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Creation of NOON states by double Fock-state/Bose-Einstein condensates

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Abstract NOON states (states of the form $|N>_a|0>_b+|0>_a|N>_b$ where a and b are single particle states) have been used for predicting violations of hidden-variable theories (Greenberger-Horne-Zeilinger violations) and are valuable in metrology for precision measurements of phase at the Heisenberg limit. We show theoretically how the use of two Fock state/Bose-Einstein condensates as sources in a modified Mach Zender interferometer can lead to the creation of the NOON state in which a and b refer to arms of the interferometer and N is the total number of particles in the two condensates. The modification of the interferometer involves making conditional "side" measurements of a few particles near the sources. These measurements put the remaining particles in a superposition of two phase states, which are converted into NOON states by a beam splitter. The result is equivalent to the quantum experiment in which a large molecule passes through two slits. The NOON states are combined in a final beam splitter and show interference. Attempts to detect through which "slit" the condensates passed destroys the interference.
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1 Introduction

NOON states are interesting and useful; they are "all-or-nothing" states, having the form

$$|\Phi\rangle = |N>_a|0>_b+|0>_a|N>_b \tag{1}$$

where the subscripts a and b represent single particle states. The superposition is of all N particles in state a and none in b, plus none in b and all in a. Such

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states have had several uses in the past: A) They are the ultimate Schrodinger cat states, sometimes called “maximally entangled”. One may use them to demonstrate the quantum interference of macroscopically distinct objects.[1] B) They have been used to study violations of quantum realism in the well-known Greenberger-Horne-Zeilinger contradictions.[2,3] C) They can be used to violate the standard quantum limit and approach the Heisenberg limit in metrology.[4,5] D) They may provide for the possibility of quantum lithography.[6]

Several methods have been proposed to create NOON states by projective measurement techniques.[7,8] The method due to Cable and Dowling[9] is quite similar to the one presented here. A two-body NOON state can be constructed by allowing two bosons to impinge one on either side of a 50-50 beam splitter. The final state will be a superposition of two-particles on either side of the splitter according to the Hong-Ou-Mandel effect.[10] Here we generalize this situation with an arbitrary number of particles in the sources and show that an appropriate preparation procedure, using state-vector reduction and a conditional preparation, leads to the creation of a NOON state. We will use two Bose-Einstein condensate/Fock states as sources for an interferometer. These number states are assumed to consist entirely of ground-state bosons, such as those available approximately in ultra-cold gas systems. The NOON state is constructed here with a and b representing two arms of the interferometer. The two components can be brought together at a beam splitter to cause the quantum interference. In this regard the system is rather like having a large molecule traveling in a superposition through two slits before interfering with itself on a screen. In analogy with two-slit problem, if an observer tries to tell through which arm the N particles has passed, the interference pattern is ruined.

2 Interferometer

The interferometer is shown in Fig. 1. Two Fock state sources of number N_α and N_β enter the interferometer. The side detectors 1 and 2, situated immediately after the sources, are a key element; by measuring m_1 and m_2 particles in these detectors, the remaining particles are put into phase states that then appear in arms 3 and 4. When these pass through the middle beam splitter the result (for suitable value of ξ) is a NOON state in arms 5 and 6. The nature of the states in these arms could be tested by examining them in detectors 5 and 6.

The destruction operators at the detectors are found by tracing back from from a detector to each source. We have

$$
\begin{aligned}
a_1 &= \frac{1}{2}\left(ie^{i\theta}a_\alpha - a_\beta\right) \qquad a_2 = \frac{1}{2}\left(-e^{i\theta}a_\alpha + ia_\beta\right) \\
a_5 &= \frac{1}{2}\left(-ie^{i\xi}a_\alpha - a_\beta\right) \qquad a_6 = \frac{1}{2}\left(-e^{i\xi}a_\alpha - ia_\beta\right)
\end{aligned}
\tag{2}
$$

We take m_1 and m_2 particles to be deflected by beam splitters into detectors 1 and 2 respectively. Subsequently if we are looking at detections in arms i and j then the amplitude for finding particle numbers $\{m_1, m_2, m_i, m_j\}$ is

