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Comments on "Full Industry Equilibrium: A Theory of the Industrial Long Run" by Arrigo Opocher and Ian Steedman (2015)

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1 Summary of the main results of this book

In *Full Industry Equilibrium: A Theory of the Industrial Long Run*, Arrigo Opocher and Ian Steedman bridge the marginalist long-run theory of the firm and the Sraffian long-period theory of production to create a unified theoretical framework explaining how firms react to exogenous shocks that result in new equilibrium positions in the whole economy. As these authors observe, the long-run theory of the firm developed in the late 1960s and the long-period theory of production in the economy as a whole both flourished after Sraffa (1960) emphasize that the forces of free competition lead to equilibrium positions of zero (extra) profit. Their central message in this book is that conventional partial equilibrium analysis to derive any law of input demand is too simple to provide correct predictions about market behavior. They share the view, consistent with these two earlier theories, that if a zero (extra-)profit equilibrium is disrupted by a change in the price of a factor, then the industry will make long-run adjustments to reach a new zero (extra-)profit equilibrium. These changes will be in the output price as well as in

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the prices of other commodities, including the produced commodity inputs of the firm/industry under consideration. As the authors note, therefore, all such effects must be taken into account simultaneously in order to derive a law of input demand.

In contrast, the conventional long-run theory of input demand and output supply turns on changing just one price at a time under a strict *ceteris paribus* stipulation – an approach the authors criticize as an inadequate foundation for any predictions of actual demand or supply. Indeed, both the long-run theory of the firm of the 1970s and the Sraffian long-period theory of production have disproved the familiar inference from the conventional analysis that an input use is inversely related to its price. During the Cambridge capital theory debates, for example, the central finding was that a change in the rate of interest may modify the aggregate capital/labor and capital/output ratios in a direction opposite to that expected on the basis of a simple law of input demand, as shown by the possibility of so-called capital reversing. Whereas critics of the conventional analysis focused on the economy as a whole or on vertically integrated sectors rather than on individual industries, Opocher and Steedman are concerned with micro-productive choices and often assume an identically zero rate of interest.

In this book, the notion of *full industry equilibrium* (FIE, hereafter) applies to any situation in which the firms forming a particular industry make maximum net profits of zero, and when there is more than one industry, all firms must be in this situation. This notion characterizes the full, long-run reaction of a firm and of an entire industry to a price shock. By "full reaction," the authors mean that the inputs and the outputs both adjust so that net profits are null, both before and after the shock, in every industry. A main conclusion in this book is that, under FIE, the reaction of primary factors to a price change is significantly different from that of produced inputs. In particular, although the reaction of primary factors may be consistent with the conventional law of input demand, the reaction of produced inputs exhibits no regularity.

The mere assumption that industrial output may be used as an input by the same industry introduces significant variations, and it shows that the input-rental/input-use relationship for a produced input is qualitatively different from the input-rental/input-use relationship for a primary input. Under FIE, the input-rental/input-use (per unit of output) relationship for a primary input is significant, and indeed the demand for a primary input is inversely related to its rental price, under the assumption that all inputs are Hicksian substitutes and that the cost function is twice-differentiable everywhere. However, under the same presumption no significant input-rental/input-use relationship exists for a commodity input.

In the more general context of multiple industries, the authors have shown repeatedly that the use of produced inputs can react to a given shock in qualitatively different ways from the use of primary inputs. This result, which is contrary to the standard theory of marginalist school, is independent of whether the rate of interest is taken as constant (and possibly null) or as variable. The authors therefore suggest that the negative result has no essential relation to capital-theoretic issues and everything to do with the more fundamental properties of FIE equilibria. At the same time, they show that the industrial capital (gross) output ratios need not be inversely related to the rate of interest.

A particularly interesting finding in this book emerges from the observation that, under FIE, conventional behavior in the economy as a whole by no means implies conventional behavior in every industry. In an industry, a small increase in the interest rate (and hence a decrease in the real wage) around the point of a switch in technique may create an increase in the capital-output ratio and a decline in employment per unit of output even though both variables behave conventionally at the level of the whole economy. The same argument applies even with a fixed null rate of interest. In such a context, one industry can exhibit a positive relationship between a primary input use and its price. These possibilities have nothing whatsoever to do with 'unequal proportions,' reswitching, capital-reversing or interest-rate effects of any kind. All that is necessary is the presumption of FIE both before and after the exogenous change.

The authors hope that the above findings will interest marginalist microeconomists and Sraffa-inspired economists alike, and that both groups will develop FIE analysis in their own ways. In particular, the authors encourage the Sraffa-inspired economists to pay much more attention both to individual industries and to the effects of exogenous changes besides changes in the interest rate.

These main results of this book deserve attention from a broader readership in economics. Indeed, the results of the comparative statics under FIE may well become one chapter of a standard textbook of microeconomic theory. Nevertheless, some qualifications are in order.

First, throughout their analysis, the authors implicitly assume that production takes time, as acknowledged in the standard literature of Sraffian economics as well as in the literature of intertemporal general equilibrium theory. Given this assumption, when they develop the comparative static analysis under FIE, they implicitly focus on stationary equilibrium. In my view, their assumption and their focus are the main source for their observations of unconventional behavior of commodity inputs.

In the time structure of production, one production period begins with investing inputs and ends when an output is produced and supplied to the market. Given this structure, the price of a commodity may generally differ, even in equilibrium, between the point where it is used as an input, before production, and the point where it is produced as an output. In this respect, the idea of stationary equilibrium is specific; where the prices of the commodity are stationary before and after production in a stationary equilibrium.

