cn + c_\nu} - \bar{d} \tag{28}$$

$$\tau^* = 1 - \frac{u^* + nb^* - \bar{\gamma} - (1-\bar{d})n - \delta}{u^* + \bar{r}b^* - (1-\bar{d})n - \delta} \tag{29}$$

Equations (28)-(29) describe long-run fiscal requirements for full-employment growth. They are necessary but not sufficient for continuous full employment: in addition to the growth condition $g = n$, the ratio of the capital stock to the labor force k must be at k^* ($\equiv e^*/u^*$).

Equation (28) has two striking implications. The long-run debt ratio, first, is inversely related to government consumption. Austerity policies which cut public consumption with the aim of reducing debt are counter-productive: if full employment is to be maintained, the result of these policies is to raise the long-run debt ratio. Intuitively, the contractionary impact of a cut in government consumption has to be countered by a reduction in taxes. With a positive balanced budget multiplier, the tax reduction must exceed the cut in government consumption, and the resulting increase in the government deficit leads to a rise in the long-run debt ratio. Second, a decline in the natural growth rate – whether because of lower productivity growth or a lower growth of the labor force – will raise the required debt ratio. Empirical findings of a negative correlation between growth and debt are consistent with this result. But the causation is from growth to debt, and the correlation does not imply that lower debt will raise the growth rate.[17]

Equation (28) describes the stationary solution for the debt ratio. This stationary solution is conditionally stable: setting $g = n$, $r = \bar{r}$ and $u = u^*$, the full-employment trajectory of the debt ratio converges to b^*. To see this, combine the equilibrium condition (27) with the dynamic equation (10) and the expression for retained earnings, equation (14), to get

$$\dot{b} = -\left(\frac{c_\nu + cn}{c}\right) b + \frac{(1-c)(u^d - \bar{\gamma}) - n - \delta - (c_\nu + cn)\bar{d}}{c} \tag{30}$$

The coefficient on b is negative, and stability follows.

2.4 Automatic stabilizers and Harrodian instability

The conditional stability result is based on restrictive assumptions: it is assumed that accumulation is at the natural rate throughout the process and that tax rates are continuously adjusted so as to maintain utilization at the desired rate. A complete stability analysis needs to relax these assumptions and consider the dynamics starting from arbitrary initial conditions.

Arbitrary starting points in combination with the inherent instability of the Harrodian process complicate policy. In the conditional stability analysis leading to (30), tax policy was adjusted to maintain $u = u^d = u^*$. This relatively simple policy no longer suffices if the initial position is one with $g \neq n$ or $e \neq e^*$. But before introducing more complex fiscal and monetary policy rules, it should be noted that, even without active short-run policy adjustment, the introduction of a public sector exerts a stabilizing influence. This happens both because of the dampening effect of taxes on the multiplier and because of the effects of public debt on consumption. In principle these automatic stabilizers could eliminate the instability but plausible parameters values suggest that this is not the case: modest values of λ are sufficient to produce instability (see Appendix B).

[17] These steady-growth results can be obtained in other settings; e.g. Schlicht (2006), Godley and Lavoie (2008), Ryoo and Skott (2013), Skott and Ryoo (2014, 2015), and Skott (2015b).

3 Policy rules

We have taken the long-run level of government consumption (as a ratio of the capital stock) to be determined by the 'needs for government provided services' (cf. section 2.2.2). There may be good reasons, however, to treat government consumption as the active instrument for short-run stabilization. Changes in tax rates, first, may be harder to implement quickly than adjustments in the timing of government consumption, and short-run variations in taxes, second, may have limited effects on consumption. Thus, we take γ as the active instrument in the short run and assume that tax rates are kept constant at the steady-growth value in (29). This assumption is not critical; analogous results could be obtained using τ as the active instrument.

We consider two different fiscal regimes: a 'functional finance' regime that targets full employment and a 'sound finance' regime that targets the debt ratio. 'Perfect' implementations of these policies maintain full employment (in the case of functional finance) or the desired debt ratio (in the case of sound finance) at all times. Deviations from the targets occur under imperfect implementation. The fiscal regimes are analyzed in two scenarios. Monetary policy is completely passive in one scenario: the real rate of interest is kept constant at $\bar{r}$. The other scenario includes a Taylor rule for monetary policy.

Our Taylor rule assumes that the nominal interest rate, i, is determined by the utilization rate u, the ratio of the capital stock to the labor force, $k = K/L_f$, and the inflation rate:

$$i = \bar{r} + \pi^T + \rho_1(u - u^*) + \rho_2(k - k^*) + \rho_3(\pi - \pi^T) \tag{31}$$

where π and π^T denote actual and target inflation; u^* and $k^* = e^*/u^*$ are the steady-growth values of u and k and e^* represents full employment. The parameters satisfy $\rho_1 \geq 0, \rho_2 \geq 0$ and $\rho_3 > 1$. The specification in equation (31) represents a slight generalization of the standard Taylor formulation in that it allows for an influence of both employment and utilization in the interest rate.[18]

Using the Fisher equation $r = i - \pi^e$ and plugging (23) into (31), the Taylor rule can written as

$$\begin{aligned} r &= \bar{r} + (\rho_1 + \eta_1\rho_3)(u - u^*) + (\rho_2 + \eta_2\rho_3)(k - k^*) + (\rho_3 - 1)(\pi^e - \pi^T) \\ &\equiv \bar{r} + \tilde{\rho}_1(u - u^*) + \tilde{\rho}_2(k - k^*) + \tilde{\rho}_3(\pi^e - \pi^T) \end{aligned} \tag{32}$$

where $\tilde{\rho}_1 \equiv \rho_1 + \eta_1\rho_3 > 0$, $\tilde{\rho}_2 \equiv \rho_2 + \eta_2\rho_3 > 0$ and $\tilde{\rho}_3 \equiv \rho_3 - 1 > 0$.

The interest rate affects investment in two ways: an increase in real interest rates has an immediate negative effect (equation (5)) as well as a delayed effect via the state variable a (equations (4) and (6)). The consumption function (22) includes a positive effect of interest rates on disposable incomes.

Functional finance Algebraically, perfect functional finance implies that

$$uk = e^* \tag{33}$$

The complete stabilization of the employment rate at e^* may be desirable but in practice unattainable.[19] The ability of policy makers to neutralize the effects of changes in accumulation (changes in a) by adjustments in government consumption seems particularly questionable. We therefore consider a specification that allows effects of a on the utilization rate:

$$u = u^* + H(a, k), \quad H_1 \geq 0, H_2 < 0, H(n, k^*) = 0 \tag{34}$$

[18]Equation (31) can be seen as a linear approximation to the following general specification:

$$\begin{aligned} i &= f(e, u, \pi) \\ &= f(uk, u, \pi) \end{aligned}$$

[19]Arguably, perfect functional finance could be achieved through employer-of-last-resort policies (Wray (2000)). These policies raise a number of issues (Palley (2001)).

The specification in equation (34) includes the perfect version as a special case (let $H(a,k) = e^*/k - u^*$).[20]

Sound finance The specification of a sound-finance rule raises several questions. In analogy with equation (33) it might seem that a perfect version of a sound-finance rule should be given by

$$\frac{B}{Y} = \frac{b}{u} = m \tag{35}$$

where $m = (B/Y)^T$ is the target debt-income ratio. This specification, however, implies that

$$\gamma = \left[\frac{1 - c(1-\tau)}{m} - c(1-\tau)r - c_\nu\right] b - (1 - c(1-\tau))(g + \delta) - [c_\nu + c(1-\tau)g]\bar{d} \tag{36}$$

For all plausible values of the parameters and the debt target, both the expression for government consumption in equation (36) and the associated budget deficit are increasing in the debt ratio b.[21] Intuitively, the debt ratio has income in the denominator, and the ratio can be brought down in the short run by an expansionary policy that raises income. This policy is not what proponents of sound finance have in mind.

An alternative rule targets the debt-capital ratio b rather than the debt-income ratio b/u. The capital stock is an indicator of potential output and targeting the debt-capital ratio corresponds to targeting the ratio of debt to potential output. The debt ratio b is a state variable, however, and no choice of γ can instantaneously bring about the equality between b and an exogenous target value b^T. Our specification of sound finance therefore allows for deviations between the target b^T and the initial value of b.[22]

Analogously with (34) we assume that

$$u = u^* + \tilde{H}(a,b); \quad \tilde{H}_a \geq 0,\ \tilde{H}_b < 0,\ \tilde{H}(n, b^T) = 0 \tag{37}$$

General fiscal rule

The functional and sound finance rules can be seen as special cases of a general specification in which

$$u = u^* + \psi(a,k,b,\pi^e,r); \qquad \psi(n, k^*, b^T, \pi^T, \bar{r}) = 0 \tag{38}$$

or, alternatively,

$$\gamma = \bar{\gamma} + \phi(a,k,b,\pi^e,r) \tag{39}$$

The general rule (39) makes government consumption a function of the state variables a, k, b, π^e (and the interest rate r which itself becomes a function of the state variables if monetary policy follows the Taylor rule (32)). The special cases correspond to particular restrictions on $\psi(a,k,b,\pi^e,r)$ and $\phi(a,k,b,\pi^e,r)$.

It should be noted that both functional and sound finance imply feedback effects from the debt ratio to government consumption. The debt ratio may not be a target under functional finance, but it affects the short-run values for utilization and employment, and these in turn influence policy.

[20] A more general specification would include effects of the debt ratio and perhaps the expected inflation rate on utilization (i.e. $u = u(a,b,k,\pi^e)$). The dimension of the system would increase, however, making it much harder to derive clearcut results.

[21] We have

$$\partial\gamma/\partial b = [1 - c(1-\tau)]/m - c(1-\tau)r - c_\nu$$

Assuming that $c \leq 0.8, \tau \geq 0.2, c_\nu \leq 0.25, r \leq 0.05$ the partial is positive for $m < 1.27$.

[22] If the target is set equal to the current debt ratio – whatever that ratio may be – the task of the rule becomes to maintain a constant debt ratio over time. This random-walk implication of a perfect sound-finance rule is optimal in orthodox benchmark models with intertemporally optimizing households; the implication is not robust, however (Portes and Wren-Lewis 2014).

4 Analysis

4.1 Dynamic system

The dynamics for accumulation, the debt ratio and the expected inflation rate are given by (6), (10) and (25), and the dynamics of k follows directly from the definition. Thus, we have a four dimensional dynamic system in the state variables a, b, k, π^e :

$$\dot{a} = \lambda(u - u^d) \tag{40}$$
$$\dot{b} = (r-g)b + \gamma - \tau^*[u - \delta - (1-\bar{d})g + rb] \tag{41}$$
$$\dot{k} = k(g-n) \tag{42}$$
$$\dot{\pi}^e = \mu\eta_1(u-u^*) + \mu\eta_2(k-k^*) \tag{43}$$

where equation (41) is derived by substituting (14) into (10); g and u^d are given by (5) and (4); τ^* can be found by substituting (28) into (29). The determination of the utilization rate u, the interest rate r, and government consumption γ depends on the fiscal and monetary policy regime.