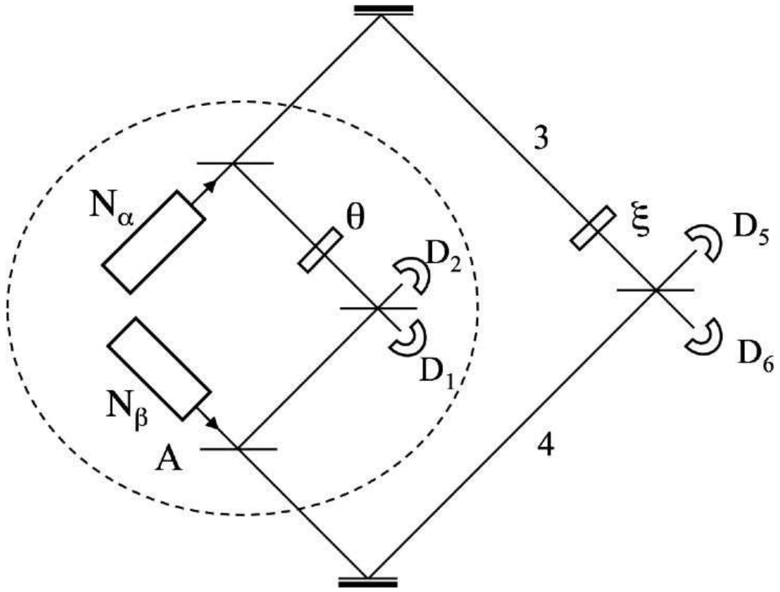

Fig. 1 Two sources have populations N_α and N_β (double Fock state). Some of the emitted particles are used for a measurement of the relative phase, with the help of two beam splitters near the sources and an interferometer with phase shift $\theta = \pi/2$ and detectors D1 and D2, recording m_1 and m_2 particles. When events where $m_1 = m_2$ are selected, the quantum projection creates a coherent superposition of states in arms 3 and 4 where the two beams have opposite relative phases. When the phase shift ξ is set to zero, after the last beam splitter this superposition becomes a NOON states in arms 5 and 6.

$$C_{m_1,m_2,m_i,m_j} = \left\langle 0 \left| \frac{a_i^{m_i} a_j^{m_j} a_1^{m_1} a_2^{m_2}}{\sqrt{m_1! m_2! m_i! m_j!}} \right| N_\alpha N_\beta \right\rangle \tag{3}$$

The double Fock state (DFS) $\left|N_\alpha N_\beta\right\rangle$ can be expanded in phase states as

$$\left|N_\alpha N_\beta\right\rangle = \sqrt{\frac{2^N N_\alpha! N_\beta!}{N!}} \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} e^{-iN_\beta \phi} \left|\phi, N\right\rangle \tag{4}$$

These states have the property that for any $a_i = v_{i\alpha} a_\alpha + v_{i\beta} a_\beta$:

$$a_i \left|\phi, N\right\rangle = \sqrt{\frac{N}{2}} (v_{i\alpha} + v_{i\beta} e^{i\phi}) \left|\phi, N-1\right\rangle \tag{5}$$

so that the state created by the interferometer measurements at 1 and 2 is

$$\left|\Gamma\right\rangle \equiv a_1^{m_1} a_2^{m_2} \left|N_\alpha N_\beta\right\rangle \sim \int_{-\pi}^{\pi} \frac{d\phi}{2\pi} e^{-iN_\beta \phi} R_{12}(\phi) \left|\phi, N-M\right\rangle \tag{6}$$

where $M = m_1 + m_2$ and

$$R_{12}(\phi) = (1 + e^{i\phi})^{m_1} (1 - e^{i\phi})^{m_2} \tag{7}$$

We have taken $\theta = \pi/2$, which places the peaks symmetrically about $\phi = 0$. We have

$$R_{12}(\phi) = (-i)^{m_2} 2^M e^{iM\phi/2} Q_{12}(\phi) \tag{8}$$

where

$$Q_{12}(\phi) = \left(\cos\frac{\phi}{2}\right)^{m_1} \left(\sin\frac{\phi}{2}\right)^{m_2} \tag{9}$$

