Old and New Formulations of the Neoclassical Theory of Aggregate Investment: A Critical Review

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by

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Abstract

This paper surveys the neoclassical theory of aggregate investment and its criticisms. We distinguish four main formulations of this theory: the traditional 'Wicksellian' investment function; the Fisherian 'array-of-opportunities' approach (as Witte Jr. called it); the Jorgensonian model; the now prevailing adjustment-costs models. With respect to other papers criticizing the neoclassical theory of investment, we do not appeal to market imperfections. We instead argue that all four formulations present serious theoretical difficulties, even conceding free competition.

Introduction

Explaining the dynamics of aggregate business investment has always been a major goal of economic analysis. Investment plays a central role in economic cycles and allows potential output and productivity to grow in the long-run. For the neoclassical (or marginalist) approach, which is nowadays still dominant among economists, the determination of investment is crucial also for a further reason. The negative dependence of investment on the interest rate is necessary, in theoretical models, for establishing a stable full-employment general equilibrium. Without that relation there would exist no 'natural' rate of interest

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capable to ensure that in the long-run investment adapts to full-capacity savings.

But how solid are the theoretical foundations of the neoclassical theory of investment? In this essay we try to answer this question by reviewing critically its four main formulations: the traditional Wicksellian investment function (to which Section 1 is dedicated), the Fisherian 'array of opportunities' approach (Section 2), the Jorgensonian model (Section 3) and the now prevailing adjustment-costs models (Section 4).

We will try to highlight the aspects that are common to these different formulations, their differences, and how they relate to one another. Particular attention will be dedicated to the criticisms that have been leveled against these theories and to unresolved issues.

Overall, this survey suggests that the neoclassical theory of investment is far from foolproof. The derivation of a negative relation between investment and the interest rate relies on the traditional conception of capital as a single homogeneous factor, that has been proved untenable by the so-called Cambridge capital controversies. Marginalist factor substitution mechanisms, on which the theory is based, have been shown to be deprived of theoretical foundations. Old and new attempts to derive the interest-elastic investment function without making recourse to factor substitution (as in the so-called 'array of opportunities' approach or in models with convex adjustment costs) are even less solid, because they rest on the mistaken assumption that expected rates of return on investment projects are independent from the interest rate.

In addition to this central problem, that affects both traditional and contemporary neoclassical theory, we will discuss some additional issues raised by contemporary models. The dominant approach, nowadays, is to derive demand for capital from the Jorgenson (1963) model, focusing on an atomistic price-taking representative firm, and append an adjustment process based on some assumption regarding the internal costs of adjustment. We will discuss the difficulties that this approach encounters in determining the optimal capital stock under constant returns to scale, and the aggregation problems that arise when the investment function of the single firm is applied to the whole economy.

With respect to other papers criticizing the neoclassical theory of investment (for example Gordon, 1992, pp. 427-437; Crotty, 1992; Stiglitz, 2011, p. 594), we will not make appeal to market imperfections or bounded rationality. These elements surely matter in real economies and neoclassical theory can certainly be criticized for not taking them seriously enough. However, if problems were only related to these aspects, it could be argued that

the neoclassical model serves as a useful benchmark, which explains what would happen under perfect competition, perfect information and rationality. More realistic formulations could then be obtained by adjusting appropriately the benchmark 'frictionless' model, relaxing one assumption or another depending on the problem at stake. Instead, our survey of the mainstream literature and of its criticisms suggests that neoclassical investment theory is unsatisfactory also when the object of analysis is a world of complete and perfectly competitive markets. Of course, if the theory encounters difficulties also without introducing market imperfections, then a stronger critique can be leveled at it, and the case for exploring alternative visions is even more compelling.

1 The traditional neoclassical approach: a 'long-period' investment function

Since the so-called 'marginal revolution' of the late 19th century, the mainstream view has been that investment adapts to savings through the equilibrating role of the interest rate.[1] The idea that investment is determined by the interest rate is closely connected to what has been called the 'long-period' version of general equilibrium theory.[2] This approach, followed by early marginalist authors such as Wicksell, Böhm Bawerk or Clark, aims to determine the normal ('natural') equilibrium prices and quantities, conceived as centres of gravitation toward which the economy tends in the long-run.[3] Given preferences, technologies and the endowment of production factors, prices and quantities are simultaneously determined in all markets by the intersection of supply and demand curves. Competition and profit-maximization imply that the remuneration of each factor is equal to its (value) marginal product.

1.1 The theory of capital and the investment function

In this early marginalist approach, the factor endowments that are taken as data of the equilibrium include the aggregate value of capital. The various produced means of production are indeed seen as embodiments of a single factor, 'capital'. While the aggregate

[1] Classical economists as Smith, Ricardo and Malthus took for granted that savings translated into investment, without any reference to the role of the interest rate (see for example the discussion in Garegnani, 1978, pp.25-28).

[2] As opposed to the Walrasian short-period version (Garegnani, 1976; Kurz and Salvadori, 1997; Petri, 1978).

[3] Wicksell (1977 [1934]) probably represents the best expression of this strand of thought.

value of capital is taken as a given, the vector of different capital goods (the *composition* of the capital stock) is endogenously determined by the tendency to a uniform (risk-adjusted) rate of return across industries. Like a fluid, capital the homogeneous factor can therefore take different forms without changing its overall quantity (Garegnani, 2012, pp.1418-1421).

As in all other markets, equilibrium in the market for capital is determined by the intersection of 'well-behaved' demand and supply curves. In particular, a stable equilibrium is ensured by the fact that demand for capital (that is, the quantity of capital that firms on aggregate want to hold) is a decreasing and sufficiently elastic function of its price, the interest rate.[4]

The negative slope of the demand function for capital is determined by direct and indirect substitution mechanisms. When the interest rate rises, firms tend to reduce the capital-labour ratio of their production processes, in order to save on the factor that became relatively costlier. Demand for capital decreases. Another force, coming from the demand side, works in the same direction. The relative prices of the more capital-intensive goods increase, so demand for them declines and their share in GDP gradually decreases, again reducing demand for capital. Of course equal and opposite processes will take place when the interest rate decreases. In this way the marginal product of capital is kept in line with its cost: when the cost of capital increases (decreases), the reduction in the K/L ratio of the economy raises (diminishes) also its marginal product.[5]

From demand for capital to investment The logic through which investment (a flow) can be derived from demand for capital (a stock) is examined at length by Garegnani (1978, pp. 346 and 352),[6] whose account is followed here. If we assume no fixed capital (i.e., all capital is circulating) and annual production cycles, it is easy to see that yearly investment is just equal to demand for capital. The presence of fixed capital makes things more complex, but the essence remains the same: investment is the flow that adds to the stock of capital. Investment will thus rise above (fall below) the level needed to compen-

[4] Early marginalist authors generally acknowledged that supply of capital (in the form of loanable, investible funds that savers are willing to provide to firms) may not be monotonically increasing in the interest rate. For this reason, the demand for capital curve has to be not only decreasing, but also significantly elastic, in order to produce a stable capital market equilibrium. The notion of savings as a supply of 'free' capital is of course a complex and problematic one. The issue is however outside the scope of the present paper, which focuses on the demand function for capital.

[5] Of course this reasoning relies on the law of diminishing returns. The latter ensures that, economy-wide, the marginal product of one factor is decreasing in its employed quantity, keeping fixed the quantities of the other factors employed in conjunction with it.

[6] An even more detailed account, expanding on that of Garegnani, is found in Petri (2004, pp. 127-135).

sate depreciation when demand for capital increases (decreases). The investment function is thus determined by the demand for capital function and (assuming no 'perverse' effect of the interest rate on the rate of depreciation) its relation with the interest rate bears the same negative sign.

In this long-period approach, technology is conceived as putty-clay: production factors are substitutable only ex-ante. This makes it possible to determine the level of investment in a given period (e.g., a year). In each year, gross investment is equal to the desired K/L ratio, multiplied by the number of workers released by the scrapping of old plants.[7]

Note that in analyzing the investment market equilibrium, traditional marginalist theory takes total labor employment as a given – determined by the conditions of equilibrium in the labor market. This is necessary to allow the optimal K/L ratio to determine univocally the optimal capital stock (that is, the long-period demand for capital). Moreover, it is this full employment assumption that justifies taking as given the quantity of labor to be employed in new plants, thus determining the gross investment level in each period on the basis of the optimal K/L ratio to be adopted in new plants.

1.2 A simple model and a graphical example

A simple formalization can help grasp the traditional neoclassical determination of investment. Take a one good economy (steel is produced by means of labour and steel)[8] with no taxes. Assume perfect ex-ante substitutability between capital and labour. As an exemplification to illustrate quantitatively the idea, we can use a Cobb-Douglas production function, which applies only to new plants in the short-run but to the whole economy in the long-run (even if technology is putty-clay, in the long-period[9] all productive plants will eventually come to adopt the optimal K/L ratio, provided that the latter remains constant for a sufficient lapse of time). We also assume, for simplicity, a rigid labor supply; the market-clearing condition thus reduces to an equality between labor employment and the

[7] This is of course a simplification, which assumes that all capital is durable. When there is circulating capital alongside fixed capital, a portion of investment will consist in circulating capital goods to be employed in 'old' productive plants. As the optimal K/L ratio in old plants is not changeable, this portion of investment will be unlikely to be influenced by changes in the optimal K/L ratio.

[8] Here steel is both the capital good and the consumption good: one unit of steel can either be consumed or employed as a means of production. Wages are obviously also paid in steel.

[9] Note that here the 'long-period' does not correspond to that horizon of time in which one can expect prices to converge to production costs, but rather to the longer time horizon in which the equilibrium K/L ratio is adopted by all production units. Some authors refer to this kind of long-period equilibrium as a fully-adjusted position (Vianello, 1985, p. 70).

given supply of labor. Profit maximization requires that the value marginal product of each factor is equal to its price. In a long-period position we thus have

$$Q = AK^{\alpha}L^{\beta} \tag{1}$$

$$w = \frac{\delta Q}{\delta L}p \tag{2}$$

$$r = \frac{\delta Q}{\delta K}p \tag{3}$$

$$p = 1 \tag{4}$$

$$L = \bar{L} \tag{5}$$

Q is output (the quantity of steel produced); K the capital stock (steel used as capital); L labour (worker-hours); w the wage; r the interest rate; p the (normalized) long-period price of steel.

Equations 1 to 4 determine the optimal K/L ratio:

$$K/L = (\alpha/\beta)(r/w)^{-1} \tag{6}$$

Eq.5 (the full labor employment condition) closes the model, allowing the interest rate to univocally determine demand for capital:

$$K^{\star} = (\frac{r\bar{L}^{-\beta}}{\alpha A})^{\frac{1}{\alpha-1}} \quad \text{and} \quad \frac{\delta K^{\star}}{\delta r} = \gamma r^{(\frac{1}{\alpha-1}-1)} \tag{7}$$

where γ is a constant.[10] Decreasing marginal productivity of capital ($\alpha < 1$) is a sufficient condition for the effect of the interest rate on the optimal capital stock to be negative ($\gamma < 0$), as long as r is non-negative.

Figure 1 illustrates an example in the K-L space with $\alpha = \beta = 0.5$. Imagine that initially the interest rate and wage rate are such that the optimal K/L ratio is equal to a. Since L is exogenously fixed, the intersection between the vertical dotted line indicating the quantity of labour and the grey slope corresponding to the optimal K/L ratio determines a definite long-period position (point 1) with a determinate optimal capital stock (K_1) and level of output (the isoquant q_1). Imagine that the interest rate increases to such a

[10] More precisely, $\gamma = \frac{(\alpha A)^{\frac{1}{\alpha-1}}\bar{L}^{-\frac{\beta}{\alpha-1}}}{\alpha-1}$

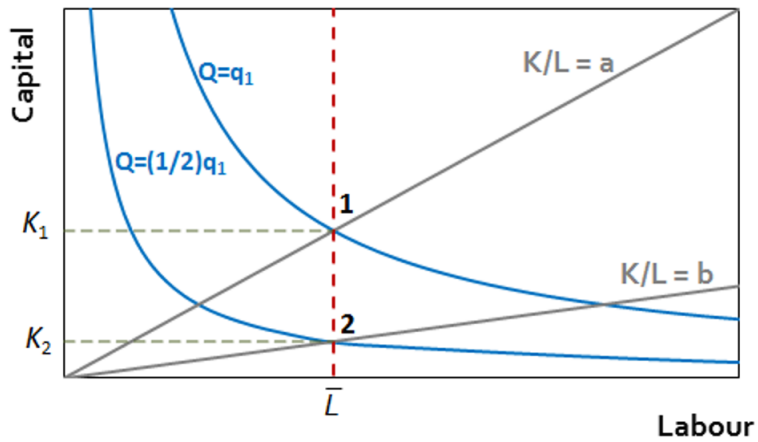

Figure 1: Determination of the optimal capital stock in the early marginalist approach - an example

level that the optimal K/L ratio decreases to b. As old plants are gradually scrapped and replaced with new plants with a lower K/L ratio, the capital-intensity of the economy will gradually decrease. Eventually, all plants would adopt the new optimal K/L ratio and the economy would reach the new long-period position (point 2). During the slow transition, a cumulated flow of net (dis)investment equal to $K_2 - K_1$ will take place. Also aggregate output decreases.