4.2 Pure monetary policy

This section examines stabilization through monetary policy. Fiscal policy is passive, and both government consumption and the tax rate are assumed to be at their steady growth values, $\gamma = \bar{\gamma}, \tau = \tau^*$.

The interest rate affects investment both in the short and the long run. Substituting (4) and (32) in (5) gives the short-run investment function:

$$g = a + [a_1 - (a_1\theta + a_2)\tilde{\rho}_1](u - u^*) - (a_1\theta + a_2)[\tilde{\rho}_2(k-k^*) + \tilde{\rho}_3(\pi^e - \pi^T)] \tag{44}$$

The dynamic element of the specification, equation (6), can also be rewritten; plugging (4) and (32) into (6), we have:

$$\dot{a} = \lambda[(1-\theta\tilde{\rho}_1)(u-u^*) - \theta\tilde{\rho}_2(k-k^*) - \theta\tilde{\rho}_3(\pi^e - \pi^T)] \tag{45}$$

The short-run solution for u can be found by substituting (32) and (44) into the equilibrium condition (26):

$$\begin{aligned} u(a,b,k,\pi^e) = \{&c(1-\tau^*)[\bar{r} - \tilde{\rho}_1 u^* + \tilde{\rho}_2(k-k^*) + \tilde{\rho}_3(\pi^e - \pi^T)]b \\ &+ c_\nu(b+\bar{d}) + [1 - c(1-\tau^*)(1-\bar{d})][a - g_1u^* - g_2(k-k^*) - g_3(\pi^e - \pi^T)] \\ &+ [1 - c(1-\tau^*)]\delta + \bar{\gamma}\}/\Delta \end{aligned} \tag{46}$$

where $\Delta \equiv 1 - c(1-\tau^*)(1+\tilde{\rho}_1 b) - [1 - c(1-\tau^*)(1-\bar{d})]g_1 > 0$, $g_1 \equiv a_1 - (a_1\theta + a_2)\tilde{\rho}_1$, $g_2 \equiv (a_1\theta + a_2)\tilde{\rho}_2$, and $g_3 \equiv (a_1\theta + a_2)\tilde{\rho}_3$.[23] [24]

[23] The signs of g_2 and g_3 are unambiguously positive. For given a_1 and a_2, however, g_1 can be negative if the response of the interest rate to utilization is strong (high ρ_1) and the desired utilization is sensitive to variations in the interest rate (high θ).

[24] The dynamic system is economically meaningful only if the short-run solution for output (utilization) is positive and stable. This in turn, requires that

$$\Delta \equiv 1 - c(1-\tau^*)(1+\tilde{\rho}_1 b) - [1 - c(1-\tau^*)(1-\bar{d})]g_1 > 0 \tag{47}$$

It follows from (47) that with an active monetary policy ($\tilde{\rho}_1 > 0$) there is an upper limit to the debt ratio. The upper limit is formally given by

$$b < \frac{1 - c(1-\tau^*) - [1 - c(1-\tau^*)(1-\bar{d})]g_1}{\tilde{\rho}_1 c(1-\tau^*)} \equiv \bar{b}$$

If the debt increases above this threshold $\bar{b}$, short-run stability is lost. We assume that this inequality condition is met.

Substituting (46) back in (32) and (44), we obtain the short-run equilibrium values of the real interest rate and the accumulation rate:

$$r = \bar{r} + \tilde{\rho}_1[u(a,b,k,\pi^e) - u^*] + \tilde{\rho}_2(k - k^*) + \tilde{\rho}_3(\pi^e - \pi^T) \equiv r(a,b,k,\pi^e) \tag{48}$$
$$g = a + g_1[u(a,b,k,\pi^e) - u^*] - g_2(k - k^*) - g_3(\pi^e - \pi^T) \equiv g(a,b,k,\pi^e) \tag{49}$$

Putting together the pieces, we have a four dimensional system in the (a,b,k,π^e) space.

$$\dot{a} = \lambda(1 - \theta\tilde{\rho}_1)(u - u^*) - \lambda\theta\tilde{\rho}_2(k - k^*) - \lambda\theta\tilde{\rho}_3(\pi^e - \pi^T) \tag{50}$$
$$\dot{b} = (r - g)b + \bar{\gamma} - \tau^*[u - (1 - \bar{d})g - \delta + rb] \tag{51}$$
$$\dot{k} = k(g - n) \tag{52}$$
$$\dot{\pi}^e = \mu\eta_1(u - u^*) + \mu\eta_2(k - k^*) \tag{53}$$

where u is given by (46), and r and g by (48) and (49).

By construction $(a,b,k,\pi^e) = (n,b^*,k^*,\pi^T)$ is a stationary point.[25] The local stability properties are determined by the Jacobian of the system and, as shown in Appendix C, monetary policy in the form of a Taylor rule stabilizes the system if

1. $\mathbf{z} \equiv n[1 - c(1 - \tau^*)] + c_\nu\tau^* - \bar{r}(1 - c)(1 - \tau^*) > 0$,
2. $\theta > \underline{\theta} \equiv [(cn + c_\nu)(1 - \tau^*)b^*]/\mathbf{z}$, and
3. ρ_2 is sufficiently large.

The stabilizing effects of monetary policy depend on the value of the debt ratio and the sensitivity of the inertial element of investment (a) to changes in the interest rate (θ). Intuitively, a monetary contraction – a rise in interest rates – increases households' net interest income and raises consumption when the debt ratio is positive; the larger the debt, the stronger this induced fiscal expansion and the concomitant, destabilizing effect on $\dot{a}$. For monetary stabilization to be successful, the contractionary effect of interest rates on investment must be sufficiently strong, that is, the sensitivity of $\dot{a}$ to changes in the interest rate (θ) must be sufficiently high. If this condition is satisfied, local stability can be achieved by choosing a large value of ρ_2. Conversely, stability cannot be obtained through monetary policy if θ is small. The critical value of θ depends on the debt ratio b^* as well as on the various parameters, including $\bar{r}$, n, c and c_ν.[26]

In interpreting this result, it should be noted that b^* – the steady growth value of the debt ratio – is itself endogenous and depends on $c, c_\nu, \bar{d}$ and $\bar{\gamma}$ (equation (28)). A fall in $\bar{\gamma}$, for instance, raises b^* but also affects the tax rate τ^*. Thus, the destabilizing effect of a reduction in $\bar{\gamma}$ and the associated rise in b^* involves two mechanisms: the net interest effect on demand is blunted by an increase in the sensitivity of total interest payments (a destabilizing effect) to changes in the interest rate and, second, for plausible parameter values a reduction in government consumption will reduce the tax rate and thereby the automatic fiscal stabilizer.[27] Monetary policy is made less powerful via the first mechanism while the second mechanism raises the need for stabilization.

[25] The equations defining the set of stationary solutions are non-linear and, in general, will have multiple solutions.

[26] The analysis has affinities with Franke's (2015) examination of monetary policy in a Harrodian model. Franke does not consider the debt ratio (there is no fiscal policy) and the ratio of capital to the labor force. By setting $b = \tau = \gamma = 0$, however, and leaving out k from the Taylor rule, our four dimensional system reduces to a two dimensional system with properties that are similar to those in Franke's 2D system. The main difference is that the stability conditions are more restrictive in our setting because of differences in the specifications of consumption and investment.

[27] If $n + c_\nu - (1-c)r > 0$ and $r > n$, a fall in γ reduces τ^*. The combination of a higher b^* and lower τ^* unambiguously increases the threshold $\underline{\theta}$. The effect on $\underline{\theta}$ of a fall in γ is ambiguous if $n + c_\nu - (1 - c)r > 0$ and $r < n$.

4.3 Functional finance with passive monetary policy

In this regime the real interest rate is kept constant at $r = \bar{r}$, and the tax rate is given by equation (29).

Perfect adjustment Consider first a 'perfect functional finance rule' without active monetary policy. In this case, neither the debt ratio nor the expected inflation rate affects the dynamics of a and k; equations (33), (4), (40) and (42) yield a two dimensional system:

$$\dot{a} = \lambda\left(\frac{e^*}{k} - u^*\right) \tag{54}$$

$$\dot{k} = k\left[a + a_1\left(\frac{e^*}{k} - u^*\right) - n\right] \tag{55}$$

In a benchmark Harrodian case with $a_1 = 0$, the system has a predator-prey structure and the dynamics produce conservative fluctuations in a, k. Somewhat paradoxically, the stabilization of the employment rate does not stabilize the state variables: starting from an arbitrary initial position with $(a, k) \neq (n, e^*/u^*)$ we get persistent fluctuations with a constant amplitude. Intuitively, keeping $uk = e^*$ leads to overshooting. A low initial ratio of capital to the labor force must be compensated by a high utilization rate to maintain full employment; high utilization leads to an increasing accumulation rate, and when the capital-labor ratio k has reached its steady growth value, the accumulation rate will be above the natural rate.

The extended Harrodian investment function introduces an instantaneous effect of utilization on investment. This effect stabilizes the system: it yields a negative feedback from k to $\dot{k}$, and (evaluated at the stationary point) the Jacobian takes the form

$$J(a,k) = \begin{pmatrix} 0 & -\lambda\frac{e^*}{k^2} \\ k & -a_1\frac{e^*}{k} \end{pmatrix} \tag{56}$$

Thus, $\det J > 0$, $\mathrm{tr}J < 0$ and the stationary point is locally stable. In fact, Olech's theorem applies in this case and we have global stability, assuming $a_1 > 0$.[28]

Imperfect functional finance Using (34), (4), (40) and (42) the dynamic system for a, k is again separable from the $b-$dynamics. The 2D system now is given by

$$\dot{a} = \lambda H(a,k); \qquad H_2 < 0 \tag{57}$$

$$\dot{k} = k[a + a_1 H(a,k) - n] \tag{58}$$

with Jacobian

$$J(a,k) = \begin{pmatrix} \lambda H_1 & \lambda H_2 \\ k(1 + a_1 H_1) & k a_1 H_2 \end{pmatrix} \tag{59}$$

H_2 is negative and the determinant $(-k\lambda H_2)$ therefore will be positive. The sign of the trace – $\mathrm{tr}J = \lambda H_1 + k a_1 H_2$ – is ambiguous, however: the destabilizing effects of a positive partial H_1 may be offset by a negative value of the partial H_2 if $a_1 > 0$.

[28] A system is globally stable if (i) $\mathrm{tr}J < 0$ and $\det J > 0$ everywhere and if either (ii) the product of the diagonal elements is positive everywhere or (iii) the product of the off-diagonal elements is negative everywhere (Olech 1963). It is readily seen that if $\tilde{k} = \log k$, conditions (i) and (iii) are satisfied for the transformed system in $(a, \tilde{k})$.