As discussed in more detail below, it seems to me that the presumption of the stationary equilibrium prices under FIE is the main source of unconventional behavior in the case of commodity inputs. Indeed, the main substantial difference of commodity inputs from primary inputs in their mathematical formulations is that the price of the former appears both in the domain of (indirect) cost functions and in their range, while the price of the latter appears only in the domain.

For elaboration on this point, let us see the equation of FIE presented in section 2.6 of this book:

$$p = c(w, r, p).$$

Here, p is the price of commodity used as an input in the production of itself, w is the wage rate, and r is the rent of land. As is obvious, p appears in both the right and the left side of this equation, unlike w and r. The presence of the same price p on both sides is due to the stationary equilibrium setting. However, in a non-stationary equilibrium, the above equation may be slightly revised to

$$p_t = c(w_t, r_t, p_{t-1})$$

where p_{t-1} is the price of the commodity at a point when it is purchased as an input by the firm and the industry while p_t is the price of the commodity at a point when it is supplied by the firm and the industry as an output after the production process. In general, $p_t \neq p_{t-1}$. This equation implies that the commodity used as an input and the commodity produced as an output are treated as different, even though they are the same type of commodity. Therefore, if we consider a non-stationary equilibrium price system under

FIE, the above-mentioned difference in mathematical formulation between commodity inputs and primary inputs would be erased, and so conventional behavior would be restored in the reaction of commodity input demands to a change in their own prices.

From this observation, I think there may be two ways to identify the time span of the full reaction of the firm and the industry to a price shock. One is to consider the time between the point of the price shock and the point of a new equilibrium, when the zero profit is reached through an adjustment process in which the new equilibrium price system is not necessarily stationary even though it was stationary before the price shock. As discussed in the literature of intertemporal general equilibrium theory, a non-stationary price system is compatible with cost minimization as well as with the zero profit condition. Therefore, the definition of FIE *per se* cannot exclude the possibility of non-stationary equilibrium prices.

The other way is what the authors presume in this book. They focus on the time span from the point of the price shock to the point of a new *stationary* equilibrium with zero profit. Under the intertemporal framework, the restoration of the stationary state requires much longer periods than does that of the 'short-period' (non-stationary) equilibrium. Therefore, it seems to me that the time span of the second approach is much longer than that of the first.

From this observation, as discussed in more detail below, I would suggest that the standard marginalist theory of input demand functions may be valid under the first type of time span but may no longer be valid if the time span of the adjustment process is presumed to be of the restoration process of the stationary state. I owe this observation to the considerable accomplishment of the authors in their analysis.

In the next section, I will develop my comments in more detail, mainly by focusing on the model in Chapter 4 in this book.

2 The main source of the unconventional observation

Among other topics, here let us focus on the analysis developed in section 4.8 of the book. Consider two industries. Each industry has three inputs: labour, its own-product and the product of another industry. Under FIE, the indirect average cost function in each industry $j = 1, 2$ is given by:

$$p_1 = c_1 \left(W, p_1 \left(1+i \right), p_2 \left(1+i \right) \right); \; p_2 = c_2 \left(W, p_1 \left(1+i \right), p_2 \left(1+i \right) \right) \quad (1)$$

where p_j is the market price of commodity $j = 1, 2$, W is the nominal wage rate, and i is the interest rate. Now, assuming the commodity 2 is the numeraire, the above equations are reduced to:

$$p = c_1 \left(w, p \left(1+i \right), \left(1+i \right) \right); \; 1 = c_2 \left(w, p \left(1+i \right), \left(1+i \right) \right) \quad (2)$$

where $w \equiv \frac{W}{p_2}$ and $p \equiv \frac{p_1}{p_2}$.

Differentiating each of the cost functions totally, we have

$$\begin{aligned} \mathrm{d}p &= \frac{\partial c_1}{\partial w}\mathrm{d}w + \frac{\partial c_1}{\partial p\left(1+i\right)}\frac{\partial p\left(1+i\right)}{\partial i}\mathrm{d}i + \frac{\partial c_1}{\partial p\left(1+i\right)}\frac{\partial p\left(1+i\right)}{\partial p}\mathrm{d}p + \frac{\partial c_1}{\partial\left(1+i\right)}\frac{\partial\left(1+i\right)}{\partial i}\mathrm{d}i \\ &= l_1\mathrm{d}w + \left(pa_{11} + a_{21}\right)\mathrm{d}i + \left(1+i\right)a_{11}\mathrm{d}p \equiv l_1\mathrm{d}w + k_1\mathrm{d}i + \left(1+i\right)a_{11}\mathrm{d}p; \end{aligned}$$

and

$$\begin{aligned} 0 &= \frac{\partial c_2}{\partial w}\mathrm{d}w + \frac{\partial c_2}{\partial p\left(1+i\right)}\frac{\partial p\left(1+i\right)}{\partial i}\mathrm{d}i + \frac{\partial c_2}{\partial p\left(1+i\right)}\frac{\partial p\left(1+i\right)}{\partial p}\mathrm{d}p + \frac{\partial c_2}{\partial\left(1+i\right)}\frac{\partial\left(1+i\right)}{\partial i}\mathrm{d}i \\ &= l_2\mathrm{d}w + \left(pa_{12} + a_{22}\right)\mathrm{d}i + \left(1+i\right)a_{22}\mathrm{d}p \equiv l_2\mathrm{d}w + k_2\mathrm{d}i + \left(1+i\right)a_{22}\mathrm{d}p, \end{aligned}$$