1.3 Theoretical flaws of the traditional neoclassical approach

This theory of capital and investment, however, presents a major theoretical problem, related to its very conception of capital. This shortcoming has not emerged in the simple model presented above, because we have assumed an economy with only one homogeneous capital good.[11] This assumption turns out to be crucial. In fact it has been demonstrated – mainly by the scholars that participated to the so-called 'Cambridge capital controversy' – that a monotonic functional relation between the optimal K/L ratio and the relative price of capital cannot be derived in an economy with several heterogeneous capital goods.

The Cambridge capital criticism We provide a more detailed exposition of this point in Appendix A, while here we limit ourselves to a brief summary of the argument.

[11] In our extremely simplified model, we have also assumed that this only capital good is homogeneous with output. A single capital good, distinct from the consumption good, would imply some complications but it would not undermine the possibility of deriving a neoclassical investment function. It is the presence of more than one capital good that undermines the main conclusions of the theory.

In an economy with heterogeneous capital goods, the aggregate capital stock must be measured in value, given that a common physical measure is not available. But then – whatever the numeraire chosen and even in the absence of any change in production methods employed and quantities produced – the quantity of capital varies with the vector of relative prices, which in turn is a function of distributive variables. The unavoidable dependence of the aggregate value of capital upon distribution undermines the neoclassical theory of capital. In the absence of a measure of the 'capital-intensity' of production techniques that is independent of distribution, it is not possible to derive a decreasing demand curve for capital based on factor substitution. It has been demonstrated that the same technique can actually be cost-minimizing at two different levels of the rental cost of capital but not in between them (a phenomenon known in the literature as 'reswitching of techniques') and that a decrease in the interest rate may well induce profit-maximizing firms to decrease their capital intensity, however the latter is defined (a phenomenon referred to as 'reverse capital deepening'), if the direct effect of the interest rate reduction is more than compensated by changes in the relative prices of the heterogeneous capital goods.[12] The reader can find a more detailed exposition of these results in Appendix A.

These findings undermine neoclassical capital theory in a fundamental sense: the very conception of capital on which the whole theoretical construct is based – capital as a single homogeneous production factor, a 'fluid' measured in value and adaptable to different forms – turns out to be untenable. And even when adopting this incorrect definition of capital, one finds that there is no general negative relation between the long-period K/L ratio and the interest rate.

The full-employment assumption While the results just summarized appear largely sufficient to undermine the traditional neoclassical theory of investment, a second line of attack, independent of the first, has been recently advanced by Petri (2015). The argument aims to prove that, even accepting the (demostrably incorrect) neoclassical determination of the K/L ratio, when endogenous changes in total labor employment are taken into account, the interest rate does not suffice to determine investment. As we have already mentioned, the neoclassical 'long-period' capital demand function is derived by assuming equilibrium in all the other markets – most importantly, in the labor market. Total labor employment

[12] The literature on this issue is ample. The reader can refer to Samuelson (1966) and Garegnani (1990) for major overviews from two leading participants in the debate belonging to opposite sides. More pedagogical discussions can be found in Petri (2004) and Lazzarini (2011)

(L) is thus taken as given, determined by a labor market clearing condition. Therefore the optimal K/L ratio, determined on the basis of capital-labor substitution, is sufficient to determine the optimal capital stock. More precisely, under putty-clay technology, it is assumed that the entire quantity of labor 'freed' by the scrapping of old plants (call this $\hat{L}$), and no more than that, will be employed in newly constructed plants. Therefore, the adoption of the optimal K/L ratio in new plants implies that the level of gross investment is fully determined by the formula $I = \frac{K}{L} \cdot \hat{L}$. Without the full employment assumption, capital-labor substitution would only determine K/L, and thus $I/\hat{L}$, but not investment, because $\hat{L}$ would not be determined.

Petri (2015), however, questions the legitimacy of the full-employment assumption. The reasoning, succinctly stated, goes as follows. The labor market clearing condition is based on the decreasing demand curve for labor. In the presence of unemployment, as neoclassical theory goes, the real wage would decrease, thus increasing employment, until full-employment is restored. However the elasticity of output to demand in the various industries, allowed by some flexibility in capacity utilization and inventory accumulation/depletion, implies that (1) total employment is influenced by desired output and (2) a multiplier effect is at work. Suppose that, in this context, the real wage decreases for exogenous reasons (for example a political decision to abolish minimum wage legislation). Given the initial level of output, this would decrease investment because of a reduction in the optimal K/L ratio, that leads to reduced investment in new plants. But the decrease in investment would have a multiplier effect. The output level would thus be reduced, and by more than the decrease in investment. A significantly lower output level would mean that, notwithstanding the decrease in the K/L ratio, total labor employment may well decrease. The sensitivity of industries' output to demand thus undermines the decreasing demand curve for labor on which the full-employment assumption is based.

According to this reasoning, the full employment assumption should be dropped and substituted with the condition that labor employment is determined by effective demand. This would give rise to a different investment function – similar, as we will see, to the so-called 'neoclassical accelerator' models – in which a major influence of demand on investment is admitted.

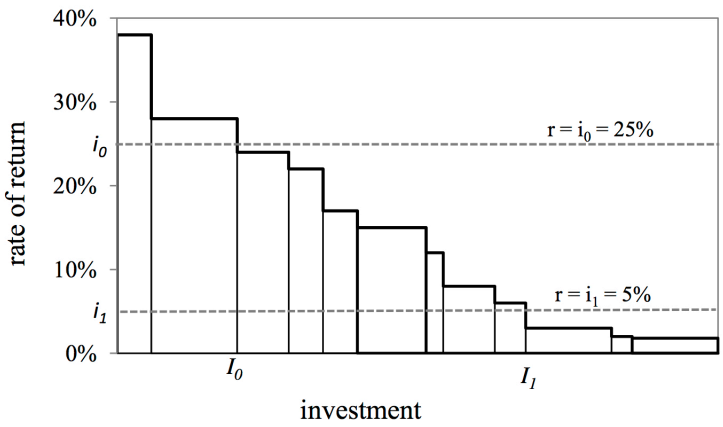

Figure 2: Investment determination in the 'array of opportunities' approach

2 The 'array of opportunities' approach

Some authors have proposed a different derivation of the neoclassical interest-elastic investment function, that does not rely on capital-labor substitution, but is instead based on what has been called the 'array of opportunities' approach.[13]

The idea is very simple. At any given point in time, each firm is presented with several possible investment projects, which can be ranked on the basis of their expected rate of return. Profit-maximization implies that the projects offering a rate of return higher than the ongoing interest rate – and thus providing positive profits – will be undertaken.[14]

Consider what happens if the interest rate decreases. For each firm, investment projects that were unfeasible will become profitable. Aggregate investment, defined as the sum of all investments realized in the economy, will thus increase. Figure 2 provides a graphical example: there are 12 potential investment projects, each defined by its expected rate of return (on the vertical axis) and its size in monetary value (on the horizontal axis). As the interest rate decreases from i_0 to i_1, the amount of investment undertaken increases from I_0 to I_1.

At closer scrutiny, however, the 'array-of-opportunities' approach appears hardly defensible. Competition implies that interest rate changes are bound to cause variations in prices, modifying the expected yields of all projects and their ranking. To grasp this point,

[13]The term is borrowed from Witte (1963, p. 445) and used also by Ackley (1978, p. 623) and Petri (2004, p. 262). This theory can be found in various authors, starting from Fisher (1930).

[14]In the presence of risk, the rate of return should be higher than the risk-adjusted interest rate.

we can note that (i) the rate of return of an investment project depends crucially on the price at which the output will be sold and (ii) the interest rate enters in the determination of prices, being one of the costs of production. Specifically, a change in the interest rate will change both the ratio between the prices of different products (i.e., relative prices) and the ratio between price and wage in each sector (i.e., rates of return).

It should now be clear that a variation in the interest rate is bound to modify the returns of all projects and their ranking. It is thus incorrect to take the array of opportunities as fixed while the interest rate varies, as done in Figure 2.[15] But this is not all: in equilibrium competition imposes a uniform normal profit rate[16] on all investments (after allowing for risk differentials), so there is no array of opportunities to start with (Ackley, 1978, p. 623). And the normal rate of return will move in step with the interest rate: because of competition, an autonomous decrease in the interest rate will result in a reduction in prices relative to wages, thus decreasing also the normal rate of return (Pivetti, 1991).[17]

3 Jorgenson's investment model

In a seminal article published in 1963, Jorgenson proposed a neoclassical investment model that proved extremely influential. The declared aim was *"to present a theory of investment behavior based on the neoclassical theory of optimal accumulation of capital"* (Jorgenson, 1963, p. 248). Jorgenson's model has indeed gained recognition as the standard neoclassical investment model.[18] In this section we provide a detailed discussion of Jorgenson's

[15] The incorrectness of this derivation of the neoclassical investment function has been generally acknowledged also by adherents to the neoclassical school. See for example Ackley (1978, pp. 623-624) and Alchian (1955, p. 942)

[16] The normal profit rate is defined as "the rate of return on capital which would be obtained by firms using dominant or generally accessible techniques, and producing output at levels regarded as normal at the time the capacity was installed" (Pivetti, 1991). It corresponds to the neoclassical equilibrium (or natural) rate of return on capital.

[17] Of course the economy can be in disequilibrium in the short run. But as long as some type of investment emerges, which is expected to yield a rate of return higher than the normal profit rate, firms will rush to exploit it, until the rate of return is pushed down to its normal value (Ackley, 1978, p. 623). In this sense the 'array of opportunities' can perhaps be seen as one of the mechanisms that contribute to ensure a tendency toward uniformity of the rate of profit over the supply prices of the various capital goods – but hardly as the mechanism that explains the rate of aggregate investment.

[18] This status is explicitly recognized, for example, in the stated motivation for the inclusion of Jorgenson's 1963 article in the list of the Top 20 most influential papers published in the first 100 years of the *American Economic Review* (Arrow et al., 2011, pp. 4-5). Indeed influential textbooks (as Romer, 2012, pp. 406-407), literature surveys (most importantly Chirinko, 1993, p. 1878 and Caballero, 1999, p. 817), theoretical treatises (notably Dixit and Pindyck, 1994, p. 5) and empirical works (recent examples are Cummins, Hassett, and Hubbard, 1994, p. 58, Fazzari, Hubbard, and Petersen, 1988, p. 144 and Bond and Xing, 2015, p. 28), all present Jorgenson's 1963 model as the baseline standard neoclassical investment model. Another exposition of

approach, as originally presented, and as interpreted and applied in the subsequent literature. We will argue that the model is not only subject to the Cambridge capital critique, like the traditional neoclassical approach, but it also presents additional fundamental difficulties.