In the benchmark Harrodian case with $a_1 = 0$, local stability would have required that $H_1 < 0$. Intuitively, to stabilize the system in this case would require an implausibly strong fiscal response to an increase in the accumulation rate: higher investment increases demand and output, and this expansionary effect must be more than offset by fiscal contraction.

Debt and inflation dynamics under functional finance The dynamics of (a, k) are independent of the debt dynamics under the functional finance rules that we have considered. Assuming stability of the (a, k) dynamics, we now examine the implications for debt and inflation.

The stabilization of a implies that $u = u^*$, and the stabilization of k that $g = n$. Using equation (26) we therefore have

$$\frac{d\gamma}{db} = -[c(1-\tau^*)\bar{r} + c_\nu] \tag{60}$$

The value of $\partial\gamma/\partial b$ is negative: in order for the utilization rate to remain at u^*, the positive demand effect from an increase in the debt ratio must be met by a reduction in government consumption.

With $g = n$, $u = u^*$ and $r = \bar{r}$, the stability condition for the debt dynamics – equation (41) – can now be written

$$\begin{aligned} \frac{\partial \dot{b}}{\partial b} &= (1-\tau^*)\bar{r} - n + \frac{\partial\gamma}{\partial b} \\ &= (1-\tau^*)(1-c)\bar{r} - n - c_\nu < 0 \end{aligned} \tag{61}$$

Standard estimates suggest that c_ν is about 0.2 (see section 2.2.3), and the negative response of γ to a rise in b is strong enough to stabilize the debt dynamics for all plausible parameter values. The stationary solution for b is given by equation (28) and $\gamma = \bar{\gamma}$ at the stationary point.[29]

Expected inflation will also be constant asymptotically (and equal to actual inflation). This follows from the exponential convergence of u to u^* and k to k^*. In general, however, the asymptotic value of π will not be equal to the target π^T.

4.4 Combining imperfect functional finance and a Taylor rule

Combining the imperfect functional finance rule (34) with the Taylor rule (32) and using (44)-(45), we get a three dimensional system:

$$\dot{a} = \lambda[(1-\theta\tilde{\rho}_1)H(a,k) - \theta\tilde{\rho}_2(k-k^*) - \theta\tilde{\rho}_3(\pi^e - \pi^T)] \tag{62}$$

$$\dot{k} = k\{a + [a_1 - (a_1\theta + a_2)\tilde{\rho}_1]H(a,k) - (a_1\theta + a_2)[\tilde{\rho}_2(k-k^*) + \tilde{\rho}_3(\pi^e - \pi^T)] - n\} \tag{63}$$

$$\dot{\pi}^e = \mu[\eta_1 H(a,k) + \eta_2(k-k^*)] \tag{64}$$

[29] The debt dynamics can be written

$$\dot{b} = (r-g)b + \gamma - \tau(u - \delta - (1-\bar{d})g + rb)$$

By construction, we have

$$0 = (r-g)b^* + \bar{\gamma} - \tau^*(u^* - \delta - (1-\bar{d})n + rb^*)$$

It follows that if $\tau = \tau^*, u = u^*$ and $g \to n$ then

$$\dot{b} \to (r(1-\tau^*) - n)(b - b^*) + (\gamma - \bar{\gamma})$$

Now, if $a = n, k = k^*$ and $H = 0$,

$$\begin{aligned} \gamma &= u^*(1 - c(1-\tau^*)) - (c(1-\tau^*)r + c_\nu)b - (1 - c(1-\tau^*))(n+\delta) - (c_\nu + c(1-\tau^*)n)\bar{d} \\ \bar{\gamma} &= u^*(1 - c(1-\tau^*)) - (c(1-\tau^*)r + c_\nu)b^* - (1 - c(1-\tau^*))(n+\delta) - (c_\nu + c(1-\tau^*)n)\bar{d} \end{aligned}$$

Hence

$$\dot{b} \to [r(1-\tau^*) - n - c(1-\tau^*)r - c_\nu](b - b^*)$$

where b^* is given by (28).

The system has a stationary solution at $(a, k, \pi^e) = (n, k^*, \pi^T)$, and if the system is stable, the last state variable b will also converge, $b \to b^*$; the analysis in section 4.3 applies here too.

The interaction between fiscal and monetary policy is quite complex, but as shown in Appendix D:

- Functional finance on its own cannot ensure asymptotic stability in the benchmark Harrodian case with predetermined accumulation ($a_1 = a_2 = 0$) if $H_1 > 0$. Adding a strong monetary policy with $\tilde{\rho}_1\theta > 1$, however, can stabilize the system in this case.
- Functional finance can stabilize the 2D system in (a, k) if the condition $\lambda H_1 + ka_1H_2 < 0$ is met and $r = \bar{r}$. But adding a Taylor rule with a strong reaction of the interest rate to utilization (a large value of the parameter $\tilde{\rho}_1$) leads to instability in this case.

Thus, depending on parameter values the introduction of the same monetary policy can be stabilizing or destabilizing. Given the complexity of the 3D system, a full intuition for this result is not obvious. As a partial insight, however, observe that the stability of the functional finance system derives from the contractionary response of fiscal policy to a rise in k (which also raises e for a given value of u); the resulting decline in utilization produces a negative feedback effect on accumulation and thereby on $\dot{k}$. This stabilizing feedback can be nullified by an active monetary policy which reacts to a decline in u by reducing the interest rate, thereby stimulating accumulation.

4.5 Sound finance

The sound-finance rule implies a two dimensional system in (a, b):

$$\begin{aligned}
\dot{a} &= \lambda\tilde{H}(a, b) && (65)\\
\dot{b} &= (\bar{r} - g)b + [1 - c(1-\tau)][u^* + \tilde{H}(a, b) - g - \delta] - [c(1-\tau)r + c_\nu]b \\
&\quad - [c_\nu + c(1-\tau)g]\bar{d} - \tau[u^* + \tilde{H}(a, b) + \bar{r}b - (1-\bar{d})g - \delta] && (66)
\end{aligned}$$

where $g = a + a_1\tilde{H}(a, b)$. The Jacobian matrix is given by

$$J(a, b) =$$

$$\begin{pmatrix}
\lambda\tilde{H}_a & \lambda\tilde{H}_b \\
\begin{array}{c}\tilde{H}_a(1-c)(1-\tau)\\ -(1 + a_1\tilde{H}_a)[b + 1 - (c + (1-c)\tau)(1-\bar{d})]\end{array} & \begin{array}{c}\bar{r}(1-\tau)(1-c) - g - c_\nu\\ +\tilde{H}_b(1-c)(1-\tau)\\ -a_1\tilde{H}_b[b + 1 - (c + (1-c)\tau)(1-\bar{d})]\end{array}
\end{pmatrix}$$

The determinant of the Jacobian matrix is

$$\det J = \lambda\{H_a[\bar{r}(1-\tau)(1-c) - g - c_\nu] + \tilde{H}_b[b + 1 - (1-\bar{d})(c + (1-c)\tau)]\} \tag{67}$$

With benchmark values of c_ν of about 0.2, the term $[\bar{r}(1-\tau)(1-c) - g - c_\nu]$ is negative for all plausible parameter values. Since $\tilde{H}_a \geq 0$ and $\tilde{H}_b < 0$, the determinant therefore becomes negative, and we get saddle-point instability.[30]

[30] The alternative definition of sound finance in footnote 22 – adjusting policy to maintain any given debt ratio – also produces instability. A stationary b requires $\dot{b}$ to be kept at zero. Using (10), we get:

$$\gamma = \tau^*[u + \bar{r}b - (1-\bar{d})g - \delta] - (\bar{r} - g)b \tag{68}$$

where g is given by $g = a + a_1(u - u^*)$. Substituting (68) in (27) and solve for u, we obtain

$$u = \frac{b + 1 - (1-\bar{d})[c + \tau(1-c)]}{(1-c)(1-\tau) - a_1[b + 1 - (1-\bar{d})(c + \tau(1-c))]}a + u_0 \tag{69}$$

where u_0 is a constant. Utilization is increasing in a which produces instability, just as in the benchmark Harrodian case.

Even if the stationary point had been stable, the system contains no mechanism to equalize the warranted and natural growth rates. To see this, note that $\tilde{H}(a,b) = 0$ and $a = g$ at a stationary point. The equalization of warranted and natural growth rates therefore requires that $\tilde{H}(n,b) = 0$. This equation determines a unique value of b. Only by a fluke – if $b^T = b^*$ – will the debt ratio be stationary for this value of b in combination with $a = g = n$ (equation 66). Targeting the right debt ratio, finally, still fails to pin down a stationary solution for the employment rate e and the ratio of capital to the labor force k. There is no feedback from the employment rate to fiscal policy. To ensure full employment at the stationary point, a fiscal rule of this kind would need to be combined with an employment sensitive monetary policy.

We may note that instability persists, even if $\tilde{H}_a = 0$ (that is, even if fiscal policy offsets the demand effects of Harrodian shifts in the investment function). In this special case, the stationary solution does imply that $b = b^T$, but the determinant of the Jacobian matrix is unambiguously negative:

$$\det J = \lambda[b^T + 1 - (1-\bar{d})(c + (1-c)\tau)]\tilde{H}'(b^T) < 0 \tag{70}$$

Intuitively, an increase in the debt ratio slows down capital accumulation ($\partial\dot{a}/\partial b < 0$) and this tends to accelerate debt accumulation ($\partial\dot{b}/\partial a < 0$).

4.6 Sound finance and monetary policy

A sound-finance rule, $u = u^* + \tilde{H}(a,b)$, in combination with the Taylor rule (32) yields the following four dimensional system:

$$\dot{a} = \lambda[(1-\theta\tilde{\rho}_1)\tilde{H}(a,b) - \theta\tilde{\rho}_2(k-k^*) - \theta\tilde{\rho}_3(\pi^e - \pi^T)] \tag{71}$$

$$\dot{b} = (r-g)b + \gamma - \tau[u^* + \tilde{H}(a,b) + rb - (1-\bar{d})g - \delta] \tag{72}$$

$$\dot{k} = k(g-n) \tag{73}$$

$$\dot{\pi}^e = \mu[\eta_1\tilde{H}(a,b) + \eta_2(k-k^*)] \tag{74}$$

where

$$\gamma = [u^* + \tilde{H}(a,b)][1 - c(1-\tau)] - [c(1-\tau)r + c_\nu]b - (1 - c(1-\tau))(g+\delta) - [c_\nu + c(1-\tau)g]\bar{d} \tag{75}$$

$$r = \bar{r} + \tilde{\rho}_1\tilde{H}(a,b) + \tilde{\rho}_2(k-k^*) + \tilde{\rho}_3(\pi^e - \pi^T) \tag{76}$$

$$g = a + [a_1 - (a_1\theta + a_2)\tilde{\rho}_1]\tilde{H}(a,b) - (a_1\theta + a_2)[\tilde{\rho}_2(k-k^*) + \tilde{\rho}_3(\pi^e - \pi^T)] \tag{77}$$

It should be noted, first, that we only have $r = \bar{r}$ and $\gamma = \bar{\gamma}$ at a stationary point if the target debt ratio b^T equals the steady-growth value b^* in equation (28).[31] Picking the wrong debt target is incompatible with full employment growth and the achievement of the desired values of interest rate and government consumption. Insofar as there is a choice of technique, the Leontief production function (equation 3) would therefore need to be modified. Disregarding this problem, the introduction of a Taylor rule can stabilize a sound-finance system; see Appendix E for details).