where l_j for $j = 1, 2$ implies the amount of labor input necessary to produce one unit of commodity j; and a_{hj} for $h, j = 1, 2$ implies the amount of commodity h input necessary to produce one unit of commodity j. Moreover, $k_1 \equiv (pa_{11} + a_{21})$ and $k_2 \equiv (pa_{12} + a_{22})$ are respectively the values of capital necessary to produce one unit of commodity $j = 1, 2$. From this computation,

$$\mathrm{d}p = \frac{l_1\mathrm{d}w + k_1\mathrm{d}i}{1 - \left(1+i\right)a_{11}},$$

and so

$$\frac{\mathrm{d}w}{\mathrm{d}i} = -\frac{\left(1+i\right)a_{22}k_1 + \left(1 - \left(1+i\right)a_{11}\right)k_2}{\left(1+i\right)a_{22}l_1 + \left(1 - \left(1+i\right)a_{11}\right)l_2} < 0.$$

Moreover,

$$\frac{\mathrm{d}p}{\mathrm{d}i} = \frac{k_1 l_2 - k_2 l_1}{\left(1+i\right)a_{22}l_1 + \left(1 - \left(1+i\right)a_{11}\right)l_2} \gtreqless 0 \Leftrightarrow \frac{k_1}{l_1} \gtreqless \frac{k_2}{l_2}.$$

By the way, note that

$$\begin{aligned}
\mathrm{d}k_1 &= \frac{\partial k_1}{\partial w}\mathrm{d}w + \frac{\partial k_1}{\partial p(1+i)}\left(\frac{\partial p(1+i)}{\partial p}\mathrm{d}p + \frac{\partial p(1+i)}{\partial i}\mathrm{d}i\right) + \frac{\partial k_1}{\partial(1+i)}\frac{\partial(1+i)}{\partial i}\mathrm{d}i \\
&= \frac{\partial^2 c_1}{\partial w \partial p(1+i)}\mathrm{d}w + \frac{\partial p a_{11}}{\partial p}\mathrm{d}p + \frac{\partial p a_{11}}{\partial p(1+i)}\frac{\partial p(1+i)}{\partial i}\mathrm{d}i + \frac{\partial a_{21}}{\partial(1+i)}\frac{\partial(1+i)}{\partial i}\mathrm{d}i \\
&= \frac{\partial^2 c_1}{\partial w \partial p(1+i)}\mathrm{d}w + a_{11}\mathrm{d}p + \left(p\frac{\partial a_{11}}{\partial p(1+i)} + \frac{\partial a_{21}}{\partial(1+i)}\right)\mathrm{d}i \\
&= \frac{\partial^2 c_1}{\partial w \partial p(1+i)}\mathrm{d}w + a_{11}\mathrm{d}p + \left(\frac{\partial^2 c_1}{\partial p(1+i)\partial p(1+i)} + \frac{\partial^2 c_1}{\partial(1+i)\partial(1+i)}\right)\mathrm{d}i \\
&= \frac{\partial^2 c_1}{\partial w \partial p(1+i)}\mathrm{d}w + a_{11}\mathrm{d}p + \frac{\partial k_1}{\partial i}\mathrm{d}i,
\end{aligned}$$

where $\frac{\partial^2 c_1}{\partial w \partial p(1+i)} > 0$ and $\frac{\partial k_1}{\partial i} = \frac{\partial^2 c_1}{\partial p(1+i)\partial p(1+i)} + \frac{\partial^2 c_1}{\partial(1+i)\partial(1+i)} \leqq 0$ follow from the assumption of Hicksian substitutes. Therefore,

$$\begin{aligned}
\frac{\mathrm{d}k_1}{\mathrm{d}i} &= \frac{\partial^2 c_1}{\partial w \partial p(1+i)}\frac{\mathrm{d}w}{\mathrm{d}i} + a_{11}\frac{\mathrm{d}p}{\mathrm{d}i} + \frac{\partial k_1}{\partial i} < 0 \\
&\Leftrightarrow \frac{k_1}{l_1} - \frac{k_2}{l_2} \text{ is negative or sufficiently small.} \quad (3)
\end{aligned}$$

Likewise, since

$$\begin{aligned}
\mathrm{d}k_2 &= \frac{\partial k_2}{\partial w}\mathrm{d}w + \frac{\partial k_2}{\partial p(1+i)}\left(\frac{\partial p(1+i)}{\partial p}\mathrm{d}p + \frac{\partial p(1+i)}{\partial i}\mathrm{d}i\right) + \frac{\partial k_2}{\partial(1+i)}\frac{\partial(1+i)}{\partial i}\mathrm{d}i \\
&= \frac{\partial^2 c_2}{\partial w \partial p(1+i)}\mathrm{d}w + \frac{\partial p a_{12}}{\partial p}\mathrm{d}p + \frac{\partial p a_{12}}{\partial p(1+i)}\frac{\partial p(1+i)}{\partial i}\mathrm{d}i + \frac{\partial a_{22}}{\partial(1+i)}\frac{\partial(1+i)}{\partial i}\mathrm{d}i \\
&= \frac{\partial^2 c_2}{\partial w \partial p(1+i)}\mathrm{d}w + a_{12}\mathrm{d}p + \left(p\frac{\partial a_{12}}{\partial p(1+i)} + \frac{\partial a_{22}}{\partial(1+i)}\right)\mathrm{d}i \\
&= \frac{\partial^2 c_2}{\partial w \partial p(1+i)}\mathrm{d}w + a_{12}\mathrm{d}p + \left(\frac{\partial^2 c_2}{\partial p(1+i)\partial p(1+i)} + \frac{\partial^2 c_2}{\partial(1+i)\partial(1+i)}\right)\mathrm{d}i \\
&= \frac{\partial^2 c_2}{\partial w \partial p(1+i)}\mathrm{d}w + a_{12}\mathrm{d}p + \frac{\partial k_2}{\partial i}\mathrm{d}i,
\end{aligned}$$

