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Addendum to “Marx’s Analysis of Ground-Rent: Theory, Examples and Applications”

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1 Introduction

In a recent paper, I had proposed an analytical framework to understand Marx's theory of ground-rent [1]. An important question had been left unaddressed in the paper: how are the price and output levels (of the agricultural commodity) determined in a way that can both take account of fluctuations in market demand and also embed profit-maximizing behaviour of the capitalist-farmers? In this note, I offer a simple way to think about the determination of equilibrium levels of price and output for the agricultural commodity that makes explicit two important dimensions: (a) the role of aggregate demand for the agricultural commodity, and (b) profit maximizing behaviour of the capitalist farmers.

2 Determination of Price and Output

Consider agricultural production under capitalist relations of production. Capitalist-farmers rent land from the class of landlords, hire labour (by purchasing labour-power) and put their capital to produce the agricultural commodity. Suppose there are N plots of land indexed by $i = 1, 2, 3, \ldots, N$, with each plot being worked by one capitalist-farmer. If N is large, each capitalist-farmer takes the price of the agricultural commodity as given and decides the level of output to maximize her profit.

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If y_i and t_i denote the level of output and the total cost of production, respectively, on plot i, then profit income is given by

$$\pi_i(y_i; p) = py_i - t_i(y_i)$$

where p denotes the exogenously given (for the individual capitalist farmer) price level and the total cost of production is a function of the level of output.

A plausible behavioral assumption is that capitalist farmers choose the level of output to maximize their profit income. Hence, the profit-maximizing (or optimal) level of output of the agricultural commodity on the i-th plot will be determined by the following condition:

$$p = m_i(y_i), \quad i = 1, 2, 3, \ldots, N, \tag{1}$$

where

$$m_i(y_i) = t_i'(y_i)$$

is the marginal cost of production. The conditions in (1) tell us that on each of the plots, the optimal level of output is determined by equating the marginal cost of production with the exogenously given price level (because of lack of monopoly power of any individual capitalist-farmer).

Since the amount of land is fixed, it is plausible to assume that each capitalist-farmer is operating with an increasing marginal cost of production technology. One way to capture this is to assume that $m_i(y_i)$ is a monotonically increasing function of the level of output, y_i. Hence, we can always find an inverse (function) of the marginal cost of production function to get the profit-maximizing level of output as

$$y_i(p) = m_i^{-1}(p), \tag{2}$$

which gives us the supply function for each capitalist-farmer, i.e. it determines the level of output that a profit-maximizing farmer will choose to produce as a function of the price level. Since the marginal cost of production is monotonically increasing, each of the individual supply functions, being the inverse of the marginal cost of production function, will be monotonically increasing as well.

The determination of the optimal level of output on the i-th plot of land is depicted on the left panel of Figure 1. Quantity of output is measured on the horizontal axis, and price is measured on the vertical axis. MC and the profit-adjusted AVC denote the marginal and the profit-adjusted average variable cost of production curves, respectively.[1] If the price level were denoted by OB (or EC), then the profit-maximizing level of output would be given by OE, the point where the MC curve is equal to the price level.

[1] The profit-adjusted AVC curve plots $(1 + \alpha) * AVC$, where α is the economy-wide average rate of profit and AVC denotes the average variable cost of production. Given the relationship between averages and marginals, the MC curve intersects the AVC at the lowest point of the latter. The minimum point of the AVC is also the minimum point of the profit-adjusted AVC. Since the MC curve is upward-sloping, it intersects the profit-adjusted curve to the right of the minimum point, as shown in Figure 1.