It may not be surprising that a sufficiently strong Taylor rule may be able to tame the intrinsic instability of sound finance. More interesting, perhaps, sound finance and Taylor rules can be complementary: a system that is unstable using a Taylor rule and passive fiscal policy can in some cases

[31] To see this, note that the stationarity of a and k requires $u = u_d$ and $g = n$, respectively, which, using the expressions for g and u_d, implies $n = a - a_2(r - \bar{r})$ and $\tilde{H}(a,b) = \theta(r - \bar{r})$. If $r = \bar{r}$ we now have $a = n$ and $\tilde{H}(n,b) = 0$. By construction, $\tilde{H}(n,b^T) = 0$ and thus we should have $b = b^T$. $b = b^T \neq b^*$, however, is inconsistent with $\gamma = \bar{\gamma}$ if the product market equilibrium and the government budget equation are to be met: (28) shows that $\gamma = \bar{\gamma}$ requires b to be b^*.

achieve local stability under a combination of sound finance and (the same) Taylor rule. Simulations also show that the introduction of sound finance can speed up the convergence process toward a stable state. The mechanism is complex, but it appears that active debt management policy in the sound finance rule can attenuate a major weakness in the pure case of monetary stabilization: the efficacy of monetary policy is blunted by the existence of debt, and bringing down the debt can be beneficial if stabilization relies exclusively on monetary policy.

The apparent benefit from a marriage of sound finance and Taylor rules should not mask dangers that are inherent to this regime. By the rules of sound finance, an economy with high debt must impose a contractionary fiscal policy (low government consumption), and the Taylor rule calls for low interest rates in response to the resulting weakness in demand, even if the private components of aggregate demand are at 'normal' levels and inflation is at the target rate. In this situation the interest rate easily hits the zero lower bound following a negative demand shock. The larger the debt and the larger the magnitude of the negative demand shock, the more likely the system hits the zero lower bound. If this happens, the economy with sound finance completely loses its stabilizing feedbacks.

5 Numerical illustrations

The analytical results in the previous section can be illustrated numerically. The benchmark parameter values in our simulation exercises are as follows:

e^*	u^*	δ	n	$\bar{r}$	c	c_ν	$\bar{d}$
0.95	0.5	0.07	0.03	0.04	0.635	0.2	0.25
$\bar{\gamma}$	a_1	a_2	θ	λ	η_1	η_2	μ
0.1	0.1	0.25	1	0.3	0.95	0.25	0.5

Table 1: Parameter values

These parameters imply in steady state the tax rate is 24.55% ($\tau^* = 0.2455$) and the debt-to-capital ratio is 25% ($b^* = 0.25$, i.e., 50% of the debt-to-GDP ratio for $u^* = 0.5$).[32] The shares of consumption, investment and government consumption in GDP on a steady growth path are 60%, 20% and 20%, respectively. The steady-growth ratio of consumption to household disposable income is 95.24%. The Phillips curve parameters, η_1 and η_2, implies that actual inflation, given the expected rate of inflation, would increase by about 0.5%p in response to a 1%p decline in the unemployment gap.[33] The parameters a_2 and θ describe the effect of the interest rate on investment and are subject to considerable uncertainty. The chosen values are probably on the high side which enhances the stabilizing potential of the interest rate policy.

We examine the implications of various fiscal and monetary policy rules for this Harrodian economy. The benchmark parameters for active policies are as follows:

The monetary parameters are chosen to fit a standard Taylor rule.[34] The value of the parameter that captures the effect of a on u is different for the functional and sound finance rules (H_1 and $\tilde{H}_a$).

[32] The tax rate may seem low, but it represents taxes net of transfers. Analogously, γ is given by $\gamma = G/K$ where G is government consumption of goods and services, not total government expenses.

[33] We have $e = uk$ and, using a first order approximation, $e - e^* \approx k^*(u - u^*) + u^*(k - k^*)$. If $e^* = 0.95, u^* = 0.5$, we have $k^* = 1.9$. Assuming $\pi = \pi^e + 0.5(e - e^*)$, we have

$$\pi \approx \pi^e + 0.5k^*(u - u^*) + 0.5u^*(k - k^*) = \pi^e + 0.95(u - u^*) + 0.25(k - k^*)$$

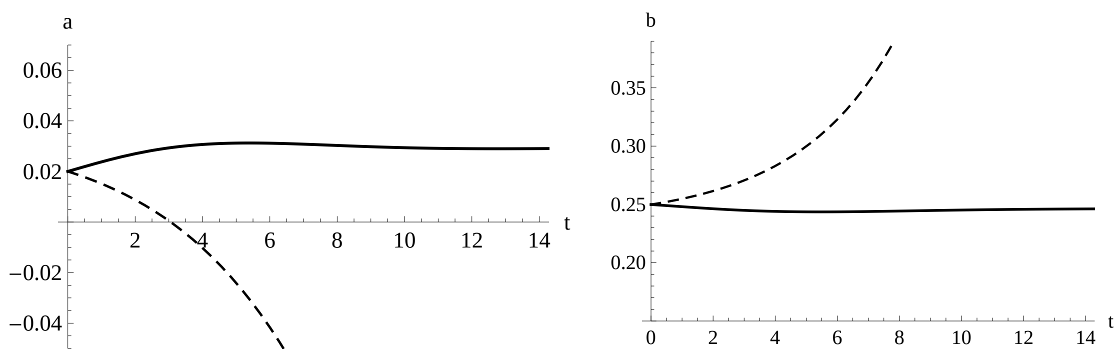
[34] Using $e - e^* \approx k^*(u - u^*) + u^*(k - k^*)$, $e^* = 0.95$ and $u^* = 0.5$, a standard rule with

$$r = \bar{r} + 0.5(e - e^*) + 0.5(\pi - \pi^T)$$

ρ_1	ρ_2	ρ_3	π^T	H_1	H_2	$\tilde{H}_a$	$\tilde{H}_b$
1	0.25	1.5	0.02	0.1	-0.2	1.40 or 0.89	-0.3

Table 2: Parameter values for active policies

In the case of sound finance, it is natural to assume that $\tilde{H}_a$ is equal to the size of the multiplier associated with a in the case with passive fiscal policy: the sound finance rule is characterized by inaction in terms of counteracting fluctuations of investment demand. Given our parameter values, the a-multiplier is 1.40 in the case without active monetary policy and 0.89 with monetary policy (see eqs. 26 and 46). For H_1, a positive value smaller than $\tilde{H}_a$ is taken ($H_1 = 0.1$), implying the adjustment of government consumption γ is used to offset, strongly but not completely, the benchmark multiplier effect of a. The value of H_2 is chosen to be smaller than the implied value of the effect of k on u in the perfect functional finance case.[35] The value of $\tilde{H}_b$ is somewhat arbitrary; the general results, however, are insensitive to variations in this value.



Notes: economy with no active policy (dashed) and with a Taylor rule(solid)

Figure 1: Stability under a Taylor rule

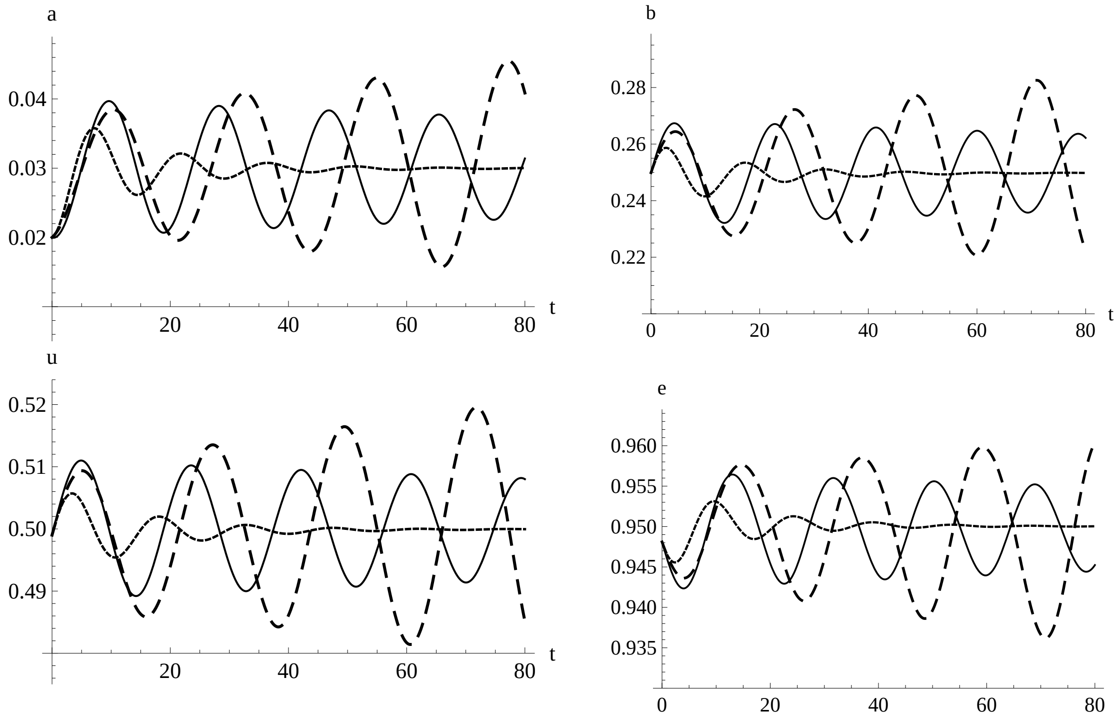
Section 2.4 (and Appendix B) describes the dynamics without active policy. Given our parameter values, the two-dimensional system in (a, b) space – (79) and (80) – is unstable: automatic stabilizers are not sufficient to eliminate Harrodian instability.[36] As the initial value of a is lowered by 0.01 from its steady state value (= 0.03), for instance, the rate of accumulation is falling indefinitely while the debt ratio keeps rising (Figure 1, dashed lines). Monetary policy alone may help contain unstable Harrodian dynamics, but, as shown in section 4.2, the efficacy of monetary policy is compromised at high levels of public debt. To illustrate this point, we compare two economies in which the steady growth debt-capital ratio is different due to the different levels of government consumption. In our benchmark parameters in Table 1 and 2, we use $\bar{\gamma} = 0.1$, which yields $b^* = 0.25$ and $\tau^* = 0.2455$. With these values, the stationary point $(n, b^*, k^*, \pi^T) = (0.03, 0.25, 1.9, 0.02)$ is locally stable (see the solid lines in Figure 1 for the path of a and b). The Taylor rule stabilizes the economy and brings it to a full employment steady growth path with a stable debt ratio. The benchmark parameters imply

corresponds to

$$r = \bar{r} + 0.95(u - u^*) + 0.25(k - k^*) + 0.5(\pi - \pi^T)$$

[35] In the case of perfect functional finance, $u = e^*/k$ and thus $\partial u/\partial k = \partial(e^*/k)/\partial k = -e^*/k^{*2} = -0.2632$ if $e^* = 0.95$ and $u^* = 0.5$.