3.1 Demand for capital in Jorgenson's model

Jorgenson (1963) starts with the assumption that the firm maximizes the present value of the sum of all future net revenues, taking all relevant prices (including the interest rate) as exogenous. He also assumes a two-good economy, with one (homogeneous) capital good and one output good, and a Cobb-Douglas production function. The firm's optimal capital stock is thus found by solving the following optimization problem

$$\begin{aligned} \max_{K_t, L_t} \qquad & V = \int_0^\infty e^{-rt} R(t) dt \\ \text{with} \qquad & R_t \equiv pQ_t - wL_t - qI_t; \\ & Q_t = F(K_t, L_t) = AK_t^\alpha L_t^\beta \\ \text{subject to} \qquad & \dot{K}_t = I_t - \delta K_{t-1} \end{aligned} \tag{8}$$

where R are net revenues,[19] r is the interest rate, p is output price, w is the wage rate, q the price of the capital good, Q the quantity of output, L the quantity of labor, δ the rate of depreciation and I is investment (measured in physical units of the capital good).[20]

the same model is provided in an often quoted empirical work by Hall and Jorgenson (1967). It may be worth noting, to avoid confusion, that in a different paper, also published in 1967, Jorgenson proposed a different model, with different and more restrictive assumptions (Jorgenson, 1967). It is however fair to say that Jorgenson's 1967 model – strongly and convincingly criticized by Tobin (1967, pp.156-158) in a comment appeared in the same volume – has not received widespread acceptance, as testified by the fact that it is not discussed at all in any major recent survey or textbook, nor in most empirical and theoretical studies on investment. Indeed, as of March 25th 2016, the 'Google Scholar' search engine reports 2,839 academic citations for Jorgenson (1963) and 1,949 for Hall and Jorgenson (1967) (the two papers presenting the model discussed here), while Jorgenson (1967) has 'only' 599 academic citations. See Petri (2004, pp. 287-90) for a thorough discussion of Jorgenson (1967), which makes it clear that the latter and Jorgenson (1963) are two very different models.

[19] This specification of the net revenue (or profit) function has been criticized by Petri (2013): the investment expenditure (I_t) is not a reduction of profits at time t, but an investment financed either by an increase in debt or by the use of previously accumulated own funds. So it will affect profits from time $t+1$ on wards, in terms of interest payments or opportunity cost of the reduction in own funds, but not those at time t. Accordingly, the net revenue function should include interest payments (or equivalently opportunity cost) on the existing capital stock, but not the investment expenditure. Surprisingly, the subsequent investment literature has followed this unorthodox definition of the net revenue function. (Specifically, Petri refers to the model in Romer (2012), that follows Jorgenson in using this definition of profits.)

[20] With respect to the formulation reported in Jorgenson (1963), we are neglecting taxes to simplify the exposition and concentrate on the important features of the model.

The solution is characterized by the marginal productivity conditions

$$\frac{\delta Q}{\delta L} = \frac{w}{p} \quad \text{and} \quad \frac{\delta Q}{\delta K} = \frac{q(r+\delta)}{p} = \frac{c}{p} \tag{9}$$

where the composite term $c = q(r + \delta)$ is labeled the 'user cost of capital'.[21]

The optimality conditions in equation 9 are of course analogous to equations 2 and 3 above (although in writing equations 2 and 3 we have assumed capital homogeneous to output – thus neglecting the price of capital relative to output – and no depreciation). The important difference is of economic interpretation: in the traditional theory the marginal productivity conditions apply at the level of the aggregate economy, while Jorgenson's model takes the single (perfectly competitive) firm as its object of analysis.

The indeterminacy problem At this point, the analysis is bound to encounter a major problem. Under constant returns to scale (CRS), the optimality conditions in equation 9 only define the optimal K/L ratio, while the optimal capital stock remains indeterminate.[22] We can call this the 'indeterminacy problem' affecting Jorgenson's model. In order to derive a demand function for capital, some other assumption must necessarily be introduced.

As discussed in Section 1, traditional neoclassical analysis surmounted this problem by studying the aggregate investment market under the assumption of equilibrium in all the other markets. In particular, assuming market-clearing equilibrium in the labor market allowed early neoclassical authors to take L as given. With L fixed, for each given level of the cost of capital, the implied optimal K/L ratio would suffice to determine the optimal capital stock. The stated aim of Jorgenson's model, as declared in its Introduction, was to present an investment model based on the traditional neoclassical theory of demand for capital (Jorgenson, 1963, pp. 247-248). However, by taking as its object of analysis the single competitive firm instead of the aggregate capital-good market, Jorgenson's model is bound to depart from the traditional neoclassical theory of capital. His model cannot possibly be 'closed' by taking labor employment as given: to the contrary a single firm operating under perfect competition is by definition able to hire any desired amount of

[21] Capital gains ($\dot{q}$) are neglected for simplicity (here and also in Jorgenson's analysis).

[22] With CRS, the profit function of a price-taking firm is linear in the capital stock, so the optimal value of the latter is not determined. This is a well-known problem in neoclassical production theory: under a linearly homogeneous production function, the optimal size of a price-taking firm is either zero, infinite or undefined, depending on whether the unit production cost is higher, lower or equal to the unit price of output.

labor at the prevailing wage rate.

Jorgenson (1963), instead, proposes to 'close' the model through the following assumption:

"*[W]e assume that output and employment on the one hand and capital stock on the other are determined by a kind of iterative process. In each period, production and employment are set at the levels given by the first marginal productivity condition and the production function with capital stock fixed at its current level; demand for capital is set at the level given by the second marginal productivity condition, given output and employment. With stationary market conditions, such a process is easily seen to converge to the desired maximum of net worth. Let $K^\star$ represent the desired amount of capital stock, if the production function is Cobb-Douglas with elasticity of output with respect to capital, [α],*[23]

$$K^\star = \alpha \frac{pQ}{c} \tag{10}$$

"(p.249)

Jorgenson thus argues that the 'indeterminacy problem' can be solved by simply taking the initial capital stock level as given and then assuming a trial-and-error process by the firm. He then jumps to the formulation of a capital demand function that, given technology, depends on output level and cost of capital. He does not provide a formal proof or a numerical example to illustrate the proposed 'iterative process' and show that it indeed yields a determinate optimal capital stock, equal to equation 10.

In fact, the iterative process proposed by Jorgenson does not appear convincing as a solution to the 'indeterminacy problem'. First, note that an optimal capital stock level cannot possibly be selected by taking both output *and* employment as given, as Jorgenson writes in the passage above. As it is easy to see from the production function in equation 8, if both L and Q are taken as fixed, no change in K is possible. The only way to make some sense of the iterative process proposed by Jorgenson is thus to assume that only output (or only employment) is taken as given in the second stage of the process, so that the firm is free to vary the K/L ratio (and consequently the capital stock) on the basis of the cost of

[23] Jorgenson denotes the elasticity of output w.r.t. capital as γ, while we have indicated it as α here for consistency with the previous sections.

capital.[24]

However, even after making this correction, the assumption that in the second step of the process, when choosing its optimal capital stock, the firm would take the output (or employment) level – determined in the previous period on the basis of inherited capital stock and marginal productivity conditions – as fixed, appears impossible to justify (as noted in Borch, 1963). A competitive firm can certainly adjust both its output and employment levels.[25]

Let us accept, for the sake of argument, the questionable assumptions of this iterative process, and assume that only output *or* employment is taken as given in the second step, to avoid immutability of the capital stock. Two questions then arise: does this process indeed converge also under CRS? is the solution to which the model iteratively converge dependent or independent of initial conditions?

It is easy to see that under decreasing returns to scale the process would converge to a solution that is independent of initial conditions, and dependent only on prices and technology. But in this case there would be little need for an iterative process to start with: the solution could be determined just on the basis of the marginal productivity conditions in equation 9. As Borch (ibid.) rightly asks (see note 25 above), why would the firm undertake the trial-and-error process described by Jorgenson, instead of optimizing K, L and Q simultaneously?

In the more interesting case of CRS, instead, the iterative process described by Jorgenson would generally not converge to an optimal capital stock. More precisely, with p greater than unitary cost, the optimal capital stock would increase at every iteration, tending towards infinite; with p equal to unitary cost, the iterative process would just bring

[24] It appears plausible to conjecture that this may be what Jorgenson actually meant. In a subsequent article, published in 1972, however, he refers to the iterative process proposed in his 1963 article, quoting almost in full the same passage that we reported above, without acknowledging or correcting the mistake of taking both employment and output as given in the second passage (Jorgenson, 1972, p. 233).

[25] Borch (1963) criticizes the iterative process proposed by Jorgenson as follows: *"Professor Jorgenson's investment theory is based on two behavioral assumptions: (1) Capital stock is taken as fixed, and the firm determines the optimal level of output. This is a very reasonable short-run assumption, (2) The output level determined under the first assumption is taken as fixed, and the optimal amount of capital stock is determined. This seems to be a rather doubtful assumption. The two assumptions together imply that investment is a trial-and-error process. Professor Jorgenson states that it is easy to see that the process will converge. However, if this is easy to see, why does not the firm see it? It seems natural to assume that intelligent management will see more than one step ahead in this process and try to optimize output level and capital stock simultaneously. This seems particularly natural under the perfect certainty assumed by Professor Jorgenson."* (p.273). Quite surprisingly, in his discussion Borch does not mention the fact that under CRS this simultaneous determination would lead to indeterminacy. Moreover, he does not note that in his article Jorgenson argues that both output and employment could be taken as given in the second step of the process.

the firm to keep K at its initial level forever (so the process converges immediately, but the solution fully depends on initial conditions); with p lower than unitary cost, K would decrease at every iteration, tending towards 0. Unsurprisingly, the 'iterative process' proposed by Jorgenson reaches the same results of standard optimization, and does not resolve the indeterminacy problem.

3.2 Two very different solutions to the indeterminacy problem

Even though explicit evaluations are in scarce supply,[26] the subsequent literature appears to generally acknowledge that the 'iterative process' proposed by Jorgenson does not solve what we called the 'indeterminacy problem'. In his survey, indeed, Chirinko (1993) states that

"the definition of $K^\star$ provided by [Jorgenson's model] has been questioned. No problem arises if the production technology exhibits decreasing returns to scale but, when returns are constant (as assumed by Jorgenson), $K_t^\star$ is not well defined" (p.1879).[27]

So how does the large literature stimulated by Jorgenson's investment model deal with this indeterminacy problem, if the iterative solution proposed by Jorgenson is (rightly) deemed unconvincing? It is possible to identify two different strands in the literature.

One way to avoid indeterminacy is to assume decreasing returns to scale. Under decreasing returns, equalization of the cost of capital with the marginal profitability of capital identifies a unique capital stock level. The profit-maximizing levels of K, Y and L are thus simultaneously determined on the basis of factors' prices and returns to scale.

A recent example of this approach can be found in the latest editions of Romer's popular macroeconomics textbook. Romer's discussion of investment theory begins by presenting a 'baseline' model for the determination of the optimal capital stock under perfect competition and no adjustment costs, assuming decreasing returns to scale (Romer, 2012, pp.405-406).[28] Another recent example, that will be discussed in some detail in Section

[26] An exception is Borch (1963), in the passage reported in note 25 above.

[27] The statement that Jorgenson assumes constant returns to scale appears in fact questionable: in his 1963 article he does not make any explicit assumption regarding returns to scale. More on this later

[28] Specifically, Romer (2012, pp. 405-406) writes profits as $\Pi(K, X_1, X_2, ..., X_n) - r_k K$, where Xs are the relevant (exogenous) prices and r_k is the Jorgensonian user cost of capital. $\Pi()$ is a value function that indicates the maximum profit the firm can achieve as a function of the capital stock level, taking prices as given. It is then assumed that $\Pi_{KK} < 0$. Here Π_K is not the marginal product of capital in the traditional sense (that is, keeping the employment of other factors fixed), but the marginal contribution of an additional unit of capital,

4.2, is found in Caballero and Engel (1999). An earlier influential paper that derives an optimal capital stock by assuming decreasing returns to scale is Lucas (1967, p. 80), while a statement signaling the diffusion of this approach can be found in Söderström (1976):

"If the production function is linearly homogeneous the [equations describing the factor demand functions] cannot be solved for the optimal levels of input but only the optimal capital/labor ratio ($K/L = k$) as determined by the factor price ratio. The step from [the marginal productivity conditions] to [the factor demand functions] thus depends on the fulfillment of a second-order condition on the concavity of the maximand. Economically this condition is usually imposed either on the production function (decreasing returns to scale) or on the revenue curve (monopolistic situations)." (p.371)

We can call the one with decreasing returns to scale the 'most neoclassical' version of the theory, because it implies that, given the production function, demand for capital depends only on prices, as in traditional neoclassical theory (although on the basis of different premises).