with $\frac{\partial^2 c_2}{\partial w \partial p(1+i)} > 0$ and $\frac{\partial k_2}{\partial i} = \frac{\partial^2 c_2}{\partial p(1+i)\partial p(1+i)} + \frac{\partial^2 c_2}{\partial(1+i)\partial(1+i)} \leqq 0$ by the Hicksian substitutes,

$$\begin{aligned}\frac{\mathrm{d}k_2}{\mathrm{d}i} &= \frac{\partial^2 c_2}{\partial w \partial p\,(1+i)}\frac{\mathrm{d}w}{\mathrm{d}i} + a_{12}\frac{\mathrm{d}p}{\mathrm{d}i} + \frac{\partial k_2}{\partial i} < 0 \\ &\Leftrightarrow \frac{k_1}{l_1} - \frac{k_2}{l_2} \text{ is negative or sufficiently small.} \quad (4)\end{aligned}$$

However, as the authors claim, the characterizations (3) and (4) are not informative at all, since they depend on the choice of the numeraire. Indeed, if we choose Commodity 1 as the numeraire as the authors do in section 4.8, then we obtain the following opposite characterization:

$$\frac{\mathrm{d}k_1}{\mathrm{d}i} < 0 \Leftrightarrow \frac{k_2}{l_2} - \frac{k_1}{l_1} \text{ is negative or sufficiently small.} \quad (5)$$

That is, if the numeraire is Commodity 1, then Industry 2 should be more capital-intensive than Industry 1 in order to ensure the inverse relationship between the capital demand and the interest rate. However, if the numeraire is Commodity 2, then Industry 1 should be more capital-intensive than Industry 2 to ensure the same inverse relationship. In summary, these characterizations suggest that there is no informative sufficient condition to ensure the conventional monotonic feature of the capital demand function unless both industries have the same capital-labor ratio: $\frac{k_2}{l_2} = \frac{k_1}{l_1}$.

The same feature is also observed for the case of inverse relationship between the labor demand and the wage rate. Indeed, the standard calculus leads us to:

$$\begin{aligned}\mathrm{d}l_1 &= \frac{\partial l_1}{\partial w}\mathrm{d}w + \frac{\partial l_1}{\partial p\,(1+i)}\left(\frac{\partial p\,(1+i)}{\partial p}\mathrm{d}p + \frac{\partial p\,(1+i)}{\partial i}\mathrm{d}i\right) + \frac{\partial l_1}{\partial\,(1+i)}\frac{\partial\,(1+i)}{\partial i}\mathrm{d}i \\ &= \frac{\partial^2 c_1}{\partial w^2}\mathrm{d}w + \frac{\partial^2 c_1}{\partial p\,(1+i)\,\partial w}\left[(1+i)\,\mathrm{d}p + p\mathrm{d}i\right] + \frac{\partial^2 c_1}{\partial\,(1+i)\,\partial w}\mathrm{d}i \\ &= \frac{\partial^2 c_1}{\partial w^2}\mathrm{d}w + (1+i)\frac{\partial^2 c_1}{\partial p\,(1+i)\,\partial w}\mathrm{d}p + \left(p\frac{\partial^2 c_1}{\partial p\,(1+i)\,\partial w} + \frac{\partial^2 c_1}{\partial\,(1+i)\,\partial w}\right)\mathrm{d}i,\end{aligned}$$

from which, we can derive the following formula:

$$\frac{\mathrm{d}l_1}{\mathrm{d}w} = \frac{\partial^2 c_1}{\partial w^2} + \left(p\frac{\partial^2 c_1}{\partial p\,(1+i)\,\partial w} + \frac{\partial^2 c_1}{\partial\,(1+i)\,\partial w}\right)\frac{\mathrm{d}i}{\mathrm{d}w} + (1+i)\frac{\partial^2 c_1}{\partial p\,(1+i)\,\partial w}\frac{\mathrm{d}p}{\mathrm{d}w}.$$

Since $\frac{\partial^2 c_1}{\partial w^2} \leqq 0$ and $\frac{\partial^2 c_1}{\partial p(1+i)\partial w} > 0 < \frac{\partial^2 c_1}{\partial(1+i)\partial w}$ by the Hicksian substitutes, and $\frac{\mathrm{d}i}{\mathrm{d}w} < 0$ holds, again the inverse relationship holds if $\frac{\mathrm{d}p}{\mathrm{d}w}$ is negative or almost close to zero. But, since

$$\frac{\mathrm{d}p}{\mathrm{d}w} = \frac{k_2 l_1 - k_1 l_2}{(1+i)\, a_{22} k_1 + (1-(1+i)\, a_{11})\, k_2} \gtreqless 0 \Leftrightarrow \frac{k_2}{l_2} \gtreqless \frac{k_1}{l_1},$$

the condition which exactly mirrors the conditions (3) and (4) for the capital demands and the interest rate is necessary and sufficient for the inverse relationship between the labor demand and the wage rate:

$$\frac{\mathrm{d}l_1}{\mathrm{d}w} < 0 \Leftrightarrow \frac{k_2}{l_2} - \frac{k_1}{l_1} \text{ is negative or sufficiently small.} \quad (6)$$

Therefore, it is almost impossible to ensure the conventional monotonic features in both the capital demand and the labor demand functions simultaneously, unless a rare case of $\frac{k_2}{l_2} = \frac{k_1}{l_1}$ exists.