[36] Given the other benchmark values, any λ greater than 0.0861 is sufficient to make the system without stabilization policy unstable.



Notes: imperfect functional finance (solid); combined with a Taylor rule ($\rho_1 = 1$, dotted; $\rho_1 = 2$, dashed)

Figure 2: Combination of imperfect functional finance and Taylor rules.

that the threshold value of θ necessary to ensure local stability is given by

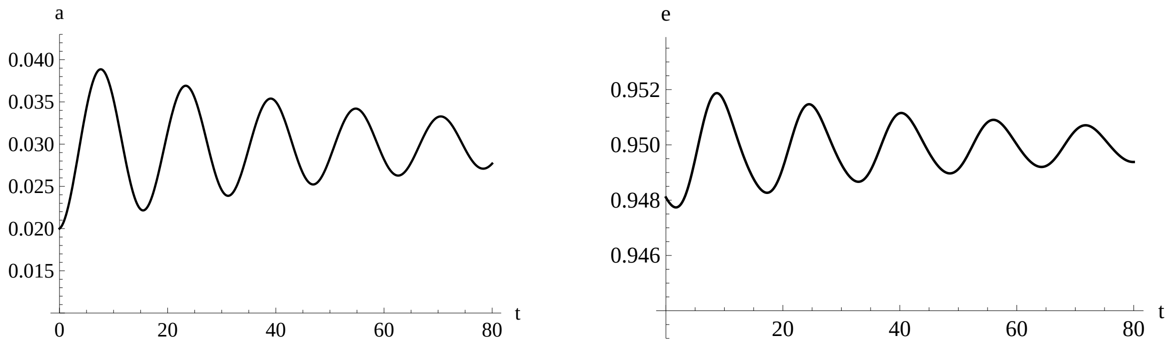
$$\theta > \underline{\theta} \equiv \frac{(cn + c_\nu)(1 - \tau^*)b^*}{n[1 - c(1 - \tau^*)] + c_\nu \tau^* - \bar{r}(1 - c)(1 - \tau^*)} = 0.7688$$

The benchmark value $\theta = 1$ satisfies this restriction.

If the steady growth debt ratio is higher, however, the system can lose its local stability. For instance, if $\bar{\gamma} = 0.05$, then $b^* = 0.333$ and $\tau^* = 0.1267$. To ensure local stability, θ now has to be above 2.4552 and with $\theta = 1$, the full employment steady growth becomes locally unstable.

In general, the four-dimensional model, (50)–(53), produces multiple steady states due to nonlinearities. Given plausible values of all other parameters, as the value of θ approaches the threshold value $\bar{\theta}$, the determinant of the system tends to vanish and an adjacent fixed point gets closer to our benchmark steady state (n, b^*, k^*, π^T). The basin of attraction around (n, b^*, k^*, π^T) tends to shrink as a result. In other words, a low value of θ compared to the threshold value has several dynamic implications. First, initial conditions become more important to ensure local stability. Second, the convergence process could be very slow, even if local stability is achieved. Third, the system becomes very fragile as it is more likely that the zero lower bound will be hit, even by a small negative demand shock.

Moving onto the case of functional finance, Figure 2 (solid lines) illustrates how an imperfect functional finance rule without active monetary policy ($H_1 = 0.1$, $H_2 = -0.2$ and $r = \bar{r}$) turns the exploding trajectories of (79) and (80) into damped fluctuations. The fluctuations, however, are highly persistent and do not quickly die out.



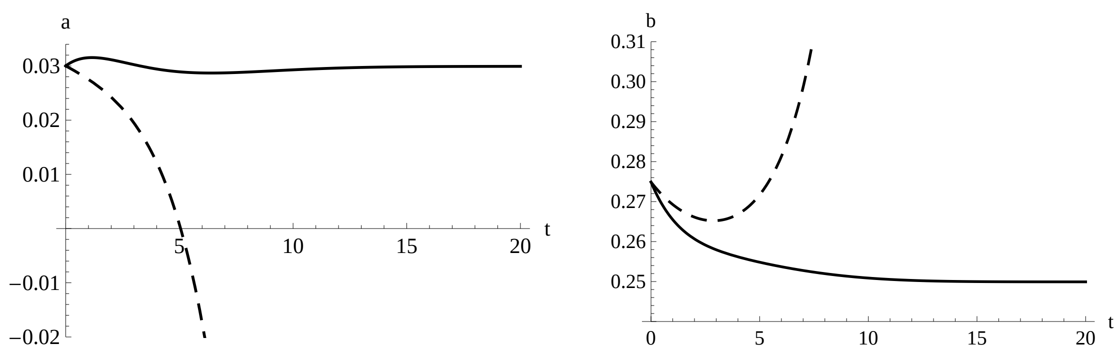
Notes: $\bar{\gamma} = 0.05$, $a_1 = a_2 = 0$ and $H_2 = -0.25$. The other parameters are the same as in Table 1 and 2.

Figure 3: Stability under the mix of fiscal and monetary policy rules which would destabilize if applied separately

The imperfect functional finance rule can benefit from the introduction of a Taylor rule. Figure 2 (dotted lines) illustrates how the combination of the Taylor and the imperfect functional finance rules can reduce the amplitude of fluctuations compared to the benchmark case with imperfect functional finance only.

The implications of monetary-fiscal interactions, however, are more complex. As our analysis suggests, the introduction of a Taylor rule can undermine the stability of the system. As the response of the interest rate to utilization becomes large ($\rho_1 = 2$), the fiscal-monetary interaction makes the benchmark steady state locally unstable, and as a result the damped oscillations are turned into exploding oscillations (Figure 2, dashed lines). Interestingly, with $\rho_1 = 2$, the Taylor rule can stabilize the system in the absence of active fiscal policy. Instability thus may come from the interaction of the fiscal and monetary rules which would stabilize the system if applied separately.

Conversely, fiscal and monetary policy rules, which would destabilize the system if applied separately, can bring about the stability of the system if combined. To see this, let us modify the values for $\bar{\gamma}$, a_1, a_2 and H_2 so that $\bar{\gamma} = 0.05$, $a_1 = a_2 = 0$ and $H_2 = -0.25$. As shown in our analysis, a Taylor rule on its own cannot stabilize the economy with high debt ratios. With the other parameters kept at the benchmark values, $\bar{\gamma} = 0.05$ is sufficient to obtain the case of local instability. We have also

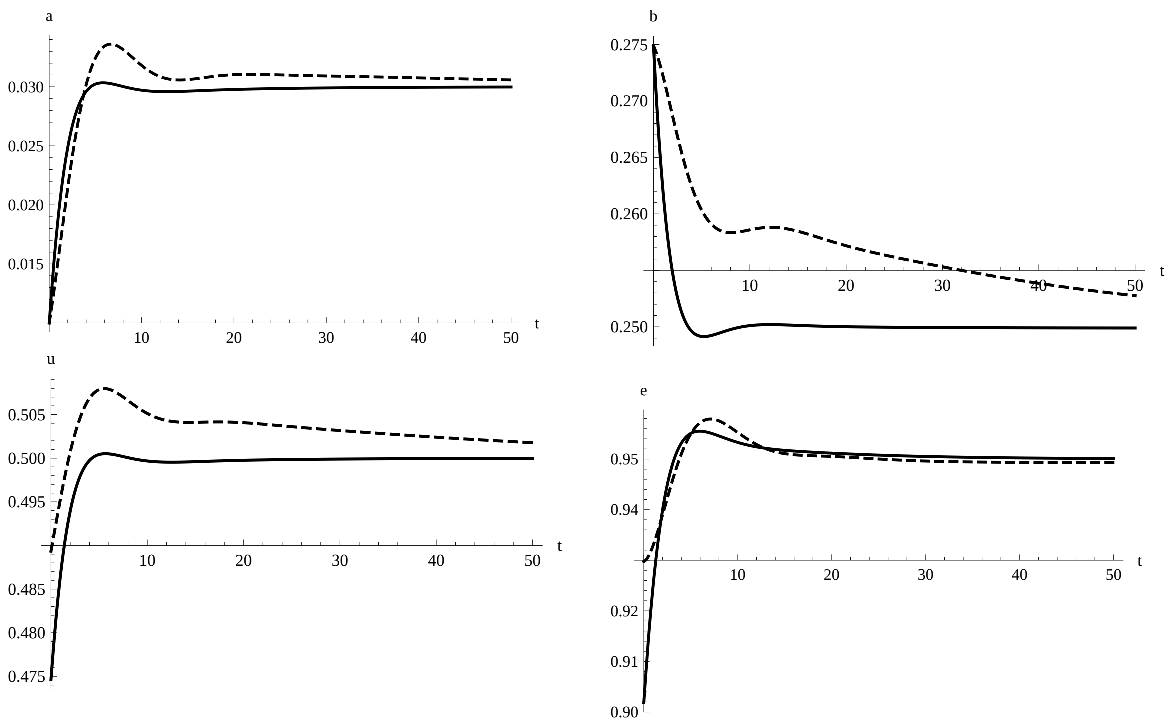


Notes: $b_0 = 0.275 > 0.25 = b^*$.

Figure 4: Imperfect sound finance with (solid) and without (dashed) a Taylor rule

shown that functional finance on its own cannot achieve stability in the benchmark Harrodian case with predetermined accumulation ($a_1 = a_2 = 0$) if $H_1 > 0$. Thus with the modified parameter values the fiscal and monetary policy rules would be destabilizing if applied separately. The same rules, if combined, however, stabilize the system around a full-employment steady growth path (Figure 3).

Turning to the case of sound finance, we have shown that the imperfect sound finance rule is unstable; stability of the system can be achieved only if sound finance is combined with a strong Taylor rule. Figure 4 illustrates the results of simulations in which the target debt ratio is calibrated to the level compatible with the natural rate of growth, $b^T = b^*$, but the initial value of b is higher than its steady growth value ($b_0 = 0.275 > b^* = 0.25$). The sound finance rule alone leads to ever-increasing debt ratios and falling accumulation rates (dashed). The benchmark parameter values in the Taylor rule are high enough to overcome the destabilizing potential in this scenario (solid).