A very different solution is to take output as exogenously given – determined by aggregate demand. This solution appears inconsistent with Jorgenson's focus on an atomistic price-taking firm, that can sell any desired amount of output at the given market price. It is however legitimate if the representative firm is interpreted, as appears more correct, as a scale copy of the whole business sector, or equivalently a hypothetical firm that would undertake all investment in the economy. It is then natural to assume that the output level is constrained by effective aggregate demand. For example the discussion of the neoclassical investment function in the well-known textbook by Dornbusch and Fischer (1993, pp.334-335) is consistent with this 'neoclassical accelerator' approach (as noted by Petri, 2015, pp. 334-336).

The 'neoclassical accelerator' model can be illustrated graphically employing the simple model presented in Section 1.2. Equations 1 to 4 define also (a simplified version of) this model. However output (instead of labor employment) is taken as fixed, thus equation 5 is substituted with $Q = \bar{Q}$. Rearranging yields the 'Jorgensonian' demand for capital function[29] $K^\star = \alpha\frac{\bar{Q}}{r}$. Figure 3 illustrates this solution (again with $\alpha = \beta = 0.5$). As in the

with other inputs optimally adjusted. CRS would thus imply $\Pi_{KK} = 0$, while $\Pi_{KK} < 0$, as assumed by Romer, implies decreasing returns to scale. See Pariboni (2010, pp. 83-86) for a detailed discussion of Romer's baseline investment model, referring to an earlier edition of the textbook, that is however identical in this part.

[29] Here the interest rate is the only component of the cost of capital, given that we are neglecting taxation

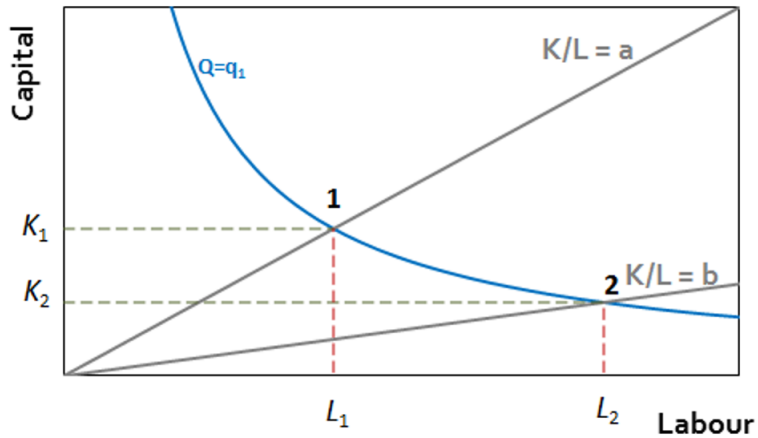

Figure 3: Determination of the optimal capital stock in the 'neoclassical accelerator' approach - an example

graphical exemplification of the traditional model (Figure 1), a given ratio of the interest rate to the wage rate determines the K/L ratio (a). But in this case L is unconstrained, while the isoquant is taken as given ($Q = q_1$) so we can determine that the economy is in point 1. In this case if the interest rate increases and lowers the optimal K/L ratio to b, the economy moves along the given isoquant and reaches point 2. Notwithstanding the putty-putty technology and perfect factor substitutability, the elasticity of demand for capital with respect to the interest rate is lower than in the traditional approach. Moreover, the decrease in capital intensity is associated with an increase in employment.

The distinction between these two ways to derive the demand for capital function from the optimality conditions is crucial: it actually defines two alternative theories of the optimal capital stock. In the first, aggregate demand exerts no autonomous influence: given returns to scale, $K^\star$ depends only on the relative cost of capital. By contrast in the second approach, taking output as given, an influence of aggregate demand (the accelerator effect) is admitted.[30]

It is not easy to assess which of these two approaches is the one actually followed by

and depreciation, and capital is homogeneous with output so their price is the same.

[30]Note that virtually all empirical works estimating the neoclassical investment function either employ the composite term $\frac{pQ}{c}$ as the independent variable (an example, besides Jorgenson, 1963 and Hall and Jorgenson, 1967, is Bernanke, 1988), or estimate separately the effects of the cost of capital and output (an example is Chirinko, Fazzari, and Meyer (1999)). In both cases output is implicitly treated as exogenous. This appears very likely to be a consequence of the universally recognized fact that output changes explains the largest fraction of investment variation in empirical data. Therefore not including it among the independent variables would result in embarrassingly low explanatory power and evident specification problems.

Jorgenson. The dominant interpretation in the literature appears to be that he assumes constant returns to scale and takes output as exogenously given, hence following the 'neoclassical accelerator' approach. This interpretation is not fully consistent with the way Jorgenson formulates the theory: in the iterative process that he describes, output is determined endogenously. He takes initial capital stock, technology and prices as the data of the iterative process by which the optimal capital stock is determined. And the iterative process would converge to a unique optimal capital stock, as claimed by Jorgenson, only under decreasing returns to scale. Moreover, his focus on a single atomistic competitive firm is not consistent with taking output as given. On the other hand, however, the Jorgensonian formula determining the optimal capital stock (equation 10), with output on the right side of the equation, implies assuming this term to be exogenous, consistently with the 'neoclassical accelerator' approach. And in the empirical estimates performed in the econometric part of the article – as well as in subsequent empirical work – the composite term $\frac{pQ}{c}$ is the independent variable, which implies considering output as exogenous.[31] In addition to this, in a later article, Jorgenson recognizes that *"[f]ailure to distinguish between the two alternative interpretations of desired capital, corresponding to constant and decreasing returns, has been an important source of confusion in the theory of investment"* and claimed that his model *"corresponds precisely to optimal production and investment policy under constant returns in production and installation"* (Jorgenson, 1972, p. 233), thus corroborating the interpretation of his model as assuming constant returns – while at the same time not noticing the inconsistency between his proposed iterative process (that he stated again in this very same 1972 article) and the assumption of CRS. In conclusion on this point, the interpretation of Jorgenson (1963) as proposing a 'neoclassical accelerator' model, while certainly not groundless, appears to neglect important inconsistencies. To be consistent, the 'neoclassical accelerator' approach must assume constant returns and interpret the representative firm as a scale copy of the business sector, not an atomistic price-taking firm. Jorgenson's 1963 model is actually a mix of two mutually incompatible approaches: it first describes an endogenous determination of output – which can yield a definite result only under decreasing returns to scale – but then derives and estimates

[31] If output was not exogenous, being determined simultaneously with the output level on the basis of prices and returns to scales, its presence in the right side of the estimated equation would result in biased estimates. Of course another large source of bias, in Jorgenson's regressions on aggregate US manufacturing data (as in virtually all the early empirical works on the investment function), is the positive influence of investment on aggregate output. As investment positively affects the composite term Y/c (through its positive effect on Y), the estimates of the impact of changes in the term Y/c on investment will suffer from upwards bias.

empirically an equation that makes sense only if output is taken as exogenously given.

3.3 From demand for capital to investment in the Jorgenson model

Having determined in this (ambiguous, as we have argued) fashion the optimal capital stock, Jorgenson (1963) deals with the problem of deriving the investment function from the capital demand function. (In section 1 we have seen how this problem has been addressed in the framework of traditional neoclassical theory.)

Jorgenson introduces delivery lags: new capital is not installed instantaneously. Net investment is therefore a distributed lag of new orders, which are made by firms in each period to fill the gap between the initial and the optimal capital stock. Given the assumption of 'radioactive' depreciation, replacement investment is proportional to the capital stock. Indicating net investment with I^N and replacement invesment with I^R, we thus have

$$I_t = I_t^N + I_t^R = \sum_{j=0}^{J} \beta_j \Delta K_{t-j}^{\star} + \delta K_t = \sum_{j=0}^{J} \beta_j \alpha \Delta(\frac{pQ_{t-j}}{c_{t-j}}) + \delta K_t \tag{11}$$

with β_s and J depending on the speed of delivery.[32] Given the assumption that in each period firms order the amount of capital goods that would fill the gap between actual and optimal capital stock, the theory also implies $\sum_{j=0}^{J} \beta_j = 1$.

Static expectations are implicitly assumed: the firm orders the quantity of capital goods that would make it reach the optimal capital stock, even if it knows that they will be delivered with some lags, as if it expected the optimal capital stock to remain the same in the following periods.[33]

3.4 Criticisms of the Jorgensonian model

Jorgenson's neoclassical investment model has sparked a lively debate in the aftermath of its publication. Two main issues have dominated this debate (as acknowledged in the early survey by Nerlove, 1972, p. 225): the assumption of a unitary elasticity of substitution and the determination of the 'speed of adjustment'.

[32] A peculiar feature of this formulation is that delivery lags do not apply to replacement investments, but only to the share of investment that enlarges the capital stock. We neglect this problem here, to focus on more fundamental issues.

[33] Alternatively, one can interpret the theory as assuming that delivery lags are unforeseen – Söderström (1976, p. 371), for example, follows this interpretation – but that would seem more far stretched.

The elasticity of substitution problem First, Jorgenson has been criticized – most famously by Robert Eisner – for having assumed a Cobb-Douglas production function. This assumption results in (i) a unitary elasticity of substitution between capital and labor and (ii) a unitary negative elasticity of desired capital and investment to the cost of capital. Indeed with a more general constant-elasticity-of-substitution (CES) production function with elasticity of substitution σ, the elasticity of the optimal capital stock to the cost of capital would be $-\sigma$.[34]

Jorgenson assumed a Cobb-Douglas production function not only in his theoretical model, but also in empirical applications. In the econometric section of his influential 1963 paper, as well as in subsequent work (most importantly the empirical analysis of the influence of tax policy on investment provided in Hall and Jorgenson, 1967), he estimated equation 11, using the composite term $\frac{pQ}{c}$ as the independent variable and imposing the constraint $\sum_{j=0}^{J} \beta_j = 1$ (Jorgenson, 1963, p. 252).[35] This amounts to assume that (i) and (ii) hold.

Critics correctly argued that Jorgenson's empirical findings, apparently favourable to the neoclassical investment function, were likely to be highly dependent upon these assumptions. In particular, by constraining the coefficients on the cost of capital and on output to be equal and unitary, he was imposing a relevant effect of the cost of capital, rather than demonstrating it. Critics like Eisner proposed, instead, to include the cost-of-capital and output terms separately, and to not constrain their coefficients, in order to test empirically whether the elasticities to output and to the cost of capital are equal and unitary, instead of assuming it.[36] These tests generally provided evidence of a near-unitary elasticity of

[34] The elasticity of desired capital to the cost of capital can be written as $E_c = \delta ln(K^\star)/\delta ln(c)$. With desired capital derived from a Cobb-Douglas and equal to equation 10, we have $E_c = -1$. As Jorgenson assumes that (barring delivery lags) net investment is equal to the change in demand for capital, also the elasticity of investment to the cost of capital is unitary. With a CES production function, the optimal capital stock would become $K^\star = \alpha(pQ)c^{-\sigma}$, where σ is the elasticity of substitution between capital and labor. In this case $E_c = -\sigma$, so σ is also the absolute value of the elasticity of desired capital (and thus of investment) to the cost of capital. The elasticity to output (if the latter is assumed to be exogenous) is instead unitary in both cases, independently of the value of σ.

[35] Note that in estimating empirically equation 11, α is not known, so it is impossible to estimate empirically both α and the β_s. While this problem could be solved by transforming variables in natural logarithms (see note 36 below), the solution employed by Jorgenson is to estimate empirically α by imposing the constraint $\sum_{j=0}^{J} \beta_j = 1$.

[36] For example this can be done by transforming variables in logarithmic terms, thus estimating an equation of the form $ln(I_t) = \alpha + \sum_{m=0}^{M}[\beta_m \Delta ln(c)_{t-m} + \gamma_m \Delta ln(Q)_{t-m} + \mu_m \Delta ln(p)_{t-m}] + \sum_{j=1}^{J} \omega_j ln(K)_{t-j} + \epsilon_t$. The hypothesis that production can be approximated by a Cobb-Douglas could thus be tested by assessing whether $\sum \beta_i = 1$. In broad terms, this is the approach followed by Eisner and Nadiri (1968). (Today of course, as applied economists have become more attentive to endogeneity problems, also this specification would raise doubts because of the simultaneous influence of investment on output growth and on the cost of capital.)

investment with respect to output, and of a much lower (usually near zero and statistically insignificant) elasticity with respect to the cost of capital,[37] therefore implying a low or null elasticity of substitution. The clearer and most forceful exposition of this critique and of these findings is provided in Eisner (1970) and Eisner and Nadiri (1968, 1970). On the basis of these findings, Eisner and his coauthors advocated the use of a more general form for the production function – CES instead of Cobb-Douglas (Eisner and Nadiri, 1970, p. 370).[38]

This critique has been essentially empirical, centering on whether the short-run elasticity of substitution between labor and capital is high or low. As confirmed by more recent surveys (Caballero, 1999; Chirinko, 1993), today this empirical controversy is still alive, but the widely held consensus is largely favorable to Eisner's thesis: most empirical studies have found the elasticity of investment to the user cost of capital to be rather low,[39] implying a low elasticity of substitution. Indeed recent presentations of the neoclassical theory of investment (for example in the already cited survey by Caballero, 1999, p. 817) tend to use a CES production function, as advocated by Eisner, so the Jorgensonian demand for capital function is usually expressed in the more general form $K^\star = \alpha(pQ)c^{-\sigma}$, where σ is the elasticity of substitution parameter in the production function.