For arbitrary m_1, m_2 Q_{12} has peaks at $\pm\phi_0 = \pm 2\arctan(m_2/m_1)$. It will be convenient for us to choose the ensemble of experiments in which we find $m_1 = m_2$ particles in the initial two branches. In that case we find the plot of Q_{12} has peaks at $\pm\pi/2$ as shown in Fig. 2. We see that we already have a superposition of phase

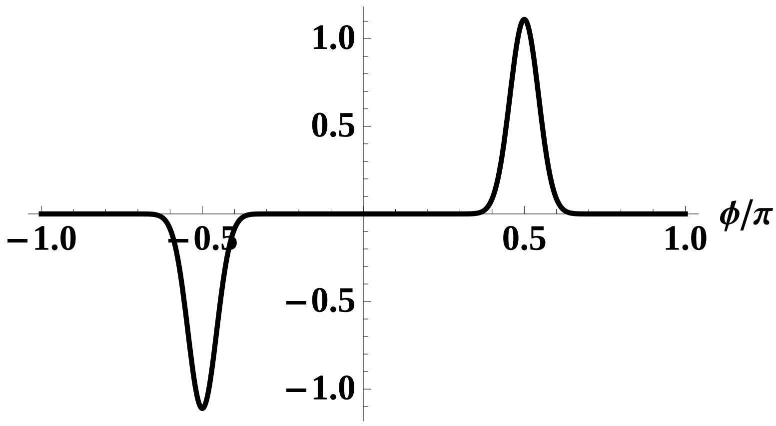

Fig. 2 Plot of $Q_{12}(\phi)$ of Eq. (9) for $m_1 = m_2 = 52$. Because we have taken $\theta = \pi/2$ and $m_1 = m_2$ we find the two symmetrical peaks at $\pm\pi/2$. For odd m_2 we have one positive and one negative peak. With m_2 even both would be positive.

states in arms 3 and 4 (but not localized in any particular arm). We have previously used these states to demonstrate macroscopic quantum interference.[11] However, now we let them pass through a middle beam splitter rather than entering detectors.

We next compute Eq. (3) for the case $i = 5$, $j = 6$. Instead of introducing the phase state (4) we expand the binomial operator forms of Eq. (2) and compute the matrix elements involving a_α and a_β, which involves appropriate δ-functions; we keep the quantity in the form of a sum resulting in the probability

$$P_{m_1,m_2m_5,m_6} = Km_5!m_6! \left| \sum_{p,q,r} \frac{e^{-i(p+q)(\xi-\pi/2)}(-1)^{p+r}}{p!(m_1-p)!q!(m_2-q)!r!(m_5-r)!} \right. \\ \left. \times \frac{1}{(N_\alpha - p - q - r)!(p+q+r+m_6-N_\alpha)!} \right|^2 \tag{10}$$

By adjusting the phase ξ to zero we get the NOON states as seen in Fig. 3 for the case $m_1 = m_2$ and for a case where this optimal situation does not occur. In each case it is shown that the probability has small peaks away from 0 or 30, but is