Notes: $b_0 = 0.275 > b^* = 0.25$ and $a_0 = 0.01 < n = 0.03$

Figure 5: A Taylor rule with (solid) and without (dashed) imperfect sound finance

Somewhat surprisingly, the debt targeting policy under the sound finance rule can enhance the efficacy of monetary policy. With substantial amounts of debt, the effect of monetary policy is blunted by induced fiscal transfers to bondholders (see section 4.2). Because of this, even if a Taylor rule achieves stability, the convergence toward a full-employment steady growth will be slow in the presence of large debt. Under these circumstances, debt management policy in the sound finance rule can speed up the convergence process (see Figure 5).

As argued in section 4.6, the combination of sound finance and a Taylor rule can be dangerous: following an adverse shock to a or π^e, the system easily hits the zero lower bound if the initial debt ratio is high, and if this happens, the destabilizing force in sound finance gains full strength. Suppose the economy has an initial debt ratio higher than its steady growth value and is hit by a negative investment shock ($b_0 > b^*$ and $a_0 < n$). Figure 6 compares the required trajectory for the interest rate in the two cases: the Taylor rule with and without sound finance. The shock is such that in the

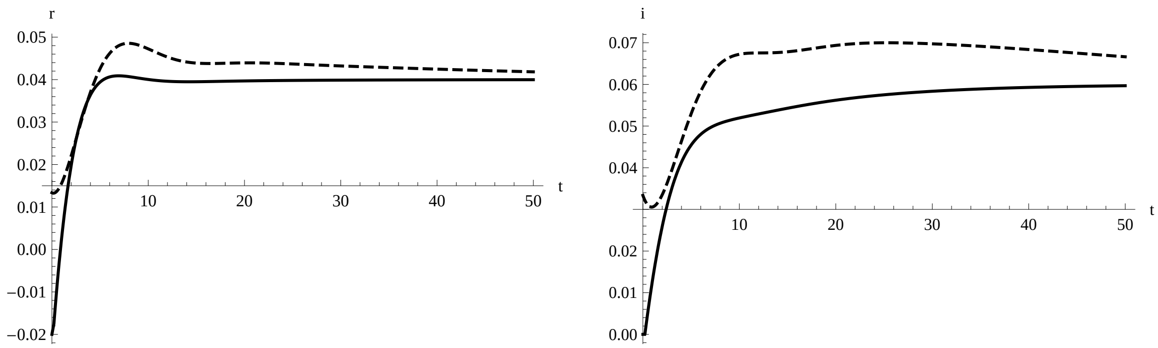
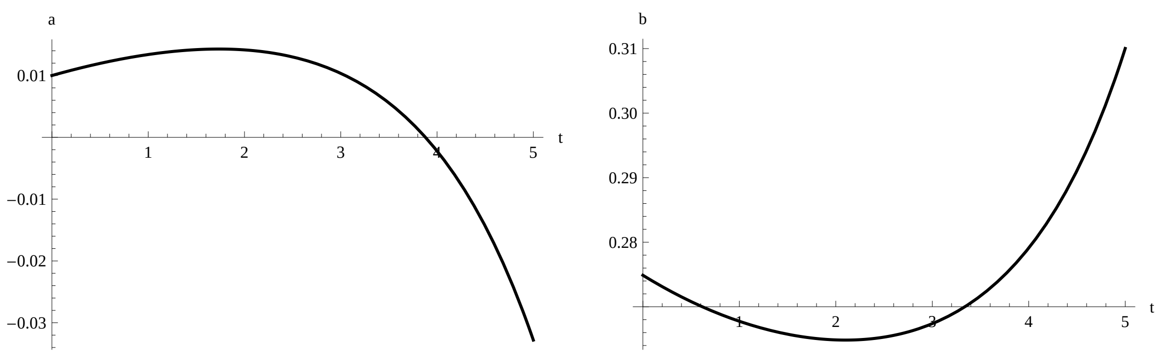


Figure 6: The required adjustment of the real and the nominal interest rates under Taylor rules with (solid) and without (dashed) sound finance

case with sound finance, the required real interest rate becomes negative and the nominal rate almost hits the zero lower bound. When the shock is so large (the initial values of a and π^e are so low), the economy gets stuck at the zero lower bound and follows unstable trajectories (see Figure 7).



Notes: $b_0 = 0.275 > b^* = 0.25$, $a_0 = 0.01 < n = 0.03$, and $\pi_0^T = 0$

Figure 7: The combination of imperfect sound finance and a Taylor rule: Binding Zero Lower Bound

6 Summary and conclusions

We have analyzed a number of policy regimes and special cases, and it may be useful to summarize the main results. In this Harrodian economy:

- Monetary policy in the form of a Taylor rule cannot stabilize the economy if the debt ratio is too high. The threshold level depends on the various parameters of the model. Assuming a sufficiently low debt ratio, stabilization becomes possible if the zero lower bound is avoided. It should be noted, however, that we have chosen parameters that are favorable to the Taylor rule (a high sensitivity of desired utilization to variations in the interest rate, for instance), and that we have omitted some potentially destabilizing effects of a Taylor rule.[37]

- A functional finance rule can stabilize the economy at full employment if investment responds quickly to changes in utilization (if the parameter a_1 is positive). In the absence of this quick response, a fiscal policy that succeeds in maintaining full employment at all times will be associated with persistent fluctuations in utilization and the rate of accumulation.

- The combination of functional finance and a Taylor rule can bring stability with steady growth, full employment, the desired capital intensity and inflation at the desired rate.

- Interactions between fiscal and monetary policy can be important. Instability can result from the combination of fiscal and monetary rules which would stabilize the system if applied separately. Conversely, rules that separately would fail to stabilize the system can in some cases lead to stability when combined.

- Sound finance fails to stabilize the economy. Moreover, the sound-finance regime provides no mechanism for aligning the warranted and natural growth rates; the two will only be equal if the right debt target is chosen.

- A combination of sound finance and a Taylor rule can stabilize the system if the debt ratio is not too high and the zero lower bound is avoided. In some cases sound finance may enhance the stabilizing potential of a Taylor rule. However, this regime also exacerbates the danger of hitting the zero lower bound. Moreover, the debt target affects the steady-growth path: a wrong target will lead to a deviation of the real rate of interest and government consumption from the desired levels.

The results clearly depend on our description of the economy, and it may not be surprising that a 'Keynesian' economy favors Keynesian policy rules. But similar remarks apply to other evaluations of fiscal policy rules: the desirability of sound-finance rules in orthodox studies is no less model dependent. The contingency of policy implications applies whether the economy is Keynesian/Harrodian as in this paper or New Keynesian as in Kirsinova and Wren-Lewis (2009) or Schmitt-Grohe and Uribe (2007). It should emphasized, however, that some of the results in this paper are independent of the specific Keynesian features. All models without Ricardian equivalence imply that picking the wrong debt target will have long-run implications for interest rates (and the choice of technique) and/or the share of government consumption in income.

We have chosen to focus on Harrodian multiplier-accelerator mechanisms as the source of potential instability. The stripped-down Harrodian model has been extended, however, in ways that seem essential for a discussion of policy. Models with public debt need to include household wealth, and if investment is the source of instability, firms' financing decisions should also be given some attention. Taylor rules, in turn, require that expected inflation be included. As a result, it is hard to avoid multiple state variables, four in our analysis. The specification of the policy rules can in some cases reduce the dimension. Thus, we suggested that reasonable specifications of both functional and sound finance could be analyzed in a two dimensional system. The advantage is obvious: the mechanisms become much clearer and it may be possible to derive global results (as in fact we did). With higher order systems, by contrast, the mechanisms can be unclear and, almost invariably, the best one can hope for is local stability results for special cases.

[37] Interest rates have distributional effects; a rise in rates tends to favor creditors at the expense of debtors. Debtors, however, may have higher propensity to consume than creditors. Rising interest costs also reduce retained earnings with possible adverse effects on investment. Partly for these reasons, a substantial Keynesian literature questions the use of interest rates as a policy instrument; contributions include Pasinetti (1981), Lavoie and Seccarecia (1999), Smithin (2007), Palley (2007), Wray (2007), Rochon and Setterfield (2007), Lima and Setterfield (2010), .

The focus on a specific source of instability and the exclusion of non-policy stabilizers helped to simplify the analysis. It may also be an advantage for the simple reason that disagreements on specification may multiply as more details are added to the basic model. As an example, Ryoo (2010) analyzes a setting that includes Harrodian and Minskian sources of instability along with stabilizing forces from the labor market. It could be interesting to add policy to this mix, but (unfortunately) the subset of the profession that buy all the elements of the Ryoo model is likely to be smaller than the subset that accept Harrodian elements. As another example Costa-Lima et al. (2014) analyze fiscal policy using an extended version of the Keen (1995) model; critics of the specific Keen model, however, may not find the analysis interesting. Against the argument for parsimony in the specification, it should be noted that quite clearly the exclusion of mechanisms that are important in real economies can be a serious problem. A neglect of the labor market may be particularly problematic. Firms' investment or employment decisions will almost certainly be affected if the economy approaches full employment, and ceilings and floors on investment or employment can affect the dynamics quite independently of policy.

In short, our approach has both strengths and weaknesses, and the present paper has many limitations. The robustness of the results, in particular, needs to be examined with respect to variations in the sources of instability and the presence of non-policy stabilizers.

Appendix A: Harrodian benchmark

Let

$$\begin{aligned} \dot{g} &= \lambda(u - u^d) \\ C &= cY \end{aligned} \tag{78}$$

where $g = \hat{K} = I/K - \delta$ is the rate of accumulation and u^d represents firms' desired utilization rate; I and δ are real (gross) investment and the capital depreciation rate; C and Y are consumption and output, and c is the propensity to consume. The equilibrium condition for the product market can be written

$$\begin{aligned} u &= \frac{Y}{K} = \frac{C}{K} + \frac{I}{K} \\ &= cu + g + \delta. \end{aligned}$$

Solving for u and substituting into the accumulation function (78), we get a dynamic equation for g,

$$\dot{g} = \frac{\lambda}{1-c}g + \lambda\left(\frac{\delta}{1-c} - u^d\right)$$

The coefficient on g is positive, implying instability. Setting $\dot{g} = 0$, the unique stationary solution – the warranted growth rate – is given by

$$g_w = (1-c)u^d - \delta$$

Appendix B: Automatic stabilizers

If $\gamma = \bar{\gamma}, \tau = \tau^*, r = \bar{r}$, equations (6), (10) and (14) define a two-dimensional system:

$$\dot{a} = \lambda(u - u^*) \tag{79}$$

$$\dot{b} = (\bar{r} - g)b + \bar{\gamma} - \tau^*[u + \bar{r}b - (1-\bar{d})g - \delta] \tag{80}$$

where u is determined by the equilibrium condition for the goods market, equation (26), and g is given by (5). Using (27) and (5) the equilibrium condition can be written

$$u = \frac{[1-c(1-\tau^*)(1-\bar{d})](a-a_1u^*)}{1-c(1-\tau^*)-a_1[1-c(1-\tau^*)(1-\bar{d})]} + \frac{[c(1-\tau^*)r+c_\nu]b+\bar{\gamma}+c_\nu\bar{d}+[1-c(1-\tau^*)]\delta}{1-c(1-\tau^*)-a_1[1-c(1-\tau^*)(1-\bar{d})]}$$

To be meaningful, the denominator $1-c(1-\tau)-a_1[1-c(1-\tau^*)(1-\bar{d})]$ must be positive; that is, the short-run Keynesian stability condition must be satisfied (the short-run effect of changes in utilization on investment must be smaller than the short-run effect on saving).