This critique has thus been very effective in demolishing the idea that the elasticity of substitution between capital and labor is empirically high. Most importantly, it has provided a great deal of empirical evidence showing that the influence of the cost of capital on investment is low or insignificant, while that of output is much higher. This has important implications for the influence of economic policy on investment (see for example

[37] Of course these findings would in turn not be considered conclusive today, especially when employing aggregate data, because they are likely to suffer from simultaneity bias due to the influence of investment on output and (possibly) on the cost of capital.

[38] Specifically, Eisner and Nadiri (1970) claim their findings to be consistent with *"the implications of a CES production functions with elasticities of substitution nearer zero than unity (...) "* (p.370).

[39] In his survey, Chirinko (1993, p. 1881) notes that *"although (...) empirical results with versions of the Neoclassical Model differ widely, they suggest (...) that output (or sales) is clearly the dominant determinant of investment spending, with the user cost having a modest effect."*. Blanchard (1986) is even clearer: *"it is well known that to get the user cost to appear at all in the investment equation, one has to display more than the usual amount of econometric ingenuity; resorting most of the time to choosing a specification that simply forces the effect to be there.* Later Bernanke and Gertler (1995) confirmed that *"empirical studies of supposedly interest sensitive components of aggregate spending have in fact had a great difficulty in identifying a quantitatively important effect of the neoclassical cost-of-capital variable"*. Even the survey by Caballero (1999), by far the one that is most optimistic about the explanatory power of neoclassical investment theories, has to admit that *"[The cost of capital] is probably not the most important explanatory variable"* (ibid., p. 815). More recent studies – for example Sharpe and Suarez (2014) and Kothari, Lewellen, and Warner (2014) – appear to generally confirm the result of a low elasticity of investment to the cost of capital.

the discussion in Fazzari, 1993). Theoretically, however, it has been a rather mild critique, that just advocates a more general form of the production function without questioning more fundamental premises of the analysis — for example the very use of an aggregate production function with some elasticity of substitution, and the conception of capital that this implies.

The 'speed of adjustment' problem A second widely debated issue has been the 'speed of adjustment' problem. This problem concerns the derivation of the investment function from the demand-for-capital function. Jorgenson assumes that adjusting the capital stock is costless (besides the cost of acquiring capital goods). He takes into account delivery lags, but assumes that firms take every change in the capital stock as permanent, so they don't influence the optimization problem. As a result, the firm makes its choices as if adjustment was instantaneous and costless. It has been argued, however, that to explain short-run investment dynamics one needs to model not only the optimal capital stock, but also the choice of the speed at which gaps between actual and optimal capital stock are filled. In order to do this, the adjustment process must be modeled explicitly (Söderström, 1976, p. 371). This point of view had been forcefully advocated by Haavelmo (1960), who argued in an often cited passage that

"The demand for investment cannot simply be derived from the demand for capital. Demand for a finite addition to the stock of capital can lead to any rate of investment, from almost zero to infinity, depending on the additional hypothesis we introduce regarding the speed of reaction of capital-users. I think the sooner this naive, and unfounded, theory of the demand-for-investment schedule is abandoned, the sooner we shall have a chance of making some real progress in constructing more powerful theories to deal with the capricious short-run variations in the rate of private investment." (p.216)

Advocates of this critique have pointed out that in the Jorgensonian approach a discrete change in the data of the equilibrium would cause a discontinuous change in the desired capital stock; in the absence of frictions, firms would therefore tend to reach the new optimal capital stock instantaneously – given the assumption of putty-putty capital implicit in the model – resulting in a rate of net investment ($\dot{K}$) that tends to infinite. This critique is at the basis of all major developments of mainstream investment theory since the Seventies, which mainly consist of attempts to enrich the neoclassical framework

by explicitly modeling adjustment costs – as we will discuss in Section 4.

Also this critique, nevertheless, however much dismissive of the Jorgenson model, does not touch its core: the Jorgensonian determination of the optimal capital stock is not questioned. The critique in fact concerns only the derivation of the short-run investment function from the long-run capital demand function. In the remainder of this section we instead discuss more fundamental critiques, that cast doubt on the Jorgensonian demand for capital function: the incorrect treatment of capital and the aggregation problem.

Treatment of capital The Jorgensonian approach - just like traditional neoclassical theory - determines the optimal capital stock on the basis of a monotonic negative relation between the K/L ratio and the relative cost of capital. However, as already discussed (Section 1.3 and Appendix A), the heterogeneity of capital goods makes it impossible to derive such a relation. This means that the use of a CES aggregate production function – as advocated by some early critics of the Jorgenson model – with capital (the homogeneous value factor) and labor as its arguments, is just as incorrect as the use of a Cobb-Douglas. The Jorgenson model, in all its versions, is thus undermined by the 'Cambridge-criticism'.[40]

The aggregation problem Jorgenson appears to take it for granted that the aggregate demand for capital function of the economy is just a scaled-up version of that of the single firm, without mentioning the problem of aggregation. But is this assumption innocuous, or does it imply some fallacy of composition? The answer appears to be that it depends.

What we have called the 'most neoclassical' version of the theory, which assumes decreasing returns to scale, certainly suffers from a fallacy of composition. Decreasing returns to scale can determine the optimal level of output and capital for each firm but not for the whole economy, in which the number of firms is free to vary (Ackley, 1978, p. 624). At the aggregate level, the marginal productivity of capital can be decreasing only if the full employment of all other factors is assumed, as done in the traditional neoclassical approach. In fact it is not even possible to define the notion of marginal product of a production factor, without taking as fixed the quantities of all other factors employed in conjunction with it.

That undermines the main implication of this approach – that aggregate investment

[40] This is pointed out for example in Petri (2004, pp. 256-291)

can be determined only on the basis of prices and technology – even without questioning the assumptions of decreasing returns to scale at the firm level and perfect substitutability. For example imagine that aggregate demand is expanding but it is not convenient for the existing price-taking firms to increase their size, because of strongly decreasing returns to scale. Other firms will be created - each one with the optimal size and K/L ratio - until the entire demand is met at cost-covering prices. So decreasing returns at the firm level will have no influence on aggregate output and on the aggregate capital stock, they will just determine the optimal size of the firm, and thus the optimal number of firms for each given level of aggregate demand. What we called the 'most neoclassical' approach is thus better interpreted as a theory of the optimal size of the competitive firm, rather than a theory of aggregate investment.

The fallacy of composition incurred by treatments of investment that rely on decreasing returns to scale is stated very effectively by Ackley (1978):

"[An] incorrect derivation [of the neoclassical investment function] rests on factors internal to a firm, which have little relevance for an economy in which the number of firms is free to vary. We assume that a firm's long-run (as well as its short-run) production is subject to diminishing returns – either because some factor of production, perhaps the contribution of the ultimate decision maker, is fixed in amount or because of inevitable diseconomies of organization, communication, or management that arise as the scale of a firm increases. This means that any firm's calculation of profitability must recognize that, beyond some point, additional investment implies less than proportionate increases in output and/or more than proportionate increases in employment. Thus, the [interest rate level] that will equate [the marginal profitability of capital] and [the interest rate] must decline in order for [investment] to increase. True enough. But this explains only the size of the firm, not the amount of aggregate investment. The number of firms is not pre-ordained." (p.624).[41]

This fact, so clearly stated by Ackley, is instead neglected in today's most popular advanced macroeconomics textbook – the one by Romer (2012). As already mentioned, in presenting the determination of the optimal capital stock in the absence of adjustment costs – a more generic version of the Jorgenson model – Romer (ibid., pp. 405-406) relies on

[41] Ackley (1978) makes it clear in a note that *"[t]he preceding comments seem applicable to the "neoclassical" investment analyses of Jorgenson and others, who derive the specifications of an investment function from a micro-economic analysis of a profit maximizing-firm, without apparent reference to problems of aggregation "*(note 15)

decreasing returns to scale at the firm level to derive a neoclassical aggregate investment function (see note 28 above).

A fallacy of composition, however, is not committed by the 'neoclassical accelerator' version of the theory. In that case the representative firm can be interpreted as a scale copy of the whole business sector (or of a single industry), because the analysis relies on factors that apply also at the aggregate level. Output must then be interpreted as normal aggregate output and it is perfectly legitimate to take it as given – that is, determined by effective demand. In other words, one can take the marginal productivity conditions as applying to the whole economy, due to both direct and indirect substitution mechanisms (as in the traditional neoclassical approach), and at the same time let aggregate output be determined by effective aggregate demand.

4 Adjustment-costs models of investment

Models with adjustment costs are nowadays the workhorse of mainstream investment theory. They are employed by virtually all of today's advanced Macroeconomics textbooks and recently published journal articles.

The approach has been developed in response to the two issues that dominated the debate in the aftermath of Jorgenson's seminal contribution. On the one hand, these models address the theoretical problem of how to derive a finite rate of investment when a change in the data causes a discrete change in the desired capital stock (the 'speed of adjustment' problem). On the other, by introducing sluggish adjustment, they attempt to reconcile neoclassical theory with the empirically observed low elasticity of investment to changes in (the neoclassical definition of) the optimal capital stock, and in particular with the weak effect of the cost of capital.

The fundamental idea is that the investment choice of a firm can be thought of as two-stage process. First, the firm selects its optimal capital stock level ($K^\star$) on the basis of the relevant data. Second, it chooses the speed at which the gap between the current and the optimal capital stock will be closed, thus determining the rate of investment. In order for the latter choice to be meaningful (that is, to not result in instantaneous adjustment), it is assumed that there are costs associated with the alteration of the capital stock, besides the costs of purchasing capital goods. These can be due to installation costs, reorganization of the plant, retraining of workers, and so on. These costs of adjustment are assumed to be

a function of the rate of investment and (possibly) of the capital stock level.[42]

Begg (1982) summarizes the reasoning as follows:

"First, we require a model determining $K^\star$, the capital stock which firms currently believe they wish to hold in the long run. (...). Secondly, we require an adjustment rule or function

$$I_t = f(K_t^\star - K_{t-1}) \tag{12}$$

which determines the rate at which I_t, [net] investment at time t, eliminates the discrepancy between K_{t-1}, the capital stock carried over from the end of the previous period, and $K_t^\star$, the capital stock to which firms currently wish to converge." (p.185-6)

The functional form of $f(.)$ – that is to say, the optimal path of investment – is determined by some assumption on the nature of adjustment costs. In general, convex adjustment-costs models tend to predict a smooth and gradual path of adjustment, while models with non-convex adjustment costs tend to predict 'lumpy' investment, that is periods of inertia followed by investment bursts. In the remainder of this section we provide a baseline version of these models to illustrate their main characteristics, before discussing criticisms.

4.1 Convex adjustment costs

The standard assumption in the literature is that adjustment-costs are convex, meaning that they are increasing at an increasing rate in I. For example, under this assumption, building two identical new plants costs to the firm more than twice as much as building just one. Adjustment costs are usually also assumed to be decreasing in K, meaning that an investment project of a given size is relatively less costly for a big firm than for a small one. The canonical model is thus written as follows:[43]

[42] With deal here only with *internal* adjustment costs, that is adjustment costs due to factors internal to the firm, while keeping fixed the supply price of capital goods. Increases in the supply price of capital goods that are due to increasing demand are instead defined as *external* adjustment costs. The recent literature, however, has focused almost exclusively on internal adjustment-costs.