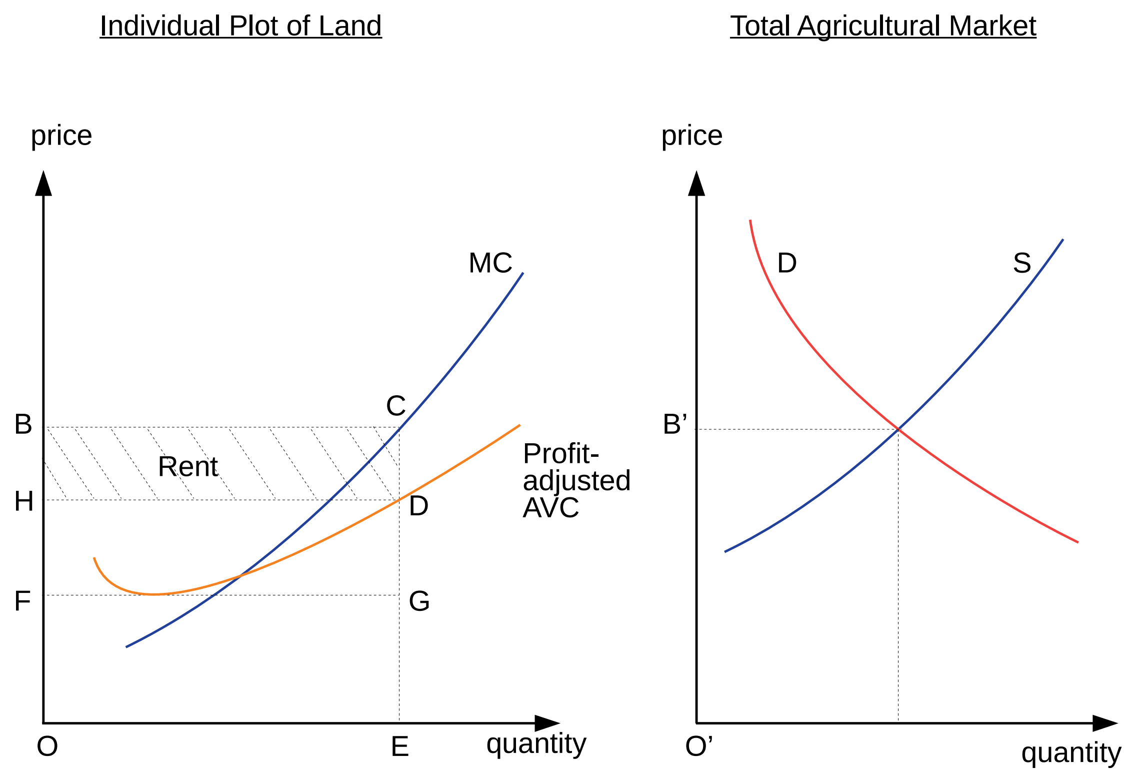

Figure 1: *Determination of equilibrium level of price and quantity in the market for the agricultural commodity. The profit-adjusted AVC curve gives* $(1 + \alpha)(c + v)$.

The total supply function for the agricultural output is the sum of the individual supply functions. Denoting the total supply function for the agricultural output as $S(p)$, we will have

$$S(p) = \sum_{i=1}^{N} y_i(p) = \sum_{i=1}^{N} m_i^{-1}(p). \tag{3}$$

with $S' > 0$ because each of the individual supply functions have $y_i'(p) > 0$.

If $D(p; z)$ denotes the total demand function for the agricultural commodity, with z denoting non-price shift factors and $\partial D/\partial p < 0$, the equilibrium level of the price will be such that demand and supply are equal

$$S(p) = D(p). \tag{4}$$

The determination of the equilibrium level of the price in the market for the agricultural commodity is depicted in the right panel of Figure 1. The upward sloping supply curve, S, is the sum of the individual supply curves given in (3). The downward sloping demand curve, D, is given by exogenous factors, z. The intersection of the two curves gives the equilibrium level of the price, which is represented by the height O'B' (= OB).

Let me summarize the main argument about the determination of the price and output levels.

- The level of demand in the economy for the agricultural commodity is given by $D(p; z)$.
- The conditions of production (technology) on each plot of land *is known* and is summarized by the total, average variable and marginal cost of production, $t_i(.)$, $k_i(.)$ and $m_i(.)$.[2]
- Given the demand conditions and the conditions of production, (2), (3) and (4) will determine the equilibrium levels of output and the price level.