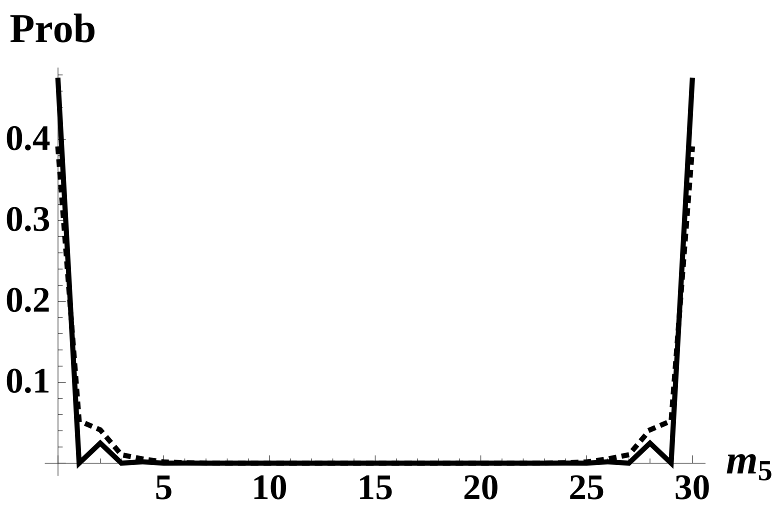

Fig. 3 Plot of $P_{m_1,m_2 m_5,m_6}$ of Eq. (10) versus m_5 ($m_6 = 30 - m_5$) for $N_\alpha = N_\beta = 30$, $\xi = 0$, and $m_1 = m_2 = 15$ (solid line). Also we show $m_1 = 18$, $m_2 = 12$ (dotted line) to show the relative insensitivity of the NOON state to the values of m_1 and m_2. A similar change is found for $N_\alpha = 26$, $N_\beta = 34$ with equal m_1, m_2.

very close to an ideal NOON state. Reference 9 proposes a "feedforward" method to make the two components of the phase superposition orthogonal, to produce a NOON state even when m_1 and m_2 are not equal.

3 Probing the state

We now let the two states pass through arms 5 and 6 to interfere at a final beam splitter and proceed to detectors 7 and 8 as shown in Fig. 4. We now need the operators

$$a_7 = \frac{1}{2\sqrt{2}} \left(u e^{i\xi} a_\alpha + v a_\beta \right) \quad a_8 = \frac{1}{2\sqrt{2}} \left(v e^{i\xi} a_\alpha - u a_\beta \right) \tag{11}$$

where

$$\begin{aligned} u &= \left(e^{i\zeta} - 1 \right) \\ v &= -i \left(e^{i\zeta} + 1 \right) \end{aligned} \tag{12}$$

We consider the phase angle $\zeta = 0$ (we also have $\theta = \pi/2$ and $\xi = 0$ as above) in which case we find

$$P_{m_1,m_2,m_7,m_8} = \frac{K}{m_7! m_8!} \left| \sum_{p=0}^{m_1} \frac{(-1)^p}{p!(m_1 - p)!(N_\alpha - p - m_8)!(m_2 + m_8 - N_\alpha + p)!} \right|^2 \tag{13}$$

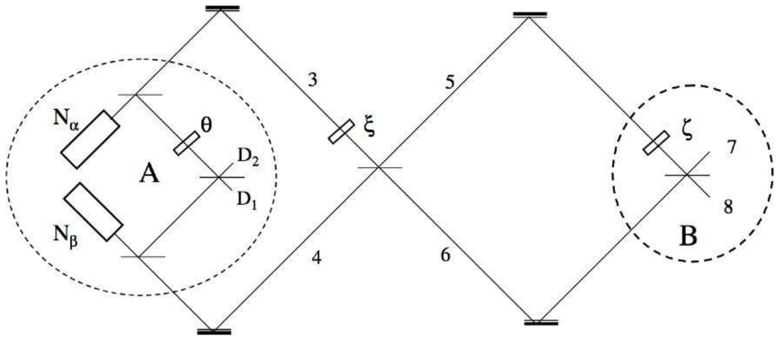

Fig. 4 The interferometer of Fig. 1 extended to allow the interference of the NOON states at the beam splitter for detectors 7 and 8. The resulting interference pattern is shown in the figure below.

A plot of this probability versus m_7 is shown in Fig. 5. The oscillations are equivalent to interference fringes and are similar to those found in Ref. 7 where the phase states shown in Fig. 2 were allowed to interfere.