[43] A formalization practically equivalent to this one is found, for example, in the influential article by Hayashi (1982).

$$
\begin{aligned}
\max_{K_t, L_t} \quad V(0) = \quad & \int_0^\infty e^{-\int_0^t r_s ds} R(t) dt \\
\text{with} \quad & R(t) \equiv Y(K_t, L_t) - G(I_t, K_t) - w_t L_t - q_t I_t; \\
& G_I > 0; \quad G_{II} > 0; \quad G_K \leq 0; \quad G(0, K) = 0; \\
\text{subject to} \quad & \dot{K} = I - \delta K; \qquad \delta > 0
\end{aligned} \tag{13}
$$

where R are revenues;[44] Y output; K capital, L labour, w the real wage; q the relative price of capital; G adjustment costs; δ the depreciation rate and r the interest rate. As usual in neoclassical models, capital is assumed to be a single homogeneous factor. Output price is taken as a numeraire and adjustment costs are expressed in terms of units of output.

It should be clear that without adjustment costs the model would be practically equivalent to the Jorgensonian formulation, thus yielding the conditions in equation 9. With adjustment costs, instead, the first order conditions for an optimum imply

$$
\lambda_t = q_t + G_I; \qquad \lambda_t = \frac{Y_K - G_K}{r + \delta} + \dot{\lambda} \tag{14}
$$

Where λ is the shadow price of capital, or marginal profitability of capital. This is defined as the increase in net revenues that would be provided by an additional unit of capital, after other inputs have been optimally adjusted.[45] The economic interpretation of this condition is that the profit-maximizing investment rate equates the marginal profitability of capital with its marginal cost. The marginal cost is comprehensive not only of the price of buying an additional unit of capital, but also of the increase in adjustment costs that its installation would generate. The marginal profitability of capital, in turn, depends on its marginal productivity, the depreciation rate, the effect of a greater capital stock on adjustment costs, the rate at which future profits are discounted, and the capital gains it will provide.

Given the assumed convex shape of $G(.)$, the adjustment-costs function, equation 14 implicitly defines the following investment function

[44] This specification of revenues suffers from the same problem discussed in note 19, however it is (quite surprisingly) the most common in the literature.

[45] Note that the shadow price of capital is thus different from the marginal product, because the latter is defined by taking the employment of other factors as fixed. The shadow price of a good is also generally defined as the maximum number of units of account that the optimizing agent is willing to pay for one additional unit of the good.

$$I = F(\lambda, q, K) \qquad \text{with} \quad F_\lambda > 0; \quad F_K \geq 0; \quad F_q < 0 \tag{15}$$

This is the general result of models with convex and symmetric[46] adjustment costs: investment is increasing in the marginal profitability of capital and in the capital stock and decreasing in the relative price of capital.[47]

Two points are worth stressing, from the perspective of this critical survey. First, unlike Jorgensonian models, convex adjustment-costs models yield definite results also under CRS, and without assuming a given level of output or labor employment. This is likely to have contributed significantly to the popularity of these models: as we have seen, the result of indeterminacy of the optimal capital stock under CRS was rightly considered as an embarrassing property of the Jorgensonian neoclassical investment model. In this sense, in view of our discussion in Section 3.2, we can interpret the assumption of convex adjustment costs as a *third* alternative solution to the problem of indeterminacy. This solution can be seen as somehow similar in nature to the assumption of decreasing returns to scale (although certainly more refined) in the sense that it relies on factors internal to the firm, that impose some cost of expansion. The difference is that, while decreasing returns to scale allow to determine an optimal capital stock level, convex adjustment costs allow to determine an optimal rate of expansion.

Second, this model implies a negative elasticity of investment to the interest rate – a fundamental relation for neoclassical macroeconomics – *even in the absence of marginalist factor substitution mechanisms*. As apparent from equation 14, a lower interest rate increases the net revenue per unit of capital, resulting in a faster optimal speed of adjustment and therefore a higher investment rate. That means that a lower interest rate would increase the optimal investment rate also with a fixed-coefficients production process. We postpone to Section 4.3 a discussion of the important objections that this derivation raises.

[46] By symmetric here it is meant that, keeping everything else fixed, the adjustment cost of increasing the capital stock by some amount is equal to the adjustment cost of decreasing it by the same quantity.

[47] Imposing additional (more restrictive) assumptions regarding the revenue function, the adjustment cost function and the efficiency of information-gathering in financial markets, the model can be shown to imply the relation proposed on the basis of intuitive arguments in Tobin and Brainard (1977) between stock market valuations and investment. In particular, indicating the stock market valuation of the firm as V and defining Tobin's q as $q = V/pK$, it can be shown that investment is a (positive) function of Tobin's q only (Hayashi, 1982). It is however more useful, for the purposes of this essay, to stick to the more general model.

4.2 Non-convex adjustment costs

It has been argued that the assumption of convex adjustment costs is unlikely to hold for most firms (Hamermesh and Pfann, 1996) and indeed microeconometric studies have pointed to a complex mix of convex and non-convex elements (Russell and Haltiwanger, 2006).[48] Therefore, while it appears fair to say that the most popular and widely adopted model remains that with convex adjustment costs, several authors have argued that non-convexities and irreversibility play an important role in influencing investment choices and have thus proposed models with non-convex adjusment costs (Caballero, 1999; Caballero and Engel, 1999; Dixit and Pindyck, 1994; Russell and Haltiwanger, 2006).

A survey of the alternative shapes of the adjustment-costs function that have been explored in the literature would be outside the scope of this paper (the interested reader can refer to Caballero, 1999; Hamermesh and Pfann, 1996; Russell and Haltiwanger, 2006). We will focus, instead, on the development which has attracted more attention in the last decade. This is a generalization of the so-called (S,s) approach, which assumes the existence of a fixed cost of adjustment. This model is broad enough to be apt to illustrate the general characteristics of non-convex adjustment-costs models.[49]

It is assumed that there is some fixed cost that firms incur every time they alter their capital stock. The size of this fixed adjustment cost is assumed to vary randomly across firms and in time. The 'frictionless' optimal capital stock is defined as the capital stock level that would maximize profits in the absence of adjustment costs. The intuition is that the larger the gap between the actual and the *frictionless* optimal capital stock, the greater the probability that the gain from adjustment outweighs the (random) fixed cost of adjustment. The probability that a firm resizes its capital stock is thus increasing in the gap between the actual and the 'frictionless' optimal capital stock.

Of course the problem of deriving the frictionless optimal capital stock is exactly equivalent to that of deriving demand for capital in Jorgenson's neoclassical model. It thus incurs in the same difficulty: the firm's 'frictionless' optimal capital stock is indeterminate under constant returns to scale. To avoid indeterminacy, the proponents of this model

[48] It should be noted that 'adjustment costs' are multiform and rather broadly defined, therefore practically impossible to observe and measure directly. Empirical studies have tried to assess them indirectly, mainly by estimating adjustment-costs models derived under different assumptions, and testing which ones work better in predicting firms' investment dynamics, given some (assumed) definition of the optimal capital stock (Russell and Haltiwanger, 2006, p. 611)

[49] Specifically, we follow the model proposed in Caballero and Engel (1999), providing a simplified version similar to the one found in Caballero (1999).

assume 'decreasing marginal profitability of capital'. The marginal profitability of capital is defined here as the marginal net revenue from an additional unit of capital, *allowing the quantities of other inputs to be optimally adjusted*, but not taking adjustment costs into account. The authors motivate this assumption as "due to decreasing returns in the technology or the presence of some degree of monopoly power" (Caballero and Engel, 1999, p. 787). However the model is difficult to reconcile with imperfect competition, given that the firm is assumed to be a price-taker, so it must probably be interpreted as assuming decreasing returns to scale – thus following what we have called the 'most neoclassical' solution to the problem of indeterminacy of the optimal capital stock.

Formally, the deviation from the frictionless optimal capital stock (the *imbalance*) is defined as $z_{it} = ln(K_{it}) - ln(K^{\star}_{it})$. The stochastic fixed cost of adjustment is ω. The probability that a firm adjusts, given its imbalance and the adjustment cost it faces, is the adjustment hazard function, generally defined as

$$\Lambda = \Lambda(z, \omega) \quad \text{with} \quad \Lambda_z > 0; \quad \Lambda_\omega < 0 \tag{16}$$

In the simple case in which adjustment costs are only fixed,[50] if the firm adjusts it does so fully, reaching instantaneously its optimal capital stock (an 'all-or-nothing' investment policy). The expected investment rate of a firm is thus equal to its imbalance times the probability of adjustment (the hazard)

$$E(I_{it}/K_{it}|z) = -z\Lambda(z) \tag{17}$$

In order to infer aggregate investment dynamics from this microeconomic model, it is assumed that the number of firms is large but fixed, so the law of large numbers can be applied. $\Lambda(z)$ is then interpreted as the share of firms with imbalance equal to z that will adjust, given the distribution function of ω. Aggregate investment is the integral of $\Lambda(z)$ over the cross-sectional distribution of z.

$$I_t/K_t = -\int_{-\infty}^{\infty} z\Lambda(z)f(z,t)dz \tag{18}$$

[50]See Caballero and Engel (1999) for the more nuanced version of the model, with both fixed and variable adjustment costs.

where $f(z,t)$ is the distribution of z across firms at time t. Intuitively, aggregate investment in the fixed set of firms is the sum of the expected adjustment of each firm.

The appeal of this approach mainly derives from its ability to rationalize the 'lumpy' investment dynamics usually observed in manufacturing plants – meaning that productive plants tend to undergo infrequent but sizable waves of investment (Doms and Dunne, 1998). The model is in this sense more consistent with the evidence with respect to the canonical convex adjustment-costs model, that would predict a gradual and smooth investment path.

4.3 Can adjustment costs explain aggregate investment?

The idea that some broadly defined costs of adjustment exist at the plant level – probably a mix of fixed and variable elements – is certainly plausible. This is a realistic assumption for the microeconomic analysis of firms' behavior in the short-run. What is less clear is whether adjustment-costs models provide new relevant insights for the theory of aggregate investment. In these respect, the theory seems to suffer from some major shortcomings.

Fallacy of composition It is generally acknowledged in the literature that non-convex adjustment costs can produce lumpy investment at the firm level without relevant implications for aggregate investment, because in the aggregate their effect is likely to 'smooth out'.[51] But the problem is, in fact, much more general than this: *all* formulations that apply the firm's investment function derived on the basis of adjustment costs to the whole economy suffer from a fallacy of composition. This is the same fallacy incurred by models that derive the aggregate investment function from the microeconomic analysis of a representative firm with decreasing returns to scale, discussed in Section 3.4 above.

This criticism is indeed not new: in his early survey, Söderström (1976, p. 386) reluctantly admitted that in models based on adjustment costs *"market equilibrium (...) may be indeterminate under free entry"*. Indeed the equilibrium rate of aggregate investment *is* left indeterminate by adjustment costs models, the moment the possibility of free entry is admitted: these models determine the optimal rate of expansion of the firm, but not the equilibrium number of firms in the economy or in an industry (Petri, 2004, pp. 279-280).

The issue is generally ignored in the recent mainstream literature: the accepted convention is to circumvent the problem by assuming that the economy consists of a fixed set

[51] See for example Thomas (2002), Khan and Thomas (2008) and Hall (2004). However Bachmann, Caballero, and Engel (2006) argue the contrary

of firms, without providing a rationale for this assumption nor acknowledging the problem. In fact, it appears difficult to justify ruling out entry in models of aggregate investment.

Neglected dependence of prices from the interest rate Another major criticism concerns the way in which adjustment-costs models derive a negative relation between investment and the interest rate. In models with convex adjustment costs, this relation is inferred from the optimality condition in equation 14. This condition states that the firm increases its investment rate until the marginal cost of capital (equal to the sum of the unit price of capital plus the marginal cost of adjustment) equals the marginal profitability of capital (defined as the present value of the increase in future net revenues that an additional unit of capital would provide). It is thus inferred that a decrease in the interest rate, keeping all other prices fixed, would raise the optimal investment rate by shifting upwards the shadow price of capital (λ) curve.