3 Determination of Rent

Once the price and output levels have been determined, we can determine the levels of rent by the procedure outlined in [1]. Let R_i denote the rent on plot i. We know, following the argument of Marx, that the magnitude of rent is the surplus profit. Hence,

$$R_i = py_i - (1 + \alpha)\, y_i k_i(y_i) \tag{5}$$

where $k_i(y_i) = \{c_i(y_i) + v_i(y_i)\} / y_i$ is the average variable cost of production (AVC), and α is the economy-wide average rate of profit.[3] This is the same expression as given in (1) in [1]. To see this, let r_i denote the *pre-rent* rate of profit realized on the i-th plot. Hence,

$$r_i = \frac{py_i}{c_i + v_i} - 1 \tag{6}$$

where c_i and v_i denote the constant and variable capital advanced on the i-th plot of land. Then, total rent on the i-th plot is given by

$$R_i = (c_i + v_i)(r_i - \alpha),$$

which is the same expression as the one given in (1) in [1].

[2]The total cost of production is given by $t_i(y) = c_i(y) + v_i(y) + R_i = k_i(y) + R_i$, where $c_i(y)$ and $v_i(y)$ denote the constant and variable capital advanced for production, and R_i denotes the rent paid to the landlord. Note that the constant and variable capitals are functions of the level of output, y, and thus, their sum give us the variable cost of production. On the other hand the rent payment is independent of the level of output and functions as a fixed cost. Hence, the marginal cost of production will not be affected by the rent.

[3]In Figure 1, we plot the profit-adjusted AVC, which is $(1 + \alpha)\, k_i(y_i)$.

We can use Figure 1 to pin down the magnitude of rent. For the situation depicted in Figure 1, the price level is given by OB and the optimal level of output on the individual plot is given by OE. The rectangle OBCE represents the total revenue, the rectangle OHDE represents the amount of income of the capitalist farmer, and the rectangle BCDH - the shaded area - represents the rent income of the landlord.

4 Components of Rent

Once we know the total rent, we can break it up into its three components: absolute rent, differential rent of the first variety (DRI), and differential rent of the second variety (DRII). To do so, we need to first arrange the plots of land according to the average variable cost of production (or, what I have earlier called, the unit cost of production), k_i. Using k_i, which is known for each plot of land, let us index the plot with the highest value of k_i as $i = 1$, the plot with the next highest value of k_i as $i = 2$, and so on, with the plot with the lowest value of k_i as $i = N$. Thus, we have now arranged plots according to an increasing order of "quality", with $i = 1$ referring to the "worst plot" and $i = N$ referring to the "best plot".

We can now decompose the total rent on the i-th plot as

$$R_i = (c_i + v_i)(r_i - \alpha) = AR + DRI_i + DRII_i$$

where the absolute rent is

$$AR = (c_1 + v_1)(r_1 - \alpha),$$

differential rent of the first variety is given by

$$DRI_i = (c_i + v_i)(r_i - r_1),$$

and differential rent of the second variety is given by

$$DRII_i = [(c_i + v_i) - (c_1 + v_1)]\,(r_1 - \alpha).$$

5 The Role of Demand and Profit Maximization

We can use Figure 1 to get some intuition about the conditions for the generation of rent and the role of demand (for the agricultural commodity) in that process. For the situation depicted in Figure 1, the rectangle BCDH represents the rent income of the landlord and the rectangle OHDE represents the amount of income of the capitalist farmer. Since the capitalist farmer would not invest his capital in agriculture unless she was ensured the economy-wide average rate of profit, α, the rectangle OHDE must be equal to the $(1 + \alpha)$ times the capital advanced (sum of constant and variable capital).