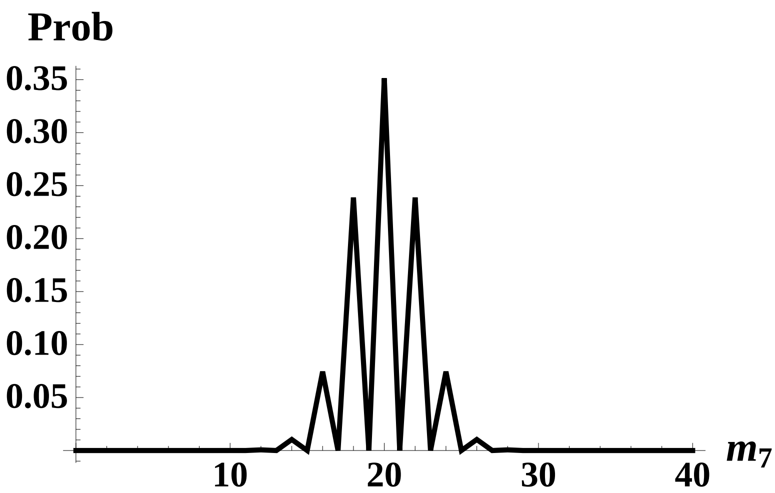

Fig. 5 Plot of $P_{m_1,m_2 m_7,m_8}$ of Eq. (13) versus m_7 for $m_1 = m_2 = 20$, $N_\alpha = N_\beta = 40$. Here $\zeta = 0$.

Leggett[12] proposed that there were two basic assumptions made implicitly by most physicists about the notion of reality at the macroscopic level.

A1) "Macroscopic realism: A macroscopic system with two or more macroscopically distinct states available to it will at all times be in one or the other of these two states."

A2) "Noninvasive measurability at the macroscopic level: It is possible, in principle, to determine the state of the system with arbitrarily small perturbation on its subsequent dynamics."

The quantum system we have studied can be used to test these rules of realism. We can attempt to demonstrate experiments that might test these assumptions using our macroscopic system. We place beam splitters in the arms 5 and 6 leading to side detectors $5'$ and $6'$ (not shown in Fig. 4 but constructed much like side detectors 1 and 2). Since the probability is that perhaps less than one particle might end in arm 5 while all the rest are in the other arm, or vice versa, we might expect that, say, detecting four particles in side detector $5'$ (and none in detector $6'$) would ruin the interference pattern, and indeed it does. The resulting plot has a single maximum with no oscillations. Moreover, if we detect even just *one* particle in detector $5'$ and none in $6'$ the interference pattern is ruined. A non-invasive measurement process is not possible in the quantum system; any probing disturbs the interference pattern, which is a not-unexpected result of decoherence in quantum mechanics. An exception to this is if we find equal numbers of particles in detectors $5'$ and $6'$; then the interference pattern is re-established. It is as if we just had a superposition of symmetrically smaller states passing through the arms; we still do not know via which arm the majority of the particles passed.

We next consider a negative experiment in which the beam splitter leading to side detector $5'$ has zero transmission probability so that any particles in arm 5 are diverted into this side detector. However, we consider *only* situations in which $m_{5'} = 0$, that is, no particles actually come into arm 5. According to A1, this corresponds to a case in which all the particles have gone the other way through arm 6. In at least half the experiments the blocking of arm 5 should have no effect at all according to the realist proposition A1. And yet in this case we get no interference pattern. Even though no particles came into arm 5, and all the particles proceeded to the beam splitter via arm 6, the interference is ruined, in contrast to assumption A1. Again this is not surprising in our quantum system.

4 Conclusion

Using double Bose condensates we have shown how to construct NOON states, which are known to have useful applications, by state-vector reduction and conditional preparation. Bose condensate interferometers, which we would require in our set-up, have already been constructed.[13] Other methods have been proposed to create NOON states by projective measurement, with a number of measurements that is an increasing function of the number of particles.[7,8] Our method makes use of a single measurement and does not even require precise values of m_1 and m_2, since the only requirement is that they should be equal (or not very different, since the method is relatively robust). Here we have considered how NOON states might be used to test experimentally the tenets of macroscopic realism and to demonstrate macroscopic quantum interference. In other work to be reported elsewhere we have also applied these states to the measurement of phase and have shown they can easily exceed the classical limit and nearly reach the Heisenberg limit.[14]

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