This derivation, however, contradicts basic microeconomics principles: one cannot vary the interest rate while keeping all relative prices fixed. (This is of course the same mistake incurred by the so-called 'array-of-opportunities' approach discussed in Section 2.) The interest rate on employed capital is a component of average costs. Assuming that the relation between output and input prices remains unchanged in the aftermath of an interest rate reduction, is tantamount to assuming that extra-profits can be earned over an infinite time-horizon. Clearly, the rate of return on capital is bound to move in step with the interest rate, especially in models that assume firms to be competitive price-takers.[52]

Sensitivity to arbitrary assumptions There is also a further, more general, aspect that appears questionable. The actual patterns of adjustment costs are likely to be irregular, specific to circumstances, and thus different in each firm and for each investment project. And there is no way to directly measure them empirically, to assess if there really is some stable relation between the marginal cost of adjustment and the investment rate. Adjustment-costs investment models thus appear bound to rely on rather arbitrary premises, and to be very sensitive to changes from one arbitrary assumption to another.

[52]This particular mistake is not incurred by the generalized (S,s) model. In this model a lower cost of capital increases investment by raising the 'frictionless' optimal capital stock. The latter, in turn, is identified by equating the marginal return on capital with its price, under the assumption of decreasing returns to scale. In these models, therefore, the interest-elastic investment function depends entirely on the assumption of decreasing returns to scale and no entry. When entry is admitted neither the optimal capital stock nor the optimal rate of investment are determined.

In this sense, the ability of adjustment-costs models to make precise predictions about the behavior of each individual firm in the very short-run appears illusory, because it depends on specific assumptions about unobservable (and probably always-changing) adjustment costs. In other words, adjustment-costs models are likely to be 'precisely wrong', while arguably it would be more reasonable for theory to focus on getting right the determination of broad tendencies, that is to say, to seek an understanding of the factors that make the aggregate capital stock tend to expand or contract.

Concluding Remarks

In this essay we have reviewed the neoclassical determination of aggregate investment and its criticisms, distinguishing between the theory of the optimal capital stock (as conceived by traditional theory and later by Jorgensonian models) and that of the short-run frictions which are thought to govern the adjustment process. As we have shown, mainstream investment theory is far from bulletproof.

The traditional 'Wickellian' investment function rests on factor substitution mechanisms, that have been undermined by the results of the Cambridge capital controversies (illustrated in Appendix A below). These have demonstrated that the neoclassical determination of the optimal capital stock can only hold in a one-capital-good economy; it falls apart when the presence of more than one capital good is admitted. The traditional approach also relies on the 'pre-Keynesian' assumption of continuous full-employment equilibrium in the labor market.

The modern 'Jorgensonian' approach to the determination of the optimal capital stock is equally based on factor substitution mechanisms, but it models the behavior of an atomistic price-taking representative firm. This model appears even less solid: not only it is subject to the Cambridge capital critique, but notwithstanding its strong assumptions it is incapable of determining the optimal capital stock in the standard setting of constant returns to scale (CRS). Specifically, even accepting the logic of the theory, under CRS only the optimal K/L ratio is determined, while demand for capital is not.

As we have shown, it is fundamental to distinguish between two distinct ways in which neoclassical theorists have solved this indeterminacy problem. The 'most neoclassical' solution is to assume decreasing returns to scale. This formulation determines investment only on the basis of relative prices and technology. It represents, however, a clear example of a

fallacy of composition: when the firm's investment function is based on a factor internal to the firm (decreasing returns to scale), it cannot be generalized to the whole economy, in which new firms can enter. The 'least neoclassical' solution to the indeterminacy problem is to take aggregate output as given, as in the 'neoclassical accelerator' model. Importantly, in this way an influence of aggregate demand on investment – the accelerator principle – is admitted.

Notwithstanding major unresolved problems, mainstream theory has evolved in the last decades as if the problem of the determination of the optimal capital stock was solved, and it only remained to study the adjustment process toward equilibrium, by modelling the frictions caused by adjustment costs. It is natural to interpret adjustment-costs as another way to solve the problem of indeterminacy, more refined with respect to the assumption of decreasing returns to scale. The fallacy of composition remains the same: adjustment-costs models of investment cannot determine aggregate investment, even if we accept all their assumptions, because the number of firms in the economy remains indeterminate.

Ultimately, the neoclassical theory of investment appears deeply unsatisfying because it studies investment behaviour as it were a choice-of-technique problem. This is profoundly misleading. It fails to take into account two fundamental functions of capital accumulation: expanding productive capacity to meet increasing demand, and incorporating discontinuous innovations which increase production efficiency and/or open up new markets, independently of the evolution of the cost of capital (which, as the Cambridge controversy revealed, bears no necessary relation with the optimal K/L ratio). There is certainly space for developing better alternatives.

A different vision may start from the idea that firms tend to exploit all investment opportunities that are expected to yield at least the normal profit rate. But the actual rate of return expected on a given investment project will be either the normal profit rate, if outlets can be found for its entire produce at production prices,[53] or less than the normal profit rate, if demand is not expected to absorb the produce. In this framework, aggregate net investment will be driven by expected demand, which determines the number and size of investment projects whose produce can be entirely sold at cost-covering prices. This vision of investment as entirely induced by demand growth is still incomplete, because it doesn't include the important roles of innovation, institutional and political factors, im-

[53] Defined as the cost-covering prices (or normal prices), with production costs including also the normal profit rate over capital goods.

perfect competition. However it appears to represent the best starting point for a more convincing, non-neoclassical, theory of aggregate investment – not least because it has plenty of empirical evidence on its side.

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Appendices

A The Cambridge capital controversy and the relation between distribution and the choice of technique

In this appendix we provide a simple exposition of the theoretical problems of early marginalist investment theory – in particular the impossibility of deriving a monotonic functional relation between the optimal K/L ratio and the relative price of capital when there is more than one capital good.[54] This critique of the marginalist determination of demand for capital was brought up during the so-called Cambridge-capital controversy, mainly by economists working at Cambridge, UK.

In this exposition we take quantities produced as given, assume that no scarce resources enter production, no joint production is undertaken and the non-substitution theorem applies.[55] Hence in the long-period prices converge, due to competitive pressure, to minimum average costs. Long-period prices can thus be determined as functions of the rate of interest, according to the system of equations

$$\mathbf{p} = \mathbf{p}(\mathbf{d} + r\mathbf{I})\mathbf{A}(r) + w\mathbf{l}(r) \tag{19}$$

where $\mathbf{p}$ is the (row) vector of product prices; $\mathbf{A}$ is the matrix of technical coefficients of non-labor inputs, r the rate of interest (here identical with the rate of profit), $\mathbf{I}$ the identity matrix, $\mathbf{d}$ a diagonal matrix which has the vector of input depreciation rates as its diagonal (depreciation is assumed to be of the 'radioactive' type), $\mathbf{l}$ the vector of technical coefficients of labor input and w the wage rate.

A.1 Dependence of the quantity of capital on distribution

The quantity of capital in the economy is an aggregate measure of the several existing capital goods. As these different capital goods are physically heterogeneous, this aggregate

[54] Obviously, I claim no originality at all for this appendix: the results summarized here are well-known.

[55] The restrictiveness of these assumptions – and of the ones that will be introduced later – is not a problem here, because our aim is to show an internal inconsistency in the neoclassical determination of the optimal K/L ratio. If this can be shown in the simplest setting, it will apply also to more nuanced models.

measure cannot be expressed in physical quantities. It must be expressed in value.[56] But then – as shown by the seminal contribution of Sraffa (1960) – independently of whatever numeraire is chosen, the value of the aggregate capital stock depends on distributive variables.

A simple representation of this result is found in the Sraffian literature (for example Petri (2004, pp. 208-210)). Consider the simplest case in which technical coefficients are taken as given (only one technique of production is available for each production process) and all capital is circulating ($d = 1$), so the price equations in eq.19 become

$$\mathbf{p} = [1 + r]\mathbf{pA} + w\mathbf{l} \tag{20}$$

The economy-wide net product is taken as a numeraire ($py = 1$, with y representing the vector of quantities produced and p the vector of prices). Furthermore, measure labor input in such a way that total labour employment L is $L = 1$ (total labor employment is exogenously determined, because we are taking technical coefficients and quantities produced as given).

Consider the system of price equations (eq. 20). The vector of prices p and the wage rate w can be determined on the basis of the interest rate r and – due to the Perron-Frobenius theorem for non-negative matrices – a wage curve $w(r)$ exists, is downward-sloping and crosses both axes[57] in the positive orthant of the w-r space[58]. We thus have a decreasing wage curve, as in Figure 4.[59] Given our assumptions and choice of numeraire, the wage rate equals both the total wage bill and the wage share. The two intercepts of the wage curve represent the extreme cases in which all income goes to labor (intercept with vertical axis: $r = 0 \Rightarrow w = 1$) or to capital (intercept with horizontal axis: $w = 0 \Rightarrow r = 1$).

We thus have the income identity $p(r)y = 1 = rk + w(r)$ and we can write k as a function of the interest rate as follows

$$k = k(r) = \frac{1 - w(r)}{r} \tag{21}$$

[56] See Petri (2004, pp. 89-91) for a thorough explanation of why capital, as conceived by the early neoclassical approach, can only be measured in value.

[57] We also assume that there exists at least one good, entering some production process as an input, that also enters directly or indirectly in its own production process. Without this assumption, the interest rate would tend to infinite as the real wage tends to zero, so the w(r) curve would not intersect the horizontal axis.

[58] See Kurz and Salvadori (1997) for the mathematical demonstration.

[59] We depict a concave wage curve, but there is no guarantee that this is the case: it may be well convex or have inflections. This is irrelevant for our exposition.

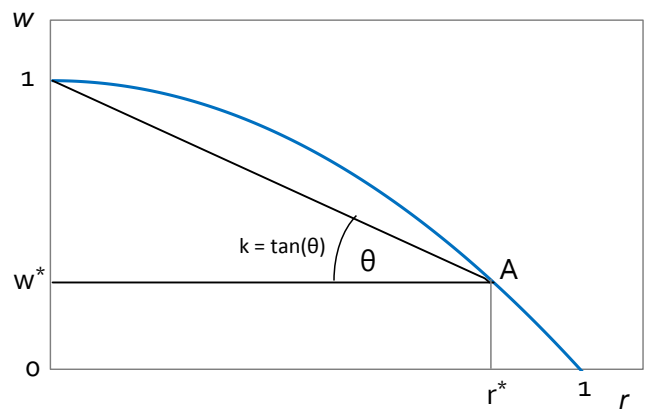

Figure 4: Relation between income distribution and the value of capital

Meaning that the value of capital is equal to the ratio between the sine and the cosine – and thus the tangent – of angle θ in Figure 4. It is thus clear that the value of capital is a function of the interest rate.

A.2 Inversion in relative price movements

As already mentioned, given the price system in eq.20, the vector of relative prices and the wage rate can be determined as functions of the interest rate. An important result demonstrated by Sraffa (1960) is that the price of a commodity in terms of another commodity need not be a *monotonic function* of the interest rate. To accomplish this, he first demonstrated that in a system like eq.19 the price of a commodity can be determined by 'reduction to dated quantity of wages'. This means that the price of a commodity can be calculated as an infinite series: the remuneration of the quantity of labor directly employed in the production of the given commodity, plus the (discounted) remuneration of the direct labor necessary to produce the inputs, plus the (twice discounted) remuneration of the direct labor necessary to produce the inputs to these inputs, and so on, in an infinite regress.