By construction $a = g = n, b = b^*$ is a stationary solution.[38] The Jacobian for the system, evaluated at this stationary point, is given by

$$J(a,b) = \begin{pmatrix} \lambda u_a & \lambda u_b \\ \\ -(1+a_1u_a)b^* - \tau^* u_a & (1-\tau^*)\bar{r} - n - a_1 u_b b^* \\ +\tau^*(1-\bar{d})(1+a_1u_a) & -\tau^* u_b[1-a_1(1-\bar{d})] \end{pmatrix}$$

where $u_a = [1-c(1-\tau^*)(1-\bar{d})]u_\gamma$, $u_b = [c_\nu + (1-\tau^*)c\bar{r}]u_\gamma$ and $u_\gamma = 1/[1-c(1-\tau^*)-a_1(1-c(1-\tau^*)(1-\bar{d}))]$. The trace and determinant of the Jacobian can be written

$$\begin{aligned} \mathrm{tr}J &= u_\gamma\{[1-c(1-\tau^*)(1-\bar{d})]\lambda \\ &\qquad - [c_\nu+(1-\tau^*)c\bar{r}][\tau^*(1-a_1(1-\bar{d}))+a_1b^*]\} + (1-\tau^*)\bar{r} - n \\ \det J &= \lambda u_\gamma\{[1-c(1-\tau^*)(1-\bar{d})][(1-\tau^*)\bar{r}-n] \\ &\qquad + [c_\nu+(1-\tau^*)c\bar{r}][b^*-\tau^*(1-\bar{d})]\} \end{aligned}$$

With one year as the unit period, plausible parameters imply that modest values of λ are sufficient to produce instability:the trace is positive for $\lambda > 0.09$ if $\tau = 0.25, \bar{d} = 0.25, r = 0.04, n = 0.03, c = 0.67, c_\nu = 0.2, a_1 = 0.1, b^* = 0.25$. Note also that if $(1-\tau^*)\bar{r} - n \leq 0$ the determinant will be negative unless $b^* \geq \tau^*(1-\bar{d})$.

Even if the 2D system for the state variables (a,b) is stable, there is an additional problem: in the absence of active policy, full employment may not be maintained. Since $e = uk$, full employment $(e = e^*)$ requires that $k = e^*/u^*$. The utilization rate will converge to u^* if the system is stable, but nothing ensures that the ratio of capital to the labor force (k) converges to the level that is required for full employment.[39]

Appendix C: Pure monetary policy

The Jacobian evaluated at (n, b^*, k^*, π^T) for the 4D system of (50), (51), (52) and (53) is given by

$$J(a,b,k,\pi^e) = \begin{pmatrix} \lambda(1-\theta\tilde{\rho}_1)u_a & \lambda(1-\theta\tilde{\rho}_1)u_b & \lambda[(1-\theta\tilde{\rho}_1)u_k - \theta\tilde{\rho}_2] & \lambda[(1-\theta\tilde{\rho}_1)u_{\pi^e} - \theta\tilde{\rho}_3] \\ \chi_a & \chi_b + (1-\tau)\bar{r} - n & \chi_k & \chi_{\pi^e} \\ k^*g_a & k^*g_b & k^*g_k & k^*g_{\pi^e} \\ \mu\eta_1u_a & \mu\eta_1u_b & \mu\eta_1u_k + \mu\eta_2 & \mu\eta_1u_{\pi^e} \end{pmatrix}$$

[38]The multiplicative ba term in the dynamic equation for the debt ratio implies that in general there will be two solutions. For present purposes, the solution with $g = a \neq n$ is irrelevant; policy should stabilize the system at the full employment path.

[39]Steady growth would be associated with full employment if the tax system has an inflation distortion (cf. footnotes 9 and 12) and the inflation dynamics (an expectations-adjusted Phillips curve) defines a unique NAIRU.

where

$$
\begin{aligned}
u_a &= [1 - c(1-\tau^*)(1-\bar{d})]u_\gamma > 0 \\
u_b &= [c(1-\tau^*)\bar{r} + c_\nu]u_\gamma > 0 \\
u_k &= \tilde{\rho}_2\{c(1-\tau^*)b^* - [1 - c(1-\tau^*)(1-\bar{d})](a_1\theta + a_2)\}u_\gamma \\
u_{\pi^e} &= \tilde{\rho}_3\{c(1-\tau^*)b^* - [1 - c(1-\tau^*)(1-\bar{d})](a_1\theta + a_2)\}u_\gamma \\
u_\gamma &= 1/\Delta = 1/\{1 - c(1-\tau^*)(1+\tilde{\rho}_1 b^*) - [1 - c(1-\tau^*)(1-\bar{d})]g_1\} \\
\chi_j &= [(1-\tau^*)r_j - g_j]b^* - \tau^*[u_j - (1-\bar{d})g_j], \quad j = a, b, k, \pi^e \\
g_a &= 1 + g_1 u_a = [1 - c(1-\tau^*)(1 + b^*\tilde{\rho}_1)]u_\gamma \\
g_b &= g_1 u_b = g_1[c(1-\tau^*)\bar{r} + c_\nu]u_\gamma \\
g_k &= g_1 u_k - g_2 = \tilde{\rho}_2\{a_1 c(1-\tau^*)b^* - [1 - c(1-\tau^*)](a_1\theta + a_2)\}u_\gamma \\
g_{\pi^e} &= g_1 u_{\pi^e} - g_3 = \tilde{\rho}_3\{a_1 c(1-\tau^*)b^* - [1 - c(1-\tau^*)](a_1\theta + a_2)\}u_\gamma \\
g_2 &= (a_1\theta + a_2)\tilde{\rho}_2 > 0 \\
g_3 &= (a_1\theta + a_2)\tilde{\rho}_3 > 0 \\
r_a &= \tilde{\rho}_1[1 - c(1-\tau^*)(1-\bar{d})]u_\gamma > 0 \\
r_b &= \tilde{\rho}_1[c(1-\tau^*)\bar{r} + c_\nu]u_\gamma > 0 \\
r_k &= \tilde{\rho}_1\tilde{u}_k + \tilde{\rho}_2 = \tilde{\rho}_2\{1 - c(1-\tau^*) - a_1[1 - c(1-\tau^*)(1-\bar{d})]\}u_\gamma > 0 \\
r_{\pi^e} &= \tilde{\rho}_1\tilde{u}_{\pi^e} + \tilde{\rho}_3 = \tilde{\rho}_3\{1 - c(1-\tau^*) - a_1[1 - c(1-\tau^*)(1-\bar{d})]\}u_\gamma > 0
\end{aligned}
$$

The Routh-Hurwitz stability conditions for local stability in this four-dimensional system are given by

$$
\begin{aligned}
& z_1 \equiv -\mathrm{tr}J > 0 \\
& z_2 \equiv \det J_{12} + \det J_{13} + \det J_{14} + \det J_{23} + \det J_{24} + \det J_{34} > 0 \\
& z_3 \equiv -(\det J_1 + \det J_2 + \det J_3 + \det J_4) > 0 \\
& z_4 \equiv \det J > 0 \\
& z_5 \equiv z_1 z_2 z_3 - z_1^2 z_4 - z_3^2 > 0
\end{aligned}
$$

where $\det J_{ij}$'s are the second-order principal minors and $\det J_i$ the third-order principal minors (the subscripts refer to the deleted columns and rows).

We show that monetary policy in the form of a Taylor rule stabilizes an unstable Harrodian economy if

1. $\mathbf{z} \equiv n[1 - c(1-\tau^*)] + c_\nu\tau^* - \bar{r}(1-c)(1-\tau^*) > 0$,
2. $\theta > \underline{\theta} \equiv [(cn + c_\nu)(1-\tau^*)b^*]/\mathbf{z}$, and
3. ρ_2 is sufficiently large.

To see the conditions for local stability, We obtain, after lengthy algebra,

$$
\begin{aligned}
z_1 &= u_\gamma k^* \tilde{\rho}_2\{(a_1\theta + a_2)[1 - c(1-\tau^*)] - a_1 c(1-\tau^*)b^*\} + z_{10} \\
z_2 &= u_\gamma k^* \tilde{\rho}_2\{\lambda\theta[1 - c(1-\tau^*)] + (a_1\theta + a_2)\mathbf{z} \\
&\qquad - [c\lambda + a_1(cn + c_\nu)](1-\tau^*)b^*\} + z_{20} \\
z_3 &= u_\gamma k^* \lambda\tilde{\rho}_2\{\mathbf{z}\theta - (cn + c_\nu)(1-\tau^*)b^*\} + z_{30} \\
z_4 &= u_\gamma k^* \lambda\eta_2\mu\tilde{\rho}_3\{\mathbf{z}\theta - (cn + c_\nu)(1-\tau^*)b^*\}
\end{aligned}
$$

where z_{10}, z_{20} and z_{30} are complicated expressions that are *independent of* $\tilde{\rho}_2$ (and therfore ρ_2). Assume that $\mathbf{z} > 0$ and define $\underline{\theta} = (cn + c_\nu)(1 - \tau^*)b^*/\mathbf{z}$. It is readily seen that $\det(J)$ $(=z_4)$ is positive if and only if $\theta > \underline{\theta}$. Moreover, straightforward algebra shows that the condition that $\theta > \underline{\theta}$ ensures that z_1, z_2 and z_3 are all increasing in ρ_2, i.e., $\partial z_i/\partial \rho_2 > 0$ $(i = 1, 2, 3)$.[40] Since an increase in ρ_2 has a linear and positive effect on z_1, z_2 and z_3 and does not affect the quantity of z_4, z_5 is cubic in ρ_2 and the coefficient associated with the cubic term is positive. Sufficiently high values of ρ_2 therefore ensure that z_1, z_2, z_3 and z_5 are all positive.

Note that if $b^* = 0$, the threshold value, $\underline{\theta}$, becomes zero. Any positive θ therefore satisfies $\theta > \underline{\theta}$. In general, $\underline{\theta}$ is increasing in the debt ratio b^* and makes the condition $\theta > \underline{\theta}$ tighter.