Thus, given that the monotone decreasing feature holds in labor demand functions, the capital reversing would likely be observed in capital demand functions under FIE. That is one of the author's main interesting findings.

In contrast, if every input is a primary factor in every industry, then the price equation under FIE is:

$$p_1 = c_1 (W, R)\, ;\, p_2 = c_2 (W, R) \quad (7)$$

where R the nominal rent of the land. Here, again, assuming Commodity 2 is the numeraire, the above equations are reduced to:

$$p = c_1 (w, r)\, ;\, 1 = c_2 (w, r) \quad (8)$$

where $r \equiv \frac{R}{p_2}$. Differentiating each of the cost functions totally, we have

$$\mathrm{d}p = \frac{\partial c_1}{\partial w}\mathrm{d}w + \frac{\partial c_1}{\partial r}\mathrm{d}r = l_1 \mathrm{d}w + t_1 \mathrm{d}r;\ 0 = \frac{\partial c_2}{\partial w}\mathrm{d}w + \frac{\partial c_2}{\partial r}\mathrm{d}r = l_2 \mathrm{d}w + t_2 \mathrm{d}r, \quad (9)$$

where t_j for $j = 1, 2$ is the input amount of land necessary to produce one unit of commodity j. Therefore, $\frac{\mathrm{d}w}{\mathrm{d}r} = -\frac{t_2}{l_2}$, and so $\frac{\mathrm{d}l_j}{\mathrm{d}w} = \frac{\partial^2 c_j}{\partial w^2} + \frac{\partial^2 c_j}{\partial w \partial r}\frac{\mathrm{d}r}{\mathrm{d}w} < 0$ and $\frac{\mathrm{d}t_j}{\mathrm{d}r} = \frac{\partial^2 c_j}{\partial r^2} + \frac{\partial^2 c_j}{\partial r \partial w}\frac{\mathrm{d}w}{\mathrm{d}r} < 0$ hold for $j = 1, 2$ by the Hicksian substitutes. Thus, in this case, the conventional features of factor demand functions are preserved.

2.1 Two implicit presumptions

As the authors repeatedly emphasize and as the above analysis shows, while the conventional features of the primary factor demand functions are preserved under FIE, the introduction of reproducible factors would cease to generate the conventional inverse relationship between the factor demand and its price. The authors insist that the unconventional features of commodity input demands has everything to do with the more fundamental properties of FIE equilibria, and all that is necessary for such observations is the presumption of FIE both before and after the exogenous change considered.

I do not disagree on this view, but I think that in their analysis of FIE the authors presume at least two more fundamental roots of these features. The first presumption, conventional in the literature of the Sraffian school, is that production takes time. The second presumption is that the full reaction of the firm and the industry to a price shock is an adjustment process, through free competition, to reach a new *stationary* equilibrium with zero profit, rather than simply a new zero-profit equilibrium.

2.1.1 Time structure of production

The assumption that production takes time is implicit in the standard literature of intertemporal general equilibrium as well as of the Sraffian production model: production takes some length of time, from the investing of commodity inputs and primary factors until the harvesting of an output. This length of time is one production period. Given such a time structure of production, the purchasing of commodity inputs in the market precedes the selling of the produced commodity in the market by one production period.

I think that this assumption about time structure is indispensable for the analysis of FIE in this book. Otherwise, it cannot be rational to consider the rental markets of capital goods. Unlike the supply of primary factors such as the land, the supply of capital as bundles of reproducible commodities can be increased by the production of those commodities. However, since producing them takes time, the supply of these reproducible factors in the market at the head of a current production period would be limited by the amount of stock produced in the preceding periods. As a result, the capital goods will be scarce relative to their potential demands, and firms will need to purchase the use of capital goods by paying a rental cost in addition to their production costs. The existence of positive interest rates in the model of Section 4 implies such an underlying scenario in capital markets, due to the time structure of production. Indeed, unless production takes time, every producer can instantaneously produce capital goods as much as he/she wishes before starting the production of final goods, and so it would be no longer necessary for him/her to pay a rental price.

Someone may wonder whether or not this scenario applies to every chapter of this book, since the authors also consider the case of null interest rate for commodity inputs. For instance, the model in Section 2.6 presumes no interest payment in the isolated industry, and the equilibrium condition is represented by

$$p = c\left(w, r, p\right).$$

This model seems to assume the time structure of production, since the price of the commodity appears in the domain of the indirect cost function. Moreover, it presumes that the firm must finance the payment of the commodity input in advance of production. Unless production takes time, the firm would not need to finance it in advance of production, since the same commodity would be produced instantaneously, and so the cost of the commodity input would be smoothly offset by the commodity output.

Indeed, in this case, the price of the commodity does not need to be included in the cost function. We can construct the equivalent cost function from the information of the cost function $C\left(Q, w, r, p\right)$, where Q is the gross products of this commodity, as

$$\mathcal{C}\left(Y, w, r\right)$$

where Y is the net products of this commodity corresponding to the gross products Q. For instance, if X amount of the commodity input is invested to produce Q amounts of this commodity, then $Y \equiv Q - X$. Therefore, this cost function represents the cost for the net products of this commodity.