Under capitalist relations of production, a plot of land will be used for cultivation only when two conditions are satisfied: (a) it generates a positive magnitude of rent for the landlord, and (b) the capital invested on it by the capitalist farmer gives her a rate of profit of α (the economy-wide average rate of profit). In terms of Figure 1, this puts a lower bound on the price level: if the price level fall below OF (shown in the left panel), then the capitalist farmer will not invest her capital (because the revenue will not cover the variable cost of production). This, in turn, puts a lower bound on the level of demand (for the agricultural commodity): if the level of demand for the agricultural commodity is such that the equilibrium price level fall below OF, then some plots of land will lie unused.

In Appendix A in [1], I had demonstrated that if the organic composition of capital in agriculture was sufficiently low and the price level was determined by the principle of zero net flow of surplus value from agriculture, then there would be positive magnitudes of rent on all plots of land. The analysis in this addendum suggests that the price level must also satisfy the additional condition that it be higher than the level represented as OF in the left panel in Figure 1.

One shortcoming of the analysis in [1] was that it did not make explicit the underlying decision making process of capitalist farmers regarding the output level. Hence it was vulnerable to the fact that the output level used to compute rent was not necessarily the one that would be chosen by profit-maximizing capitalist farmers. Thus, it was possible that the magnitudes of rent computed by landlords, and incorporated into rental lease agreements, were not based on optimal levels of output for profit-maximizing capitalist farmers. This meant that the decision making process underlying the computation of rent and those that related to the actual production process were not tightly linked together by conditions of mutual compatibility.

Consider the discussion in Appendix B in [1]. It was argued in the appendix that Marx's calculations about DRII were incorrect. Marx had argued that when there is a rise in demand, the price of the agricultural commodity would have to rise to 3.5 to make sure that the capitalist farmer of plot B had the incentive to produce the extra amount of output. I had argued that the price would remain at 3 (because the average cost of production - including the average rate of profit - on the worst plot, which was A, was still 3).[4] This reasoning has one problem: it is no longer optimal for the capitalist on plot B to produce the full 9.5 unit of output. This is because she can make a surplus profit of 0.5 - after paying the rent of 4 - by producing 3.5 units of output with a capital investment of 5.[5] But this is an anomalous situation because it means that the rent specified in the lease contract did not account for all of the surplus profit. This means that the calculation of rent and the production decisions of capitalist farmers have not been made mutually compatible.[6]

By explicitly incorporating profit-maximizing behavior, this note addresses that lacuna. Once rents are computed on the basis of output levels that satisfy the marginal cost conditions given in (1), then there will be no incentive for capitalist farmers to deviate from those output levels. This will imply that there will be no possibility for surplus profit to remain unallocated to rent. Thus, the computation of rent (by landlords) will be mutually

[4] See panel C in Table 4 in [1] for details.

[5] Revenue is $3 * 3.5$, and cost of production, including rent, is $5 * 1.2 + 4$. Subtracting the latter from the former leaves 0.5.

[6] I would like to thank Debarshi Das for pointing out this problem in the argument in Appendix B in [1].

consistent with profit-maximizing behavior (by capitalist farmers).

6 Some Comparative Statics

There are two interesting comparative static exercises that could be easily carried out with the help of Figure 1. First, if there is an exogenous increase in demand, other things remaining constant, there will be an increase in the magnitude of rent. To see this, note that the exogenous increase in demand will shift the demand curve in the right panel of Figure 1 rightward. The result will be an increase in both the equilibrium price and quantity. This will increase the area of the rectangle BCDH, which shows that the magnitude of rent will increase.

Second, if there is technological progress in agriculture, it will lead to an ambiguous change in the magnitude of rent. To see this, note that technological progress will lead to a downward movement of the marginal cost curves, so that the total supply curve in the right panel of Figure 1 will shift downward. This will lead to an increase in the quantity of output and a fall in its price. Depending on the elasticity of the supply curve, this can lead to either a fall or a rise in the magnitude of rent. More inelastic the demand, higher is the possibility of a fall in the rent.

References

[1] Basu, D. (2018). Marx's Analysis of Ground-Rent: Theory, Examples and Applications, Working Paper 2018-04, *Department of Economics, University of Massachusetts Amherst*.