Appendix D: Imperfect functional finance and a Taylor rule

The Jacobian of the system is given by

$$J(a, k, \pi^e) = \begin{pmatrix} \lambda(1 - \tilde{\rho}_1\theta)H_1 & \lambda[(1 - \tilde{\rho}_1\theta)H_2 - \tilde{\rho}_2\theta] & -\lambda\theta\tilde{\rho}_3 \\ k[1 + (a_1 - (a_1\theta + a_2)\tilde{\rho}_1)H_1] & k[a_1H_2 - (a_1\theta + a_2)(\tilde{\rho}_1H_2 + \rho_2)] & -k(a_1\theta + a_2)\tilde{\rho}_3 \\ \mu\eta_1H_1 & \mu(\eta_1H_2 + \eta_2) & 0 \end{pmatrix}$$

The Routh-Hurwitz conditions for local stability are

$$\begin{aligned} \mathrm{tr}J &= \lambda(1 - \tilde{\rho}_1\theta)H_1 + k[a_1H_2 - (a_1\theta + a_2)(\tilde{\rho}_1H_2 + \tilde{\rho}_2)] < 0 \\ \det J &= \lambda k\mu\tilde{\rho}_3[\eta_2(a_2H_1 - \theta) - \theta\eta_1H_2] < 0 \\ \sum_{i=1}^{3}\det J_i &= \lambda k\{-a_2\tilde{\rho}_2H_1 - (1 - \tilde{\rho}_1\theta)H_2 + \theta\tilde{\rho}_2\} \\ &\qquad + \tilde{\rho}_3\mu[\lambda\theta\eta_1H_1 + (a_1\theta + a_2)(\eta_1H_2 + \eta_2)k] > 0 \\ -\mathrm{tr}J\sum_{i=1}^{3}\det J_i &+ \det J > 0 \end{aligned}$$

where J_i is the second-order submatrix obtained by deleting ith column and row of J.

Consider, first, the benchmark Harrodian case with predetermined accumulation $(a_1 = a_2 = 0)$. Functional finance on its own cannot ensure asymptotic stability in this case and if $H_1 > 0$, the stationary point becomes unstable. Adding a strong monetary policy with $\tilde{\rho}_1\theta > 1$ stabilizes the system in this case: if $H_1 > 0$, $H_2 < 0$, $\eta_1H_2 + \eta_2 > 0$, and $a_1 = a_2 = 0$, there exists a value of $\tilde{\rho}_1 \in (1/\theta, \tilde{\rho}_1^*)$ that satisfies the Routh-Hurwitz conditions for local stability where $\tilde{\rho}_1^* \equiv 1/\theta + [(\tilde{\rho}_2k + \tilde{\rho}_3\mu\eta_1H_1)/k|H_2|]$.

Large values of the parameter ρ_1 (a strong reaction of the interest rate to utilization) can be destabilizing under other conditions, however. The effects of an increase in the monetary parameters

[40] Note that

$$\begin{aligned} \left.\frac{\partial z_1}{\partial \rho_2}\right|_{\theta=\underline{\theta}} &= \tilde{u}_\gamma k^*\{(a_1\underline{\theta} + a_2)[1 - c(1 - \tau^*)] - a_1c(1 - \tau^*)b^*\} \\ &= \tilde{u}_\gamma k^*\left\{\frac{a_1(1 - c)(1 - \tau^*)[c(1 - \tau^*)\bar{r} + c_\nu]b^*}{\mathbf{z}} + a_2[1 - c(1 - \tau^*)]\right\} > 0 \\ \left.\frac{\partial z_2}{\partial \rho_2}\right|_{\theta=\underline{\theta}} &= \tilde{u}_\gamma k^*\{\lambda\underline{\theta}[1 - c(1 - \tau^*)] + (a_1\underline{\theta} + a_2)\mathbf{z} - [c\lambda + a_1(cn + c_\nu)](1 - \tau^*)b^*\} \\ &= \tilde{u}_\gamma k^*\left\{\frac{\lambda(1 - c)(1 - \tau^*)[c(1 - \tau^*)\bar{r} + c_\nu]b^*}{\mathbf{z}} + a_2\mathbf{z}\right\} > 0 \end{aligned}$$

ρ_1 and ρ_2 on tr$J$ and $\sum \det J_i$ are given by

$$\begin{aligned}
\frac{\partial \mathrm{tr} J}{\partial \rho_1} &= -\lambda\theta H_1 - k(a_1\theta + a_2)H_2 \\
\frac{\partial \mathrm{tr} J}{\partial \rho_2} &= -k(a_1\theta + a_2) \leq 0 \\
\frac{\partial \sum \det J_i}{\partial \rho_1} &= \lambda k\theta H_2 \leq 0 \\
\frac{\partial \sum \det J_i}{\partial \rho_2} &= -\lambda k a_2 H_1 + \lambda k\theta
\end{aligned}$$

The signs of two of these derivatives are ambiguous but it is readily seen that if the imperfect functional finance system is stable ($\lambda H_1 + k a_1 H_2 < 0$), a rise in ρ_1 must increase the trace and reduce $\sum \det J_i$, i.e. an increase in the responsiveness of the real interest rate to changes in utilization is destabilizing. Moreover, if $H_1 > 0$ and the interest effect on desired utilization is sufficiently small ($\theta < a_2 H_1$), a rise in ρ_2 will reduce $\sum \det J_i$.

Appendix E: Combinations of sound finance with a Taylor rule

The Jacobian matrix around a steady state is given by

$$J(a,b,k,\pi^e) = \begin{pmatrix} \lambda(1-\theta\tilde{\rho}_1)\tilde{H}_a & \lambda(1-\theta\tilde{\rho}_1)\tilde{H}_b & -\lambda\theta\tilde{\rho}_2 & -\lambda\theta\tilde{\rho}_3 \\ \zeta_a & \zeta_b & \zeta_k & \zeta_{\pi^e} \\ k^*(1+g_1\tilde{H}_a) & k^* g_1 \tilde{H}_b & -k^*(a_1\theta + a_2)\tilde{\rho}_2 & -k^*(a_1\theta + a_2)\tilde{\rho}_3 \\ \mu\eta_1\tilde{H}_a & \mu\eta_1\tilde{H}_b & \mu\eta_2 & 0 \end{pmatrix}$$

where

$$\begin{aligned}
\zeta_a &\equiv -(1+g_1\tilde{H}_a)\{b+1-(1-\bar{d})[c(1-\tau)+\tau]\} + \tilde{H}_a(1-\tau)(1-c)(1+b\tilde{\rho}_1) \\
\zeta_b &\equiv -[c_\nu + n - (r+\tilde{H}_b)(1-c)(1-\tau)] \\
&\qquad + \tilde{H}_b\{(1-\tau)(1-c)\tilde{\rho}_1 b - g_1[b+1-(1-\bar{d})(c(1-\tau)+\tau)]\} \\
\zeta_k &\equiv (1-\tau)(1-c)\tilde{\rho}_2 b + (a_1\theta + a_2)\tilde{\rho}_2\{b+1-(1-\bar{d})[c(1-\tau)+\tau]\} > 0 \\
\zeta_{\pi^e} &\equiv (1-\tau)(1-c)\tilde{\rho}_3 b + (a_1\theta + a_2)\tilde{\rho}_3\{b+1-(1-\bar{d})[c(1-\tau)+\tau]\} > 0
\end{aligned}$$

The Routh-Hurwitz stability conditions for local stability in this four-dimensional system are given by

$$\begin{aligned}
z_1 &\equiv -\mathrm{tr} J > 0 \\
z_2 &\equiv \det J_{12} + \det J_{13} + \det J_{14} + \det J_{23} + \det J_{24} + \det J_{34} > 0 \\
z_3 &\equiv -(\det J_1 + \det J_2 + \det J_3 + \det J_4) > 0 \\
z_4 &\equiv \det J > 0 \\
z_5 &\equiv z_1 z_2 z_3 - z_1^2 z_4 - z_3^2 > 0
\end{aligned}$$

where $\det J_{ij}$'s are the second-order principal minors and $\det J_i$ the third-order principal minors (the subscripts refer to the deleted columns and rows).

We show that if (i) $\tilde{\mathbf{z}}_0 \equiv c_\nu + n - (r+\tilde{H}_b)(1-c)(1-\tau) > 0$ and (ii) θ and ρ_2 are sufficiently large, the stationary point is locally stable. Note that since $\tilde{H}_b < 0$, the condition (i), $\tilde{\mathbf{z}}_0 > 0$, is weaker than the condition $c_\nu + n - r(1-c)(1-\tau) > 0$, which itself is almost certainly met under plausible parameter values. Under the condition that $\tilde{\mathbf{z}}_\mathbf{0} > 0$, a large value of θ can ensure that

$$\tilde{\mathbf{z}}_\mathbf{1} \equiv \{\theta \tilde{\mathbf{z}}_\mathbf{0} - \tilde{H}_b b(1-c)(1-\tau)\} - a_2 \tilde{H}_a [c_\nu + n - r(1-c)(1-\tau)] > 0$$
$$\tilde{\mathbf{z}}_\mathbf{2} \equiv (\theta - a_2 \tilde{H}_a)\lambda + (a_1\theta + a_2)\tilde{\mathbf{z}}_\mathbf{0} - a_1 \tilde{H}_b (1-c)(1-\tau) > 0$$

Note that the required value of θ depends positively on the size of $\tilde{H}_a$. Let us now examine the Routh-Hurwitz stability conditions:

$$z_1 = -\lambda(1-\theta\tilde{\rho}_1)\tilde{H}_a - \zeta_b + k^*(a_1\theta + a_2)\tilde{\rho}_2 = k^*(a_1\theta + a_2)\tilde{\rho}_2 + \tilde{z}_{10}$$
$$z_2 = k^* \tilde{\mathbf{z}}_\mathbf{2} \tilde{\rho}_2 + \tilde{z}_{20}$$
$$z_3 = k^* \lambda \tilde{\mathbf{z}}_\mathbf{1} \tilde{\rho}_2 + \tilde{z}_{30}$$
$$z_4 = k^* \lambda \eta_2 \mu \tilde{\mathbf{z}}_\mathbf{1} \tilde{\rho}_3 > 0$$
$$z_5 = {k^*}^3 \lambda (a_1\theta + a_2) \tilde{\mathbf{z}}_\mathbf{1} \tilde{\mathbf{z}}_\mathbf{2} \tilde{\rho}_2^3 + \text{other terms of lower degree}$$

where z_{10}, z_{20} and z_{30} are the terms that are independent of ρ_2. z_1, z_2 and z_3 are linear and increasing in ρ_2. z_4 is positive if $\tilde{\mathbf{z}}_\mathbf{1} > 0$, and independent of ρ_2. z_5 is cubic in ρ_2 and the coefficient of the cubic term is positive. Therefore we can choose a sufficiently large value of ρ_2 to ensure that z_1, z_2 and z_3 are all positive.

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