In summary, I think that the authors should consider the economic environments where production takes time throughout the whole of this book. We acknowledge that in the static model of the neoclassical general equilibrium, where no budget constraint is presented in the firm's profit maximization problem, the time structure of production is not necessarily presumed, and so all commodities are produced as if instantaneously. Such an alternative underlying scenario would lie outside the authors' framework.

2.1.2 The presumption of stationary equilibrium

Given the time structure of production as discussed in the previous subsection, it is clear that the system of equations (2) represents the stationary equilibrium price system. To see how the presumption of the stationary equilibrium is crucial to produce the unconventional features of reproducible factor demands, let us examine the case in which a non-stationary equilibrium is allowed to be the position reached by the full adjustment to a price shock. If a FIE admits a non-stationary price system, the system of equations (2) should be revised as follows:

$$p = c_1\left(w, q_1\left(1+i\right), q_2\left(1+i\right)\right); \; 1 = c_2\left(w, q_1\left(1+i\right), q_2\left(1+i\right)\right) \quad \text{(2')}$$

where Commodity 2 supplied as an output after the production is the numeraire; q_1 is the price of Commodity 1 at the point when it is purchased as an input before the production; and q_2 is the price of Commodity 2 at the point when it is purchased as an input before the production. In this case, as in the literature of the neoclassical intertemporal general equilibrium theory, the commodity j as an input before production and the same commodity as an output after production can be treated separately, even when $\frac{q_1}{q_2}$ and p happen to be identical.

In this setting, there may be two possible comparative statics. The first one is to assume that the full reaction of the firm and the industry to a price shock has been completed within a production period, and so a new equilibrium is established within the same production period as the timing of the price shock. This assumption is based on the standard view of the intertemporal general equilibrium, i.e., that the price adjustment process through market competition is completed within a production period, and so in every production period the economy reaches an equilibrium path. In this case, it is typically assumed that the prices of q_1 and q_2 are realized at the end of the previous production period; and that, therefore, those prices cannot be changed as the same time as the wage rate or the interest rate in the current production period. Changes in the wage rate and the interest rate may involve a change in commodity price p, but they do not involve a change of $\frac{q_1}{q_2}$. Note that this setting is compatible with the zero profit condition, as in the standard literature of the intertemporal general equilibrium theory.[1]

[1] For instance, see Malinvaud (1972) and Dumenil and Levy (1985). The neoclassical intertemporal general equilibrium theory examines the equilibrium path, which is shown

In such a case, the appropriate method of comparative statics is to derive the following equations by the total differentiation:

$$\begin{aligned} \mathrm{d}p &= \frac{\partial c_1}{\partial w}\mathrm{d}w + \frac{\partial c_1}{\partial q_1(1+i)}\frac{\partial q_1(1+i)}{\partial i}\mathrm{d}i + \frac{\partial c_1}{\partial q_2(1+i)}\frac{\partial q_2(1+i)}{\partial i}\mathrm{d}i \\ &= l_1\mathrm{d}w + (q_1a_{11} + q_2a_{21})\,\mathrm{d}i \equiv l_1\mathrm{d}w + k_1\mathrm{d}i; \quad (10) \end{aligned}$$

and

$$\begin{aligned} 0 &= \frac{\partial c_2}{\partial w}\mathrm{d}w + \frac{\partial c_2}{\partial q_1(1+i)}\frac{\partial q_1(1+i)}{\partial i}\mathrm{d}i + \frac{\partial c_2}{\partial q_2(1+i)}\frac{\partial q_2(1+i)}{\partial i}\mathrm{d}i \\ &= l_2\mathrm{d}w + (q_1a_{12} + q_2a_{22})\,\mathrm{d}i \equiv l_2\mathrm{d}w + k_2\mathrm{d}i, \quad (11) \end{aligned}$$

where $k_1 \equiv (q_1a_{11} + q_2a_{21})$ and $k_2 \equiv (q_1a_{12} + q_2a_{22})$ are the values of capital necessary to produce one unit of commodity $j = 1, 2$ respectively. Obviously, the system (10) and (11) has essentially the same structure as the system (9) of the case of the production with two primary factors. Indeed, we can have $\frac{\mathrm{d}w}{\mathrm{d}i} = -\frac{k_2}{l_2}$, and so

$$\frac{\mathrm{d}l_j}{\mathrm{d}w} = \frac{\partial^2 c_j}{\partial w^2} + \frac{\partial^2 c_j}{\partial w \partial i}\frac{\mathrm{d}i}{\mathrm{d}w} < 0 \text{ and } \frac{\mathrm{d}k_j}{\mathrm{d}i} = \frac{\partial^2 c_j}{\partial i^2} + \frac{\partial^2 c_j}{\partial i \partial w}\frac{\mathrm{d}w}{\mathrm{d}i} < 0 \quad (12)$$

will hold for $j = 1, 2$ by the Hicksian substitutes. Thus, in this case, the conventional features of factor demand functions are preserved. In this setting, we can find a rational foundation for the marginalist theory of factor demand functions, and to do so would be a possible reaction from the neoclassical school to the analysis of this book.