Formally, this can be proved by applying the Perron-Frobenius theorem, which implies that the following equality holds

$$[I - (1+r)A]^{-1} = I + (1+r)A + (1+r)^2A^2 + (1+r)^3A^3 + ... \tag{22}$$

We can rewrite the price equations in eq.20 as $p = w\ell[I - (1+r)A]^{-1}$ and then apply eq.22 to obtain

$$p = w\ell[I + (1+r)A + (1+r)^2A^2 + (1+r)^3A^3 + ...] \tag{23}$$

Let us now introduce the notation $L_{i(t)}$ to indicate the dated quantities of labor that must be applied at each time period $T-t$, for the production process of a good i to happen at time T. $L_{i(0)}$ is thus the quantity of labor directly applied in the production process that generates good i at time T; $L_{i(1)}$ is the quantity of labor directly applied at time $T-1$ in the production processes that will produce the intermediate goods that are necessary to produce i ; $L_{i(2)}$ is the quantity of labor directly applied at time $T-2$ to produce the means of production that at period $T-1$ are employed to produce the inputs necessary to produce i, and so on. And of course we will have $L_{i(0)} = \ell$; $L_{i(1)} = \ell A$; $L_{i(2)} = \ell A^2$; $L_{i(3)} = \ell A^3$, ... and in general $L_{i(t)} = \ell A^t$. Applying this notation to eq.23, we can express the price of a commodity as an infinite series of discounted dated wages

$$p_i = wL_{i(0)} + wL_{i(1)}(1+r) + wL_{i(2)}(1+r)^2 + wL_{i(3)}(1+r)^3 + ... \tag{24}$$

Sraffa (1960, p. 37) provides the example of two commodities – 'a' and 'b' – which differ in three of their 'dated labour' terms, all the others being equal. In other words, the series of dated wage payments determining the prices of a and b are identical, except for three terms. Commodity 'a' implies a greater number of work hours by 20 units at time $T-8$, while commodity 'b ' necessitates 19 more work hours at the time of production T and 1 more unit applied 25 years earlier. (Sraffa notes that this can resemble the classical example of 'wine aged in the cellar' and of 'old oak made into a chest'.) The difference between the prices of these two commodities is thus equal to

$$p_a - p_b = 20w(1+r)^8 - [19w + w(1+r)^{25}] \tag{25}$$

Sraffa takes the 'Standard Commodity' as the numeraire.[60] This implies that the wage-curve is linear and equal to $w = 1 - \frac{r}{R}$, where R is the maximum rate of profit. (But note that this choice of numeraire – and so the particular shape of the wage curve – are not

[60]Given a certain economy (defined by its price equations), the Standard Commodity is a composite commodity, constructed in such a way that if the net product of an economy was constituted by the Standard Commodity, the set of non-wage inputs necessary for its production would present exactly the same composition. Sraffa takes as the unit of measure of the Standard Commodity the quantity of it whose production would necessitate the same quantity of labor (per period) employed by the economy under study (Sraffa, 1960, pp.18-25).

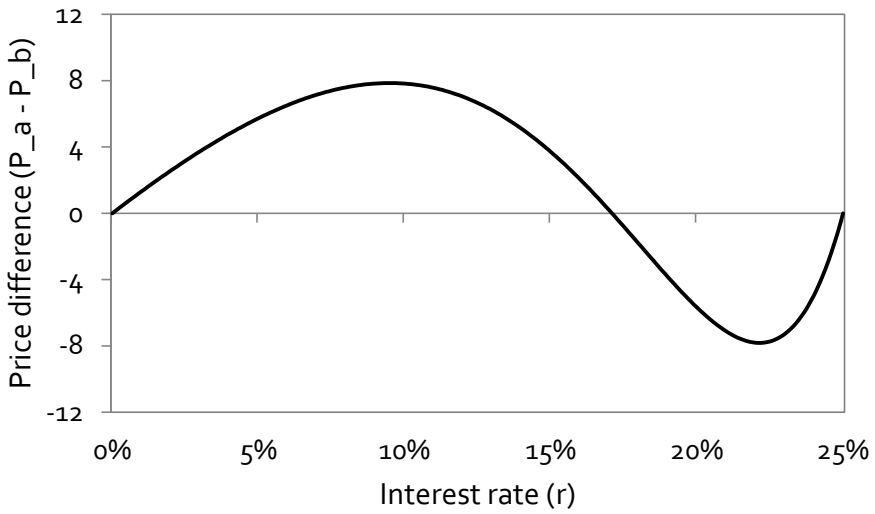

Figure 5: Difference between the prices of commodities a and b as a function of the interest rate

Source: Sraffa (1960)

necessary for proving the point, which would hold under any numeraire.) By assuming a maximum rate of profit of 25%, we can appreciate the dynamics of $p_a - p_b$ as a function of the interest rate (Figure 5).

We thus see that the price of a commodity relative to another can well be a non-monotonic function of the interest rate. Specifically, in this example, the price of a relative to b increases as the interest rate rises from 0% to 9%, then decreases as r passes from 9% to 22%, and then rises again when r increases from 22% to 25%.

From the point of view of neoclassical analysis, this means that for some values of the interest rate ($r < 9\%$ and $r > 22\%$) a is more 'capital-intensive' than b, but for others ($9\% < r < 22\%$) it is b that is more capital-intensive than a. So the relative capital intensity of the two commodities changes *even if their production methods are held constant.*

A.3 Reswitching of techniques

The result just presented undermines the mechanisms of factor substitution on which the neoclassical capital demand function is based (discussed in Sec.1.1 of this chapter). Indirect substitution predicts that the composition of demand would shift towards more capital intensive goods as the interest rate decreases. It thus needs a *purely technical* measure of capital intensity, independent of distribution. But the latter, as the example above demon-

strates, does not in fact exist. If some arbitrary criterion is adopted to establish that commodity a or commodity b is more capital-intensive irrespective of distribution, we will have the paradoxical (from the point of view of neoclassical theory) result that, in some relevant ranges, increases in the cost of capital will reduce the price of the more capital intensive technique, thus *increasing* demand for capital.

By interpreting 'a' and 'b' as two alternative techniques for producing the same commodity, instead of two different commodities, it is possible to show that also the mechanism of direct substitution is based on shaky theoretical foundations. p_a and p_b are thus now interpreted as the unitary costs of production of a and b, and of course cost-minimizing firms will adopt the technique with lower p. Following Petri (2004, pp. 214-218), the point can be shown very simply by modifying Sraffa's example in such a way that the $p_a - p_b$ curve (eq.25 and Fig.5) is shifted downwards by some small amount. (This can be done by simply assuming uniformly smaller production coefficients for commodity a – see ibid., p. 215). The point of this modification is to yield an example in which the $p_a - p_b$ curve crosses twice the horizontal axis, as in Figure 6, thus showing how is it possible for a technique to be cost-minimizing at relatively low and high values of the interest rate, but not at intermediate values. This phenomenon is known in the literature as *reswitching of techniques*. Besides the one that we just considered (taken from Petri (ibid., p. 215)), other numerical examples of reswitching have been presented for example by Samuelson (1966) and Pasinetti (1966) . Notably, Han and Schefold (2006) provide empirical (as opposed to theoretical) examples of reswitching, using the OECD input-output tables dataset.

A.4 Reverse capital deepening

Reverse capital deepening occurs when a rise in the interest rate leads to an increase in the ratio of aggregate capital (measured in value)[61] to total labor employment. It is straightforward to see that reswitching of techniques leads to reverse capital deepening. Consider the example with one commodity and two available techniques (a and b) presented in the previous section and depicted in Figure 6. The corresponding wage curves associated with the two techniques are displayed in Figure 7. For each given level of the interest rate, the cost-minimizing technique is the one associated with the most external wage curve, i.e.,

[61] Of course the discussion in the preceding sections implies that the conception of capital as a single factor measurable in value independently of distribution is flawed. In this section we adopt this mistaken definition only to show that it brings to results that run counter to neoclassical capital theory.

the one that lies on the outer envelope.[62]. Again we see that technique a is more profitable at low and high levels of the interest rate, but not at 'intermediate' values. Recall from Sec. A.1 (in particular Fig.4) that the value of capital can be inferred from the prevailing wage curve.[63] We can thus appreciate the evolution of the value of capital per worker as a function of the interest rate in this example. This is depicted in Figure 8. A discrete increase in the interest rate from (for example) 1% to 4% would have in this case the effect predicted by neoclassical theory: capital intensity would decrease; however a further interest rate increase, for example from 4% to 10% (of from 10% to 15%, or from 15% to 20%), would cause an *increase* in capital intensity.

But while reswitching of techniques is a sufficient condition for reverse capital deepening, it is not a necessary one. Reverse capital deepening can occur also in the absence of reswitching. A trivial (but possibly empirically relevant) case is the following. Imagine that the same technique is cost-minimizing for all relevant values of the interest rate. This is equivalent to (but more general than) the case in which only one production technique is available. Suppose, furthermore, that the wage curve generated by this production method is concave, like the wage curve associated with technique b in Figure 7. The corresponding relation between the interest rate and the value of capital per worker will be monotonically *increasing*, like in the segment of Figure 8 in which technique b remains dominant. This is due to the so-called *price Wicksell effects*: in the absence of any switch of technique, the aggregate value of capital is altered by the price changes caused by interest rate variations. Note that also in the presence of multiple switches of techniques that are all well-behaved from a neoclassical point of view (that is, interest rate increases are always associated with switches toward less capital intensive techniques), positive price Wicksell effects may still dominate them and produce a non-neoclassical demand curve for aggregate capital.

In addition to the cases just discussed, based on price Wicksell effects, reverse capital deepening in the absence of reswitching can be also be caused by technique switches that run counter to marginalist theory. This can happen when there are more than two available techniques. Then it is possible that two techniques cross more than once, but only one of the switches is on the outer envelope. In this case we would have reverse capital deepening whenever there is a switch between two techniques whose wage curves have already crossed

[62] This is a well-known result, which demonstration can be found for example in Kurz and Salvadori (1997).

[63] Note that in this case, with different wage curves compared in the same diagram, we do not set the intercept of both wage curves at $w = y = 1$: the real wage associated with $r = 0$ (that is, y) is different for the two techniques. So the formula determining the value of capital is, more generally, $k = (y - w)/r$.

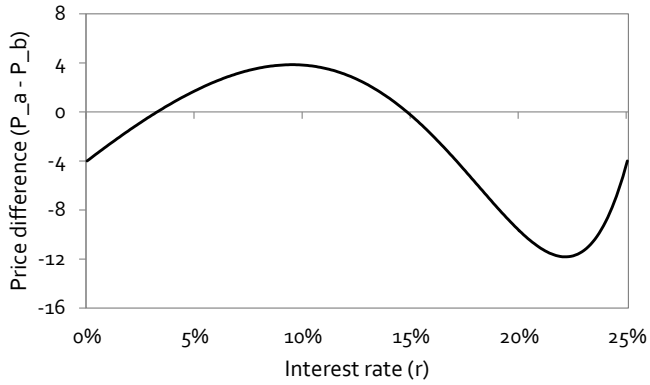

Figure 6: Difference between the unit production cost of techniques a and b as a function of the interest rate

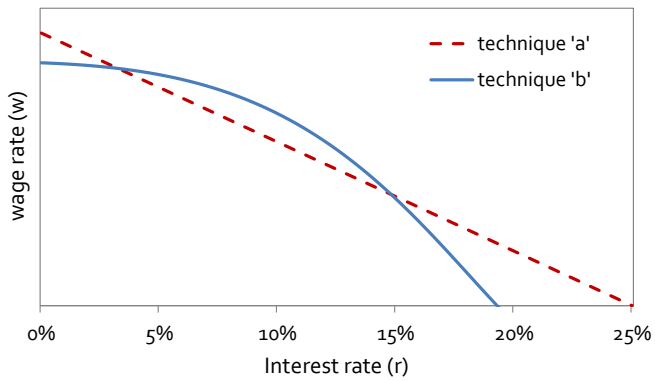

Figure 7: Reswitching between techniques a and b in terms of wage curves

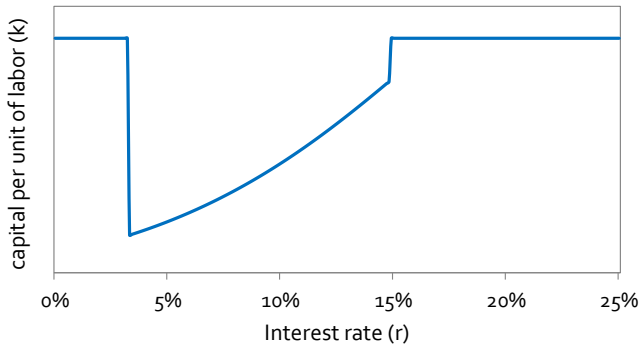

Figure 8: Capital intensity as a function on the interest rate, resulting from reswitching between production techniques a and b

Source: Our own elaboration on Petri (2004, pp. 215-217) (Sraffa example modified to yield reswitching)

at a lower interest rate. And whenever this first intersection between the two wage curves is situated below the outer envelope (while the second, at a higher interest rate, is on the outer envelope) reverse capital deepening happens in the absence of reswitching.

The bottom line is that capital, defined as the set of all commodities that are employed as means of production, cannot be conceived as a single homogeneous factor, that will be employed more intensively in production whenever its price decreases. Capital cannot be conceived as a single homogeneous factor because there is no way to measure its quantity in 'technical' units – that is, independently of distributive variables. And even if that erroneous conception of capital is adopted, there is no guarantee that the resulting demand curve for capital is monotonically decreasing in the interest rate.