The second possible comparative statics is to assume that the full reaction of the firm and the industry to a price shock may proceed beyond one production period, and so the shock of the interest rate in the current production period may involve a change of prices for commodity inputs in the next production period. But, unlike the case of the authors' implicit assumption, the full reaction of the firm and the industry would be completed before the price system reaches to a stationary state, and so the realized new equilibrium prices can be non-stationary. Thus, a change in the interest rate may involve the change of $\frac{q_1}{q_2}$, but $p = \frac{q_1}{q_2}$ is not required for a new equilibrium.

to converge to a stationary equilibrium state. However, when the economy is on the equilibrium path, it implies that the zero-profit price system is established in each period as per the definition of the intertemporal competitive equilibrium. For a more detailed explanation about this view, see Dumenil and Levy (1985).

In such a case, the appropriate method of comparative statics is to derive the following equations by the total differentiation:

$$\begin{aligned}
\mathrm{d}p &= \frac{\partial c_1}{\partial w}\mathrm{d}w + \frac{\partial c_1}{\partial q_1(1+i)}\left(\frac{\partial q_1(1+i)}{\partial i}\mathrm{d}i + \frac{\partial q_1(1+i)}{\partial q_1}\mathrm{d}q_1\right) \\
&\quad + \frac{\partial c_1}{\partial q_2(1+i)}\left(\frac{\partial q_2(1+i)}{\partial i}\mathrm{d}i + \frac{\partial q_2(1+i)}{\partial q_2}\mathrm{d}q_2\right) \\
&= l_1\mathrm{d}w + (q_1a_{11} + q_2a_{21})\,\mathrm{d}i + (1+i)(a_{11}\mathrm{d}q_1 + a_{21}\mathrm{d}q_2) \\
&\equiv l_1\mathrm{d}w + k_1\mathrm{d}i + (1+i)(a_{11}\mathrm{d}q_1 + a_{21}\mathrm{d}q_2)\,; \quad (13)
\end{aligned}$$

and

$$\begin{aligned}
0 &= \frac{\partial c_2}{\partial w}\mathrm{d}w + \frac{\partial c_2}{\partial q_1(1+i)}\left(\frac{\partial q_1(1+i)}{\partial i}\mathrm{d}i + \frac{\partial q_1(1+i)}{\partial q_1}\mathrm{d}q_1\right) \\
&\quad + \frac{\partial c_2}{\partial q_2(1+i)}\left(\frac{\partial q_2(1+i)}{\partial i}\mathrm{d}i + \frac{\partial q_2(1+i)}{\partial q_2}\mathrm{d}q_2\right) \\
&= l_2\mathrm{d}w + (q_1a_{12} + q_2a_{22})\,\mathrm{d}i + (1+i)(a_{12}\mathrm{d}q_1 + a_{22}\mathrm{d}q_2) \\
&\equiv l_2\mathrm{d}w + k_2\mathrm{d}i + (1+i)(a_{12}\mathrm{d}q_1 + a_{22}\mathrm{d}q_2)\,. \quad (14)
\end{aligned}$$

Then, from (14), we have:

$$\frac{\mathrm{d}w}{\mathrm{d}i} = -\frac{k_2}{l_2} < 0;\ \frac{\mathrm{d}q_1}{\mathrm{d}i} = -\frac{k_2}{(1+i)\,a_{12}} < 0;\ \frac{\mathrm{d}q_2}{\mathrm{d}i} = -\frac{k_2}{(1+i)\,a_{22}} < 0. \quad (15)$$

Now, note that

$$\begin{aligned}
\mathrm{d}k_1 &= \frac{\partial k_1}{\partial w}\mathrm{d}w + \frac{\partial k_1}{\partial q_1(1+i)}\left(\frac{\partial q_1(1+i)}{\partial q_1}\mathrm{d}q_1 + \frac{\partial q_1(1+i)}{\partial i}\mathrm{d}i\right) \\
&\quad + \frac{\partial k_1}{\partial q_2(1+i)}\left(\frac{\partial q_2(1+i)}{\partial q_2}\mathrm{d}q_2 + \frac{\partial q_2(1+i)}{\partial i}\mathrm{d}i\right) \\
&= \frac{\partial^2 c_1}{\partial w\partial q_1(1+i)}\mathrm{d}w + a_{11}\mathrm{d}q_1 + q_1\frac{\partial a_{11}}{\partial q_1(1+i)}\mathrm{d}i + a_{21}\mathrm{d}q_2 + q_2\frac{\partial a_{21}}{\partial q_2(1+i)}\mathrm{d}i \\
&= \frac{\partial^2 c_1}{\partial w\partial q_1(1+i)}\mathrm{d}w + a_{11}\mathrm{d}q_1 + a_{21}\mathrm{d}q_2 + \left(q_1\frac{\partial a_{11}}{\partial q_1(1+i)} + q_2\frac{\partial a_{21}}{\partial q_2(1+i)}\right)\mathrm{d}i \\
&= \frac{\partial^2 c_1}{\partial w\partial q_1(1+i)}\mathrm{d}w + a_{11}\mathrm{d}q_1 + a_{21}\mathrm{d}q_2 \\
&\quad + \left(\frac{\partial^2 c_1}{\partial q_1(1+i)\,\partial q_1(1+i)} + \frac{\partial^2 c_1}{\partial q_2(1+i)\,q_2(1+i)}\right)\mathrm{d}i \\
&= \frac{\partial^2 c_1}{\partial w\partial q_1(1+i)}\mathrm{d}w + a_{11}\mathrm{d}q_1 + a_{21}\mathrm{d}q_2 + \frac{\partial k_1}{\partial i}\mathrm{d}i, \quad (16)
\end{aligned}$$

where $\frac{\partial^2 c_1}{\partial w \partial q_1 (1+i)} > 0$ and $\frac{\partial k_1}{\partial i} = \frac{\partial^2 c_1}{\partial q_1 (1+i) \partial q_1 (1+i)} + \frac{\partial^2 c_1}{\partial q_2 (1+i) q_2 (1+i)} \leqq 0$ by the Hicksian substitutes. Therefore, from (15) and (16), we can obtain:

$$\frac{\mathrm{d}k_1}{\mathrm{d}i} = \frac{\partial^2 c_1}{\partial w \partial q_1 (1+i)} \frac{\mathrm{d}w}{\mathrm{d}i} + a_{11} \frac{\mathrm{d}q_1}{\mathrm{d}i} + a_{21} \frac{\mathrm{d}q_2}{\mathrm{d}i} + \frac{\partial k_1}{\partial i} < 0. \quad (17)$$

A similar argument can be applied to $\frac{\mathrm{d}k_2}{\mathrm{d}i}$ and $\frac{\mathrm{d}l_j}{\mathrm{d}w}$, and thus, the conventional features of factor demand functions are preserved in this setting, too. Here, we allow the price changes of commodity inputs as the reaction to the change in the interest rate, but we do not assume that a new equilibrium must be stationary. Such a case also preserves the conventional theory of factor demand functions, including the case of reproducible factors.

By the observations of (12) and (17), we may say that the essential source of the unconventional features of factor demand functions under FIE would be the presumption of the stationary equilibrium as a new equilibrium state reached through the full reaction of the firm and the industry to a price shock. Such a conclusion does not imply that the authors' analysis under the presumption of the stationary equilibrium is inappropriate; rather, their analysis may make a clear bridge between the neoclassical theory of factor demand functions and the Sraffian capital theory resulted from the Cambridge capital debate.

Our above analysis combined with the main result of this book implies that the difference between the neoclassical and the Sraffian theories is the difference in the time span each theory assumes for its equilibrium analysis. When considering a change of equilibrium due to a shock, the neoclassical school would allow that a new equilibrium price system is not necessarily stationary. They may do so because the time span of their comparative static analysis suits a shift from a short-period 'temporal' equilibrium to another short-period 'temporal' equilibrium under the (implicit) intertemporal framework,[2] even though each 'temporary' equilibrium has a long-run fea-

[2] Of course, it does not deny the possibility of a shift from an one-period 'temporary' equilibrium associated with a stationary price system to another one-period 'temporary' equilibrium associated with another stationary price system. For instance, if all agents at the head of every production period are assumed to have the *stationary expectation* about the commodity prices which would be realized at the end of this period, as in Roemer (1980), then the temporary equilibrium under such an institution would be associated with a stationary price system. However, the stationary expectation is just one specific type of price-expectation, and there is no reason to focus our attention only upon the temporary equilibrium with this specific expectation.

ture in the sense that a zero-profit condition is established in every industry through the market competition.

In contrast, the Sraffian school would be interested in the full reaction of the firm and the industry to such a shock through price adjustments until it reaches a new stationary equilibrium. Therefore, the time span of Opoche and Steedmans' comparative static analysis suits a shift from a stationary equilibrium to another stationary equilibrium. In such a longer time span, the downward sloping of factor demand curves, regularly observed in neoclassical equilibrium shifts, may no longer be a regular feature.

3 Conclusion

As Opocher and Steedman formulate it in this book, the notion of Full Industrial Equilibrium (FIE) implies that when an equilibrium is disrupted by a shock, the full reaction of the firm and the industry through price adjustment under free competition to the shock reaches a new equilibrium, characterized by zero profit in all firms and all industries. With this notion, the authors develop various versions of comparative statics, all of which give us the clear message that while the downward-sloping features are regularly observed for the demands of primary inputs as the conventional theory predicts, they are not so for produced inputs.

Given the main results of their book, the point of my comments is that in their comparative statics under FIE, the authors implicitly presume that production takes time, and that the newly reached equilibrium is always associated with a stationary price system. I have also developed the arguments that these presumptions seem to be the main source of discrepancy between the behavior of primary inputs and that of produced inputs. To argue this point, I have shown that if a new equilibrium with a zero-profit condition is allowed to be associated with a non-stationary price system, the conventional neoclassical theory of factor demands would hold even for the case of produced inputs. Given these observations, I suggest that the neoclassical and the Sraffian schools assume different time spans in regard to the equilibrium shifts, the latter focusing mainly on the shift from a stationary equilibrium to another stationary equilibrium, the former not necessarily so.

It is regrettable that almost all of the lessons from the Cambridge capital debates and all of the Sraffian critiques against the neoclassical marginalist theories have been neglected in the standard textbook of microeconomics.

This neglect may be partly because the essential reasoning in critical observations, with awareness of reswitching and of capital reversing, has not been clearly analyzed but simply been treated as a minor exception, at least by neoclassical scholars. By contrast, Opocher and Steedman show that the presence of produced inputs is crucial for the unconventional behavior of input demand functions. Their work leads us to a view that the main difference between the two schools is the difference between the time spans that each presumes. If this view is not inappropriate, it seems to me that the conventional factor demand theory and the Sraffian theory of production and capital can coexist and be worth mentioning in standard microeconomics textbooks, once these different underlying settings are clarified. In this respect, Opocher and Steedman contribute greatly to filling in the gap between the neoclassical and the Sraffian theories.

4 References

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